Note of equation of state, Taylor expansion up to $O(\mu^{12})$

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Pressure (p) and its derivatives in terms of $\mu = \mu_{\rm q} a$

$$\frac{p}{T^4} = \frac{1}{VT^3}\ln \mathcal{Z}, \qquad \frac{\partial^n (p/T^4)}{\partial(\mu_{\rm q}/T)^n} = \frac{1}{VT^3}\frac{\partial^n \ln \mathcal{Z}}{\partial(\mu_{\rm q}/T)^n} = \frac{N_t^{3-n}}{N_s^3}\frac{\partial^n \ln \mathcal{Z}}{\partial \mu^n}. \tag{1}$$

Partition function

$$\mathcal{Z} = \int \mathcal{D}U (\det M)^{N_{\rm f}/4} e^{-S_g}. \tag{2}$$

Derivatives of $\langle \mathcal{O} \rangle$

$$\begin{aligned}
\langle \mathcal{O} \rangle &= \frac{1}{\mathcal{Z}} \int \mathcal{D}U \mathcal{O} (\det M)^{N_{\rm f}/4} e^{-S_g} && (3)\\
\frac{\partial \langle \mathcal{O} \rangle}{\partial \mu} &= \frac{1}{\mathcal{Z}} \int \mathcal{D}U \frac{\partial \mathcal{O}}{\partial \mu} (\det M)^{N_{\rm f}/4} e^{-S_g} \\
&+ \frac{1}{\mathcal{Z}} \int \mathcal{D}U \mathcal{O} \frac{N_{\rm f}}{4} \frac{\partial (\ln \det M)}{\partial \mu} (\det M)^{N_{\rm f}/4} e^{-S_g} \\
&- \frac{1}{\mathcal{Z}^2} \int \mathcal{D}U \mathcal{O} (\det M)^{N_{\rm f}/4} e^{-S_g} \int \mathcal{D}U \frac{N_{\rm f}}{4} \frac{\partial (\ln \det M)}{\partial \mu} (\det M)^{N_{\rm f}/4} e^{-S_g} \\
&= \left\langle \frac{\partial \mathcal{O}}{\partial \mu} \right\rangle + \left\langle \mathcal{O} \frac{N_{\rm f}}{4} \frac{\partial (\ln \det M)}{\partial \mu} \right\rangle - \langle \mathcal{O} \rangle \left\langle \frac{N_{\rm f}}{4} \frac{\partial (\ln \det M)}{\partial \mu} \right\rangle, && (4)
\end{aligned}$$

Derivatives of $\ln \mathcal{Z}$; Denoting

$$\mathcal{D}_n = \frac{N_{\rm f}}{4} \frac{\partial^n (\ln \det M)}{\partial \mu^n}, \tag{5}$$

We define

$$\begin{aligned}
\mathcal{A}_1 &= \langle \mathcal{D}_1 \rangle, && (6)\\
\mathcal{A}_2 &= \langle \mathcal{D}_2 \rangle + \left\langle \mathcal{D}_1^2 \right\rangle, && (7)\\
\mathcal{A}_3 &= \langle \mathcal{D}_3 \rangle + 3 \langle \mathcal{D}_2 \mathcal{D}_1 \rangle + \left\langle \mathcal{D}_1^3 \right\rangle, && (8)\\
\mathcal{A}_4 &= \langle \mathcal{D}_4 \rangle + 4 \langle \mathcal{D}_3 \mathcal{D}_1 \rangle + 3 \left\langle \mathcal{D}_2^2 \right\rangle + 6 \left\langle \mathcal{D}_2 \mathcal{D}_1^2 \right\rangle + \left\langle \mathcal{D}_1^4 \right\rangle, && (9)\\
\mathcal{A}_5 &= \langle \mathcal{D}_5 \rangle + 5 \langle \mathcal{D}_4 \mathcal{D}_1 \rangle + 10 \langle \mathcal{D}_3 \mathcal{D}_2 \rangle + 10 \left\langle \mathcal{D}_3 \mathcal{D}_1^2 \right\rangle + 15 \left\langle \mathcal{D}_2^2 \mathcal{D}_1 \right\rangle + 10 \left\langle \mathcal{D}_2 \mathcal{D}_1^3 \right\rangle + \left\langle \mathcal{D}_1^5 \right\rangle, && (10)\\
\mathcal{A}_6 &= \langle \mathcal{D}_6 \rangle + 6 \langle \mathcal{D}_5 \mathcal{D}_1 \rangle + 15 \langle \mathcal{D}_4 \mathcal{D}_2 \rangle + 10 \left\langle \mathcal{D}_3^2 \right\rangle + 15 \left\langle \mathcal{D}_4 \mathcal{D}_1^2 \right\rangle + 60 \langle \mathcal{D}_3 \mathcal{D}_2 \mathcal{D}_1 \rangle + 15 \left\langle \mathcal{D}_2^3 \right\rangle \\
&\quad + 20 \left\langle \mathcal{D}_3 \mathcal{D}_1^3 \right\rangle + 45 \left\langle \mathcal{D}_2^2 \mathcal{D}_1^2 \right\rangle + 15 \left\langle \mathcal{D}_2 \mathcal{D}_1^4 \right\rangle + \left\langle \mathcal{D}_1^6 \right\rangle, && (11)\\
\mathcal{A}_7 &= \langle \mathcal{D}_7 \rangle + 7 \langle \mathcal{D}_6 \mathcal{D}_1 \rangle + 21 \langle \mathcal{D}_5 \mathcal{D}_2 \rangle + 35 \langle \mathcal{D}_4 \mathcal{D}_3 \rangle + 21 \left\langle \mathcal{D}_5 \mathcal{D}_1^2 \right\rangle + 105 \langle \mathcal{D}_4 \mathcal{D}_2 \mathcal{D}_1 \rangle \\
&\quad + 70 \left\langle \mathcal{D}_3^2 \mathcal{D}_1 \right\rangle + 105 \left\langle \mathcal{D}_3 \mathcal{D}_2^2 \right\rangle + 35 \left\langle \mathcal{D}_4 \mathcal{D}_1^3 \right\rangle + 210 \left\langle \mathcal{D}_3 \mathcal{D}_2 \mathcal{D}_1^2 \right\rangle + 105 \left\langle \mathcal{D}_2^3 \mathcal{D}_1 \right\rangle \\
&\quad + 35 \left\langle \mathcal{D}_3 \mathcal{D}_1^4 \right\rangle + 105 \left\langle \mathcal{D}_2^2 \mathcal{D}_1^3 \right\rangle + 21 \left\langle \mathcal{D}_2 \mathcal{D}_1^5 \right\rangle + \left\langle \mathcal{D}_1^7 \right\rangle, && (12)\\
\mathcal{A}_8 &= \langle \mathcal{D}_8 \rangle + 8 \langle \mathcal{D}_7 \mathcal{D}_1 \rangle + 28 \langle \mathcal{D}_6 \mathcal{D}_2 \rangle + 56 \langle \mathcal{D}_5 \mathcal{D}_3 \rangle + 35 \left\langle \mathcal{D}_4^2 \right\rangle + 28 \left\langle \mathcal{D}_6 \mathcal{D}_1^2 \right\rangle + 168 \langle \mathcal{D}_5 \mathcal{D}_2 \mathcal{D}_1 \rangle \\
&\quad + 280 \langle \mathcal{D}_4 \mathcal{D}_3 \mathcal{D}_1 \rangle + 210 \left\langle \mathcal{D}_4 \mathcal{D}_2^2 \right\rangle + 280 \left\langle \mathcal{D}_3^2 \mathcal{D}_2 \right\rangle + 56 \left\langle \mathcal{D}_5 \mathcal{D}_1^3 \right\rangle + 420 \left\langle \mathcal{D}_4 \mathcal{D}_2 \mathcal{D}_1^2 \right\rangle
\end{aligned}$$

$$
\begin{aligned}
&+280\left\langle \mathcal{D}_3^2\mathcal{D}_1^2\right\rangle + 840\left\langle \mathcal{D}_3\mathcal{D}_2^2\mathcal{D}_1\right\rangle + 105\left\langle \mathcal{D}_2^4\right\rangle + 70\left\langle \mathcal{D}_4\mathcal{D}_1^4\right\rangle + 560\left\langle \mathcal{D}_3\mathcal{D}_2\mathcal{D}_1^3\right\rangle \\
&+420\left\langle \mathcal{D}_2^3\mathcal{D}_1^2\right\rangle + 56\left\langle \mathcal{D}_3\mathcal{D}_1^5\right\rangle + 210\left\langle \mathcal{D}_2^2\mathcal{D}_1^4\right\rangle + 28\left\langle \mathcal{D}_2\mathcal{D}_1^6\right\rangle + \left\langle \mathcal{D}_1^8\right\rangle, \qquad (13)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{A}_9 &= \langle \mathcal{D}_9\rangle + 9\langle \mathcal{D}_8\mathcal{D}_1\rangle + 36\langle \mathcal{D}_7\mathcal{D}_2\rangle + 84\langle \mathcal{D}_6\mathcal{D}_3\rangle + 126\langle \mathcal{D}_5\mathcal{D}_4\rangle + 36\left\langle \mathcal{D}_7\mathcal{D}_1^2\right\rangle \\
&+252\langle \mathcal{D}_6\mathcal{D}_2\mathcal{D}_1\rangle + 504\langle \mathcal{D}_5\mathcal{D}_3\mathcal{D}_1\rangle + 315\left\langle \mathcal{D}_4^2\mathcal{D}_1\right\rangle + 378\left\langle \mathcal{D}_5\mathcal{D}_2^2\right\rangle + 1260\langle \mathcal{D}_4\mathcal{D}_3\mathcal{D}_2\rangle \\
&+280\left\langle \mathcal{D}_3^3\right\rangle + 84\left\langle \mathcal{D}_6\mathcal{D}_1^3\right\rangle + 756\left\langle \mathcal{D}_5\mathcal{D}_2\mathcal{D}_1^2\right\rangle + 1260\left\langle \mathcal{D}_4\mathcal{D}_3\mathcal{D}_1^2\right\rangle + 1890\left\langle \mathcal{D}_4\mathcal{D}_2^2\mathcal{D}_1\right\rangle \\
&+2520\left\langle \mathcal{D}_3^2\mathcal{D}_2\mathcal{D}_1\right\rangle + 1260\left\langle \mathcal{D}_3\mathcal{D}_2^3\right\rangle + 126\left\langle \mathcal{D}_5\mathcal{D}_1^4\right\rangle + 1260\left\langle \mathcal{D}_4\mathcal{D}_2\mathcal{D}_1^3\right\rangle + 840\left\langle \mathcal{D}_3^2\mathcal{D}_1^3\right\rangle \\
&+3780\left\langle \mathcal{D}_3\mathcal{D}_2^2\mathcal{D}_1^2\right\rangle + 945\left\langle \mathcal{D}_2^4\mathcal{D}_1\right\rangle + 126\left\langle \mathcal{D}_4\mathcal{D}_1^5\right\rangle + 1260\left\langle \mathcal{D}_3\mathcal{D}_2\mathcal{D}_1^4\right\rangle + 1260\left\langle \mathcal{D}_2^3\mathcal{D}_1^3\right\rangle \\
&+84\left\langle \mathcal{D}_3\mathcal{D}_1^6\right\rangle + 378\left\langle \mathcal{D}_2^2\mathcal{D}_1^5\right\rangle + 36\left\langle \mathcal{D}_2\mathcal{D}_1^7\right\rangle + \left\langle \mathcal{D}_1^9\right\rangle, \qquad (14)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{A}_{10} &= \langle \mathcal{D}_{10}\rangle + 10\langle \mathcal{D}_9\mathcal{D}_1\rangle + 45\langle \mathcal{D}_8\mathcal{D}_2\rangle + 120\langle \mathcal{D}_7\mathcal{D}_3\rangle + 210\langle \mathcal{D}_6\mathcal{D}_4\rangle + 126\left\langle \mathcal{D}_5^2\right\rangle + 45\left\langle \mathcal{D}_8\mathcal{D}_1^2\right\rangle \\
&+360\langle \mathcal{D}_7\mathcal{D}_2\mathcal{D}_1\rangle + 840\langle \mathcal{D}_6\mathcal{D}_3\mathcal{D}_1\rangle + 1260\langle \mathcal{D}_5\mathcal{D}_4\mathcal{D}_1\rangle + 630\left\langle \mathcal{D}_6\mathcal{D}_2^2\right\rangle + 2520\langle \mathcal{D}_5\mathcal{D}_3\mathcal{D}_2\rangle \\
&+1575\left\langle \mathcal{D}_4^2\mathcal{D}_2\right\rangle + 2100\left\langle \mathcal{D}_4\mathcal{D}_3^2\right\rangle + 120\left\langle \mathcal{D}_7\mathcal{D}_1^3\right\rangle + 1260\left\langle \mathcal{D}_6\mathcal{D}_2\mathcal{D}_1^2\right\rangle + 2520\left\langle \mathcal{D}_5\mathcal{D}_3\mathcal{D}_1^2\right\rangle \\
&+1575\left\langle \mathcal{D}_4^2\mathcal{D}_1^2\right\rangle + 3780\left\langle \mathcal{D}_5\mathcal{D}_2^2\mathcal{D}_1\right\rangle + 12600\langle \mathcal{D}_4\mathcal{D}_3\mathcal{D}_2\mathcal{D}_1\rangle + 2800\left\langle \mathcal{D}_3^3\mathcal{D}_1\right\rangle + 3150\left\langle \mathcal{D}_4\mathcal{D}_2^3\right\rangle \\
&+6300\left\langle \mathcal{D}_3^2\mathcal{D}_2^2\right\rangle + 210\left\langle \mathcal{D}_6\mathcal{D}_1^4\right\rangle + 2520\left\langle \mathcal{D}_5\mathcal{D}_2\mathcal{D}_1^3\right\rangle + 4200\left\langle \mathcal{D}_4\mathcal{D}_3\mathcal{D}_1^3\right\rangle + 9450\left\langle \mathcal{D}_4\mathcal{D}_2^2\mathcal{D}_1^2\right\rangle \\
&+12600\left\langle \mathcal{D}_3^2\mathcal{D}_2\mathcal{D}_1^2\right\rangle + 12600\left\langle \mathcal{D}_3\mathcal{D}_2^3\mathcal{D}_1\right\rangle + 945\left\langle \mathcal{D}_2^5\right\rangle + 252\left\langle \mathcal{D}_5\mathcal{D}_1^5\right\rangle + 3150\left\langle \mathcal{D}_4\mathcal{D}_2\mathcal{D}_1^4\right\rangle \\
&+2100\left\langle \mathcal{D}_3^2\mathcal{D}_1^4\right\rangle + 12600\left\langle \mathcal{D}_3\mathcal{D}_2^2\mathcal{D}_1^3\right\rangle + 4725\left\langle \mathcal{D}_2^4\mathcal{D}_1^2\right\rangle + 210\left\langle \mathcal{D}_4\mathcal{D}_1^6\right\rangle + 2520\left\langle \mathcal{D}_3\mathcal{D}_2\mathcal{D}_1^5\right\rangle \\
&+3150\left\langle \mathcal{D}_2^3\mathcal{D}_1^4\right\rangle + 120\left\langle \mathcal{D}_3\mathcal{D}_1^7\right\rangle + 630\left\langle \mathcal{D}_2^2\mathcal{D}_1^6\right\rangle + 45\left\langle \mathcal{D}_2\mathcal{D}_1^8\right\rangle + \left\langle \mathcal{D}_1^{10}\right\rangle, \qquad (15)
\end{aligned}
$$

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\begin{aligned}
\mathcal{A}_{11} &= \langle \mathcal{D}_{11}\rangle + 11\langle \mathcal{D}_{10}\mathcal{D}_1\rangle + 55\langle \mathcal{D}_9\mathcal{D}_2\rangle + 165\langle \mathcal{D}_8\mathcal{D}_3\rangle + 330\langle \mathcal{D}_7\mathcal{D}_4\rangle + 462\langle \mathcal{D}_6\mathcal{D}_5\rangle \\
&+55\left\langle \mathcal{D}_9\mathcal{D}_1^2\right\rangle + 495\langle \mathcal{D}_8\mathcal{D}_2\mathcal{D}_1\rangle + 1320\langle \mathcal{D}_7\mathcal{D}_3\mathcal{D}_1\rangle + 2310\langle \mathcal{D}_6\mathcal{D}_4\mathcal{D}_1\rangle + 1386\left\langle \mathcal{D}_5^2\mathcal{D}_1\right\rangle \\
&+990\left\langle \mathcal{D}_7\mathcal{D}_2^2\right\rangle + 4620\langle \mathcal{D}_6\mathcal{D}_3\mathcal{D}_2\rangle + 6930\langle \mathcal{D}_5\mathcal{D}_4\mathcal{D}_2\rangle + 4620\left\langle \mathcal{D}_5\mathcal{D}_3^2\right\rangle + 5775\left\langle \mathcal{D}_4^2\mathcal{D}_3\right\rangle \\
&+165\left\langle \mathcal{D}_8\mathcal{D}_1^3\right\rangle + 1980\left\langle \mathcal{D}_7\mathcal{D}_2\mathcal{D}_1^2\right\rangle + 4620\left\langle \mathcal{D}_6\mathcal{D}_3\mathcal{D}_1^2\right\rangle + 6930\left\langle \mathcal{D}_5\mathcal{D}_4\mathcal{D}_1^2\right\rangle \\
&+6930\left\langle \mathcal{D}_6\mathcal{D}_2^2\mathcal{D}_1\right\rangle + 27720\langle \mathcal{D}_5\mathcal{D}_3\mathcal{D}_2\mathcal{D}_1\rangle + 17325\left\langle \mathcal{D}_4^2\mathcal{D}_2\mathcal{D}_1\right\rangle + 23100\left\langle \mathcal{D}_4\mathcal{D}_3^2\mathcal{D}_1\right\rangle \\
&+6930\left\langle \mathcal{D}_5\mathcal{D}_2^3\right\rangle + 34650\left\langle \mathcal{D}_4\mathcal{D}_3\mathcal{D}_2^2\right\rangle + 15400\left\langle \mathcal{D}_3^3\mathcal{D}_2\right\rangle + 330\left\langle \mathcal{D}_7\mathcal{D}_1^4\right\rangle \\
&+4620\left\langle \mathcal{D}_6\mathcal{D}_2\mathcal{D}_1^3\right\rangle + 9240\left\langle \mathcal{D}_5\mathcal{D}_3\mathcal{D}_1^3\right\rangle + 5775\left\langle \mathcal{D}_4^2\mathcal{D}_1^3\right\rangle + 20790\left\langle \mathcal{D}_5\mathcal{D}_2^2\mathcal{D}_1^2\right\rangle \\
&+69300\left\langle \mathcal{D}_4\mathcal{D}_3\mathcal{D}_2\mathcal{D}_1^2\right\rangle + 15400\left\langle \mathcal{D}_3^3\mathcal{D}_1^2\right\rangle + 34650\left\langle \mathcal{D}_4\mathcal{D}_2^3\mathcal{D}_1\right\rangle + 69300\left\langle \mathcal{D}_3^2\mathcal{D}_2^2\mathcal{D}_1\right\rangle \\
&+17325\left\langle \mathcal{D}_3\mathcal{D}_2^4\right\rangle + 462\left\langle \mathcal{D}_6\mathcal{D}_1^5\right\rangle + 6930\left\langle \mathcal{D}_5\mathcal{D}_2\mathcal{D}_1^4\right\rangle + 11550\left\langle \mathcal{D}_4\mathcal{D}_3\mathcal{D}_1^4\right\rangle \\
&+34650\left\langle \mathcal{D}_4\mathcal{D}_2^2\mathcal{D}_1^3\right\rangle + 46200\left\langle \mathcal{D}_3^2\mathcal{D}_2\mathcal{D}_1^3\right\rangle + 69300\left\langle \mathcal{D}_3\mathcal{D}_2^3\mathcal{D}_1^2\right\rangle + 10395\left\langle \mathcal{D}_2^5\mathcal{D}_1\right\rangle \\
&+462\left\langle \mathcal{D}_5\mathcal{D}_1^6\right\rangle + 6930\left\langle \mathcal{D}_4\mathcal{D}_2\mathcal{D}_1^5\right\rangle + 4620\left\langle \mathcal{D}_3^2\mathcal{D}_1^5\right\rangle + 34650\left\langle \mathcal{D}_3\mathcal{D}_2^2\mathcal{D}_1^4\right\rangle + 17325\left\langle \mathcal{D}_2^4\mathcal{D}_1^3\right\rangle \\
&+330\left\langle \mathcal{D}_4\mathcal{D}_1^7\right\rangle + 4620\left\langle \mathcal{D}_3\mathcal{D}_2\mathcal{D}_1^6\right\rangle + 6930\left\langle \mathcal{D}_2^3\mathcal{D}_1^5\right\rangle + 165\left\langle \mathcal{D}_3\mathcal{D}_1^8\right\rangle + 990\left\langle \mathcal{D}_2^2\mathcal{D}_1^7\right\rangle \\
&+55\left\langle \mathcal{D}_2\mathcal{D}_1^9\right\rangle + \left\langle \mathcal{D}_1^{11}\right\rangle, \qquad (16)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{A}_{12} &= \langle \mathcal{D}_{12}\rangle + 12\langle \mathcal{D}_{11}\mathcal{D}_1\rangle + 66\langle \mathcal{D}_{10}\mathcal{D}_2\rangle + 220\langle \mathcal{D}_9\mathcal{D}_3\rangle + 495\langle \mathcal{D}_8\mathcal{D}_4\rangle + 792\langle \mathcal{D}_7\mathcal{D}_5\rangle \\
&+462\left\langle \mathcal{D}_6^2\right\rangle + 66\left\langle \mathcal{D}_{10}\mathcal{D}_1^2\right\rangle + 660\langle \mathcal{D}_9\mathcal{D}_2\mathcal{D}_1\rangle + 1980\langle \mathcal{D}_8\mathcal{D}_3\mathcal{D}_1\rangle + 3960\langle \mathcal{D}_7\mathcal{D}_4\mathcal{D}_1\rangle \\
&+5544\langle \mathcal{D}_6\mathcal{D}_5\mathcal{D}_1\rangle + 1485\left\langle \mathcal{D}_8\mathcal{D}_2^2\right\rangle + 7920\langle \mathcal{D}_7\mathcal{D}_3\mathcal{D}_2\rangle + 13860\langle \mathcal{D}_6\mathcal{D}_4\mathcal{D}_2\rangle + 8316\left\langle \mathcal{D}_5^2\mathcal{D}_2\right\rangle
\end{aligned}
$$

$$
\begin{aligned}
&+9240\left\langle \mathcal{D}_6\mathcal{D}_3^2\right\rangle + 27720\left\langle \mathcal{D}_5\mathcal{D}_4\mathcal{D}_3\right\rangle + 5775\left\langle \mathcal{D}_4^3\right\rangle + 220\left\langle \mathcal{D}_9\mathcal{D}_1^3\right\rangle + 2970\left\langle \mathcal{D}_8\mathcal{D}_2\mathcal{D}_1^2\right\rangle \\
&+7920\left\langle \mathcal{D}_7\mathcal{D}_3\mathcal{D}_1^2\right\rangle + 13860\left\langle \mathcal{D}_6\mathcal{D}_4\mathcal{D}_1^2\right\rangle + 8316\left\langle \mathcal{D}_5^2\mathcal{D}_1^2\right\rangle + 11880\left\langle \mathcal{D}_7\mathcal{D}_2^2\mathcal{D}_1\right\rangle \\
&+55440\left\langle \mathcal{D}_6\mathcal{D}_3\mathcal{D}_2\mathcal{D}_1\right\rangle + 83160\left\langle \mathcal{D}_5\mathcal{D}_4\mathcal{D}_2\mathcal{D}_1\right\rangle + 55440\left\langle \mathcal{D}_5\mathcal{D}_3^2\mathcal{D}_1\right\rangle + 69300\left\langle \mathcal{D}_4^2\mathcal{D}_3\mathcal{D}_1\right\rangle \\
&+13860\left\langle \mathcal{D}_6\mathcal{D}_2^3\right\rangle + 83160\left\langle \mathcal{D}_5\mathcal{D}_3\mathcal{D}_2^2\right\rangle + 51975\left\langle \mathcal{D}_4^2\mathcal{D}_2^2\right\rangle + 138600\left\langle \mathcal{D}_4\mathcal{D}_3^2\mathcal{D}_2\right\rangle \\
&+15400\left\langle \mathcal{D}_3^4\right\rangle + 495\left\langle \mathcal{D}_8\mathcal{D}_1^4\right\rangle + 7920\left\langle \mathcal{D}_7\mathcal{D}_2\mathcal{D}_1^3\right\rangle + 18480\left\langle \mathcal{D}_6\mathcal{D}_3\mathcal{D}_1^3\right\rangle \\
&+27720\left\langle \mathcal{D}_5\mathcal{D}_4\mathcal{D}_1^3\right\rangle + 41580\left\langle \mathcal{D}_6\mathcal{D}_2^2\mathcal{D}_1^2\right\rangle + 166320\left\langle \mathcal{D}_5\mathcal{D}_3\mathcal{D}_2\mathcal{D}_1^2\right\rangle + 103950\left\langle \mathcal{D}_4^2\mathcal{D}_2\mathcal{D}_1^2\right\rangle \\
&+138600\left\langle \mathcal{D}_4\mathcal{D}_3^2\mathcal{D}_1^2\right\rangle + 83160\left\langle \mathcal{D}_5\mathcal{D}_2^3\mathcal{D}_1\right\rangle + 415800\left\langle \mathcal{D}_4\mathcal{D}_3\mathcal{D}_2^2\mathcal{D}_1\right\rangle + 184800\left\langle \mathcal{D}_3^3\mathcal{D}_2\mathcal{D}_1\right\rangle \\
&+51975\left\langle \mathcal{D}_4\mathcal{D}_2^4\right\rangle + 138600\left\langle \mathcal{D}_3^2\mathcal{D}_2^3\right\rangle + 792\left\langle \mathcal{D}_7\mathcal{D}_1^5\right\rangle + 13860\left\langle \mathcal{D}_6\mathcal{D}_2\mathcal{D}_1^4\right\rangle \\
&+27720\left\langle \mathcal{D}_5\mathcal{D}_3\mathcal{D}_1^4\right\rangle + 17325\left\langle \mathcal{D}_4^2\mathcal{D}_1^4\right\rangle + 83160\left\langle \mathcal{D}_5\mathcal{D}_2^2\mathcal{D}_1^3\right\rangle + 277200\left\langle \mathcal{D}_4\mathcal{D}_3\mathcal{D}_2\mathcal{D}_1^3\right\rangle \\
&+61600\left\langle \mathcal{D}_3^3\mathcal{D}_1^3\right\rangle + 207900\left\langle \mathcal{D}_4\mathcal{D}_2^3\mathcal{D}_1^2\right\rangle + 415800\left\langle \mathcal{D}_3^2\mathcal{D}_2^2\mathcal{D}_1^2\right\rangle + 207900\left\langle \mathcal{D}_3\mathcal{D}_2^4\mathcal{D}_1\right\rangle \\
&+10395\left\langle \mathcal{D}_2^6\right\rangle + 924\left\langle \mathcal{D}_6\mathcal{D}_1^6\right\rangle + 16632\left\langle \mathcal{D}_5\mathcal{D}_2\mathcal{D}_1^5\right\rangle + 27720\left\langle \mathcal{D}_4\mathcal{D}_3\mathcal{D}_1^5\right\rangle \\
&+103950\left\langle \mathcal{D}_4\mathcal{D}_2^2\mathcal{D}_1^4\right\rangle + 138600\left\langle \mathcal{D}_3^2\mathcal{D}_2\mathcal{D}_1^4\right\rangle + 277200\left\langle \mathcal{D}_3\mathcal{D}_2^3\mathcal{D}_1^3\right\rangle + 62370\left\langle \mathcal{D}_2^5\mathcal{D}_1^2\right\rangle \\
&+792\left\langle \mathcal{D}_5\mathcal{D}_1^7\right\rangle + 13860\left\langle \mathcal{D}_4\mathcal{D}_2\mathcal{D}_1^6\right\rangle + 9240\left\langle \mathcal{D}_3^2\mathcal{D}_1^6\right\rangle + 83160\left\langle \mathcal{D}_3\mathcal{D}_2^2\mathcal{D}_1^5\right\rangle \\
&+51975\left\langle \mathcal{D}_2^4\mathcal{D}_1^4\right\rangle + 495\left\langle \mathcal{D}_4\mathcal{D}_1^8\right\rangle + 7920\left\langle \mathcal{D}_3\mathcal{D}_2\mathcal{D}_1^7\right\rangle + 13860\left\langle \mathcal{D}_2^3\mathcal{D}_1^6\right\rangle + 220\left\langle \mathcal{D}_3\mathcal{D}_1^9\right\rangle \\
&+1485\left\langle \mathcal{D}_2^2\mathcal{D}_1^8\right\rangle + 66\left\langle \mathcal{D}_2\mathcal{D}_1^{10}\right\rangle + \left\langle \mathcal{D}_1^{12}\right\rangle . \qquad (17)
\end{aligned}
$$

Then,

$$\frac{\partial \mathcal{A}_n}{\partial \mu} = \mathcal{A}_{n+1} - \mathcal{A}_n\mathcal{A}_1. \qquad (18)$$

Because $\mathcal{A}_1 = (\partial \ln \mathcal{Z})/(\partial \mu)$, we get

$$
\begin{aligned}
\frac{\partial \ln \mathcal{Z}}{\partial \mu} &= \mathcal{A}_1, &(19)\\
\frac{\partial^2 \ln \mathcal{Z}}{\partial \mu^2} &= \mathcal{A}_2 - \mathcal{A}_1^2, &(20)\\
\frac{\partial^3 \ln \mathcal{Z}}{\partial \mu^3} &= \mathcal{A}_3 - 3\mathcal{A}_2\mathcal{A}_1 + 2\mathcal{A}_1^3, &(21)\\
\frac{\partial^4 \ln \mathcal{Z}}{\partial \mu^4} &= \mathcal{A}_4 - 4\mathcal{A}_3\mathcal{A}_1 - 3\mathcal{A}_2^2 + 12\mathcal{A}_2\mathcal{A}_1^2 - 6\mathcal{A}_1^4, &(22)\\
\frac{\partial^5 \ln \mathcal{Z}}{\partial \mu^5} &= \mathcal{A}_5 - 5\mathcal{A}_4\mathcal{A}_1 - 10\mathcal{A}_3\mathcal{A}_2 + 20\mathcal{A}_3\mathcal{A}_1^2 + 30\mathcal{A}_2^2\mathcal{A}_1 - 60\mathcal{A}_2\mathcal{A}_1^3 + 24\mathcal{A}_1^5, &(23)\\
\frac{\partial^6 \ln \mathcal{Z}}{\partial \mu^6} &= \mathcal{A}_6 - 6\mathcal{A}_5\mathcal{A}_1 - 15\mathcal{A}_4\mathcal{A}_2 - 10\mathcal{A}_3\mathcal{A}_3 + 30\mathcal{A}_4\mathcal{A}_1^2 + 120\mathcal{A}_3\mathcal{A}_2\mathcal{A}_1 \\
&\quad +30\mathcal{A}_2^3 - 120\mathcal{A}_3\mathcal{A}_1^3 - 270\mathcal{A}_2^2\mathcal{A}_1^2 + 360\mathcal{A}_2\mathcal{A}_1^4 - 120\mathcal{A}_1^6, &(24)\\
\frac{\partial^7 \ln \mathcal{Z}}{\partial \mu^7} &= \mathcal{A}_7 - 7\mathcal{A}_6\mathcal{A}_1 - 21\mathcal{A}_5\mathcal{A}_2 - 35\mathcal{A}_4\mathcal{A}_3 + 42\mathcal{A}_5\mathcal{A}_1^2 + 210\mathcal{A}_4\mathcal{A}_2\mathcal{A}_1 \\
&\quad +140\mathcal{A}_3\mathcal{A}_3\mathcal{A}_1 + 210\mathcal{A}_3\mathcal{A}_2^2 - 210\mathcal{A}_4\mathcal{A}_1^3 - 1260\mathcal{A}_3\mathcal{A}_2\mathcal{A}_1^2 - 630\mathcal{A}_2^3\mathcal{A}_1 \\
&\quad +840\mathcal{A}_3\mathcal{A}_1^4 + 2520\mathcal{A}_2^2\mathcal{A}_1^3 - 2520\mathcal{A}_2\mathcal{A}_1^5 + 720\mathcal{A}_1^7, &(25)\\
\frac{\partial^8 \ln \mathcal{Z}}{\partial \mu^8} &= \mathcal{A}_8 - 8\mathcal{A}_7\mathcal{A}_1 - 28\mathcal{A}_6\mathcal{A}_2 - 56\mathcal{A}_5\mathcal{A}_3 - 35\mathcal{A}_4^2 + 56\mathcal{A}_6\mathcal{A}_1^2 + 336\mathcal{A}_5\mathcal{A}_2\mathcal{A}_1
\end{aligned}
$$

$$
\begin{aligned}
&+560\mathcal{A}_4\mathcal{A}_3\mathcal{A}_1+420\mathcal{A}_4\mathcal{A}_2^2+560\mathcal{A}_3^2\mathcal{A}_2-336\mathcal{A}_5\mathcal{A}_1^3-2520\mathcal{A}_4\mathcal{A}_2\mathcal{A}_1^2\\
&-1680\mathcal{A}_3^2\mathcal{A}_1^2-5040\mathcal{A}_3\mathcal{A}_2^2\mathcal{A}_1-630\mathcal{A}_2^4+1680\mathcal{A}_4\mathcal{A}_1^4+13440\mathcal{A}_3\mathcal{A}_2\mathcal{A}_1^3\\
&+10080\mathcal{A}_2^3\mathcal{A}_1^2-6720\mathcal{A}_3\mathcal{A}_1^5-25200\mathcal{A}_2^2\mathcal{A}_1^4+20160\mathcal{A}_2\mathcal{A}_1^6-5040\mathcal{A}_1^8 \qquad (26)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^9\ln\mathcal{Z}}{\partial\mu^9} &= \mathcal{A}_9-9\mathcal{A}_8\mathcal{A}_1-36\mathcal{A}_7\mathcal{A}_2-84\mathcal{A}_6\mathcal{A}_3-126\mathcal{A}_5\mathcal{A}_4\\
&+72\mathcal{A}_7\mathcal{A}_1^2+504\mathcal{A}_6\mathcal{A}_2\mathcal{A}_1+1008\mathcal{A}_5\mathcal{A}_3\mathcal{A}_1+630\mathcal{A}_4^2\mathcal{A}_1+756\mathcal{A}_5\mathcal{A}_2^2\\
&+2520\mathcal{A}_4\mathcal{A}_3\mathcal{A}_2+560\mathcal{A}_3^3-504\mathcal{A}_6\mathcal{A}_1^3-4536\mathcal{A}_5\mathcal{A}_2\mathcal{A}_1^2-7560\mathcal{A}_4\mathcal{A}_3\mathcal{A}_1^2\\
&-11340\mathcal{A}_4\mathcal{A}_2^2\mathcal{A}_1-15120\mathcal{A}_3^2\mathcal{A}_2\mathcal{A}_1-7560\mathcal{A}_3\mathcal{A}_2^3+3024\mathcal{A}_5\mathcal{A}_1^4+30240\mathcal{A}_4\mathcal{A}_2\mathcal{A}_1^3\\
&+20160\mathcal{A}_3^2\mathcal{A}_1^3+90720\mathcal{A}_3\mathcal{A}_2^2\mathcal{A}_1^2+22680\mathcal{A}_2^4\mathcal{A}_1-15120\mathcal{A}_4\mathcal{A}_1^5-151200\mathcal{A}_3\mathcal{A}_2\mathcal{A}_1^4\\
&-151200\mathcal{A}_2^3\mathcal{A}_1^3+60480\mathcal{A}_3\mathcal{A}_1^6+272160\mathcal{A}_2^2\mathcal{A}_1^5-181440\mathcal{A}_2\mathcal{A}_1^7+40320\mathcal{A}_1^9 \qquad (27)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^{10}\ln\mathcal{Z}}{\partial\mu^{10}} &= \mathcal{A}_{10}-10\mathcal{A}_9\mathcal{A}_1-45\mathcal{A}_8\mathcal{A}_2-120\mathcal{A}_7\mathcal{A}_3-210\mathcal{A}_6\mathcal{A}_4-126\mathcal{A}_5^2+90\mathcal{A}_8\mathcal{A}_1^2\\
&+720\mathcal{A}_7\mathcal{A}_2\mathcal{A}_1+1680\mathcal{A}_6\mathcal{A}_3\mathcal{A}_1+2520\mathcal{A}_5\mathcal{A}_4\mathcal{A}_1+1260\mathcal{A}_6\mathcal{A}_2^2+5040\mathcal{A}_5\mathcal{A}_3\mathcal{A}_2\\
&+3150\mathcal{A}_4^2\mathcal{A}_2+4200\mathcal{A}_4\mathcal{A}_3^2-720\mathcal{A}_7\mathcal{A}_1^3-7560\mathcal{A}_6\mathcal{A}_2\mathcal{A}_1^2-15120\mathcal{A}_5\mathcal{A}_3\mathcal{A}_1^2\\
&-9450\mathcal{A}_4^2\mathcal{A}_1^2-22680\mathcal{A}_5\mathcal{A}_2^2\mathcal{A}_1-75600\mathcal{A}_4\mathcal{A}_3\mathcal{A}_2\mathcal{A}_1-16800\mathcal{A}_3^3\mathcal{A}_1\\
&-18900\mathcal{A}_4\mathcal{A}_2^3-37800\mathcal{A}_3^2\mathcal{A}_2^2+5040\mathcal{A}_6\mathcal{A}_1^4+60480\mathcal{A}_5\mathcal{A}_2\mathcal{A}_1^3+100800\mathcal{A}_4\mathcal{A}_3\mathcal{A}_1^3\\
&+226800\mathcal{A}_4\mathcal{A}_2^2\mathcal{A}_1^2+302400\mathcal{A}_3^2\mathcal{A}_2\mathcal{A}_1^2+302400\mathcal{A}_3\mathcal{A}_2^3\mathcal{A}_1+22680\mathcal{A}_2^5\\
&-30240\mathcal{A}_5\mathcal{A}_1^5-378000\mathcal{A}_4\mathcal{A}_2\mathcal{A}_1^4-252000\mathcal{A}_3^2\mathcal{A}_1^4-1512000\mathcal{A}_3\mathcal{A}_2^2\mathcal{A}_1^3\\
&-567000\mathcal{A}_2^4\mathcal{A}_1^2+151200\mathcal{A}_4\mathcal{A}_1^6+1814400\mathcal{A}_3\mathcal{A}_2\mathcal{A}_1^5+2268000\mathcal{A}_2^3\mathcal{A}_1^4\\
&-604800\mathcal{A}_3\mathcal{A}_1^7-3175200\mathcal{A}_2^2\mathcal{A}_1^6+1814400\mathcal{A}_2\mathcal{A}_1^8-362880\mathcal{A}_1^{10} \qquad (28)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^{11}\ln\mathcal{Z}}{\partial\mu^{11}} &= \mathcal{A}_{11}-11\mathcal{A}_{10}\mathcal{A}_1-55\mathcal{A}_9\mathcal{A}_2-165\mathcal{A}_8\mathcal{A}_3-330\mathcal{A}_7\mathcal{A}_4-462\mathcal{A}_6\mathcal{A}_5\\
&+110\mathcal{A}_9\mathcal{A}_1^2+990\mathcal{A}_8\mathcal{A}_2\mathcal{A}_1+2640\mathcal{A}_7\mathcal{A}_3\mathcal{A}_1+4620\mathcal{A}_6\mathcal{A}_4\mathcal{A}_1+2772\mathcal{A}_5^2\mathcal{A}_1\\
&+1980\mathcal{A}_7\mathcal{A}_2^2+9240\mathcal{A}_6\mathcal{A}_3\mathcal{A}_2+13860\mathcal{A}_5\mathcal{A}_4\mathcal{A}_2+9240\mathcal{A}_5\mathcal{A}_3^2+11550\mathcal{A}_4^2\mathcal{A}_3\\
&-990\mathcal{A}_8\mathcal{A}_1^3-11880\mathcal{A}_7\mathcal{A}_2\mathcal{A}_1^2-27720\mathcal{A}_6\mathcal{A}_3\mathcal{A}_1^2-41580\mathcal{A}_5\mathcal{A}_4\mathcal{A}_1^2-41580\mathcal{A}_6\mathcal{A}_2^2\mathcal{A}_1\\
&-166320\mathcal{A}_5\mathcal{A}_3\mathcal{A}_2\mathcal{A}_1-103950\mathcal{A}_4^2\mathcal{A}_2\mathcal{A}_1-138600\mathcal{A}_4\mathcal{A}_3^2\mathcal{A}_1-41580\mathcal{A}_5\mathcal{A}_2^3\\
&-207900\mathcal{A}_4\mathcal{A}_3\mathcal{A}_2^2-92400\mathcal{A}_3^3\mathcal{A}_2+7920\mathcal{A}_7\mathcal{A}_1^4+110880\mathcal{A}_6\mathcal{A}_2\mathcal{A}_1^3+221760\mathcal{A}_5\mathcal{A}_3\mathcal{A}_1^3\\
&+138600\mathcal{A}_4^2\mathcal{A}_1^3+498960\mathcal{A}_5\mathcal{A}_2^2\mathcal{A}_1^2+1663200\mathcal{A}_4\mathcal{A}_3\mathcal{A}_2\mathcal{A}_1^2+369600\mathcal{A}_3^3\mathcal{A}_1^2\\
&+831600\mathcal{A}_4\mathcal{A}_2^3\mathcal{A}_1+1663200\mathcal{A}_3^2\mathcal{A}_2^2\mathcal{A}_1+415800\mathcal{A}_3\mathcal{A}_2^4-55440\mathcal{A}_6\mathcal{A}_1^5\\
&-831600\mathcal{A}_5\mathcal{A}_2\mathcal{A}_1^4-1386000\mathcal{A}_4\mathcal{A}_3\mathcal{A}_1^4-4158000\mathcal{A}_4\mathcal{A}_2^2\mathcal{A}_1^3-5544000\mathcal{A}_3^2\mathcal{A}_2\mathcal{A}_1^3\\
&-8316000\mathcal{A}_3\mathcal{A}_2^3\mathcal{A}_1^2-1247400\mathcal{A}_2^5\mathcal{A}_1+332640\mathcal{A}_5\mathcal{A}_1^6+4989600\mathcal{A}_4\mathcal{A}_2\mathcal{A}_1^5\\
&+3326400\mathcal{A}_3^2\mathcal{A}_1^5+24948000\mathcal{A}_3\mathcal{A}_2^2\mathcal{A}_1^4+12474000\mathcal{A}_2^4\mathcal{A}_1^3-1663200\mathcal{A}_4\mathcal{A}_1^7\\
&-23284800\mathcal{A}_3\mathcal{A}_2\mathcal{A}_1^6-34927200\mathcal{A}_2^3\mathcal{A}_1^5+6652800\mathcal{A}_3\mathcal{A}_1^8+39916800\mathcal{A}_2^2\mathcal{A}_1^7\\
&-19958400\mathcal{A}_2\mathcal{A}_1^9+3628800\mathcal{A}_1^{11}, \qquad (29)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^{12}\ln\mathcal{Z}}{\partial\mu^{12}} &= \mathcal{A}_{12}-12\mathcal{A}_{11}\mathcal{A}_1-66\mathcal{A}_{10}\mathcal{A}_2-220\mathcal{A}_9\mathcal{A}_3-495\mathcal{A}_8\mathcal{A}_4-792\mathcal{A}_7\mathcal{A}_5-462\mathcal{A}_6^2\\
&+132\mathcal{A}_{10}\mathcal{A}_1^2+1320\mathcal{A}_9\mathcal{A}_2\mathcal{A}_1+3960\mathcal{A}_8\mathcal{A}_3\mathcal{A}_1+7920\mathcal{A}_7\mathcal{A}_4\mathcal{A}_1+11088\mathcal{A}_6\mathcal{A}_5\mathcal{A}_1\\
&+2970\mathcal{A}_8\mathcal{A}_2^2+15840\mathcal{A}_7\mathcal{A}_3\mathcal{A}_2+27720\mathcal{A}_6\mathcal{A}_4\mathcal{A}_2+16632\mathcal{A}_5^2\mathcal{A}_2+18480\mathcal{A}_6\mathcal{A}_3^2\\
&+55440\mathcal{A}_5\mathcal{A}_4\mathcal{A}_3+11550\mathcal{A}_4^3-1320\mathcal{A}_9\mathcal{A}_1^3-17820\mathcal{A}_8\mathcal{A}_2\mathcal{A}_1^2-47520\mathcal{A}_7\mathcal{A}_3\mathcal{A}_1^2\\
&-83160\mathcal{A}_6\mathcal{A}_4\mathcal{A}_1^2-49896\mathcal{A}_5^2\mathcal{A}_1^2-71280\mathcal{A}_7\mathcal{A}_2^2\mathcal{A}_1-332640\mathcal{A}_6\mathcal{A}_3\mathcal{A}_2\mathcal{A}_1\\
&-498960\mathcal{A}_5\mathcal{A}_4\mathcal{A}_2\mathcal{A}_1-332640\mathcal{A}_5\mathcal{A}_3^2\mathcal{A}_1-415800\mathcal{A}_4^2\mathcal{A}_3\mathcal{A}_1-83160\mathcal{A}_6\mathcal{A}_2^3\\
&-498960\mathcal{A}_5\mathcal{A}_3\mathcal{A}_2^2-311850\mathcal{A}_4^2\mathcal{A}_2^2-831600\mathcal{A}_4\mathcal{A}_3^2\mathcal{A}_2-92400\mathcal{A}_3^4+11880\mathcal{A}_8\mathcal{A}_1^4\\
&+190080\mathcal{A}_7\mathcal{A}_2\mathcal{A}_1^3+443520\mathcal{A}_6\mathcal{A}_3\mathcal{A}_1^3+665280\mathcal{A}_5\mathcal{A}_4\mathcal{A}_1^3+997920\mathcal{A}_6\mathcal{A}_2^2\mathcal{A}_1^2
\end{aligned}
$$

$$
\begin{aligned}
&+3991680\mathcal{A}_5\mathcal{A}_3\mathcal{A}_2\mathcal{A}_1^2 + 2494800\mathcal{A}_4^2\mathcal{A}_2\mathcal{A}_1^2 + 3326400\mathcal{A}_4\mathcal{A}_3^2\mathcal{A}_1^2 + 1995840\mathcal{A}_5\mathcal{A}_2^3\mathcal{A}_1 \\
&+9979200\mathcal{A}_4\mathcal{A}_3\mathcal{A}_2^2\mathcal{A}_1 + 4435200\mathcal{A}_3^3\mathcal{A}_2\mathcal{A}_1 + 1247400\mathcal{A}_4\mathcal{A}_2^4 + 3326400\mathcal{A}_3^2\mathcal{A}_2^3 \\
&-95040\mathcal{A}_7\mathcal{A}_1^5 - 1663200\mathcal{A}_6\mathcal{A}_2\mathcal{A}_1^4 - 3326400\mathcal{A}_5\mathcal{A}_3\mathcal{A}_1^4 - 2079000\mathcal{A}_4^2\mathcal{A}_1^4 \\
&-9979200\mathcal{A}_5\mathcal{A}_2^2\mathcal{A}_1^3 - 33264000\mathcal{A}_4\mathcal{A}_3\mathcal{A}_2\mathcal{A}_1^3 - 7392000\mathcal{A}_3^3\mathcal{A}_1^3 - 24948000\mathcal{A}_4\mathcal{A}_2^3\mathcal{A}_1^2 \\
&-49896000\mathcal{A}_3^2\mathcal{A}_2^2\mathcal{A}_1^2 - 24948000\mathcal{A}_3\mathcal{A}_2^4\mathcal{A}_1 - 1247400\mathcal{A}_2^6 + 665280\mathcal{A}_6\mathcal{A}_1^6 \\
&+11975040\mathcal{A}_5\mathcal{A}_2\mathcal{A}_1^5 + 19958400\mathcal{A}_4\mathcal{A}_3\mathcal{A}_1^5 + 74844000\mathcal{A}_4\mathcal{A}_2^2\mathcal{A}_1^4 \\
&+99792000\mathcal{A}_3^2\mathcal{A}_2\mathcal{A}_1^4 + 199584000\mathcal{A}_3\mathcal{A}_2^3\mathcal{A}_1^3 + 44906400\mathcal{A}_2^5\mathcal{A}_1^2 \\
&-3991680\mathcal{A}_5\mathcal{A}_1^7 - 69854400\mathcal{A}_4\mathcal{A}_2\mathcal{A}_1^6 - 46569600\mathcal{A}_3^2\mathcal{A}_1^6 - 419126400\mathcal{A}_3\mathcal{A}_2^2\mathcal{A}_1^5 \\
&-261954000\mathcal{A}_2^4\mathcal{A}_1^4 + 19958400\mathcal{A}_4\mathcal{A}_1^8 + 319334400\mathcal{A}_3\mathcal{A}_2\mathcal{A}_1^7 + 558835200\mathcal{A}_2^3\mathcal{A}_1^6 \\
&-79833600\mathcal{A}_3\mathcal{A}_1^9 - 538876800\mathcal{A}_2^2\mathcal{A}_1^8 + 239500800\mathcal{A}_2\mathcal{A}_1^{10} - 39916800\mathcal{A}_1^{12}. \qquad (30)
\end{aligned}
$$

At $\mu = 0$, because $(\partial^n(\ln\det M)/\partial\mu^n)$ is real for n even, purely imaginary for n odd, $\mathcal{A}_n = 0$ for n odd.

$$
\begin{aligned}
\frac{\partial^2 \ln \mathcal{Z}}{\partial\mu^2} &= \mathcal{A}_2, \\
\frac{\partial^4 \ln \mathcal{Z}}{\partial\mu^4} &= \mathcal{A}_4 - 3\mathcal{A}_2^2, \\
\frac{\partial^6 \ln \mathcal{Z}}{\partial\mu^6} &= \mathcal{A}_6 - 15\mathcal{A}_4\mathcal{A}_2 + 30\mathcal{A}_2^3, \\
\frac{\partial^8 \ln \mathcal{Z}}{\partial\mu^8} &= \mathcal{A}_8 - 28\mathcal{A}_6\mathcal{A}_2 - 35\mathcal{A}_4^2 + 420\mathcal{A}_4\mathcal{A}_2^2 - 630\mathcal{A}_2^4, \\
\frac{\partial^{10} \ln \mathcal{Z}}{\partial\mu^{10}} &= \mathcal{A}_{10} - 45\mathcal{A}_8\mathcal{A}_2 - 210\mathcal{A}_6\mathcal{A}_4 + 1260\mathcal{A}_6\mathcal{A}_2^2 + 3150\mathcal{A}_4^2\mathcal{A}_2 - 18900\mathcal{A}_4\mathcal{A}_2^3 + 22680\mathcal{A}_2^5, \\
\frac{\partial^{12} \ln \mathcal{Z}}{\partial\mu^{12}} &= \mathcal{A}_{12} - 66\mathcal{A}_{10}\mathcal{A}_2 - 495\mathcal{A}_8\mathcal{A}_4 - 462\mathcal{A}_6^2 + 2970\mathcal{A}_8\mathcal{A}_2^2 + 27720\mathcal{A}_6\mathcal{A}_4\mathcal{A}_2 + 11550\mathcal{A}_4^3 \\
&\quad -83160\mathcal{A}_6\mathcal{A}_2^3 - 311850\mathcal{A}_4^2\mathcal{A}_2^2 + 1247400\mathcal{A}_4\mathcal{A}_2^4 - 1247400\mathcal{A}_2^6, \qquad (31)
\end{aligned}
$$

and

$$
\frac{\partial^n \ln \mathcal{Z}}{\partial\mu^n} = 0 \qquad (32)
$$

for n odd.

Derivatives of $\ln\det M$**;** $\partial^n(\ln\det M)/\partial\mu^n$**.**

$$
\begin{aligned}
\frac{\partial \ln\det M}{\partial\mu} &= \mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}\right), \qquad (33) \\
\frac{\partial^2 \ln\det M}{\partial\mu^2} &= \mathrm{tr}\left(M^{-1}\frac{\partial^2 M}{\partial\mu^2}\right) - \mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}\right), \qquad (34) \\
\frac{\partial^3 \ln\det M}{\partial\mu^3} &= \mathrm{tr}\left(M^{-1}\frac{\partial^3 M}{\partial\mu^3}\right) - 3\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^2 M}{\partial\mu^2}\right) \\
&\quad +2\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}\right), \qquad (35) \\
\frac{\partial^4 \ln\det M}{\partial\mu^4} &= \mathrm{tr}\left(M^{-1}\frac{\partial^4 M}{\partial\mu^4}\right) - 4\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^3 M}{\partial\mu^3}\right) \\
&\quad -3\mathrm{tr}\left(M^{-1}\frac{\partial^2 M}{\partial\mu^2}M^{-1}\frac{\partial^2 M}{\partial\mu^2}\right) + 12\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^2 M}{\partial\mu^2}\right)
\end{aligned}
$$

$$-6\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}\right), \tag{36}$$

$$\begin{aligned}
\frac{\partial^5 \ln\det M}{\partial\mu^5} &= \mathrm{tr}\left(M^{-1}\frac{\partial^5 M}{\partial\mu^5}\right) - 5\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^4 M}{\partial\mu^4}\right) \\
&-10\mathrm{tr}\left(M^{-1}\frac{\partial^2 M}{\partial\mu^2}M^{-1}\frac{\partial^3 M}{\partial\mu^3}\right) + 20\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^3 M}{\partial\mu^3}\right) \\
&+30\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^2 M}{\partial\mu^2}M^{-1}\frac{\partial^2 M}{\partial\mu^2}\right) - 60\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^2 M}{\partial\mu^2}\right) \\
&+24\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}\right),
\end{aligned} \tag{37}$$

$$\begin{aligned}
\frac{\partial^6 \ln\det M}{\partial\mu^6} &= \mathrm{tr}\left(M^{-1}\frac{\partial^6 M}{\partial\mu^6}\right) - 6\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^5 M}{\partial\mu^5}\right) \\
&-15\mathrm{tr}\left(M^{-1}\frac{\partial^2 M}{\partial\mu^2}M^{-1}\frac{\partial^4 M}{\partial\mu^4}\right) - 10\mathrm{tr}\left(M^{-1}\frac{\partial^3 M}{\partial\mu^3}M^{-1}\frac{\partial^3 M}{\partial\mu^3}\right) \\
&+30\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^4 M}{\partial\mu^4}\right) + 60\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^2 M}{\partial\mu^2}M^{-1}\frac{\partial^3 M}{\partial\mu^3}\right) \\
&+60\mathrm{tr}\left(M^{-1}\frac{\partial^2 M}{\partial\mu^2}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^3 M}{\partial\mu^3}\right) + 30\mathrm{tr}\left(M^{-1}\frac{\partial^2 M}{\partial\mu^2}M^{-1}\frac{\partial^2 M}{\partial\mu^2}M^{-1}\frac{\partial^2 M}{\partial\mu^2}\right) \\
&-120\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^3 M}{\partial\mu^3}\right) \\
&-180\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^2 M}{\partial\mu^2}M^{-1}\frac{\partial^2 M}{\partial\mu^2}\right) \\
&-90\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^2 M}{\partial\mu^2}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^2 M}{\partial\mu^2}\right) \\
&+360\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^2 M}{\partial\mu^2}\right) \\
&-120\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}\right),
\end{aligned} \tag{38}$$

$$\begin{aligned}
\frac{\partial^7 \ln\det M}{\partial\mu^7} &= \mathrm{tr}\left(M^{-1}\frac{\partial^7 M}{\partial\mu^7}\right) - 7\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^6 M}{\partial\mu^6}\right) \\
&-21\mathrm{tr}\left(M^{-1}\frac{\partial^2 M}{\partial\mu^2}M^{-1}\frac{\partial^5 M}{\partial\mu^5}\right) - 35\mathrm{tr}\left(M^{-1}\frac{\partial^3 M}{\partial\mu^3}M^{-1}\frac{\partial^4 M}{\partial\mu^4}\right) \\
&+42\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^5 M}{\partial\mu^5}\right) + 105\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^2 M}{\partial\mu^2}M^{-1}\frac{\partial^4 M}{\partial\mu^4}\right) \\
&+105\mathrm{tr}\left(M^{-1}\frac{\partial^2 M}{\partial\mu^2}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^4 M}{\partial\mu^4}\right) + 140\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^3 M}{\partial\mu^3}M^{-1}\frac{\partial^3 M}{\partial\mu^3}\right) \\
&+210\mathrm{tr}\left(M^{-1}\frac{\partial^2 M}{\partial\mu^2}M^{-1}\frac{\partial^2 M}{\partial\mu^2}M^{-1}\frac{\partial^3 M}{\partial\mu^3}\right) - 210\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^4 M}{\partial\mu^4}\right) \\
&-420\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^2 M}{\partial\mu^2}M^{-1}\frac{\partial^3 M}{\partial\mu^3}\right) \\
&-420\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^2 M}{\partial\mu^2}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^3 M}{\partial\mu^3}\right)
\end{aligned}$$

$$
\begin{aligned}
&-420\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^3 M}{\partial\mu^3}M^{-1}\frac{\partial^2 M}{\partial\mu^2}\right)\\
&-630\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^2 M}{\partial\mu^2}M^{-1}\frac{\partial^2 M}{\partial\mu^2}M^{-1}\frac{\partial^2 M}{\partial\mu^2}\right)\\
&+840\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^3 M}{\partial\mu^3}\right)\\
&+1260\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^2 M}{\partial\mu^2}M^{-1}\frac{\partial^2 M}{\partial\mu^2}\right)\\
&+1260\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^2 M}{\partial\mu^2}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^2 M}{\partial\mu^2}\right)\\
&-2520\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^2 M}{\partial\mu^2}\right)\\
&+720\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}\right), \qquad (39)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^8 \ln\det M}{\partial\mu^8} &= \mathrm{tr}\left(M^{-1}\frac{\partial^8 M}{\partial\mu^8}\right)-8\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^7 M}{\partial\mu^7}\right)\\
&-28\mathrm{tr}\left(M^{-1}\frac{\partial^2 M}{\partial\mu^2}M^{-1}\frac{\partial^6 M}{\partial\mu^6}\right)-56\mathrm{tr}\left(M^{-1}\frac{\partial^3 M}{\partial\mu^3}M^{-1}\frac{\partial^5 M}{\partial\mu^5}\right)\\
&-35\mathrm{tr}\left(M^{-1}\frac{\partial^4 M}{\partial\mu^4}M^{-1}\frac{\partial^4 M}{\partial\mu^4}\right)+56\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^6 M}{\partial\mu^6}\right)\\
&+168\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^2 M}{\partial\mu^2}M^{-1}\frac{\partial^5 M}{\partial\mu^5}\right)+168\mathrm{tr}\left(M^{-1}\frac{\partial^2 M}{\partial\mu^2}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^5 M}{\partial\mu^5}\right)\\
&+280\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^3 M}{\partial\mu^3}M^{-1}\frac{\partial^4 M}{\partial\mu^4}\right)+280\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^4 M}{\partial\mu^4}M^{-1}\frac{\partial^3 M}{\partial\mu^3}\right)\\
&+420\mathrm{tr}\left(M^{-1}\frac{\partial^2 M}{\partial\mu^2}M^{-1}\frac{\partial^2 M}{\partial\mu^2}M^{-1}\frac{\partial^4 M}{\partial\mu^4}\right)+560\mathrm{tr}\left(M^{-1}\frac{\partial^2 M}{\partial\mu^2}M^{-1}\frac{\partial^3 M}{\partial\mu^3}M^{-1}\frac{\partial^3 M}{\partial\mu^3}\right)\\
&-336\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^5 M}{\partial\mu^5}\right)\\
&-840\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^2 M}{\partial\mu^2}M^{-1}\frac{\partial^4 M}{\partial\mu^4}\right)\\
&-840\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^2 M}{\partial\mu^2}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^4 M}{\partial\mu^4}\right)\\
&-840\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^4 M}{\partial\mu^4}M^{-1}\frac{\partial^2 M}{\partial\mu^2}\right)\\
&-1120\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^3 M}{\partial\mu^3}M^{-1}\frac{\partial^3 M}{\partial\mu^3}\right)\\
&-560\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^3 M}{\partial\mu^3}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^3 M}{\partial\mu^3}\right)\\
&-1680\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^2 M}{\partial\mu^2}M^{-1}\frac{\partial^2 M}{\partial\mu^2}M^{-1}\frac{\partial^3 M}{\partial\mu^3}\right)
\end{aligned}
$$

$$
\begin{aligned}
&-1680\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^2 M}{\partial\mu^2}M^{-1}\frac{\partial^3 M}{\partial\mu^3}M^{-1}\frac{\partial^2 M}{\partial\mu^2}\right)\\
&-1680\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^3 M}{\partial\mu^3}M^{-1}\frac{\partial^2 M}{\partial\mu^2}M^{-1}\frac{\partial^2 M}{\partial\mu^2}\right)\\
&-630\mathrm{tr}\left(M^{-1}\frac{\partial^2 M}{\partial\mu^2}M^{-1}\frac{\partial^2 M}{\partial\mu^2}M^{-1}\frac{\partial^2 M}{\partial\mu^2}M^{-1}\frac{\partial^2 M}{\partial\mu^2}\right)\\
&+1680\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^4 M}{\partial\mu^4}\right)\\
&+3360\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^2 M}{\partial\mu^2}M^{-1}\frac{\partial^3 M}{\partial\mu^3}\right)\\
&+3360\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^2 M}{\partial\mu^2}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^3 M}{\partial\mu^3}\right)\\
&+3360\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^3 M}{\partial\mu^3}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^2 M}{\partial\mu^2}\right)\\
&+3360\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^3 M}{\partial\mu^3}M^{-1}\frac{\partial^2 M}{\partial\mu^2}\right)\\
&+5040\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^2 M}{\partial\mu^2}M^{-1}\frac{\partial^2 M}{\partial\mu^2}M^{-1}\frac{\partial^2 M}{\partial\mu^2}\right)\\
&+5040\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^2 M}{\partial\mu^2}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^2 M}{\partial\mu^2}M^{-1}\frac{\partial^2 M}{\partial\mu^2}\right)\\
&-6720\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^3 M}{\partial\mu^3}\right)\\
&-10080\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^2 M}{\partial\mu^2}M^{-1}\frac{\partial^2 M}{\partial\mu^2}\right)\\
&-10080\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^2 M}{\partial\mu^2}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^2 M}{\partial\mu^2}\right)\\
&-5040\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^2 M}{\partial\mu^2}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^2 M}{\partial\mu^2}\right)\\
&+20160\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial^2 M}{\partial\mu^2}\right)\\
&-5040\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}\right). \quad (40)
\end{aligned}
$$