Strangeness Fluctuations in QCD at High Temperature and Non-Zero Baryon Number Density

M. Cheng[a], N. H. Christ[a], S. Datta[b], J. van der Heide[c], C. Jung[b], F. Karsch[b,c], O. Kaczmarek[c], E. Laermann[c], R. D. Mawhinney[a], C. Miao[c], P. Petreczky[b,d], K. Petrov[e], C. Schmidt[b] and T. Umeda[b]
[a] *Physics Department, Columbia University, New York, NY 10027, USA*
[b] *Physics Department, Brookhaven National Laboratory, Upton, NY 11973, USA*
[c] *Fakultät für Physik, Universität Bielefeld, D-33615 Bielefeld, Germany*
[d] *RIKEN-BNL Research Center, Brookhaven National Laboratory, Upton, NY 11973, USA*
[e] *Niels Bohr Institute, University of Copenhagen, Blegdamsvej 17, DK-2100 Copenhagen, Denmark*

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We present results on strangeness fluctuations in QCD at non-zero baryon number density and temperature obtained from lattice calculations with almost physical up and down quark masses and a physical value for the strange quark mass. We show that strangeness fluctuations are enhanced at non-zero baryon number density and correlated to fluctuations of the latter. Nonetheless, in a dense medium with overall vanishing strangeness as it is created in heavy ion collisions strangeness fluctuations are significantly smaller than baryon number fluctuations. We also comment on strangeness fluctuations at vanishing baryon number density as well as in flavor symmetric dense matter.

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INTRODUCTION

Abundances of strange particles and their fluctuations are generally considered to be sensitive indicators for the thermal properties of the dense matter created in heavy ion collisions. While the overall strangeness as well as the number of strange quarks and antiquarks vanishes in the initial stage of such a collision strange quark antiquark pairs will be created early on and eventually will equilibrate with the dense system created in the collision. The net strangeness number, however, will remain zero during the short life time of such a system. This is in contrast to the net baryon number density that can be controlled through the center of mass energy, $\sqrt{s}$, in a heavy ion collision.

While at vanishing baryon number density the transition from the low temperature hadronic phase to the high temperature plasma phase is a rapid but continuous crossover it is expected that this transition is a true phase transition at non-vanishing baryon number density [1]. In the vicinity of the critical point that controls this second order phase transition fluctuation of e.g. baryon number and electric charge are expected to be large. In fact, at any non-vanishing value of the net baryon number density these fluctuations will diverge at least in the chiral limit and they will also diverge for non-zero quark masses if a critical point indeed exists in the QCD phase diagram at non-zero baryon number density.

Experimentally one hopes to learn about critical behavior at non-zero baryon number density from studies of event-by-event fluctuations of e.g. baryon number and strangeness or from studies of abundances of strange hadrons. In fact, the non monotonic behavior of the K/π ratio as function of $\sqrt{s}$ is sometimes taken as indication for critical behavior at non-zero baryon number density [2].

However, to what extend strangeness fluctuations are correlated with the large baryon number fluctuations in the vicinity of a critical point although the net strangeness number vanishes is a question on properties of QCD at non-zero baryon chemical potential μ_B which so far has not been analyzed in any detail. From lattice calculations it is well known that baryon number and strangeness fluctuations increase continuously with increasing temperature at vanishing baryon number density, or equivalently at $\mu_B = 0$, [3–5] and it also has been shown that for $\mu_B > 0$ the baryon number fluctuations increase and start to show a pronounced peak in the transition region from low to high temperature [4]. There are also indications from lattice calculations with dynamical up and down quarks and a valence (quenched) strange quark sector [6] that at $\mu_B = 0$ baryon number and strangeness are strongly correlated. However, so far nothing is known about strangeness fluctuations at non-zero μ_B

We present here first results from lattice calculations on strangeness fluctuations in QCD with dynamical light and strange quark degrees of freedom. The results are based on calculations with an improved staggered fermion action that strongly reduces lattice cut-off effects in bulk thermodynamics. The values of the quark masses used in this calculation are almost physical; the strange quark mass is fixed to its physical value while the light up and down quark mass are taken to be degenerate and about twice as large as the average up and down quark masses realized in nature.

STRANGENESS FLUCTUATIONS

As a direct lattice calculations at non-zero chemical potential are not possible various approaches have been designed to circumvent this problem [7]. We will here use here a Taylor expansion of thermodynamic quantities in terms of quark number and strangeness chemical potentials, μ_q and μ_s, respectively. We will analyze the density dependence of light and strange quark fluctuations using a Taylor expansion around the limit of vanishing $\hat\mu_{q,s} \equiv \mu_{q,s}/T$. We start with an expansion of the pressure,

$$\frac{p}{T^4} \equiv \frac{1}{VT^3} \ln Z(V,T,\mu_q,\mu_s) = \sum_{k,l} c_{k,l} \hat\mu_q^k \hat\mu_s^l \ , \qquad (1)$$

with Taylor expansion coefficients,

$$c_{k,l} = \frac{1}{k!l!} \frac{\partial^{k+l} p/T^4}{\partial \hat\mu_q^k \partial \hat\mu_q^l} \ . \qquad (2)$$

We note that only expansion coefficients with $k+l$ even contribute to Eq. 1. From this one obtains Taylor expansions for observables that are given in terms of derivatives of p/T^4, e.g.

$$O_{ij} = \frac{\partial^{i+j} p/T^4}{\partial \hat\mu_q^i \partial \hat\mu_q^j} \ . \qquad (3)$$

In particular one obtains the light and strange quark number densities as $n_q/T^3 = O_{10}$ and $n_s/T^3 = O_{01}$ as well as the corresponding susceptibilities, $\chi_q/T^2 = O_{20}$ and $\chi_s/T^2 = O_{02}$. The leading order corrections $\chi_{q,s}$ at non-zero $\mu_{q,s}$ are then given by

$$\begin{aligned} \chi_q/T^2 &= 2c_{20} + 6c_{31}\hat\mu_q\hat\mu_s + \\ & \quad 2c_{22}\hat\mu_s^2 + 12c_{40}\hat\mu_q^2 + \mathcal{O}(\hat\mu_{q,s}^4) \ ; \qquad (4) \\ \chi_s/T^2 &= 2c_{02} + 6c_{13}\hat\mu_q\hat\mu_s + \\ & \quad 2c_{22}\hat\mu_q^2 + 12c_{04}\hat\mu_s^2 + \mathcal{O}(\hat\mu_{q,s}^4) \ . \qquad (5) \end{aligned}$$

The constraint of vanishing strangeness, $n_s \equiv 0$, which is appropriate for situations met in dense matter created in heavy ion collisions, determines the dependence of the strange quark chemical potential on μ_q,

$$\begin{aligned} \hat\mu_s &= \sum_{n=0}^{\infty} d_n \hat\mu_q^{2n+1} = -\frac{c_{11}}{c_{02}}\hat\mu_q + \\ & \quad \frac{2c_{04}c_{11}^3 - 3c_{02}c_{11}^2c_{13} + 4c_{02}^2c_{11}c_{22} - 4c_{02}^3c_{31}}{8c_{02}^4}\hat\mu_q^3 \\ & \quad + \mathcal{O}(\hat\mu_q^5) \ . \qquad (6) \end{aligned}$$

We note that to all orders the expansion coefficients d_n are proportional to off-diagonal expansion coefficients of the pressure, *i.e.* c_{ij} with $i > 0$ and $j > 0$. These expansion coefficients vanish in perturbation theory up to $\mathcal{O}(g^6 \ln 1/g)$ [4, 8] and are generally small relative to the leading order terms c_{20} and c_{02}, respectively. Setting $\mu_s = 0$ thus is expected to be a good approximation at high temperature whereas it should be non-zero in the hadronic phase of QCD.

We have calculated all terms contributing to the expansion of p/T^4 and needed to satisfy the constraint of vanishing strangeness up to $\mathcal{O}(\mu_q^i \mu_s^j)$ with $(i+j) \le 6$ on lattices of size $16^3 \times 4$ and $24^3 \times 6$ using a tree-level improved gauge action and the p4fat3 staggered fermion action as described in [9]. All calculations are performed with a strange quark mass adjusted to its physical value by tuning the strange pseudo-scalar meson mass, $m_{ß}$ to a value of about 470 MeV. The light quark masses are chosen to be degenerate, $m_q = 0.1 m_s$, which is about twice as large as the physical value for the average of up and down quark masses. The choice of the relevant bare quark mass values is based of extensive zero temperature calculations performed on lattices of size $16^3 \times 32$ and $24^3 \times 32$ [10]. We have performed calculations in a temperature range $0.8 \lesssim T/T_c \lesssim 2$. At the various values of the gauge coupling we used 300 to 700 gauge field configurations separated by at least 300 trajectories to calculated the expansion coefficients. The latter have been calculated using unbiased random estimators [11]. While at high temperature about 50 random sources per configuration were sufficient to get reliable estimates for the various expansion coefficients we used up to 400 random sources below the transition temperature. xxxxxxxx ($N_\tau = 6$, c_6 ??)

The temperature dependence of quark number fluctuations at vanishing chemical potential is well known since quite some time. We show in the upper part of Fig. 1 our result for strangeness fluctuations at $\mu_q,\ \mu_s = 0$, *i.e.* $\chi_s(0,0) = 2c_{02}$. The dominant correction to strangeness fluctuations at non-vanishing μ_q arise from terms that do not vanish for $\mu_s = 0$, $\chi_s(\mu_q,0)/T^2 = 2c_{02} + 2c_{22}\mu_q^2 + 2c_{42}\mu_q^4$. In fact, all additional terms that contribute at non-zero μ_q due to the constraint of vanishing overall strangeness, $n_s = 0$, vanish in the high temperature ideal gas limit and arise in higher orders in perturbation theory xxxx. We find that all terms contributing to $\chi_s(\mu_q, \mu_s(\mu_q)$ which arise from a non-vanishing of the strangeness chemical potential, contribute less than xx% to $\chi_s(\mu_q,\mu_s(\mu_q))$ at temperatures $0.8T_c \lesssim T \lesssim T_c$ and are negligible for $T \ge 1.1T_c$.

While the leading contribution gives, of course, the strangeness fluctuation at vanishing baryon number density, $c_{02} = \langle N_s^2 \rangle$, the dominant contribution to the $\mathcal{O}(\mu_q^2)$ correction is related to fluctuations of the strangeness-

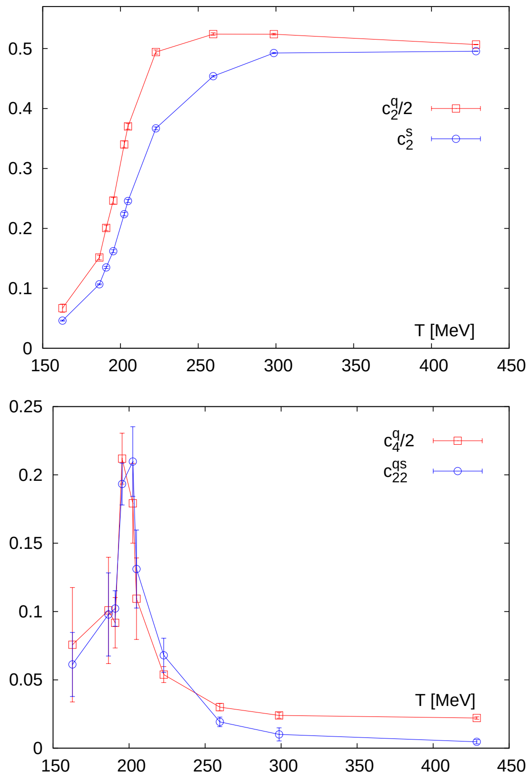

FIG. 1: Leading order and next to leading order expansion coefficients for the light quark and strangeness number susceptibilities.

baryon number correlations,

$$c_{22} \simeq \langle N_s^2 N_q^2 \rangle - \langle N_s^2 \rangle \langle N_q^2 \rangle - 2\langle N_s N_q \rangle^2 \qquad (7)$$

We find that the last term contributing to c_{22} is, in fact, small compared to the two others. The leading contribution to the second order Taylor expansion coefficient thus reflects the correlation between the squares of baryon number and strangeness. The temperature dependence of c_{22} is shown in the lower part of Fig. 1.

Fig. 1 shows that at vanishing chemical potential the correlation between strange and light quark numbers, N_s^2 and N_q^2, is strongly enhanced. In fact, c_{22} behaves very similar to the quartic fluctuations of the light quark number, $c_{40} \sim \partial^4 \ln Z / \partial \hat{\mu}_q^4$, which in the chiral limit is dominated by the contribution from the singular part of the free energy at vanishing chemical potential, $f/T = -V^{-1} \ln Z$.

A comparison of the two leading order expansion coefficients for χ_s shows that the ratio c_{22}/c_{02} never becomes larger than unity. The ratio of the expansion coefficients for p/T^4, on the other hand, becomes larger than unity for $T \lesssim T_c$. This suggests that at all temperatures the expansion of χ_s is well behaved up to the radius of convergence for the pressure. xxxxx

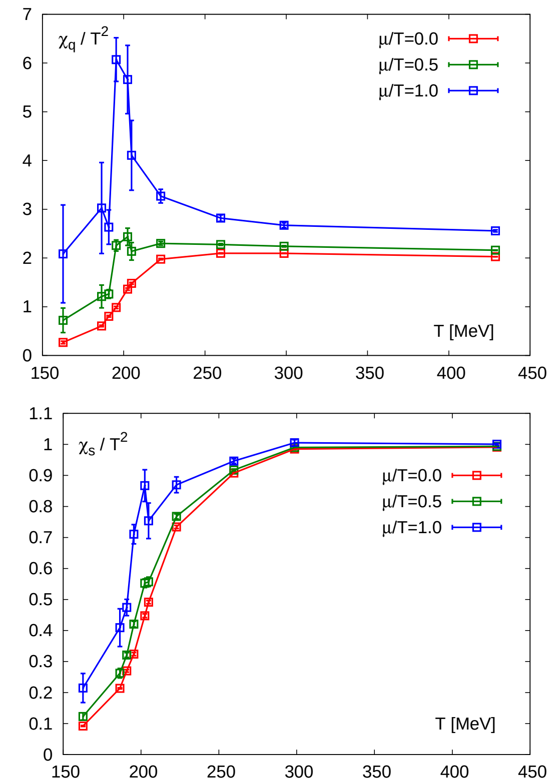

FIG. 2: Light quark and strangeness number fluctuations as function of the light quark chemical potential.

We show in Fig. 2 results for strangeness and light quark number fluctuations at non-zero chemical potential. xxxx

[1] Stephanov
[2] Marek's horn
[3] MILC susceptibility
[4] our c6 paper
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