Statistical Systems with Finite Bath: A Description of an Event Generator

Michael Hauer[1]

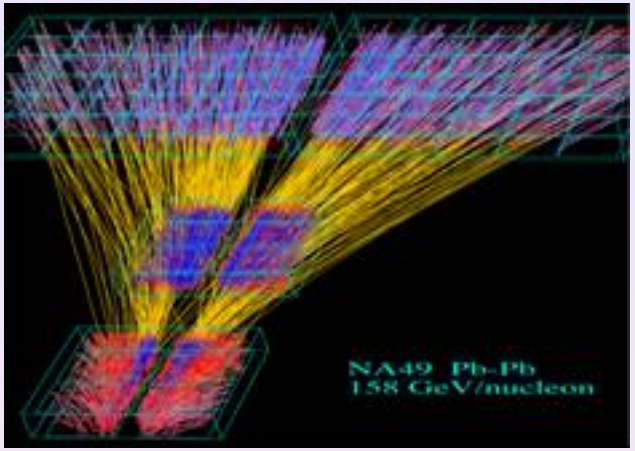
[1] Helmholtz Research School, University of Frankfurt, Frankfurt, Germany

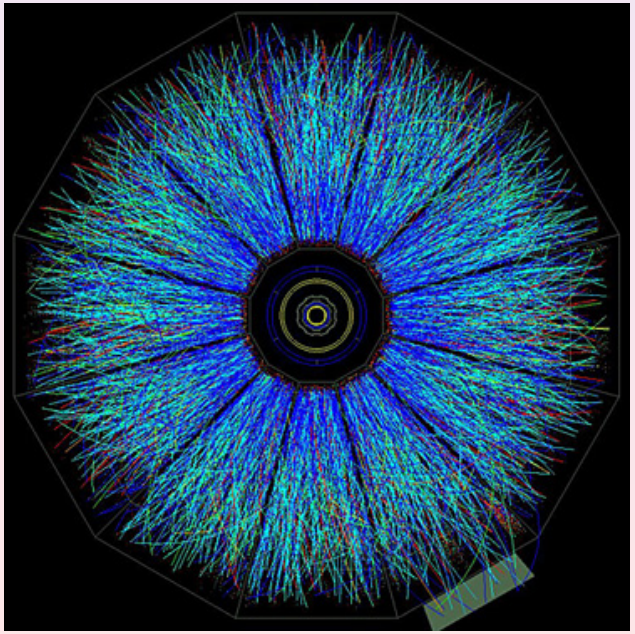
CPOD, BNL, June 8, 2009

H-QM Helmholtz Research School Quark Matter Studies

Outline

1. Statistical Ensembles
2. Monte Carlo Event Generator
3. Grand Canonical Ensemble
4. Extrapolating to the MCE
5. Multiplicity Fluctuations
6. Conclusion





Motivation

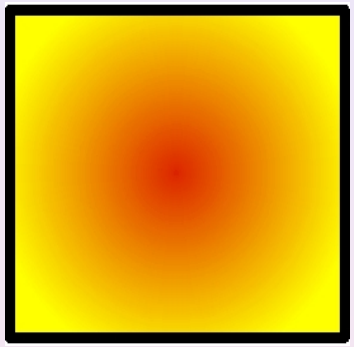
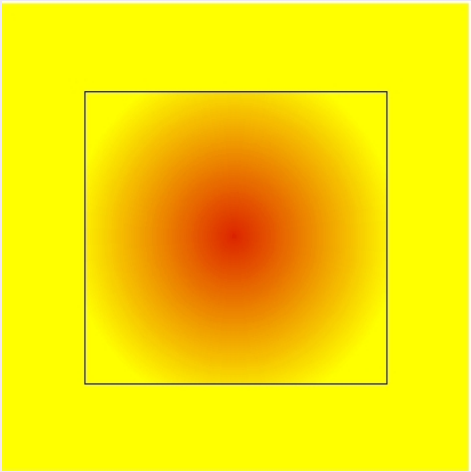
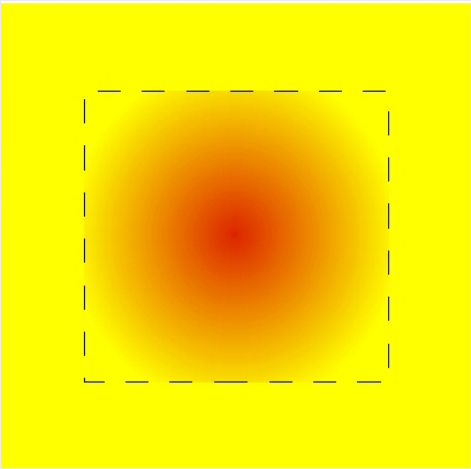
Aim:

Study (multiplicity) fluctuations and correlations for a final state hadron resonance gas in limited acceptance in the microcanonical limit.

Tool: Monte Carlo

- We first sample the GCE, then
- extrapolate to the MCE ‘sample and reject’ limit.
- This allows for inclusion of resonance decay effects, and
- the for analysis of ‘real events’.

Textbook Statistical Ensembles

MCE	CE	GCE
		
$Z(V,E,Q)$	$Z(V,T,Q)$	$Z(V,T,\mu)$
	$\sum_E e^{-E/T}\, Z(V,E,Q)$	$\sum_Q e^{Q\mu/T}\, Z(V,T,Q)$

$\Longleftarrow$ simpler to calculate and to sample $\Longrightarrow$

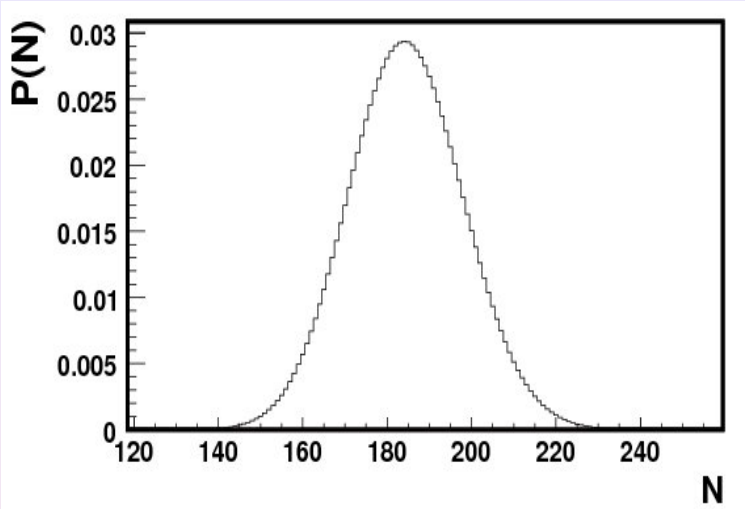
$\Longleftarrow$ more interesting $\Longrightarrow$

Sampling the GCE (Boltzmann)

1. Multiplicity

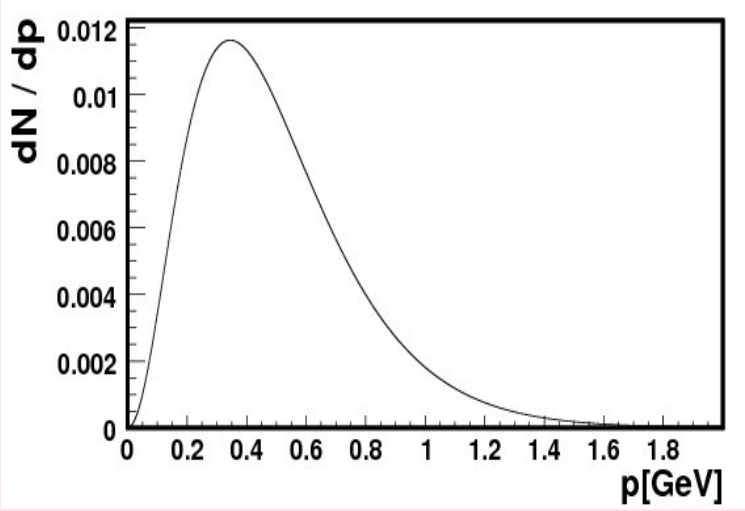
$$P(N_j) = \frac{z_j^{N_j}}{N_j!} e^{-z_j}, \quad \text{where}$$

$$z_j = \frac{g_j V}{2\pi^2} m_j^2 \, T \, K_2\left(\frac{m_j}{T}\right) e^{\mu_j/T}$$



2. Momentum Spectrum

$$\frac{dN}{dp_j} \propto p_j^2 \; e^{-\sqrt{m_j^2+p_j^2}/T}$$



Particles are sampled independently and are isotropically distributed in momentum space.

Sampling the GCE (Boltzmann)

3. Resonance Decay

- Perform 2 and 3 body decays
- Allow for successive decay of unstable daughters
- Consider only strong decays, and omit weak and electro magnetic decays

See also: THERMINATOR package

4. Calculate Extensive Quantities

Calculate for each event n:

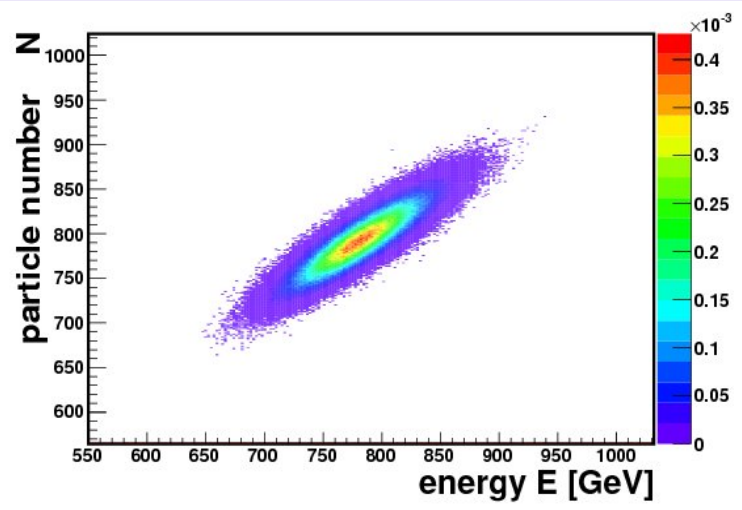
- $\mathcal{Q}^l_{1,n} = \sum_{\text{particles } i_n} \mathfrak{q}^l_{i_n}$, where
- $\mathfrak{q}^l_{i_n} = \left(b_{i_n} , s_{i_n} , q_{i_n} , \varepsilon_{i_n} , p_{x,i_n} , p_{y,i_n} , p_{z,i_n} \right)$

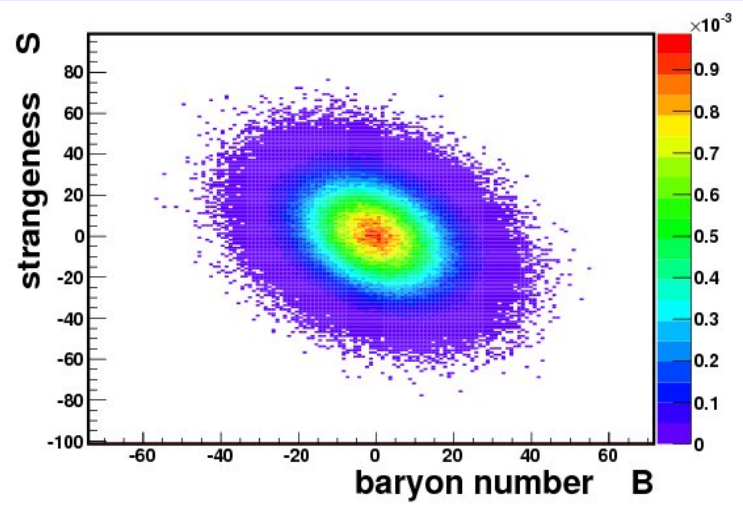
is the 'charge vector' of particle i in event n.

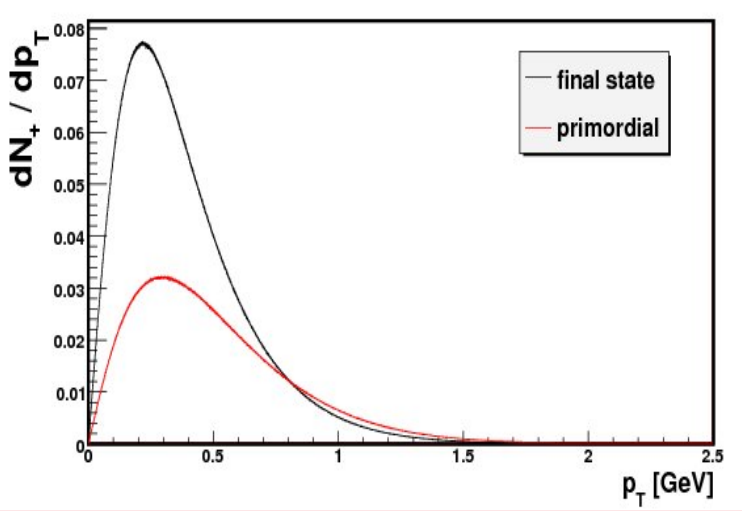
Momentum Spectra and Distributions

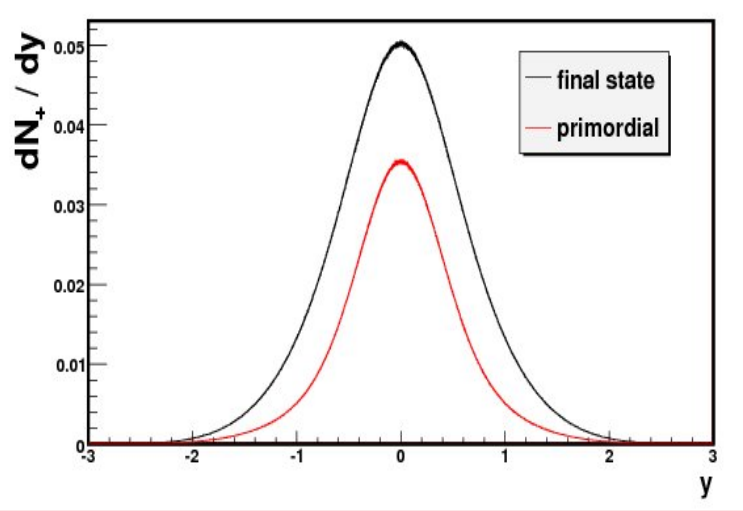
$V_1 = 2000 fm^3 \quad T = 0.16 GeV \quad \mu = 0.0 GeV$

full hadron gas









Moments of a Distribution

Moments of a Distribution

$$\langle X^n Y^m \rangle \equiv \sum_{X,Y} X^n \, Y^m \, P(X,Y)$$

Variance

$$\langle (\Delta X)^2 \rangle = \langle X^2 \rangle - \langle X \rangle^2$$

Covariance

$$\langle \Delta X \Delta Y \rangle = \langle XY \rangle - \langle X \rangle \langle Y \rangle$$

Scaled Variance

$$\omega_X = \frac{\langle (\Delta X)^2 \rangle}{\langle X \rangle}$$

Correlation Coefficient

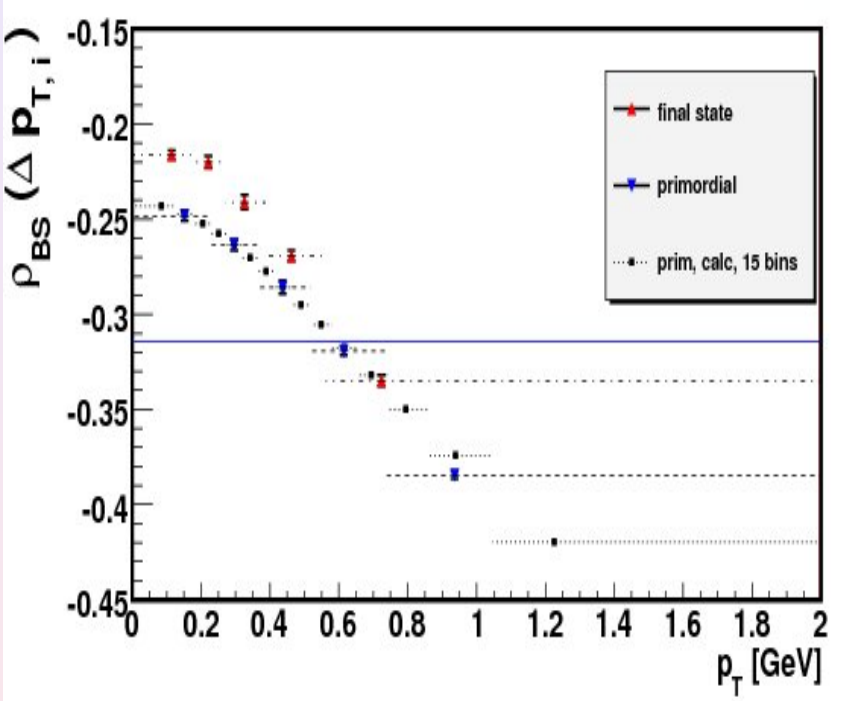
$$\rho_{XY} = \frac{\langle \Delta X \Delta Y \rangle}{\sqrt{\langle (\Delta X)^2 \rangle \langle (\Delta Y)^2 \rangle}}$$

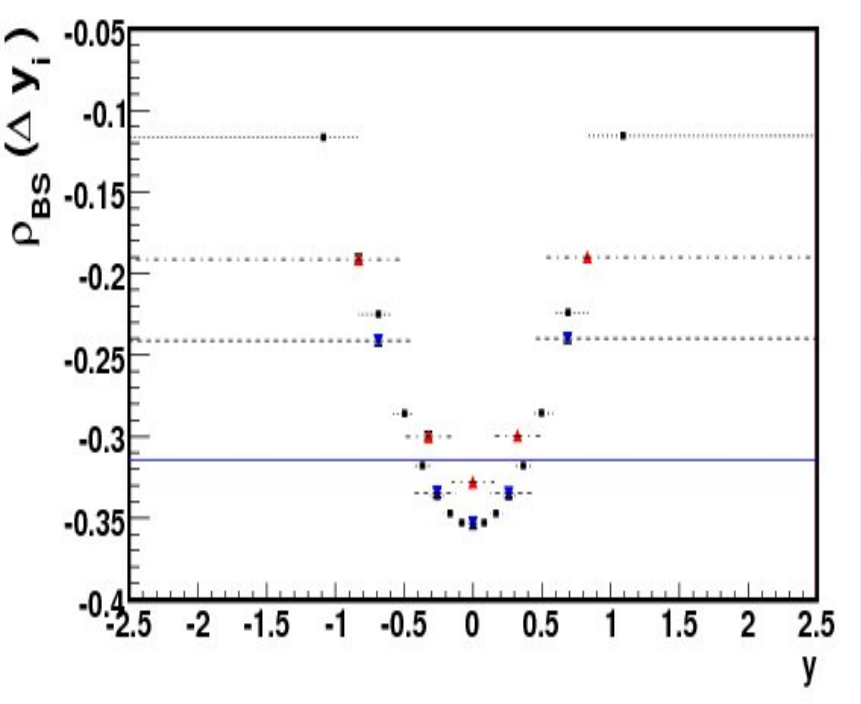
GCE Correlations between Charges

$V_1 = 2000 fm^3 \quad T = 0.16 GeV \quad \mu = 0.0 GeV$

full hadron gas

Correlation Coefficient ρ_{BS} in Limited Acceptance





- The correlation is sensitive to one's acceptance window.
- The Δp_T and Δy dependence is caused by different hadron masses.

Resonance decay:

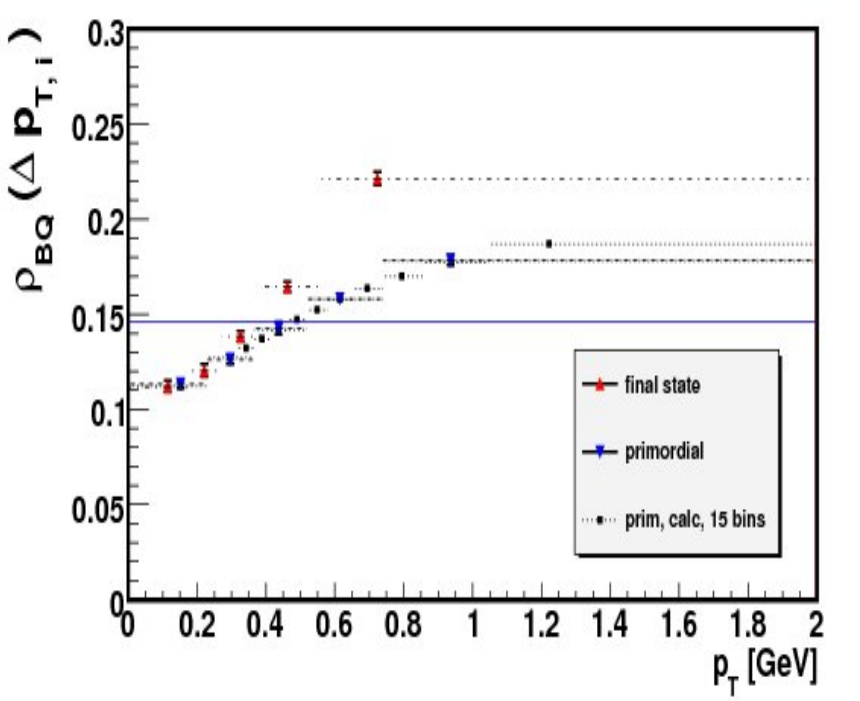
- does not change trends.
- could, however, wash out the signal!

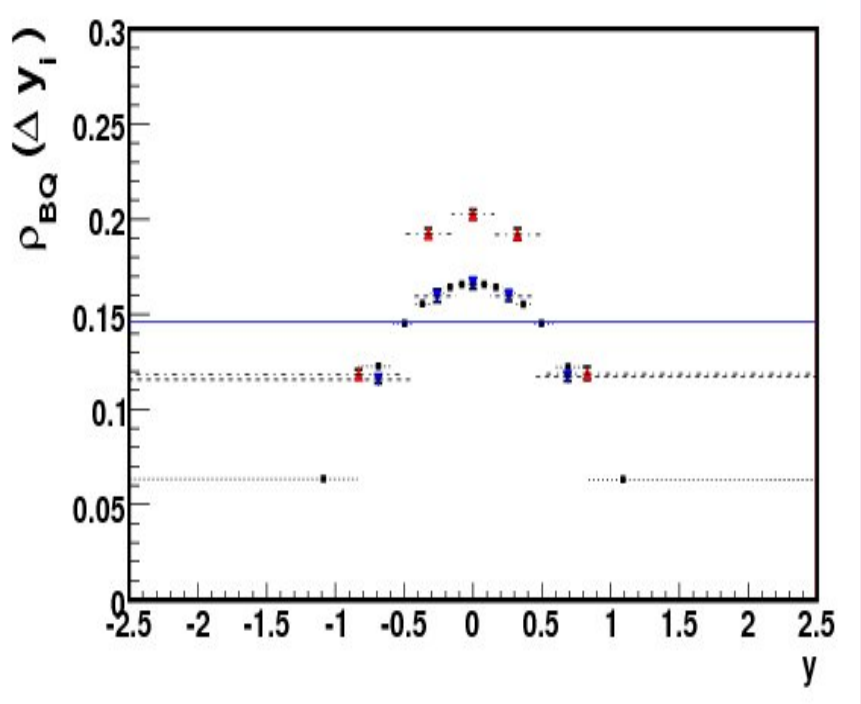
GCE Correlations between Charges

$V_1 = 2000 fm^3 \quad T = 0.16 GeV \quad \mu = 0.0 GeV$

full hadron gas

Correlation Coefficient ρ_{BQ} in Limited Acceptance





- The correlation is sensitive to one's acceptance window.
- The Δp_T and Δy dependence is caused by different hadron masses.

Resonance decay:

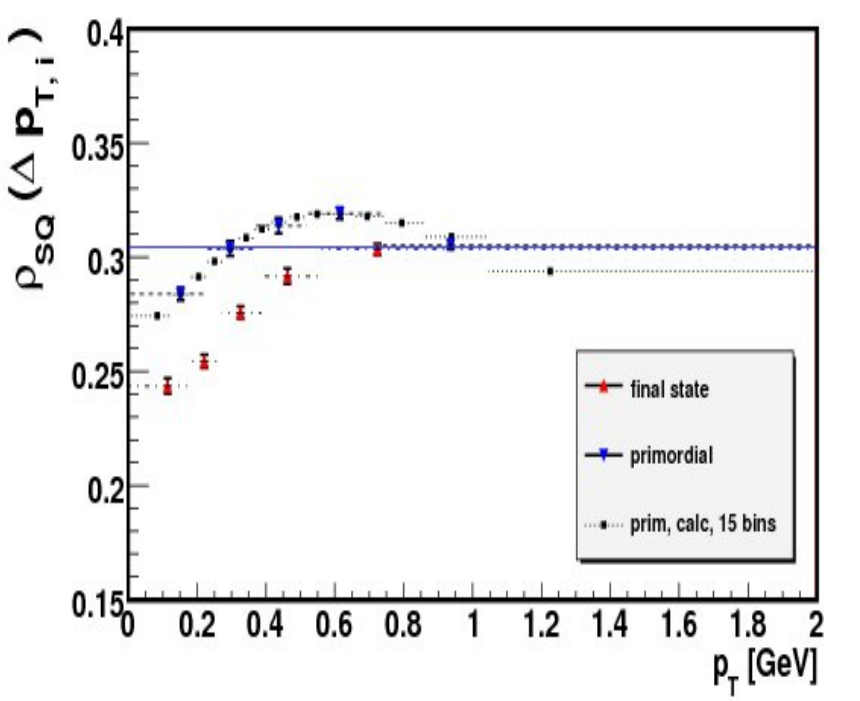
- does not change trends.
- could, however, wash out the signal!

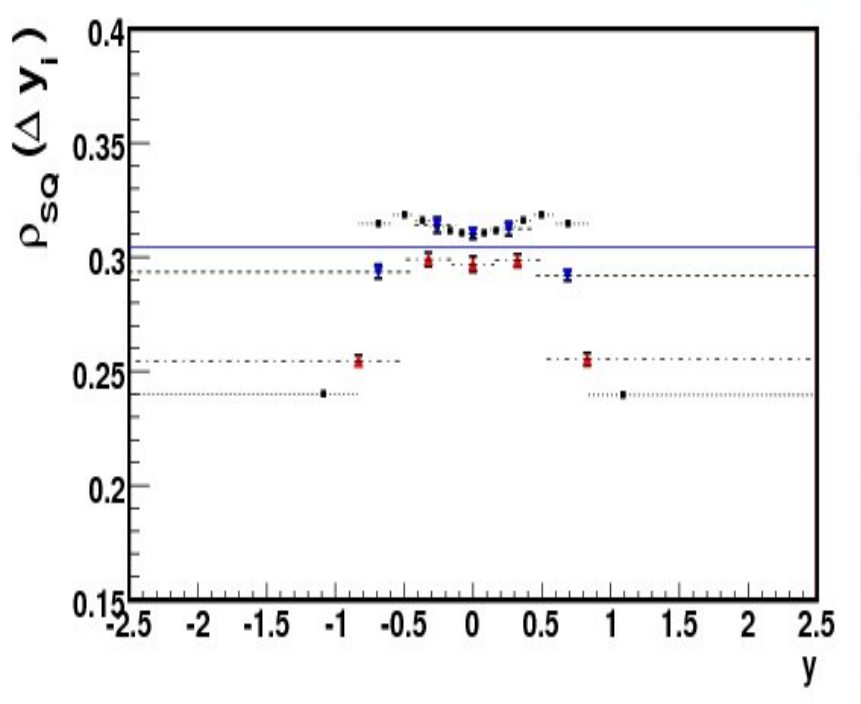
GCE Correlations between Charges

$V_1 = 2000 fm^3 \quad T = 0.16 GeV \quad \mu = 0.0 GeV$

full hadron gas

Correlation Coefficient ρ_{SQ} in Limited Acceptance



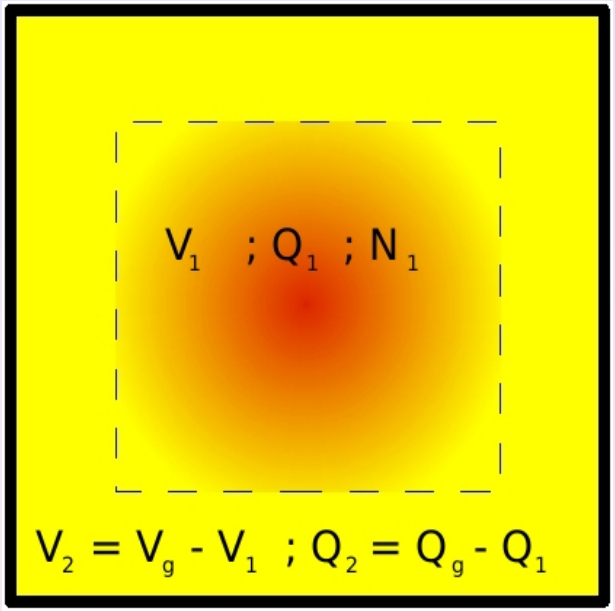


- The correlation is sensitive to one's acceptance window.
- The Δp_T and Δy dependence is caused by different hadron masses.

Resonance decay:

- does not change trends.
- could, however, wash out the signal!

Systems With Finite Bath



System 1

$$Z_{N_1}(V_1, Q_1)$$

System 2

$$Z(V_2, Q_2)$$

Impose Contstraints

$$V_g = V_1 + V_2 \quad \text{and} \quad Q_g = Q_1 + Q_2$$

Then for the combined System

$$Z(V_g, Q_g) = \sum_{Q_1} \sum_{N_1} Z(V_g - V_1, Q_g - Q_1) Z_{N_1}(V_1, Q_1)$$

Probability Distribution

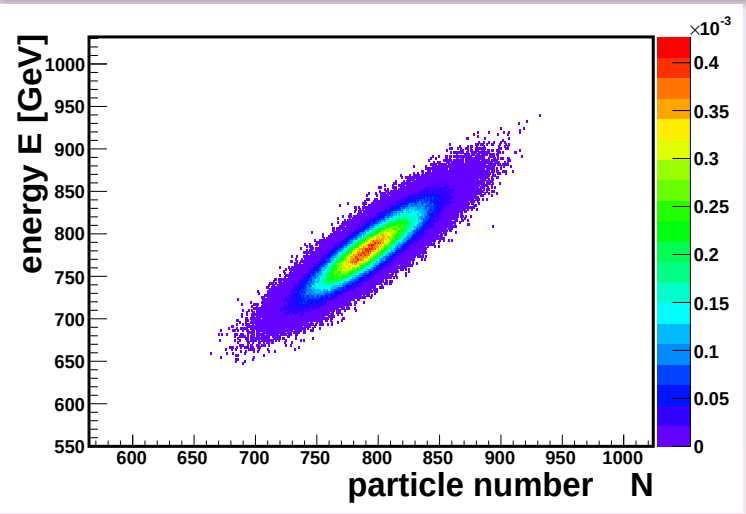
$$\begin{aligned} P(Q_1, N_1) &= \frac{Z(V_g - V_1, Q_g - Q_1)}{Z(V_g, Q_g)} \; Z_{N_1}(V_1, Q_1) \\ &= W(V_1, Q_1; V_g, Q_g) \; Z_{N_1}(V_1, Q_1) \\ &= \mathcal{W}_P^{Q_1; Q_g}(V_1; V_g | \beta, \mu) \; P_{gce}(Q_1, N_1) \end{aligned}$$

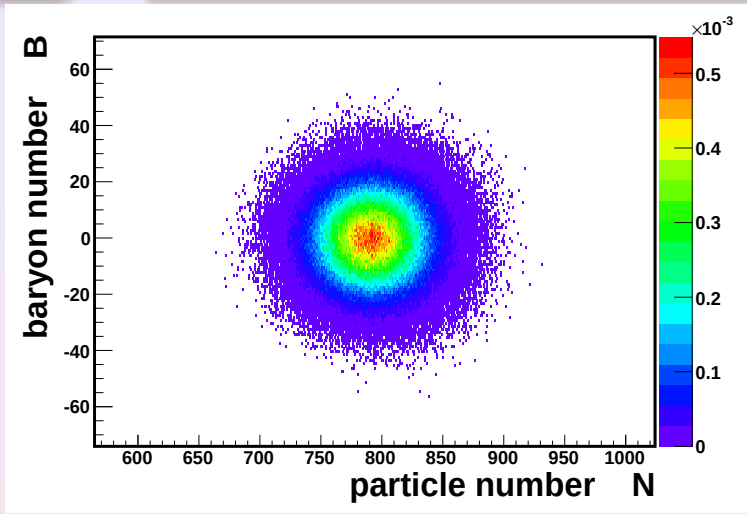
- Defines a ‘family’ of thermodynamically equivalent ensembles (V_1 very large)
- Effectively a recipe for a Monte Carlo scheme

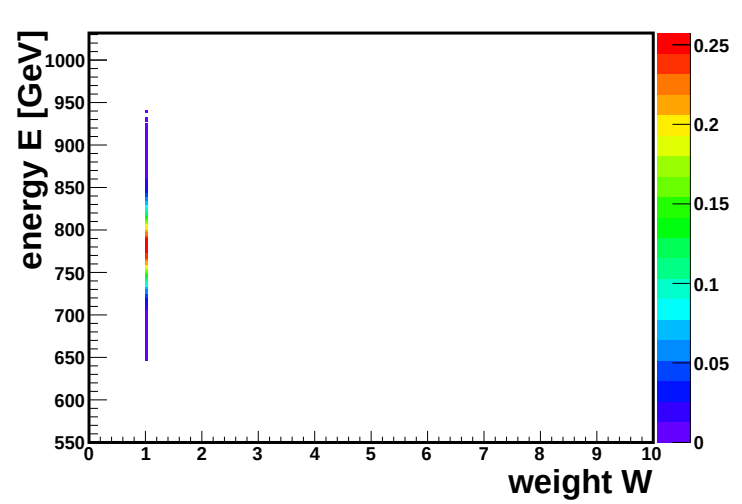
Statistical Distributions and Weight Factor

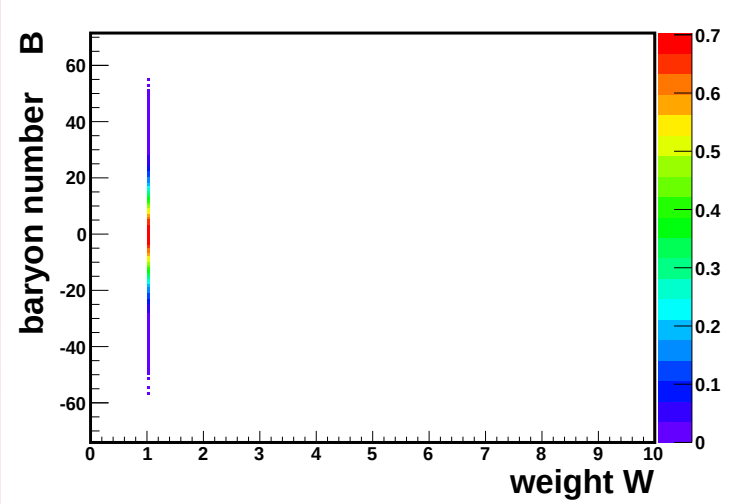
$V_1 = 2000 fm^3 \quad T = 0.16 GeV \quad \mu = 0.0 GeV$

$\lambda = V_1 / V_g = 0.000$





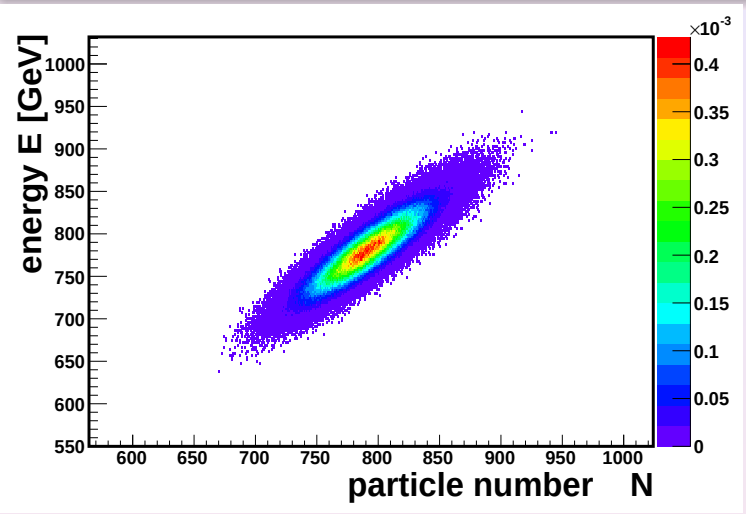


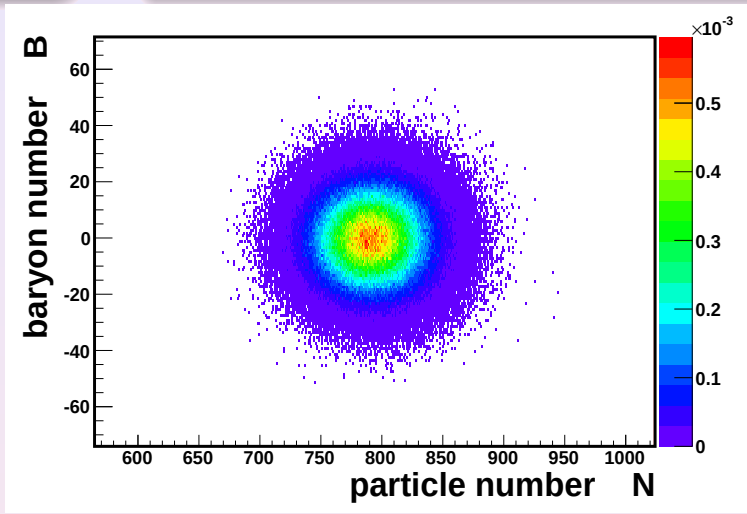


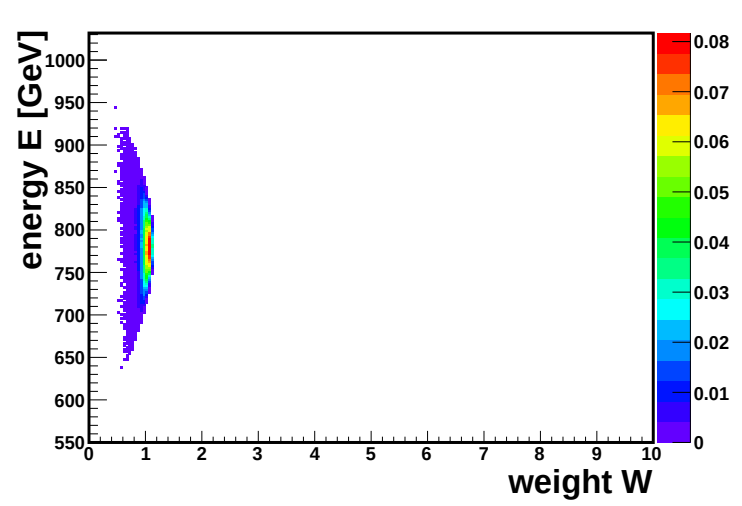
Statistical Distributions and Weight Factor

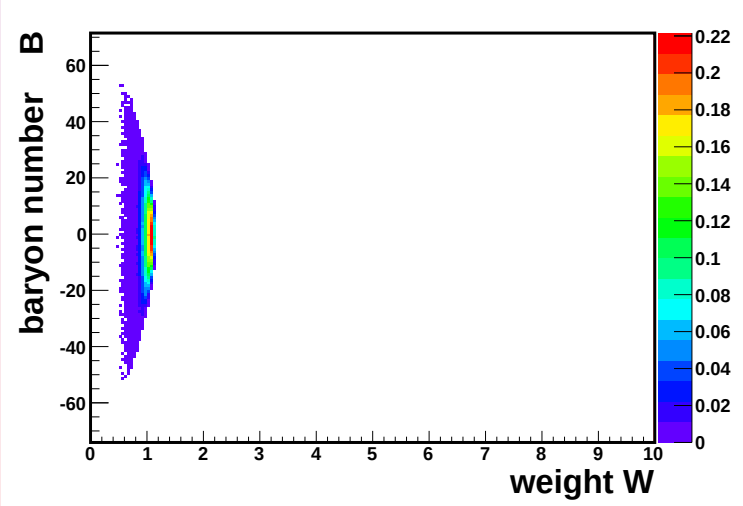
$V_1 = 2000 fm^3 \quad T = 0.16 GeV \quad \mu = 0.0 GeV$

$\lambda = V_1 / V_g = 0.050$





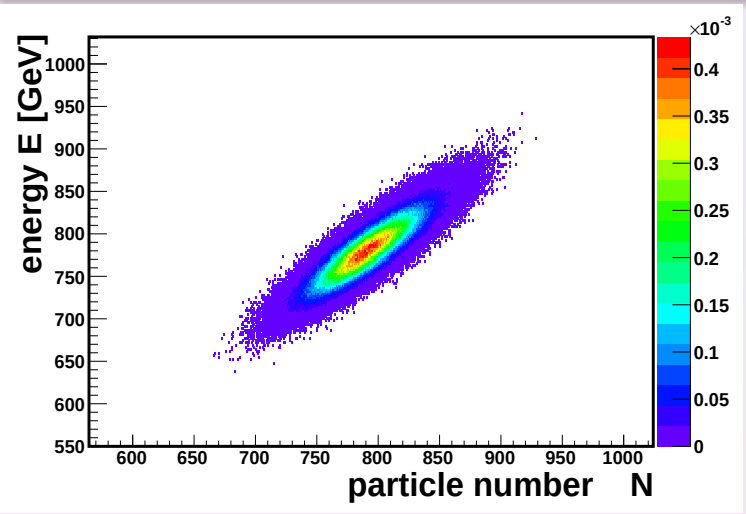


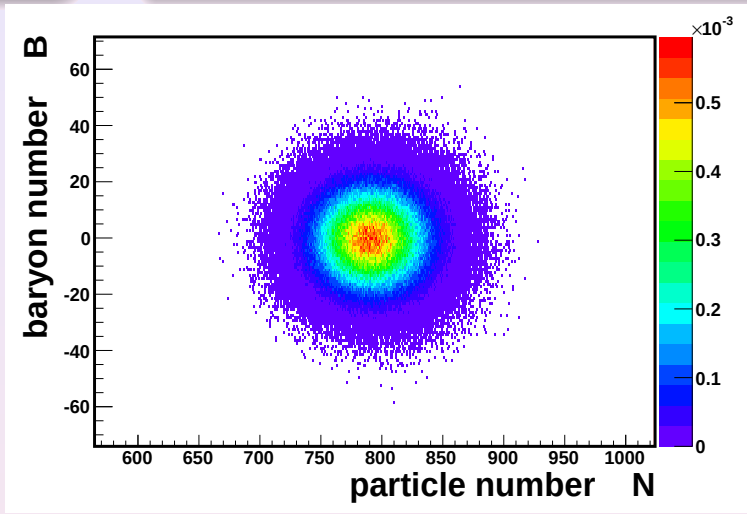


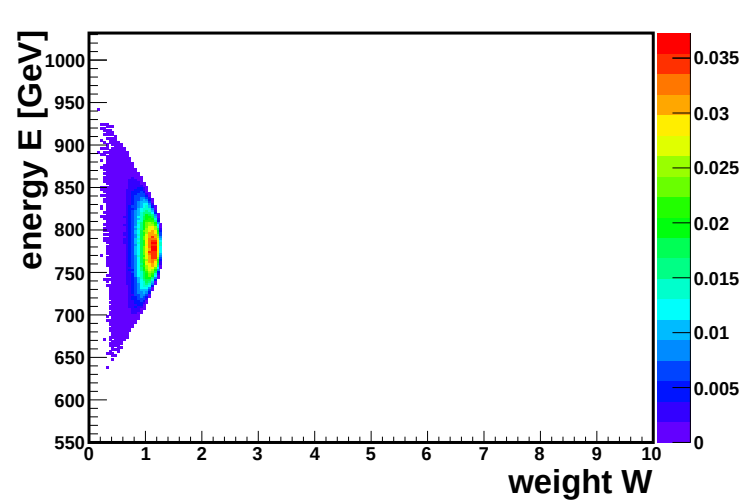
Statistical Distributions and Weight Factor

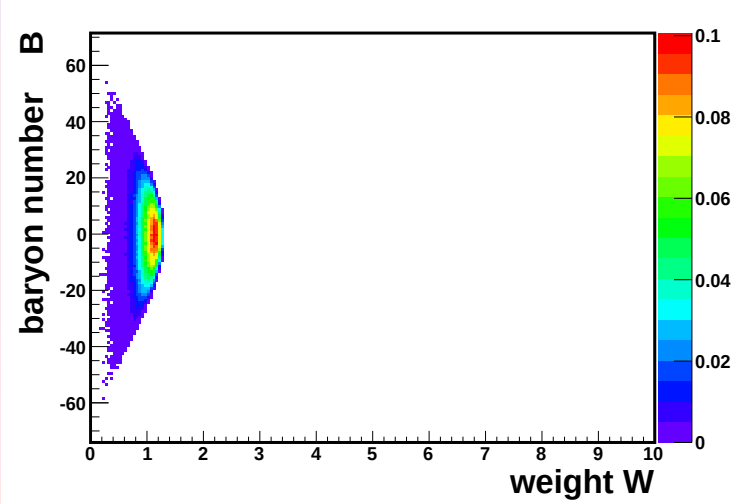
$V_1 = 2000 fm^3 \quad T = 0.16 GeV \quad \mu = 0.0 GeV$

$\lambda = V_1 / V_g = 0.100$





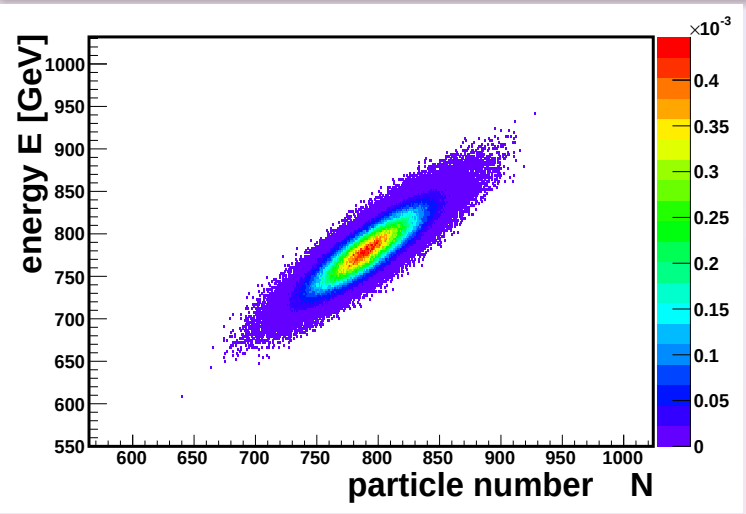


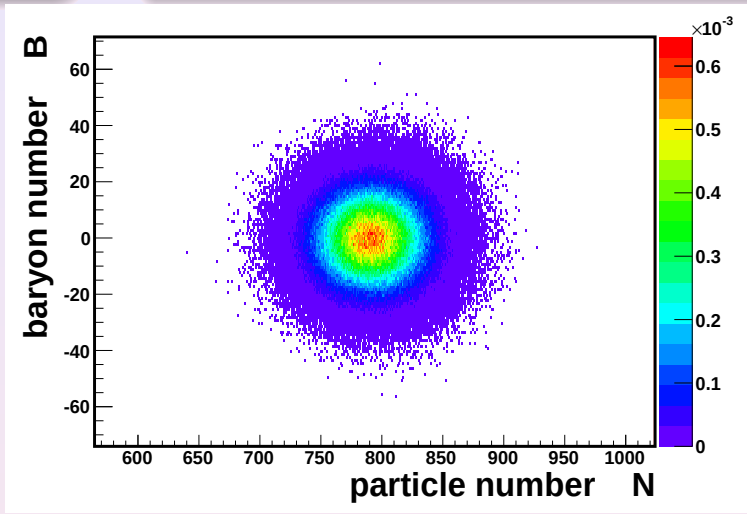


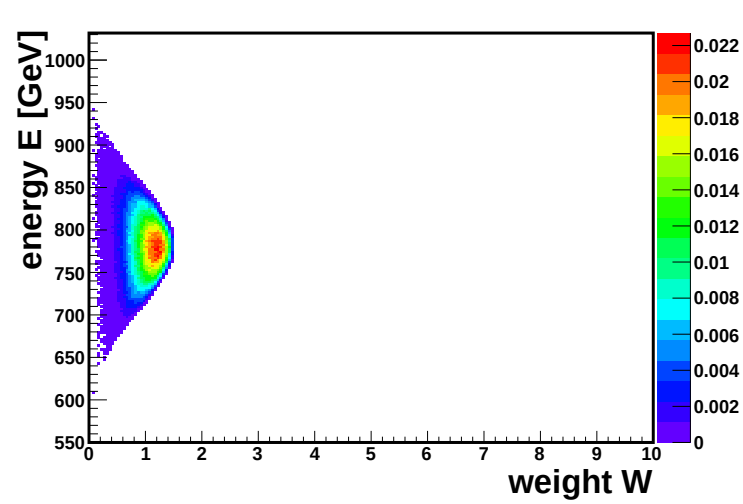
Statistical Distributions and Weight Factor

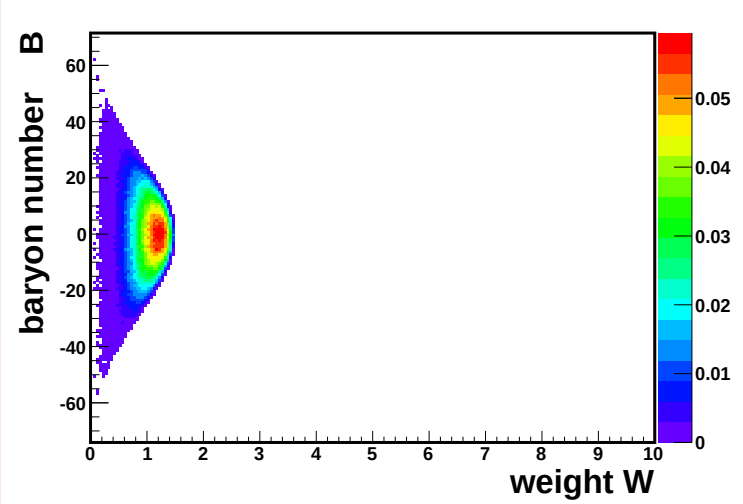
$V_1 = 2000 fm^3 \quad T = 0.16 GeV \quad \mu = 0.0 GeV$

$\lambda = V_1 / V_g = 0.150$





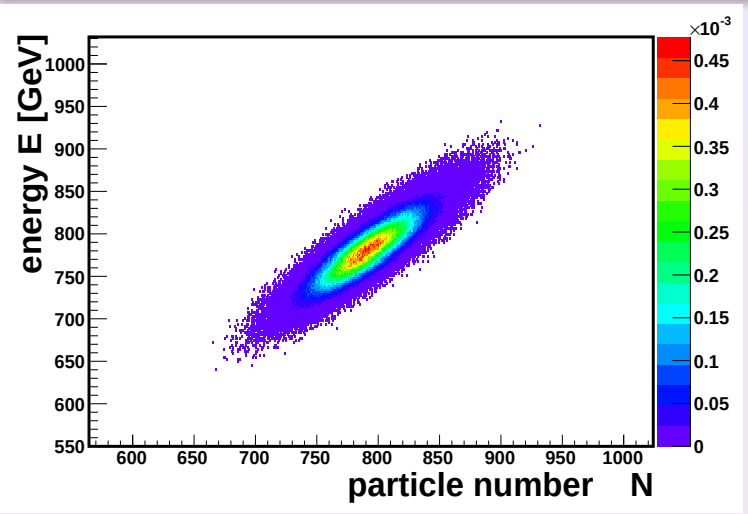


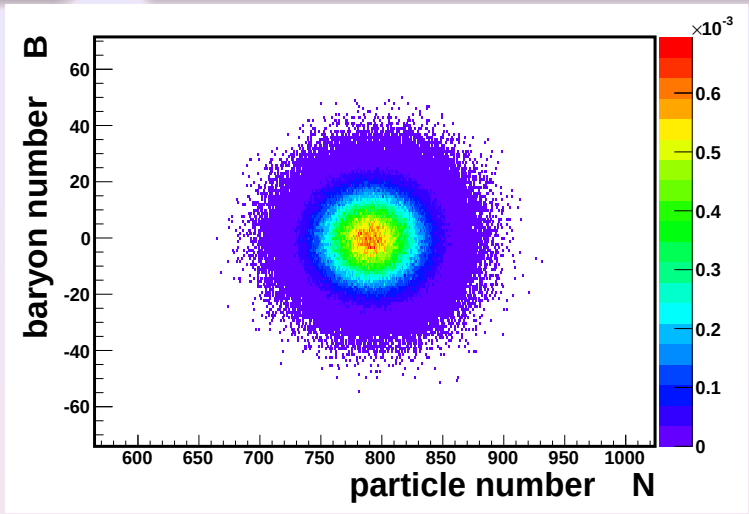


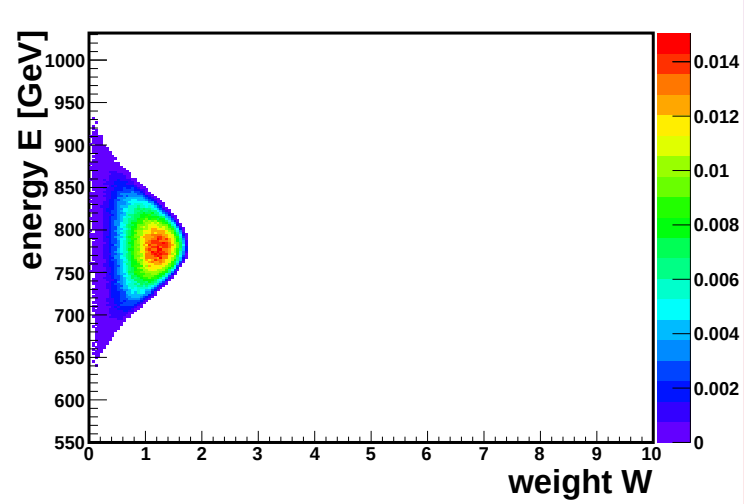
Statistical Distributions and Weight Factor

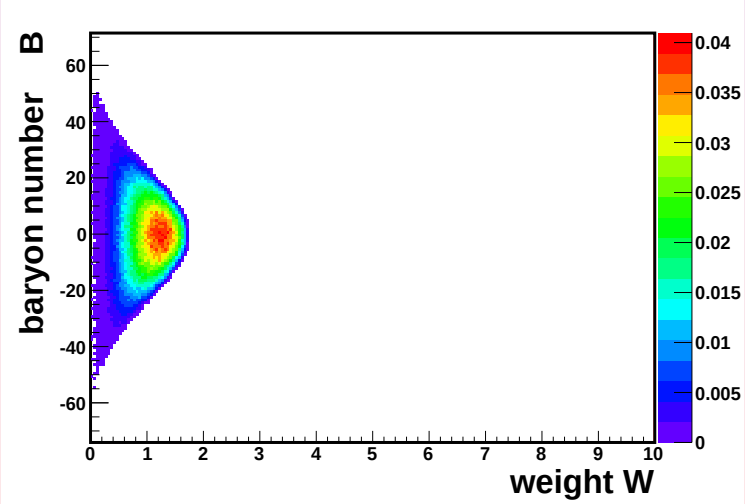
$V_1 = 2000 fm^3 \quad T = 0.16 GeV \quad \mu = 0.0 GeV$

$\lambda = V_1 / V_g = 0.200$





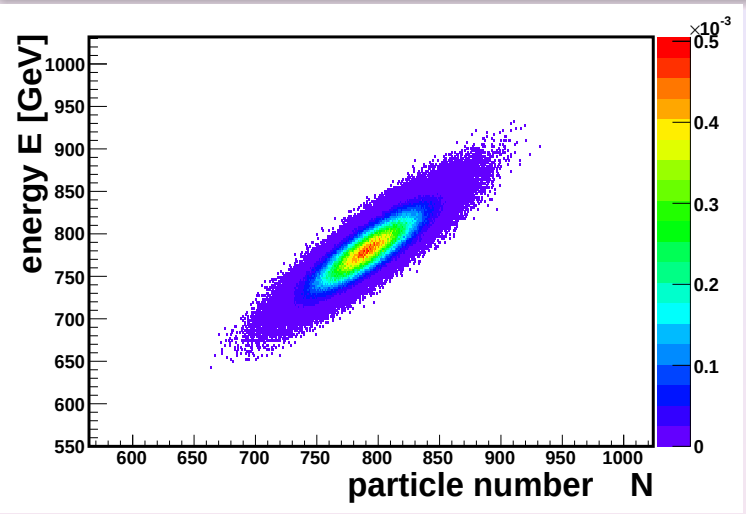


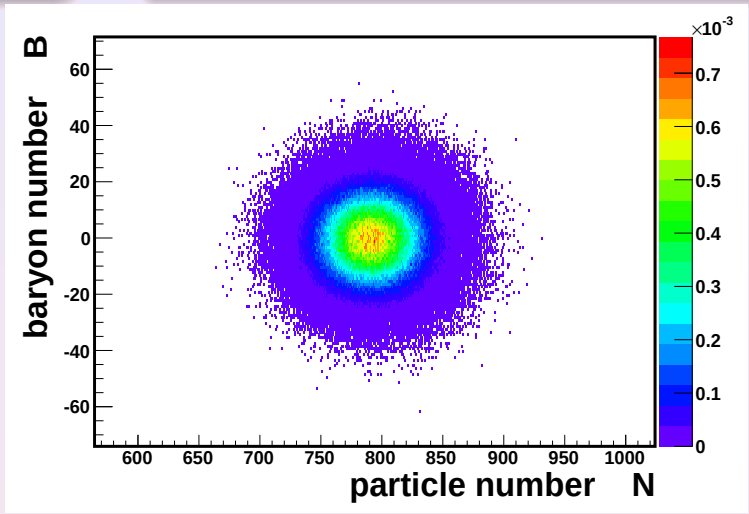


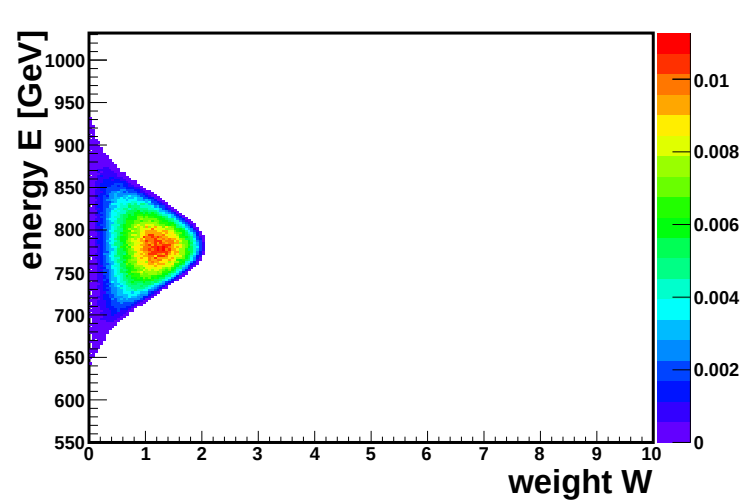
Statistical Distributions and Weight Factor

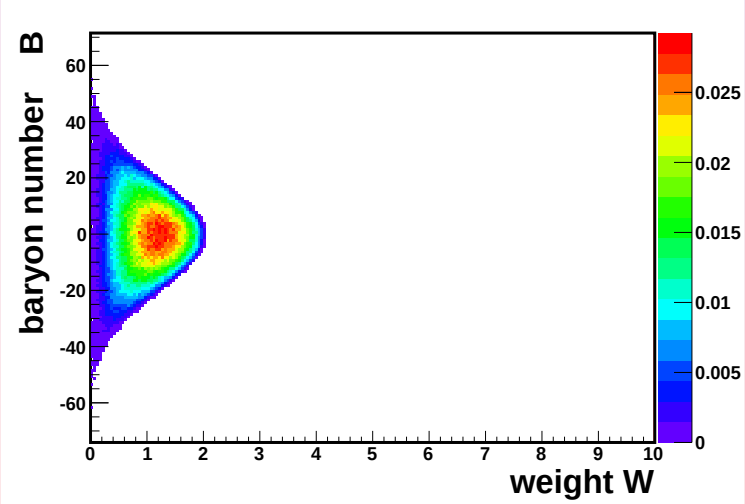
$V_1 = 2000 fm^3 \quad T = 0.16 GeV \quad \mu = 0.0 GeV$

$\lambda = V_1 / V_g = 0.250$





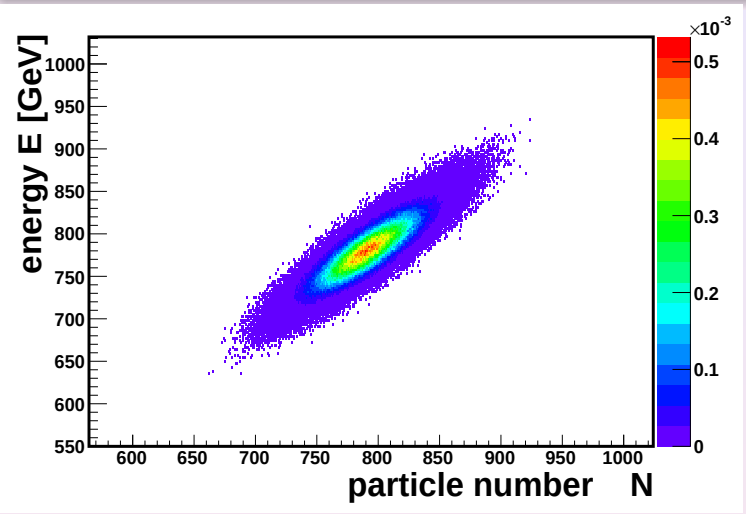


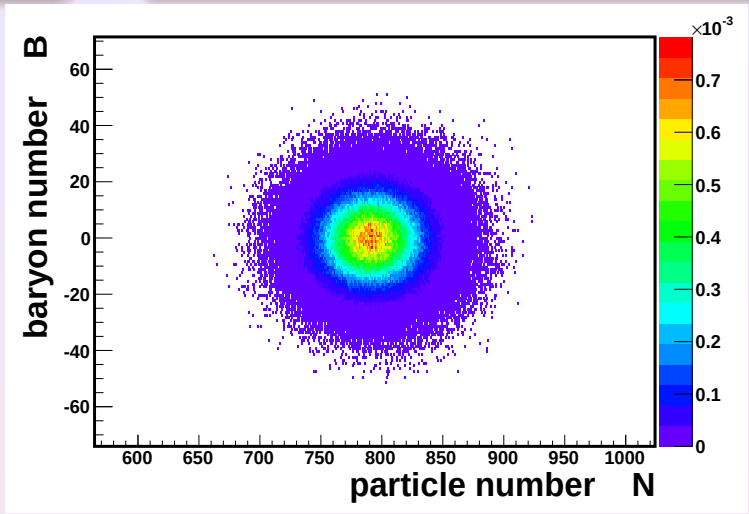


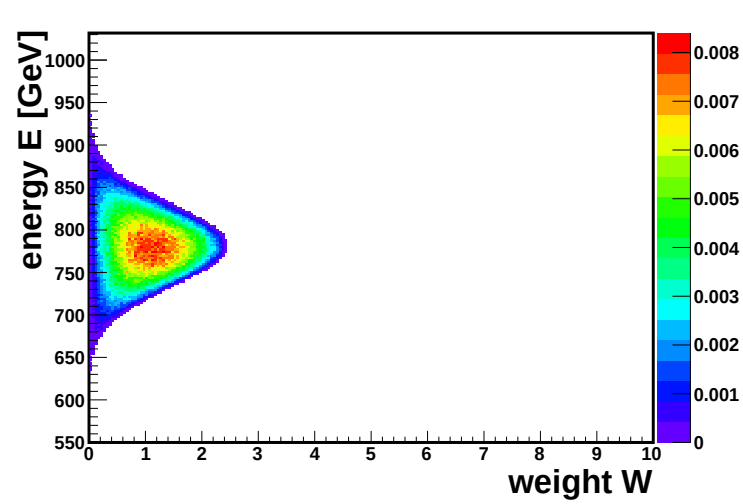
Statistical Distributions and Weight Factor

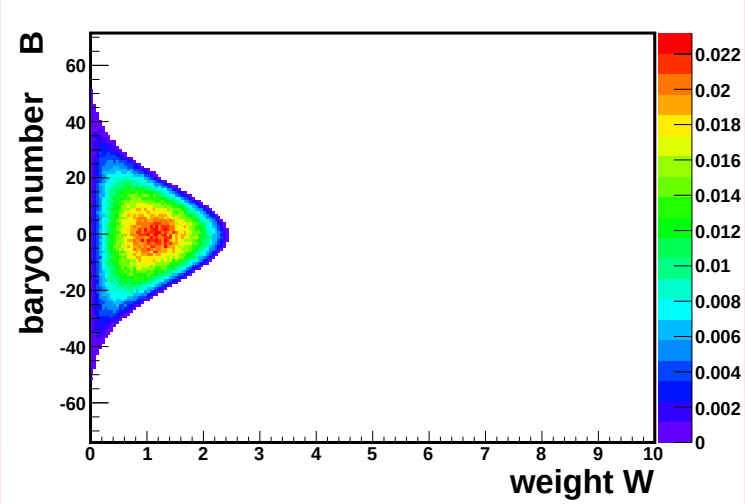
$V_1 = 2000 fm^3 \quad T = 0.16 GeV \quad \mu = 0.0 GeV$

$\lambda = V_1 / V_g = 0.300$





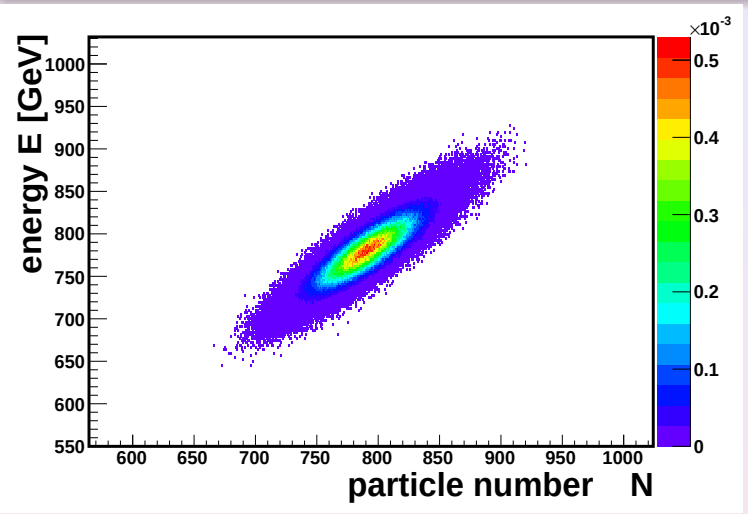


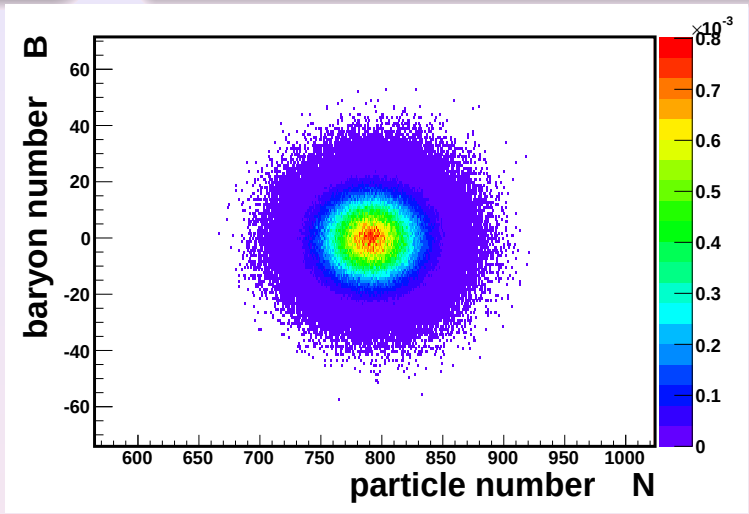


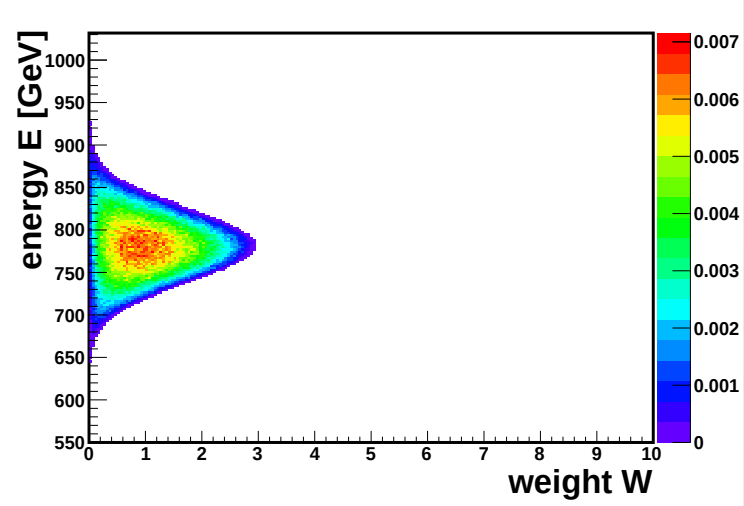
Statistical Distributions and Weight Factor

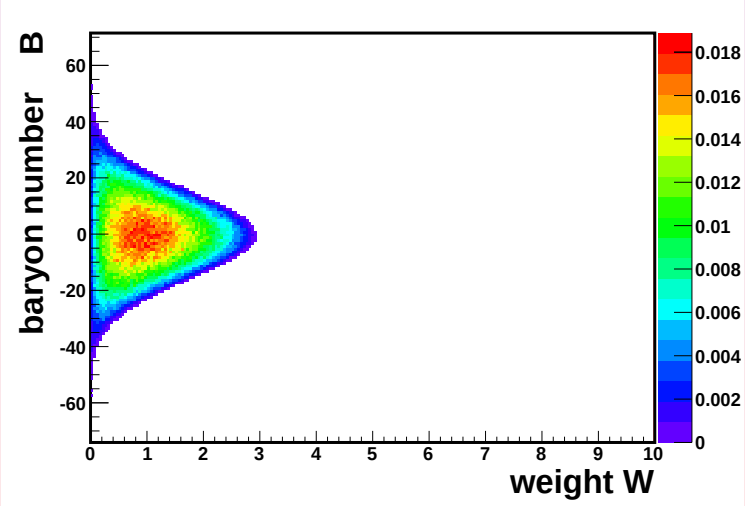
$V_1 = 2000 fm^3 \quad T = 0.16 GeV \quad \mu = 0.0 GeV$

$\lambda = V_1 / V_g = 0.350$





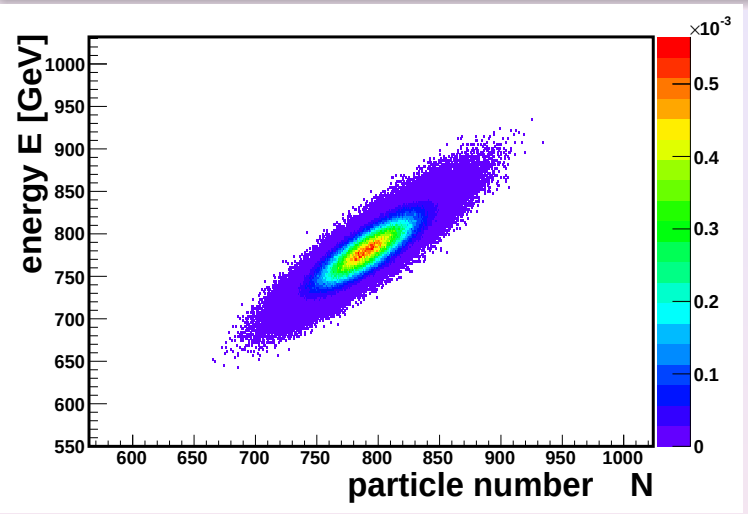


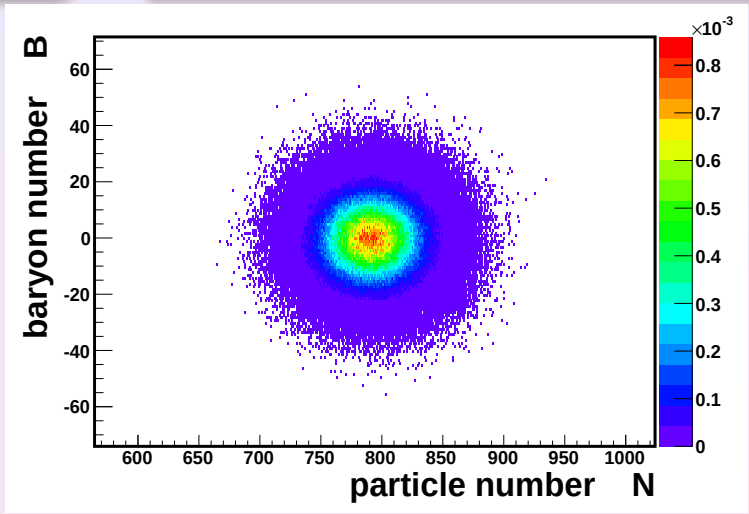


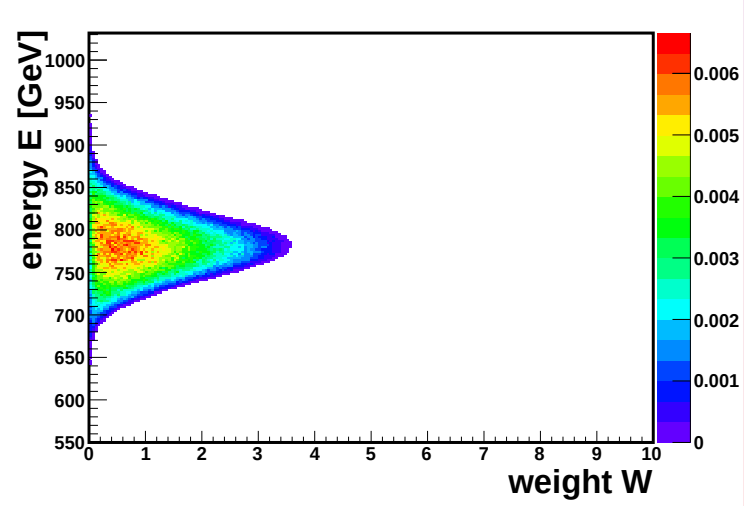
Statistical Distributions and Weight Factor

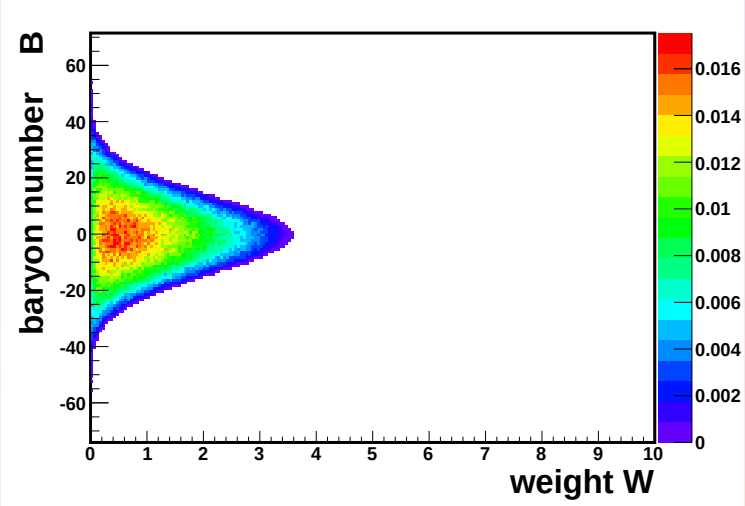
$V_1 = 2000 fm^3 \quad T = 0.16 GeV \quad \mu = 0.0 GeV$

$\lambda = V_1 / V_g = 0.400$





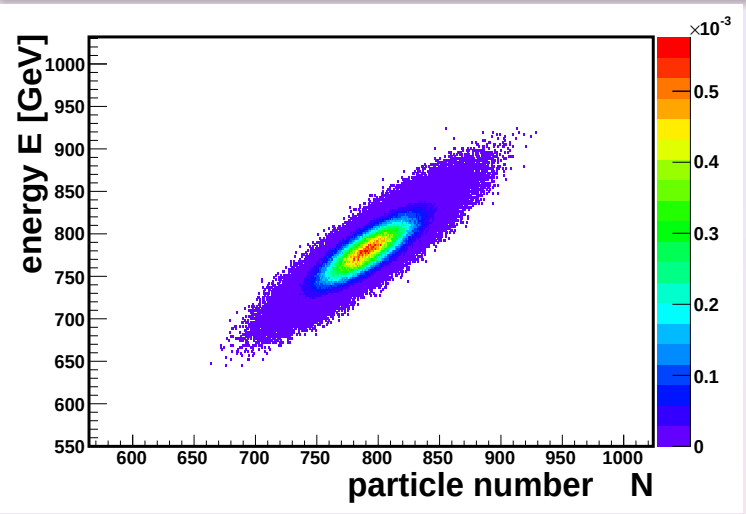


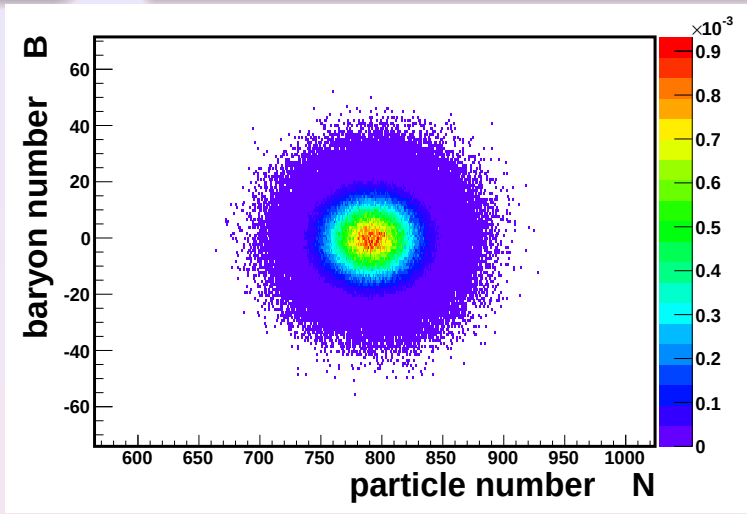


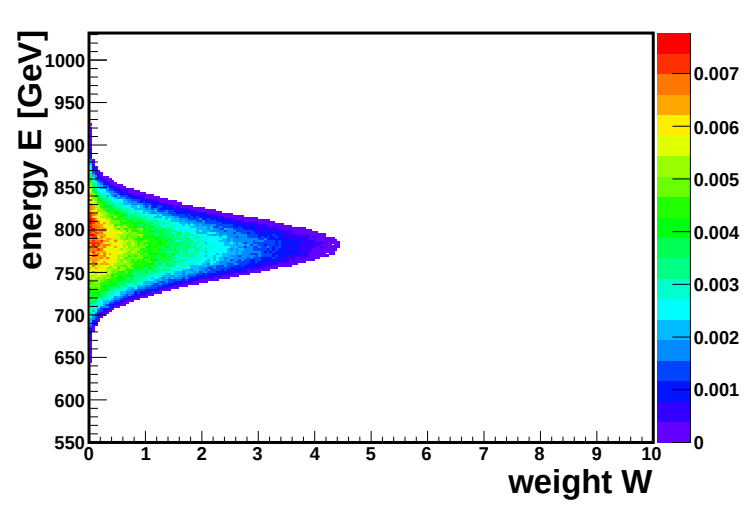
Statistical Distributions and Weight Factor

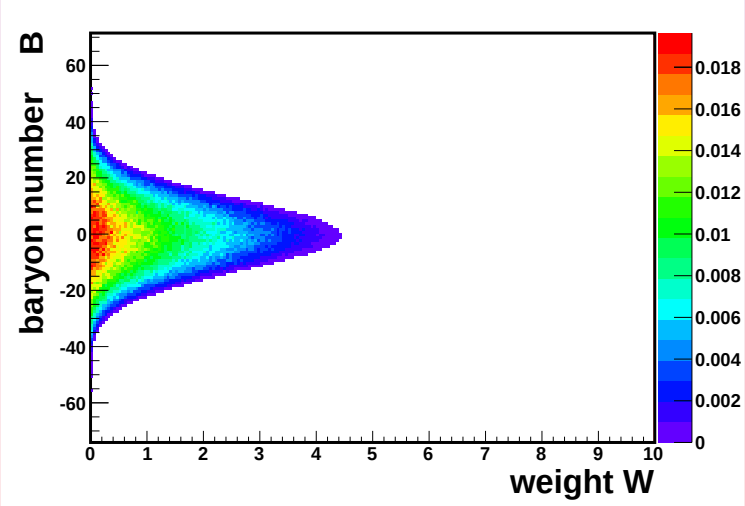
$V_1 = 2000 fm^3 \quad T = 0.16 GeV \quad \mu = 0.0 GeV$

$\lambda = V_1 / V_g = 0.450$





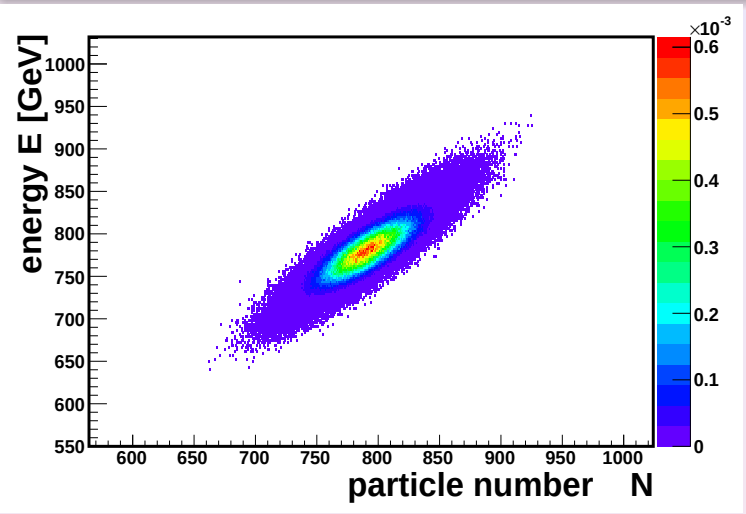


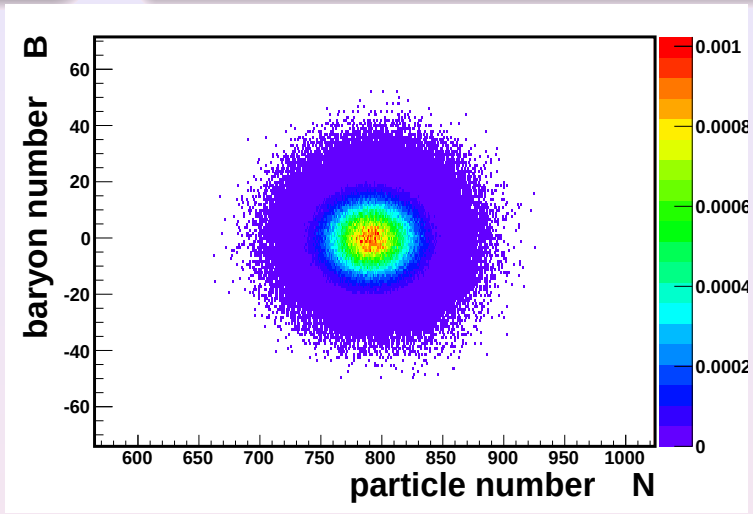


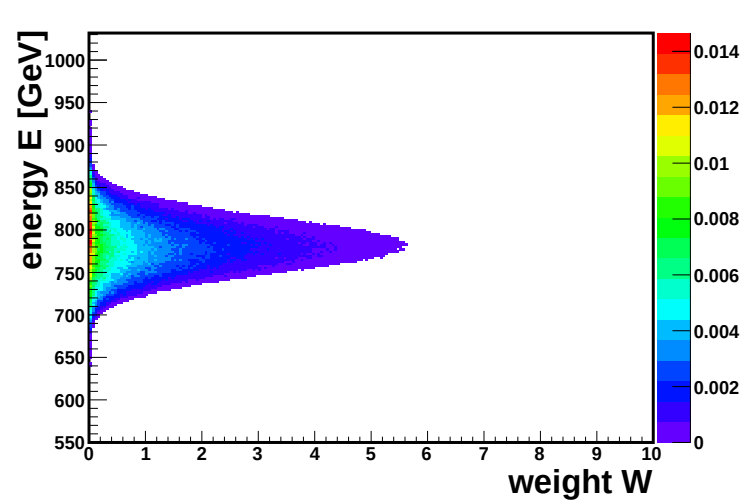
Statistical Distributions and Weight Factor

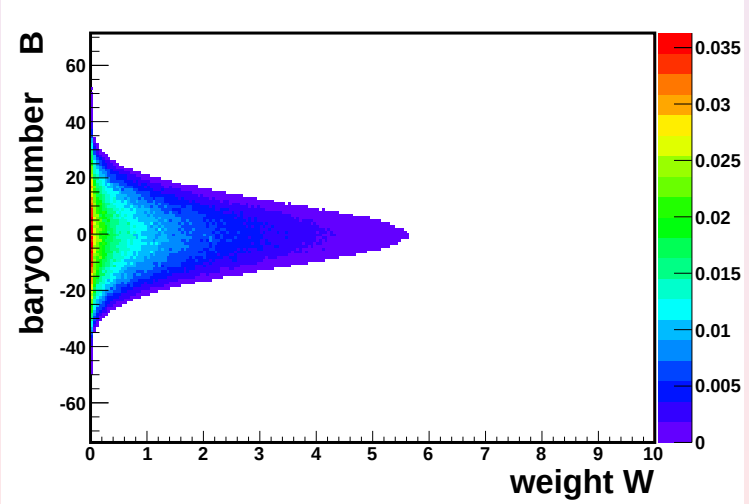
$V_1 = 2000 fm^3 \quad T = 0.16 GeV \quad \mu = 0.0 GeV$

$\lambda = V_1 / V_g = 0.500$





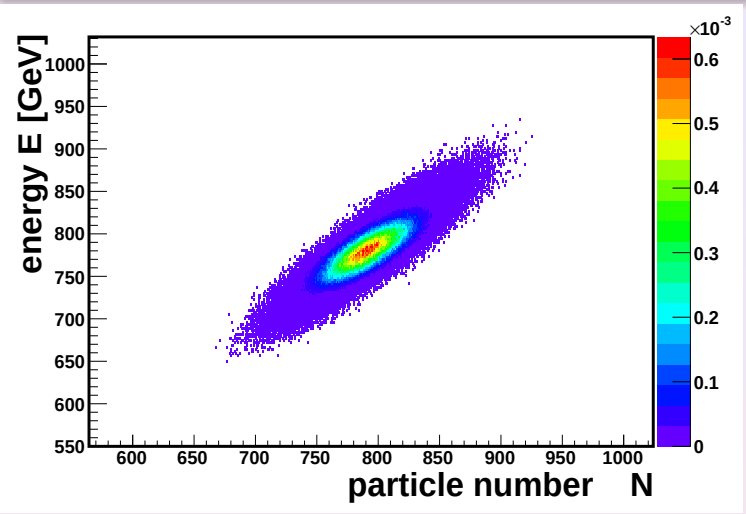


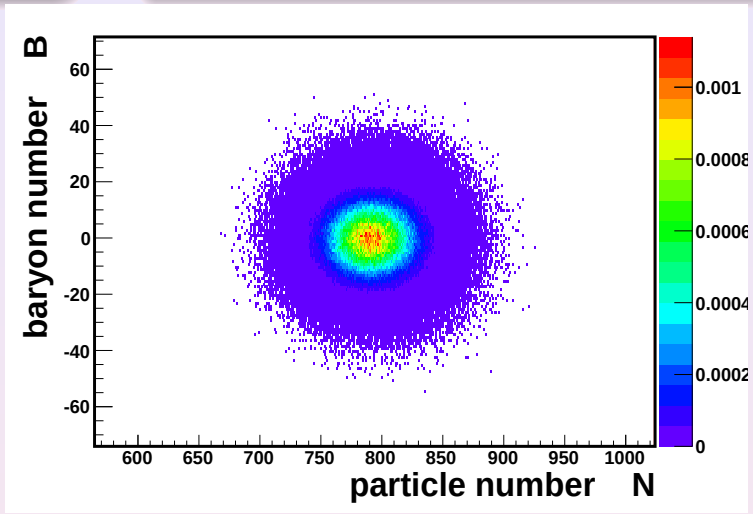


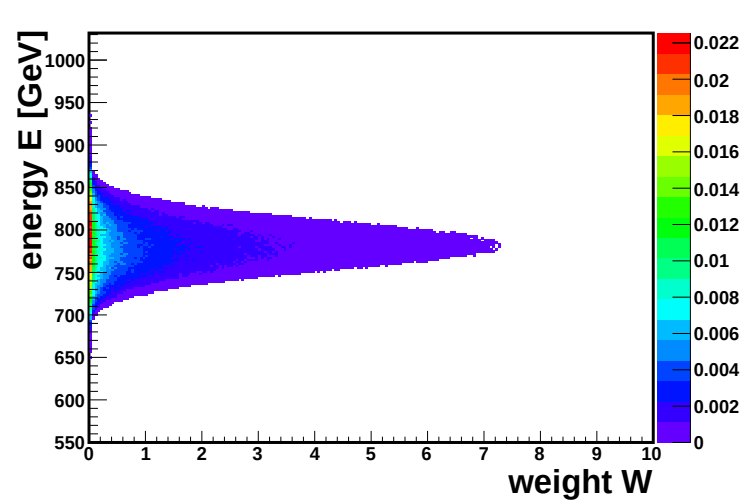
Statistical Distributions and Weight Factor

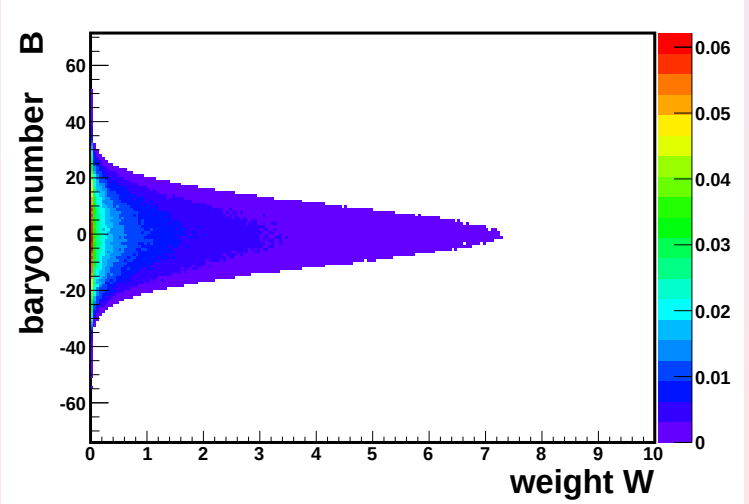
$V_1 = 2000 fm^3 \quad T = 0.16 GeV \quad \mu = 0.0 GeV$

$\lambda = V_1 / V_g = 0.550$





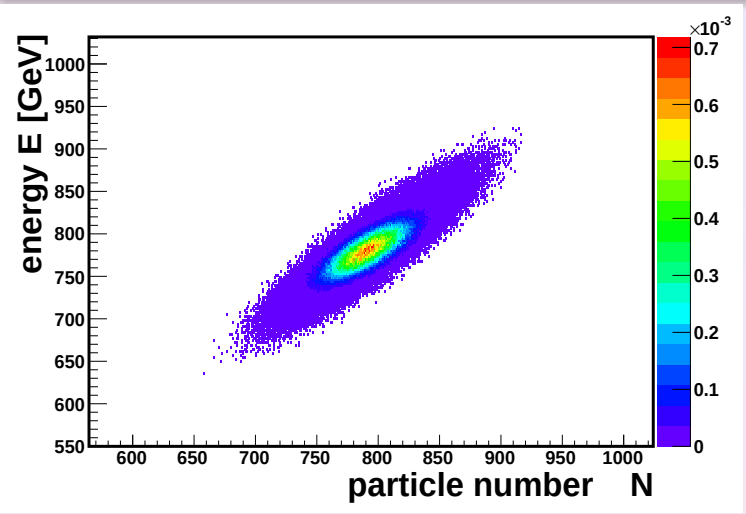


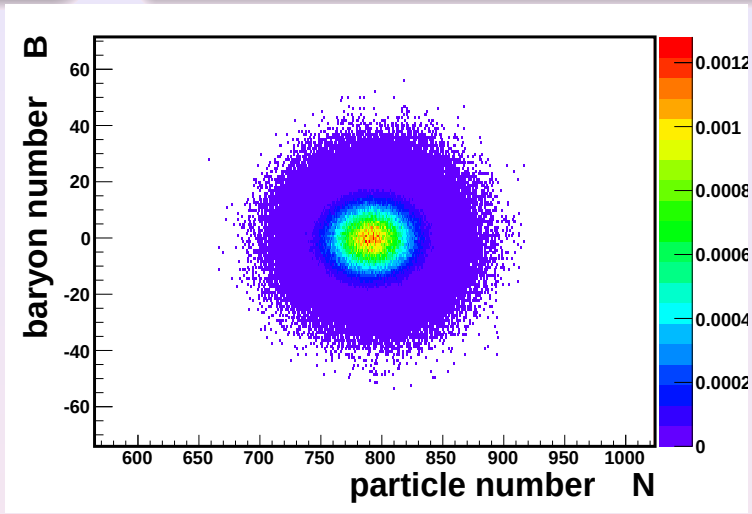


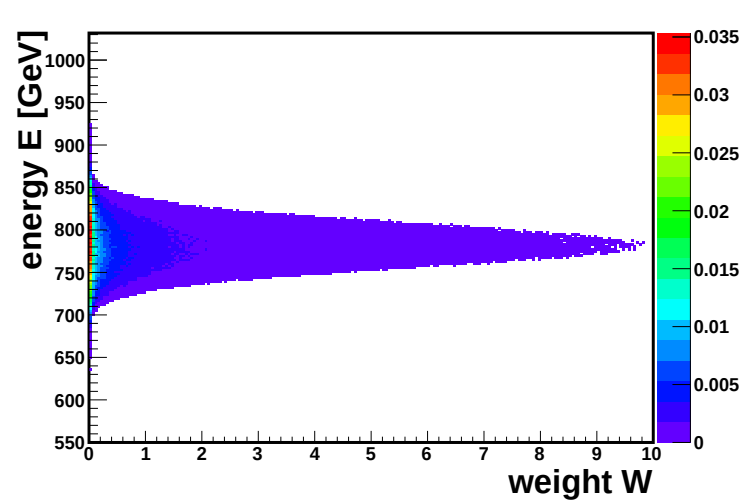
Statistical Distributions and Weight Factor

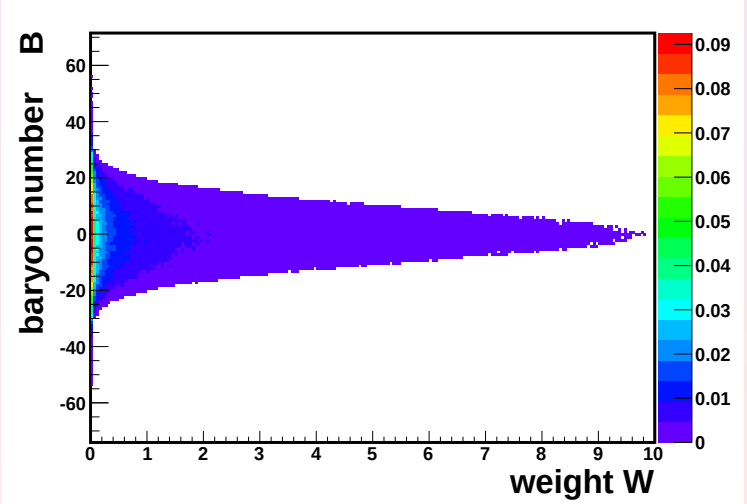
$V_1 = 2000 fm^3 \quad T = 0.16 GeV \quad \mu = 0.0 GeV$

$\lambda = V_1 / V_g = 0.600$





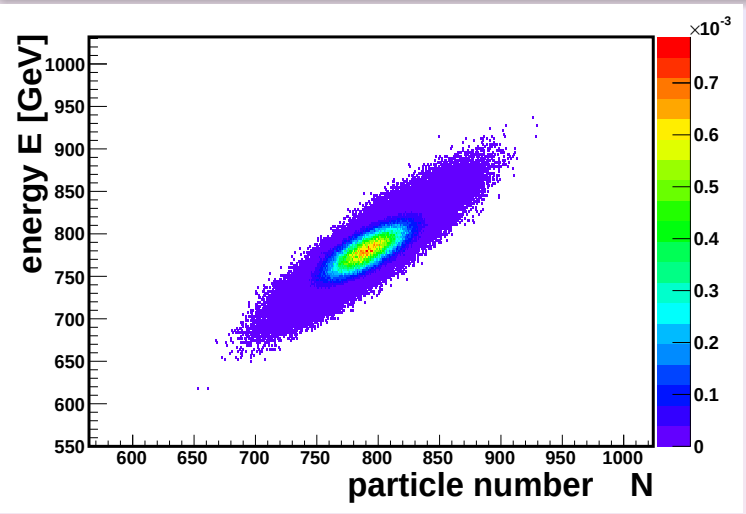


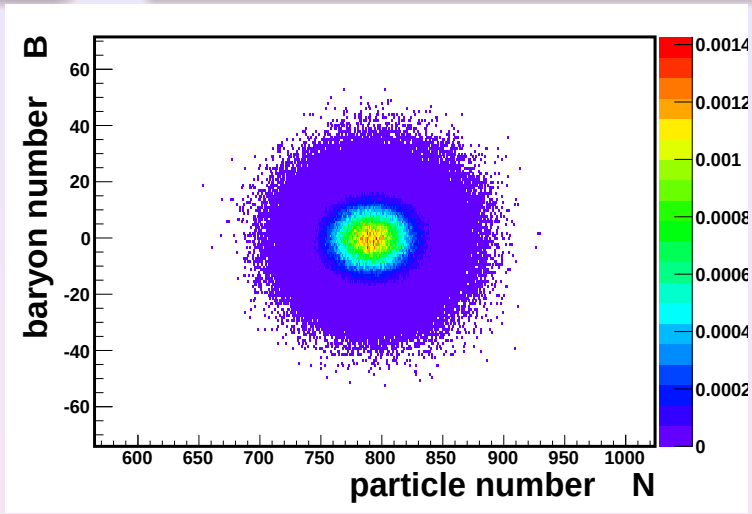


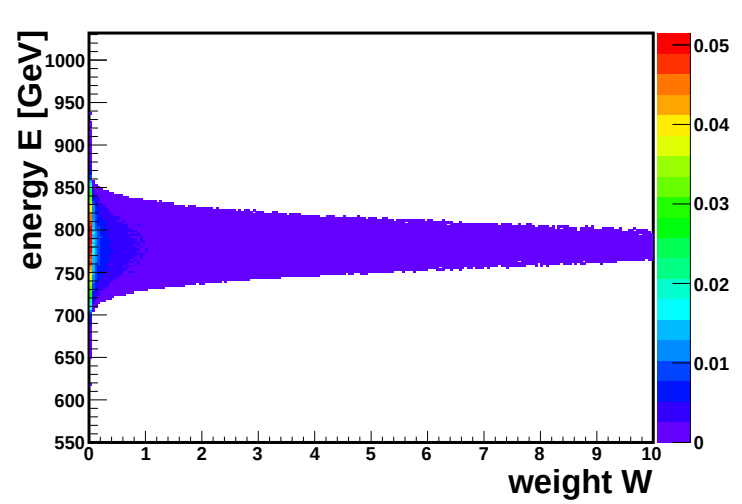
Statistical Distributions and Weight Factor

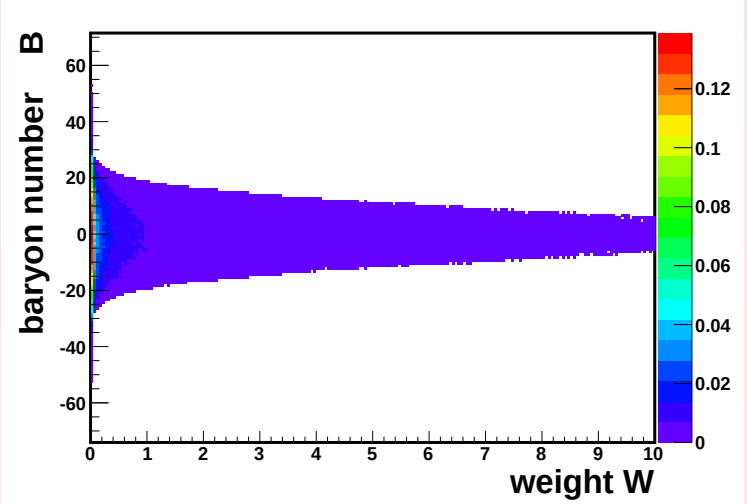
$V_1 = 2000 fm^3 \quad T = 0.16 GeV \quad \mu = 0.0 GeV$

$\lambda = V_1 / V_g = 0.650$





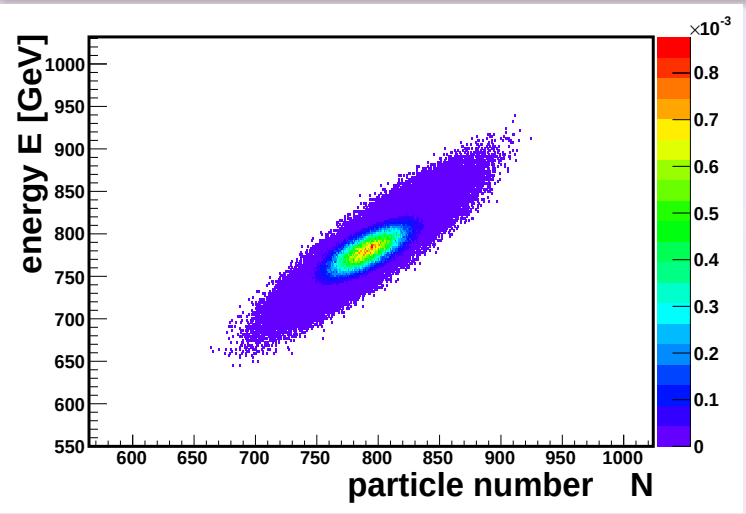


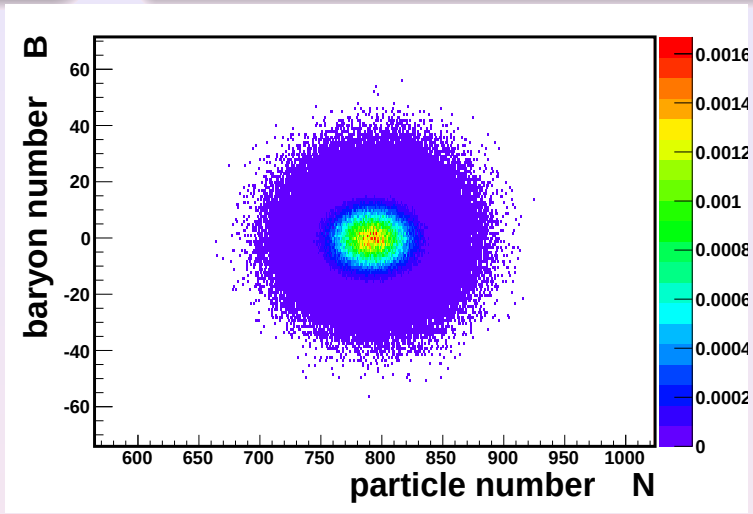


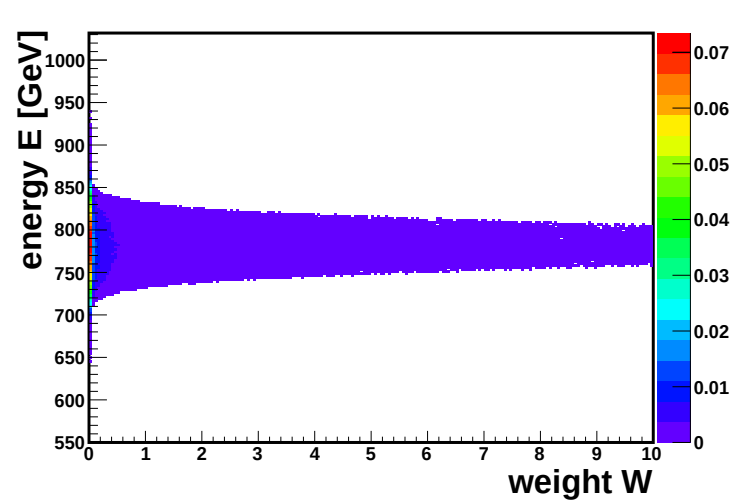
Statistical Distributions and Weight Factor

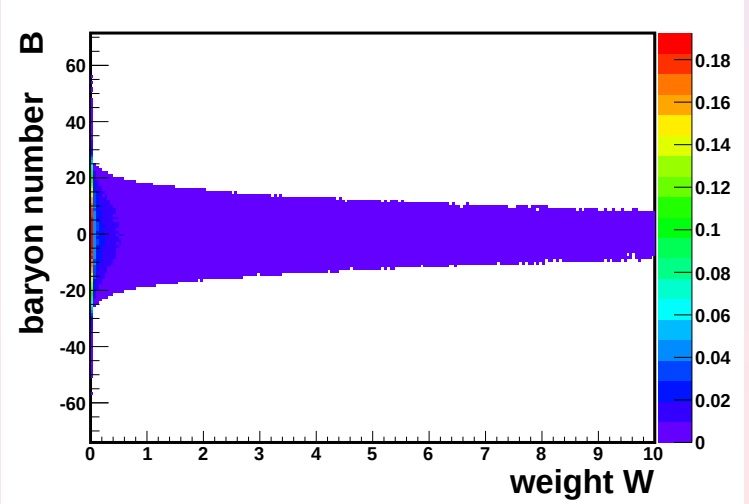
$V_1 = 2000 fm^3 \quad T = 0.16 GeV \quad \mu = 0.0 GeV$

$\lambda = V_1 / V_g = 0.700$





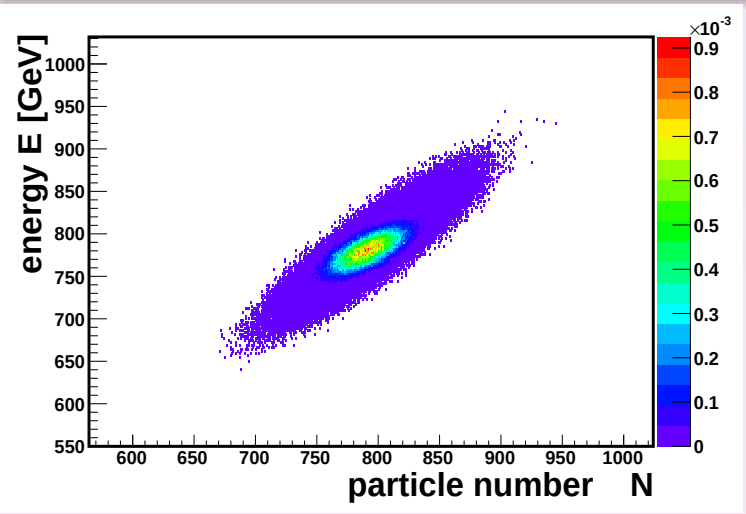


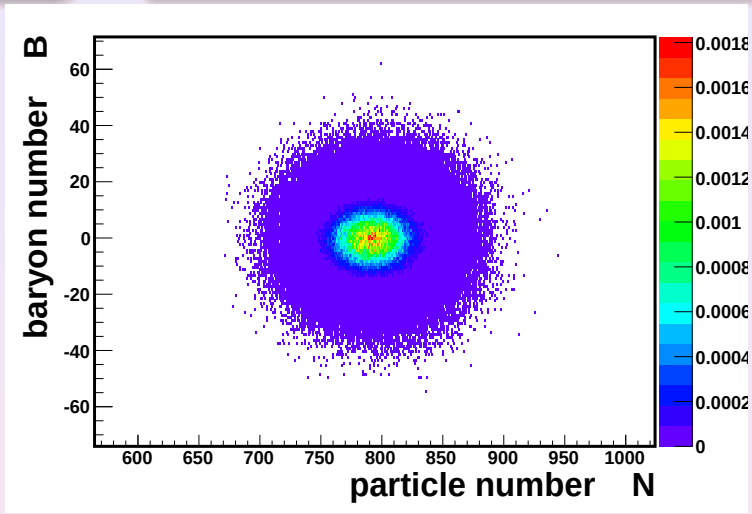


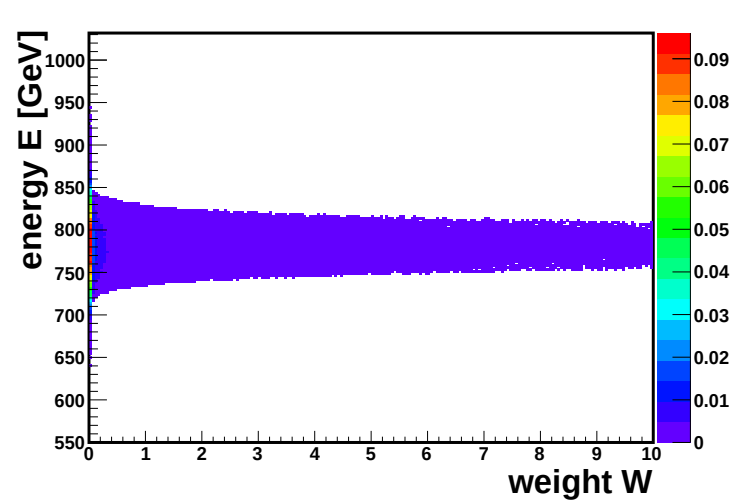
Statistical Distributions and Weight Factor

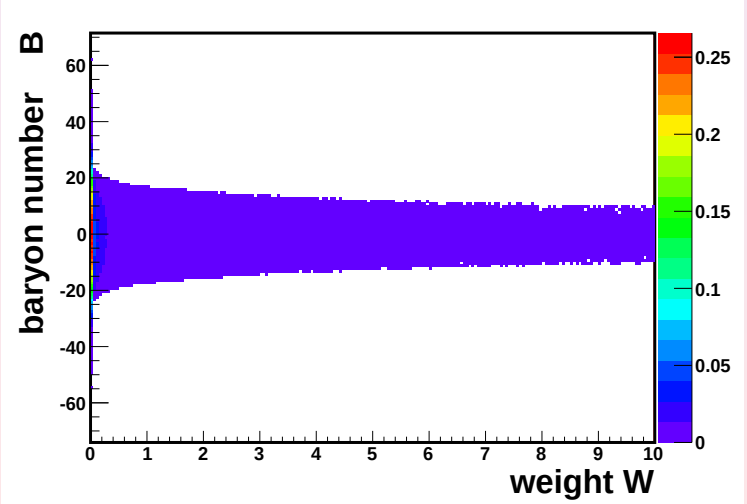
$V_1 = 2000 fm^3 \quad T = 0.16 GeV \quad \mu = 0.0 GeV$

$\lambda = V_1 / V_g = 0.750$





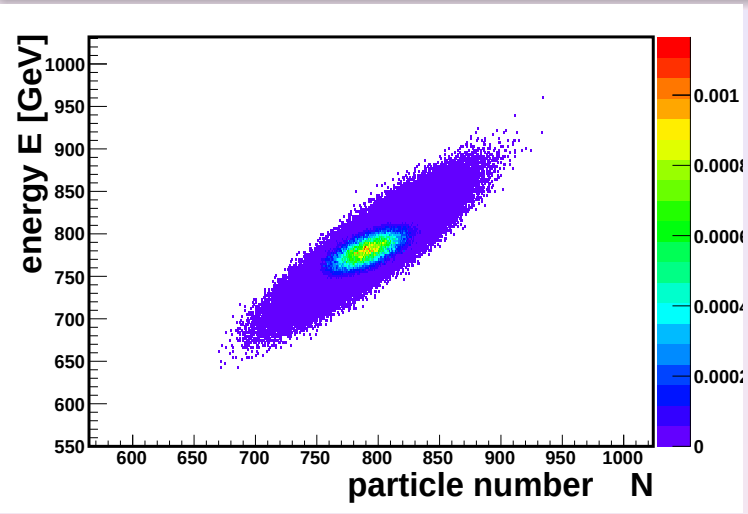


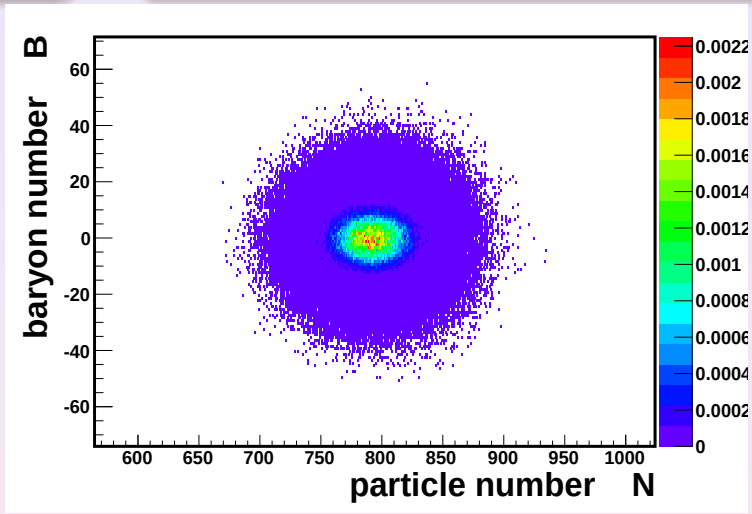


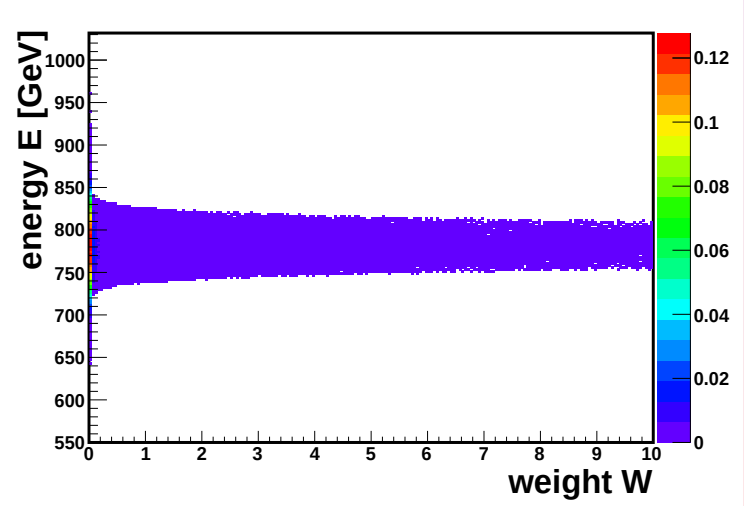
Statistical Distributions and Weight Factor

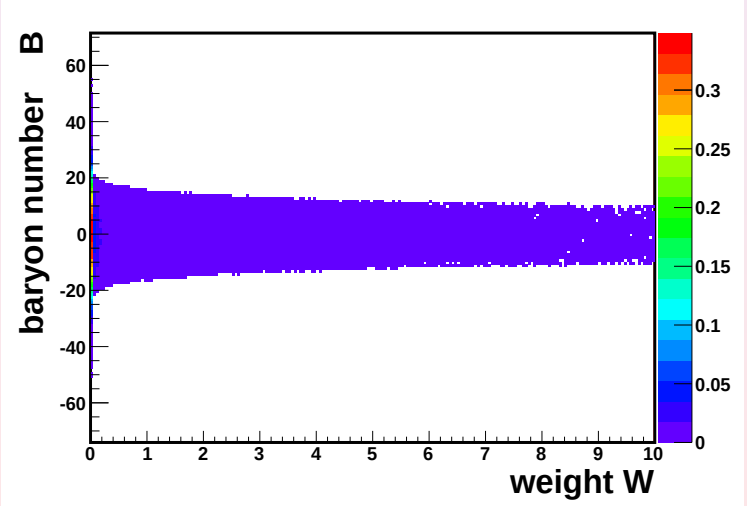
$V_1 = 2000 fm^3 \quad T = 0.16 GeV \quad \mu = 0.0 GeV$

$\lambda = V_1 / V_g = 0.800$





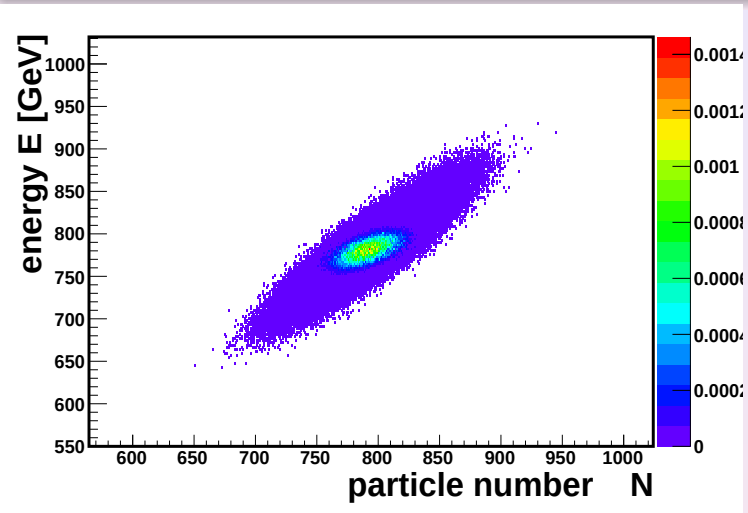


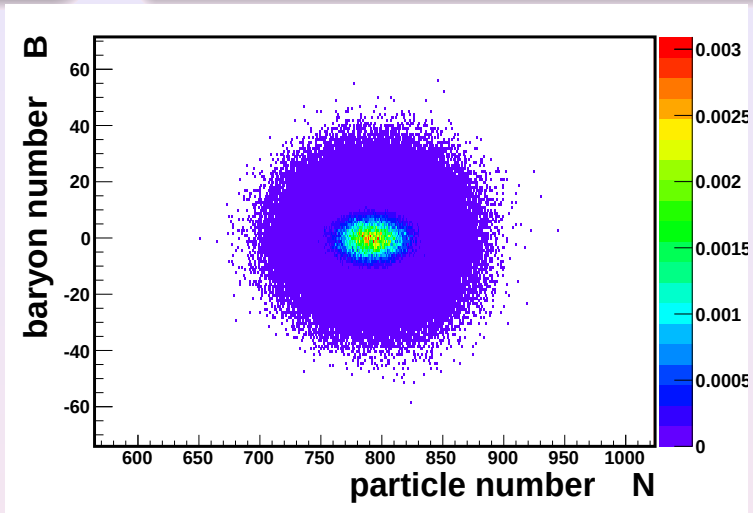


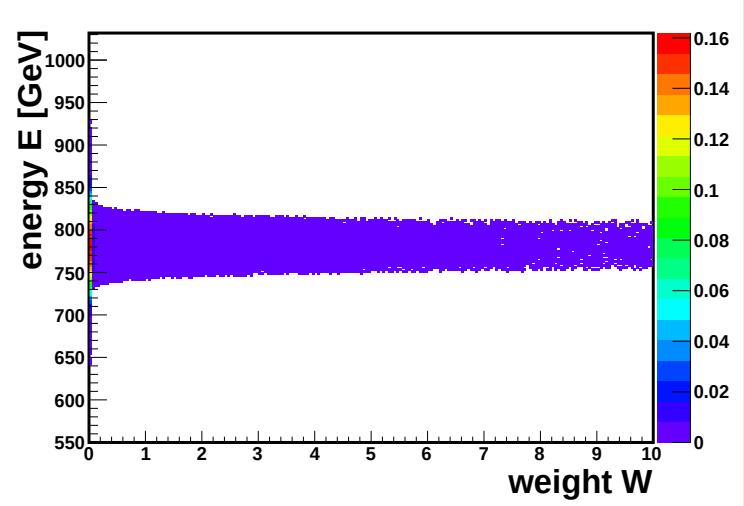
Statistical Distributions and Weight Factor

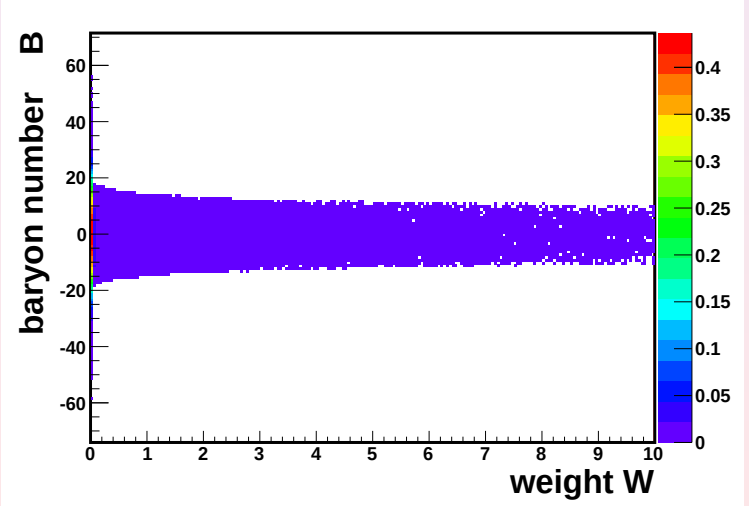
$V_1 = 2000 fm^3 \quad T = 0.16 GeV \quad \mu = 0.0 GeV$

$\lambda = V_1 / V_g = 0.850$





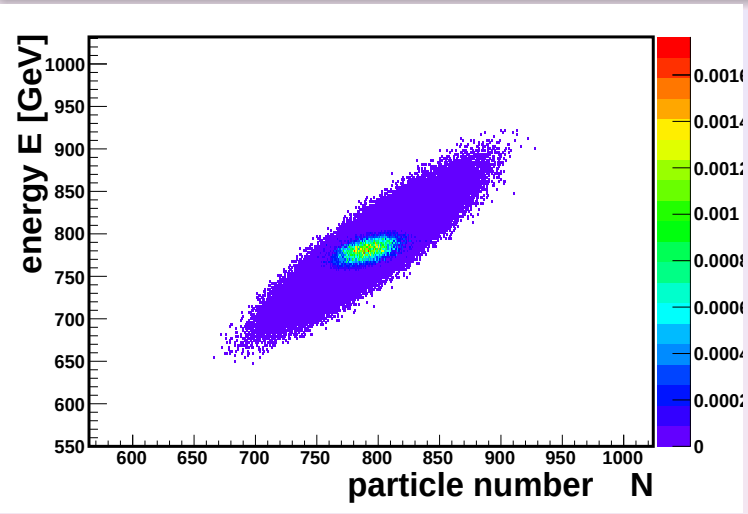


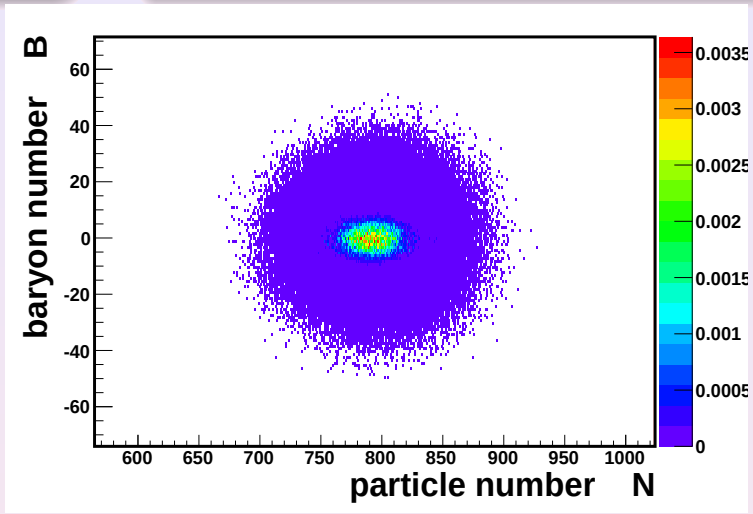


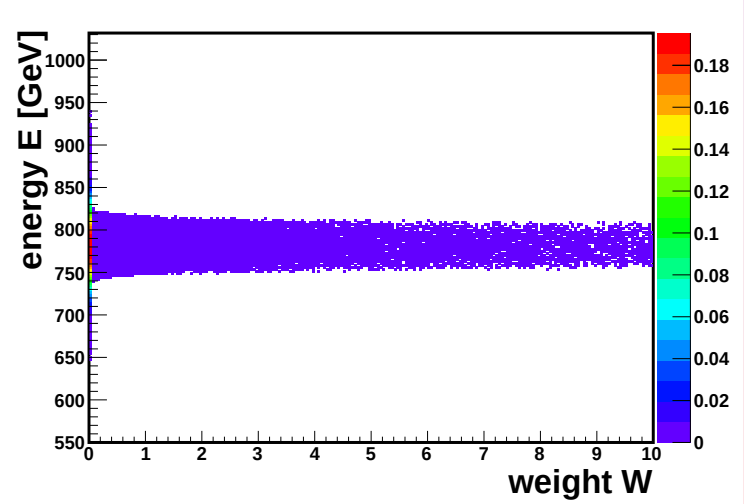
Statistical Distributions and Weight Factor

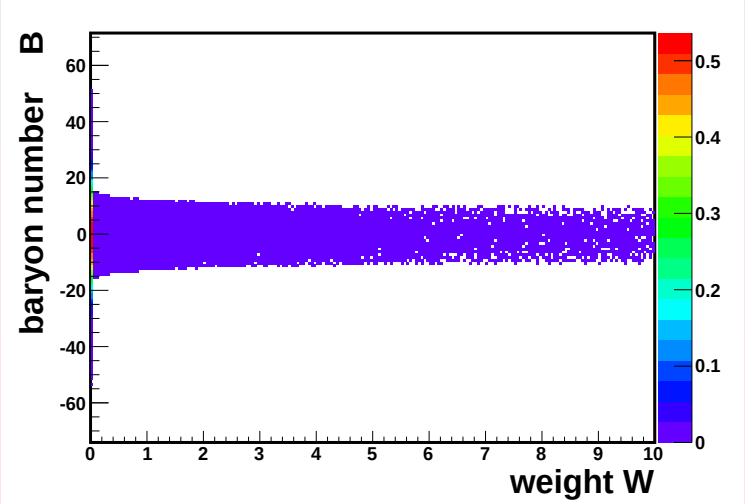
$V_1 = 2000 fm^3 \quad T = 0.16 GeV \quad \mu = 0.0 GeV$

$\lambda = V_1 / V_g = 0.900$





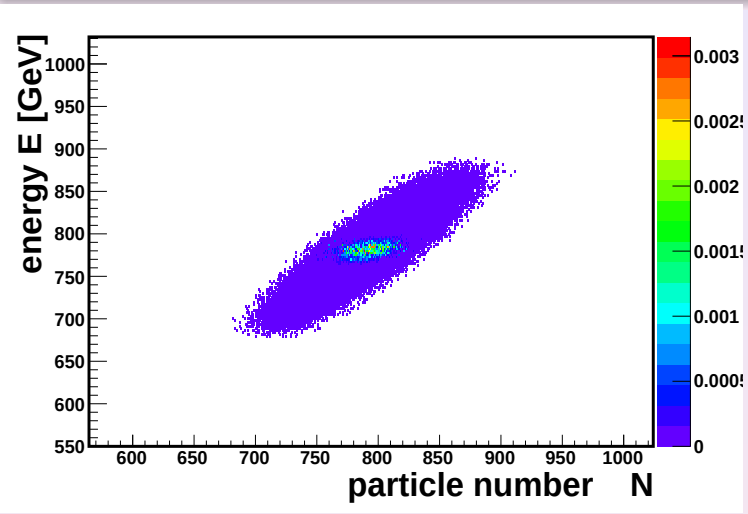


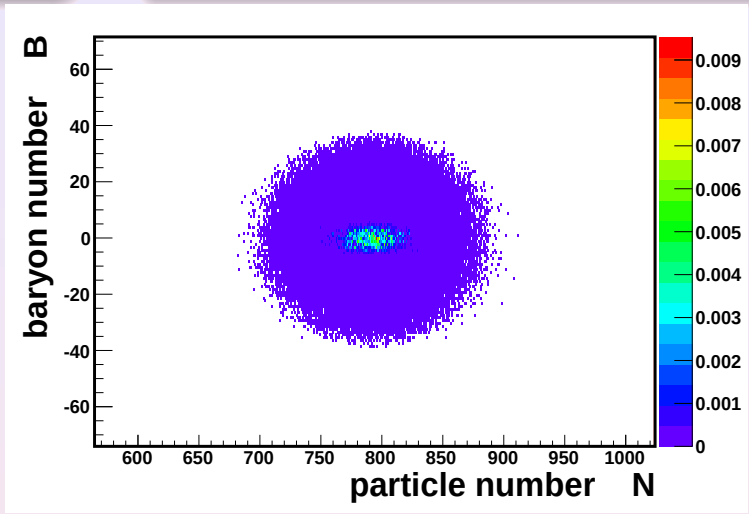


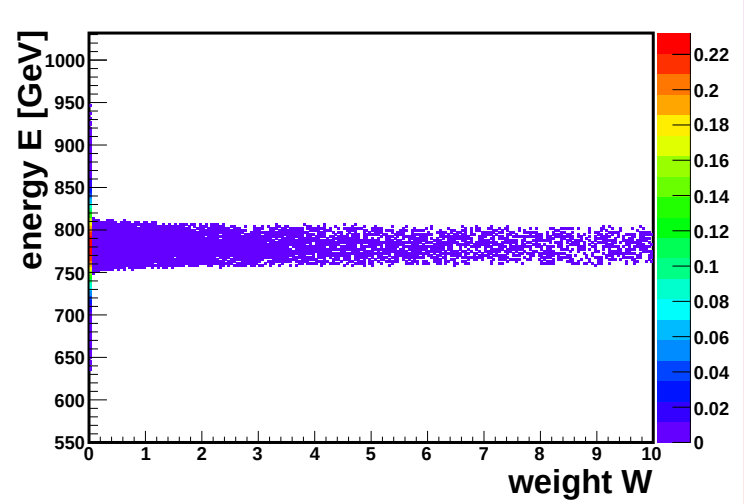
Statistical Distributions and Weight Factor

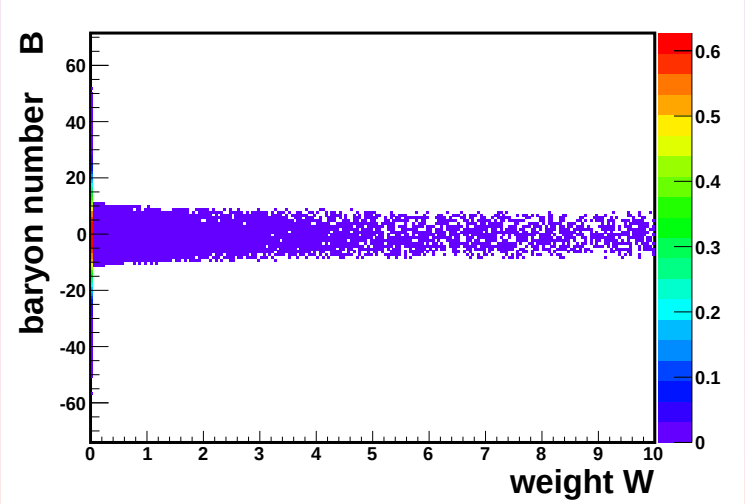
$V_1 = 2000 fm^3 \quad T = 0.16 GeV \quad \mu = 0.0 GeV$

$\lambda = V_1 / V_g = 0.950$





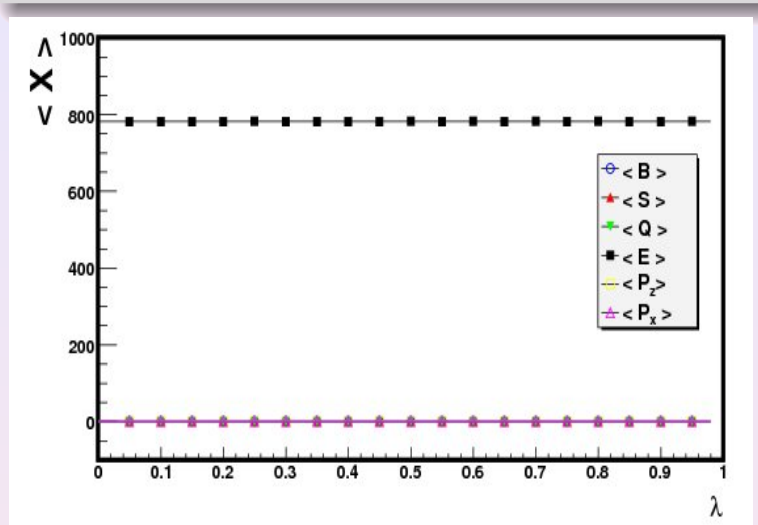


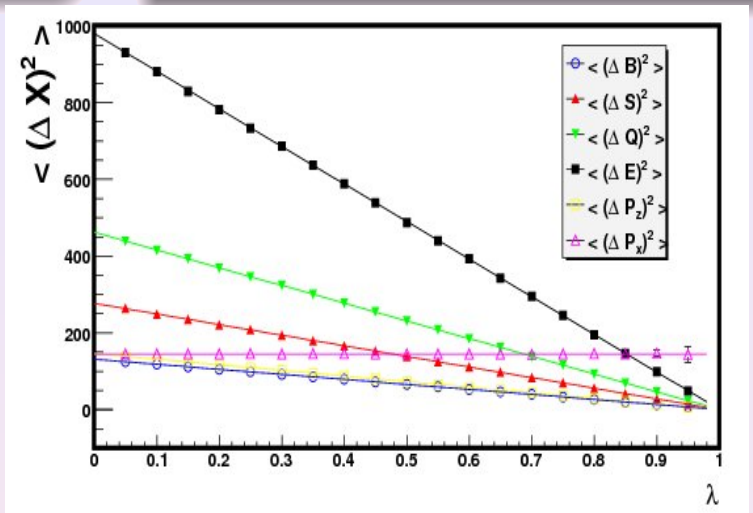


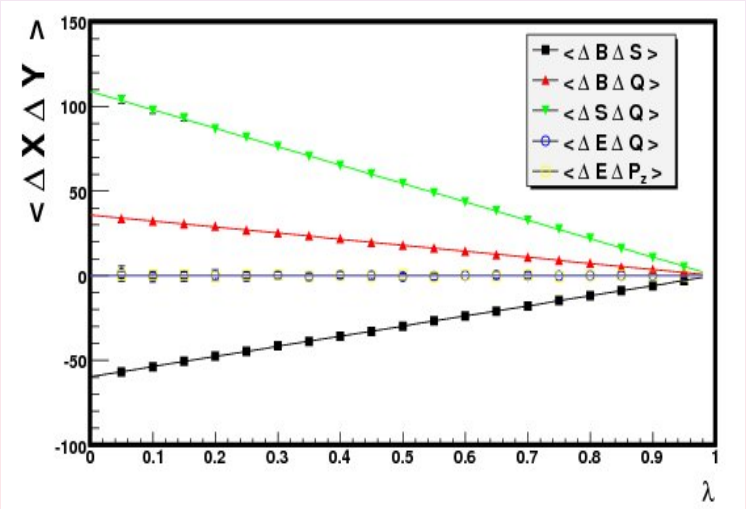
As a function of 'the size of the bath' λ

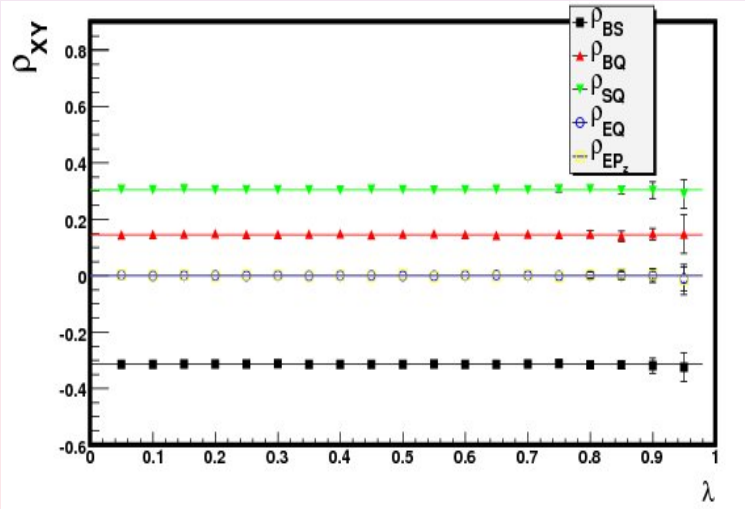
$V_1 = 2000 fm^3 \quad T = 0.16 GeV \quad \mu = 0.0 GeV$

$\lambda = V_1 / V_g$









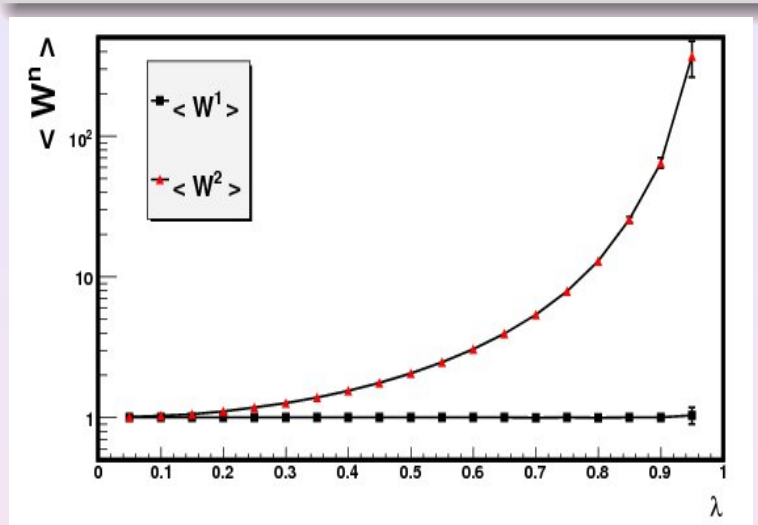
$\langle X \rangle$ and ρ_{XY} remain const.

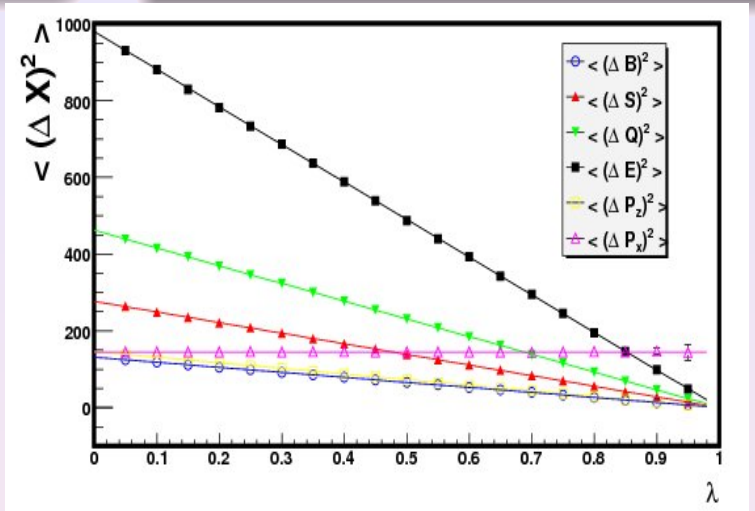
$\langle (\Delta X)^2 \rangle$ and $\langle \Delta X \Delta Y \rangle$ converge linearly.

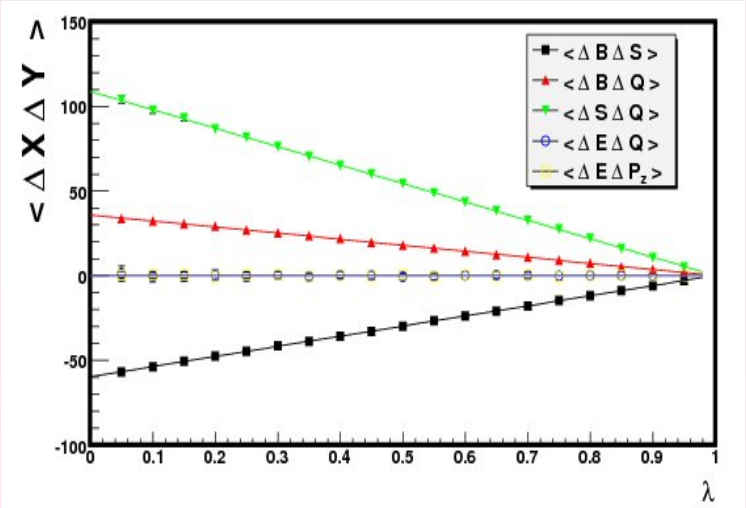
As a function of 'the size of the bath' λ

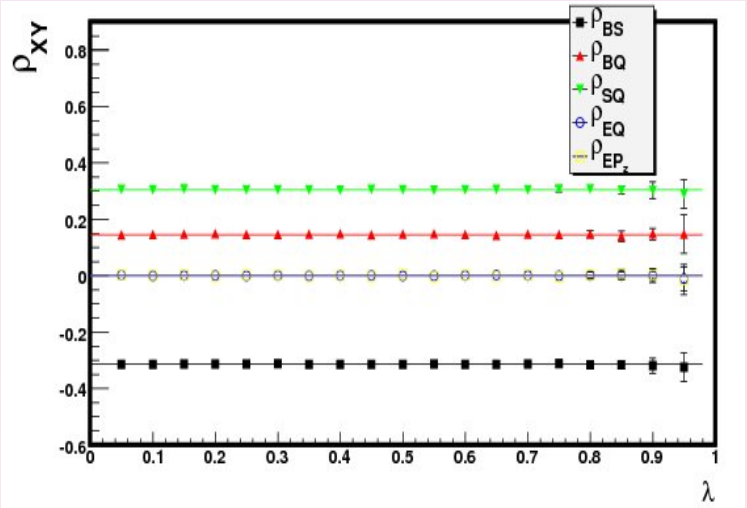
$V_1 = 2000 fm^3 \quad T = 0.16 GeV \quad \mu = 0.0 GeV$

$\lambda = V_1 / V_g$









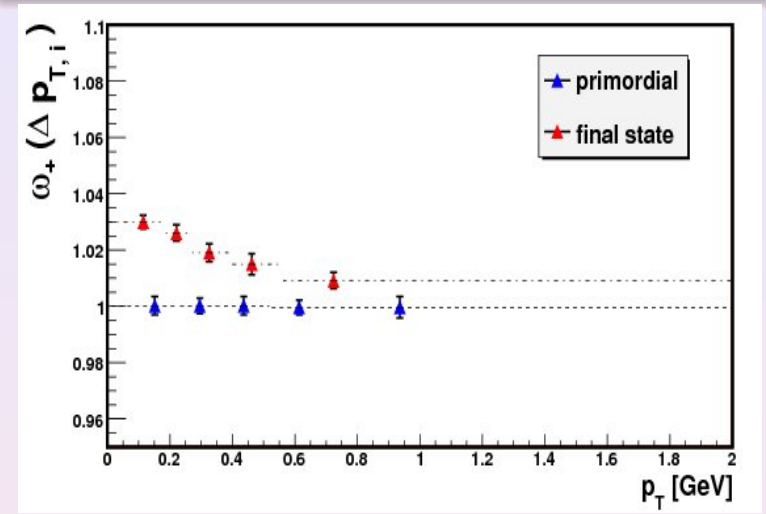
The av. weight $\langle \mathcal{W} \rangle = 1$

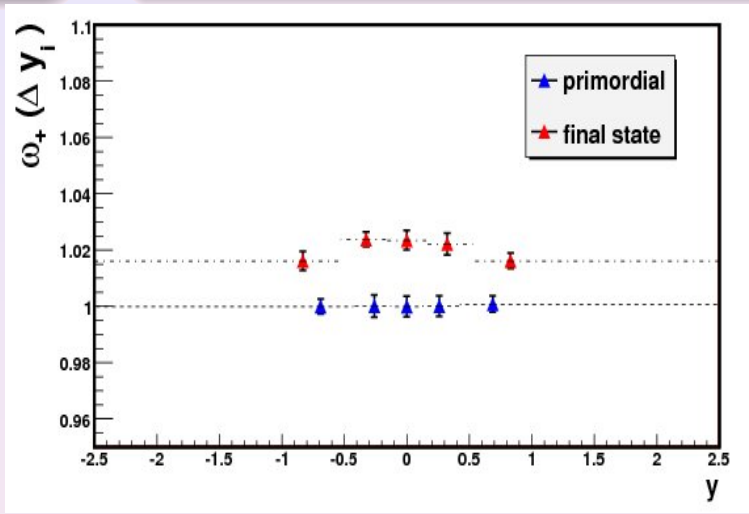
However: $\langle (\Delta \mathcal{W})^2 \rangle$ diverges as $\lambda \to 1$.

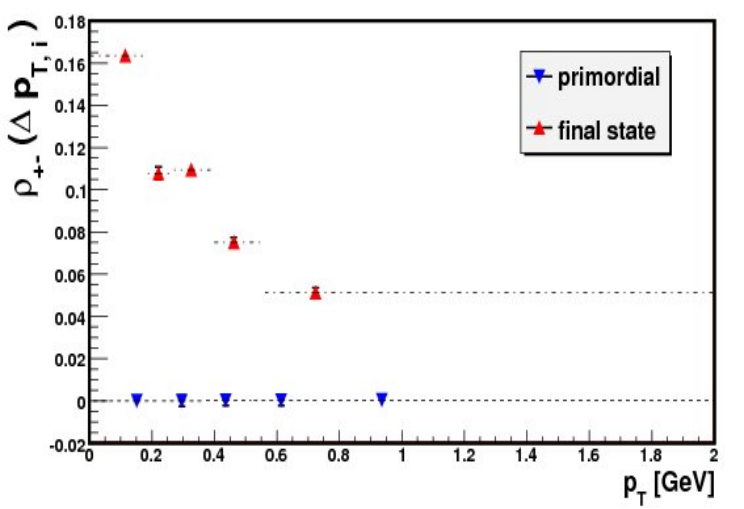
Grand Canonical Ensemble

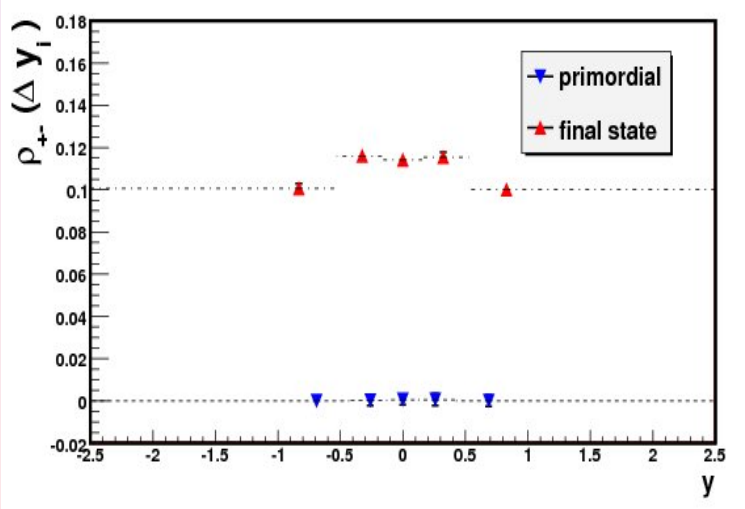
$V_1 = 2000 fm^3 \quad T = 0.16 GeV \quad \mu = 0.0 GeV$

GCE







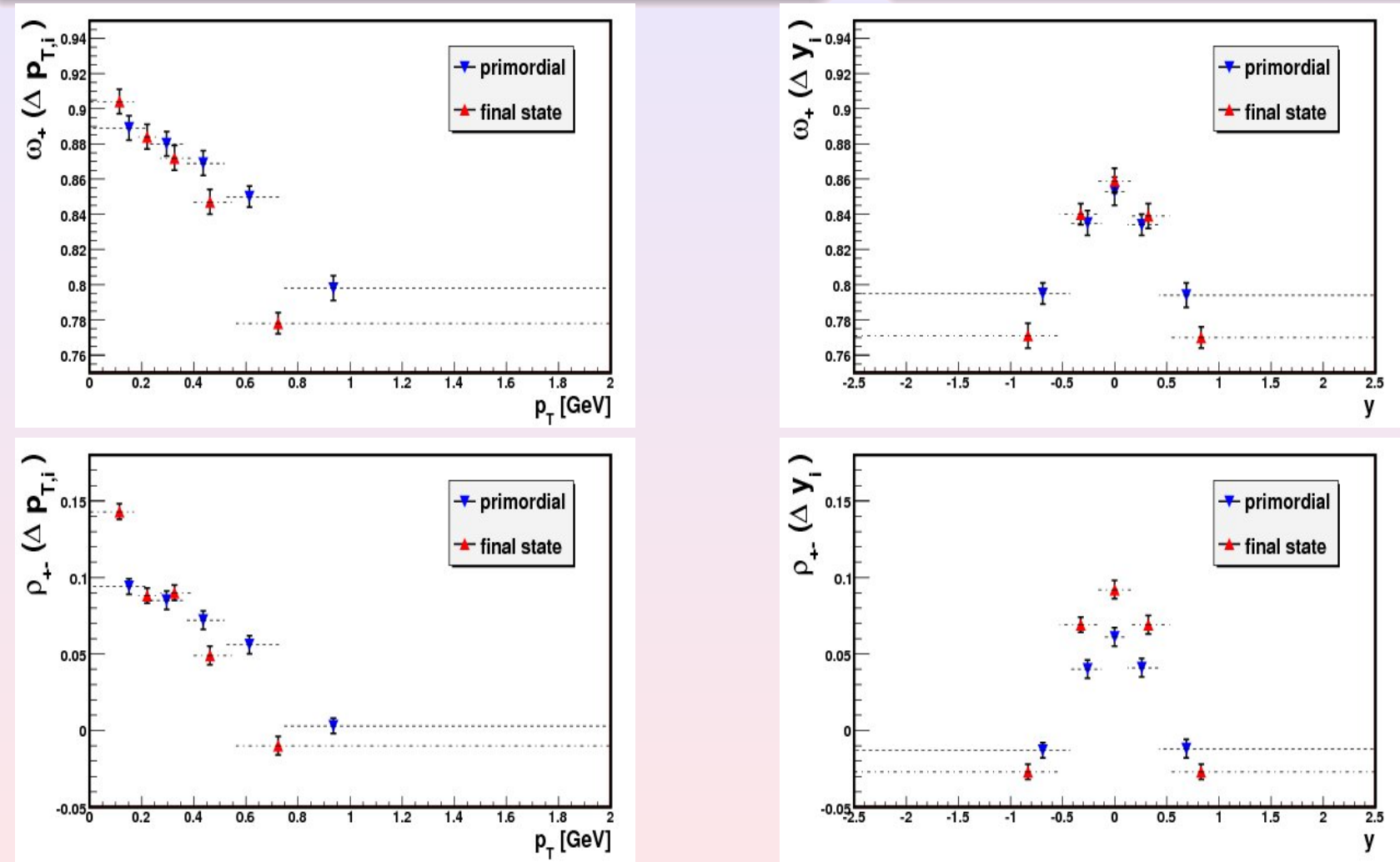


Resonance decay is the only source of correlation.

Microcanonical Ensemble

$V_1 = 2000 fm^3 \quad T = 0.16 GeV \quad \mu = 0.0 GeV$

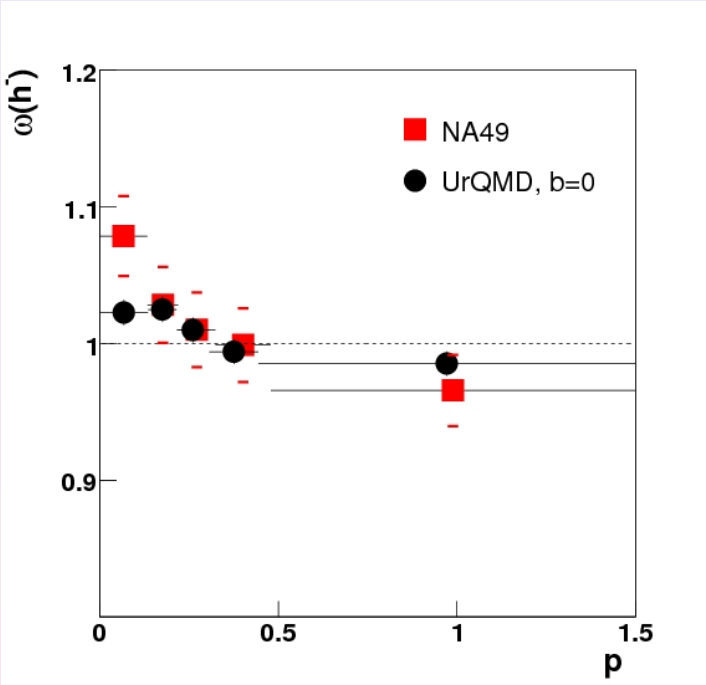
MCE

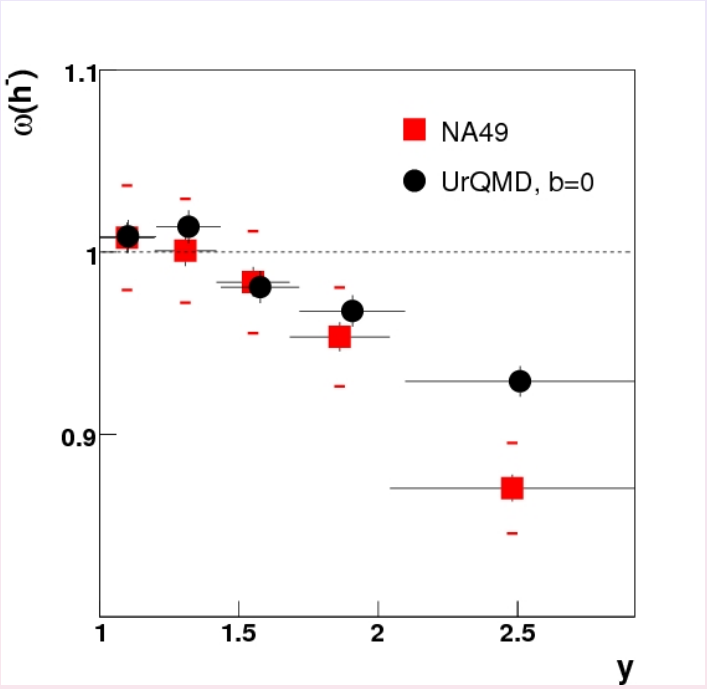


Extrapolated results, re-weighting B, S, Q, E, P_z.

Momentum Cuts in NA49 Data

UrQMD vs. NA49 158AGeV Pb-Pb data (1% most central)





Rapidity and transverse momentum dependence also seen in data!

MCE effects are of similar magnitude as proposed enhancement due to a phase transition / critical point!

B. Lungwitz *et al.* [NA49 Collaboration], PoS C **POD07**, 023 (2007)

Conclusion

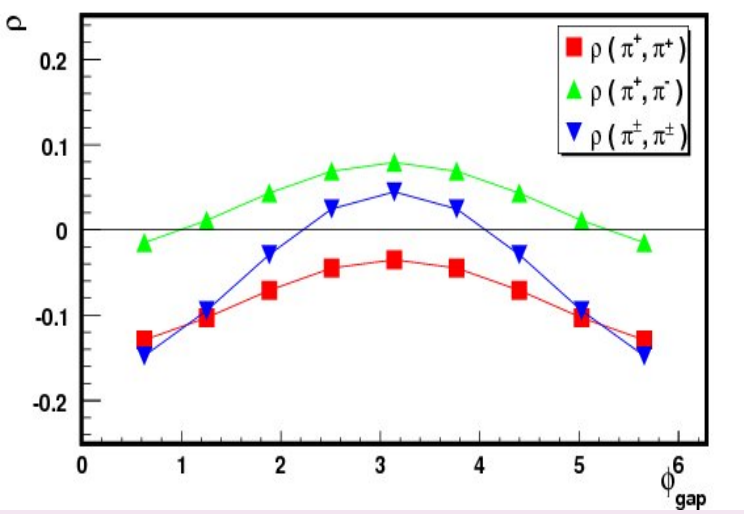
1. We have an developed a thermal model Monte Carlo event generator capable of accounting for globally applied conservation laws in the large volume limit.
2. Fluctuations and Correlations are ensembles specific.
3. Fluctuations and Correlations are sensitive to what part of a system is observed.
4. NA49 data, UrQMD as well as MCE show suppressed multiplicity fluctuations in momentum bins with high p_T and y.
5. Fluctuations are an important test for the statistical hadronization model.

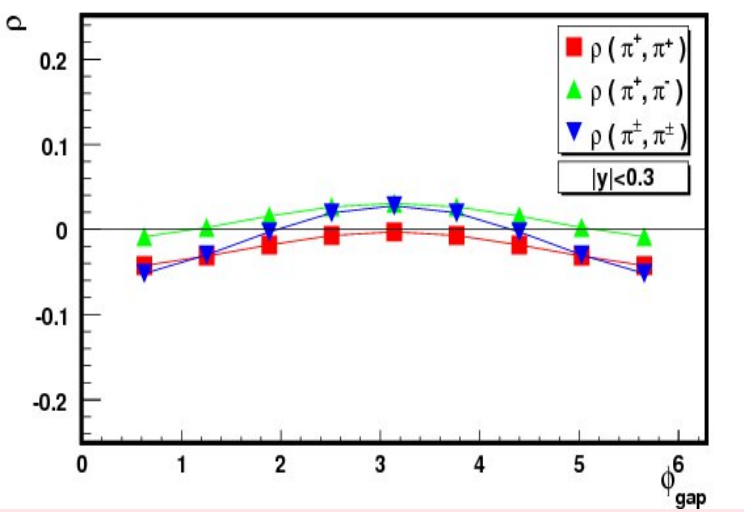
Conclusion

1. We have an developed a thermal model Monte Carlo event generator capable of accounting for globally applied conservation laws in the large volume limit.
2. Fluctuations and Correlations are ensembles specific.
3. Fluctuations and Correlations are sensitive to what part of a system is observed.
4. NA49 data, UrQMD as well as MCE show suppressed multiplicity fluctuations in momentum bins with high p_T and y.
5. Fluctuations are an important test for the statistical hadronization model. (Or any other model, in fact.)

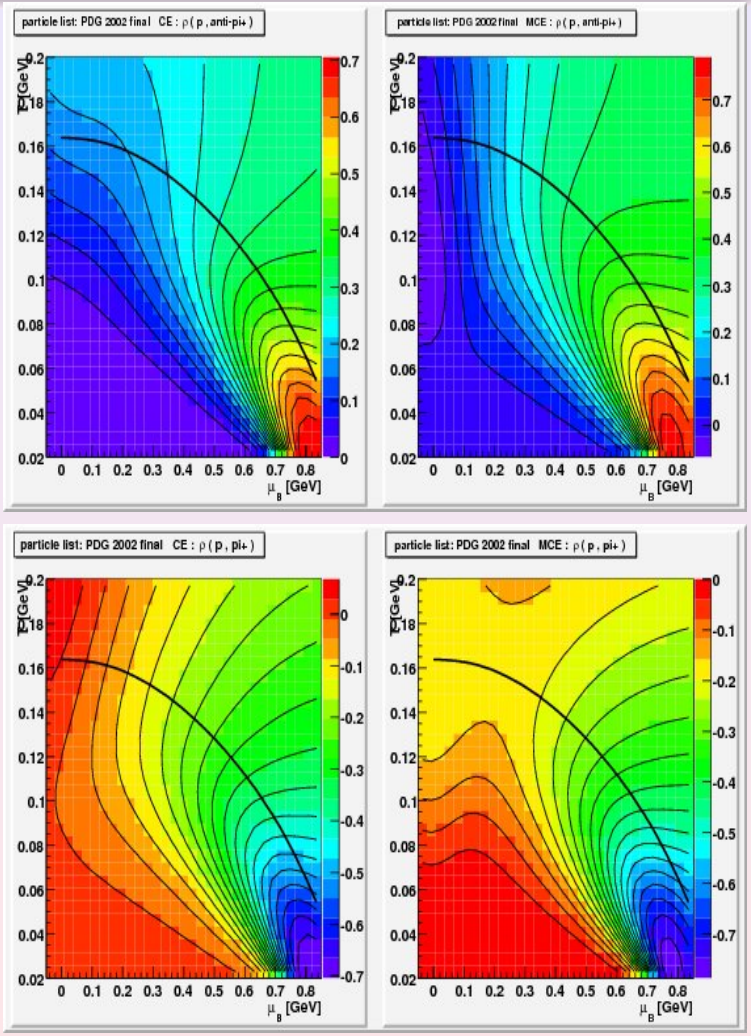
Outlook

Study Angular Correlations





'Explore' the phase diagram



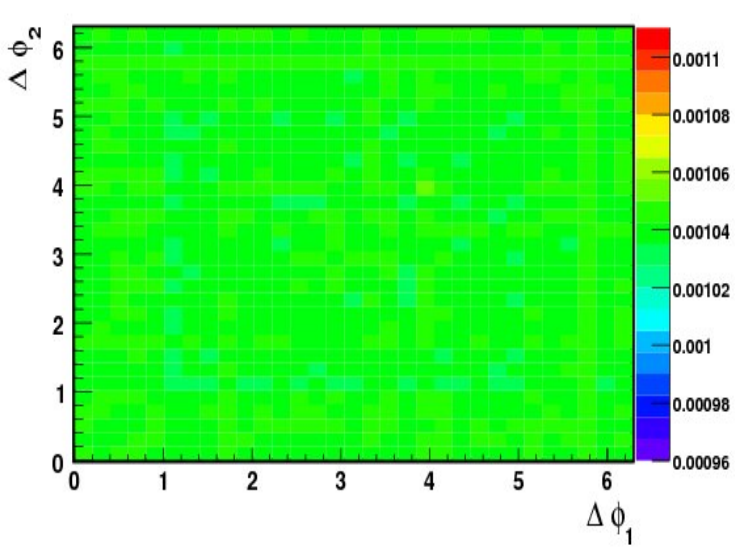
After I have written a thesis, my boss says.

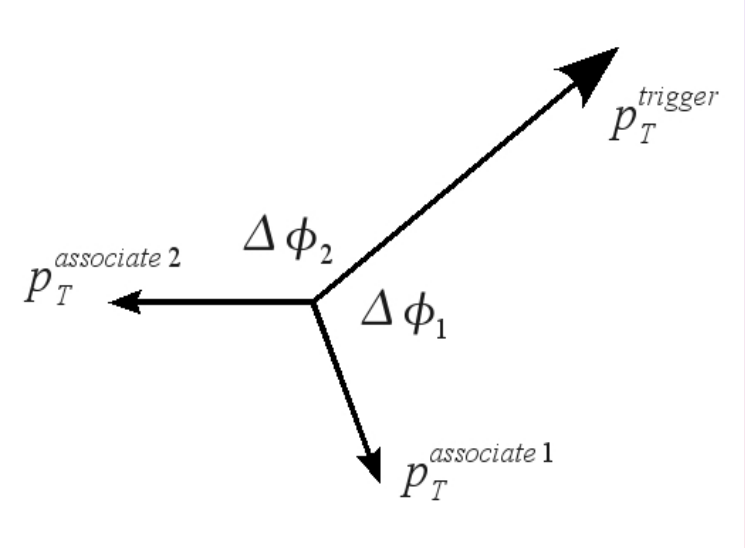
Correlation Function

A simple Boltzmann pion gas

$V_1 = 1000 fm^3 \quad T = 0.16 GeV \quad \mu_Q = 0.0 GeV$

$\lambda = V_1 / V_g = 0.000$





$0.0 GeV < p_T^{asso} < 1.0 GeV$ and $1.0 GeV < p_T^{trigger} < 4.0 GeV$

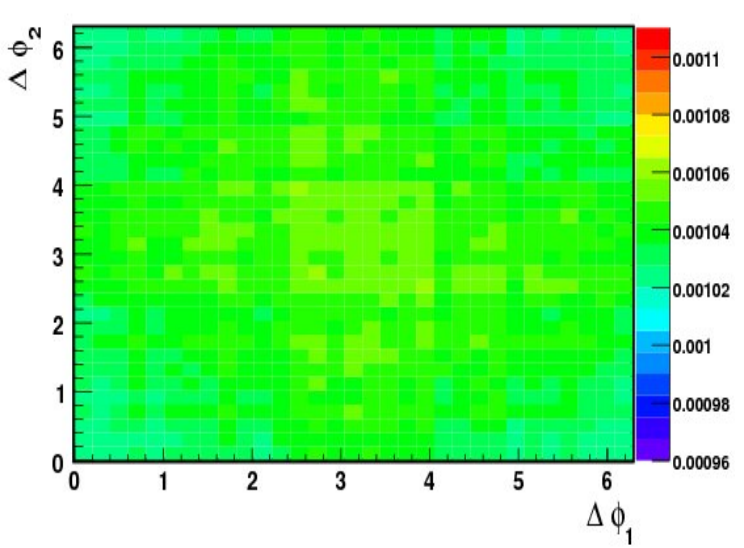
The re-weighting was done w.r.t. Q, E, P_x, P_y, (but not P_z)

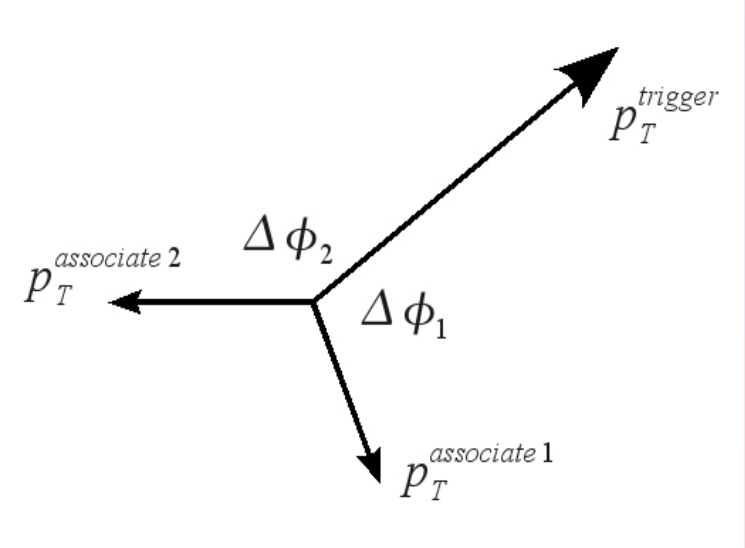
Correlation Function

A simple Boltzmann pion gas

$V_1 = 1000 fm^3 \quad T = 0.16 GeV \quad \mu_Q = 0.0 GeV$

$\lambda = V_1 / V_g = 0.250$





$0.0 GeV < p_T^{asso} < 1.0 GeV$ and $1.0 GeV < p_T^{trigger} < 4.0 GeV$

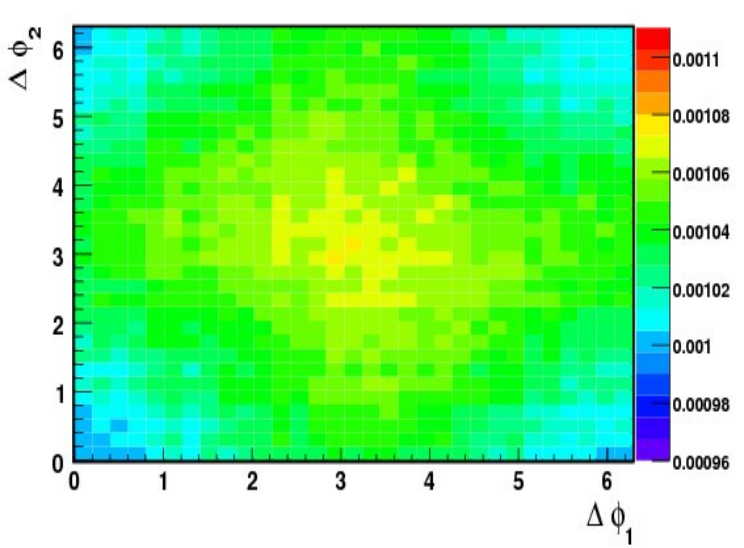
The re-weighting was done w.r.t. Q, E, P_x, P_y, (but not P_z)

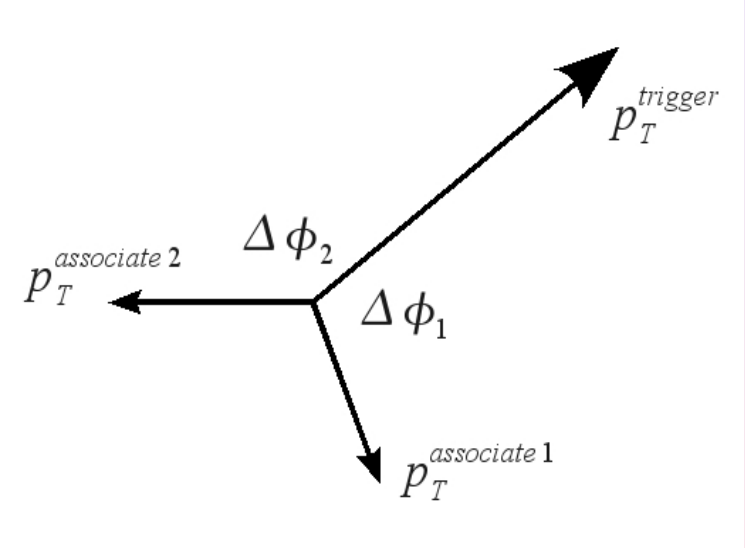
Correlation Function

A simple Boltzmann pion gas

$V_1 = 1000 fm^3 \quad T = 0.16 GeV \quad \mu_Q = 0.0 GeV$

$\lambda = V_1 / V_g = 0.500$





$0.0 GeV < p_T^{asso} < 1.0 GeV$ and $1.0 GeV < p_T^{trigger} < 4.0 GeV$

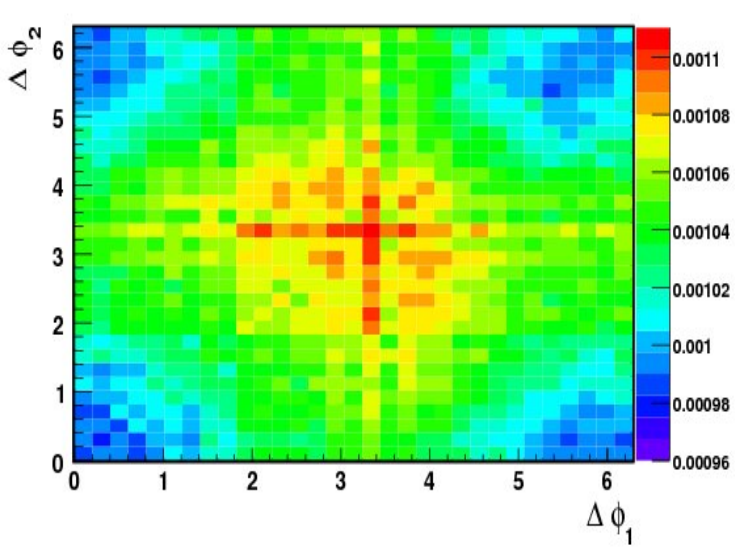
The re-weighting was done w.r.t. Q, E, P_x, P_y, (but not P_z)

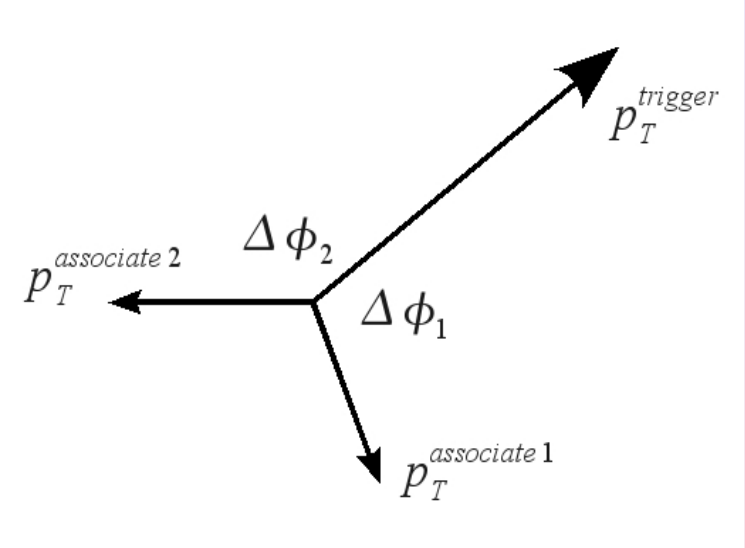
Correlation Function

A simple Boltzmann pion gas

$V_1 = 1000 fm^3 \quad T = 0.16 GeV \quad \mu_Q = 0.0 GeV$

$\lambda = V_1 / V_g = 0.750$





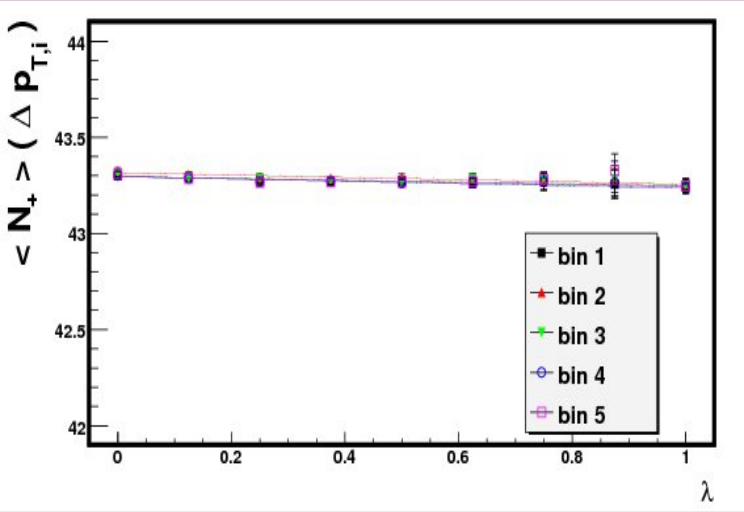
$0.0 GeV < p_T^{asso} < 1.0 GeV$ and $1.0 GeV < p_T^{trigger} < 4.0 GeV$

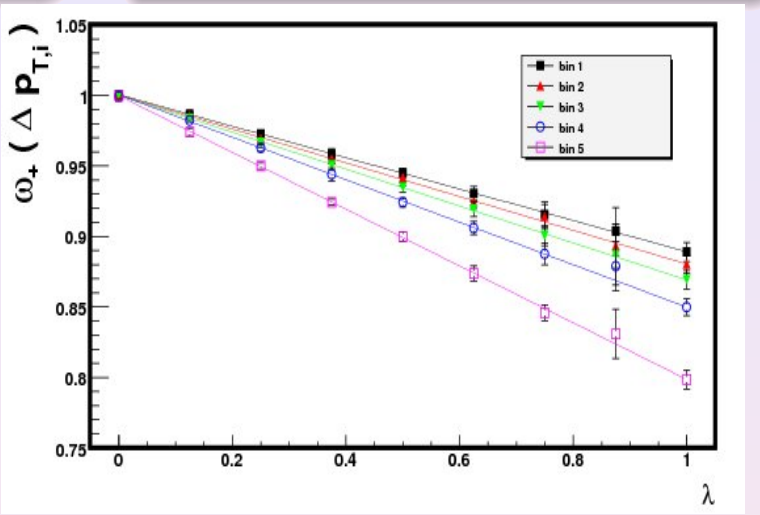
The re-weighting was done w.r.t. Q, E, P_x, P_y, (but not P_z)

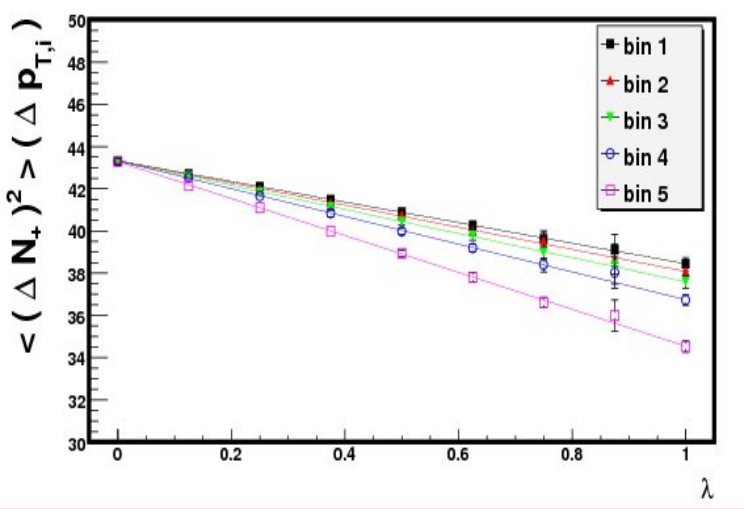
Multiplicity Distributions

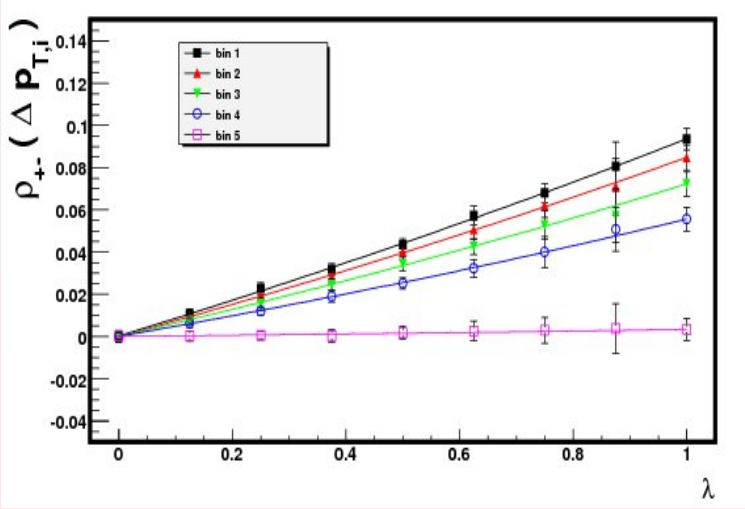
$V_1 = 2000 fm^3 \quad T = 0.16 GeV \quad \mu = 0.0 GeV$

full hadron gas







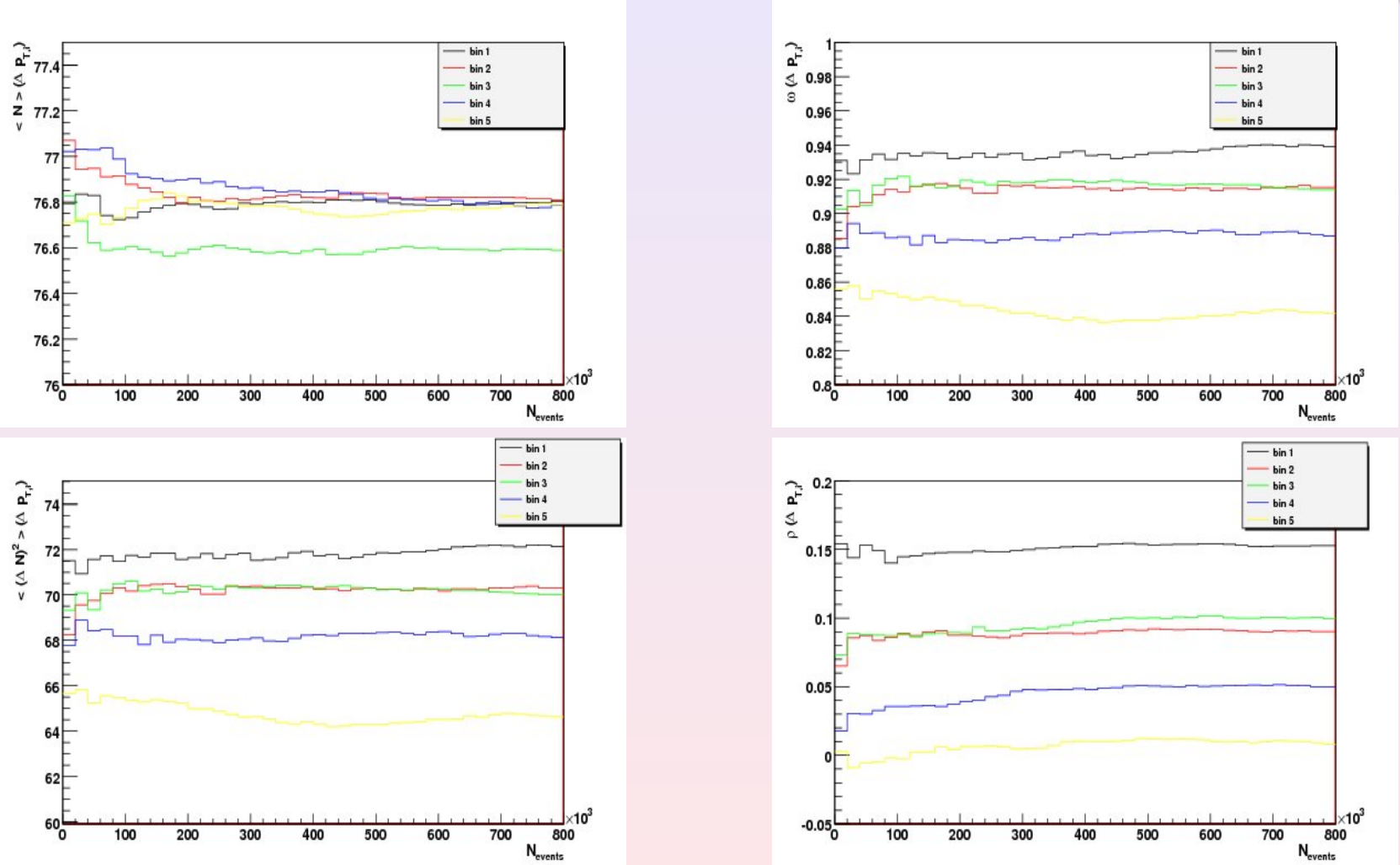


Primordial Transverse Momentum Dependence

Multiplicity Distributions

$V_1 = 2000 fm^3 \quad T = 0.16 GeV \quad \mu = 0.0 GeV$

$\lambda = V_1 / V_g = 0.75$



Convergence with the sample size