QCD Thermodynamics with effective models

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QCD Phase Transitions

QCD $\longrightarrow$ two phase transitions:

1. restoration of chiral symmetry

$$SU_{L+R}(N_f) \longrightarrow SU_L(N_f) \times SU_R(N_f)$$

order parameter:

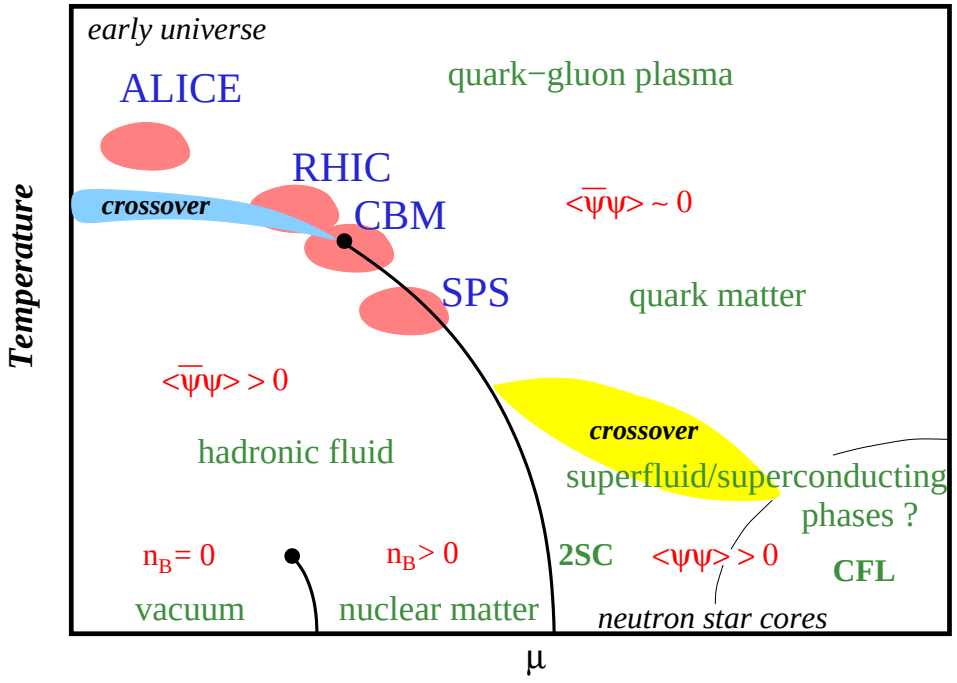
$$\langle \bar{q}q \rangle \begin{cases} > 0 \Leftrightarrow \text{symmetry broken}, \; T < T_c \\ = 0 \Leftrightarrow \text{symmetric phase}, \; T > T_c \end{cases}$$

2. de/confinement

order parameter: Polyakov loop variable

$$\Phi \begin{cases} = 0 \Leftrightarrow \text{confined phase}, \quad T < T_c \\ > 0 \Leftrightarrow \text{deconfined phase}, \; T > T_c \end{cases}$$

$$\Phi = \left\langle \mathrm{tr}_c \mathcal{P} \exp\left(i \int_0^\beta d\tau A_0(\tau, \vec{x}) \right) \right\rangle / N_c$$



At densities/temperatures of interest only model calculations available

effective models:

1. Quark-meson model — or other models e.g. NJL
2. Polyakov–quark-meson model — or PNJL models

Outline

- Three-Flavor Quark-Meson Model
- ...with Polyakov loop dynamics
- Finite density extrapolations

$N_f = 3$ Quark-Meson (QM) model

- Model Lagrangian: $\mathcal{L}_{\text{qm}} = \mathcal{L}_{\text{quark}} + \mathcal{L}_{\text{meson}}$

a) Quark part with Yukawa coupling g:

$$\mathcal{L}_{\text{quark}} = \bar{q}(i\partial\!\!\!/ - g\frac{\lambda_a}{2}(\sigma_a + i\gamma_5\pi_a))q$$

b) Meson part: scalar σ_a and pseudoscalar π_a nonet

fields: $\phi = \sum_{a=0}^{8} \frac{\lambda_a}{2}(\sigma_a + i\pi_a)$

$$\begin{aligned}\mathcal{L}_{\text{meson}} = {} & \text{tr}[\partial_\mu\phi^\dagger\partial^\mu\phi] - m^2\text{tr}[\phi^\dagger\phi] - \lambda_1(\text{tr}[\phi^\dagger\phi])^2 \\ & -\lambda_2\text{tr}[(\phi^\dagger\phi)^2] + c\,[\det(\phi) + \det(\phi^\dagger)] \\ & +\text{tr}[H(\phi + \phi^\dagger)]\end{aligned}$$

explicit sym. breaking matrix: $H = \sum_a \frac{\lambda_a}{2} h_a$

Phase diagram $N_f = 3$ $\quad (\mu \equiv \mu_q = \mu_s)$

[BJS, M. Wagner, '09]

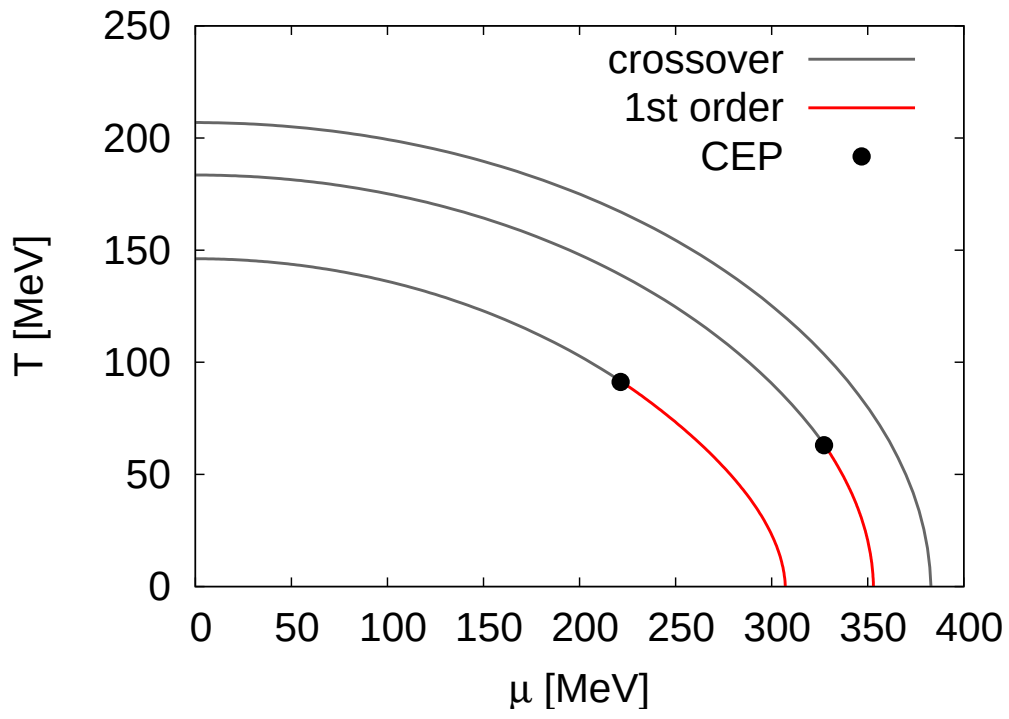
Model parameter fitted to (pseudo)scalar meson spectrum:

PDG: $f_0(600)$ mass=$(400 \ldots 1200)$ MeV $\rightarrow$ broad resonance

➜ influence of existence of CEP!

Example: $m_\sigma = 600$ MeV (lower lines), 800 and 900 MeV (here Mean-field approximation)

with $U(1)_A$



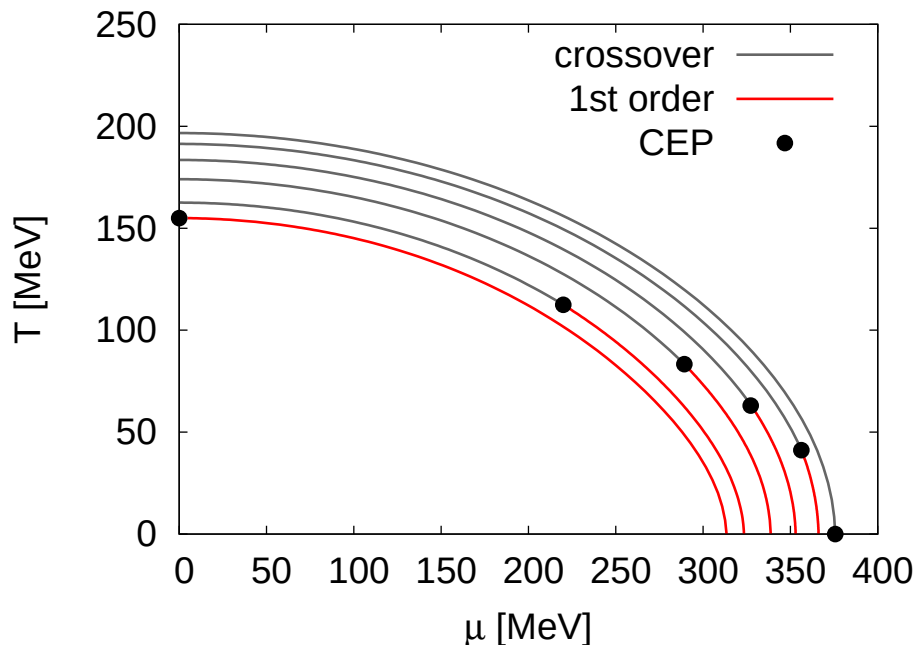
Mass sensitivity

Chiral limit: RG arguments $\rightarrow$ for $N_f = 3$ first-order ✔ [Pisarski, Wilczek '84]

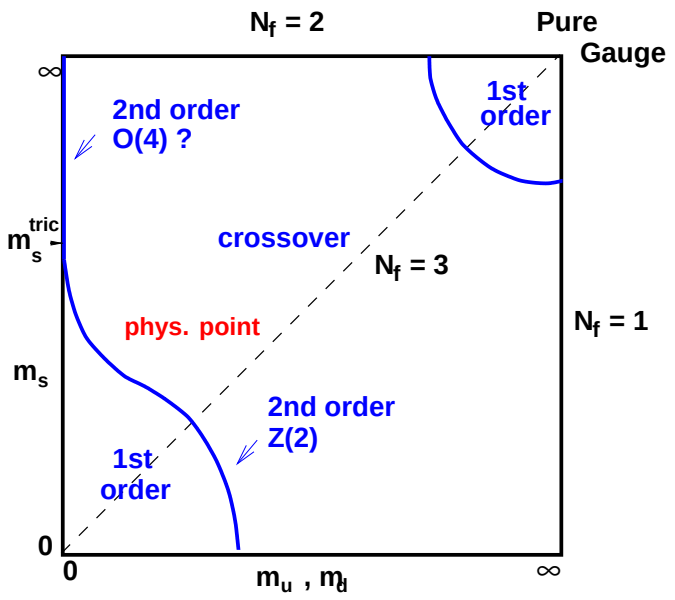
- $m_\pi / m_\pi^* = 0.49$ (lower line), $0.6, 0.8 \ldots, 1.36$ (upper line)

  $m_\pi^* = 138$ MeV , $m_K^* = 496$ MeV , ratio $m_\pi / m_K = m_\pi^* / m_K^*$ fixed

with $U(1)_A$, $m_\sigma = 800$ MeV



Columbia plot [Brown et al. '90]



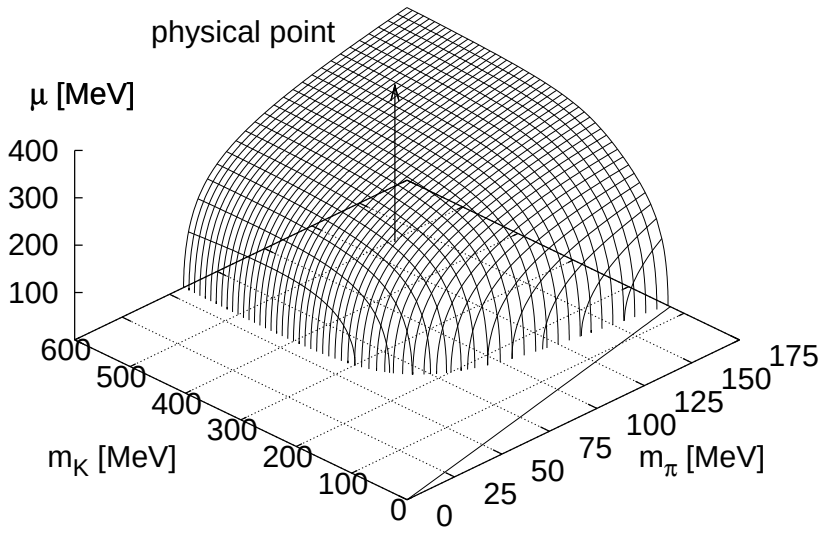
Chiral critical surface $(m_\sigma = 800$ MeV)

[BJS, M. Wagner, '09]

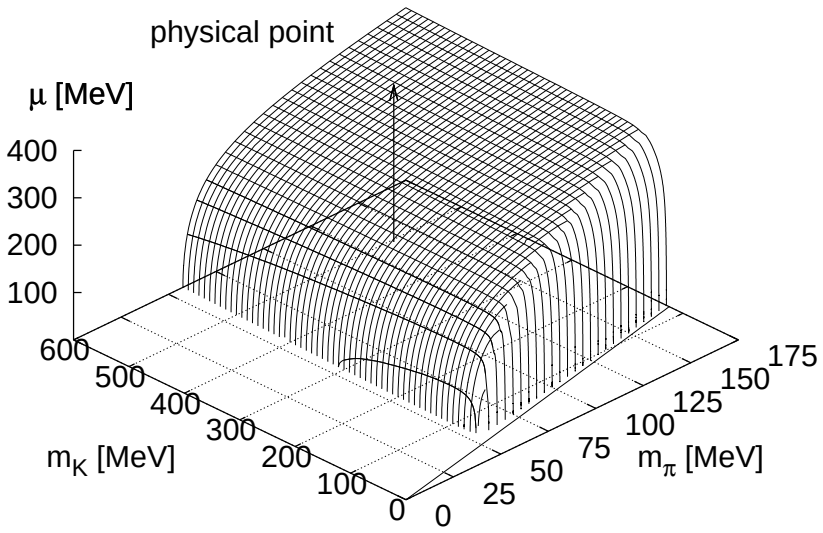
- chiral critical surface in (m_π, m_K)-plane

  $\rightarrow$ standard scenario for $m_\sigma = 800$ MeV (as expected)

with $U(1)_A$



without $U(1)_A$



Chiral critical surface for different m_σ

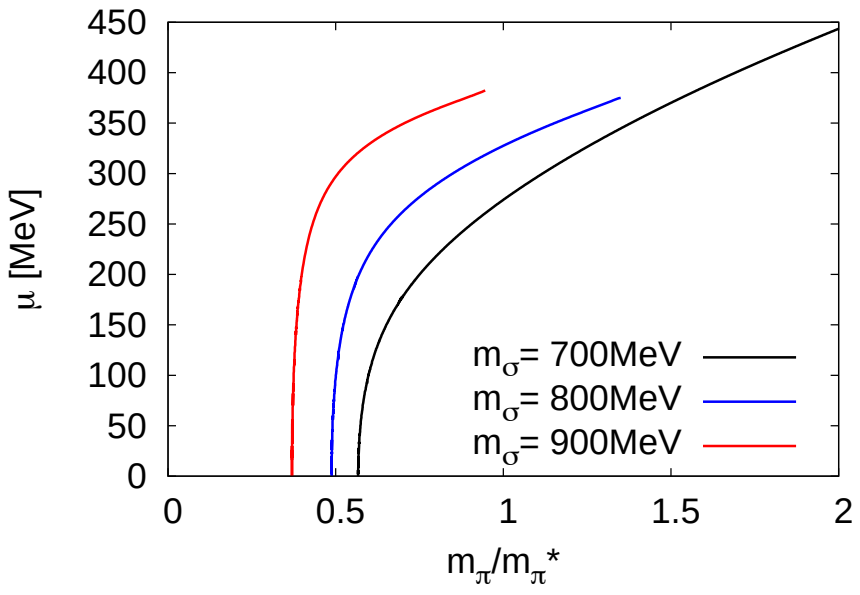
[BJS, M. Wagner, '09]

- chiral critical surface in (m_π, m_K)-plane for different m_σ

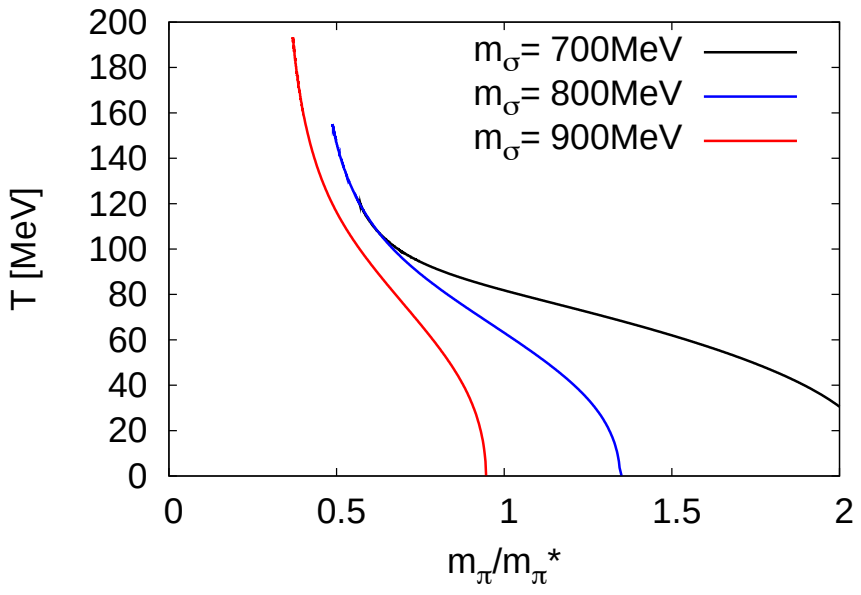
▷ CEP vanishes for $m_\sigma > 800$ MeV $\rightarrow$ possible non-standard scenario?

➜ three cuts of critical surface along fixed m_π/m_K ratio through physical point

critical μ_c



critical T_c



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Polyakov–quark-meson (PQM) model

- Lagrangian $\mathcal{L}_{\text{PQM}} = \mathcal{L}_{\text{qm}} + \mathcal{L}_{\text{pol}}$ with $\mathcal{L}_{\text{pol}} = -\bar{q}\gamma_0 A_0 q - \mathcal{U}(\phi, \bar{\phi})$

1 polynomial Polyakov loop potential:

Polyakov 1978
Meisinger 1996
Pisarski 2000

$$\frac{\mathcal{U}(\phi, \bar{\phi})}{T^4} = -\frac{b_2(T, T_0)}{2}\phi\bar{\phi} - \frac{b_3}{6}\left(\phi^3 + \bar{\phi}^3\right) + \frac{b_4}{16}\left(\phi\bar{\phi}\right)^2$$

with

$$b_2(T, T_0) = a_0 + a_1(T_0/T) + a_2(T_0/T)^2 + a_3(T_0/T)^3$$

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2 logarithmic potential:

Rössner et al. 2007

$$\frac{\mathcal{U}_{\text{log}}}{T^4} = -\frac{1}{2}a(T)\bar{\phi}\phi + b(T)\ln\left[1 - 6\bar{\phi}\phi + 4\left(\phi^3 + \bar{\phi}^3\right) - 3\left(\bar{\phi}\phi\right)^2\right]$$

with

$$a(T) = a_0 + a_1(T_0/T) + a_2(T_0/T)^2 \quad \text{and} \quad b(T) = b_3(T_0/T)^3$$

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3 Fukushima

Fukushima 2008

$$\mathcal{U}_{\text{Fuku}} = -bT\left\{54e^{-a/T}\phi\bar{\phi} + \ln\left[1 - 6\bar{\phi}\phi + 4\left(\phi^3 + \bar{\phi}^3\right) - 3\left(\bar{\phi}\phi\right)^2\right]\right\}$$

with

a controls deconfinement b strength of mixing chiral & deconfinement

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in presence of dynamical quarks: $T_0 = T_0(N_f)$ BJS, Pawlowski, Wambach, 2007

N_f	0	1	2	2 + 1	3
T_0 [MeV]	270	240	208	187	178

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$\mu \neq 0$: $T_0 = T_0(N_f, \mu)$ $\bar{\phi} \neq \phi^*$

Phase diagram $N_f = 2 + 1$

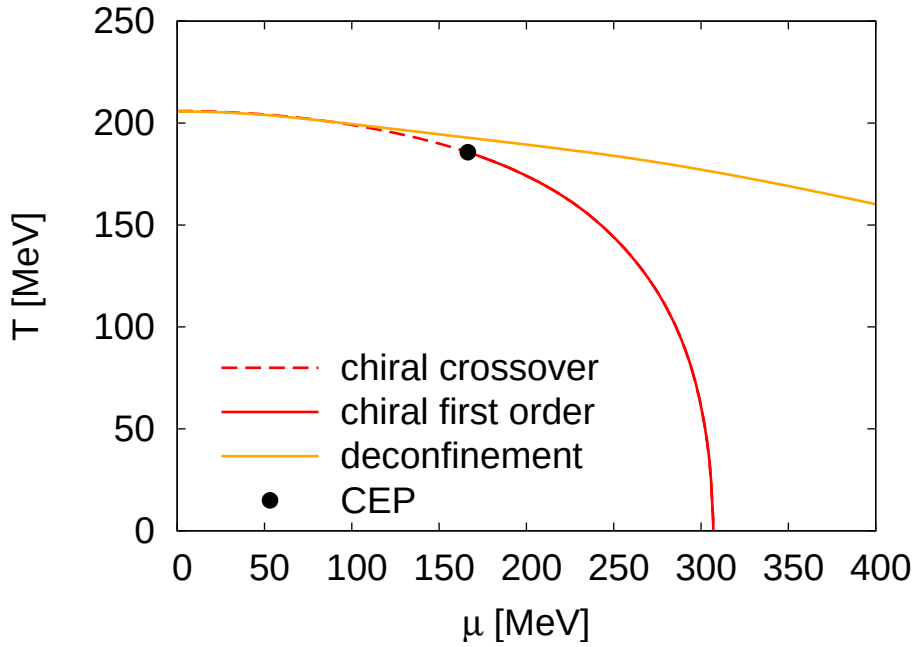
[BJS, M. Wagner; in preparation '09]

influence of Polyakov loop

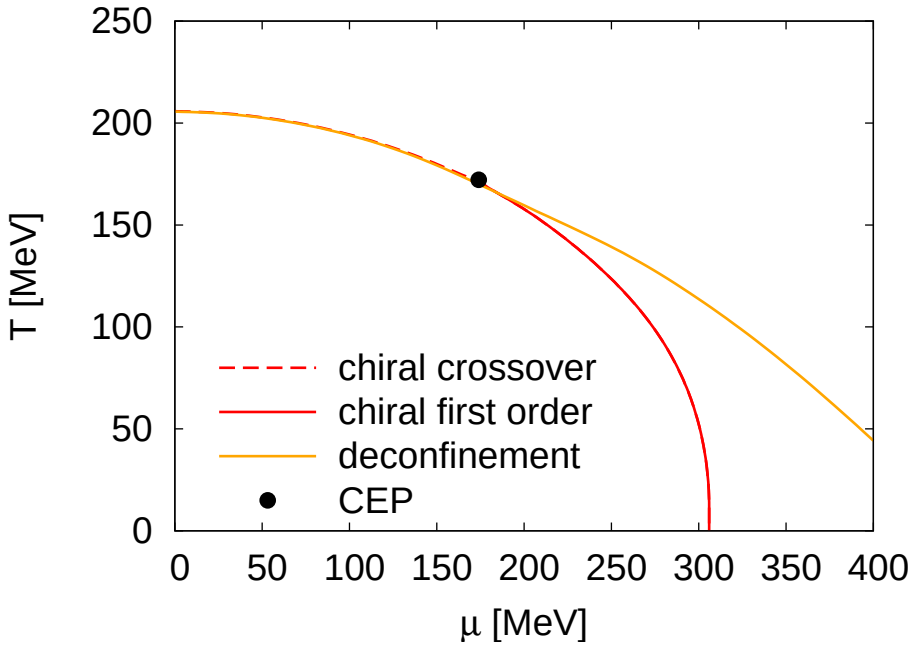
Logarithmic Polyakov loop potential

Mean-field approximation

$T_0 = 270$ MeV (constant)



$T_0(\mu)$ (i.e. with μ corrections)



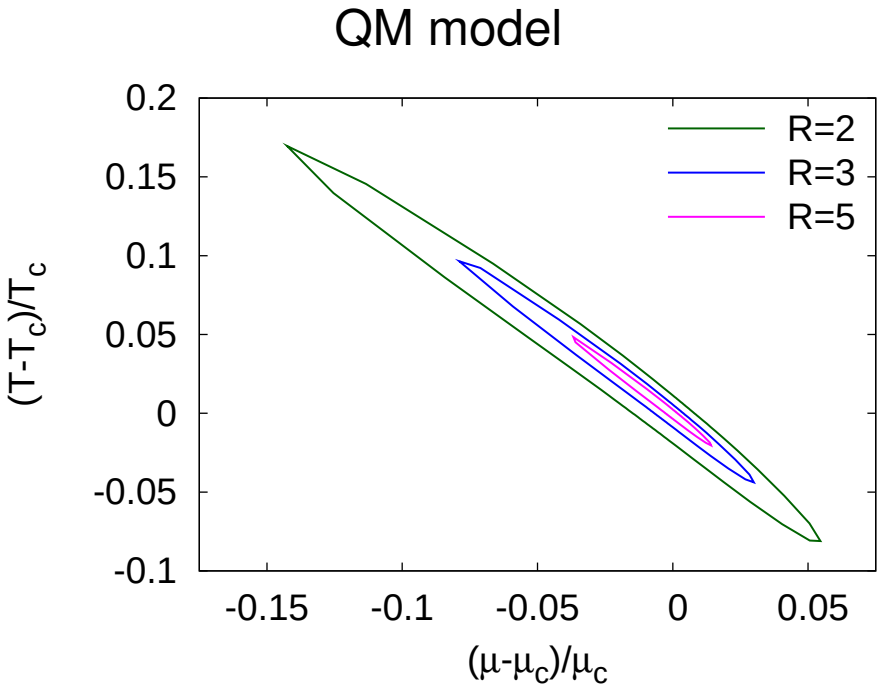
Critical region $N_f = 2 + 1$

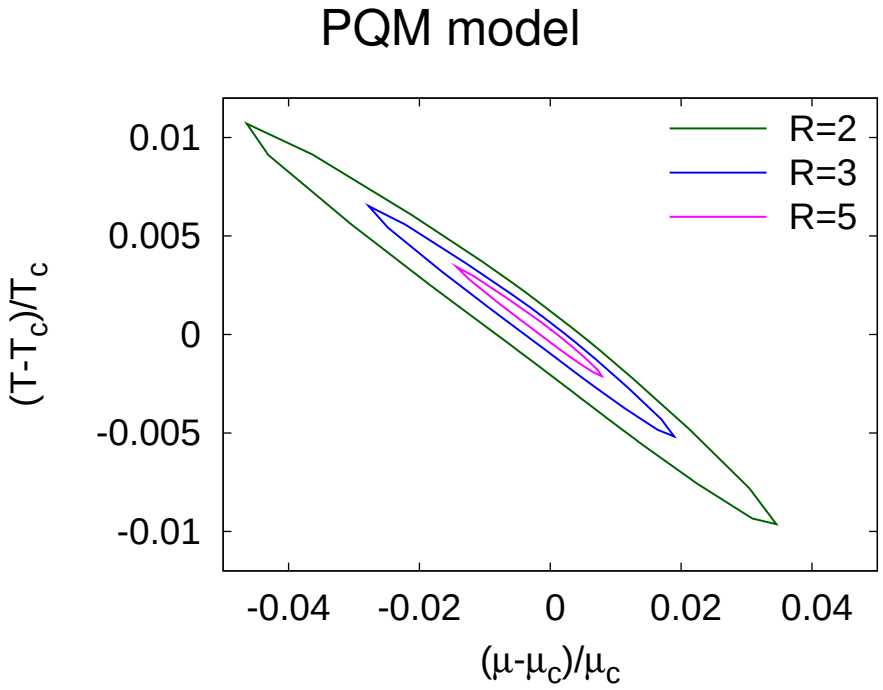
[BJS, M. Wagner; in preparation '09]

contour plot of size of the critical region around CEP

defined via fixed ratio of susceptibilities: $R = \chi_q / \chi_q^{\text{free}}$

➜ compressed with Polyakov loop





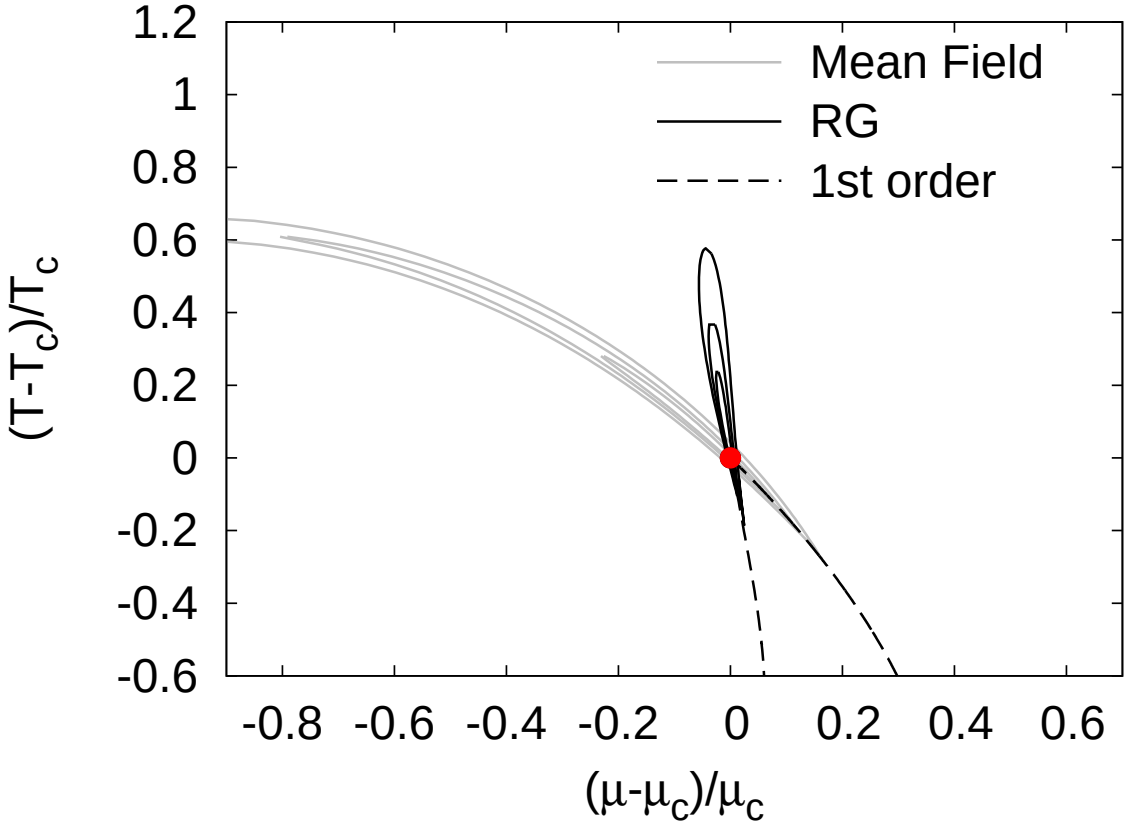
Critical region $N_f = 2 + 1$

[BJS, J. Wambach '06]

similar conclusion if fluctuations (via RG techniques) are included

example: $N_f = 2$ QM model

Mean Field $\leftrightarrow$ RG analysis



Isentropes $s/n = const$ and Focussing

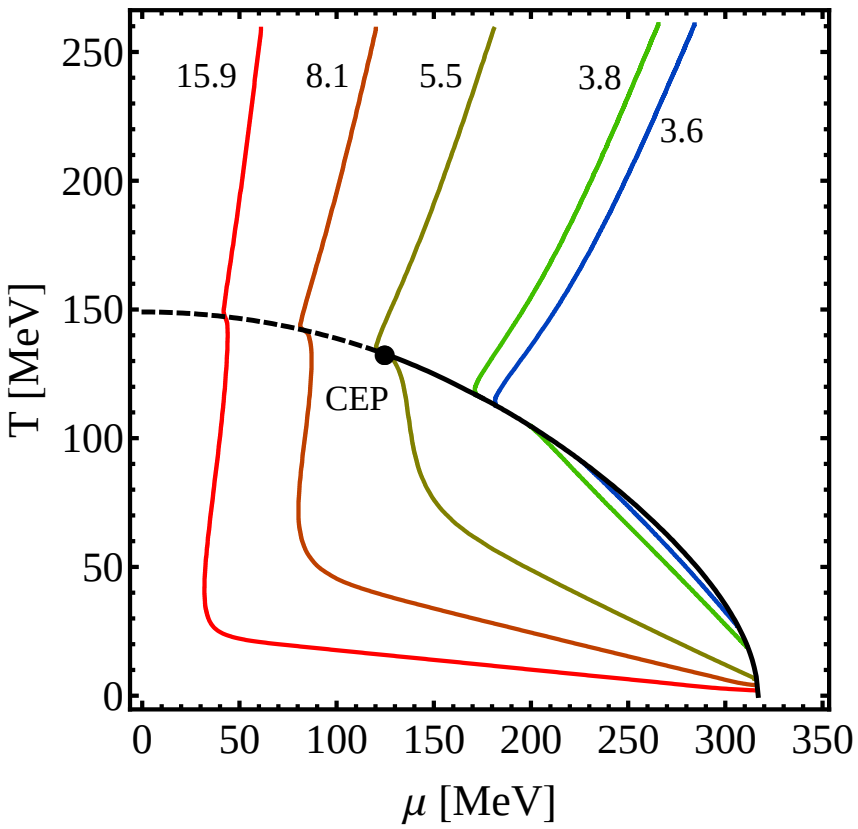
[E. Nakano, BJS, B.Stokic, B.Friman, K.Redlich; arXiv:0906.xxxx]

here: $N_f = 2$ QM model: kink in MFA are washed out in FRG

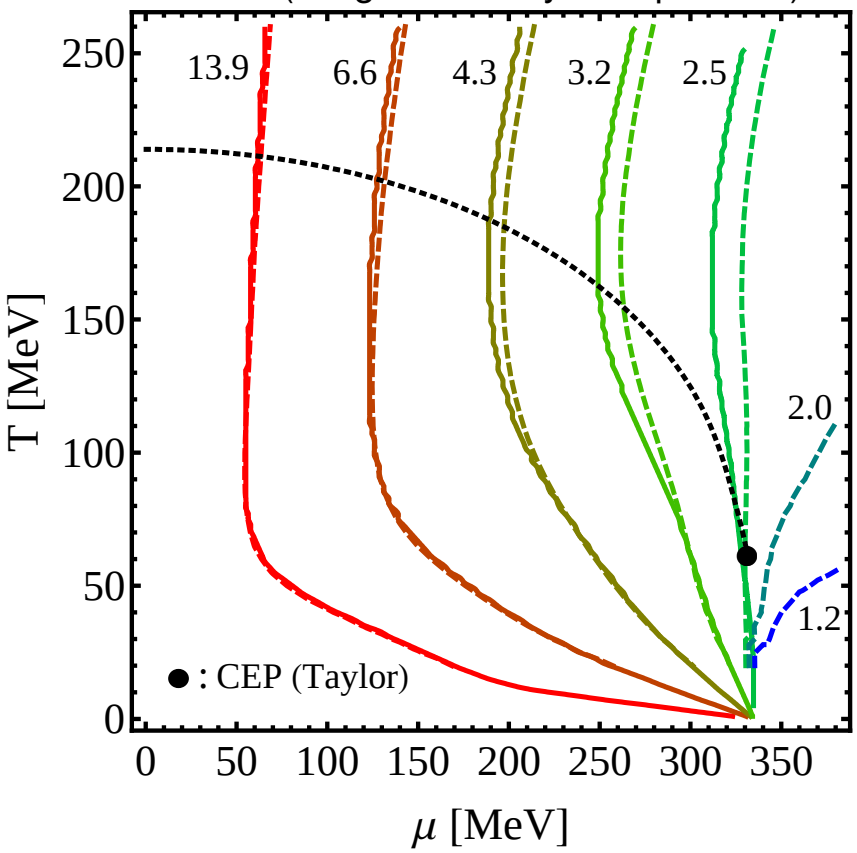
$\rightarrow$ no focussing if fluctuations taken into account

a) influence of Dirac term b) smallnest of crit region

MFA (no Dirac term)



FRG (full grid and Taylor expansion)

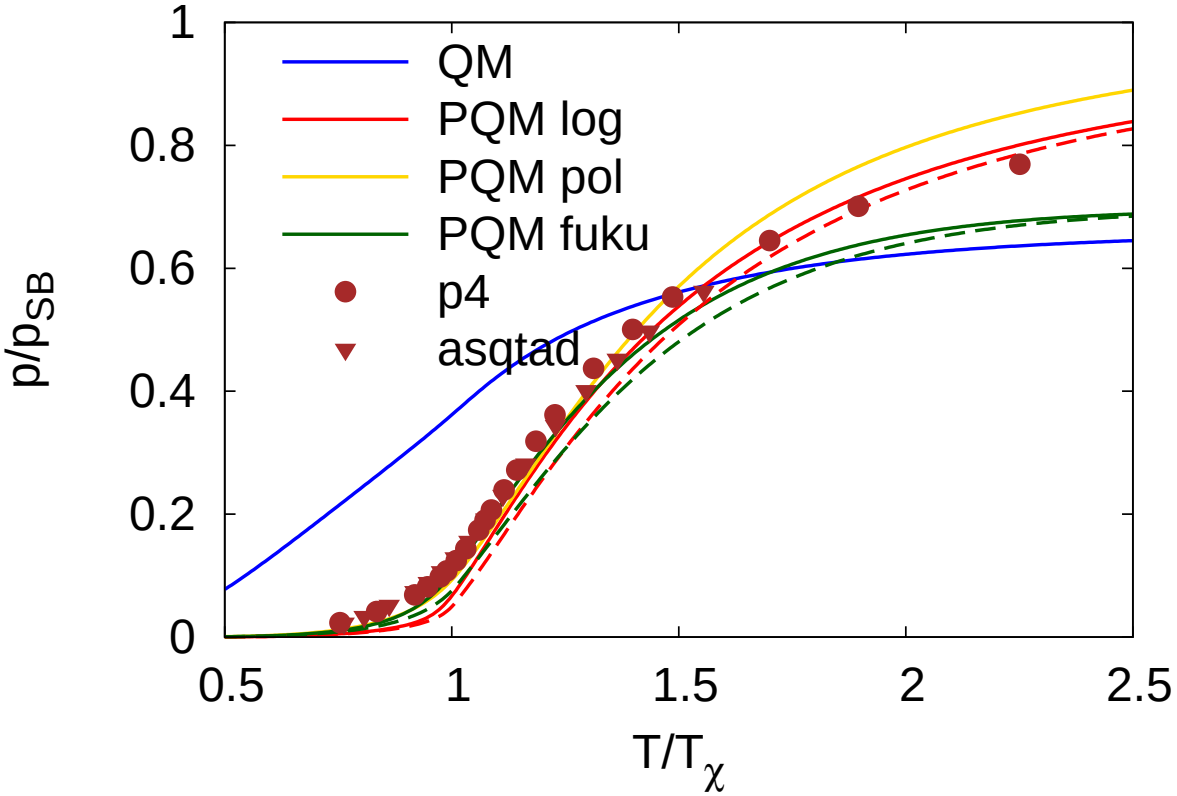


QCD Thermodynamics $N_f = 2 + 1$

[BJS, M. Wagner, J. Wambach; in preparation '09]

SB limit: $\frac{p_{\rm SB}}{T^4} = 2(N_c^2 - 1)\frac{\pi^2}{90} + N_f N_c \frac{7\pi^2}{180}$

model versus QCD lattice simulations



- dashed lines:
  PQM with lattice masses
  $m_\pi \sim 220, m_K \sim 503$ MeV
- solid lines:
  (P)QM with realistic masses

lattice data: [Bazavov et al. '09]

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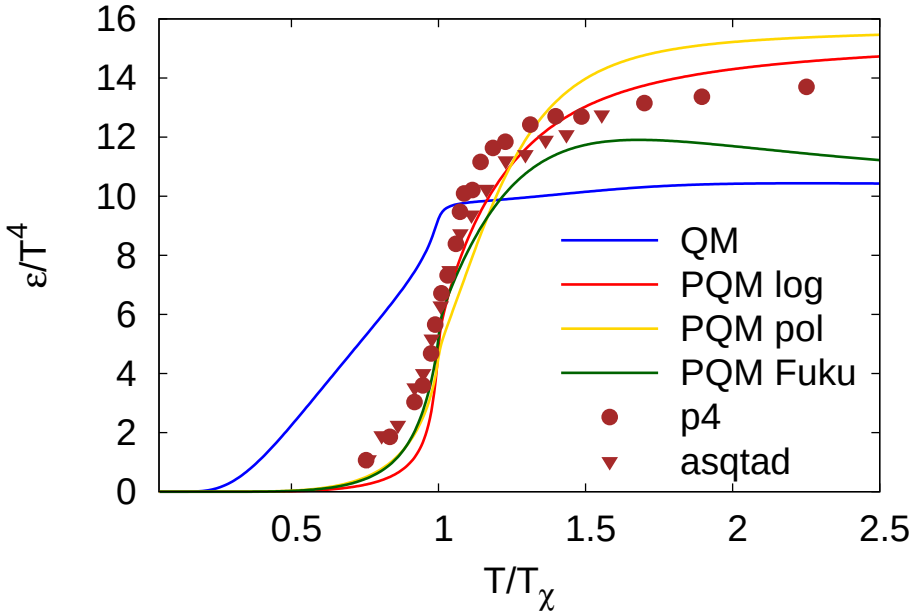

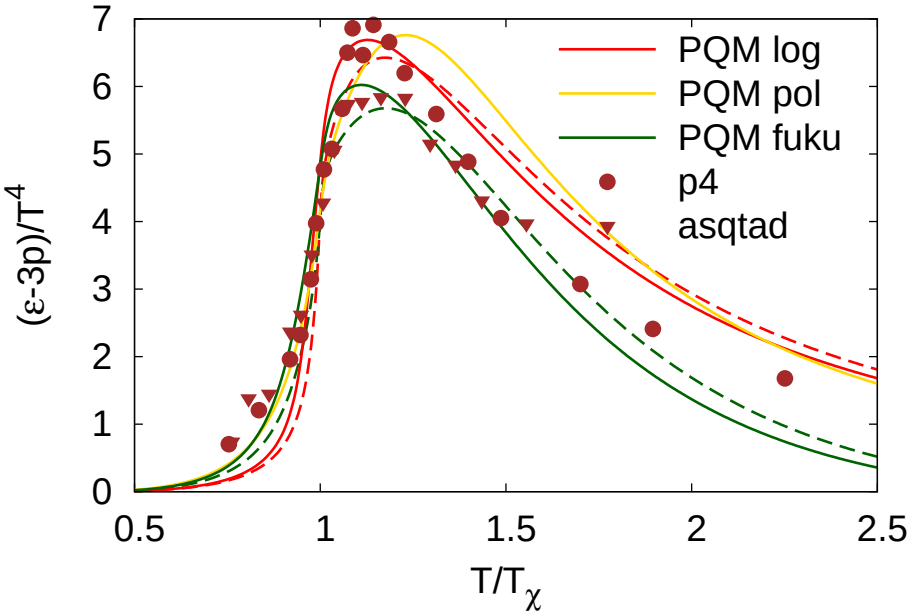

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[Bazavov et al. '09]

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Finite density extrapolations $N_f = 2 + 1$

[F. Karsch, BJS, M. Wagner, J. Wambach; in preparation '09]

Taylor expansion:

$$\frac{p(T,\mu)}{T^4} = \sum_{n=0}^{\infty} c_n(T) \left(\frac{\mu}{T}\right)^n \quad \text{with} \quad c_n(T) = \frac{1}{n!} \frac{\partial^n \left(p(T,\mu)/T^4\right)}{\partial \left(\mu/T\right)^n}\bigg|_{\mu=0}$$

high temperature limits:

$$\begin{aligned}
c_0(T \to \infty) &= \frac{7 N_c N_f \pi^2}{180} \ , \\
c_2(T \to \infty) &= \frac{N_c N_f}{6} \ , \\
c_4(T \to \infty) &= \frac{N_c N_f}{12\pi^2} \\
c_n(T \to \infty) &= 0 \text{ for } n > 4.
\end{aligned}$$

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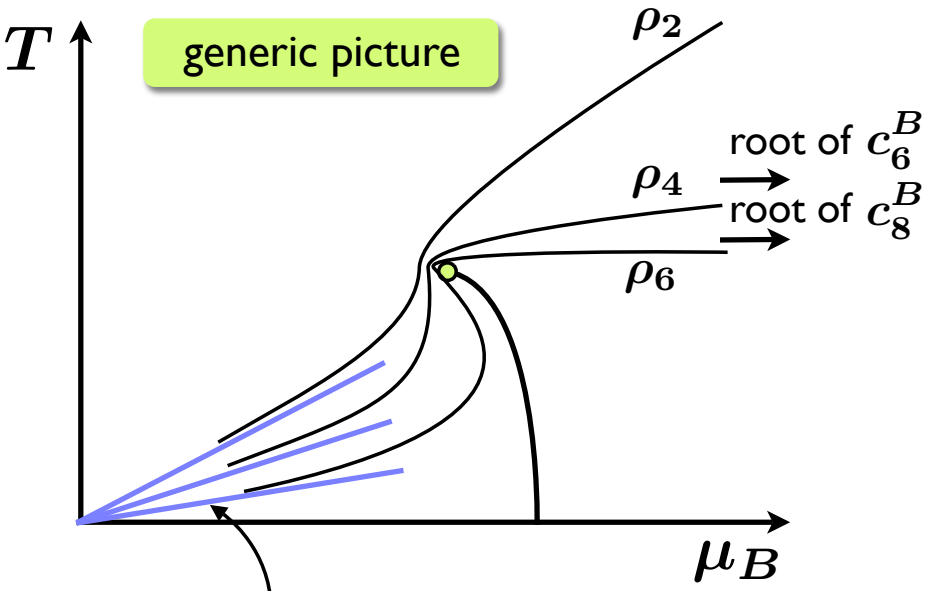

[C. Schmidt '08]

convergence radii:

limited by first-order line?

$$\rho_{2n} = \left|\frac{c_2}{c_{2n}}\right|^{1/(2n-2)}$$

or

$$r_{2n} = \left|\frac{c_{2n}}{c_{2n+2}}\right|^{1/2}$$

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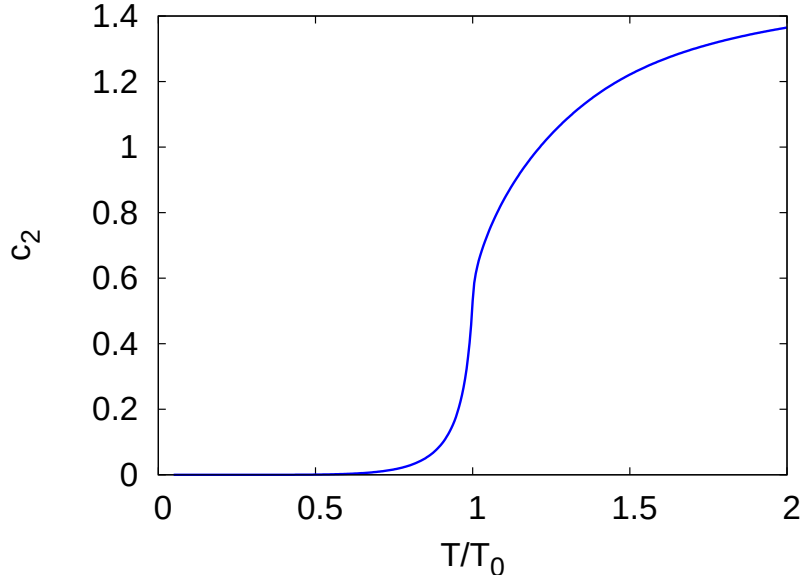
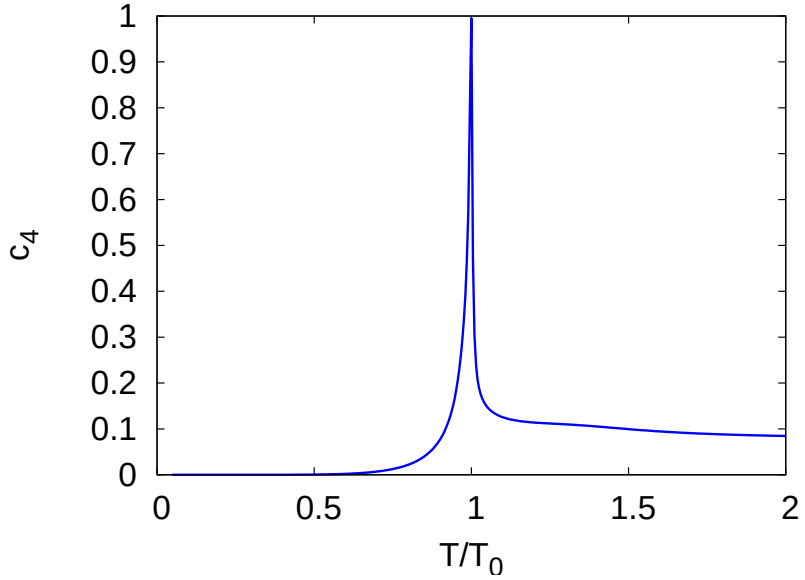
first three coefficients:

c_0: pressure at $\mu = 0$





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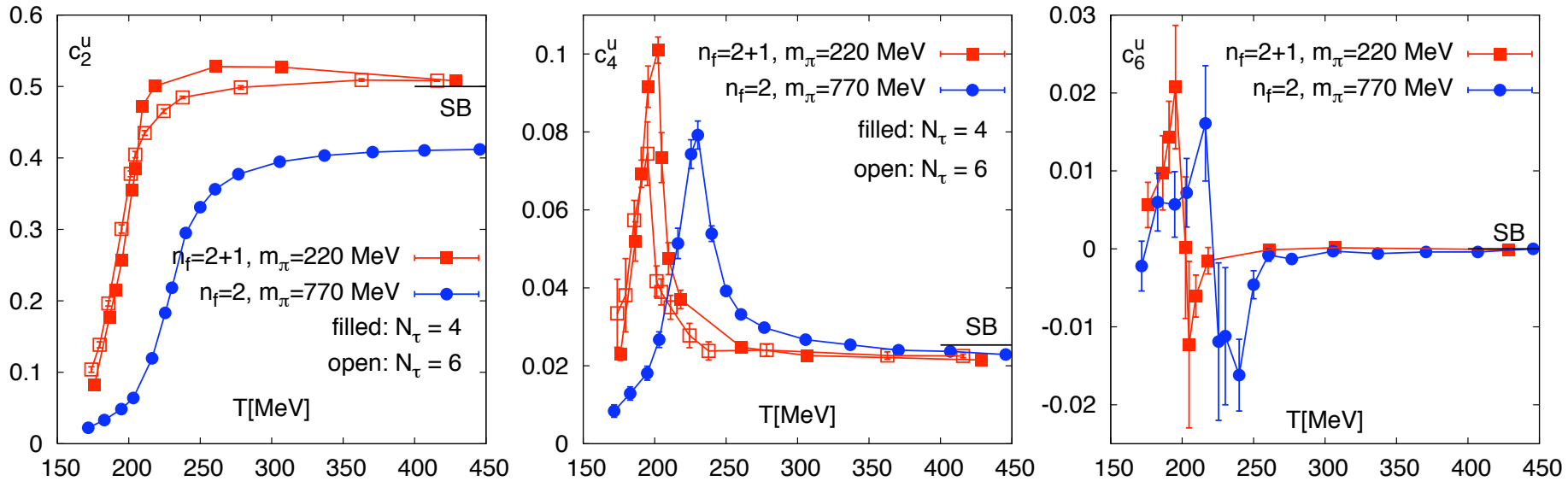
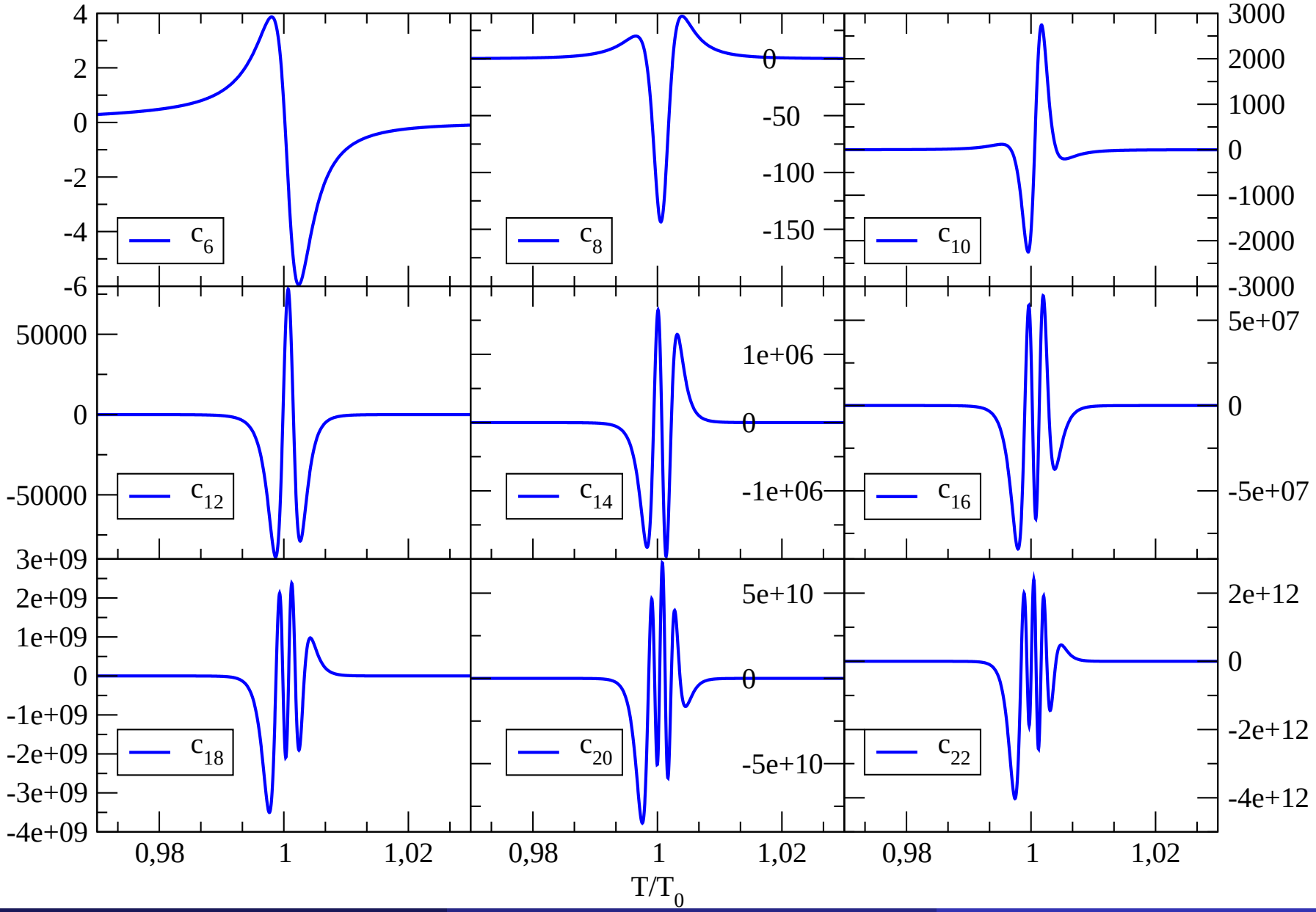


[Miao et al. '08]

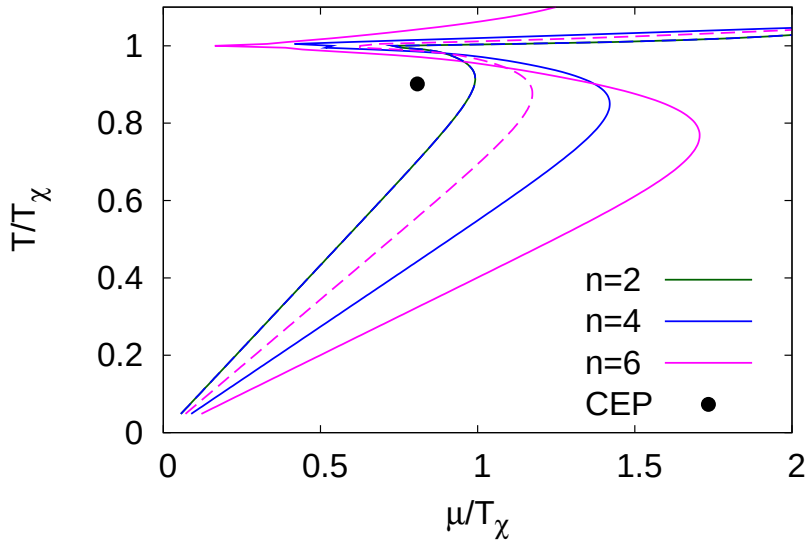
novel AD technique: [M. Wagner, A. Walther, BJS; in preparation '09]

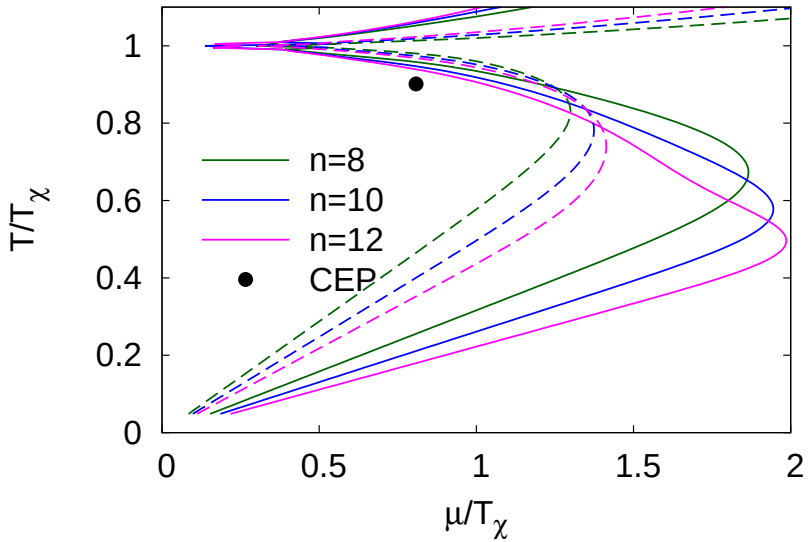


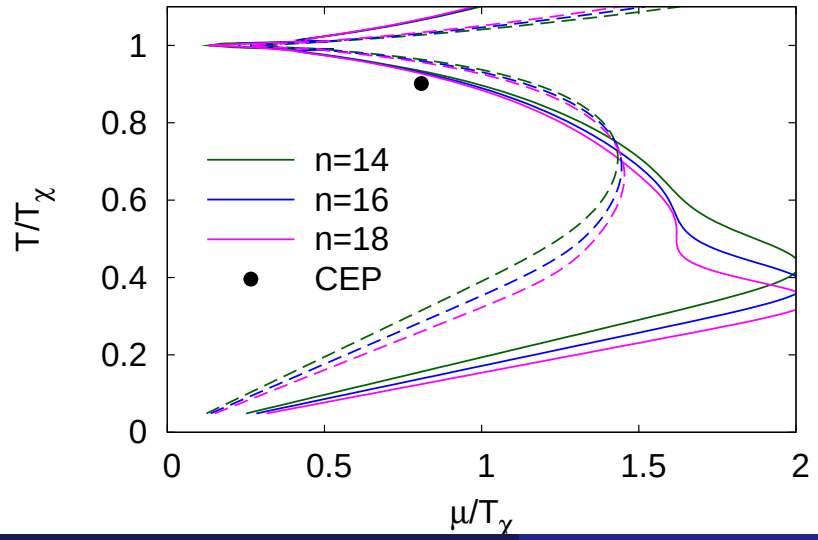
Convergence radius PQM

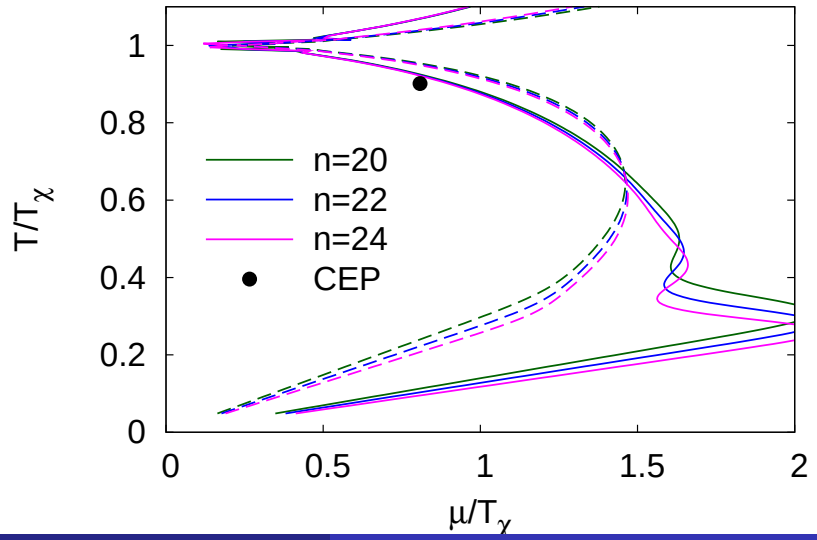
r_n : solid lines

ρ_n : dashed lines

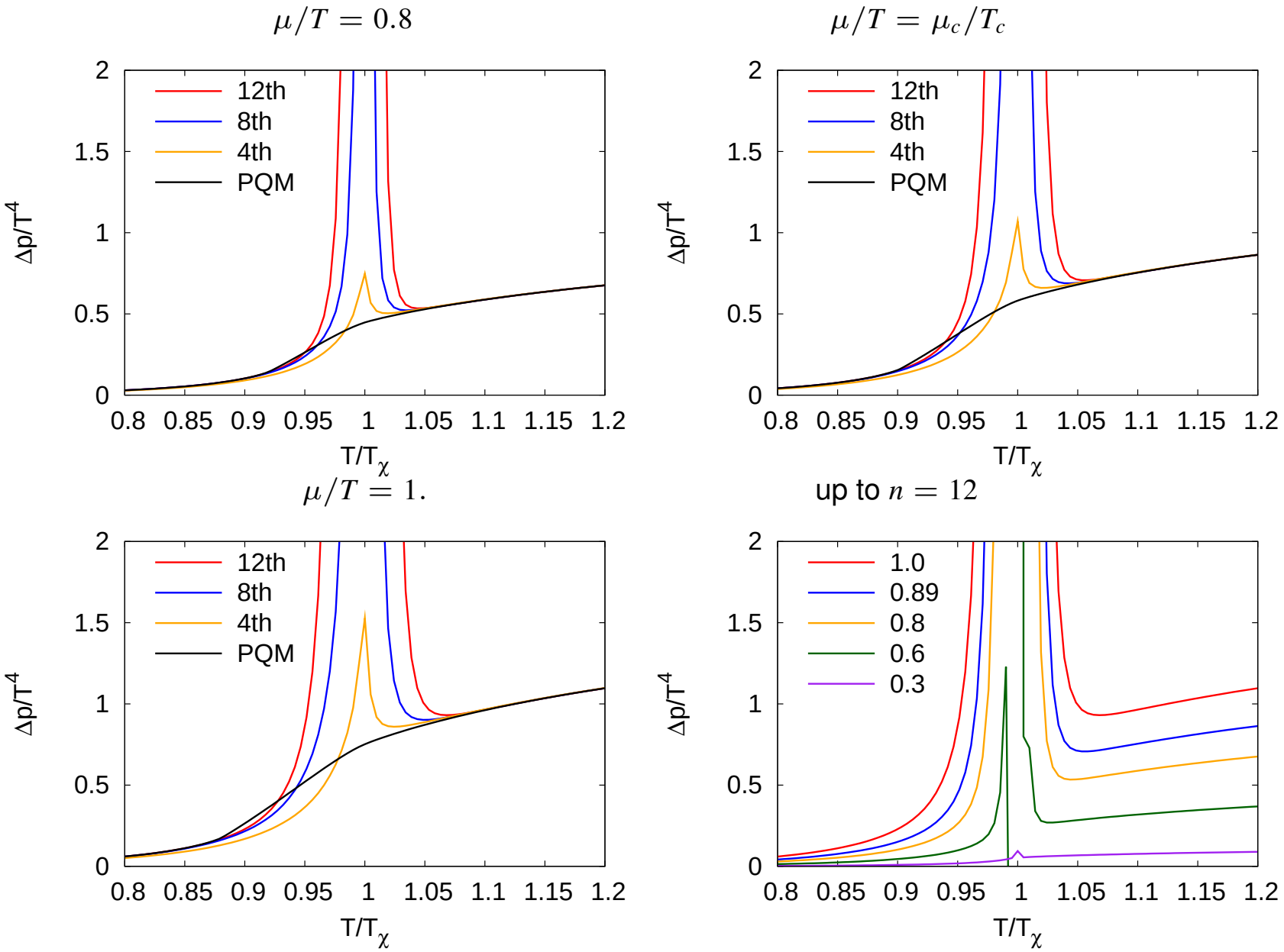




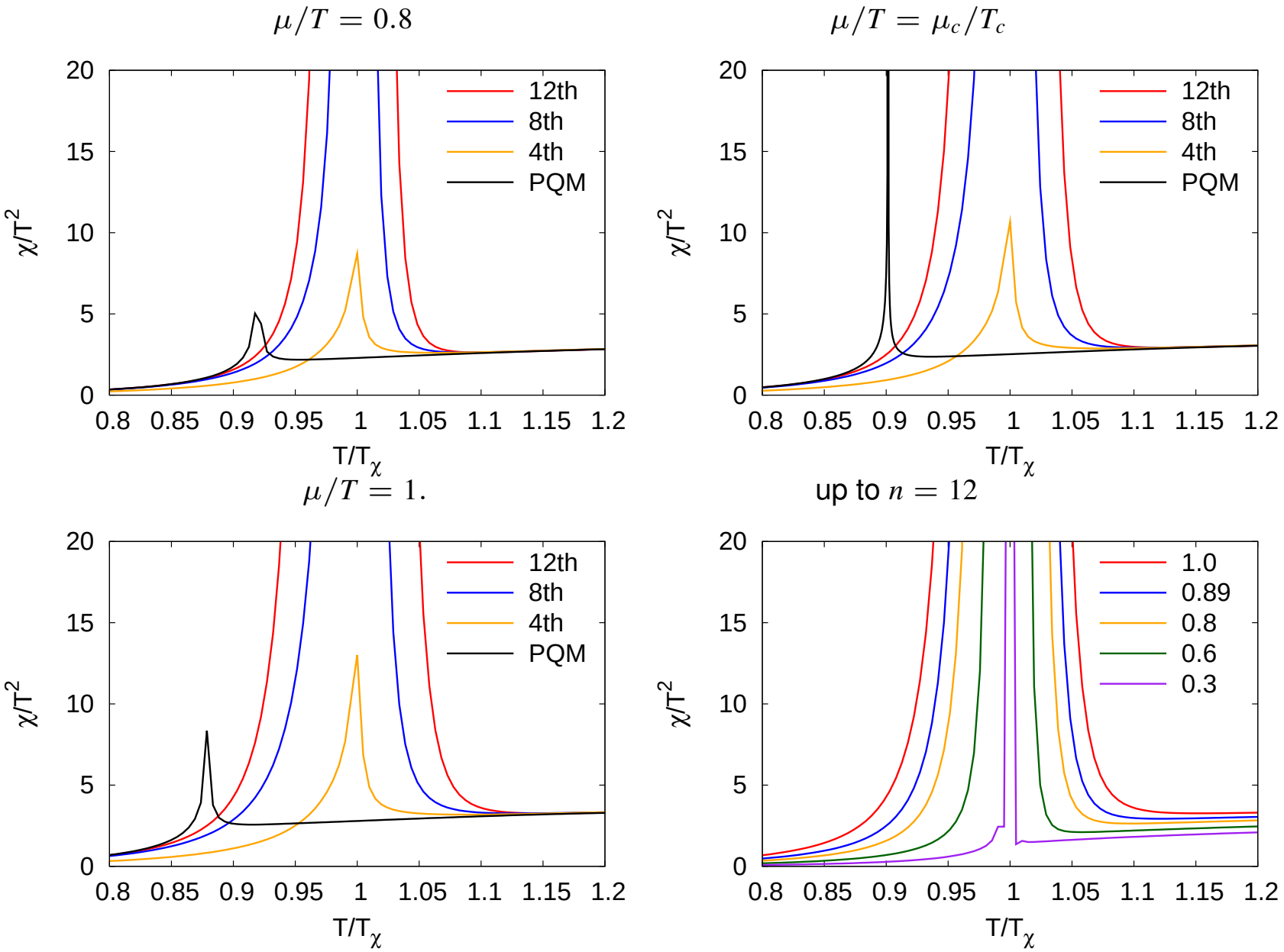




Pressure $N_f = 2 + 1$ PQM



Susceptibility $N_f = 2 + 1$ PQM



Summary

- $N_F = 3$ chiral (Polyakov)-quark-meson model study

  ➜ Mean-field approximation

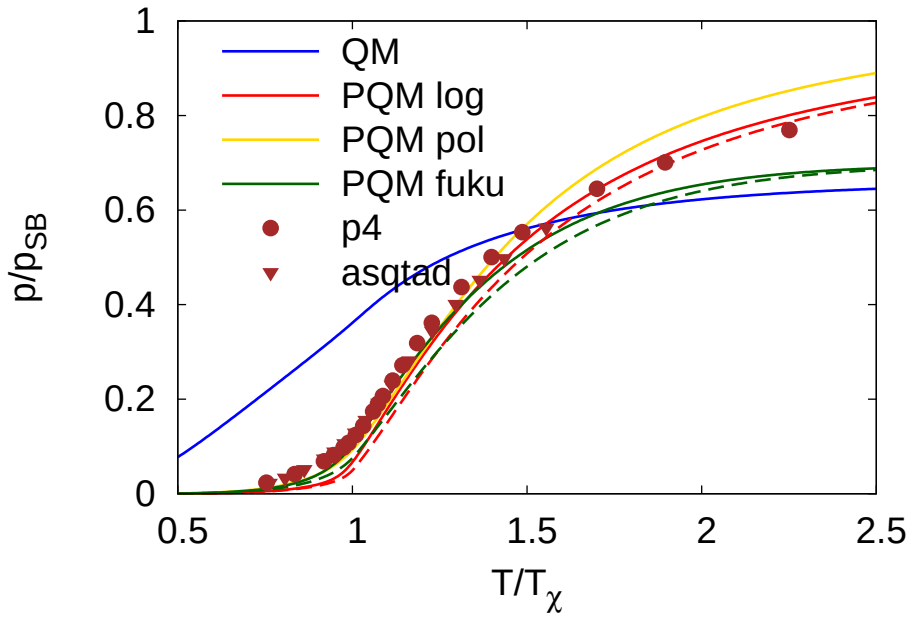
  with and without axial anomaly

- novel AD technique: high order Taylor coefficients, here: $c_{n=24}(T)$

Findings:

- ▷ Parameter in Polyakov loop potential: $T_0 \Rightarrow T_0(N_f, \mu)$
- ▷ Chiral & deconfinement transition coincide
- ▷ Mean-field approximation encouraging

  Quark-meson model is renormalizable
  $\longrightarrow$ no UV cutoff parameter (cf. PNJL model)
- ▷ Taylorcoefficient $c_n(T) \longrightarrow$ high order
- ▷ useful to develop general arguments to determine CEP location



Outlook:

- include Polyakov loop dynamics in PQM model with FRG
- include glue dynamics with FRG $\longrightarrow$ full QCD (step by step)