MSSM renormalization and radiative corrections

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J.G., W. Hollik, Joan Solà, JHEP **0210** (2002) 040 [hep-ph/0207364];

J. A. Coarasa, D. Garcia, J. G., R. A. Jiménez and J. Solà, Eur. Phys. J. C **2** (1998) 373 [hep-ph/9607485];

J.G. hep-ph/9906517

Outline

- Introduction
- The problem
  - Mixing renormalization
- Squark sector
- Chargino/neutralino sector
- SUSY couplings
- Comments on the Higgs sector/couplings
- Summary

Introduction

Low Energy Supersymmetry

- Supersymmetry (SUSY) is a *symmetry* that relates bosons with fermions.
- Its introduction solves several theoretical *problems*:
  - The hierarchy problem.
  - Maximal S-matrix symmetry
  - Dark matter
  - Supergravity
  - Radiative EW symmetry breaking
  - proton decay
  - $SU(5)$ unification ($\sin^2\theta_W$)
- Phenomenological consequence:

  Existence of *scalar partners* for fermions (sfermions), and of *fermionic partners* for gauge and Higgs bosons (charginos, neutralinos, gluino)

- Low Energy Supersymmetry: these SUSY partners have a mass around the Electroweak (EW) scale ($\lesssim 1 TeV$).
  - Existence of a light Higgs boson, $M_{h^0} \lesssim 130\,\mathrm{GeV}$.

M. Carena *et al.*, *Nucl. Phys.* **B 580** (2000) 29 [hep-ph/0001002].
J.R. Espinosa, R. Zhang, *JHEP* **0003** (2000) 026, [hep-ph/9912236].

- No direct evidence for SUSY partners has been found.
  - SUSY is (softly) broken

    $\Rightarrow$ Soft-SUSY-breaking parameters (masses).
- SUSY models predictions are not in contradiction with high precision EW data.
- If SUSY exists $\Longrightarrow$ Test SUSY at the same level as the Standard Model.

  $\Rightarrow$ Need of loop corrections (also from the EW sector!)
- Radiative effects (masses or other) will give insight to the SUSY breaking mechanism (SUGRA, AMSB, GMSB, . . .)
- Experimental side: $500 - 1000\,\mathrm{GeV}\ e^+e^-$ Linear Collider
  - $\lesssim$ few % measurements of masses, cross-sections, branching ratios, . . .

    $\Rightarrow$ translation to model parameters ($\rightarrow$ See T. Barklow talk)
- Theoretical side: model parameters
  - computation of masses, cross-sections, branching ratios, . . .

    $\Rightarrow$ at $\lesssim$ few % level!

- SUSY theories do not introduce new fundamental problems in the renormalization/computation procedure
  - BUT: they DO introduce technical problems not present (or easily avoidable) in the Standard Model.
- Regularization:
  - At one loop: Dimensional reduction (preserve SUSY).
  - Higher orders: Dimensional regularization + SUSY restoration counterterms.

W. Hollik *et al.* Nucl.Phys.B639 (2002) 3, hep-ph/0204350

- This talk:
  - One loop
  - CP-conserving MSSM

The program

1.- Write down the relevant tree-level Lagrangian

2.- Perform the renormalization procedure

- Choose the independent input parameters
- Find suitable renormalization conditions

3.- Draw and compute the tree-level diagrams

4.- Draw and compute the one-loop diagrams

- self-energies and some vertices $\Longrightarrow$ counterterms and wave function renormalization
- Process dependent diagrams

5.- Put everything in a computer program

- Reads input parameters, computes derived quantities, computes tree-level and loop

6.- Obtain numbers!

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6.- Obtain numbers!

7.- Check everything again!

The problems

- Large number of particles
- Parameters: Definitions; relations; SUSY relations
- Mixing (careful renormalization)

Large number of particles

- SUSY models have (more than) twice degrees of freedom than the SM

  ⇒ large number of diagrams

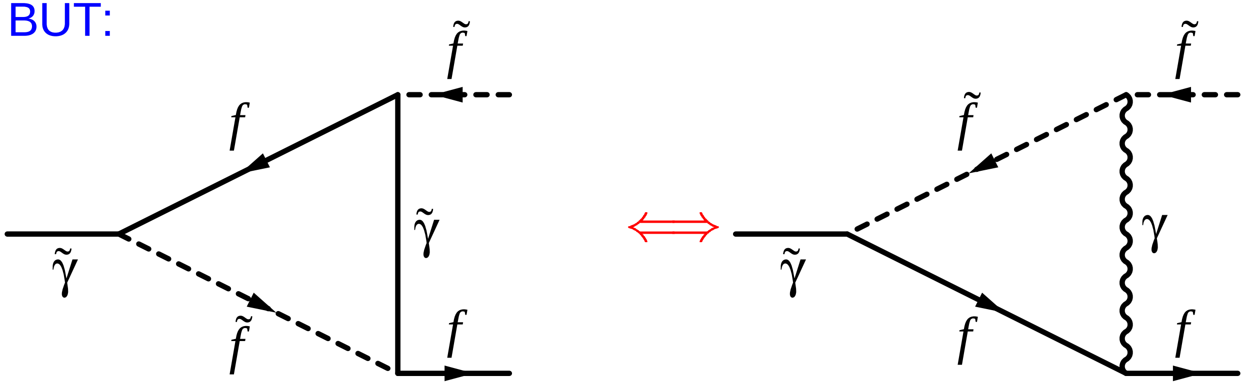
  – Whenever possible: separate particle sectors in the loops

    ∗ SM/non-SM

      · quark/lepton/gauge-Higgs/squark/slepton/charginos/neutralinos

    ∗ fermion-sfermion/Gauge-Higgs-charginos-neutralinos, . . .

    ∗ BUT:



AND: we don't have $\tilde{\gamma}$ but $\chi^0_{\{1...4\}} \equiv \tilde{\gamma}, \tilde{z}, \tilde{h}$

- Use tools!
  - FeynArts-FormCalc

$\rightarrow$ See T. Hahn talk

  - MSSM model file for FeynArts 3

T. Hahn, C. Schappacher, CPC143(2002)68, hep-ph/0105349

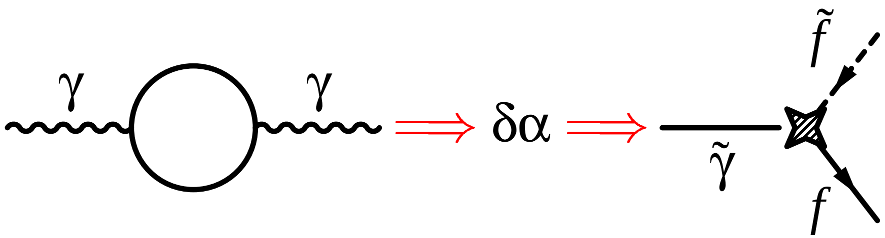
  - Any tool: Many advantages, but also drawbacks:
    - Learn to use the tool.
    - Adapt yourself to the tool or write new model file.
  - Thoroughly checking continues to be necessary!

Parameters

- Definition (renormalization scheme):
  - New parameters $\Rightarrow$ new definitions:

    $\Rightarrow$ Not obvious ($\tan\beta$?, mixing angles?, trilinear couplings?, use $\overline{MS}$?)
  - What are your input parameters?

    $\Rightarrow$ SSB parameters/physical mass, mixing angle/trilinear coupling, ...
- Relations:
  - A large number of parameter relations in different sectors:
    - $\tan\beta$, left SSB up-down sfermion masses, chargino-neutralino (μ, M), ...
    - Easy to overlook relations $\Longrightarrow$ overconstrain the system (too many ren. conditions) ($\downarrow$)
    - Small errors in computations lead to inconsistent (e.g. divergent) results ($\uparrow$)
- SUSY relations: Mix the SUSY and non-SUSY sectors in a non-trivial way (e.g. counterterms):



Mixing and Renormalization

$$\Phi_{Int} = R\Phi_{mass} \quad , \quad R^{\dagger}AR = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \; , \; R = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \; , \; A = \begin{pmatrix} a & b \\ b & c \end{pmatrix}$$

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Ren. before rotation

- Introduce counterterms in the interaction basis:

$$A^0 = A + \delta A = \begin{pmatrix} a+\delta a & b+\delta b \\ b+\delta b & c+\delta c \end{pmatrix}$$

- Perform a rotation θ'

$$\Phi^0_{Int} = R'\Phi^0_{mass} \, ,$$

$$R'^\dagger A^0 R' = \begin{pmatrix} m_1+\delta m_1 & \delta m_{12} \\ \delta m_{21} & m_2+\delta m_2 \end{pmatrix}$$

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Ren. after rotation

- Perform the rotation using bare fields

$$\Phi^0_{Int} = R^0\Phi^0_{mass} \; , \; R^{\dagger 0} A^0 R^0 = \begin{pmatrix} m_1^0 & 0 \\ 0 & m_2^0 \end{pmatrix}$$

- Introduce counterterms in mass basis

$$\theta^0 = \theta + \delta\theta \; , \; m_1^0 = m_1 + \delta m_1 \; , \; m_2^0 = m_2 + \delta m_2$$

- Fix conveniently the counterterms, e.g.

$$\delta\theta = \frac{1}{2}\frac{\Sigma^{12}(m_2) + \Sigma^{21}(m_1)}{m_2 - m_1}$$

- Compute the counterterms of the interaction basis, according to

$$A^0 = R^0 \begin{pmatrix} m_1^0 & 0 \\ 0 & m_2^0 \end{pmatrix} R^{\dagger 0}$$

- Two different renormalization schemes

Renormalization before rotation	Renormalization after rotation
Always possible $\uparrow$ Gauge invariant $\uparrow$ Complicated expression $\downarrow$	Not always possible $\downarrow$ Not gauge invariant $\downarrow$ Simpler $\uparrow$

  – Equivalent (up to higher order effects)

  – Different Meaning of the parameters

- Still need of wave function mixing renormalization!

$\Rightarrow$ Introduce mixing wave function renormalization counterterms in the Lagrangian

$$\Phi_i^0 = (1 + \frac{1}{2}\delta Z_i)\Phi_i + \delta Z_{ij}\Phi_j \quad (i \neq j)$$

$\Rightarrow$ OR include external wave function constants in the amplitude

$$T^{\mathrm{one-loop}}(\Phi_i) = T^{\mathrm{1PI}}(\Phi_i) + T^{\mathrm{counter}}(\Phi_i) + \mathcal{Z}_{ij}T^{\mathrm{tree}}(\Phi_j)$$

Squark renormalization

Squark renormalization

- We will perform: renormalization after rotation with wave function renormalization counterterms.

$$\mathcal{M}^2_{\tilde q} = \begin{pmatrix} {M_{\tilde q_L}}^2 + m_q^2 + c_{2\beta}(T_3^{q_L} - Q_q s_W^2) M_Z^2 & m_q M_{LR}^q \\ m_q M_{LR}^q & {M_{\tilde q_R}}^2 + m_q^2 + c_{2\beta} Q_q s_W^2 M_Z^2 \end{pmatrix} \Rightarrow \begin{pmatrix} M_{11}^2 & M_{12}^2 \\ M_{12}^2 & M_{22}^2 \end{pmatrix}$$

$$M_{LR}^u = A_u - \mu \cot\beta \, , \; M_{LR}^d = A_d - \mu \tan\beta \, .$$

- For each $\tilde q_{(1,2)}$ there are three input parameters: $\boxed{m_{\tilde q_1} \, , \, m_{\tilde q_2} \, , \, \theta_{\tilde q}}$

- Trilinear couplings: $A_b = \mu \tan\beta + \frac{m^2_{\tilde b_2} - m^2_{\tilde b_1}}{2\, m_b} \sin 2\theta_{\tilde b} \, ; \quad A_t = \mu \cot\beta + \frac{m^2_{\tilde t_2} - m^2_{\tilde t_1}}{2\, m_t} \sin 2\theta_{\tilde t} \, ,$

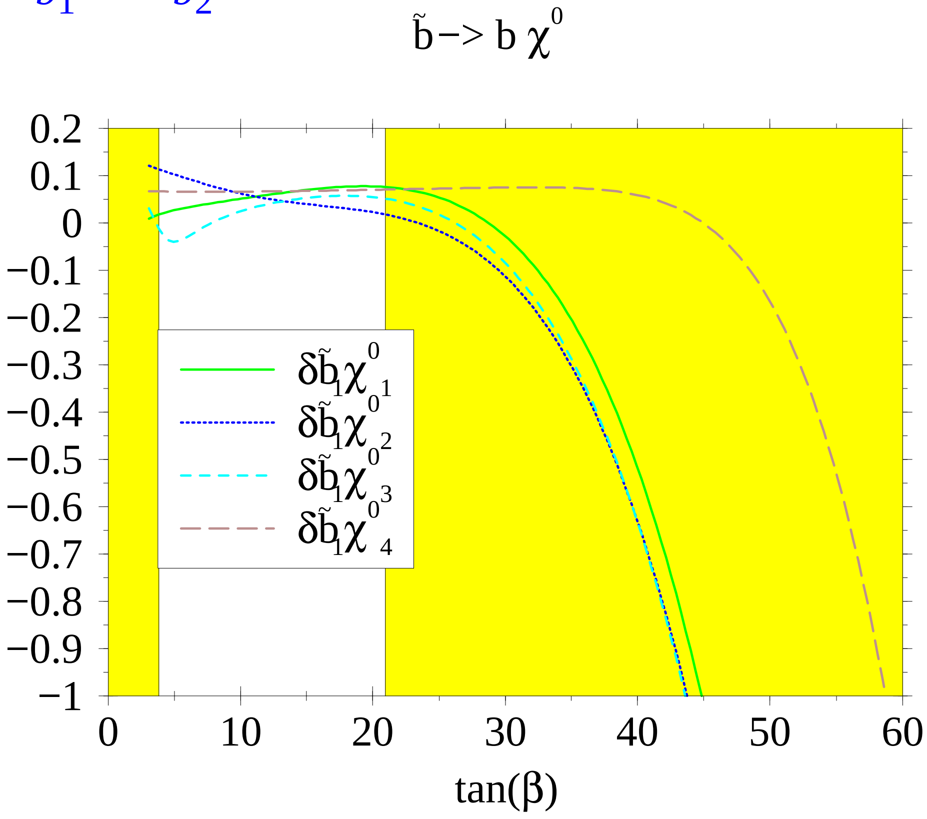
- $A_{\tilde q} \lesssim 3 m_{\tilde q}$: non-existence of charge and color breaking vacua.

  $\Rightarrow$ Huge correlation $\theta_{\tilde b} - \tan\beta - \mu$!!

Problems with perturbativity ?

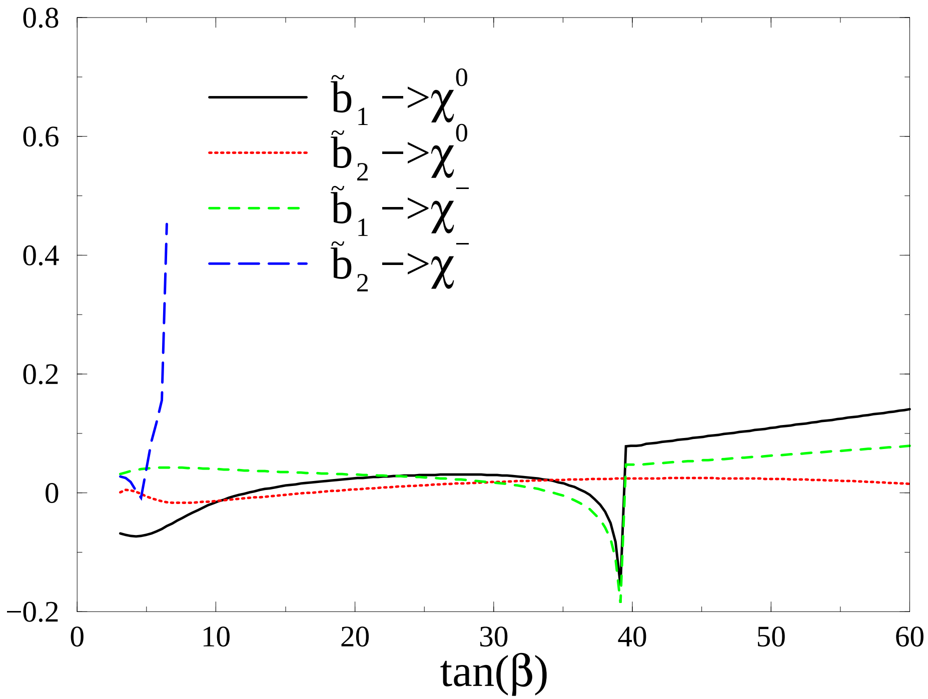
$\boxed{\Gamma(\tilde{b} \to \chi^0 b) \;\; ; \;\; \Gamma(\tilde{b} \to \chi^- t)}$: EW corrections ($\delta\Gamma/\Gamma$)

$m_{\tilde{b}_1} - m_{\tilde{b}_2} = 5\,\mathrm{GeV}$:



- No problems if A_q is maintained fixed

$\Rightarrow$ Use Soft-SUSY-breaking parameters as input!



Wave function and mixing angle

- Asymmetric wave function counterterms:

$$\tilde{q}^0_a = (1+\frac{1}{2}\delta Z_a)\tilde{q}_a + \delta Z_{ab}\tilde{q}_b \quad (a \neq b)$$

- Inverse propagator:

$$\boxed{\begin{aligned} \Delta^{-1}_{11}(k^2) &= (k^2 - m^2_{\tilde{b}_1})(1+\delta Z^1) - \delta m_{\tilde{b}_1} - \Sigma^{11}(k^2)\,, \\ \Delta^{-1}_{12}(k^2) &= (\delta Z^{21} + \delta Z^{12})\,k^2 - m^2_{\tilde{b}_2}\,\delta Z^{21} - m^2_{\tilde{b}_1}\,\delta Z^{12} - \Sigma^{12}(k^2)\,, \end{aligned}}$$

- On-Shell conditions $\oplus \Delta^{-1}_{ab} = 0 \Longrightarrow \boxed{\delta Z^{ab} = \frac{\Sigma^{ab}(m^2_{\tilde{b}_b})}{m^2_{\tilde{b}_b} - m^2_{\tilde{b}_a}}\,.}$

  δZ^{ab} represents the running mixing angle at the squark pole mass scale.

- Mixing angle: No canonical definition
  - $\delta\theta_{\tilde{q}}(Q^2) = \frac{\Sigma^{12}(Q^2)}{m^2_{\tilde{q}_2} - m^2_{\tilde{q}_1}}$
  - $\delta\theta_{\tilde{q}} = \frac{1}{2}\frac{\Sigma^{12}(m_{\tilde{q}_2}) + \Sigma^{21}(m_{\tilde{q}_1})}{m^2_{\tilde{q}_2} - m^2_{\tilde{q}_1}}$

Mass corrections

- Sfermion doublet: Same $M_{\tilde{q}_L}$
  - Translate sbottom pole mass to EW basis:

$$(\mathcal{M}^2_{\tilde{f}})^{(0)} = R^{(f)(0)} \begin{pmatrix} (m^2_{\tilde{f}_1})^{(0)} & 0 \\ 0 & (m^2_{\tilde{f}_2})^{(0)} \end{pmatrix} (R^{(f)(0)})^{\dagger} \, .$$

  - Compute Soft-SUSY-breaking counterterm:

$$\delta M^2_{\tilde{b}_L} = \delta(\mathcal{M}^2_{\tilde{b}})_{11} - 2m_b \delta m_b - c_{2\beta}(T^b_3 - Q^b s^2_W)\delta M^2_Z - M^2_Z \left(\delta c_{2\beta}(T^b_3 - Q^b s^2_W) - c_{2\beta} Q^b \delta s^2_W \right) \, ,$$

  - Compute stop mass counterterm in EW basis:

$$\delta(\mathcal{M}^2_{\tilde{t}})_{11} = \delta M^2_{\tilde{b}_L} + 2m_t \delta m_t + c_{2\beta}(T^t_3 - Q^t s^2_W)\delta M^2_Z + M^2_Z \left(\delta c_{2\beta}(T^t_3 - Q^t s^2_W) - c_{2\beta} Q^t \delta s^2_W \right) \, .$$

  - Compute physical stop mass counterterm:

$$\delta m^2_{\tilde{t}_1} = \frac{1}{(R^{(t)}_{11})^2} \left(\delta(\mathcal{M}^2_{\tilde{t}})_{11} - (R^{(t)}_{21})^2 \delta m^2_{\tilde{t}_2} - 2m^2_{\tilde{t}_1} R^{(t)}_{11} \delta R^{(t)}_{11} - 2m^2_{\tilde{t}_2} R^{(t)}_{12} \delta R^{(t)}_{12} \right) \, .$$

  - Mass correction:

$$(m^2_{\tilde{t}_1})^{\mathrm{os}} = (m^2_{\tilde{t}_1})^{tree} + \delta m^2_{\tilde{t}_1} + \Sigma_{\tilde{t}_1}(m^2_{\tilde{t}_1}) \, .$$

  - $\Delta m^{QCD}_{\tilde{q}_1} \simeq 0.1\% - 5\%$, $\Delta m^{EW}_{\tilde{q}_1} \simeq 0.01\% - 0.5\%$

Chargino/neutralino sector

Chargino/neutralino sector

- We will perform: renormalization before rotation with wave function renormalization constants.

$$\mathcal{M} = \begin{pmatrix} M & \sqrt{2}M_W s_\beta \\ \sqrt{2}M_W c_\beta & \mu \end{pmatrix}, \mathcal{M}^0 = \begin{pmatrix} M' & 0 & M_Z c_\beta s_W & -M_Z s_\beta s_W \\ 0 & M & -M_Z c_\beta c_W & M_Z s_\beta c_W \\ M_Z c_\beta s_W & -M_Z c_\beta c_W & 0 & -\mu \\ -M_Z s_\beta s_W & M_Z s_\beta c_W & -\mu & 0 \end{pmatrix}.$$

- convenient to use positive definite masses:

$$\begin{aligned} U^* \mathcal{M} V^\dagger &= \mathcal{M}_D = \mathrm{diag}\,(M_1, M_2)\ (0 < M_1 < M_2)\,, \\ N^* \mathcal{M}^0 N^\dagger &= \mathcal{M}_D^0 = \mathrm{diag}\left(M_1^0, M_2^0, M_3^0, M_4^0\right)\ (0 < M_1^0 < M_2^0 < M_3^0 < M_4^0)\,. \end{aligned}$$

- 3 parameters (M, M', μ) and 6 particles.

$\Rightarrow$ It is not possible to build on-shell conditions for all of them.

1. Choose 3 “input” particles $(\chi^\pm_{1,2}, \chi^0_1)$
2. Ask for on-shell conditions for them

3. Invert the relations, and find counterterms for the original parameters (M, M', μ).

4. Compute the mass-counterterms for the other particles *and* the mixing self-energies.

   – it is non-trivial to check the finiteness of the computed masses.

- Ex: Chargino sector

  M_1, M_2: Chargino Masses

$$\boxed{\begin{aligned} M_1\,\delta M_1 + M_2\,\delta M_2 &= M\,\delta M + \mu\,\delta\mu + \delta M_W^2 \\ M_1 M_2\,(M_1\,\delta M_2 + M_2\,\delta M_1) &= \left(M\mu - M_W^2 \sin(2\beta)\right)[M\,\delta\mu + \mu\,\delta M \\ &\quad - M_W^2\,\delta\sin(2\beta) - \sin(2\beta)\,\delta M_W^2] \end{aligned}}$$

  On Shell condition:

$$\delta M_i = -\frac{1}{2}\left(\Sigma_L^i(M_i^2) + \Sigma_R^i(M_i^2)\right) - \Sigma_S^i(M_i^2)\,,$$

- neutralino mass corrections: $0.1\% - 2 - 3\%$

Other Approaches:

H. Eberl, M. Kincel, W. Majerotto, Y. Yamada , hep-ph/0104109, Phys.Rev.D64:115013,2001

W. Hollik, T. Fritzsche, hep-ph/0203159,Eur.Phys.J.C24:619-629,2002

Wave function

- Introduce usual fermion wave function counterterms, fixed by on-shell conditions:

$$(\chi_s)^{(0)} \equiv \chi_s + \tfrac{1}{2}\delta Z_L^s P_L \chi_s + \tfrac{1}{2}\delta Z_R^s P_R \chi_s \quad , \quad \chi_s \equiv \chi_i^\pm, \chi_\alpha^0 \,.$$

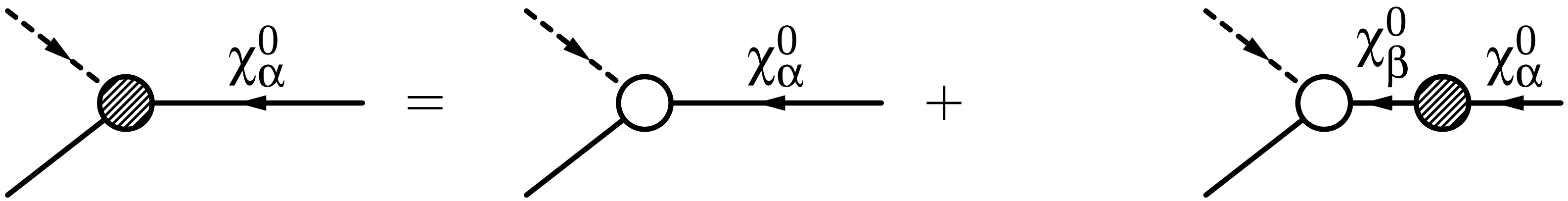
- The mixing self-energies are non-diagonal:

$$-i\hat{\Sigma}^{\alpha\beta}(k^2) = -i\left(\hat{\Sigma}_L^{\alpha\beta}(k^2) k P_L + \hat{\Sigma}_R^{\alpha\beta}(k^2) k P_R + \hat{\Sigma}_{SL}^{\alpha\beta}(k^2) P_L + \hat{\Sigma}_{SR}^{\alpha\beta}(k^2) P_R\right) \;,\; \alpha \neq \beta \,,$$

$$\hat{\Sigma}^{\alpha\beta}_{\{L,R\}} = \Sigma^{\alpha\beta}_{\{L,R\}} \,,\; \hat{\Sigma}^{\alpha\beta}_{SL} = \Sigma^{\alpha\beta}_{SL} - N_{\alpha\gamma}\delta\mathcal{M}^{0*}_{\gamma\lambda}N_{\beta\lambda} \,,\; \hat{\Sigma}^{\alpha\beta}_{SR} = \Sigma^{\alpha\beta}_{SR} - N^*_{\alpha\gamma}\delta\mathcal{M}^{0}_{\gamma\lambda}N^*_{\beta\lambda} \,,$$

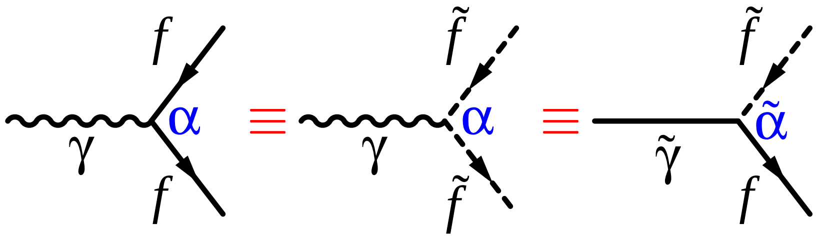
- External mixing wave function normalization:

$$T_\alpha = \bar{u}\tilde{T}_\alpha v_\alpha + \sum_{\beta\neq\alpha} \bar{u}\tilde{T}_\beta (Z_L^{0\beta\alpha} P_L + Z_R^{0\beta\alpha} P_R) v_\alpha \,,$$

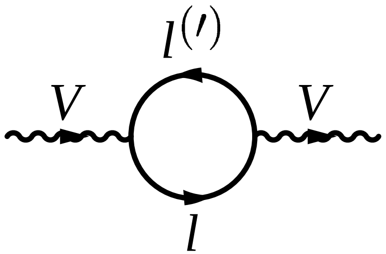


$$Z_R^{0\beta\alpha} = \frac{M^0_\beta \hat{\Sigma}^{\beta\alpha}_{SL}({M^0_\alpha}^2) + M^0_\alpha \hat{\Sigma}^{\beta\alpha}_{SR}({M^0_\alpha}^2) + M^0_\beta M^0_\alpha \hat{\Sigma}^{\beta\alpha}_{L}({M^0_\alpha}^2) + {M^0_\alpha}^2 \hat{\Sigma}^{\beta\alpha}_{R}({M^0_\alpha}^2)}{{M^0_\alpha}^2 - {M^0_\beta}^2} \,,$$

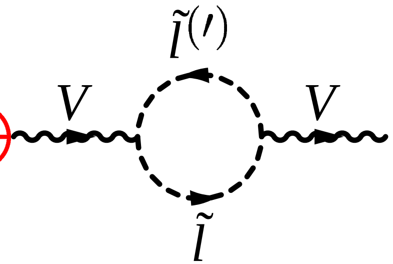
SUSY couplings



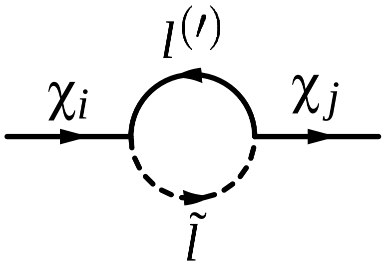
- $\delta\tilde{\alpha} \equiv \delta\alpha$ BUT $\Sigma_\gamma \neq \Sigma_{\tilde{\gamma}} \Longrightarrow$ non-decoupling effects.



$$\sim \Delta_\varepsilon - \log\left(\frac{m_l}{\mu_\varepsilon}\right) \oplus$$



$$\sim \Delta_\varepsilon - \log\left(\frac{M_{\tilde{l}}}{\mu_\varepsilon}\right) \Rightarrow \delta g\ ,\ \delta M_V^2$$



$$\sim \Delta_\varepsilon - \log\left(\frac{M_{\tilde{l}}}{\mu_\varepsilon}\right) \Rightarrow \delta Z_i\ ,\ \delta\mu,\ \delta M\ \ldots\ \Sigma_{ij}$$

- The Δ_ε must cancel between the VV and $\chi\chi$ diagrams $\Longrightarrow$ A term $\log\left(\frac{M_{\tilde{l}}}{m_l}\right)$ appears!!
- Non-decoupling of *gauginos* is known

A. Djouadi, W. Hollik, C. Jünger, *Phys. Rev.* **D55** (1997) 6975, `hep-ph/9609419`

Renormalization Group: SUSY-QED

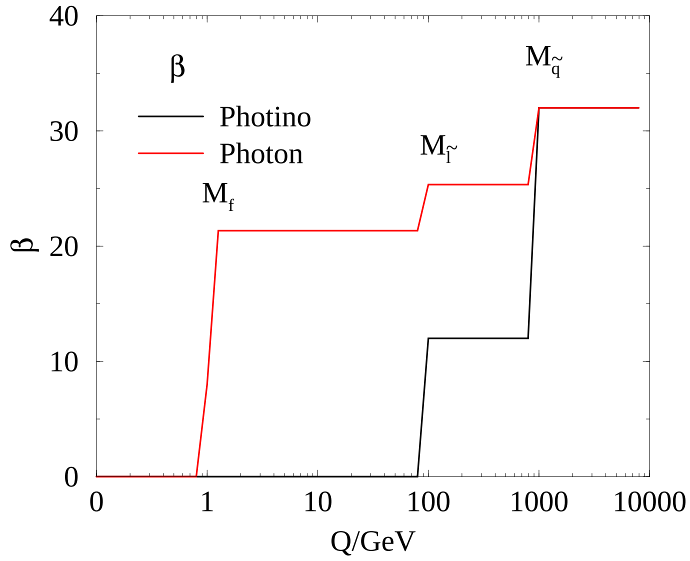
$$\boxed{e^+e^-\gamma \equiv \alpha \quad , \quad e^+\tilde{e}^-\tilde{\gamma} \equiv \tilde{\alpha}}$$

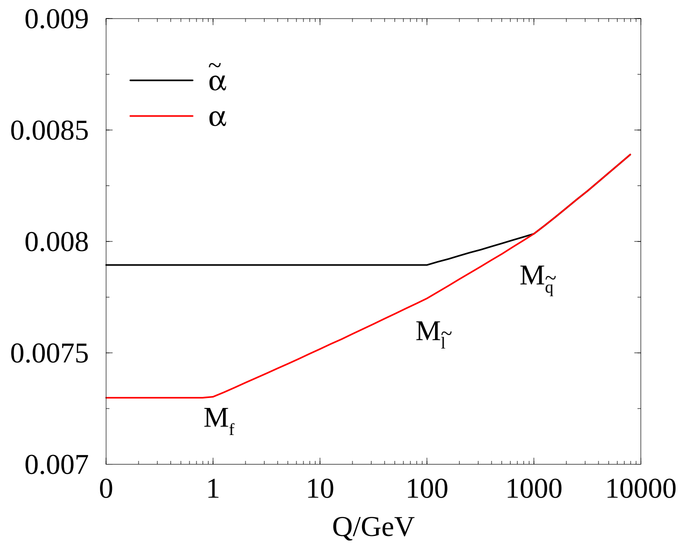
- Assume: $M_{\tilde{l}} < M_{\tilde{q}}$, take only q-$\tilde{q}$ loops into account
  - $Q > M_{\tilde{q}} \Longrightarrow \tilde{\alpha}(Q) = \alpha(Q)$
  - $Q = M_{\tilde{q}}$:
    - $\ast$ $\tilde{q}$ decouple from α
    - $\ast$ $\tilde{q}$ and q decouple from $\tilde{\alpha}$

    $\Rightarrow$ $\tilde{\alpha}$ is frozen at $M_{\tilde{q}}$
  - $Q' < M_{\tilde{q}} \Longrightarrow \tilde{\alpha}(Q') = \alpha(M_{\tilde{q}})$

    $$\Rightarrow \frac{\tilde{\alpha}(Q')}{\alpha(Q')} - 1 = \frac{\alpha(M_{\tilde{q}})}{\alpha(Q')} - 1 \propto \log\left(\frac{M_{\tilde{q}}}{Q'}\right)$$
- On-Shell renormalization:

$$\frac{\tilde{\alpha}(Q')}{\alpha(M_{Weak})} - 1 \propto \log\left(\frac{M_{\tilde{q}}}{M_{Weak}}\right)$$





MSSM EW sector: effective coupling matrices

- $\tilde{U} = U + \Delta U \ , \ \tilde{V} = V + \Delta V \ , \ \tilde{N} = N + \Delta N \ ,$

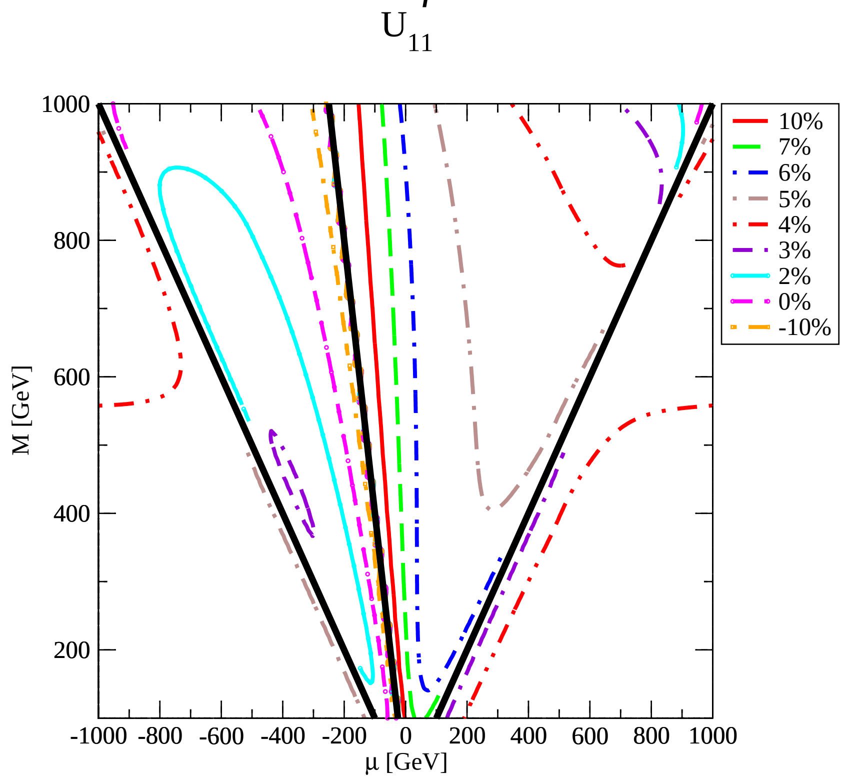
$$\Delta U_{i1} \equiv U_{i1} \left(\frac{\delta g}{g} + \frac{\delta Z_R^{-i}}{2} \right) + U_{j1} Z_R^{-ji} \, , \ \Delta U_{i2} \equiv U_{i2} \left(\frac{\delta g}{g} + \frac{\delta Z_R^{-i}}{2} - \frac{1}{2} \frac{\delta M_W^2}{M_W^2} - \frac{\delta \cos\beta}{\cos\beta} \right) + U_{j2} Z_R^{-ji} \, ,$$

- Only the fermion-sfermion contributions are finite
- Universal Corrections (*Super-oblique*) Hikasa, Nakamura ’96; Katz, Randall, Su ’98
- limit $m_e \ll M_Z \ll M_{\tilde{e}}$, heavy masses log terms:

$$\begin{aligned} \Delta U_{i1}^{(f)} &= \frac{\alpha}{4\pi s_W^2} \log\left(\frac{M_{\tilde{L}}^2}{M_W^2}\right) \left[\frac{U_{i1}^3}{6} - U_{i2} \frac{\sqrt{2} M_W \left(M c_\beta + \mu s_\beta\right)}{3\left(M^2 - \mu^2\right) \left(M_1^2 - M_2^2\right)^2} \left(M^4 - M^2 \mu^2 + 3 M^2 M_W^2 + \right.\right. \\ &+ \left.\left. \mu^2 M_W^2 + M_W^4 + M_W^4 c_{4\beta} + (\mu^2 - M^2) M_i^2 + 4 M \mu M_W^2 s_{2\beta}\right) \right] \, , \end{aligned}$$

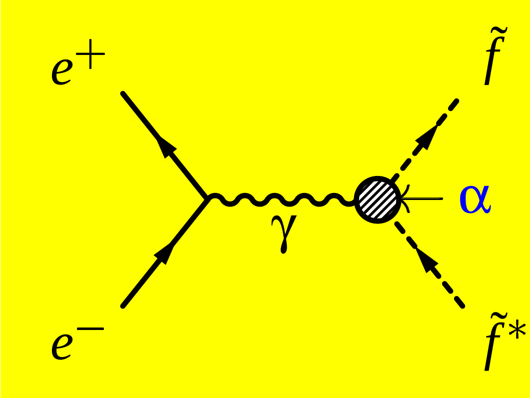
- Mismatch between
  - U-matrix that diagonalizes the chargino
  - U'-matrix that couples fermion-sfermion-chargino
- Finite shift between
  - Charge in the Compton limit ($e^+e^-\gamma$)
  - Yukawa coupling of fermion-sfermion-photino ($e^+\tilde{e}^-\tilde{\gamma}$)

- fermion-sfermion contributions to $\Delta U_{11}/U_{11}$, ($M_{\tilde{f}} = 1\,TeV$)

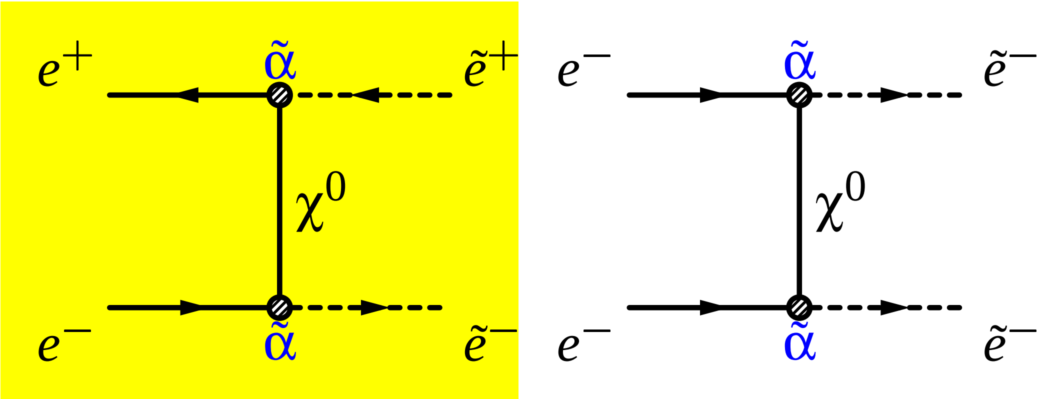


- All particles show non-decoupling effects (χ^-, χ^0, $H^\pm$)
  - They don't admit simple expressions (need to add one-particle irreducible 3-point functions to obtain a finite result).
- non-decoupling: SUSY is (softly) broken, and we are testing a SUSY relation!

- some other observables (e.g. S-channel pair production) do not test SUSY relations, and exhibit decoupling:
  Here α is the sfermion gauge coupling, and only fixed by $\tilde{f}$ quantum numbers (not by SUSY relations)



- Other observables that also test SUSY relations and exhibit the same non-decoupling effects:



- Consequences:
  - All sectors of the model MUST be taken into account ($\downarrow$)
    $\Rightarrow$ All parameters of the model MUST be given (we need $\tilde{q}$ parameters to compute $\tilde{l}$ obs.)
  - We can separate between Universal/non-Universal corrections ($\uparrow$)
    $\Rightarrow$ $\tilde{q}$ parameters are encoded in effective coupling matrices

Comments on the Higgs sector/couplings

- Comments on $\tan\beta$ renormalization: $\tan\beta$ is a mixing angle $\Longrightarrow$ Same kind of discussion

$\rightarrow$ See A. Freitas talk

- Renormalization before rotation with gauge invariant renormalization counterterms

A. Dabelstein Z.Phys.C67:495-512,1995,hep-ph/9409375.

P. Chankowski, S. Pokorski, J. Rosiek, Nucl.Phys.B423:437-496,1994, hep-ph/9303309

- Two independent parameters: $\tan\beta$, M_{A^0} $(M_{H^\pm})$

- Two independent wave function counterterms: δZ_{H_1}, δZ_{H_2}

- conditions:
  - On-shell condition for A^0 (or $H^\pm$)

    $\Rightarrow$ $\delta M_{A^0}, \delta Z_{A^0} \equiv s_\beta^2 \delta Z_{H_1} + c_\beta^2 \delta Z_{H_2}$
  - $\frac{\delta v_2}{v_2} = \frac{\delta v_1}{v1} \oplus$ no $A^0 - Z$ mixing

    $\Rightarrow$ $\frac{\delta \tan\beta}{\tan\beta} = \frac{1}{2}(\delta Z_{H_2} - \delta Z_{H_1}) = -\frac{1}{2 s_\beta c_\beta M_Z^2} \Sigma^{AZ}(M_{A^0}^2)$

    $\Rightarrow$ Ward Identity: $\hat{\Sigma}^{AG}(M_{A^0}) = 0$

- Large corrections in the Higgs/higgsino-bottom/sbottom couplings due to:
  - running bottom mass
  - SUSY threshold corrections

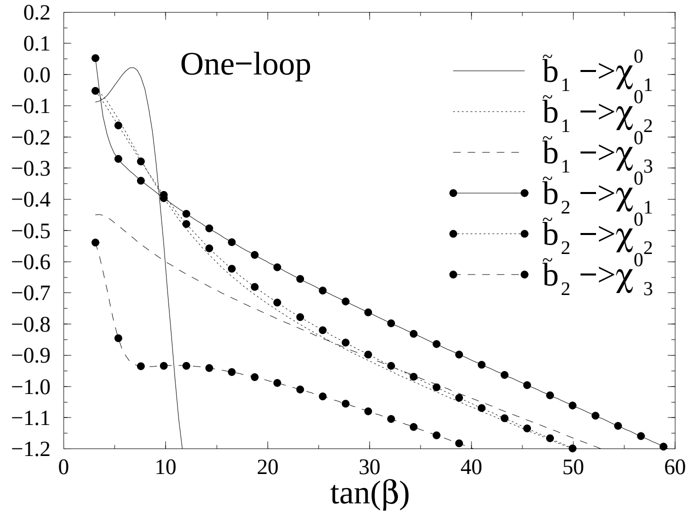
$\Longrightarrow$ Use resummed expressions:

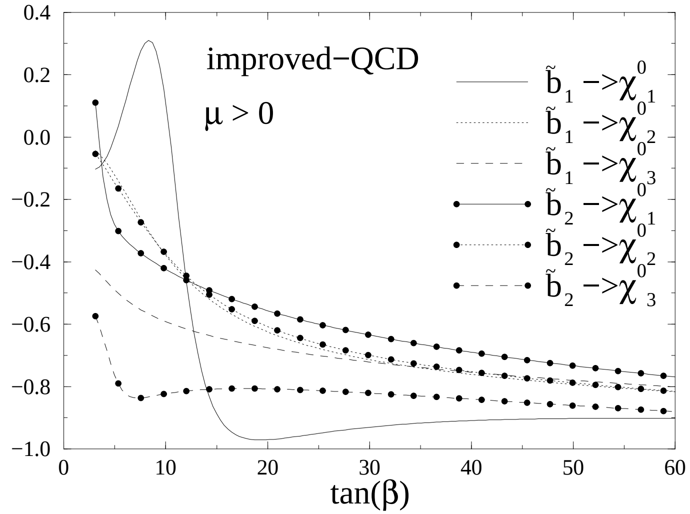
M. Carena, D. Garcia, U. Nierste, C. Wagner, NPB577 (2000) 88, hep-ph/9912516
J. Guasch, P. Häfliger, M. Spira, hep-ph/0305101

$$h_b^{eff} \equiv \frac{m_b^{eff}}{v_1} \equiv \frac{m_b(Q)}{v_1\,(1+\Delta_b)}\,,$$

$$A^{Full} = A^{Resum} + \left(A^{1-loop} - A^{(1-loop\;eff)}\right)$$

$\delta\Gamma(\tilde{b} \to \chi^0 b)/\Gamma$ $(\mu > 0)$





Summary

- SUSY loop computations introduce technical problems due to
  - Large number of particles
  - Parameter relations
  - Mixing among particles
- Different ways of performing the renormalization
  - Different meaning of the parameters
- Two explicit examples have been worked out
- Proper renormalization leads to physical insight
- Eventually: Translate to $\overline{MS}$ to compare with Model building (SUGRA, . . .)

$$\boxed{\text{Be always very systematic}}$$

$$\boxed{\text{Be always very careful}}$$

$$\boxed{\text{Check anything you can}}$$

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