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H-QM Helmholtz Research School Quark Matter Studies

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

Kinetic Derivation of Fluid Dynamics for Mixtures

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Contents

- Motivation/Introduction
- Approach
- Kinetic Corrections
- Conclusions/Perspectives

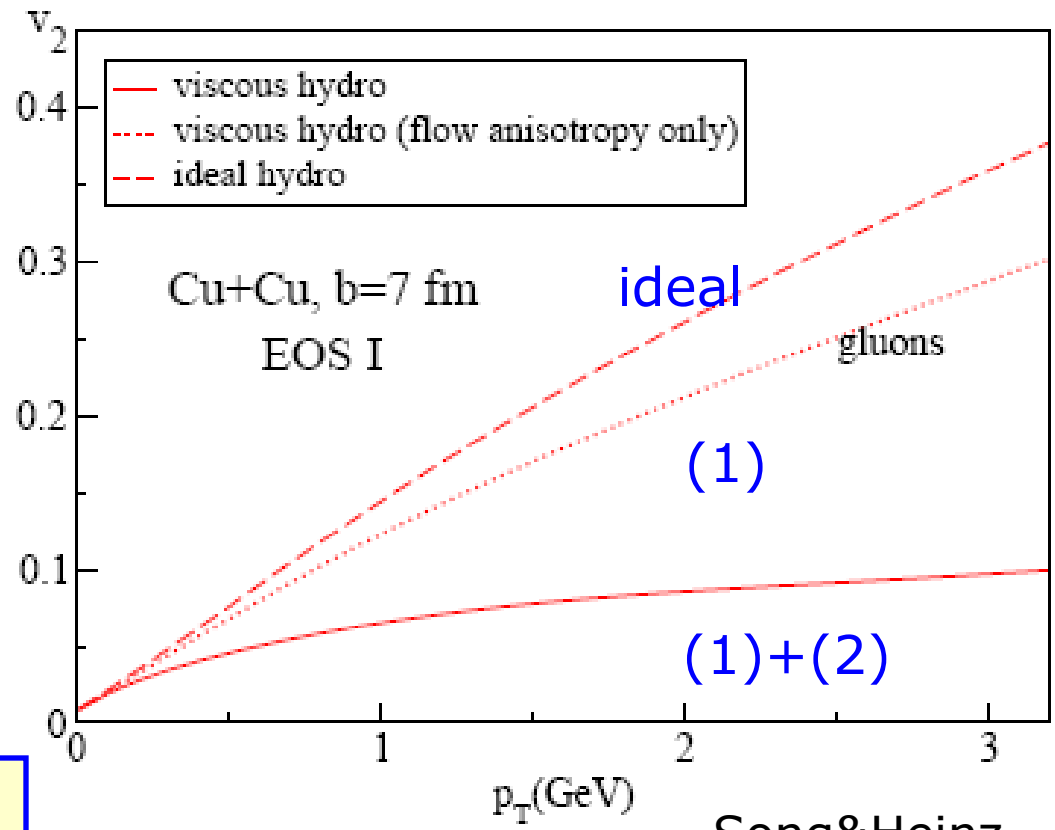
Effects of Viscosity

Two different effects :

1) Flow of the fluid

2) Momentum distribution function

$$f_{(i)} = f_{0(i)} \quad \delta f_{(i)}$$



Song&Heinz

Several Approaches

De Groot's book

- BGK

Bhatnagar, Gross and Kook (1954)
Marle (1964)

- Chapman-Enskog-Hilbert

Hilbert (1912), Chapman (1916) and Enskog (1917)
Marle (1964)

- Variational Methods

Robinson, Bernstein (1952), Van Leeuwen (1971)

- Moments Method

Maxwell (1867), Grad (1949) , Mintzner(1965)
Chernikov (1963), Stewart (1969), Marle (1966/69)
Israel&Stewart(1978)

Several Approaches

Usually applied in HIC

- Moments Method

  Maxwell (1867), Grad (1949) , Mintzner(1965)
  Chernikov (1963), Stewart (1969), Marle (1966/69)
  Israel&Stewart(1978)

Several Approaches

However, the method was constructed for a single component fluid

Usually applied in HIC

- Moments Method

  Maxwell (1867), Grad (1949) , Mintzner(1965)
  Chernikov (1963), Stewart (1969), Marle (1966/69)
  Israel&Stewart(1978)

What we investigate

- Moments Method for a mixture
- What are the differences ?

Basics

Boltzmann Equation

$$K^{\mu}_{(i)}\partial_{\mu}f_{(i)} = C\left[\{f\}\right]$$

Conserved Currents

$$T^{\mu\nu} = \sum_i T^{\mu\nu}_{(i)} \qquad N^{\mu}_{r} = \sum_i q^{r}_{(i)} N^{\mu}_{(i)}$$

Currents for each particle specie

$$T^{\mu\nu}_{(i)} = \int d\omega_{(i)} K^{\mu}_{(i)} K^{\nu}_{(i)} f_{(i)}$$

$$N^{\mu}_{(i)} = \int d\omega_{(i)} K^{\mu}_{(i)} f_{(i)}$$

$$d\omega_{(i)} \equiv \frac{d^3K}{(2\pi)^3 K^0{}_{(i)}}$$

Basics

The separation

$$K^{\mu}_{(i)} = u^{\mu} E_{(i)} + \Delta^{\mu\alpha} K_{\alpha(i)}$$

$$E_{(i)} \equiv u^{\mu} K_{\mu(i)}$$

General form of the currents

$$T^{\mu\nu}_{(i)} = \varepsilon_{(i)} u^{\mu} u^{\nu} - \Delta^{\mu\nu} (P_{0(i)} + \Pi_{(i)}) + 2 q^{(\mu}_{(i)} u^{\nu)} + \pi^{\mu\nu}_{(i)}$$

$$N^{\mu}_{(i)} = n_{(i)} u^{\mu} + \nu^{\mu}_{(i)}$$

Matching Conditions

$$\varepsilon_{(i)} = \varepsilon_{0(i)} \qquad n_{(i)} = n_{0(i)}$$

Basics

The separation

$$K^{\mu}_{(i)} = u^{\mu} E_{(i)} + \Delta^{\mu\alpha} K_{\alpha(i)}$$

$$E_{(i)} \equiv u^{\mu} K_{\mu(i)}$$

General form of the currents

$$T^{\mu\nu}_{(i)} = \varepsilon_{(i)} u^{\mu} u^{\nu} - \Delta^{\mu\nu} (P_{0(i)} + \Pi_{(i)}) + 2 q^{(\mu}_{(i)} u^{\nu)} + \pi^{\mu\nu}_{(i)}$$

$$N^{\mu}_{(i)} = n_{(i)} u^{\mu} + \nu^{\mu}_{(i)}$$

Bulk

Energy Difusion

Shear

Particle Difusion

for each specie

Irreversible Currents

$$\Pi_{(i)} = -\frac{1}{3} m_{(i)}^2 \int d\omega_{(i)} \delta f_{(i)}$$

$$\pi_{(i)}^{\mu\nu} = \int d\omega_{(i)} \Delta^{\mu\nu\alpha\beta} K_{\alpha(i)} K_{\beta(i)} \delta f_{(i)}$$

$$q_{(i)}^{\mu} = \int d\omega_{(i)} E_{(i)} \Delta^{\mu\nu} K_{\nu(i)} \delta f_{(i)}$$

$$\nu_{(i)}^{\mu} = \int d\omega_{(i)} \Delta^{\mu\nu} K_{\nu(i)} \delta f_{(i)}$$

$$\Delta^{\alpha\beta} = g^{\alpha\beta} - u^{\alpha} u^{\beta}$$

$$\Delta^{\mu\nu\alpha\beta} = \Delta^{<\mu\nu} \Delta^{\alpha\beta>}$$

$\downarrow$

Symmetric

$+$

Traceless

Local Eq. distribution function

$$f_{0(i)} = \left(\exp(\beta_0 E_{(i)} - \alpha_{0(i)}) \pm 1\right)^{-1}$$

Deviation

$$\delta f_{(i)} = f_{(i)} - f_{0(i)}$$

Total Currents

$$T^{\mu\nu} = \varepsilon u^{\mu} u^{\nu} - \Delta^{\mu\nu}(P_0 + \Pi) + 2q^{(\mu} u^{\nu)} + \pi^{\mu\nu}$$

$$N_r^{\mu} = n_r u^{\mu} + \nu_r^{\mu}$$

$$\Pi = \sum_{(i)} \Pi_{(i)}$$

$$\pi^{\mu\nu} = \sum_{(i)} \pi_{(i)}^{\mu\nu}$$

$$q^{\mu} = \sum_{(i)} q_{(i)}^{\mu}$$

$$\nu_r^{\mu} = \sum_{(i)} q_{(i)}^{r} \nu_{(i)}^{\mu}$$

$$T^{\mu\nu} = \sum_i T_{(i)}^{\mu\nu}$$

$$N_r^{\mu} = \sum_i q_{(i)}^{r} N_{(i)}^{\mu}$$

Diffusion relative to the net number density

Grad's Method (by IS)

Assume

$$f_{(i)} = \left(\exp(y_{(i)}) \pm 1\right)^{-1}$$

The following expansion is applied

$$y_{(i)} - y_{0(i)} = \varepsilon(E_{(i)}) + \varepsilon_{\mu}(E_{(i)})\Delta^{\mu\nu}K_{\nu(i)} + \varepsilon_{\mu\nu}(E_{(i)})\Delta^{\mu\nu\alpha\beta}K_{\alpha(i)}K_{\beta(i)}$$

Expand in Taylor

$$\varepsilon(E_{(i)}) = \varepsilon_{0(i)} + \varepsilon_{1(i)}E_{(i)} + \varepsilon_{2(i)}E_{(i)}^2$$

$$\varepsilon_{\mu}(E_{(i)}) = \varepsilon_{0\mu(i)} + \varepsilon_{1\mu(i)}E_{(i)}$$

$$\varepsilon_{\mu\nu}(E_{(i)}) = \varepsilon_{0\mu\nu}$$

14 Moments Approx.

(finite Bulk)

Grad's Method (by IS)

Assume

$$f_{(i)} = \left(\exp(y_{(i)}) \pm 1\right)^{-1}$$

The following expansion is applied

$$y_{(i)} - y_{0(i)} = \varepsilon(E \ldots \Lambda^{\mu\nu\alpha\beta} K_{\alpha(i)} K_{\beta(i)}$$

$$y_{(i)} - y_{0(i)} = \varepsilon_{(i)} + \varepsilon_{\mu(i)} K^{\mu}_{(i)} + \varepsilon_{\mu\nu(i)} K^{\mu}_{(i)} K^{\nu}_{(i)}$$

$$\varepsilon(E \ldots$$

$$\varepsilon_{\mu}(E_{(i)}) \ldots$$

$$\varepsilon_{\mu\nu}(E_{(i)}) = \varepsilon_{0\mu\nu}$$

prox.

Grad's Method (by IS)

The expansion variables are determined using

Matching Cond.

$$u_\mu u_\nu (T^{\mu\nu} - T_0^{\mu\nu}) = 0$$

$$u_\mu (N^\mu - N_0^\mu) = 0$$

Definition of Currents

$$\nu^\mu_{(i)} = \Delta^{\mu\nu} N_{\nu(i)}$$

$$q^\mu_{(i)} = u^\alpha \Delta^{\mu\beta} T_{\alpha\beta(i)}$$

$$\pi^{\mu\nu}_{(i)} = \Delta^{\mu\nu\alpha\beta} T_{\alpha\beta(i)}$$

$$\Pi_{(i)} = -\frac{1}{3}\Delta_{\alpha\beta}\left(T^{\alpha\beta}_{(i)} - T^{\alpha\beta}_{0(i)}\right)$$

$$f_{(i)} = \left(\exp(y_{(i)}) \pm 1\right)^{-1}$$

And expanding

$$f_{(i)} \approx f_{0(i)} + f_{0(i)}(1 \mp f_{0(i)})\left(y_{(i)} - y_{0(i)}\right)$$

Grad's Method (by IS)

$$
\begin{aligned}
\epsilon_{(0)} J_{00(i)} + \epsilon_{(1)} J_{10(i)} + \epsilon_{(2)} J_{20(i)} &= -\frac{3\Pi_{(i)}}{m^2_{(i)}}, \\
\epsilon_{(0)} J_{10(i)} + \epsilon_{(1)} J_{20(i)} + \epsilon_{(2)} J_{30(i)} &= 0, \\
\epsilon_{(0)} J_{20(i)} + \epsilon_{(1)} J_{30(i)} + \epsilon_{(2)} J_{40(i)} &= 0,
\end{aligned}
$$

$$
\begin{aligned}
\Delta^{\mu\nu} \epsilon_{\mu(0)} J_{21(i)} + \Delta^{\mu\nu} \epsilon_{\mu(1)} J_{31(i)} + \Delta^{\mu\nu} \epsilon_{\mu(2)} J_{41(i)} &= -n^{\nu}_{(i)} \\
\Delta^{\mu\nu} \epsilon_{\mu(0)} J_{31(i)} + \Delta^{\mu\nu} \epsilon_{\mu(1)} J_{41(i)} + \Delta^{\mu\nu} \epsilon_{\mu(2)} J_{51(i)} &= -q^{\nu}_{(i)}
\end{aligned}
$$

$$
2\Delta^{\mu\nu\lambda\sigma} \epsilon_{\lambda\sigma(0)} J_{42(i)} = \pi^{\mu\nu}_{(i)}
$$

Definition of L.R.F

We use the Landau picture

$$u_\nu (T^{\mu\nu} - T_0^{\mu\nu}) = 0$$

Then,

$$q_\nu = \sum_i q_{\nu(i)} = 0$$

But,

$$q_{\nu(i)} \neq 0$$

The energy flow for each particle specie cannot be neglected

$$q_{\nu(i)} \qquad v_{\nu(i)}$$

Solution

$$f_{(i)}(p) = f_{0(i)}\left[1 + (1 \mp f_{0(i)})\left(\varepsilon_{(i)} + \varepsilon_{\mu(i)} p^{\mu}_{(i)} + \varepsilon_{\mu\nu(i)} p^{\mu}_{(i)} p^{\nu}_{(i)}\right)\right]$$

$$\varepsilon_{(i)} = E_{0(i)} \Pi_{(i)}$$

$$\varepsilon^{\mu}_{(i)} = D_{0(i)} \Pi_{(i)} u^{\mu} + D_{1(i)} q^{\mu}_{(i)} + D_{2(i)} \nu^{\mu}_{(i)}$$

$$\varepsilon^{\mu\nu}_{(i)} = B_{0(i)} \left(\Delta^{\mu\nu} - 3u^{\mu}u^{\nu}\right) \Pi_{(i)} + 2B_{1(i)} u^{(\mu} q^{\nu)}_{(i)} + 2B_{3(i)} u^{(\mu} \nu^{\nu)}_{(i)} + B_{2(i)} \pi^{\mu\nu}_{(i)}$$

Solution

$$f_{(i)}(p) = f_{0(i)}\left[1 + (1 \mp f_{0(i)})\left(\varepsilon_{(i)} + \varepsilon_{\mu(i)} p^{\mu}_{(i)} + \varepsilon_{\mu\nu(i)} p^{\mu}_{(i)} p^{\nu}_{(i)}\right)\right]$$

$$\varepsilon_{(i)} = E_{0(i)} \Pi_{(i)}$$

$$\varepsilon^{\mu}_{(i)} = D_{0(i)} \Pi_{(i)} u^{\mu} + D_{1(i)} q^{\mu}_{(i)} + D_{2(i)} \nu^{\mu}_{(i)}$$

$$\varepsilon^{\mu\nu}_{(i)} = B_{0(i)} \left(\Delta^{\mu\nu} - 3u^{\mu}u^{\nu}\right) \Pi_{(i)} + 2B_{1(i)} u^{(\mu} q^{\nu)}_{(i)} + 2B_{3(i)} u^{(\mu} \nu^{\nu)}_{(i)} + B_{2(i)} \pi^{\mu\nu}_{(i)}$$

$$B_{2(i)} = \frac{1}{2J_{42(i)}} \longrightarrow$$ shear viscosity

$$J_{nq(i)} = \frac{1}{(2q-1)!!} \int d\omega_{(i)} E^{n-2q}_{(i)} K^{2q} f_{0(i)} \left(1 \quad a f_{0(i)}\right) \qquad -$$

Solution

$$f_{(i)}(p) = f_{0(i)}\left[1 + (1 \mp f_{0(i)})\left(\varepsilon_{(i)} + \varepsilon_{\mu(i)} p^{\mu}_{(i)} + \varepsilon_{\mu\nu(i)} p^{\mu}_{(i)} p^{\nu}_{(i)}\right)\right]$$

$$\varepsilon_{(i)} = E_{0(i)} \Pi_{(i)}$$

$$\varepsilon^{\mu}_{(i)} = D_{0(i)} \Pi_{(i)} u^{\mu} + D_{1(i)} q^{\mu}_{(i)} + D_{2(i)} \nu^{\mu}_{(i)}$$

$$\varepsilon^{\mu\nu}_{(i)} = B_{0(i)} \left(\Delta^{\mu\nu} - 3u^{\mu}u^{\nu}\right) \Pi_{(i)} + 2B_{1(i)} u^{(\mu} q^{\nu)}_{(i)} + 2B_{3(i)} u^{(\mu} \nu^{\nu)}_{(i)} + B_{2(i)} \pi^{\mu\nu}_{(i)}$$

$$B_{2(i)} = \frac{1}{2J_{42(i)}} \longrightarrow$$ shear viscosity

Boltzmann Gas

$$B_{2(i)} = \frac{1}{2\left(\varepsilon_{(i)} + P_{(i)}\right) T^{2}}$$

$$J_{nq(i)} = \frac{1}{(2q-1)!!} \int d\omega_{(i)} E^{n-2q}_{(i)} K^{2q} f_{0(i)} \left(1 \quad a f_{0(i)}\right)$$

-

Comment

$$\delta f_{(i)}^{shear} = f_{0(i)} \frac{1}{2(\varepsilon_{(i)} + P_{(i)})T^2} \pi_{\mu\nu(i)} K_{(i)}^{\mu} K_{(i)}^{\nu}$$

Solution

$$\delta f_{(i)}^{shear} = f_{0(i)} \frac{\sum_j \pi_{\mu\nu(j)}}{2T^2 \sum_j (\varepsilon_{(j)} + P_{(j)})} K_{(i)}^{\mu} K_{(i)}^{\nu}$$

used so far

$$\frac{\pi_{\mu\nu(i)}}{\varepsilon_{(i)} + P_{(i)}} \approx \frac{\sum_j \pi_{\mu\nu(j)}}{\sum_j (\varepsilon_{(j)} + P_{(j)})}$$

True ?

Solution

$$f_{(i)}(p) = f_{0(i)}\left[1 + (1 \mp f_{0(i)})\left(\varepsilon_{(i)} + \varepsilon_{\mu(i)} p^{\mu}_{(i)} + \varepsilon_{\mu\nu(i)} p^{\mu}_{(i)} p^{\nu}_{(i)}\right)\right]$$

$$\varepsilon_{(i)} = E_{0(i)} \Pi_{(i)}$$

$$\varepsilon^{\mu}_{(i)} = D_{0(i)} \Pi_{(i)} u^{\mu} + D_{1(i)} q^{\mu}_{(i)} + D_{2(i)} \nu^{\mu}_{(i)}$$

$$\varepsilon^{\mu\nu}_{(i)} = B_{0(i)} \left(\Delta^{\mu\nu} - 3u^{\mu}u^{\nu}\right) \Pi_{(i)} + 2B_{1(i)} u^{(\mu} q^{\nu)}_{(i)} + 2B_{3(i)} u^{(\mu} \nu^{\nu)}_{(i)} + B_{2(i)} \pi^{\mu\nu}_{(i)}$$

$$D_{1(i)} = \frac{-J_{31(i)}}{J_{31(i)} J_{31(i)} - J_{41(i)} J_{21(i)}}$$

$$D_{2(i)} = \frac{J_{41(i)}}{J_{31(i)} J_{31(i)} - J_{41(i)} J_{21(i)}}$$

Energy difusion

Particle difusion

$$B_{1(i)} = \frac{J_{21(i)}}{J_{31(i)} J_{31(i)} - J_{41(i)} J_{21(i)}}$$

$$B_{3(i)} = \frac{-J_{31(i)}}{J_{31(i)} J_{31(i)} - J_{41(i)} J_{21(i)}}$$

Solution

$$f_{(i)}(p) = f_{0(i)}\left[1 + (1 \mp f_{0(i)})\left(\varepsilon_{(i)} + \varepsilon_{\mu(i)} p^{\mu}_{(i)} + \varepsilon_{\mu\nu(i)} p^{\mu}_{(i)} p^{\nu}_{(i)}\right)\right]$$

$$\varepsilon_{(i)} = E_{0(i)} \Pi_{(i)}$$

$$\varepsilon^{\mu}_{(i)} = D_{0(i)} \Pi_{(i)} u^{\mu} + D_{1(i)} q^{\mu}_{(i)} + D_{2(i)} \nu^{\mu}_{(i)}$$

$$\varepsilon^{\mu\nu}_{(i)} = B_{0(i)} \left(\Delta^{\mu\nu} - 3u^{\mu}u^{\nu}\right)\Pi_{(i)} + 2B_{1(i)} u^{(\mu} q^{\nu)}_{(i)} + 2B_{3(i)} u^{(\mu} \nu^{\nu)}_{(i)} + B_{2(i)} \pi^{\mu\nu}_{(i)}$$

$$D_{0(i)} = -3C_{2(i)} B_{0(i)}, \quad E_{0(i)} = -3C_{1(i)} B_{0(i)}$$

$$B_{0(i)} = \frac{-1}{3C_{1(i)} J_{21(i)} + 3C_{2(i)} J_{31(i)} + 3J_{41(i)} + 5J_{42(i)}}$$

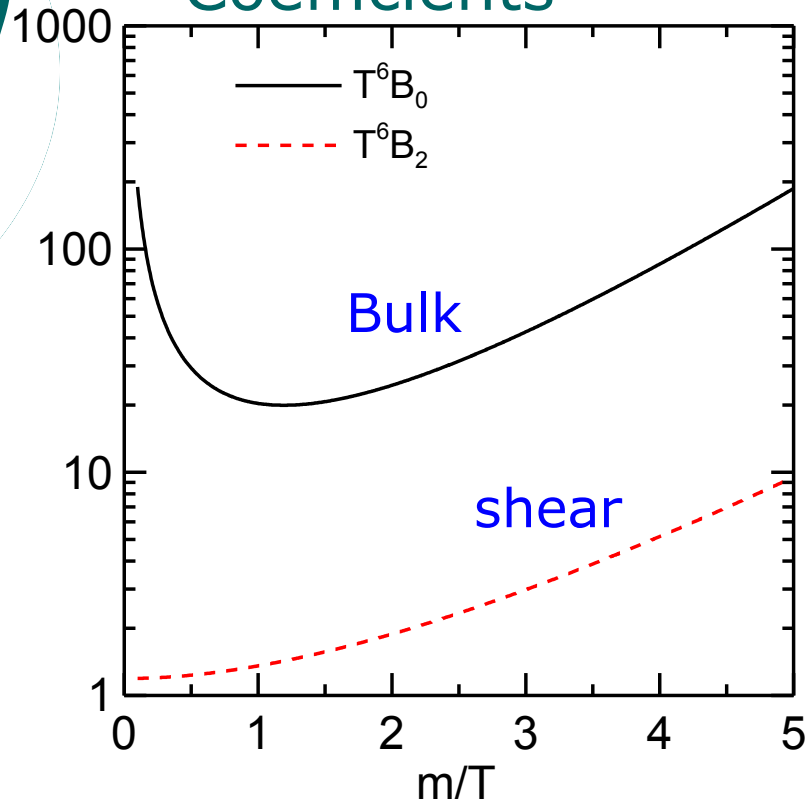
$$C_{1(i)} = -m^2_{(i)} - 4\frac{J_{31(i)} J_{30(i)} - J_{41(i)} J_{20(i)}}{J_{30(i)} J_{10(i)} - J_{20(i)} J_{20(i)}}$$

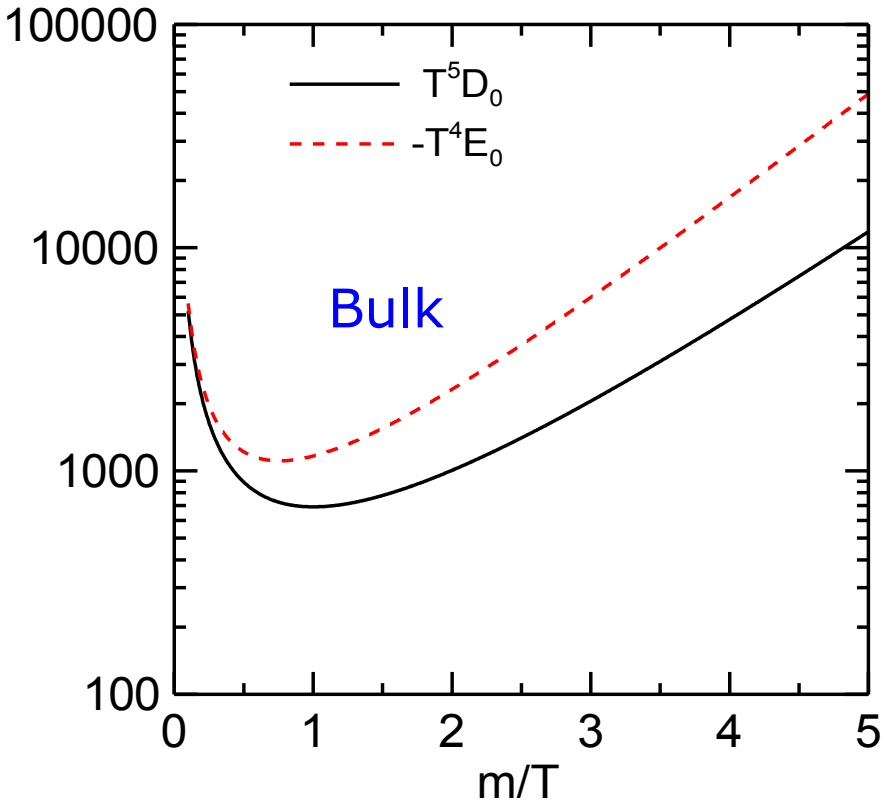
$$C_{2(i)} = 4\frac{J_{31(i)} J_{20(i)} - J_{41(i)} J_{10(i)}}{J_{30(i)} J_{10(i)} - J_{20(i)} J_{20(i)}}$$

bulk viscosity

Grad's Method

Coefficients





Closed Equations – Bulk and Shear

$$\frac{d\Pi_{(i)}}{d\tau} + \frac{\Pi_{(i)}}{\tau_{\Pi(i)}} + \sum_{j\neq i} \frac{\Pi_{(j)}}{\tau_{\Pi(i)(j)}} = -\left(\beta_{\zeta(i)} + \zeta_{\Pi\Pi(i)}\Pi_{(i)}\right)\theta + \zeta_{\Pi\pi(i)}\pi^{\mu\nu}_{(i)}\sigma_{\mu\nu}$$
$$-\zeta_{\Pi n(i)}\partial_\mu n^\mu - \alpha_{\Pi n(i)} n^\mu_{(i)}\dot{u}_\mu - \beta_{\Pi n(i)} n^\mu_{(i)}\nabla_\mu \alpha_{0(i)}$$
$$-\zeta_{\Pi q(i)}\partial_\mu q^\mu - \alpha_{\Pi q(i)} q^\mu_{(i)}\dot{u}_\mu - \beta_{\Pi q(i)} q^\mu_{(i)}\nabla_\mu \alpha_{0(i)}$$

$$\frac{d\pi^{\langle\mu\nu\rangle}_{(i)}}{d\tau} + \frac{\pi^{\mu\nu}_{(i)}}{\tau_{\pi(i)}} + \sum_{j\neq i} \frac{\pi^{\mu\nu}_{(j)}}{\tau_{\pi(i)(j)}} = 2\left(\beta_{\eta(i)} + \eta_{\pi\Pi(i)}\Pi_{(i)}\right)\sigma^{\mu\nu} - 2\eta_{\pi\pi(i)}\pi^{\langle\mu}_{\alpha(i)}\sigma^{\nu\rangle\alpha} + 2\pi^{\langle\mu}_{\alpha(i)}\omega^{\nu\rangle\alpha} - \left(\frac{1}{2} + \frac{7}{6}\eta_{\pi\pi(i)}\right)\pi^{\mu\nu}_{(i)}\theta$$
$$+2\eta_{\pi n(2)(i)}\nabla^{\langle\mu} n^{\nu\rangle}_{(i)} + 2\beta_{\pi n(i)} n^{\langle\mu}_{(i)}\nabla^{\nu\rangle}\alpha_{0(i)} - 2\alpha_{\pi n(i)} n^{\langle\mu}_{(i)}\dot{u}^{\nu\rangle}$$
$$+2\eta_{\pi q(2)(i)}\nabla^{\langle\mu} q^{\nu\rangle}_{(i)} + 2\beta_{\pi q(i)} q^{\langle\mu}_{(i)}\nabla^{\nu\rangle}\alpha_{0(i)} - 2\alpha_{\pi q(i)} q^{\langle\mu}_{(i)}\dot{u}^{\nu\rangle}$$

Transport coefficient β is independent of the approximation

only τ_R^{-1} depends on the collision term

Closed Equations – Bulk and Shear

$$\frac{d\Pi_{(i)}}{d\tau} + \frac{\Pi_{(i)}}{\tau_{\Pi(i)}} + \sum_{j\neq i} \frac{\Pi_{(j)}}{\tau_{\Pi(i)(j)}} = -\left(\beta_{\zeta(i)} + \zeta_{\Pi\Pi(i)}\Pi_{(i)}\right)\theta + \zeta_{\Pi\pi(i)}\pi^{\mu\nu}_{(i)}\sigma_{\mu\nu}$$
$$-\zeta_{\Pi n(i)}\partial_\mu n^\mu - \alpha_{\Pi n(i)} n^\mu_{(i)}\dot{u}_\mu - \beta_{\Pi n(i)} n^\mu_{(i)}\nabla_\mu \alpha_{0(i)}$$
$$-\zeta_{\Pi q(i)}\partial_\mu q^\mu - \alpha_{\Pi q(i)} q^\mu_{(i)}\dot{u}_\mu - \beta_{\Pi q(i)} q^\mu_{(i)}\nabla_\mu \alpha_{0(i)}$$

$$\frac{d\pi^{\langle\mu\nu\rangle}_{(i)}}{d\tau} + \frac{\pi^{\mu\nu}_{(i)}}{\tau_{\pi(i)}} + \sum_{j\neq i} \frac{\pi^{\mu\nu}_{(j)}}{\tau_{\pi(i)(j)}} = 2\left(\beta_{\eta(i)} + \eta_{\pi\Pi(i)}\Pi_{(i)}\right)\sigma^{\mu\nu} - 2\eta_{\pi\pi(i)}\pi^{\langle\mu}_{\alpha(i)}\sigma^{\nu\rangle\alpha} + 2\pi^{\langle\mu}_{\alpha(i)}\omega^{\nu\rangle\alpha} - \left(\frac{1}{2} + \frac{7}{6}\eta_{\pi\pi(i)}\right)\pi^{\mu\nu}_{(i)}\theta$$
$$+2\eta_{\pi n(2)(i)}\nabla^{\langle\mu} n^{\nu\rangle}_{(i)} + 2\beta_{\pi n(i)} n^{\langle\mu}_{(i)}\nabla^{\nu\rangle}\alpha_{0(i)} - 2\alpha_{\pi n(i)} n^{\langle\mu}_{(i)}\dot{u}^{\nu\rangle}$$
$$+2\eta_{\pi q(2)(i)}\nabla^{\langle\mu} q^{\nu\rangle}_{(i)} + 2\beta_{\pi q(i)} q^{\langle\mu}_{(i)}\nabla^{\nu\rangle}\alpha_{0(i)} - 2\alpha_{\pi q(i)} q^{\langle\mu}_{(i)}\dot{u}^{\nu\rangle}$$

Eqs. will depend on $\Pi_{(i)}, q^\mu_{(i)}, \pi^{\mu\nu}_{(i)}, \nu^\mu_{(i)}$

$\Pi, q^\mu, \pi^{\mu\nu}, \nu^\mu_r$ (possible?)

Relaxation Times

Massless limit, Boltzmann Gas, Constant Cross Sections

$$\tau_{\eta(i)}^{-1} = \frac{3}{5}\sigma_{ii} n_{0(i)} + \frac{4}{5}\sum_{j\neq i} n_{0(j)}\sigma_{ij},$$

$$\tau_{\eta(ij)}^{-1} = -\frac{1}{5}\sigma_{ij} n_{0(j)}.$$

If all cross sections are the same we return to a degenerate gas

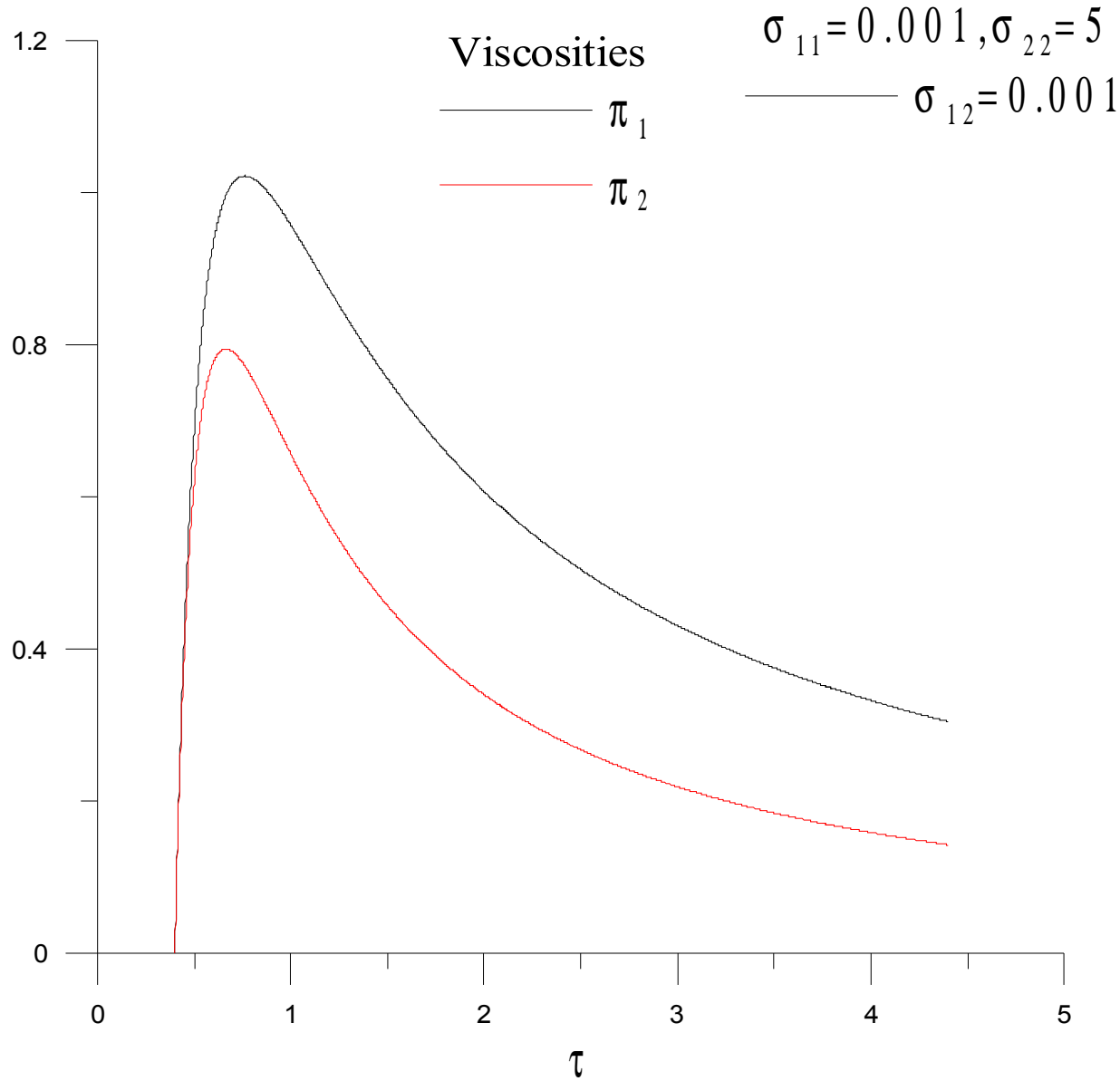
2 Components

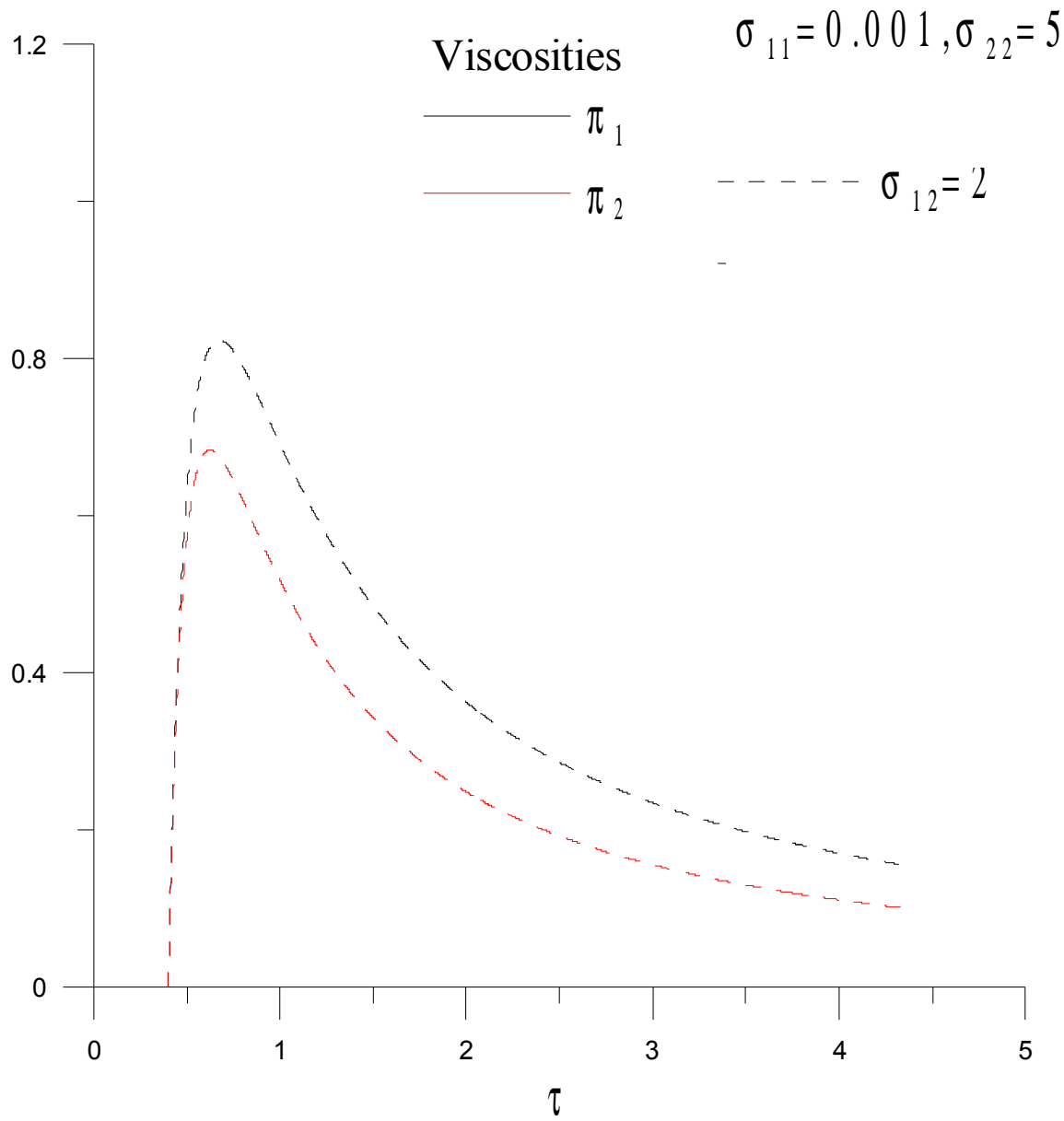
Massless limit, Boltzmann Gas, Constant Cross Sections

Bjorken Scalling

$$\frac{d\pi_1}{d\tau} + \frac{1}{\tau_{\eta(1)}}\pi_1 + \frac{1}{\tau_{\eta(12)}}\pi_2 = \frac{4P_{0(1)}}{5}\frac{4}{3\tau} - \left(\frac{31}{7}\right)\frac{4\pi_1}{9\tau}.$$

$$\frac{d\pi_2}{d\tau} + \frac{1}{\tau_{\eta(2)}}\pi_2 + \frac{1}{\tau_{\eta(21)}}\pi_1 = \frac{4P_{0(2)}}{5}\frac{4}{3\tau} - \left(\frac{31}{7}\right)\frac{4\pi_2}{9\tau}.$$


Viscosities
π_1
π_2
$\sigma_{11}=0.001, \sigma_{22}=5$
$\sigma_{12}=0.001$
1.2
0.8
0.4
0
0
1
2
3
4
5
τ


Viscosities
$\sigma_{11} = 0.001, \sigma_{22} = 5$
π_1
π_2
$\sigma_{12} = 2$
1.2
0.8
0.4
0
0
1
2
3
4
5
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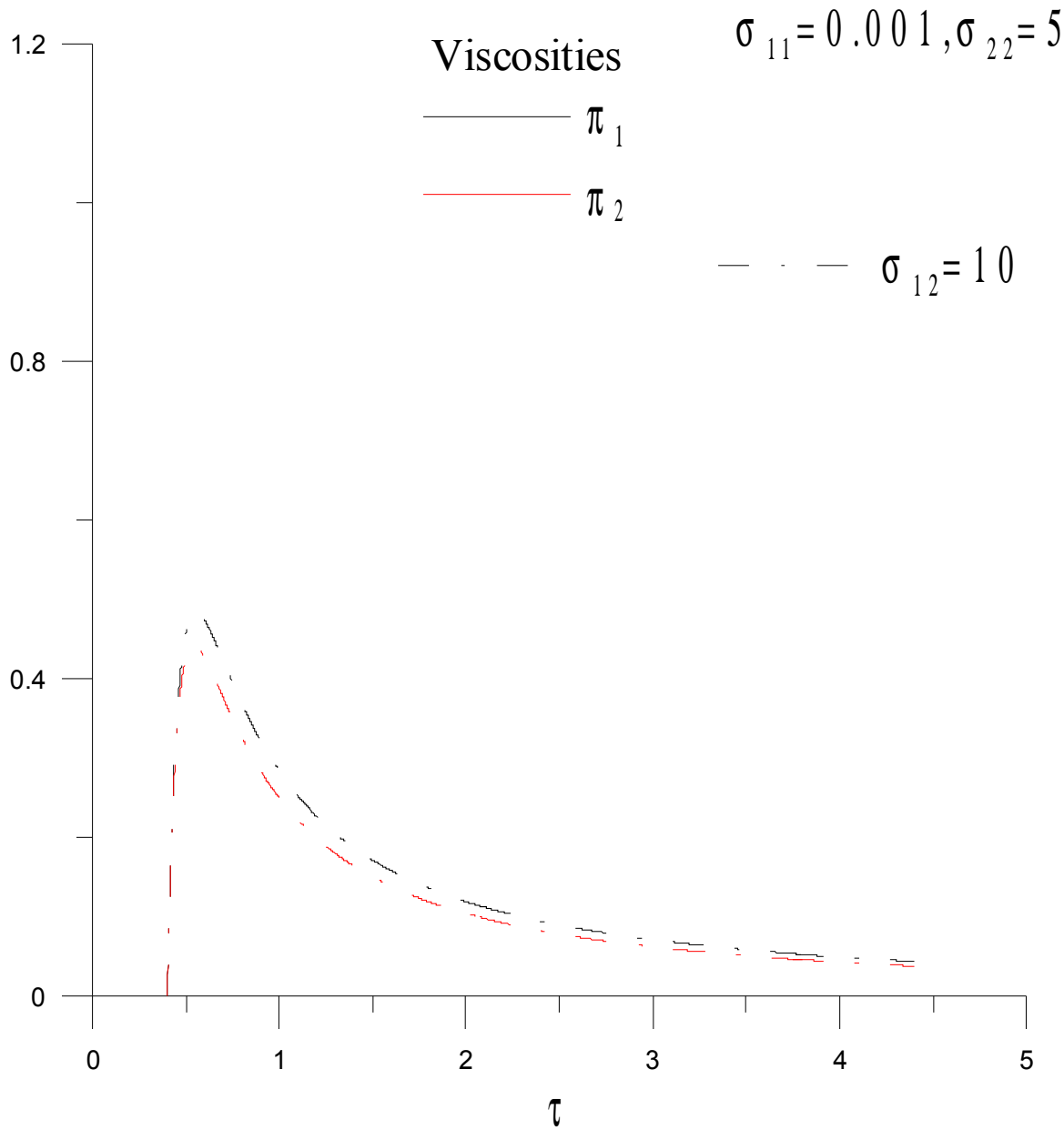
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π_1
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1.2
0.8
0.4
0
0
1
2
3
4
5
τ

Conclusions

- Corrections to the equilibrim distribution function will depend on the irreversible currents for each particle species
- Same will happen for the equations of motion

Can we obtain hydro?