Third order dissipative hydrodynamics

arXiv:0907.4500

Andrej El
Zhe Xu
Carsten Greiner

Goethe University Frankfurt am Main

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Motivation

Recent attempts to quantify and understand deviations between dissipative hydrodynamics and kinetic transport:

> *D.Molnar, P.Huovinen arXiv:0808.0953, arXiv:0907.5014*

> *I.Bouras et al, Phys.Rev.Lett.103:032301,2009 + paper in preparation also talk by H.Niemi*

Applicability limits of dissipative hydrodynamics:

> *M. Martinez , M.Strickland, Phys.Rev.C79:044903,2009*

In *arXiv:0907.4500* we derive a third-order equation for shear tensor using Grad's ansatz for off-equilibrium distribution function

Entropy current

In Grad's approach $f(x,p,\pi)=f_0(1+\phi)=f_0\left(1+\epsilon_{\mu\nu}\frac{p^\mu p^\nu}{T^2}\right)$

In particular for massless Boltzmann particles, in 1D:

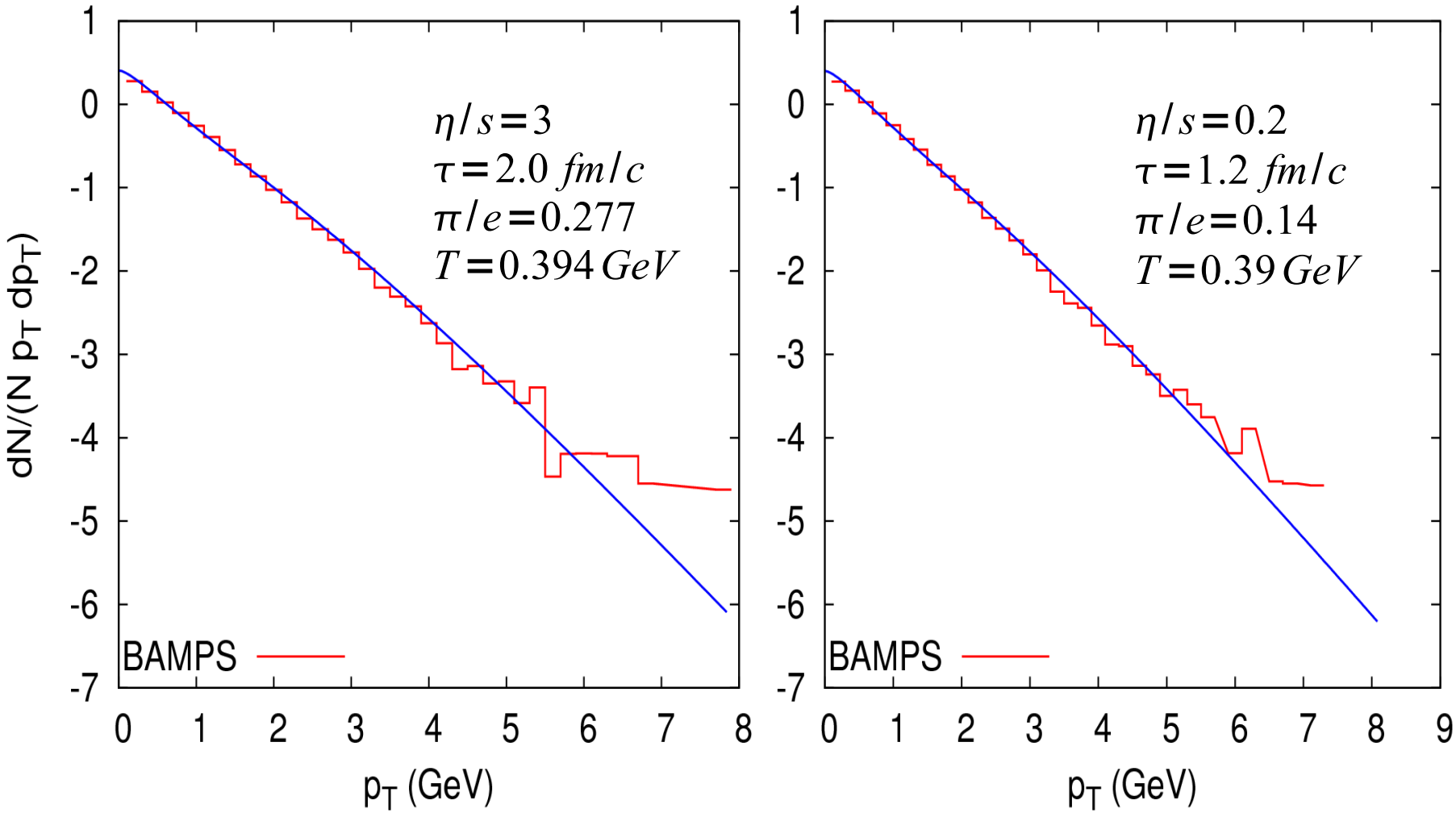
$$\left.\begin{aligned}\delta T^{00}&=\int p_0^2 f_0(1+\phi)\,d\Gamma=0\\ \delta T^{33}&=\int p_z^2 f_0(1+\phi)\,d\Gamma=-\pi\end{aligned}\right\}\longrightarrow \epsilon_{\mu\nu}=\frac{3}{8e}diag(0,\pi/2,\pi/2,-\pi)$$

$$\longrightarrow \quad f(x,p,\pi)=f_0\left(1+\frac{3}{8\mathrm{eT}^2}\pi_{\mu\nu}p^\mu p^\nu\right)$$

$$s^\mu=s_0u^\mu-\beta_2\pi_{\mu\nu}\pi^{\mu\nu}\frac{u^\mu}{2T}+\alpha\beta_2^2\pi_{\alpha\beta}\pi^\alpha_\sigma\pi^{\beta\sigma}\frac{u^\mu}{T}$$

Off-equilibrium distribution

Transverse spectra dN/ (N p_T dp_T) from 1D BAMPS calculations



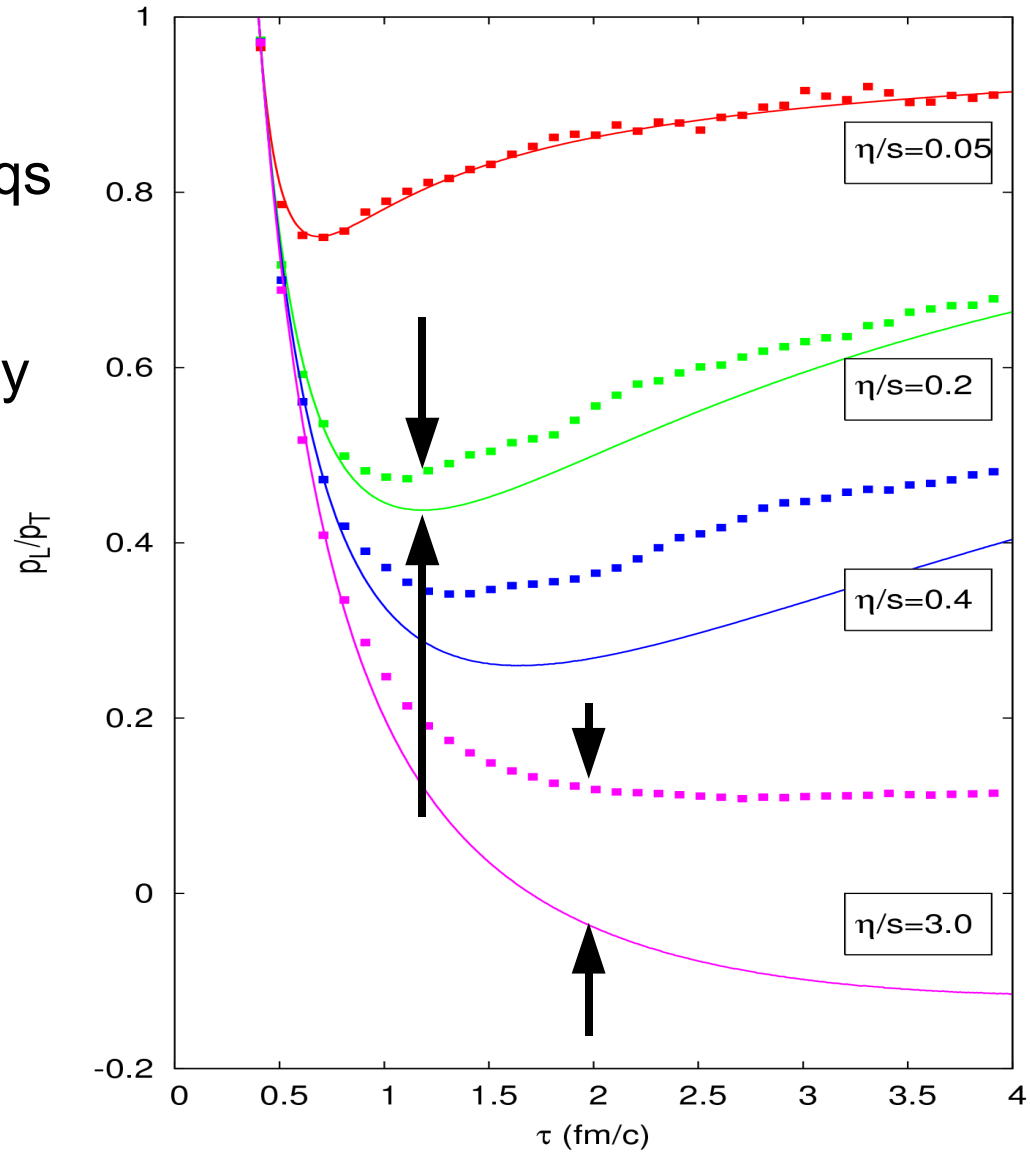
Analytic solution: $f(x,p,\pi)=f_0\left(1+\frac{3}{8eT^2}\pi_{\mu\nu}p^\mu p^\nu\right)$ ← $T,\pi^{\mu\nu}$ from BAMPS

Off-equilibrium distribution

BAMPS
vs
Israel-Stewart eqs

pressure isotropy

$$\frac{p_L}{p_T} = \frac{p-\pi}{p+\pi/2}$$



Israel-Stewart equations have to be corrected

Third order hydrodynamics

$$s^\mu = s_0 u^\mu - \beta_2 \pi_{\mu\nu}\pi^{\mu\nu}\frac{u^\mu}{2T} + \alpha\beta_2^2 \pi_{\alpha\beta}\pi^\alpha_\sigma \pi^{\beta\sigma}\frac{u^\mu}{T}$$

$$T\partial_\mu s^\mu = \pi_{\mu\nu}\overbrace{\left(\sigma^{\mu\nu} - \beta_2 \dot{\pi}^{\mu\nu} - \frac{1}{2}T\partial_\alpha\left(\frac{\beta_2}{T}u^\alpha\right)\pi^{\mu\nu}\right)}^{\textit{Israel-Stewart}} +$$

$$+\alpha T \pi_{\alpha\beta}\pi^\alpha_\sigma \pi^{\beta\sigma}\partial_\mu\left(\beta_2^2\frac{u^\mu}{T}\right) + 3\alpha\beta_2^2 \pi_{\alpha\beta}\pi^\alpha_\sigma \dot{\pi}^{\beta\sigma}$$

with

$\sigma^{\mu\nu} = \nabla^{<\mu}u^{\nu>}$ and $\dot{\pi}^{\alpha\beta} = u^\mu \partial_\mu \pi^{\alpha\beta}$

Enforcing the second law of thermodynamics $\partial_\mu s^\mu \geqslant 0$,

one obtains a third-order evolution equation for $\pi^{\mu\nu}$

(s. arXiv:0907.4500 for details)

Second law

Neglecting parts which are always positive in the entropy production:

$$\pi_{\mu\nu}\left(\sigma^{\mu\nu}-\beta_2\dot{\pi}^{\mu\nu}-\frac{1}{2}T\,\partial_\alpha\left(\frac{\beta_2}{T}u^\alpha\right)\pi^{\mu\nu}+\alpha\,T\,\pi_\sigma^{<\mu}\pi^{\nu>\sigma}\partial_\mu\left(\beta_2^2\frac{u^\mu}{T}\right)+3\,\alpha\,\beta_2^2\theta\,\tau_\pi\pi_\sigma^{<\mu}\dot{\pi}^{\nu>\sigma}\right)\geqslant 0$$

Israel-Stewart eq.

$$\pi_{\mu\nu}=2\,\eta\left(\sigma^{\mu\nu}-\beta_2\dot{\pi}^{\mu\nu}-\frac{1}{2}T\,\partial_\alpha\left(\frac{\beta_2}{T}u^\alpha\right)\pi^{\mu\nu}+\alpha\,T\,\pi_\sigma^{<\mu}\pi^{\nu>\sigma}\partial_\mu\left(\beta_2^2\frac{u^\mu}{T}\right)+3\,\alpha\,\beta_2^2\theta\,\tau_\pi\pi_\sigma^{<\mu}\dot{\pi}^{\nu>\sigma}\right)$$

Fixing the coefficients

Entropy current

$$s^{\mu} = s_0 u^{\mu} - \beta_2 \pi_{\mu\nu} \pi^{\mu\nu} \frac{u^{\mu}}{2T} + \alpha \beta_2^2 \pi_{\alpha\beta} \pi^{\alpha}_{\sigma} \pi^{\beta\sigma} \frac{u^{\mu}}{T}$$

Calculate LRF entropy density from kinetic definition:

$$s = \int p_0 f_0 (1+\phi) \left(\ln\left[f_0 (1+\phi) \right] - 1 \right) d\Gamma$$

$$\phi = \frac{3}{8eT^2} \pi_{\mu\nu} p^{\mu} p^{\nu}$$

expanding the integrand up to *thrid order in* $\boldsymbol{\pi}$

$$\beta_2 = \frac{9}{4e} \qquad \alpha = -\frac{8}{9}$$

Third order hydrodynamics

$$\pi_{\mu\nu} = 2\eta\left(\sigma^{\mu\nu} - \beta_2 \dot{\pi}^{\mu\nu} - \frac{1}{2} T \partial_\alpha\left(\frac{\beta_2}{T} u^\alpha\right)\pi^{\mu\nu} + \alpha T \pi_\sigma^{<\mu} \pi^{\nu>\sigma} \partial_\mu\left(\beta_2^2 \frac{u^\mu}{T}\right) + 3\alpha\beta_2^2 \theta \tau_\pi \pi_\sigma^{<\mu} \dot{\pi}^{\nu>\sigma}\right)$$

for 1-Dim boost-invariant expansion :

$$T^{\mu\nu} = T^{\mu\nu}_{eq} + \pi^{\mu\nu} = \begin{pmatrix} e & 0 & 0 & 0 \\ 0 & p+\pi/2 & 0 & 0 \\ 0 & 0 & p+\pi/2 & 0 \\ 0 & 0 & 0 & p-\pi \end{pmatrix}$$

$\longrightarrow$ energy density

$\longrightarrow$ transverse pressure

$\longrightarrow$ longitudinal pressure

$$v = \frac{z}{t} \longrightarrow u^\mu = (\cosh\eta, 0, 0, \sinh\eta) \longrightarrow \partial_\mu u^\mu = 1/\tau$$

$$\dot{\pi} = -\frac{\pi}{\tau_\pi} - \frac{4}{3}\frac{\pi}{\tau} + \frac{8}{27}\frac{e}{\tau} - 3\frac{\pi^2}{e\tau} - 3\frac{\tau_\pi}{\tau}\frac{\pi}{e}\dot{\pi}$$

$$\tau_\pi = 2\eta\beta_2$$

Third-order theory: 1-Dim expansion

$$\dot{\pi} = -\frac{\pi}{\tau_\pi} - \frac{4}{3}\frac{\pi}{\tau} + \frac{8}{27}\frac{e}{\tau} - 3\frac{\pi^2}{e\tau}$$

Third order equation for 1-Dim system

$$\dot{e} = -\frac{4}{3}\frac{e}{\tau} + \frac{\pi}{\tau}$$

$$\dot{n} = -\frac{n}{\tau}$$

Hydro vs BAMPS

BAMPS: Z.Xu, C.Greiner, PRC 71 (2005) 064901; Talks by Z.Xu, H.Niemi

BAMPS :

$$\eta = \frac{6}{5}\frac{T}{\sigma_{22}} \longrightarrow \sigma_{22} = \frac{6}{5}\frac{T}{\frac{\eta}{s}\cdot(4n - n\ln\lambda)}$$

For hydro:

$$\eta = \frac{\eta}{s}\cdot(4n - n\ln\lambda)$$

Isotropic elastic cross section, thermal initial conditions

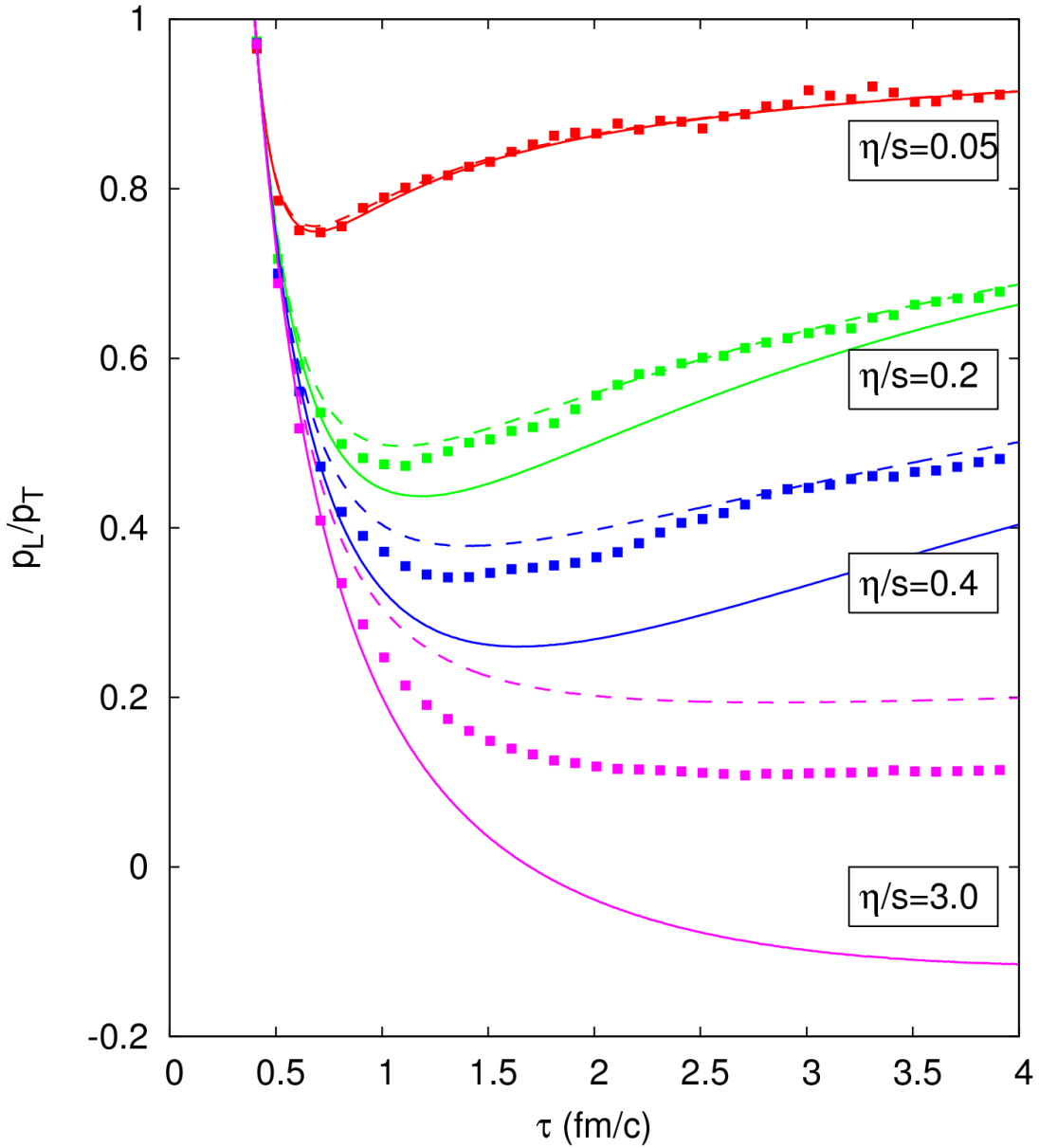
Hydro vs BAMPS

Third order is much closer to kinetic transport than IS

Oscillating behavior of higher-oder corrections?

Third order: deviations at early times (dissipation at largest there)
→ higher orders still important

No negative longitudinal pressure



Third-order theory: 1-Dim expansion

$$\dot{\pi} = -\frac{\pi}{\tau_\pi} - \frac{4}{3}\frac{\pi}{\tau} + \frac{8}{27}\frac{e}{\tau} - 3\frac{\pi^2}{e\tau}$$

Third order equation for 1-Dim system

Way to estimate all higher-order corrections:

We make the *ansatz*

$$\dot{\pi} = -\frac{\pi}{\tau_\pi} - \frac{4}{3}\frac{\pi}{\tau} + \frac{8}{27}\frac{e}{\tau} - x\frac{\pi^2}{e\tau} \equiv -\frac{\pi}{\tau_\pi} - \frac{4}{3}\frac{\pi}{\tau} + \frac{8}{27}\frac{e}{\tau} - \frac{\pi^2}{e\tau}\left(x_0 + x_1\frac{\pi}{e} + x_2\frac{\pi^2}{e^2} + \ldots\right)$$

x contains all corrections
(third order and higher)

If all corrections are included, it should be possible to solve the $\sigma = 0$, $\eta \to \infty$ limit!

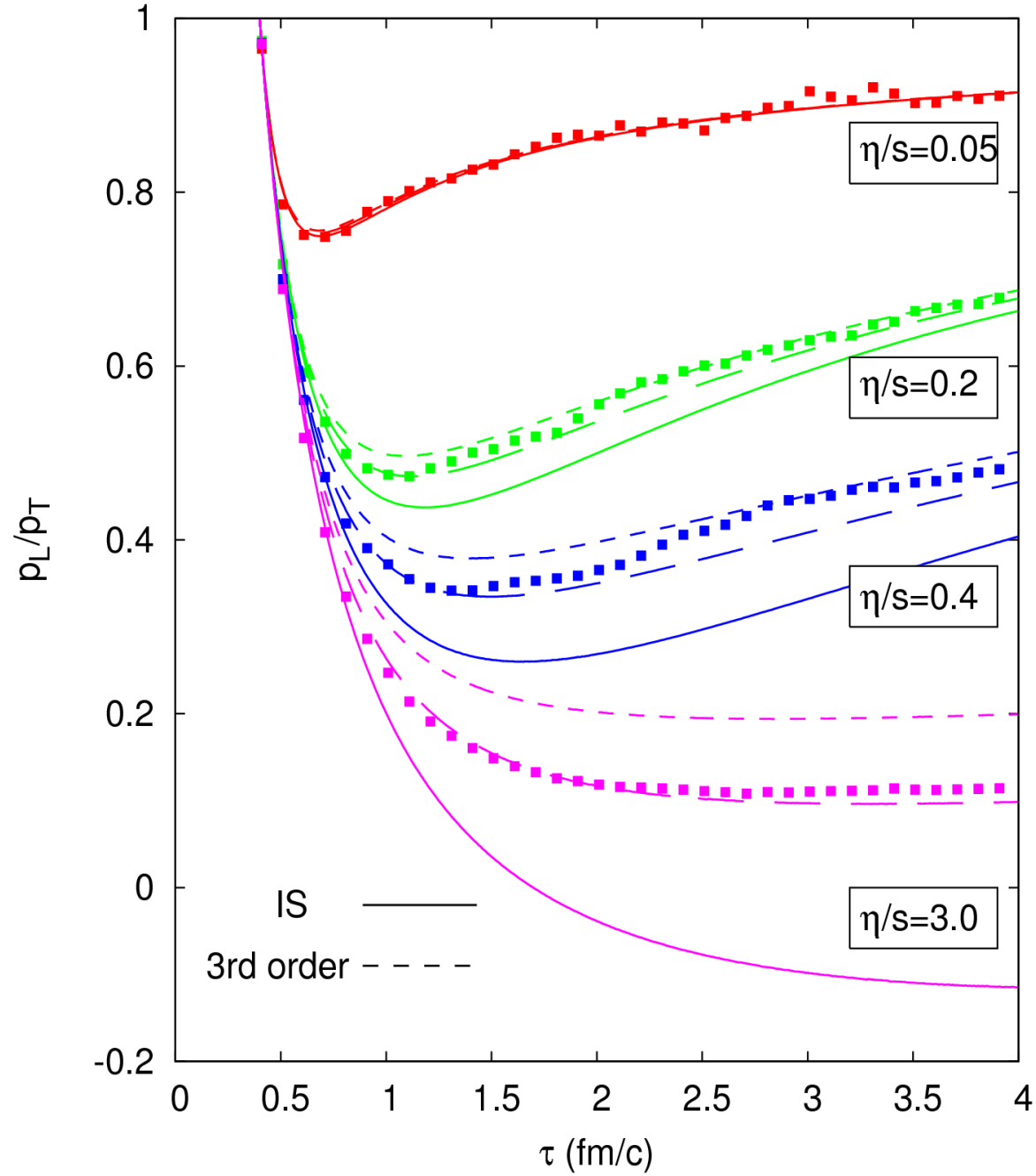
I.e., free streaming: $\dot{e} = -\frac{e}{\tau}$ and $\pi = p = \frac{e}{3}$ (i.e. $p_L = 0$) $\Rightarrow \dot{\pi} = -\frac{\dot{e}}{3}$

$\Rightarrow$ $x = \frac{5}{3}$ this *x includes* all higher order corrections by their maximal values

Hydro vs BAMPS

$$\dot{\pi} = -\frac{\pi}{\tau_\pi} - \frac{4}{3}\frac{\pi}{\tau} + \frac{8}{27}\frac{e}{\tau} - 3\frac{\pi^2}{e\tau}$$

$$\dot{\pi} = -\frac{\pi}{\tau_\pi} - \frac{4}{3}\frac{\pi}{\tau} + \frac{8}{27}\frac{e}{\tau} - \frac{5}{3}\frac{\pi^2}{e\tau}$$



Including “higher-orders”:

HO terms are taken by maximum value → good agreement at maximum dissipation

Conclusions

> Beyond Israel-Stewart equations needed to reduce deviations from kinetic transport theory

> Third order equations derived using Grad's ansatz for distribution function.

> Knowledge of dissipative corrections to the distribution function is crucial!

> Higher than 3rd orders are still important

arXiv:0907.4500

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To P. Huovinen, H. Niemi and D. Rischke for discussions

BACKUP

Third-order theory: second law

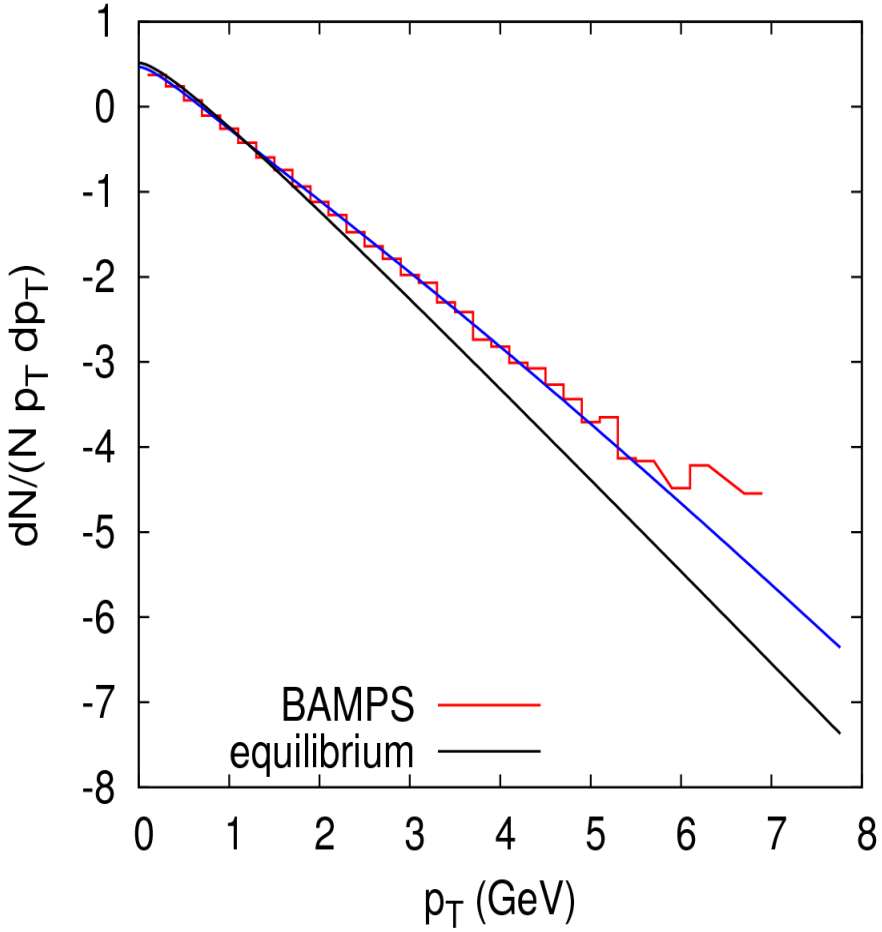
$$T\,\partial_\mu s^\mu = \pi_{\mu\nu}\left(\sigma^{\mu\nu} - \beta_2\,\dot{\pi}^{\mu\nu} - \frac{1}{2}T\,\partial_\alpha\left(\frac{\beta_2}{T}u^\alpha\right)\pi^{\mu\nu}\right) + \underline{\alpha\,T\,\pi_{\alpha\beta}\pi^\alpha_\sigma\pi^{\beta\sigma}\,\partial_\mu\left(\beta_2^2\frac{u^\mu}{T}\right)} +$$

$$+\underline{3\,\alpha\,\beta_2^2\,\theta\,\tau_\pi\,\pi_{\alpha\beta}\pi^\alpha_\sigma\dot{\pi}^{\beta\sigma}} + \underbrace{3\,\alpha\,\beta_2^2\left(1-\theta\,\tau_\pi\right)\pi_{\alpha\beta}\pi^\alpha_\sigma\dot{\pi}^{\beta\sigma}}_{\text{always > 0}} \geqslant 0$$

$$\pi_{\mu\nu}\left(\sigma^{\mu\nu} - \beta_2\,\dot{\pi}^{\mu\nu} - \frac{1}{2}T\,\partial_\alpha\left(\frac{\beta_2}{T}u^\alpha\right)\pi^{\mu\nu} + \underline{\alpha\,T\,\pi^{<\mu}_\sigma\pi^{\nu>\sigma}\,\partial_\mu\left(\beta_2^2\frac{u^\mu}{T}\right)} + 3\,\underline{\alpha\,\beta_2^2\,\theta\,\tau_\pi\,\pi^{<\mu}_\sigma\dot{\pi}^{\nu>\sigma}}\right) \geqslant 0$$

$$\pi_{\mu\nu} = 2\,\eta\left(\overbrace{\sigma^{\mu\nu} - \beta_2\,\dot{\pi}^{\mu\nu} - \frac{1}{2}T\,\partial_\alpha\left(\frac{\beta_2}{T}u^\alpha\right)\pi^{\mu\nu}}^{\text{Israel-Stewart eq.}} + \alpha\,T\,\pi^{<\mu}_\sigma\pi^{\nu>\sigma}\,\partial_\mu\left(\beta_2^2\frac{u^\mu}{T}\right) + 3\,\alpha\,\beta_2^2\,\theta\,\tau_\pi\,\pi^{<\mu}_\sigma\dot{\pi}^{\nu>\sigma}\right)$$

Since then $T\,\partial_\mu s^\mu = \frac{1}{2\,\eta}\pi_{\mu\nu}\pi^{\mu\nu} + 3\,\alpha\,\beta_2^2\left(1-\theta\,\tau_\pi\right)\pi_{\alpha\beta}\pi^\alpha_\sigma\dot{\pi}^{\beta\sigma} \geqslant 0$


1
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-1
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-3
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-6
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-8
0
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2
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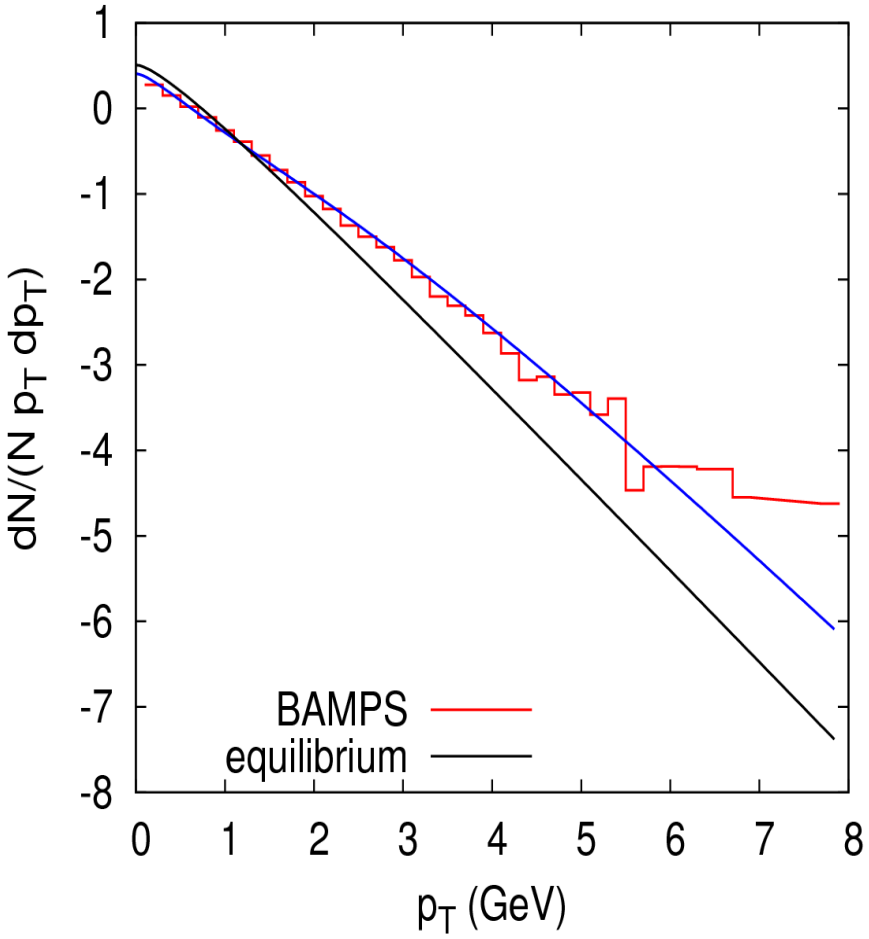
5
6
7
8
dN/(N p_T dp_T)
p_T (GeV)
BAMPS
equilibrium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8
dN/(N p_T dp_T)
p_T (GeV)
BAMPS
equilibrium