State of the art Equation of State

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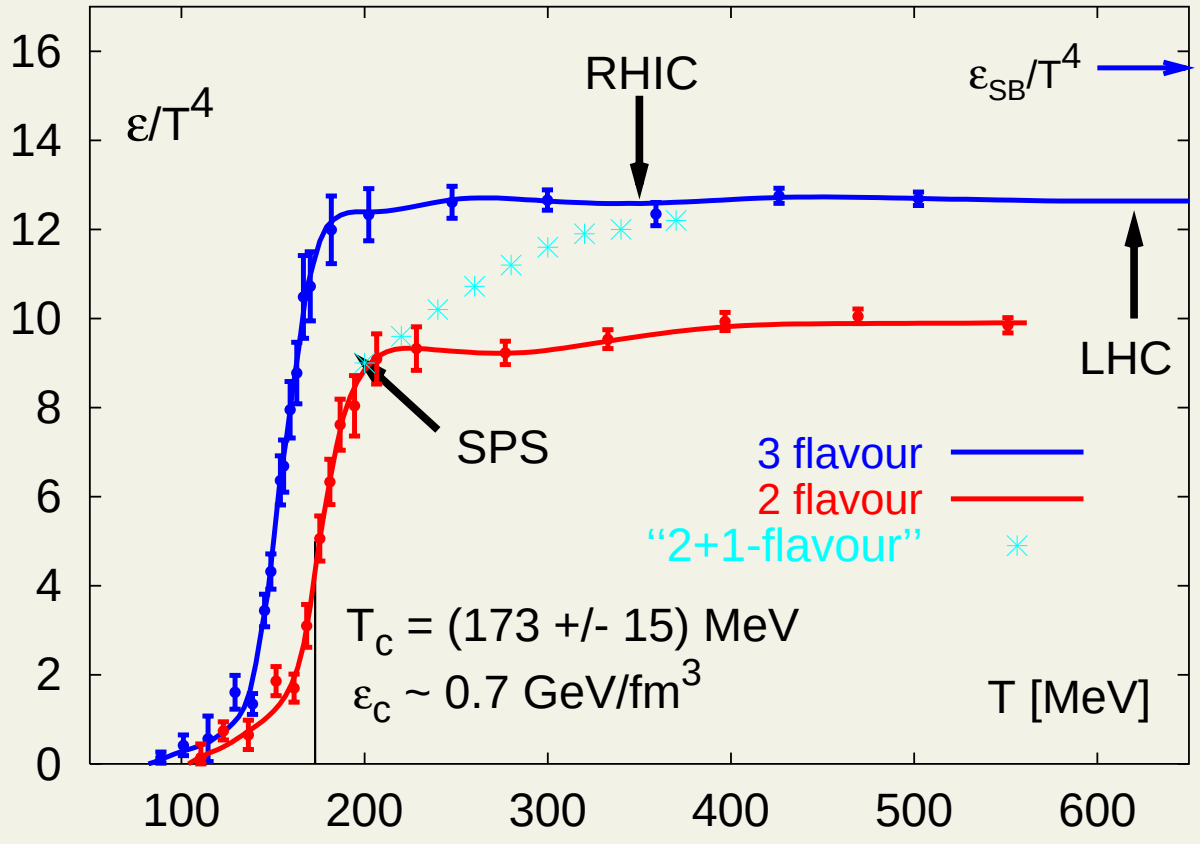
in collaboration with Peter Petreczky at BNL

based on arXiv:0912.2541

QCD equation of state

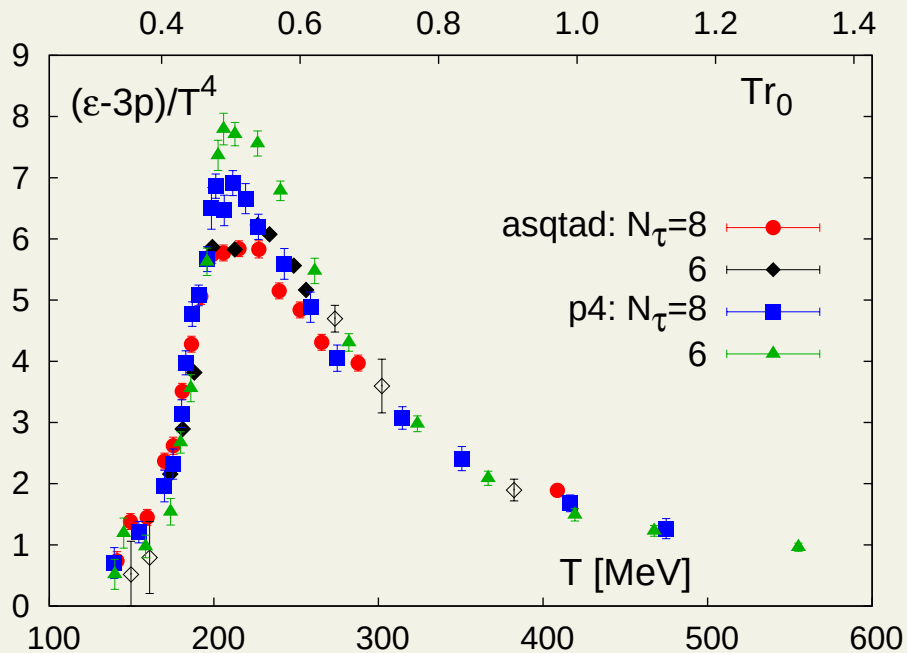
lattice QCD (Karsch & Laermann, hep-lat/0305025):

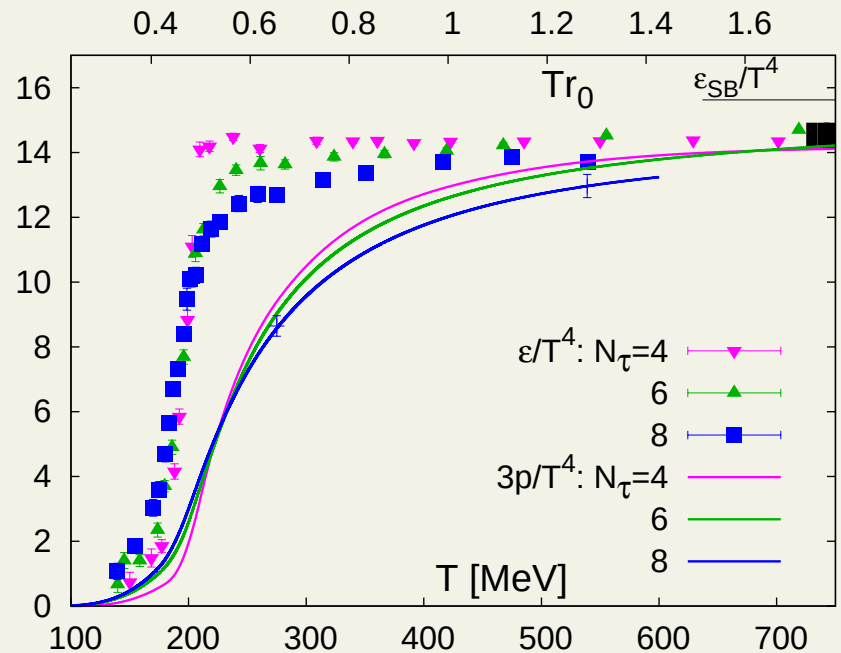


- **EoS from first principles**

EoS by hotQCD collaboration

Bazavov *et al.* arXiv:0903.4379 [hep-lat]





- **evaluate interaction measure** $(\epsilon - 3P)/T^4$
- **obtain pressure via**

$$\frac{P}{T^4} - \frac{P_0}{T_0^4} = \int_{T_0}^{T} \mathrm{d}T' \, \frac{\epsilon - 3P}{T'^5}$$

- **What is** $P(T_0)$**?**
- **What is** $(\epsilon(T_0) - 3P(T_0))/T_0^4$**?**
- **How good is lattice below** T_c**?**

Energy conservation

Cooper-Frye:

$$\int_{\Sigma_{\rm fo}} {\rm d}\Sigma_\mu \, T^{\mu 0}(u_f, T_f) = E = \int_{\Sigma_{\rm fo}} {\rm d}\Sigma_\mu \, p^\mu p^0 f(p \cdot u_p, T_p)$$

- **same EoS for fluid and particles**
- **or** $T_{\rm fluid} \neq T_{\rm particles}$ **and** $\mathbf{v}_{\rm fluid} \neq \mathbf{v}_{\rm particles}$

Hadron Resonance Gas model

- **EoS of interacting hadron gas well approximated by non-interacting gas of hadrons and resonances**

$$P(T) = \sum_i \int \mathrm{d}^3p \, \frac{p^2}{3E} \, f(p,T)$$

- **valid when**

- interactions mediated by resonances

- resonances have zero width

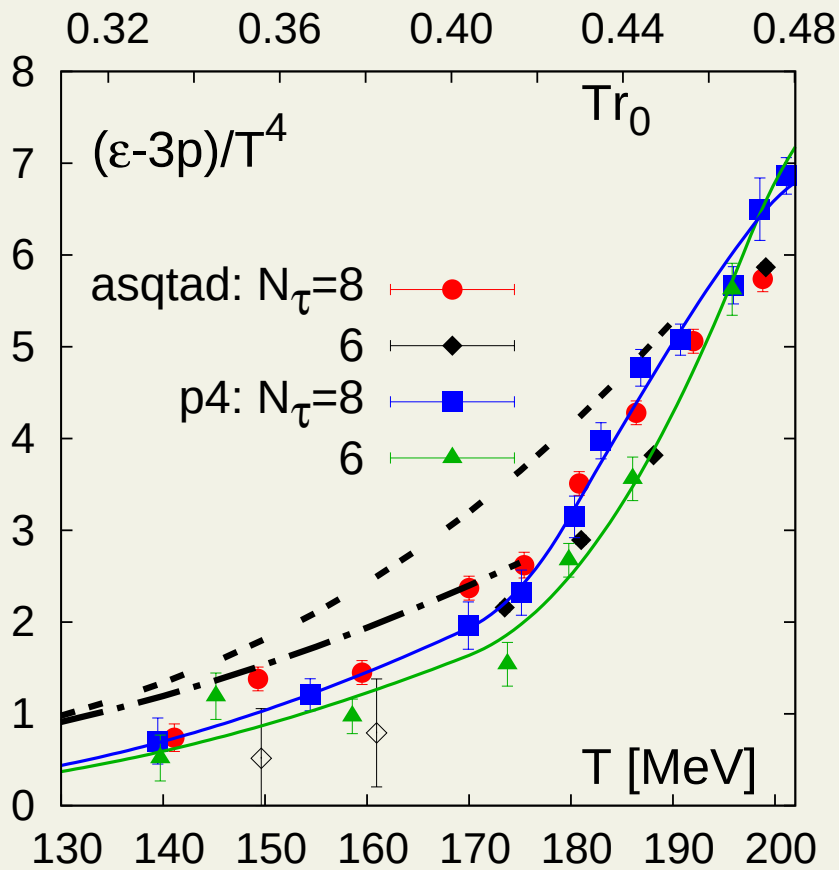
- **Prakash & Venugopalan, NPA546, 718 (1992): experimental phase shifts**
- **Gerber & Leutwyler, NPB321, 387 (1989): chiral perturbation theory**

$\Rightarrow$ **HRG good approximation at low temperatures**

$\rightarrow$ **lattice should reproduce HRG at** $T \leq 120 - 140$ **MeV**

Interaction measure below T_c

Bazavov *et al* arXiv:0903.4379



- **Lattice EoS $\neq$ Hadron Resonance Gas EoS**

Hadrons on lattice

- Hadron masses depend on lattice cutoff

$\Rightarrow$ i.e. on temperature:

E.g. for pseudoscalar mesons

$$
\begin{aligned}
m^2_{\mathrm{ps}_i} &= m^2_{\mathrm{ps}_0} + \frac{1}{r_1^2}\frac{a^i_{\mathrm{ps}}x + b^i_{\mathrm{ps}}x^2}{(1+c^i_{\mathrm{ps}}x)^{\beta_i}} \\
x &= (a/r_1)^2 \\
a &= \frac{1}{N_\tau T}
\end{aligned}
$$

+ 16 pseudoscalar mesons on lattice

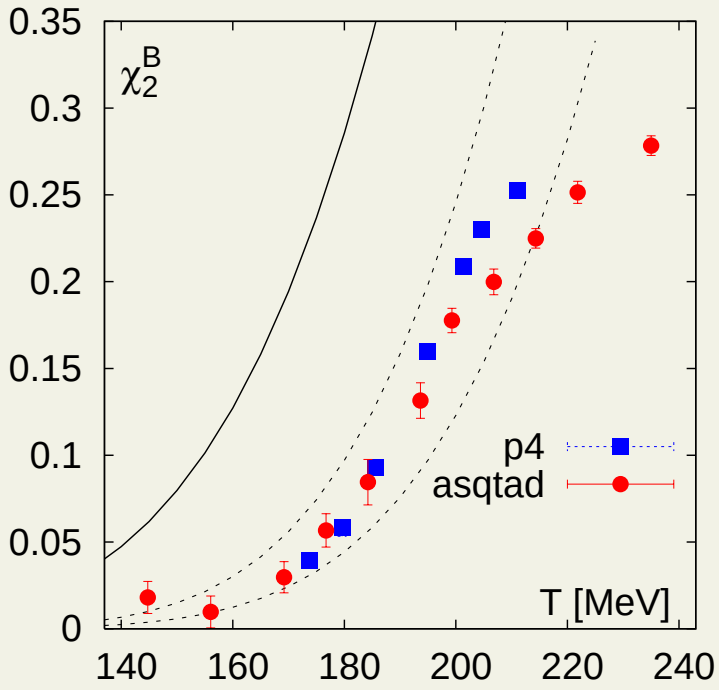
- HRG with lattice mass spectrum?

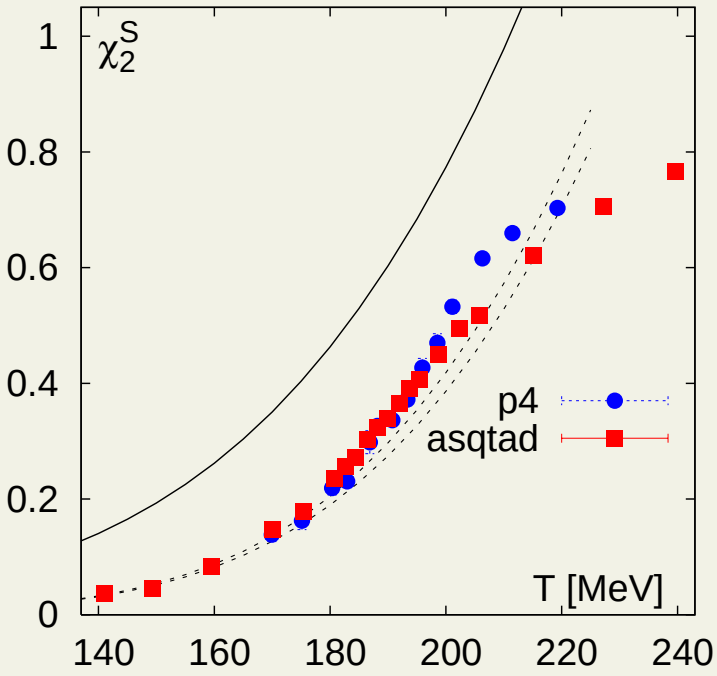
Hadronic fluctuations

i.e. baryon number, strangeness and charge susceptibilities

$$\chi_2^x = \frac{1}{VT^3}\frac{\partial^2 \ln Z}{\partial(\mu_x/T)^2} = \frac{1}{T^2}\frac{\partial^2 P}{\partial \mu_x^2},$$

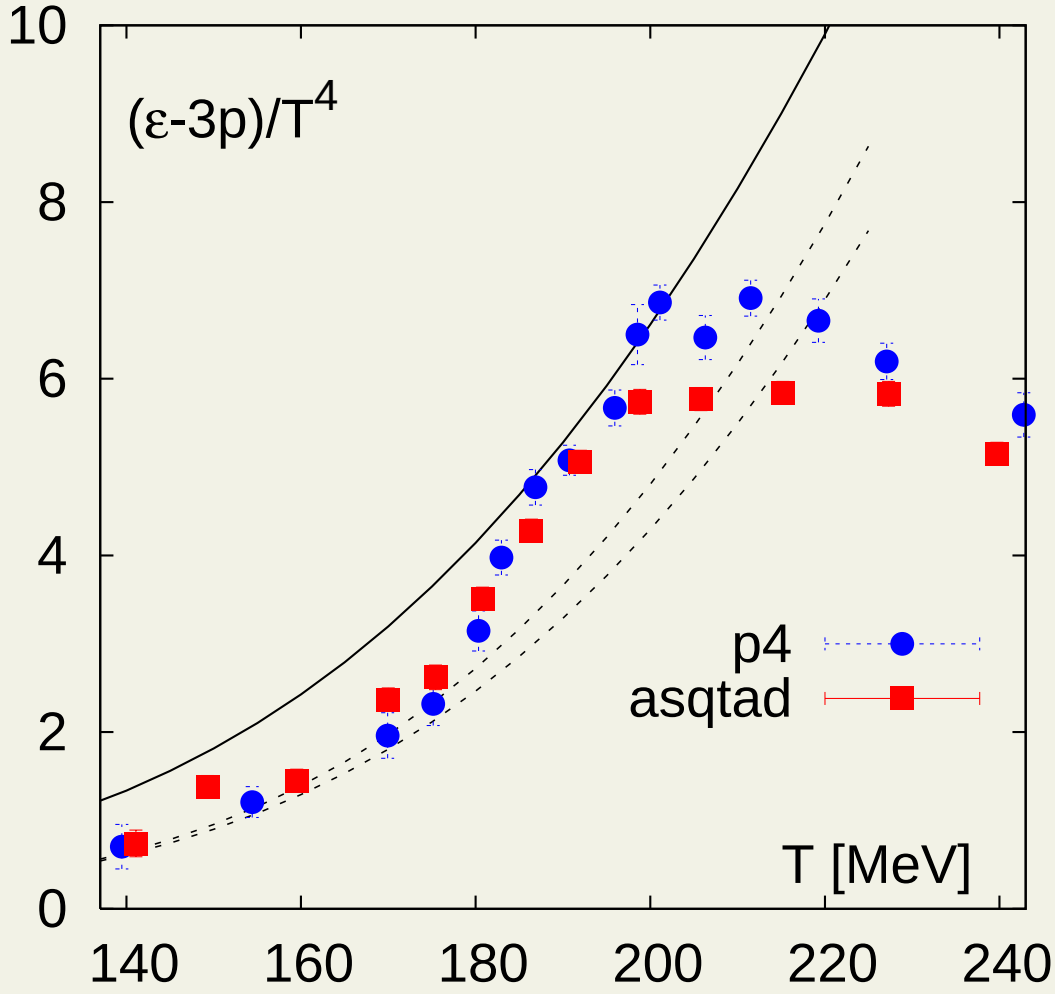
where $\mu_x = \mu_B, \mu_S$ **or** μ_Q





- **Lattice masses** $\rightarrow$ **fluctuations in resonance gas and lattice similar**

Interaction measure

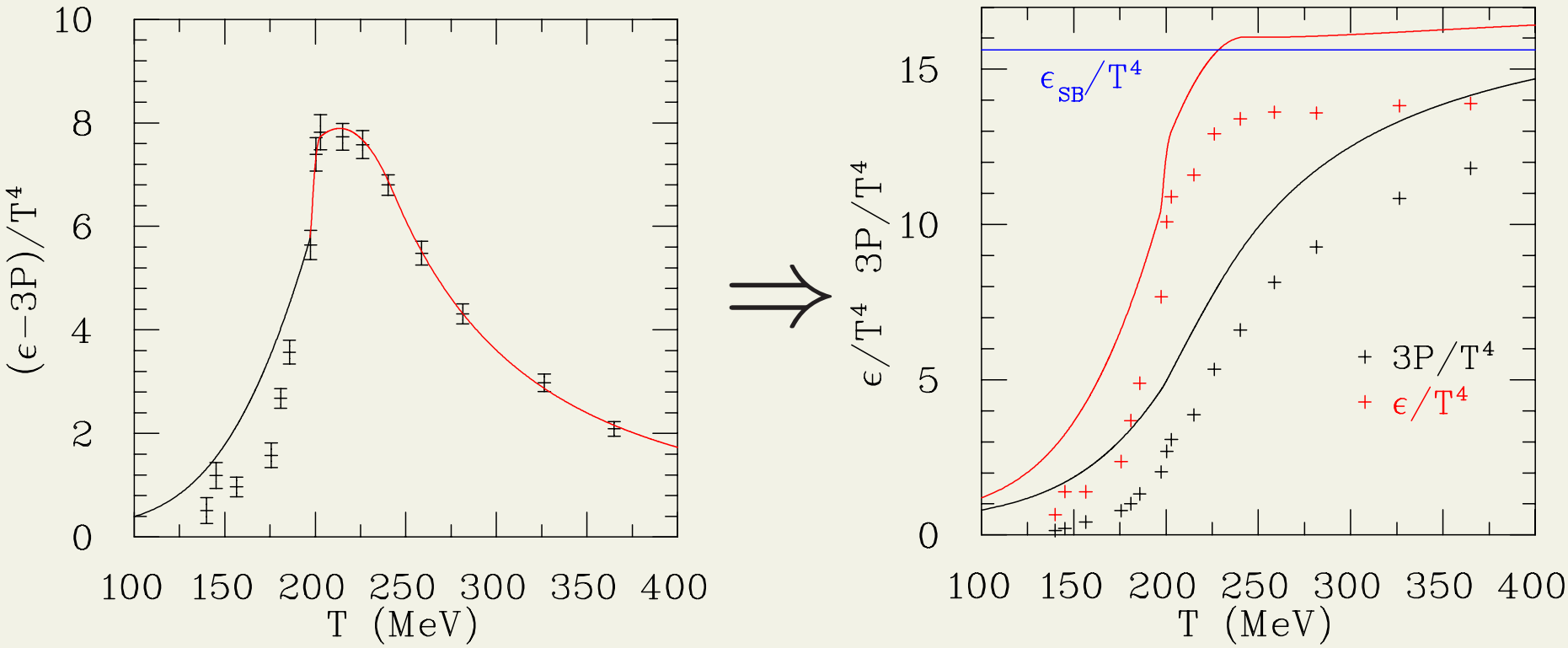


- very little room for modifications in hadron gas
- BUT, what is physical mass spectrum?
- conservative estimate: free particle masses

Phenomenological EoS

- $T < T_{sw}$: **HRG interaction measure (black)**
- $T > T_{sw}$: **Lattice interaction measure (red)**

- lattice $N_\tau = 6$ data, Cheng *et al.* Phys. Rev. D 77, 014511 (2008)



- ϵ **and** P **overshoot Stefan-Boltzmann limit!**
- **Interaction measure too large • peak region sensitive to** N_τ

Procedure for EoS

- **HRG below** $T \approx 180 - 190$ **MeV**
- χ^2 **fit to lattice above** $T = 250$ **MeV using:**

$$\frac{\epsilon - 3P}{T^4} = \frac{d_2}{T^2} + \frac{d_4}{T^4} + \frac{c_1}{T^{n_1}} + \frac{c_2}{T^{n_2}}$$

- **Require that:**

$$\left.\frac{\epsilon - 3P}{T^4}\right|_{T_0}, \quad \left.\frac{\mathrm{d}}{\mathrm{d}T}\frac{\epsilon - 3P}{T^4}\right|_{T_0}, \quad \left.\frac{\mathrm{d}^2}{\mathrm{d}T^2}\frac{\epsilon - 3P}{T^4}\right|_{T_0} \qquad \text{are continuous}$$

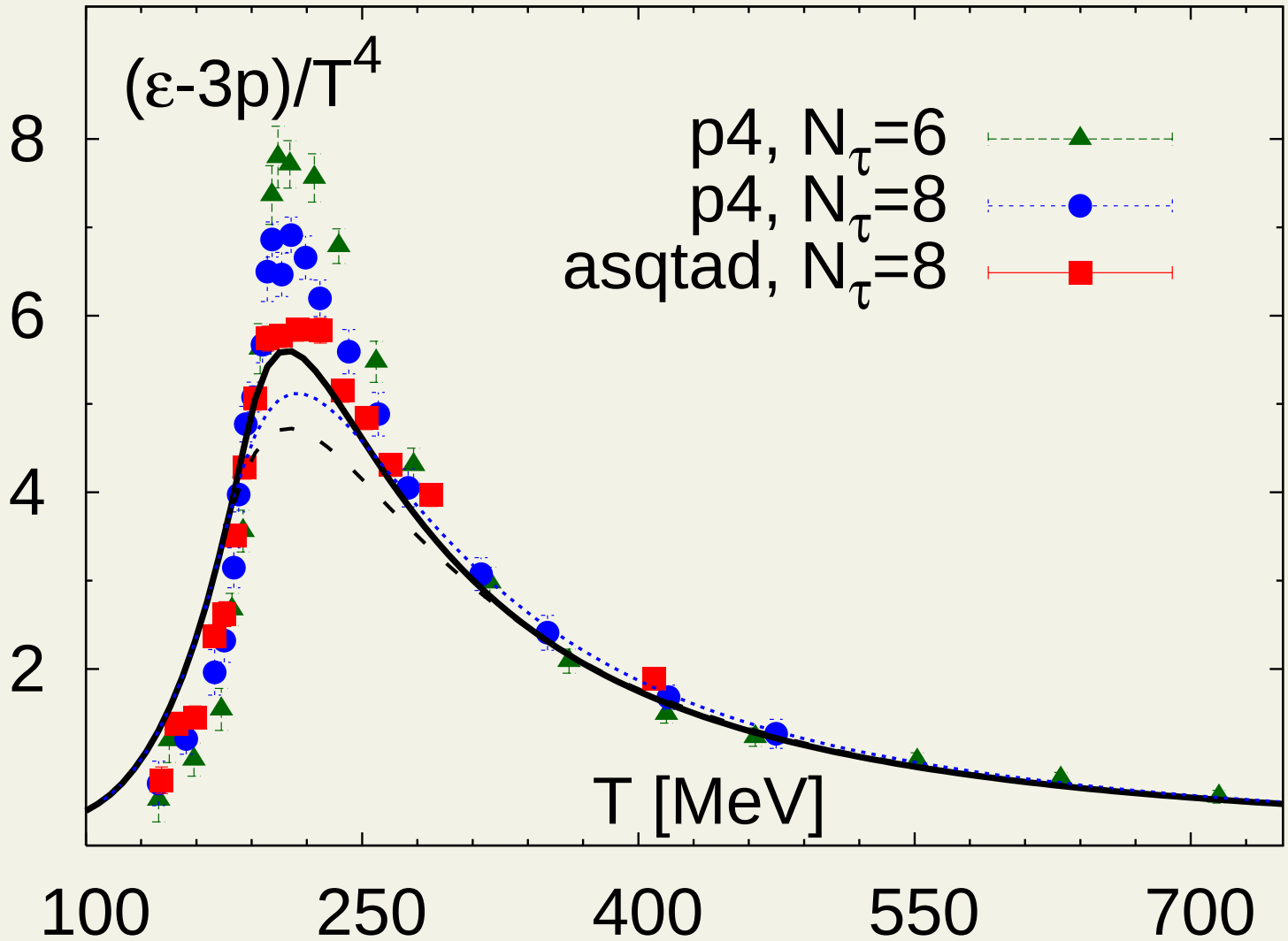
$$\left.\frac{s}{T^3}\right|_{T=800\mathrm{MeV}} = (90\% - 95\%)\,\frac{s_{\mathrm{SB}}}{T^4}$$

$\Longrightarrow$ T_0, d_4, c_1, c_2 **fixed,** d_2, n_1, n_2 **from fit**

- **Or: additional point at** $T = 206$ **MeV**
- **Or: fixed** $T_0 = 170$ **MeV**

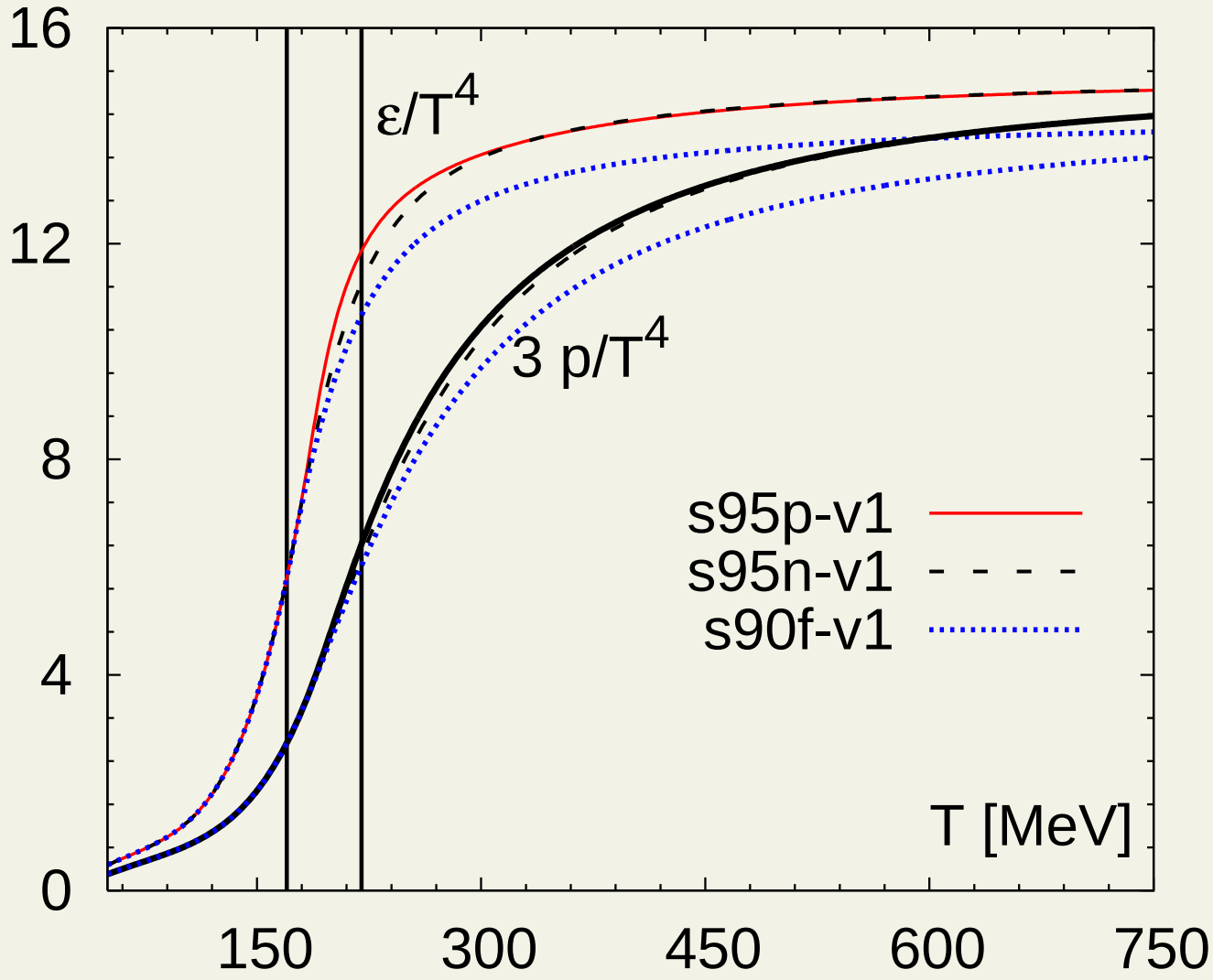
- **see arXiv:0912.2541**

Interaction measure



- **For the 95% $s_{\rm SB}$ limit we get**
  $T_0 = 171.8$ **MeV**, $d_2 = 0.2654$, $d_4 = 6.563 \cdot 10^{-3}$,
  $c_1 = -4.370 \cdot 10^{-5}$, $c_2 = 5.774 \cdot 10^{-6}$, $n_1 = 8$, $n_2 = 9$

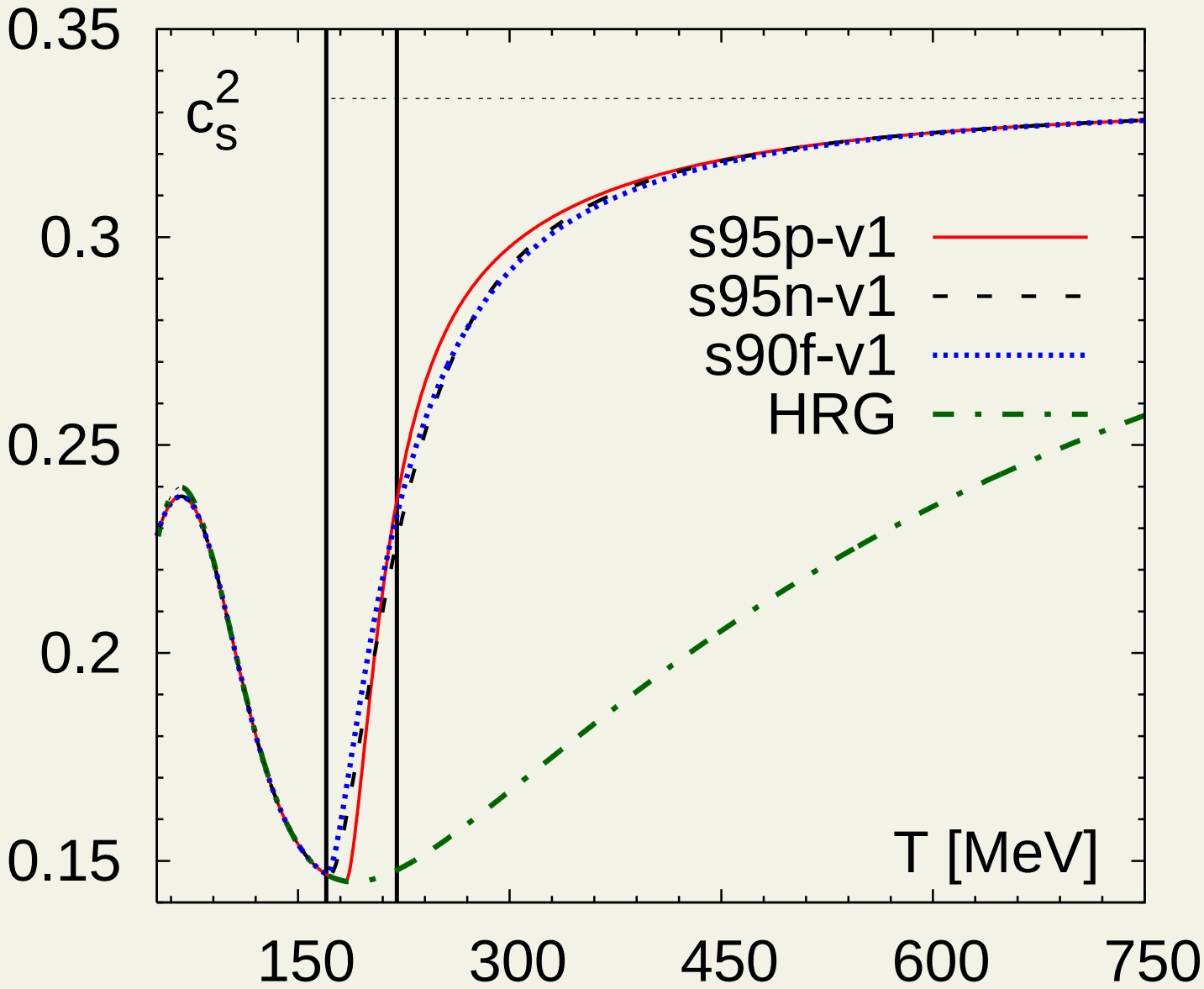
Phenomenological EoS



- **pressure from**

$$\frac{P}{T^4} - \frac{P_0}{T_0^4} = \int_{T_0}^{T} \mathrm{d}T' \, \frac{\epsilon - 3P}{T'^5}$$

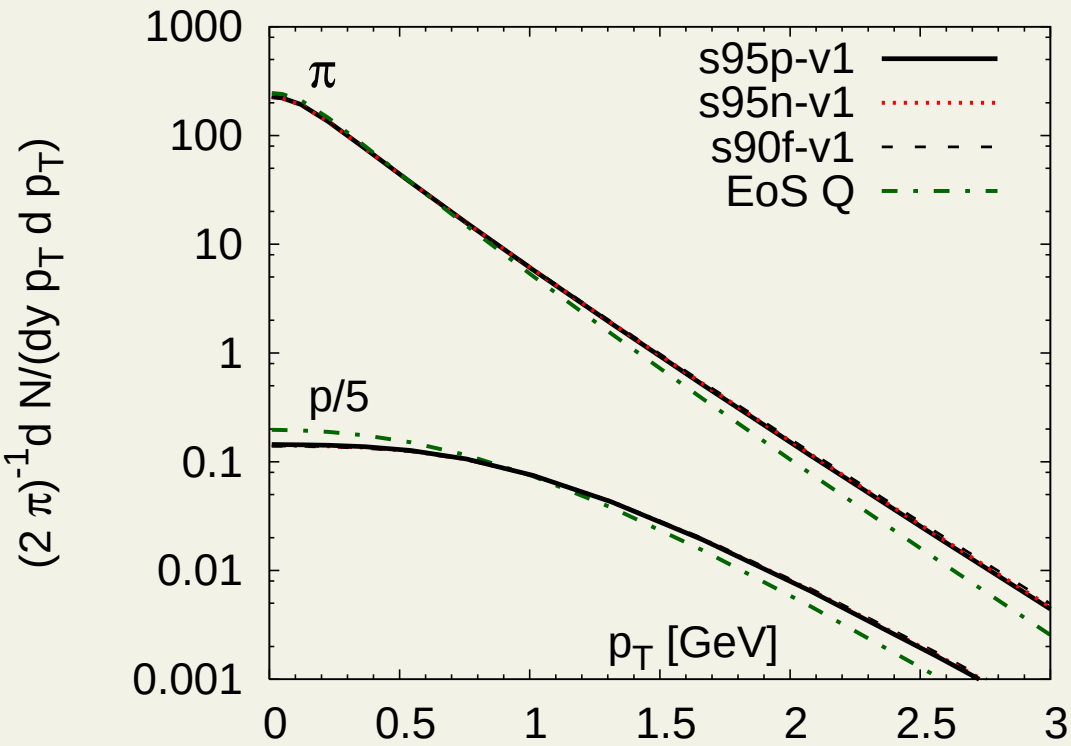
Speed of sound

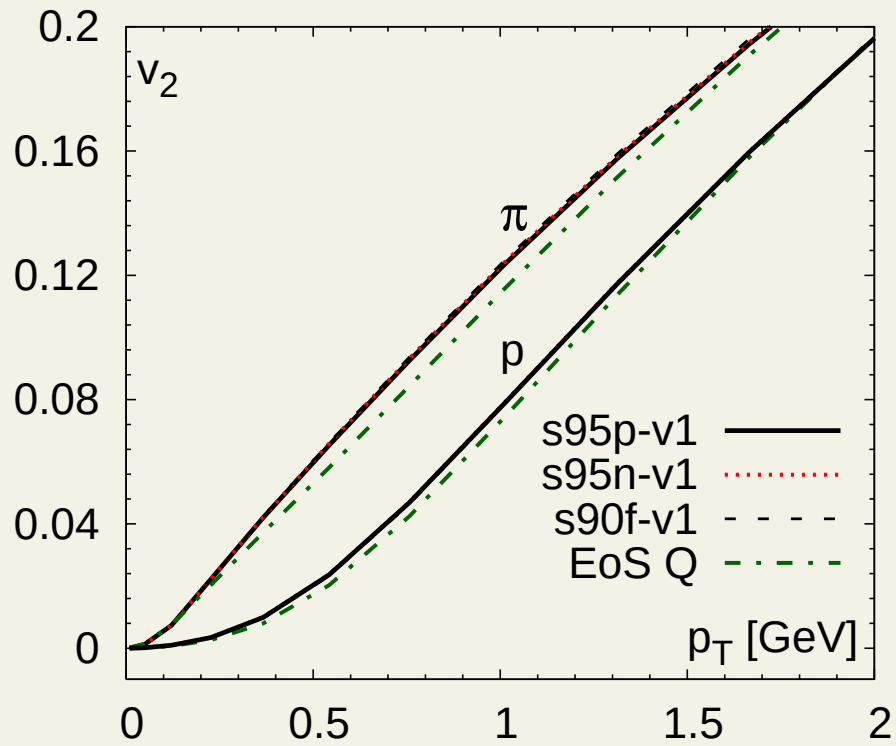


- **no softening below the HRG!**

Effect on flow I

- **ideal fluid,** $b = 7$ **fm**
- **keep everything fixed:**
- $\tau_0 = 0.6$ **fm/**c**,** $T_{dec} = 125$ **MeV**

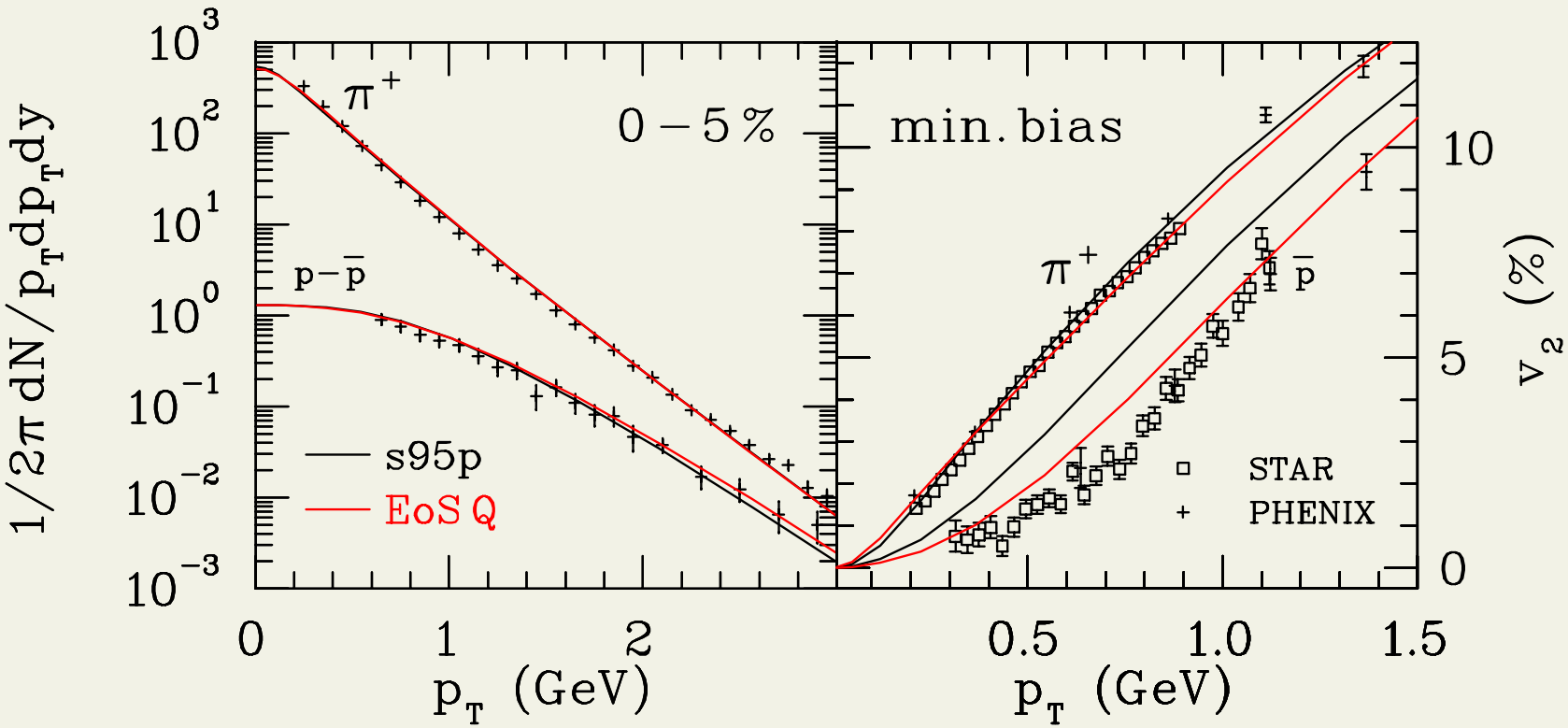




$\Longrightarrow$ **harder EoS, flatter spectra**

Effect on flow II

- **ideal hydro, Au+Au at** $\sqrt{s_{NN}} = 200$ **GeV**
- **chemical equilibrium**



- **s95p:** $T_{dec} = 140$ **MeV**
- **EoS Q: first order phase transition at** $T_c = 170$ **MeV,** $T_{dec} = 125$ **MeV**

Conclusions

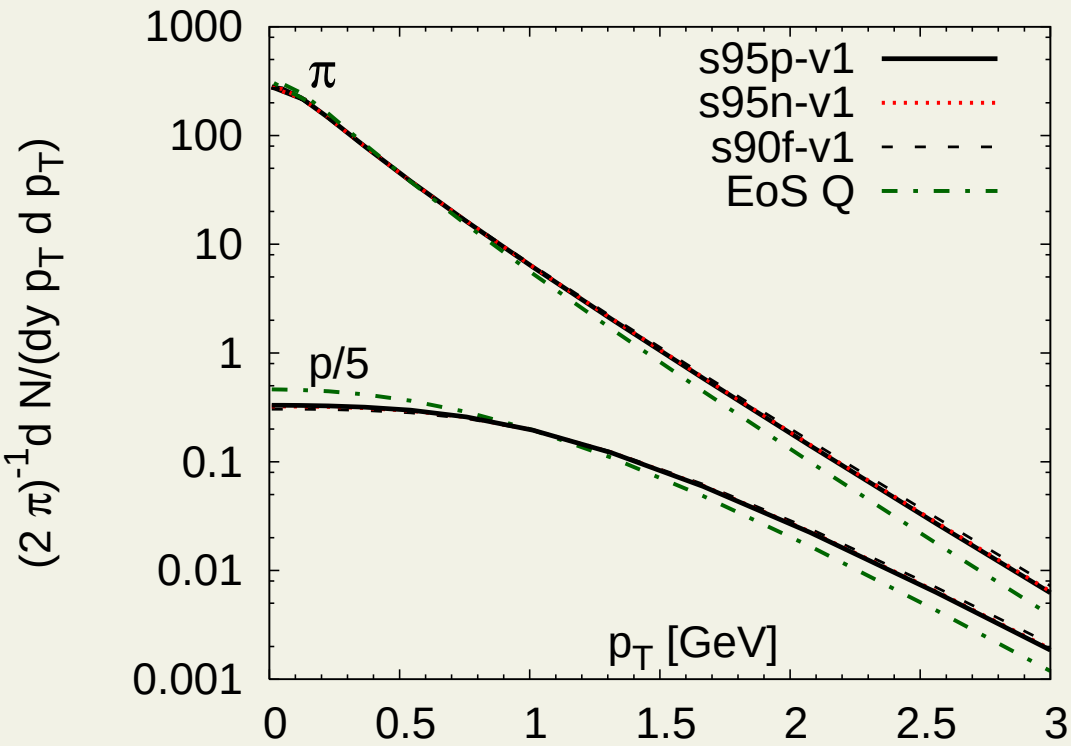
- below T_c lattice and HRG differ because of hadron mass spectrum

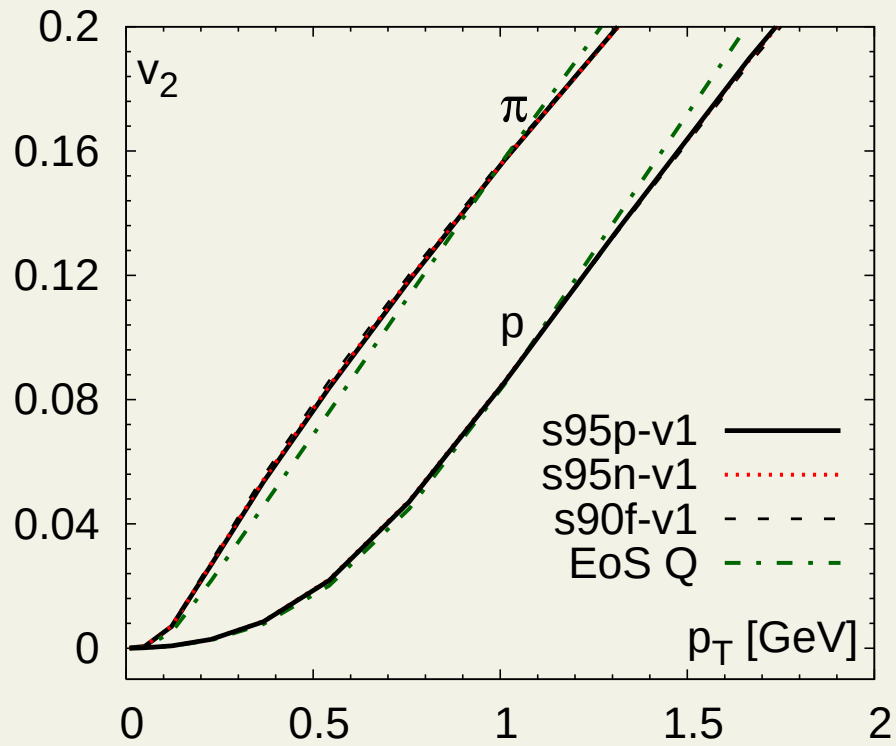
$\Rightarrow$ HRG good description below T_c

- parametrization for EoS
- uncertainty in trace anomaly doesn't matter
- proton $v_2(p_T)$ sensitive to EoS
- but initial state has larger effect
- can we measure η/s and EoS simultaneously?

Chemical non-equilibrium

- **ideal fluid,** $b = 7$ **fm**
- **keep everything fixed:**
- $\tau_0 = 0.2$ **fm/**c**,** $T_{chem} = 150$ **MeV,** $T_{dec} = 120$ **MeV**

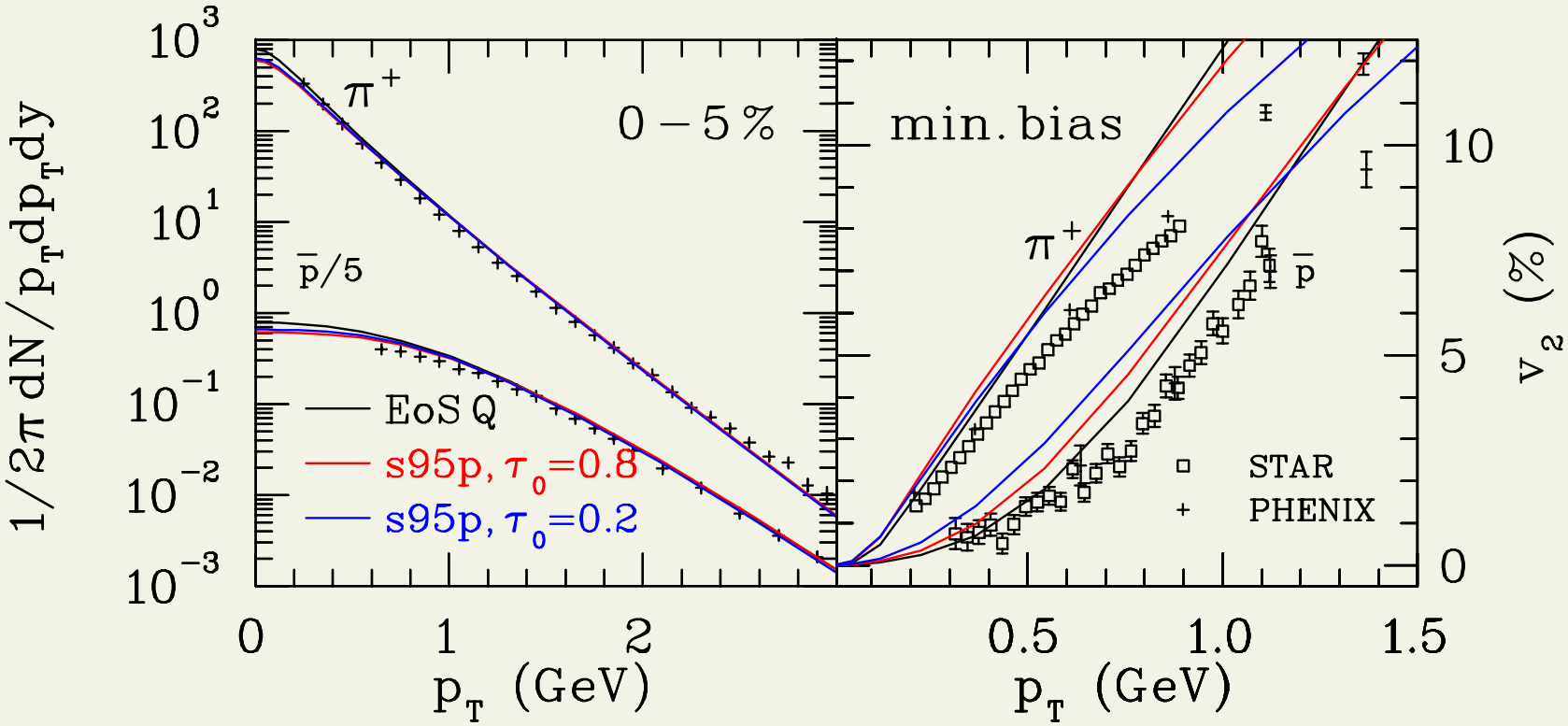




$\Longrightarrow$ **harder EoS, flatter spectra**

Effect on flow III

- **ideal hydro, Au+Au at** $\sqrt{s_{NN}} = 200$ **GeV**
- $T_{\rm chem} = 150$ **MeV**



- **EoS Q:** $T_{dec} = 120$ **MeV**, $s_{\rm ini} \propto N_{bin}$ $\tau_0 = 0.2$ **fm**$/c$
- **s95p,** $\tau_0 = 0.8$**:** $T_{dec} = 120$ **MeV**, $s_{\rm ini} \propto N_{bin}$, $\tau_0 = 0.8$ **fm**$/c$
- **s95p,** $\tau_0 = 0.2$**:** $T_{dec} = 120$ **MeV**, $s_{\rm ini} \propto N_{bin} + N_{part}$, $\tau_0 = 0.2$ **fm**$/c$