Transport Coefficients and Relaxation Times: Hadron Resonance Gas

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References, Inspirations & Requests

References:

- M. Prakash, Manju Prakash, R. Venugopalan & G. Welke,
  Phys. Rep., 227 (1993) 321.
- M. Prakash, Manju Prakash, R. Venugopalan & G. Welke,
  Phys. Rev. Lett., 70 (1993) 1228.
- S. Postnikov & M. Prakash,
  arXiv: 0902.2384v1 [quant.ph] 13 Feb 2009.
- A. Wiranata & M. Prakash,
  QM09 Proceedings; arXiv: 0906.5592v2 [nucl-th] 16 Sep 2009.

Inspirations (Representative, not exhaustive!):

- Classic & modern references in the above articles.
- Current & ongoing work.

Requests:

- Theorists: Please alert us of your work with references.
- Experimentalists: Please talk to us about your ideas.

Contents

Shear viscosity:

- Temperature dependence from general considerations
- Working formulas & results: One- & multi-component gases
- Role of inelastic collisions: Formalism ready; results being readied!

Bulk viscosity:

- Working formulas & results: One- & multi-component gases
- Roles of relativity & inelastic collisions: Results for the former

Relaxation times

Tasks to accomplish

Temperature dependence of η

Estimates from kinetic theory:

$$\eta \approx \frac{1}{3}\frac{\bar{p}}{\bar{\sigma}_t(T)}\,; \quad \bar{\sigma}_t(T) = \frac{1}{3}\int_0^\infty dg\; g^7 e^{-g^2} \int d\theta\; \sin^2\theta\,\sigma\left(\frac{2g}{\sqrt{m\beta}},\theta\right)$$

$\bar{p}$: Mean momentum, g : Scaled (w.r.t. $p_{m.p.}$) relative momentum.
$\bar{\sigma}_t(T)$:= $T(=\beta^{-1})-$dependent transport cross section.

Relativistic expressions are more complicated, but have similar content.

$$\eta \approx \#\,\frac{\bar{p}}{\bar{\sigma}_t(T)} = \#\,\frac{\hbar}{\lambda\,\bar{\sigma}_t(T)} := \frac{\text{action}}{\text{physical volume}}$$

Above, λ is the thermal de-Broglie wavelength.

$$\lambda\,\&\,\eta \;\propto\; \begin{cases} \frac{1}{\sqrt{mT}} \;\&\; \frac{\sqrt{mT}}{\bar{\sigma}_t(T)} & \text{for NR} \\ \frac{1}{T} \;\&\; \frac{T}{\bar{\sigma}_t(T)} & \text{for UR} \end{cases}$$

Illustration with the delta-shell gas

Delta-shell potential : $V(r) = -v\,\delta(r-R)$

Strength & extent : v & R Dilution parameter : nR^3

Single dimesionless physical parameter : $g = (2\mu v\hbar^2)\, R$

Delta-shell scattering length a_{sl} and range parameter r_0:

Cross section at low energies: $\sigma = 4\pi a_{sl}^2$

$$a_{sl} = \frac{Rg}{g-1} \qquad \text{and} \qquad r_0 = \frac{2R}{3}\left(1+\frac{1}{g}\right)$$

Interesting cases :

$$g = \ \to \left\{ \begin{array}{ll} 1 & \text{unitarity limit} \\ 2l+1 & \text{resonances} \\ -\infty & \text{hard-sphere} \end{array} \right.$$

The QM two-body problem analytically solvable (Gottfried, 1966).
For thermal and transport properties, see Postnikov & Prakash (2009).

Asymptotic trends of delta-shell viscosity

$$\frac{\eta}{\tilde{\eta}} \to \begin{cases} \left(\frac{1-g}{g}\right)^2 \left(T/\tilde{T}\right)^{1/2} & \text{for } g \neq 1,3 \text{ (non} - \text{resonant region)} \\ 6\pi \left(T/\tilde{T}\right)^{3/2} & \text{for } g = 1 \text{ (unitarity limit)} \\ \frac{16}{111} \left(T/\tilde{T}\right)^{1/2} & \text{for } g = 3 \; (l = 1 \text{ resonance})\,. \end{cases}$$

Characteristic temperature:

$$\tilde{T} \equiv \frac{2\pi\hbar^2}{k_B m a^2} \text{ or } \frac{T}{\tilde{T}} = \left(\frac{a}{\lambda}\right)^2$$

Effective physical volumes

g	$mn\mathcal{D}$	η	$mn\mathcal{D}/\eta$
1	$\frac{3\sqrt{2}\pi}{4}\frac{\hbar}{\lambda^3}$	$\frac{15\sqrt{2}\pi}{16}\frac{\hbar}{\lambda^3}$	$\frac{4}{5} = 0.80$
3	$\frac{9}{104\sqrt{2}}\frac{\hbar}{\lambda\ R^2}$	$\frac{5}{111\sqrt{2}}\frac{\hbar}{\lambda\ R^2}$	$\frac{999}{520} = 1.92$
$\neq 1, 3$	$\frac{3\sqrt{2}}{16}\frac{\hbar}{\lambda\ a_{sl}^2}$	$\frac{5\sqrt{2}}{32}\frac{\hbar}{\lambda\ a_{sl}^2}$	$\frac{6}{5} = 1.20$

Table 1: First order coefficients of diffusion (times mn), shear viscosity, and their ratios for $T \ll \tilde{T}$ for select g's .

Expectations for relativistic particles

$$\eta \approx \#\, \hbar/(\lambda\, \bar{\sigma}_t(T))$$

Unitary limit : For infinitely strong coupling, $\bar{\sigma}_t(T) \to \infty$, but η remains finite as λ^2 replaces $\bar{\sigma}_t(T)$. Consequently,

$$\eta = \#\, \frac{\hbar}{\lambda^3} = \#\, T^3$$

(see also, Danielewicz & Gyulassy (1985) & earlier works).
Pion gas: For both chiral pions ($m_\pi = 0$) and for massive pions treated using current algebra, $\sigma \propto E^2_{c.m}$; thus, $\sigma_t(T) \propto T^2$. Hence,

$$\eta = \#\, \frac{\hbar T}{T^2} = \#\, \frac{\hbar}{T}$$

With experimental cross sections featuring the $\rho-$resonance prominently,

$$\eta = \#\, \frac{(\sqrt{m_\pi T} \text{ to } T)}{\sigma_t(T)}$$

Example total cross sections: σ's

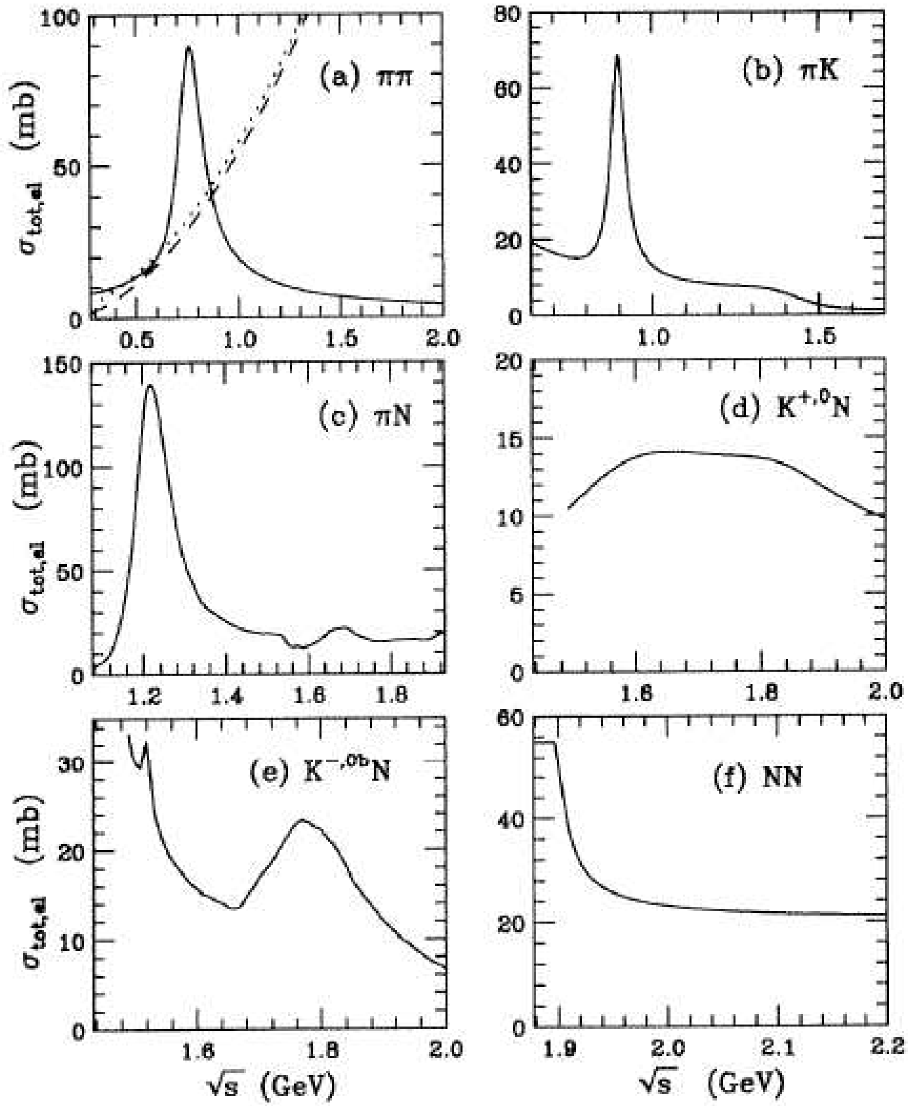


Fig. 16. Isospin averaged total elastic cross-sections versus center of mass energy. (a) $\pi\pi$, eq. (A.3) (solid curve); current algebra result, eq. (A.5) (dashed line); and corresponding chiral result (dotted curve). (b) πK, eq. (A.4). (c) πN, from eq. (A.6). (d) $(K^+, K^0 N)$, from expression analogous to eq. (A.6). (e) $(K^-, \overline{K}^0 N)$, from expression analogous to eq. (A.6). (f) NN, from eq. (A.10).

Viscosities for relativistic particles

First order transport coefficients:

$$
\begin{aligned}
\eta_v^{(1)} &= (9/8)kT\,[zh(\gamma - 5/3) + \gamma)]^2/\omega_0^{(2)} \\
\eta_s^{(1)} &= (5/8)kT\,h^2/(\omega_2^{(2)} - z^{-1}\omega_1^{(2)} + z^{-2}\omega_0^{(2)}/3)
\end{aligned}
$$

with the relativistic omega–integrals $\omega_i^{(s)}$ given by

$$
\begin{aligned}
\omega_i^{(s)} &= \frac{2\pi z^3 c}{K_2^2(z)} \int_0^\infty d\psi \sinh^7\psi \cosh^i\psi K_j(2z\cosh\psi) \\
&\times \int_0^\pi d\Theta \sin\Theta \sigma(\psi,\Theta)(1 - \cos^{(s)}\Theta) \\
j &= 5/2 + (-1)^i/2, \qquad i = 0, \pm 1, \pm 2, \cdots \qquad s = 2, 4, 6, \cdots
\end{aligned}
$$

Rel. mom. $g = mc\sinh\psi$ & and tot. mom. $P = 2mc\cosh\psi$.

Reduced enthalpy $h = K_3(z)/K_2(z)$ & ratio of sp. heats $\gamma = c_p/c_v$.

Inelastic collisions & shear viscosity

Inelastic collisions can induce transitions to excited states or result in new species of particles. For the general formalism, see e.g., Kapusta (2008). In the nonrelativistic case (applicable to heavy resonances) of $i+j \rightarrow k+l$ (Wang et al., 1964),

$$
\begin{aligned}
\frac{1}{\eta} &= \frac{8}{5(\pi m k T)^{1/2}} \frac{1}{(\sum_i e^{-\epsilon_i})^2} \sum_{i,j,k,l} e^{-\epsilon_i-\epsilon_j} \times (I_1+I_2) \\
I_1 &= \int\int\int d\gamma\, e^{-\gamma^2}\gamma^7\, \sigma_{i,j}^{k,l}(\gamma,\theta,\phi) \sin^3\theta\, d\theta\, d\phi \\
I_2 &= \frac{2}{3}\int\int\int d\gamma\, e^{-\gamma^2}\gamma^5\, \Delta\epsilon \left(1-\frac{3}{2}\sin^2\theta\right)\, \sigma_{i,j}^{k,l} \sin\theta\, d\theta\, d\phi \\
\epsilon_i &= \frac{E_i}{kT}; \qquad \Delta\epsilon = \epsilon_k+\epsilon_l-\epsilon_i-\epsilon_j \\
\gamma_{i,j} &= \left(\frac{\mu_{i,j}}{2kT}\right)^{1/2} \left(\frac{p_j}{m_j}-\frac{p_i}{m_i}\right)
\end{aligned}
$$

Role of inelastic collisions in bulk viscosity

Internal excitations and creation of new species of particles contribute to bulk viscosity. For nonrelativistic particles (Wang et al., (1964)),

$$\eta_v = \frac{nk^2T}{c_v^2} \sum_l c_v^{(l)} \tau_l$$

For each species l, the relaxation time τ is

$$\frac{1}{\tau} = 2nc_v^{int} \left(\frac{kT}{\pi m}\right)^{1/2} \frac{1}{(\sum_i e^{-\epsilon_i})^2} \sum_{i,j,k,l} (\Delta\epsilon)^2 \, e^{-\epsilon_i - \epsilon_j} \times I$$

$$I = \int\int\int d\gamma \, e^{-\gamma^2} \gamma^3 \, \sigma_{i,j}^{k,l}(\gamma, \theta, \phi) \sin\theta \, d\theta \, d\phi$$

Generalization to the relativistic case in van Weert et al. (1973).
Numerical results for HIP under progress.

Results for chiral & current algebra σ's

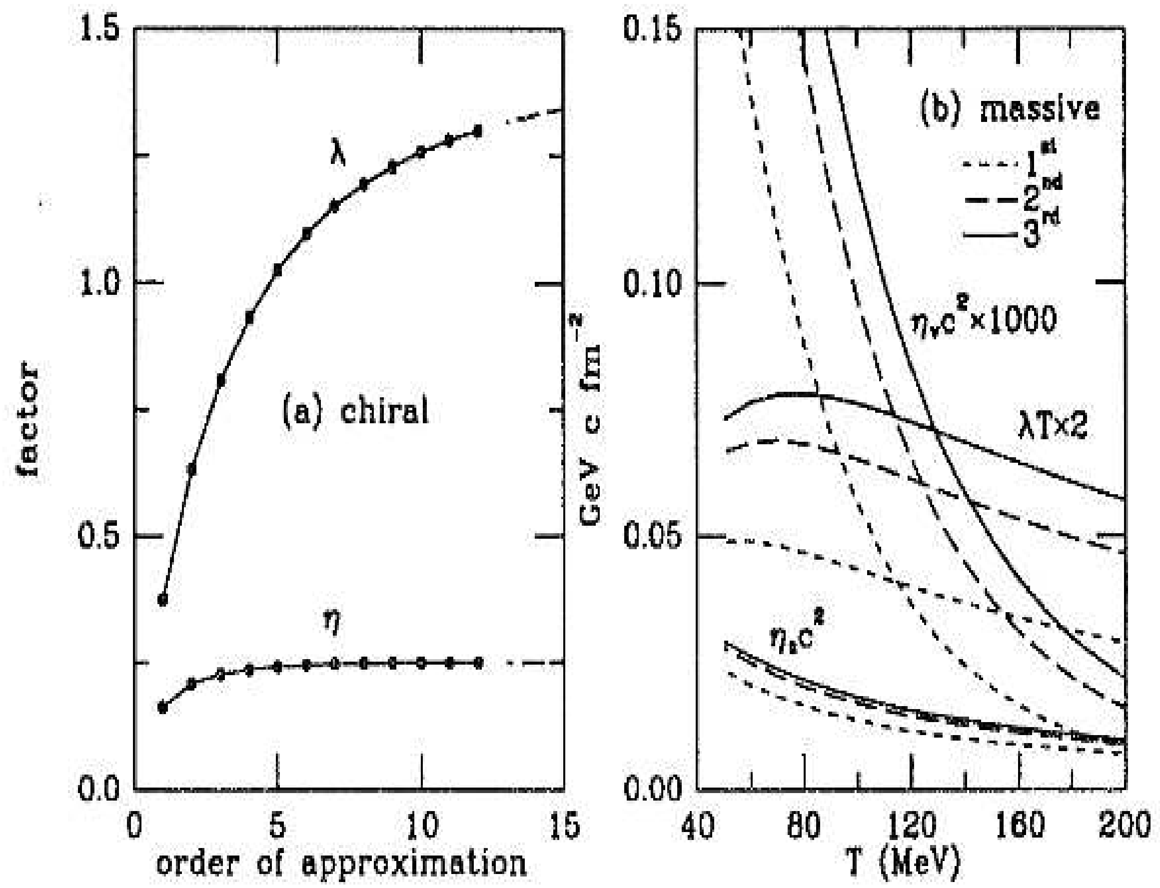


Fig. 8. Convergence of successive approximations to the transport coefficients of (a) chiral pions [see eq. (4.20) for T dependence] and (b) of massive pions using the current algebra result [eq. (A.5)] for $\pi\pi$ scattering.

$$\eta_s c^2 \simeq (0.25) \cdot \left(\frac{f_\pi}{\hbar c}\right)^2 \left(\frac{f_\pi}{kT}\right) f_\pi c \simeq 7.5 \left(\frac{100\ \mathrm{MeV}}{kT}\right)\ \mathrm{MeV\ fm^{-2}\ c}$$

Pion gas: results with experimental σ's

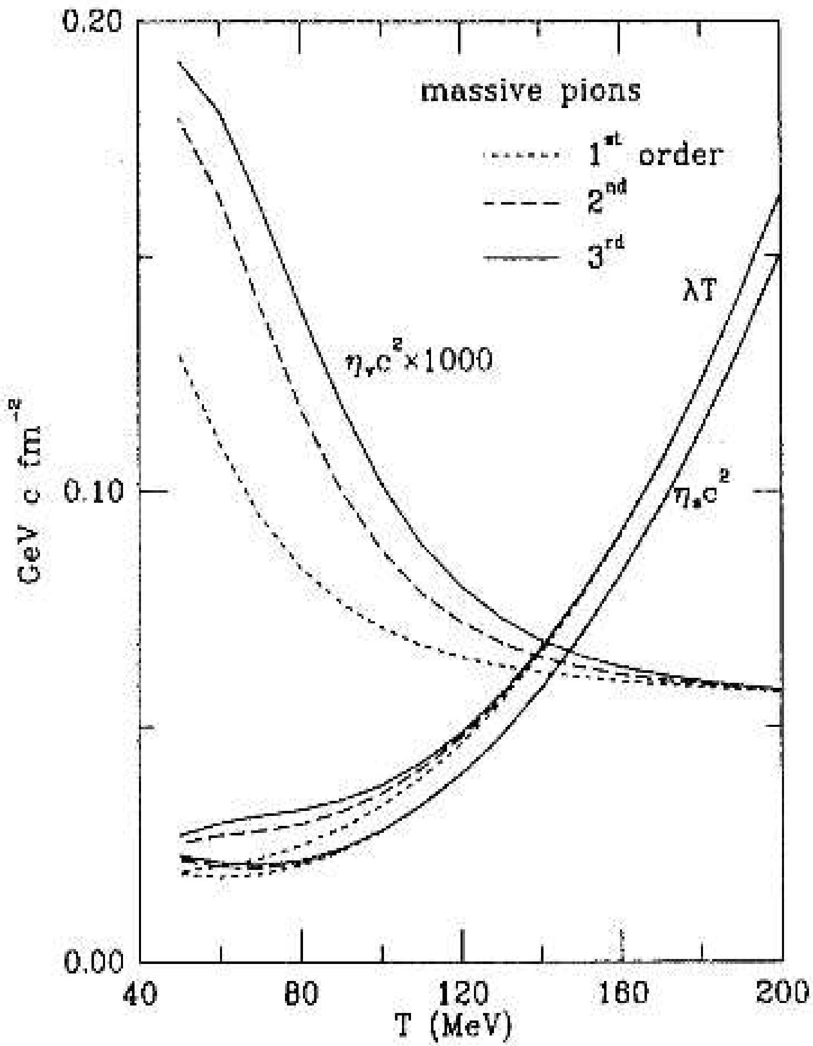


Fig. 9. Bulk viscosity $\eta_v c^2$, shear viscosity $\eta_s c^2$, and heat conductivity times temperature λT of massive pions, from eq. (C.1) through eq. (C.9), up to the third order of the Chapman–Enskog approximation using the experimental cross-sections [eq. (A.3)] for $\pi\pi$ scattering.

Note $T-$dependence & convergence of results with experimental σ's.

Formulas for binary mixtures

Bulk viscosity:

$$\eta_v^{(1)} = \frac{kTM^2n^2}{16\rho_1\rho_2}\frac{\alpha_2^2}{\omega_{1200}^{(1)}(\sigma_{12})}\,,$$

where $\alpha_2 = x_1[\gamma_1 - \gamma]/[\gamma_1 - 1]$.

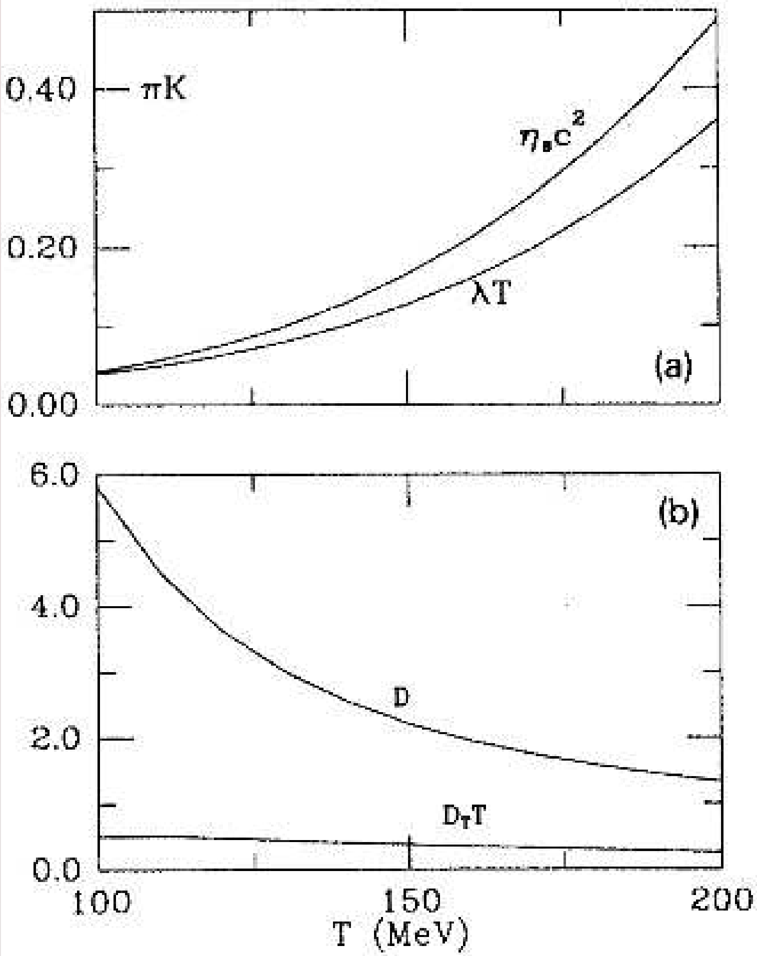
Diffusion coefficient:

$$D^{(1)} = \frac{3MkT}{16nm_1m_2}\frac{1}{\left(\omega_{1100}^{(1)} - 3(z_1+z_2)^{-1}\omega_{1200}^{(1)}\right)}\,.$$

Shear viscosity:

$$\begin{aligned}
\eta_s^{(1)} &= \frac{\rho^2k^2T^2}{10}\left(c_{-1}\gamma_{-1} + c_1\gamma_1\right)\,,\\
\gamma_1 &= -10c_1h_1/c^2\,, \qquad \gamma_{-1} = -10c_2h_2/c^2\,.
\end{aligned}$$

Results for binary mixtures



. 10. Transport coefficients in a πK mixture using
. (C.6) through eq. (C.9). (a) Shear viscosity and
rmal conductivity. (b) Diffusion and thermal diffusion
fficients.

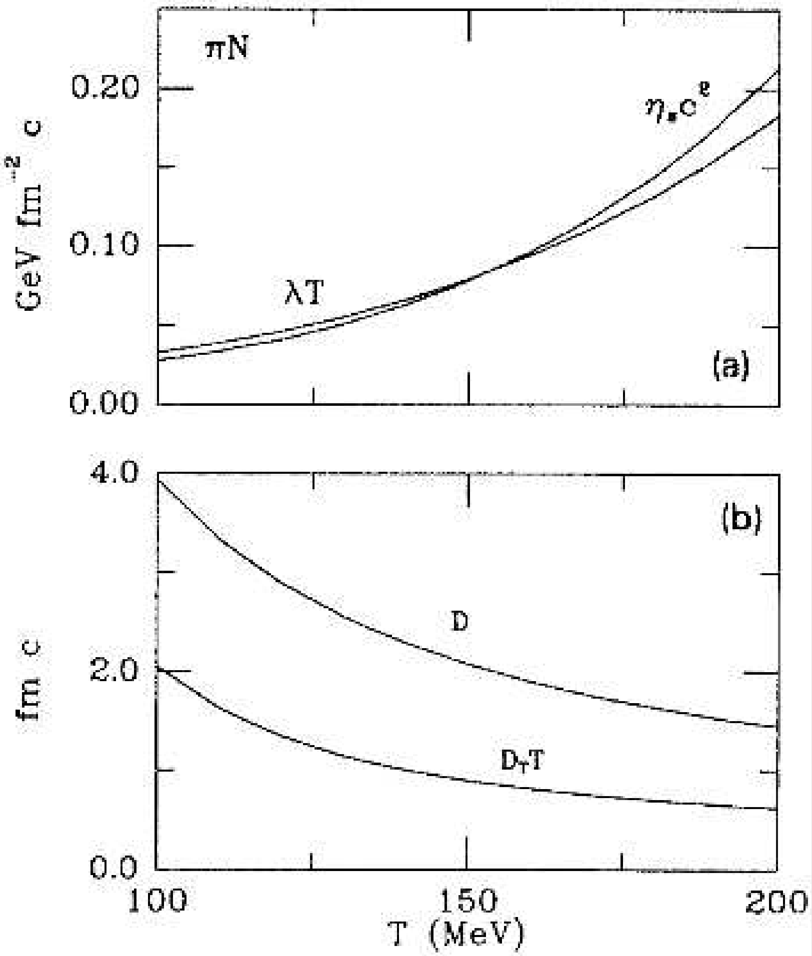


Fig. 11. Transport coefficients in a πN mixture. (a) Shear viscosity and thermal conductivity. (b) Diffusion and thermal diffusion coefficients.

Formulas for relaxation times

From the relativistic generalization of Grad's moment method, the coefficients of the time derivatives of the various fluxes yield

$$\tau_v = \frac{\eta_v}{nkT}\,\alpha_0'\,, \qquad \tau_\lambda = \frac{\lambda T c^{-2}}{nkT}\,\beta_1'\,, \qquad \tau_\eta = \frac{2\eta_s}{nkT}\,\gamma_2'\,,$$

where

$$\begin{aligned}
\alpha_0' &= \frac{(10-7\gamma)\hat{h} + z^2(5/3-\gamma)}{\left[(5/3-\gamma)\hat{h}-\gamma\right]^2} \\
\beta_1' &= \left[(\gamma-1)/\gamma\hat{h}\right]\left[5(\gamma-1)\hat{h}^2/\gamma - z^2\right]\,, \\
\gamma_2' &= (3+z^2/2\hat{h})/\hat{h}\,.
\end{aligned}$$

Results for relaxation times

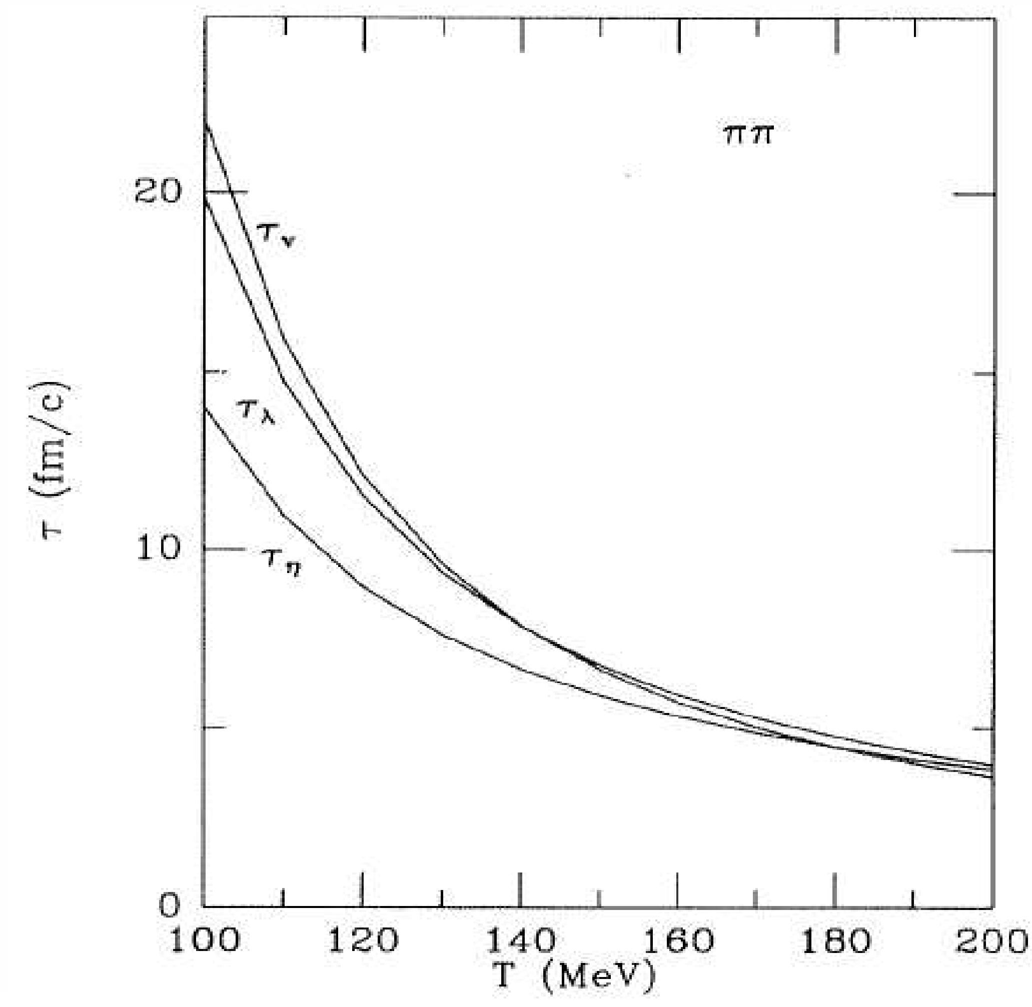


Fig. 12. Relaxation times of bulk and shear viscous flow, and also heat flow in a gas of interacting pions from eq. (5.2).

Note: The relaxation times converge at high T, but diverge at low T.

Momentum relaxation times in a mixture

For the kth component,

$$\tau_k^\eta = 2\eta_s \tau'_{k,\eta} \equiv \frac{2\eta_s}{nkT} \frac{\left[3 + (z_k^2/2\hat{h}_k)\right]}{\left(\sum_{k=1}^N x_k \hat{h}_k\right)} .$$

Above, η_s is the shear viscosity of the mixture.

For shear flow,

$$R^\eta = \frac{\tau_1^\eta}{\tau_2^\eta} = \frac{6 + z_1 K_2(z_1)/K_3(z_1)}{6 + z_2 K_2(z_2)/K_3(z_2)} , \qquad z_i = \frac{m_i}{kT}$$

For $z_i \ll 6$, $R^\eta \to 1$, whereas for $z_i \gg 6$, $R^\eta \to m_1/m_2$.

Relaxation times for mixtures

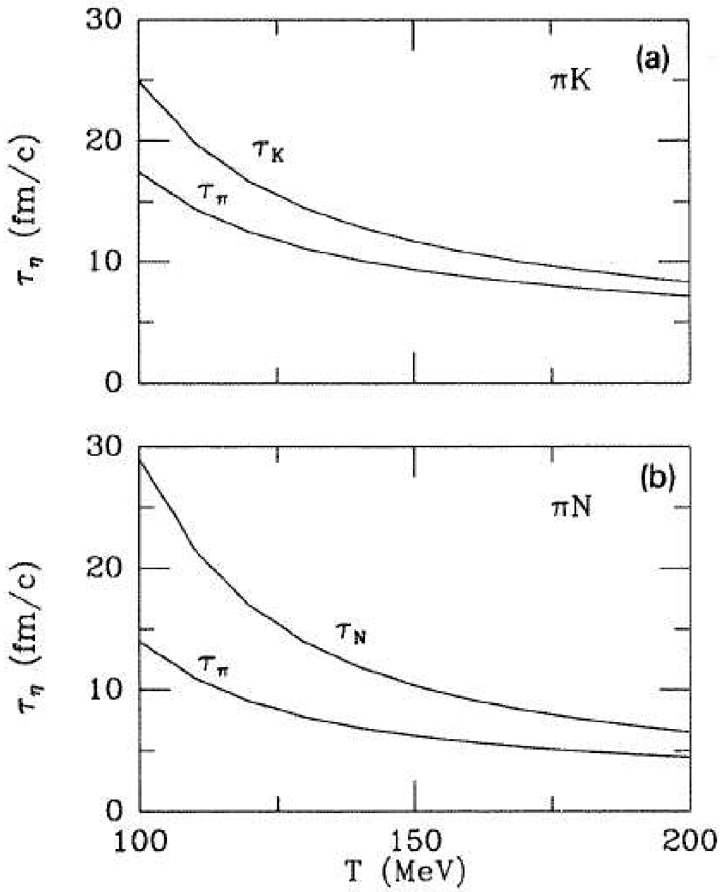


Fig. 13. Relaxation times of heat flow in (a) a πK mixture and (b) a πN mixture from eq. (5.4).

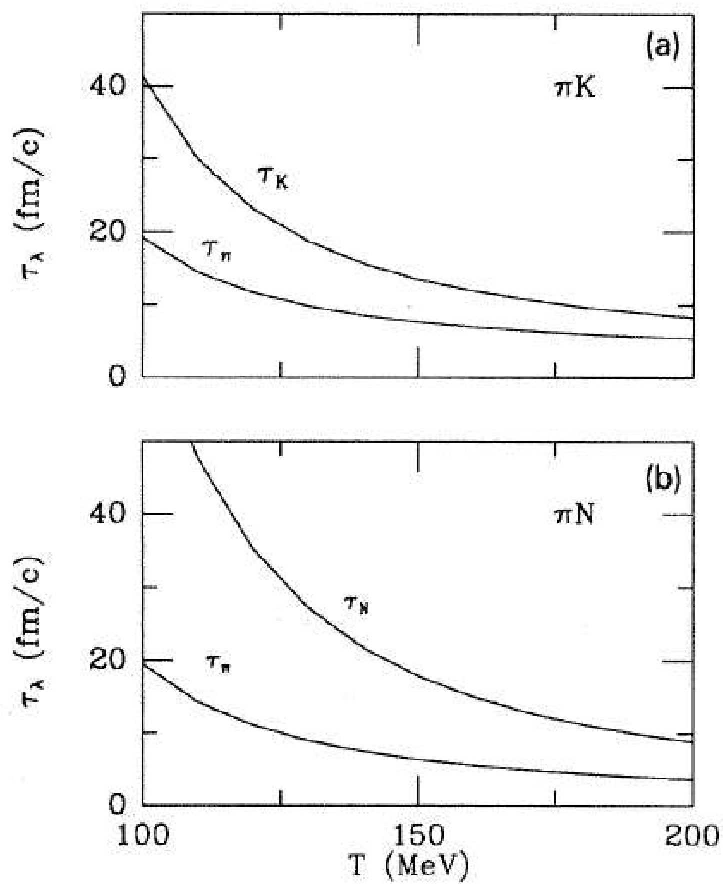


Fig. 14. Relaxation times of shear viscous flow in (a) a πK mixture and (b) a πN mixture from eq. (5.6).

Note: The ratio of nucleon to kaon relaxation times is nearly unity at $T = 200$ MeV, whereas at $T = 100$ MeV, the ratio is nearly m_N/m_K.

Tasks to accomplish

- Extension to N-component mixtures; formalism ready
  (calculations for hadronic mixtures await completion.)

- Comparison of Chapman-Enskog & Greek-Kubo calculations
  (collaboration with Duke Univ. forged).

- Preparation of tables for use in hydrodynamic calculations
  (require inputs from hydrodynamicists).