Long Range Rapidity Correlations

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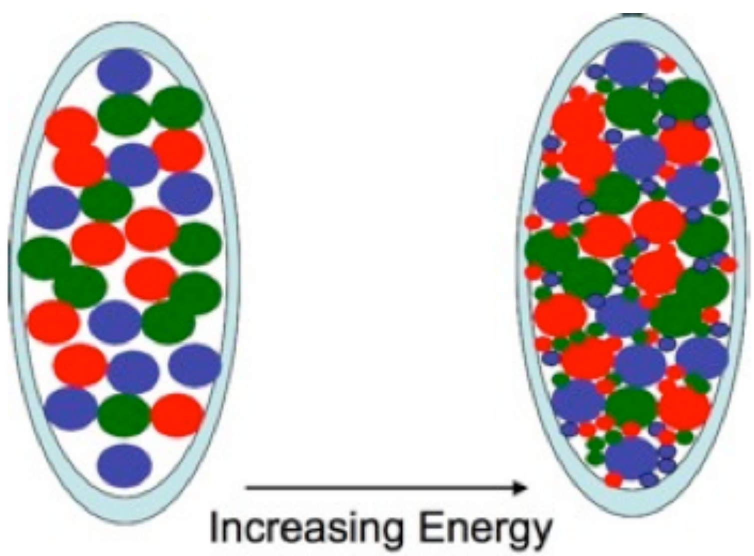
Contents

K.D., Francois Gelis, Tuomas Lappi, Raju Venugopalan

arXiv:0911.2720

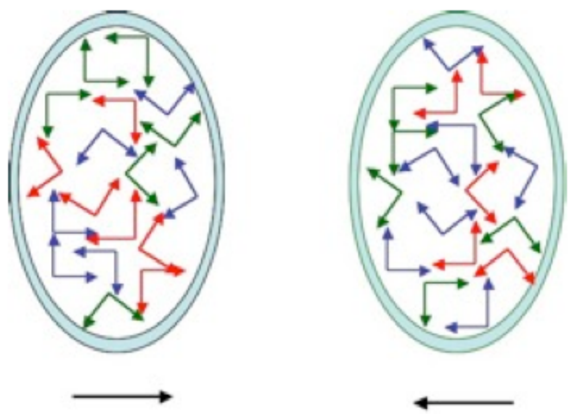
Color Glass Condensate

- Parton of size $\dfrac{1}{k_\perp^2}$ has cross section $\sigma \sim \alpha_s \dfrac{1}{k_\perp^2}$
- For a hadron of size $S_\perp$ geometric overlap occurs when $\dfrac{dN}{dy}\sigma \sim S_\perp$
- Saturation momentum: $k_{\perp,\mathrm{max}}^2 \equiv Q_S^2 \sim \dfrac{\alpha_s}{S_\perp}\dfrac{dN}{dy}$

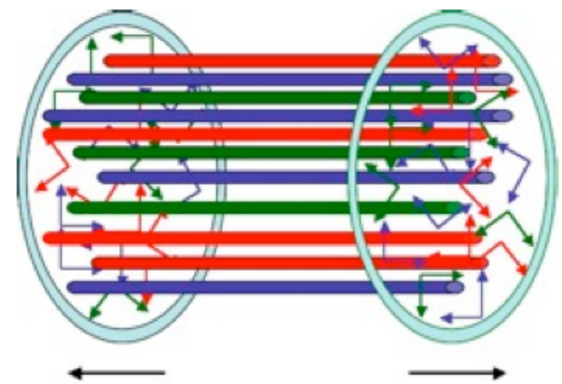


Glasma Flux Tubes

- Before



After



$$N_{\text{f.t.}} \sim \alpha_s \frac{dN}{dy}$$

~ 300 in Au-Au

These flux tubes generate long range 2 particle correlations.

$$\tau_f = \tau_{\text{f.o.}} \exp\left(-\frac{1}{2}\Delta y\right)$$

Previous Calculations

- Two and three particle correlations in glasma flux tube model yield a geometric interpretation of correlation strength

$$\overline{C}_2 \sim \frac{\kappa_2}{S_\perp Q_S^2}$$

Dumitru, Gelis, McLerran, Venugopalan
NPA, aX:0804.3858
κ_2 computed in: Lappi, Srednyak, Venugopalan, aX:0911.2069

$$\overline{C}_3 \sim \frac{\kappa_3}{(S_\perp Q_S^2)^2}$$

KD, D. Fernandez-Fraile, R. Venugopalan
NPA, aX:0902.4435

- Generalized to $\overline{C}_N$ *Gelis, Lappi, McLerran, NPA, aX:0905.3234*

- Collimation of signal into the near-side ridge generated by transverse flow *George Moschelli (NEXT TALK), Sean Gavin*

However, all these works have no rapidity dependence.
Goal of this work: address rapidity dependence.

Multi-particle production at leading log order

- Leading log calculation $\sum_n \alpha_s^n \ln^n\left(\frac{1}{x}\right)$
- Inclusive observables can be expressed in factorized form

$$\left\langle\mathcal{O}\right\rangle_{\text{LLog}} = \int \left[D\Omega_1(\bar{y},\mathbf{x}_\perp) D\Omega_2(\bar{y},\mathbf{x}_\perp)\right] W\big[\Omega_1(\bar{y},\mathbf{x}_\perp)\big] W\big[\Omega_2(\bar{y},\mathbf{x}_\perp)\big]\, \mathcal{O}_{\text{LO}}$$

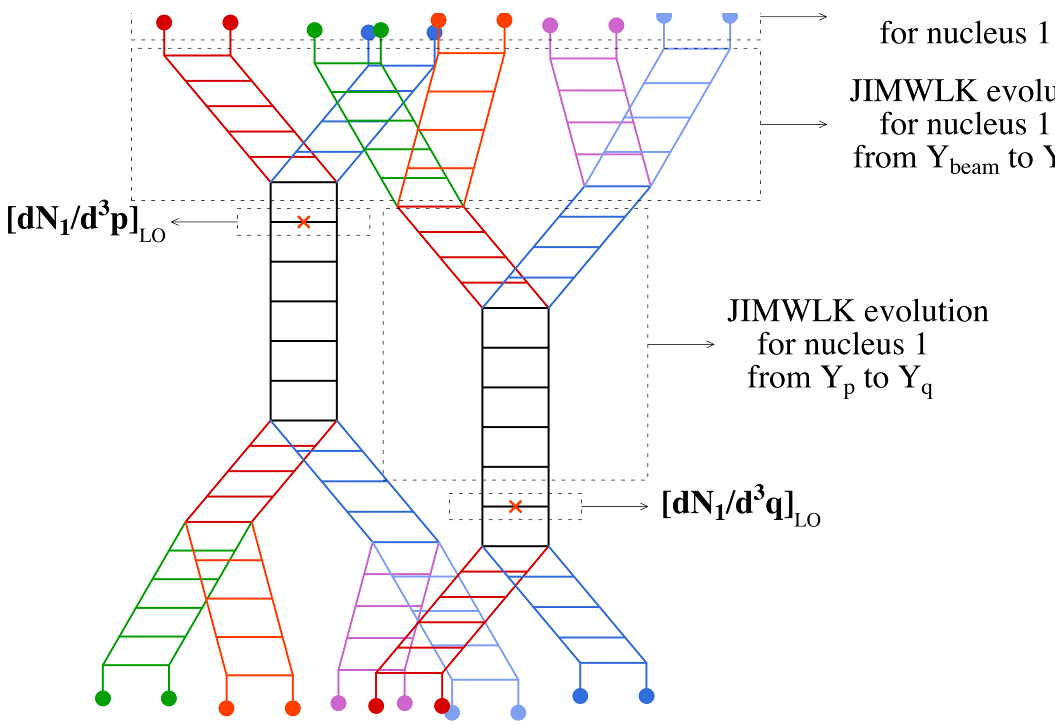
$$\Omega_{1,2}(\bar{y},\mathbf{x}_\perp) \equiv \mathrm{P}\,\exp ig \int_0^{x_y^\mp} \mathrm{d}z^\mp \,\frac{1}{\nabla_\perp^2}\rho_{1,2}(z^\mp,\mathbf{x}_\perp)$$

F. Gelis, T. Lappi and R. Venugopalan,
High energy factorization and long range rapidity correlations in the Glasma I, II, III
PRD, aX:0810.4829 [hep-ph]

Double Inclusive Spectra

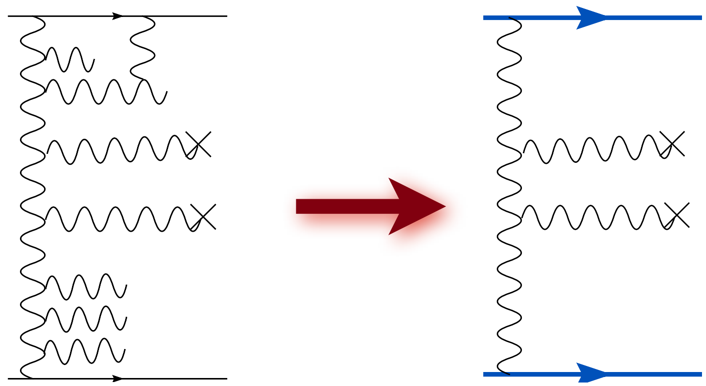
- Factorized form for double inclusive spectra

$$\left\langle \frac{dN_n}{d^2p_{\perp,1}dy_1\cdots d^2p_{\perp,n}dy_n} \right\rangle = \int [d\rho_A d\rho_B] W_{y_{\rm beam}-Y}[\rho_A] W_{y_{\rm beam}+Y}[\rho_B]$$
$$\times \frac{dN_{\rm LO}}{d^2p_{\perp,1}dy_1}(\rho_A,\rho_B)\cdots\frac{dN_{\rm LO}}{d^2p_{\perp,n}dy_n}(\rho_A,\rho_B)$$



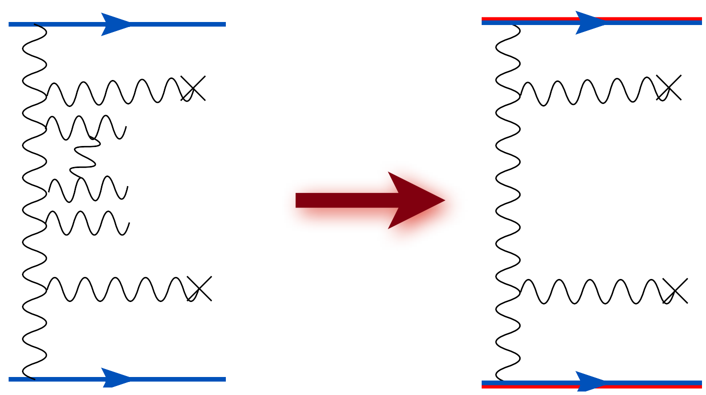
Quantum Evolution

$$\left\langle \frac{d^2N}{d^2p_{\perp,1}dy_1 d^2p_{\perp,2}dy_2} \right\rangle = \int [d\rho_A d\rho_B] W^A_{y_{\rm beam}-y_1} W^B_{y_{\rm beam}+y_2} \frac{dN_{\rm LO}}{d^2p_{\perp,1}dy_1} \frac{dN_{\rm LO}}{d^2p_{\perp,n}dy_n}$$



$$\Delta y \ll 1/\alpha_S$$

$$\left\langle \frac{d^2N}{d^2p_{\perp,1}dy_1 d^2p_{\perp,2}dy_2} \right\rangle = \int [d\rho_A d\rho_B d\rho_A d\rho_B] W^A_{y_{\rm beam}-y_1} W^B_{y_{\rm beam}+y_2} \mathcal{G}_{y_1,y_2} \mathcal{G}_{y_2,y_1} \frac{dN_{\rm LO}}{d^2p_{\perp,1}dy_1} \frac{dN_{\rm LO}}{d^2p_{\perp,n}dy_n}$$



Leading Order form

- Leading order single inclusive spectra with fixed sources

$$\left.\frac{\mathrm{d}N_1\left[\rho_1,\rho_2\right]}{\mathrm{d}^2\mathbf{p}_\perp\mathrm{d}y_p}\right|_{\mathrm{LO}} = \frac{1}{16\pi^3}\lim_{x_0,y_0\to+\infty}\int \mathrm{d}^3\mathbf{x}\,\mathrm{d}^3\mathbf{y}\; e^{ip\cdot(x-y)}\,(\partial_x^0 - iE_p)(\partial_y^0 + iE_p)$$
$$\times\sum_{\lambda,a}\epsilon_\lambda^\mu(\mathbf{p})\epsilon_\lambda^\nu(\mathbf{p})\;A_\mu^a(x)[\rho_1,\rho_2]\;A_\nu^a(y)[\rho_1,\rho_2]\;.$$

- at large $\mathrm{k_T}$ use perturbative solution to YM eqn.

$$p^2 A_a^\mu(\mathbf{p}) = -if_{abc}\,g^3\int\frac{\mathrm{d}^2\mathbf{k}_\perp}{(2\pi)^2}\,L^\mu(\mathbf{p},\mathbf{k}_\perp)\,\frac{\tilde{\rho}_1^b(\mathbf{k}_\perp)\tilde{\rho}_2^c(\mathbf{p}_\perp-\mathbf{k}_\perp)}{\mathbf{k}_\perp^2\,(\mathbf{p}_\perp-\mathbf{k}_\perp)^2}$$

- with non-local source correlations

$$\left\langle\widetilde{\rho}^a(\mathbf{k}_\perp)\widetilde{\rho}^b(\mathbf{k}'_\perp)\right\rangle = (2\pi)^2\mu_A^2(y)\,\delta^{ab}\delta(\mathbf{k}_\perp-\mathbf{k}'_\perp)$$

Final Result

- Color sources related to UGD

$$\phi_A(y, \mathbf{k}_\perp) = g^2 \pi (\pi R_A^2)(N_c^2 - 1) \frac{\mu_A^2(y, \mathbf{k}_\perp)}{\mathbf{k}_\perp^2}$$

- Final result is

$$\boxed{\begin{aligned} C(\mathbf{p}, \mathbf{q}) = \; & \frac{\alpha_s^2}{16\pi^{10}} \frac{N_c^2(N_c^2-1)S_\perp}{d_A^4 \; \mathbf{p}_\perp^2 \mathbf{q}_\perp^2} \int \mathrm{d}^2\mathbf{k}_{1\perp} \times \\ & \Big\{ \Phi_{A_1}^2(y_p, \mathbf{k}_{1\perp}) \Phi_{A_2}(y_p, \mathbf{p}_\perp - \mathbf{k}_{1\perp}) \left[\Phi_{A_2}(y_q, \mathbf{q}_\perp + \mathbf{k}_{1\perp}) + \Phi_{A_2}(y_q, \mathbf{q}_\perp - \mathbf{k}_{1\perp}) \right] \\ & + \Phi_{A_2}^2(y_q, \mathbf{k}_{1\perp}) \Phi_{A_1}(y_p, \mathbf{p}_\perp - \mathbf{k}_{1\perp}) \left[\Phi_{A_1}(y_q, \mathbf{q}_\perp + \mathbf{k}_{1\perp}) + \Phi_{A_1}(y_q, \mathbf{q}_\perp - \mathbf{k}_{1\perp}) \right] \Big\} \end{aligned}}$$

- and probes both target and projectile

$$\begin{aligned} x_{1p} &= \frac{p_\perp}{\sqrt{s}} e^{-y_p} \;\; ; \;\; x_{1q} = \frac{q_\perp}{\sqrt{s}} e^{-y_q} \\ x_{2p} &= \frac{p_\perp}{\sqrt{s}} e^{+y_p} \;\; ; \;\; x_{2q} = \frac{q_\perp}{\sqrt{s}} e^{+y_q} \end{aligned}$$

Nuclear wave-function & evolution

- large N_c: adj. UGD written in terms of fund. Wilson lines

$$\Phi_{A_{1,2}}(x,k_\perp) = \frac{\pi N_c k_\perp^2}{2\,\alpha_s} \int_0^{+\infty} r_\perp \mathrm{d}r_\perp \; J_0(k_\perp r_\perp)\; \left[1 - T_{A_{1,2}}(r_\perp, \ln(1/x))\right]^2$$

- Dipole scattering amplitude given from BK equation

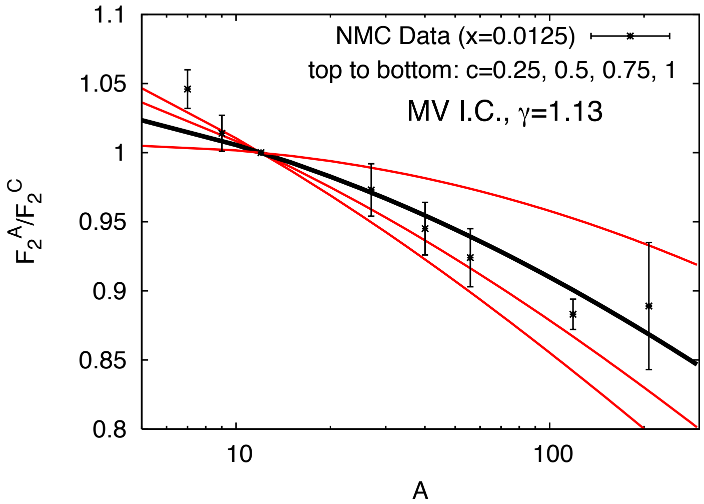
$$\frac{\partial T(\mathbf{r},Y)}{\partial Y} = \int \mathrm{d}\mathbf{r}_1 \; \mathcal{K}_{\mathrm{LO}}(\mathbf{r},\mathbf{r}_1,\mathbf{r}_2) \times$$
$$\left[T(\mathbf{r}_1,Y) + T(\mathbf{r}_2,Y) - T(\mathbf{r},Y) - T(\mathbf{r}_1,Y)\,T(\mathbf{r}_2,Y)\right]$$

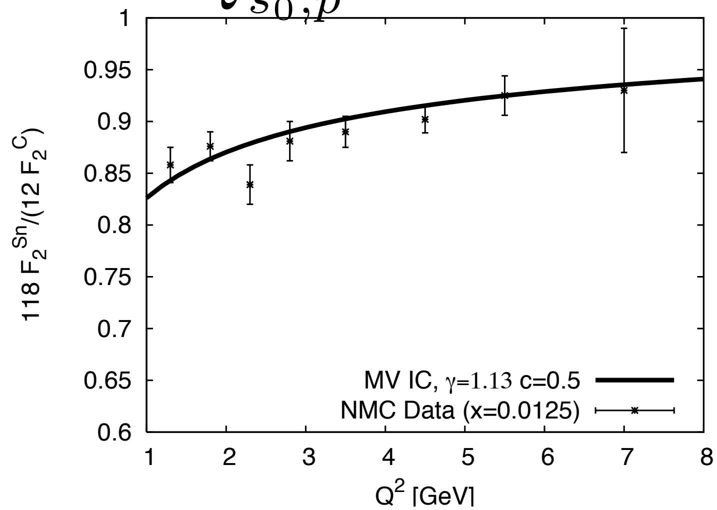
- Leading order Kernal

$$\mathcal{K}_{\mathrm{LO}}(\mathbf{r},\mathbf{r}_1,\mathbf{r}_2) = \frac{\alpha_s N_c}{2\pi^2}\,\frac{\mathbf{r}^2}{\mathbf{r}_1^2\mathbf{r}_2^2}$$

Initial Condition

$$Q_{s_0}^2 = c\,A^{1/3} Q_{s_0,p}^2$$





$$N(r, Y=0) = 1 - \exp\left[-\left(\frac{Q_{s_0}^2 r^2}{4}\right)^{\gamma} \ln\left(\frac{1}{r^2\Lambda_{QCD}^2} + e\right)\right]$$

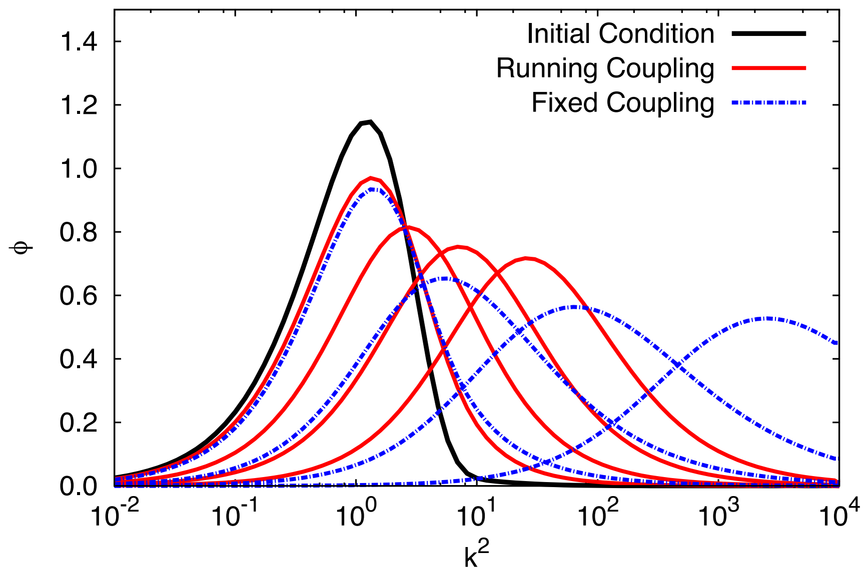
Initial condition consistent with DIS data at x≈0.01

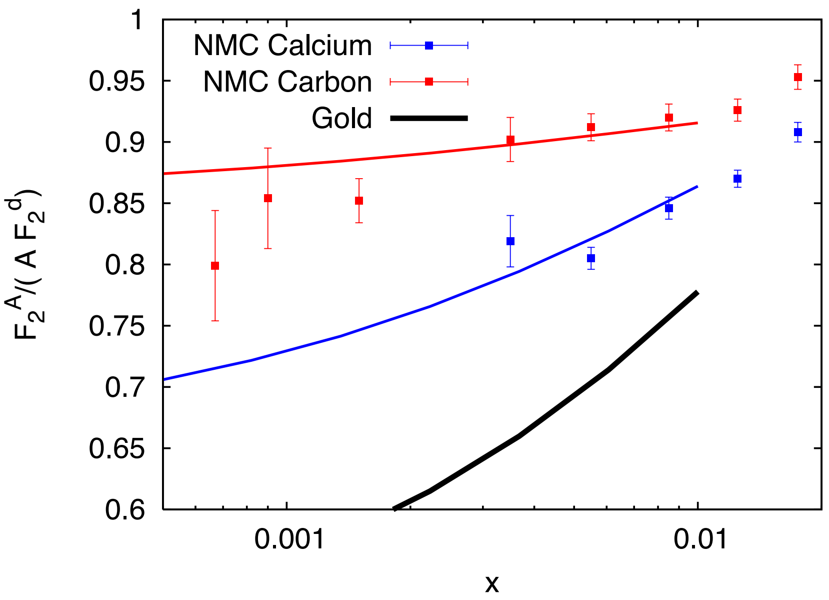
Evolution

- NLO BK: use "Balitsky prescription"

$$\mathcal{K}_{\text{Bal.}}(\mathbf{r},\mathbf{r}_1,\mathbf{r}_2) = \frac{\alpha_s(\mathbf{r})N_c}{\pi}\left[\frac{\mathbf{r}^2}{\mathbf{r}_1^2\mathbf{r}_2^2} + \frac{1}{\mathbf{r}_1^2}\left(\frac{\alpha_s(\mathbf{r}_1^2)}{\alpha_s(\mathbf{r}_2^2)} - 1\right) + \frac{1}{\mathbf{r}_2^2}\left(\frac{\alpha_s(\mathbf{r}_2^2)}{\alpha_s(\mathbf{r}_1^2)} - 1\right)\right]$$

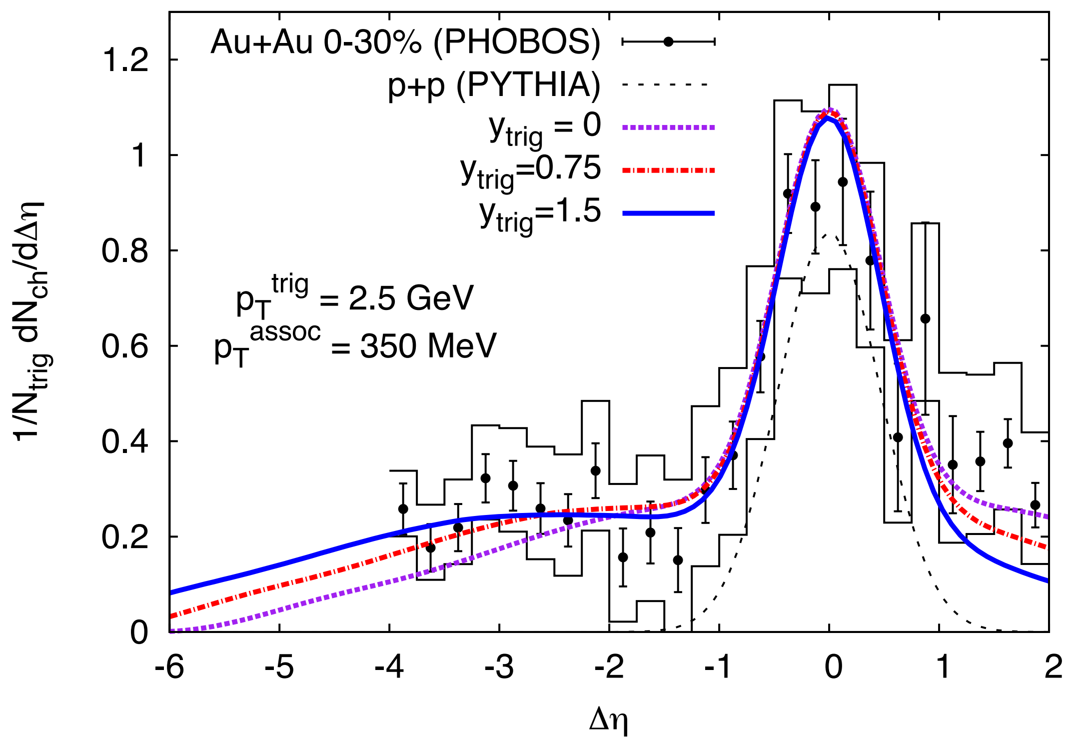
I. Balitsky, G.A. Chirilli, Phys. Rev. D 77, 014019 (2008).
J.L. Albacete, N. Armesto, J.G. Milhano, C.A. Salgado, Phys. Rev. D 80, 034031





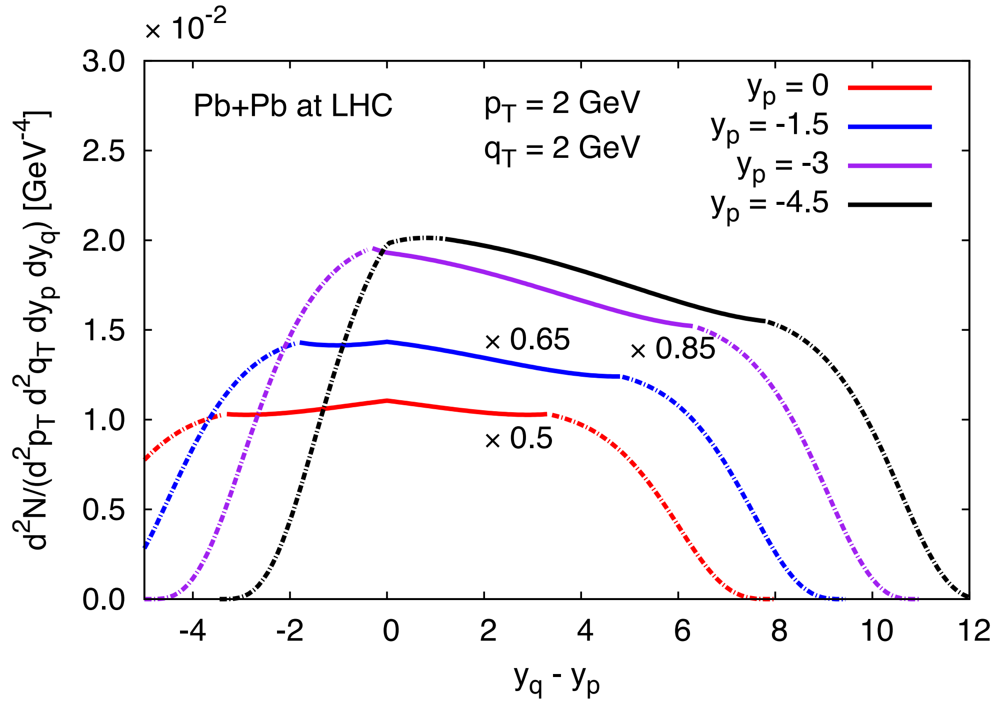
Initial and evolution of dipole-nucleus scattering amplitude consistent with measured nuclear DIS data.

Results for PHOBOS



LRC seen to the kinematic extent of RHIC

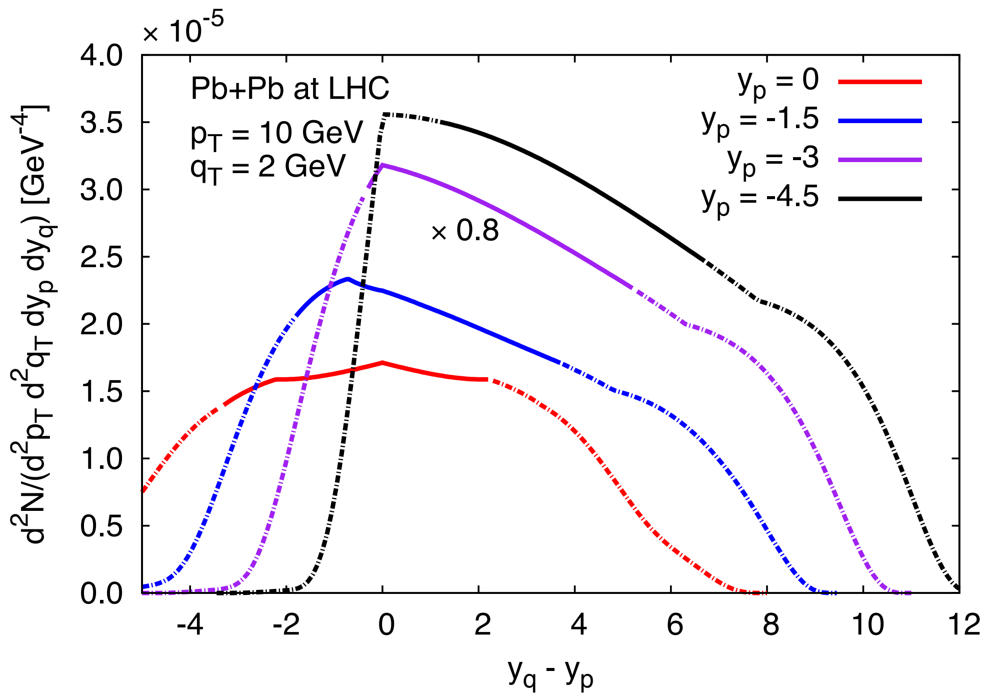
Predictions for LHC I



LRC up to 10 units in rapidity at LHC.

7 units probe the small x evolution of the nucleon wave-function.

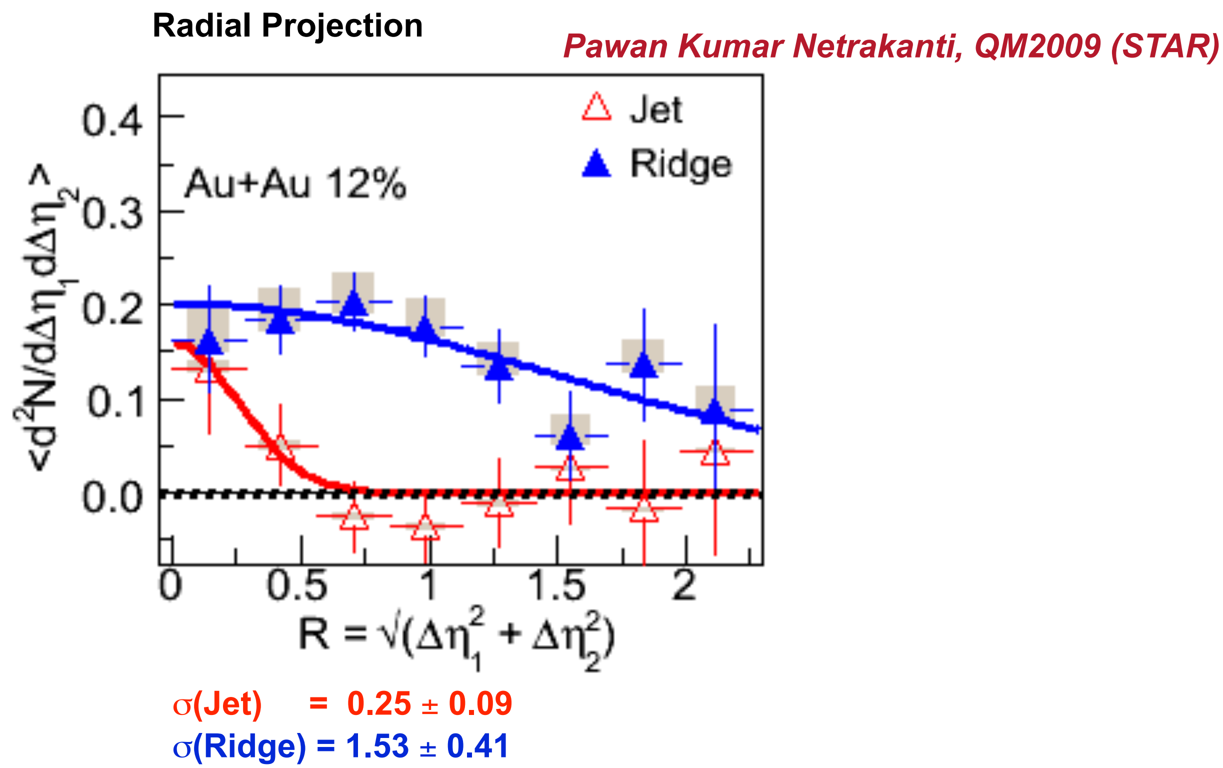
Predictions for LHC II



LRC up to 6 units in rapidity at LHC.

5 units probe the small x evolution of the nucleon wave-function.

Three Particle Correlation



- Three particle correlation flat in rapidity
- The same formalism could be used to calculate $\sigma(Ridge)$

Conclusions

- Computed two particle correlation at arbitrary rapidity separations

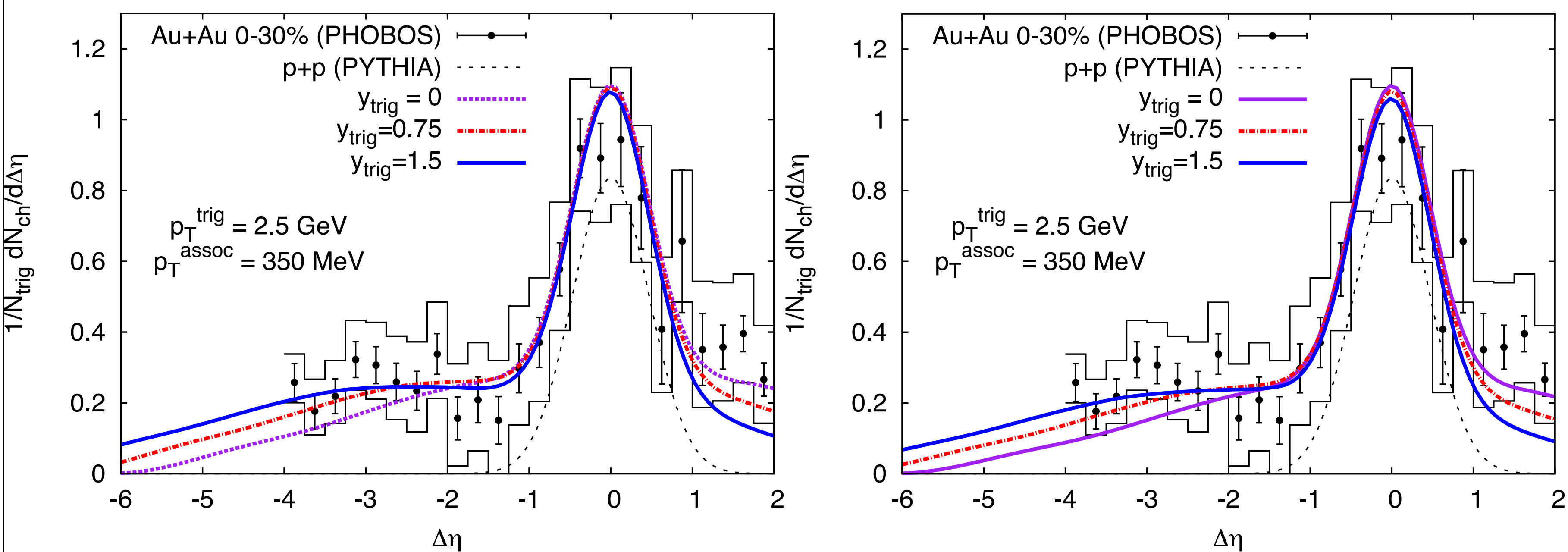
- Glasma Flux Tube explanation of ridge is becoming Quantitative
  - Have quantitative agreement with rapidity dependence at STAR and PHOBOS

- LRC at the LHC can probe the high energy evolution of the nuclear wave-function

Backup

Rapidity Dependence of Flow

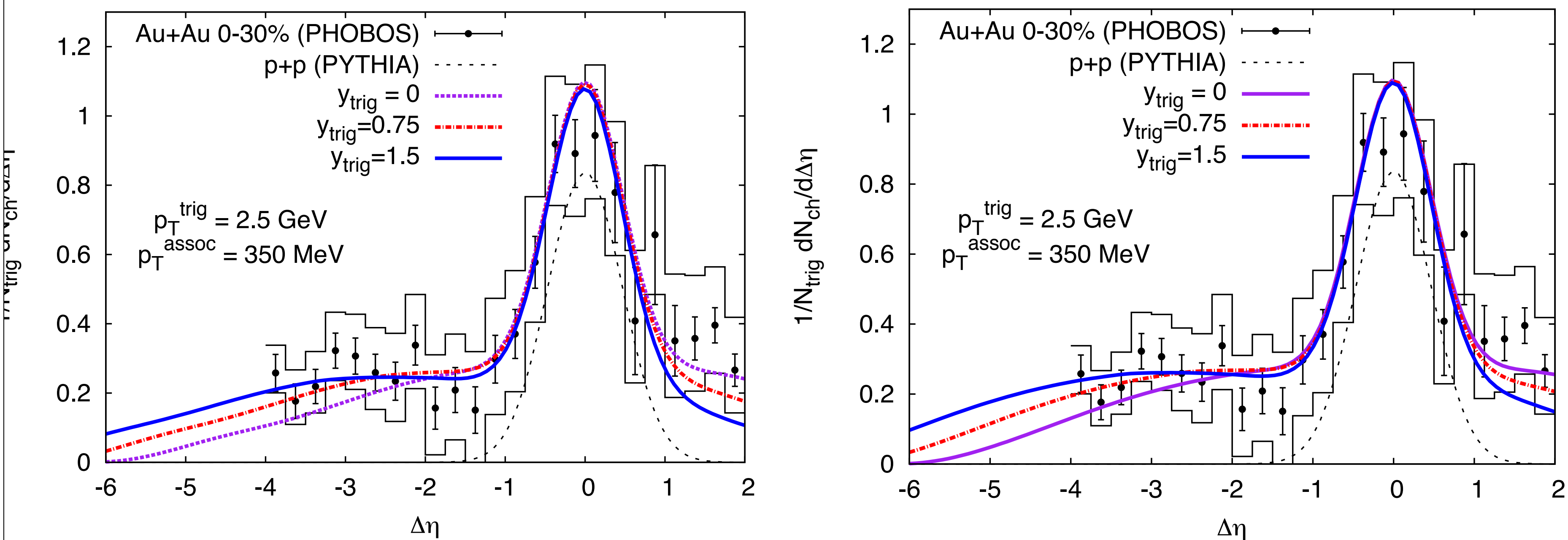
$$\frac{1}{N_{\text{trig.}}}\frac{\mathrm{d}N}{\mathrm{d}\Delta\eta} \approx \mathcal{V}_{ps}^{\text{assoc.}} F(\Delta\phi_{pq}=0)\,\frac{C(p_\perp^{\text{trig.}}, p_\perp^{\text{assoc.}}, y_{\text{trig.}}, y_{\text{assoc.}} = y_{\text{trig.}} + \Delta\eta, \Delta\phi_{pq}=0)}{\mathrm{d}N_1(p_\perp^{\text{trig.}}, y_{\text{trig.}})}$$



$$F(\Delta\phi_{pq}=0) = \cosh(\tanh^{-1}\beta)$$

- From a crude fit to BRAHMS data $\beta(\eta) = 0.72 - 0.04|\eta|$

Scale Dependence



DIS

$$F_2^A(x,Q^2) = \frac{Q^2}{4\pi^2\alpha_{em}}\left(\sigma_{_A}^T + \sigma_{_A}^L\right)$$

$$\sigma_{_A}^{T,L}(x,Q^2) = \int_0^1 \mathrm{d}z \int \mathrm{d}^2\mathbf{b}\; \mathrm{d}^2\mathbf{r} \mid Psi_{T,L}(z,Q^2,\mathbf{r})|^2 \mathcal{N}_{_A}(\mathbf{b},\mathbf{r},x)$$

$$\mathcal{N}_{_A}(\mathbf{b},\mathbf{r},x) = 2\mathcal{T}_{_A}(\mathbf{b})\,\mathcal{N}_{_A}(\mathbf{r},x) \qquad\qquad \sigma_{_A} = 2\int \mathrm{d}^2\mathbf{b}\; \mathcal{T}_{_A}(\mathbf{b})$$

$$\sigma_{_A}^{T,L}(x,Q^2) = \sigma_{_A}\int_0^1 \mathrm{d}z \int \mathrm{d}^2\mathbf{r}\; |\Psi_{T,L}(z,Q^2,\mathbf{r})|^2 \mathcal{N}_{_A}(\mathbf{r},x)$$

JIMWLK

$$W_{\Lambda'^+}\left[\Omega_1'(y,\boldsymbol{x}_\perp)\right] \equiv \Big[1+\underbrace{\ln\left(\frac{\Lambda^+}{\Lambda'^+}\right)}_{\mathrm{d}y}\mathcal{H}_{\Lambda^+}\Big]W_{\Lambda^+}\left[\Omega_1(y,\boldsymbol{x}_\perp)\right]$$

$$\Big\langle \mathcal{O}_{\text{LO}} + \underbrace{\mathcal{O}_{\text{NLO}}}_{\Lambda'^{\pm}<k^{\pm}<\Lambda^{\pm}} \Big\rangle = \Big\langle \mathcal{O}_{\text{LO}} \Big\rangle'$$