Recent lattice results on heavy quarks at $T > 0$

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Outline & setup

What we do

- **Thermodynamics** of charm quarks
  - EoS
  - charm number susceptibility
  - higher order susceptibilities
- **Flavor correlations** with light quarks
- **Spatial** mesonic correlations

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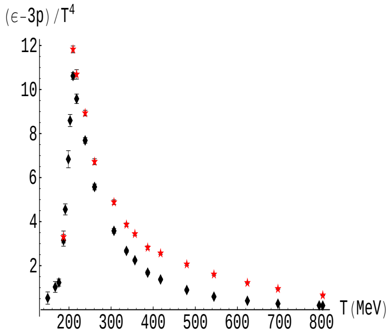
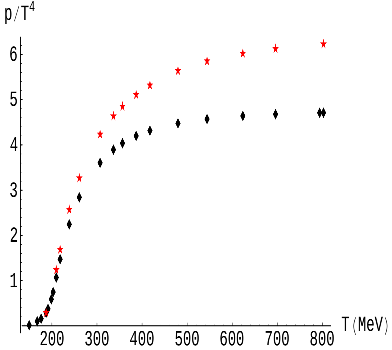
How we do

- **Partially quenched**
  - $N_f = 2 + 1$ dynamical
  - improved staggered fermions

$$
\begin{aligned}
m_\pi &\approx 220\ \mathrm{MeV} \\
m_K &\approx 500\ \mathrm{MeV} \\
m_{\eta_c} &\approx 2980\ \mathrm{MeV} \\
m_\Psi &\approx 3097\ \mathrm{MeV}
\end{aligned}
$$

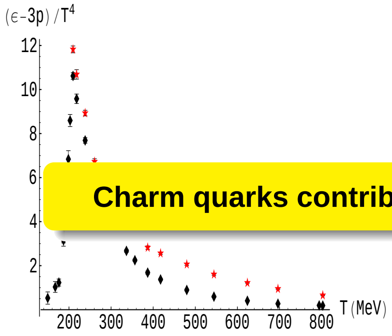
EoS

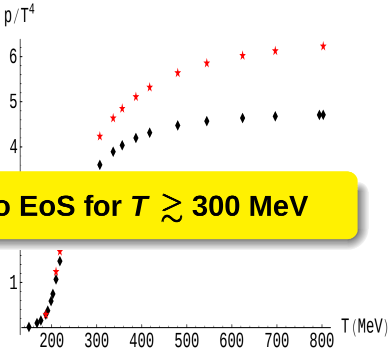




M. Cheng *et. al*, arXiv:0710.4357 [hep-lat]

EoS

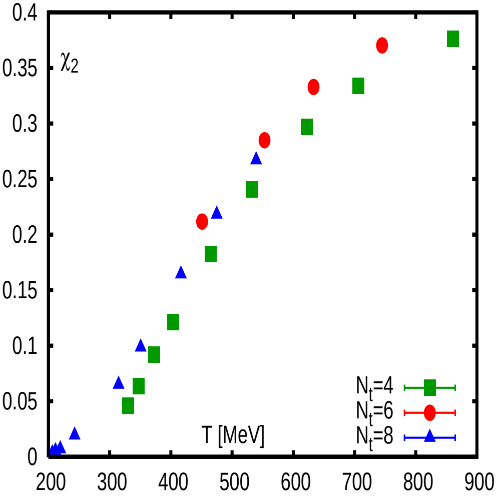




Charm quarks contribute to EoS for $T \gtrsim 300$ MeV

M. Cheng *et. al*, arXiv:0710.4357 [hep-lat]

Charm number susceptibility



$$\chi_2 = \frac{1}{2T^2}\langle \mathcal{N}_c \mathcal{N}_c \rangle$$

- no charmonia

$$\rho_{tt}(\omega, k) \sim \frac{\omega D k^2 \chi_2}{\omega^2 + (Dk^2)^2}$$

- $\omega \longrightarrow 0, k \longrightarrow 0$

Petreczky, Hegde, Velytsky, arXiv:0911.0196 [hep-lat]

Quasi-particle: effective mass

- Free theory

$$\rho_{tt}(\omega, k) \sim \left(\frac{T}{m_{eff}}\right) \chi_2\, \omega\delta(\omega)$$

$$\searrow \left\langle v_{th}^2 \right\rangle$$

$$\frac{P}{T^4} = \frac{6}{\pi^2}\left(\frac{m_{eff}}{T}\right)^2 \sum_{l=1}^{\infty}(-1)^{l+1} l^{-2} K_2\left(\frac{l m_{eff}}{T}\right) \cosh\left(\frac{l\mu_c}{T}\right)$$

$$\chi_2 = \frac{1}{2}\frac{6}{\pi^2}\left(\frac{m_{eff}}{T}\right)^2 \sum_{l=1}^{\infty}(-1)^{l+1} K_2\left(\frac{l m_{eff}}{T}\right)$$

$$\chi_4 = \frac{1}{24}\frac{6}{\pi^2}\left(\frac{m_{eff}}{T}\right)^2 \sum_{l=1}^{\infty}(-1)^{l+1} l^2 K_2\left(\frac{l m_{eff}}{T}\right)$$

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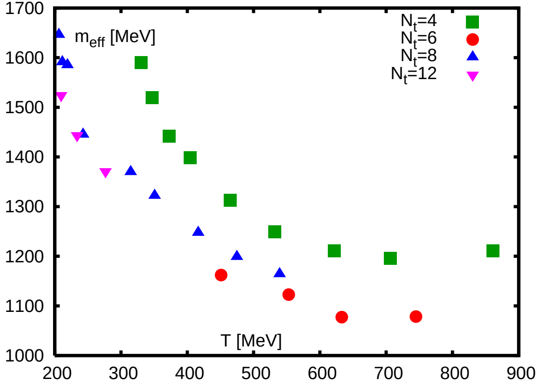

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Higher order susceptibilities

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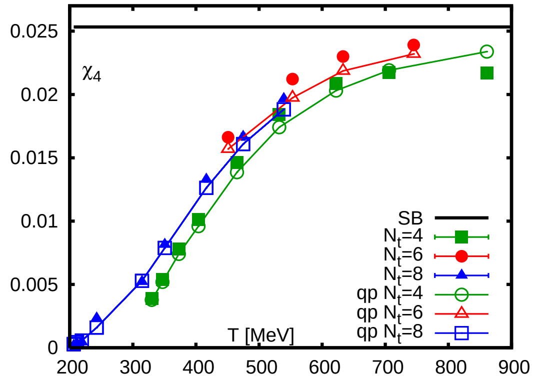
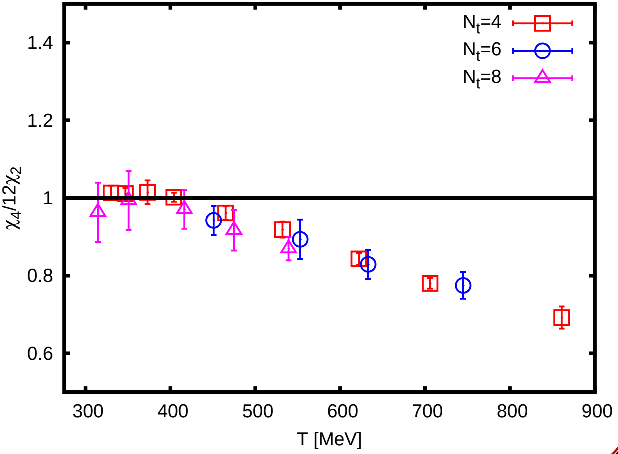


Higher order susceptibilities

- $I = 0$

$$\chi_4 = 12\,\chi_2$$



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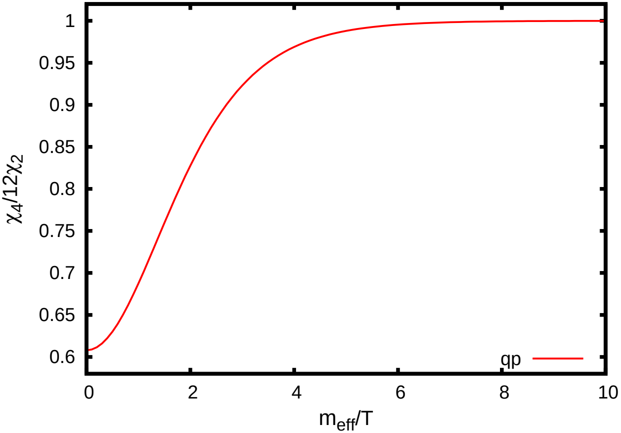

Flavor correlations

$$\chi_2^{(f,g)} = \frac{1}{2T^2} \langle \mathcal{N}_f \mathcal{N}_g \rangle \qquad \chi_2^{(g)} = \frac{1}{2T^2} \langle \mathcal{N}_g \mathcal{N}_g \rangle$$

$T \to 0$	*all hadrons with f & g*	*all hadrons with g*
$T \to \infty$ $(m = 0)$	$0 + \# g^6 \ln g$	$\# + \# g^2$

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$$\boldsymbol{\chi_2^{(f,g)} / \chi_2^{(g)} = \langle \mathcal{N}_f \mathcal{N}_g \rangle \,/\, \langle \mathcal{N}_g \mathcal{N}_g \rangle}$$

- $T \to 0$ $\quad -1$
- $T \to \infty$ $\quad 0 + \# g^6 \ln g$

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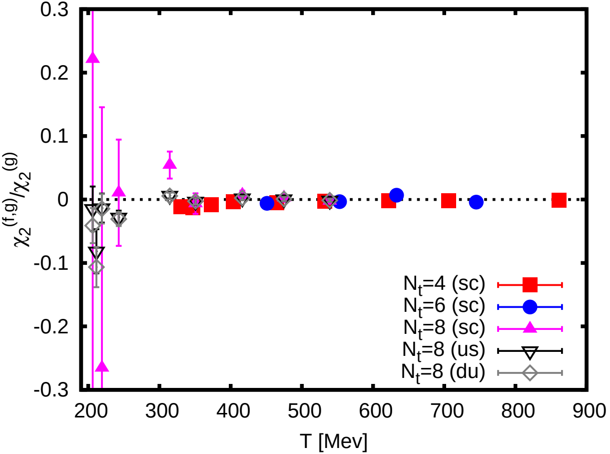

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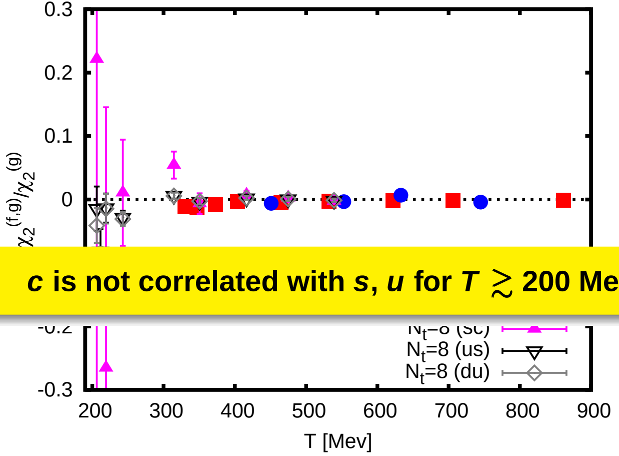
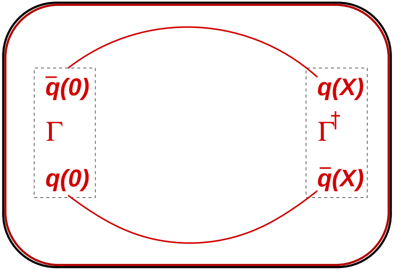


c is not correlated with s, u for $T \gtrsim 200$ MeV

Spatial correlations of charmonia

$$C(z) = \sum_{x,y,\tau} \left\langle \mathcal{M}^\dagger(\boldsymbol{x}) \mathcal{M}(\boldsymbol{0}) \right\rangle$$

- $\mathcal{M}(\boldsymbol{x}) = \bar{q}(\boldsymbol{x}) \Gamma q(\boldsymbol{x})$
- PS: $\Gamma = \gamma_5$, V: $\Gamma = \gamma_{1,2}$



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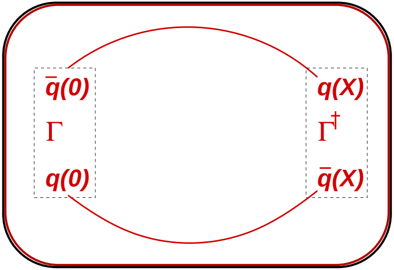
- **Screening Mass**

$$C(z) = \exp\left(-M\, z\right)$$



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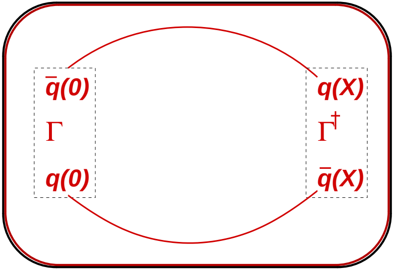

$$C(z) = \exp\left(-M\, z\right)$$

$$C(z) \sim \rho_{zz}(\omega, k) \sim \frac{\omega^2}{k^2} \rho_{tt}(\omega, k)$$

Spatial correlations of charmonia

$$T \longrightarrow 0 ::: M = m_{had}$$

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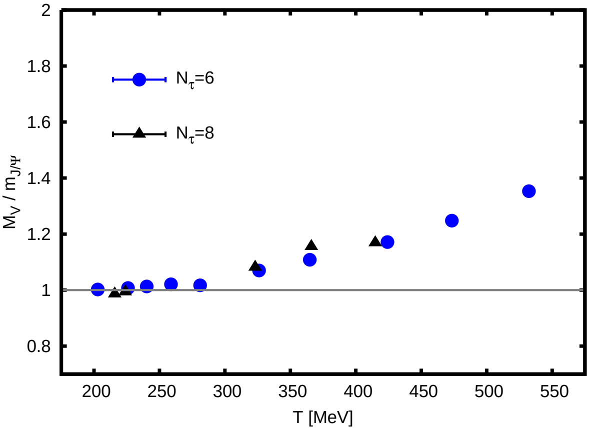

SM, arXiv:0810.2906 [hep-lat]

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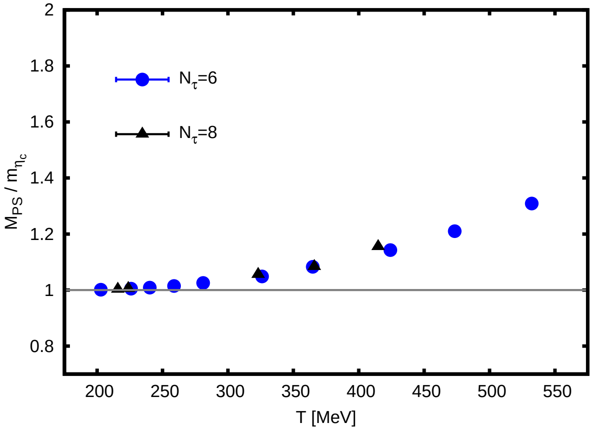

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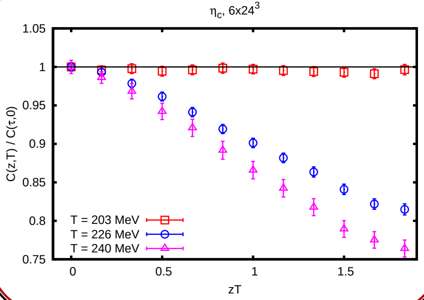

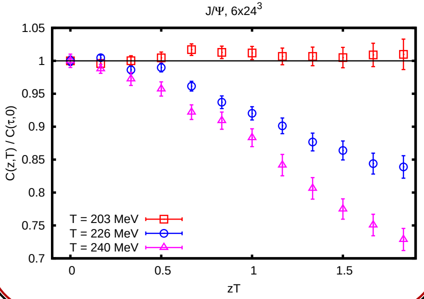

Spatial correlations of charmonia

$$T \to \infty ::: M = 2\sqrt{(\pi T)^2 + m_c^2}$$

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