# On local $\mathcal{P}$-violation in strongly interacting matter

Swagato Mukherjee
and
Dmitri Kharzeev

Brookhaven National Laboratory

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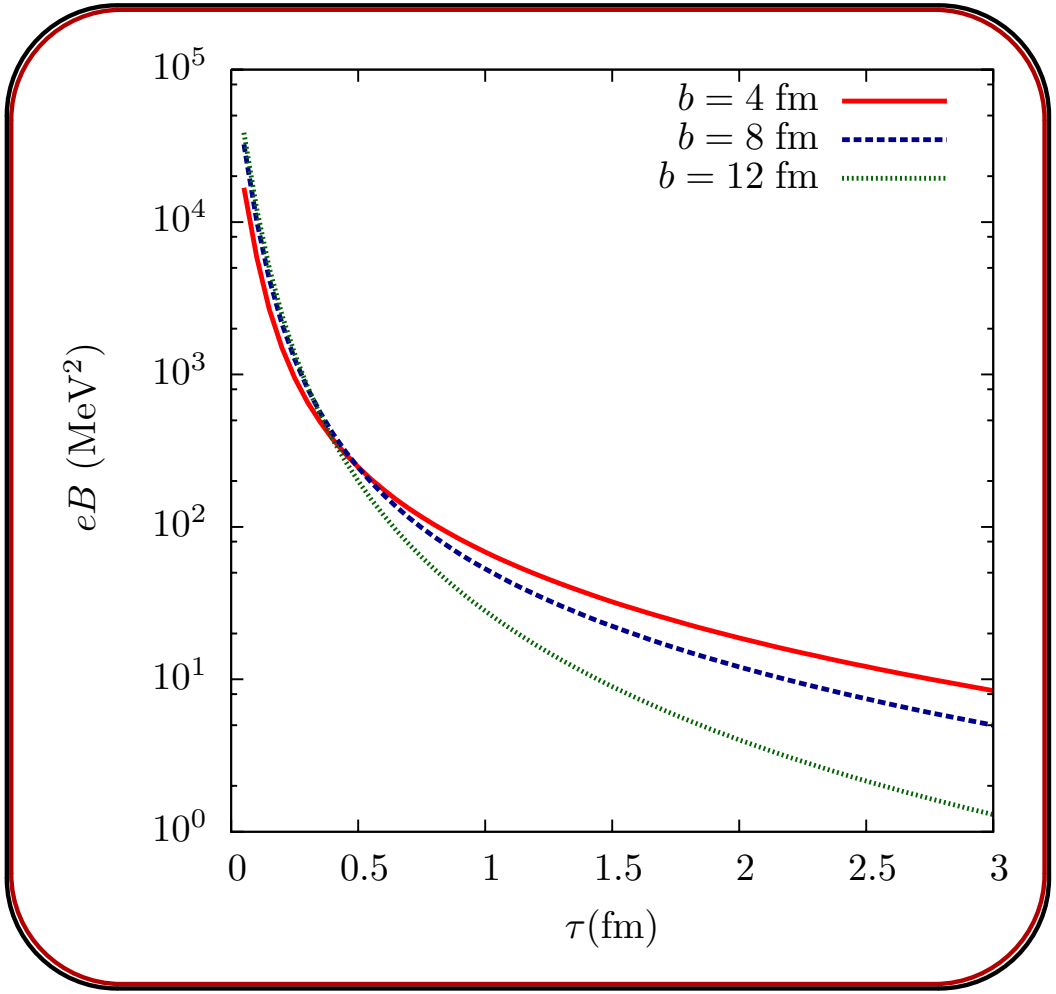
# Chiral magnetic effect

Kharzeev, McLerran, Warringa

$eB \sim 10^{19}$ G



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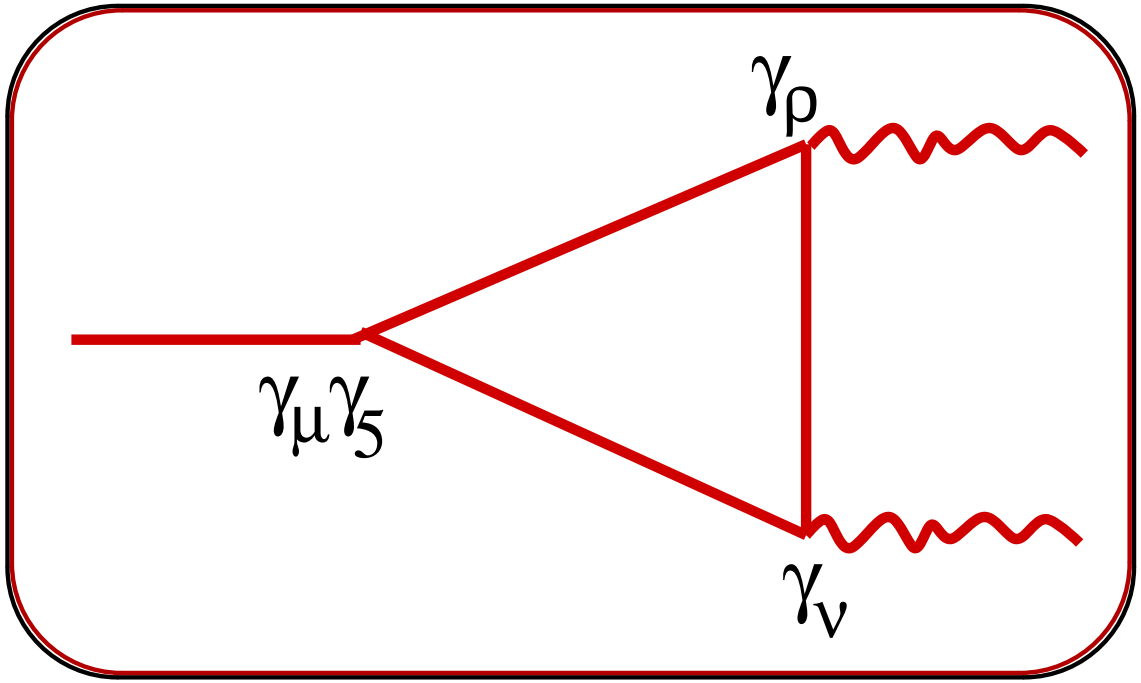

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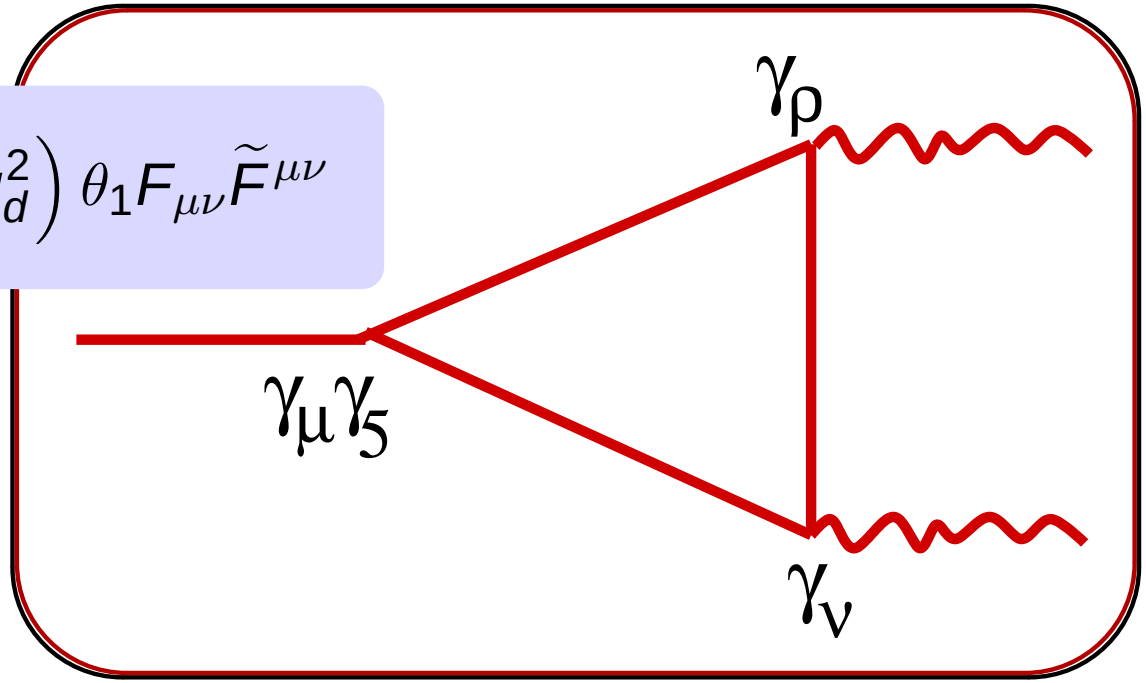

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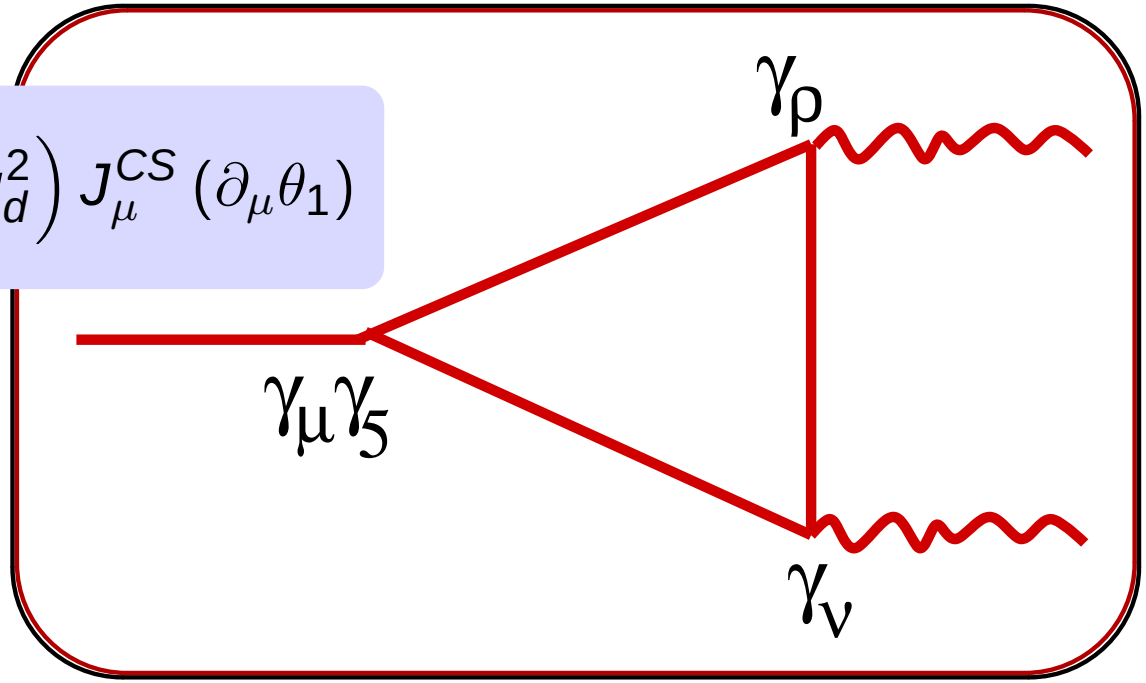

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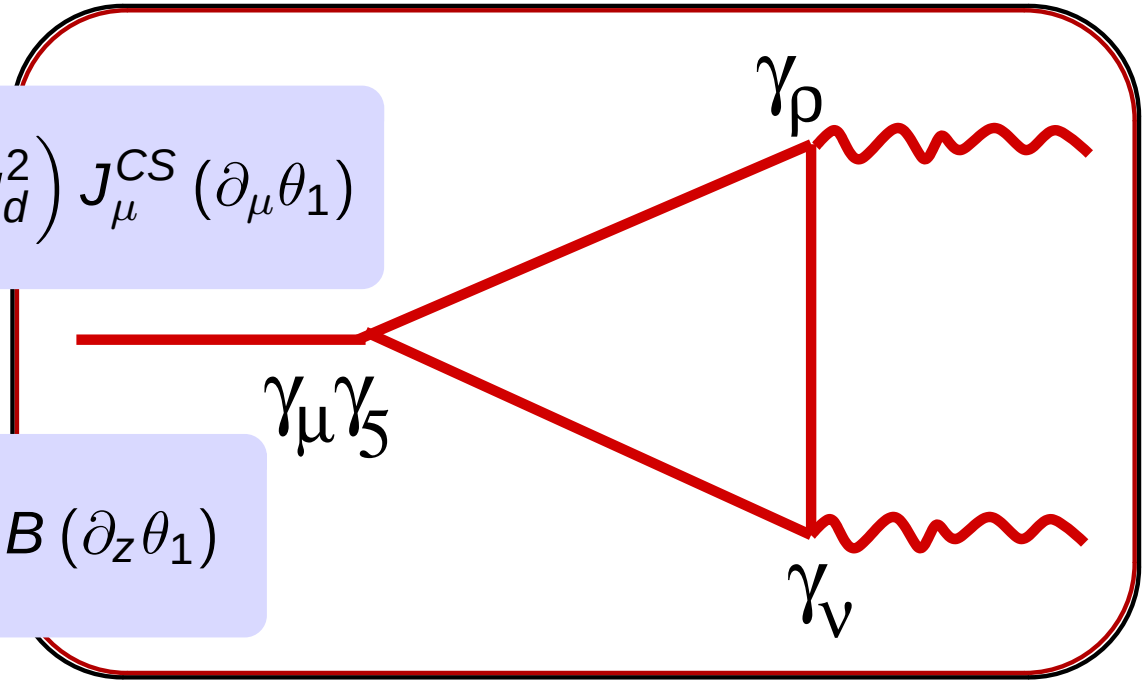

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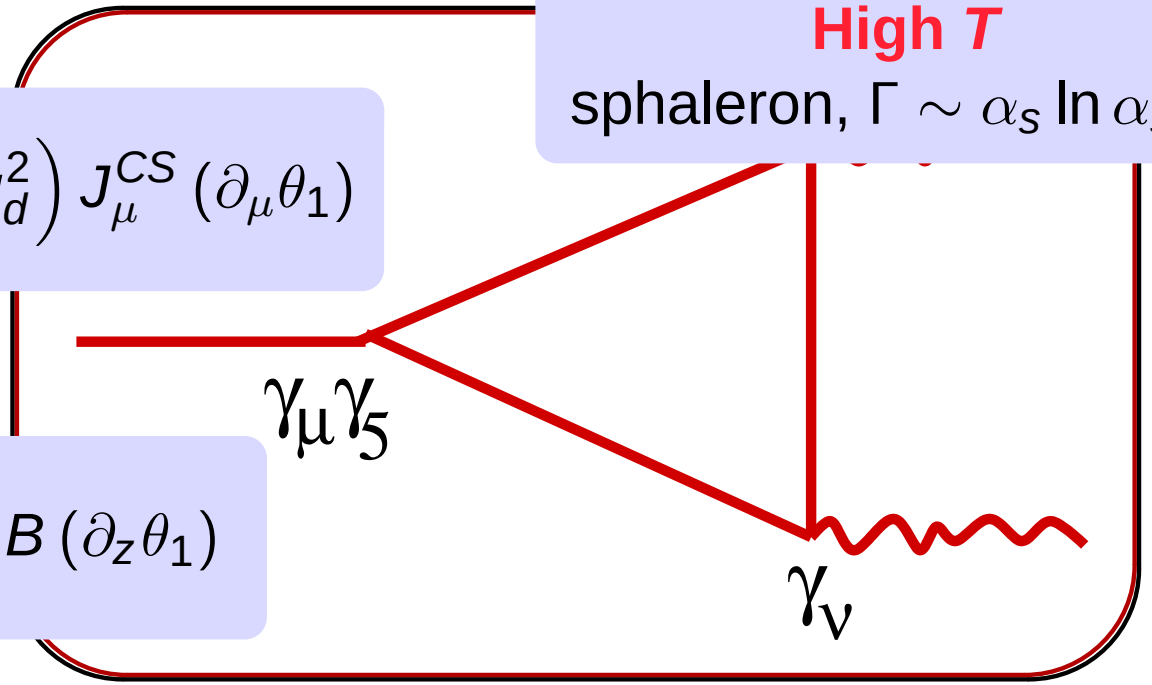
**High *T***

sphaleron, $\Gamma \sim \alpha_s \ln \alpha_s T^4$




**Charge separation due to local $\mathcal{P}$ violation**

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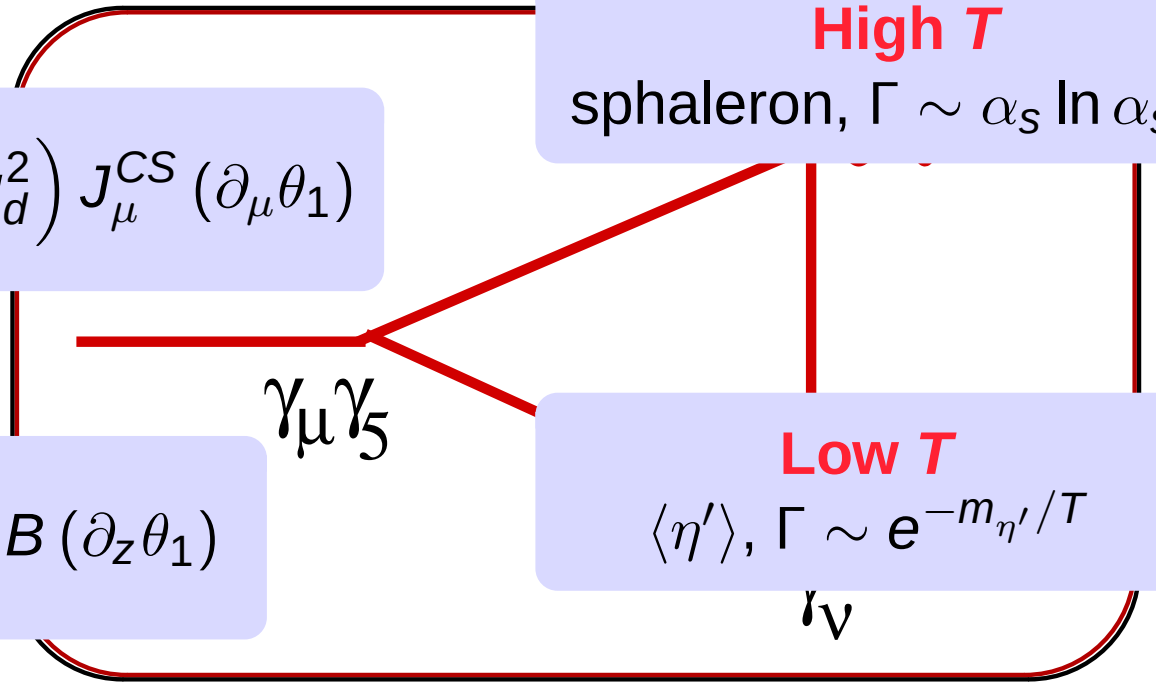


**High $T$**

sphaleron, $\Gamma \sim \alpha_s \ln \alpha_s T^4$

**Low $T$**

$\langle \eta' \rangle$, $\Gamma \sim e^{-m_{\eta'}/T}$

**Charge separation due to local $\mathcal{P}$ violation**

# QCD inequality in presence of EM

$$\mathcal{D} = \gamma_\mu \left(\partial_\mu + \imath g A_{\alpha\mu} t_\alpha + \imath e \mathcal{Q} A_\mu\right) + \mathcal{M}$$

- $\mathcal{Q} = \mathrm{diag}\,(2/3, -1/3)$
- $\mathcal{M} = \mathrm{diag}\,(m, m)$
- $\mathcal{Q} = \mathcal{I}_3 + \mathcal{B}/2,\ \mathcal{I}_3 = \tau_3/2,\ \mathcal{B} = \mathbb{1}/3$
- $(\gamma_5 \tau_{1,2})\, \mathcal{D}\, (\gamma_5 \tau_{1,2})^\dagger \neq \mathcal{D}^\dagger$

$$(\gamma_5 \tau_3)\, \mathcal{D}\, (\gamma_5 \tau_3)^\dagger = \mathcal{D}^\dagger$$

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- $\underline{I = 1}$: $\qquad M(\boldsymbol{x}) = \bar{\Psi}(\boldsymbol{x}) \Gamma \Psi(\boldsymbol{x})$

$$\left\langle M(\boldsymbol{x}) M^\dagger(\boldsymbol{0}) \right\rangle = \left\langle \mathrm{Tr}\, S(\boldsymbol{x}, \boldsymbol{0}) \Gamma\, (\gamma_5 \tau_3)\, S^\dagger(\boldsymbol{x}, \boldsymbol{0})\, (\gamma_5 \tau_3)\, \bar{\Gamma} \right\rangle \leq \left\langle \mathrm{Tr}\, S(\boldsymbol{x}, \boldsymbol{0}) S^\dagger(\boldsymbol{x}, \boldsymbol{0}) \right\rangle$$

- $S(\boldsymbol{x}, \boldsymbol{0}) = \mathcal{D}^{-1}_{\boldsymbol{x}, \boldsymbol{0}}, \qquad \bar{\Gamma} = \gamma_0 \Gamma \gamma_0$

$$\boldsymbol{\Gamma = \gamma_5 \tau_3}$$

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- $(\gamma_5 \tau$

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**lowest energy state contains $\pi^0$**

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# Locally non-zero $\pi^0$ condensate

$$\mathcal{L}_\chi = \frac{f_\pi}{4} \mathrm{Tr}\, \mathcal{D}_\mu \mathcal{U}^\dagger \mathcal{D}^\mu \mathcal{U} + \mathrm{Tr}\left(\mathcal{M}\mathcal{U}^\dagger + \mathcal{M}^\dagger \mathcal{U}\right)$$

- $\mathcal{D}_\mu \mathcal{U} = \partial_\mu \mathcal{U} + \imath e A_\mu \left[\mathcal{Q}, \mathcal{U}\right]$
- $\mathcal{U} = \frac{1}{f_\pi}\left(\sigma + \imath\tau_1\pi_1 + \imath\tau_2\pi_2 + \imath\tau_3\pi^0\right)$
- $\sigma^2 + \pi_1^2 + \pi_2^2 + \left(\pi^0\right)^2 = f_\pi^2$

- $\sigma = f_\pi \cos\chi \cos\theta$
- $\pi^0 = f_\pi \cos\chi \sin\theta$
- $\pi_1 = f_\pi \sin\chi \cos\phi$
- $\pi_2 = f_\pi \sin\chi \sin\phi$

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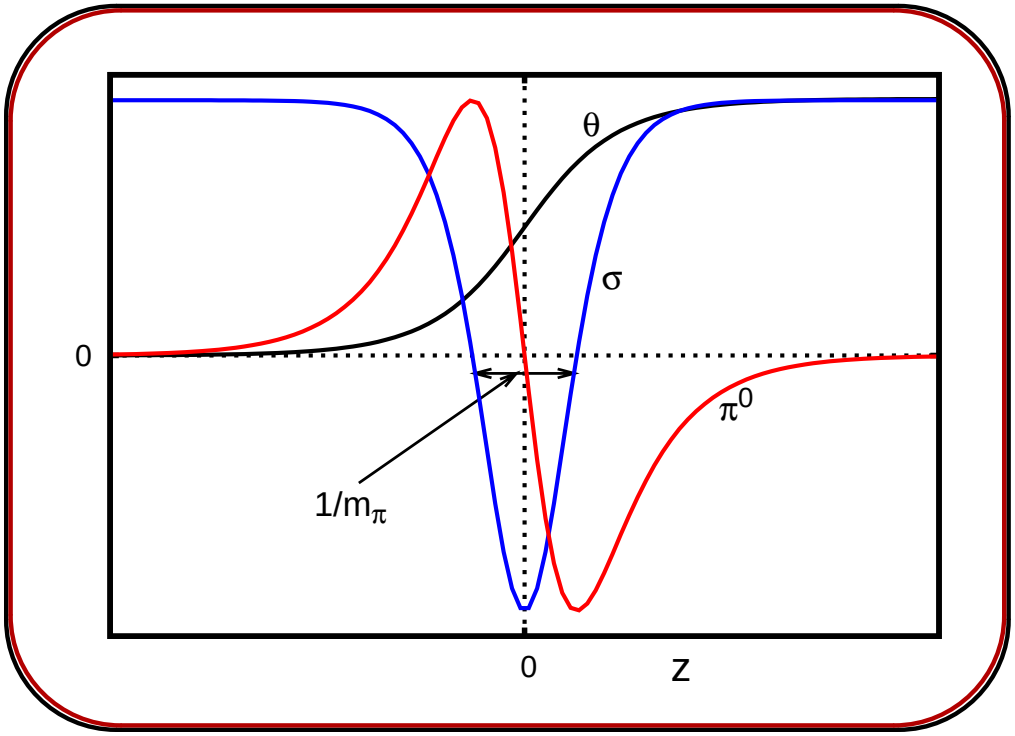


$$\theta(z) = 4\tan^{-1}\exp\left[m_\pi z\right]$$

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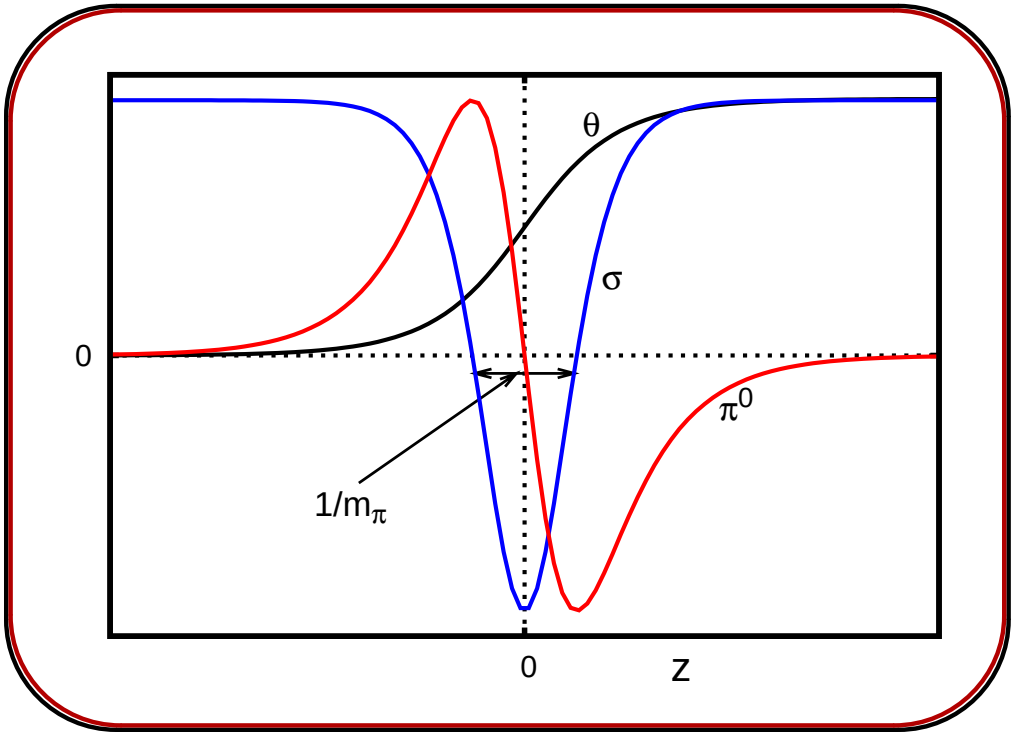


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**Unstable**

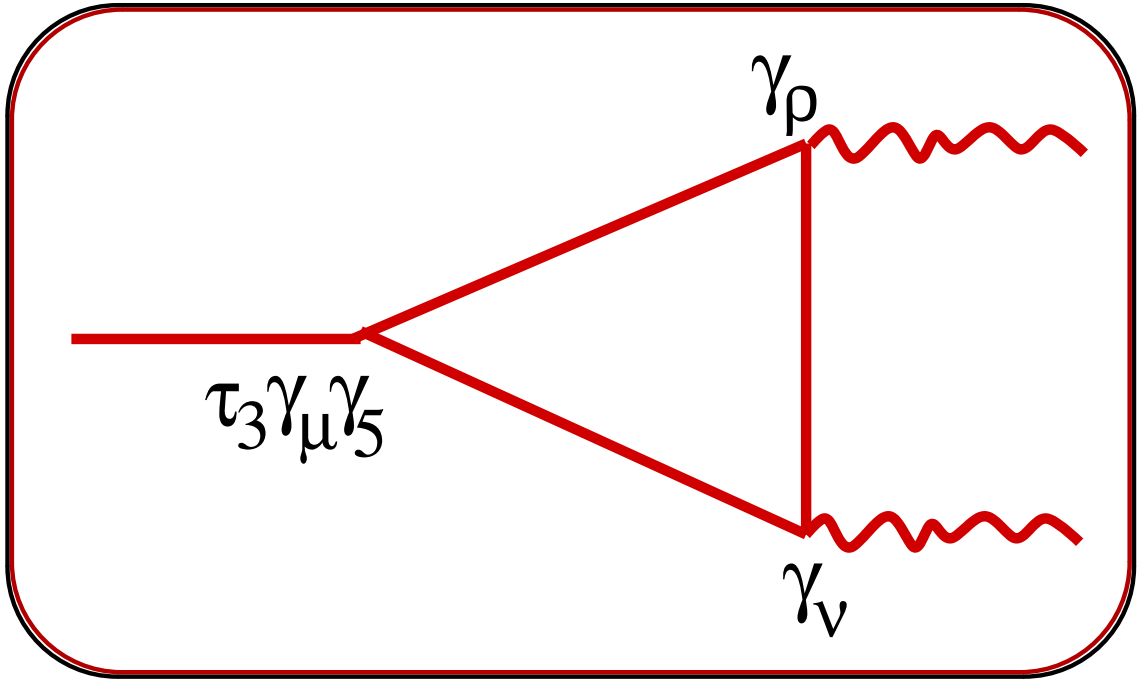
$$E \sim m_\pi,\ \Gamma \sim e^{-m_\pi/T}$$

- **locally stable : $eB \gtrsim 3m_\pi^2 \simeq 10^{19}$ G**

Son & Stephanov, PRD 77, 014021 (2008)

# Charge separation at low $T$

$$\partial_\mu J^{\mu 5,3} = -\frac{3e^2}{16\pi^2}\left(q_u^2 - q_d^2\right) F_{\mu\nu}\widetilde{F}^{\mu\nu}$$



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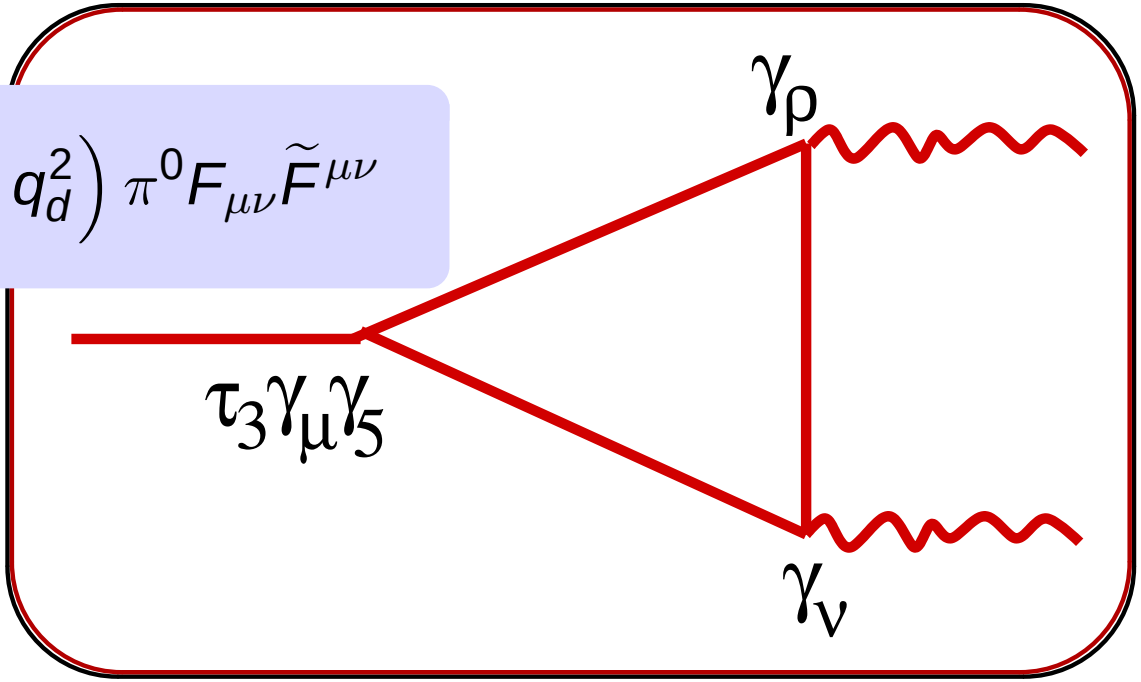

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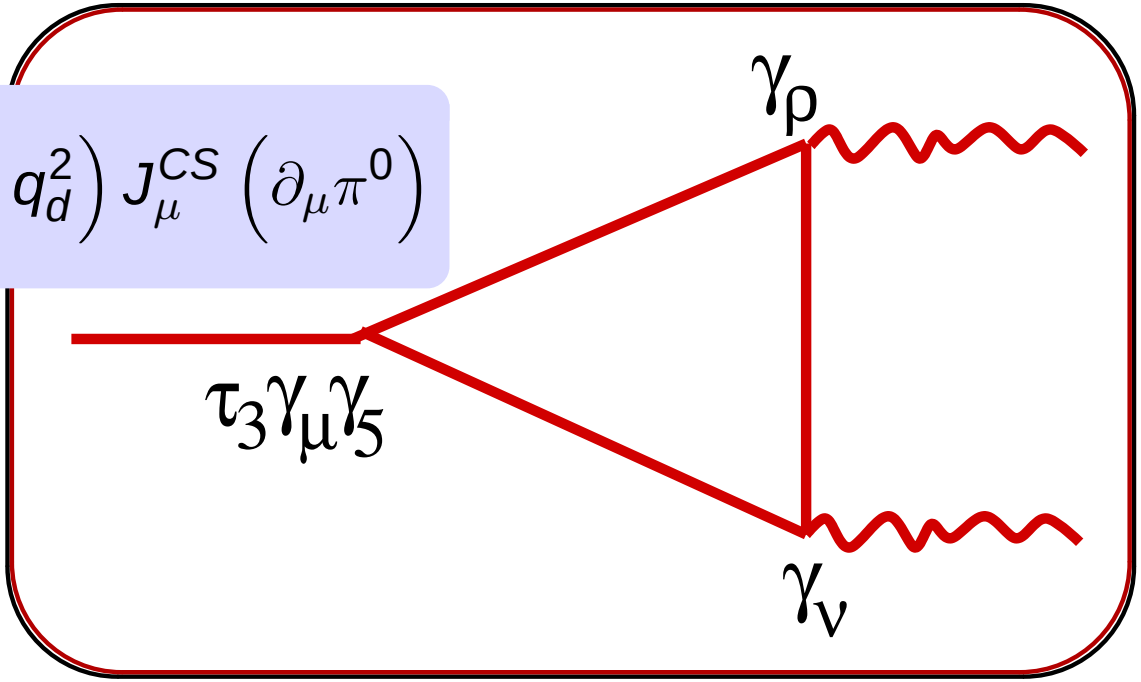

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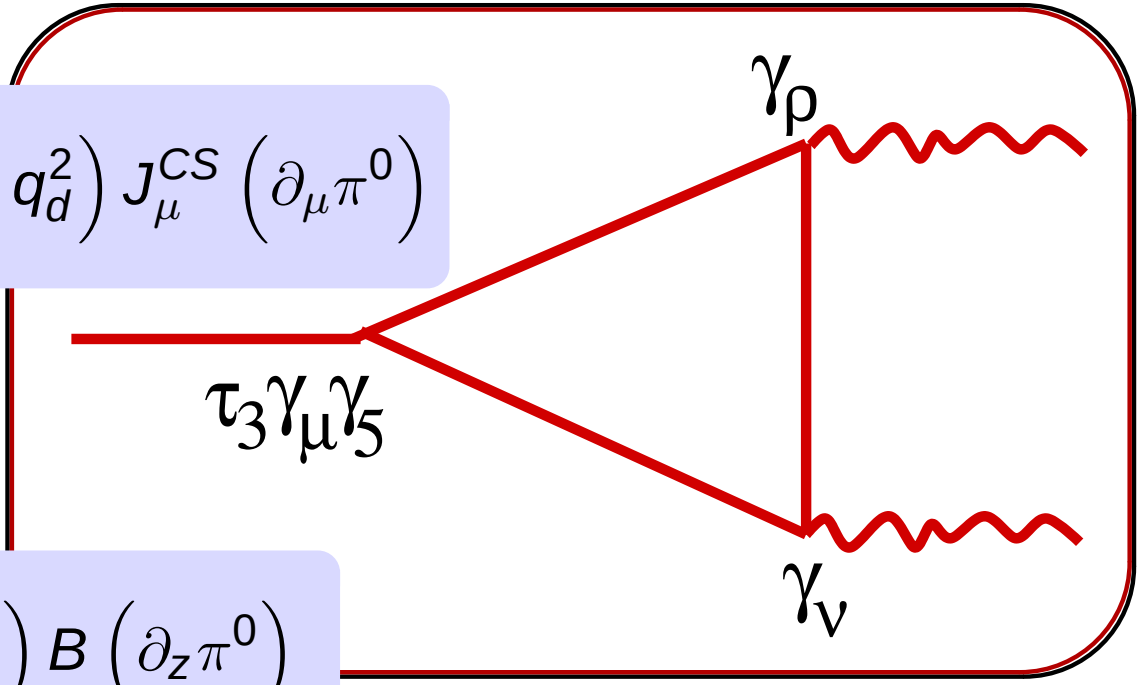


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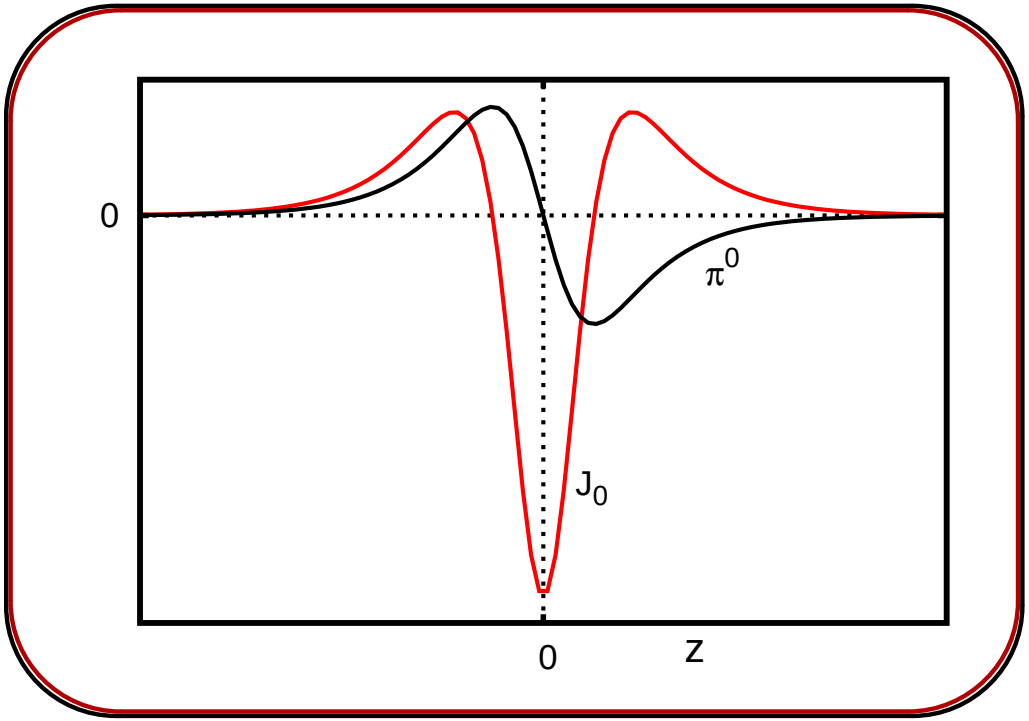
# Charge separation at low $T$

$$J_0 = \frac{3e^2 \boldsymbol{m_\pi}}{2\pi^2}\left(q_u^2 - q_d^2\right)\frac{B\cos\theta\, e^{m_\pi z}}{1+e^{2m_\pi z}}$$

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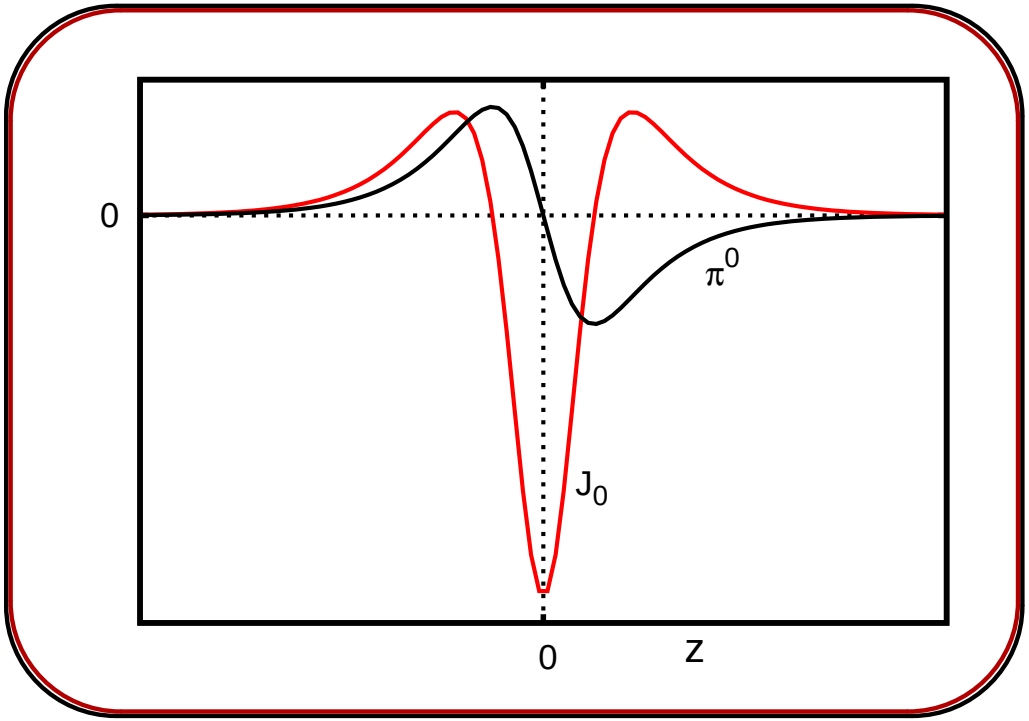


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$$edm \sim d \cdot J_0 \sim \frac{1}{m_\pi} \cdot m_\pi \neq 0$$

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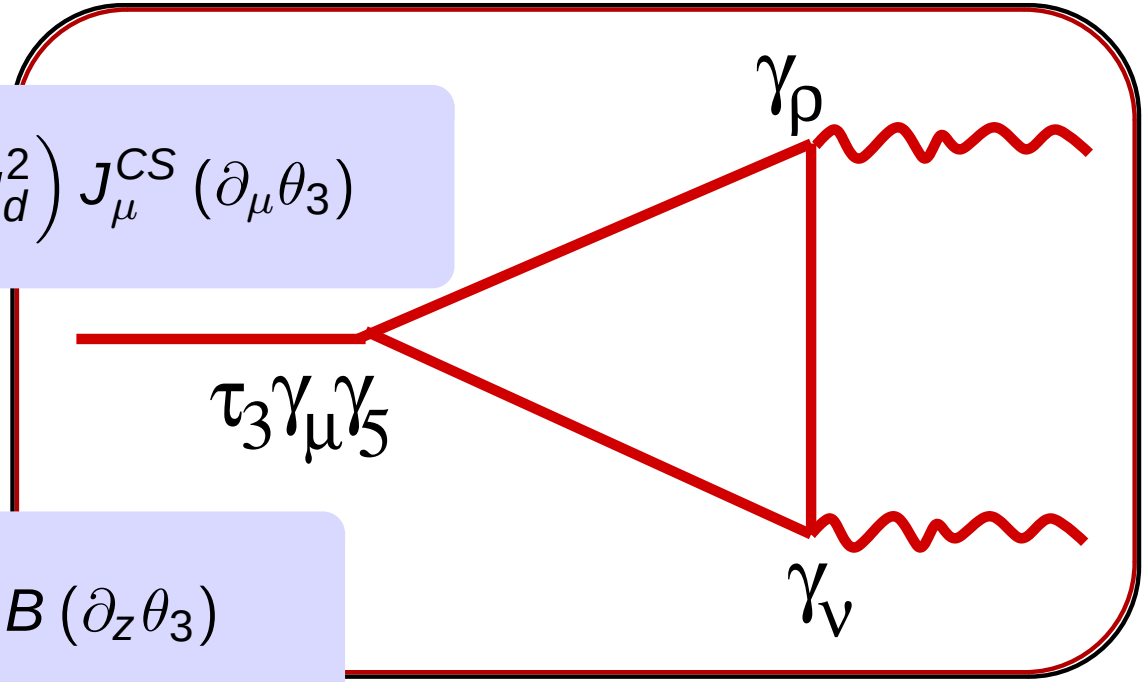

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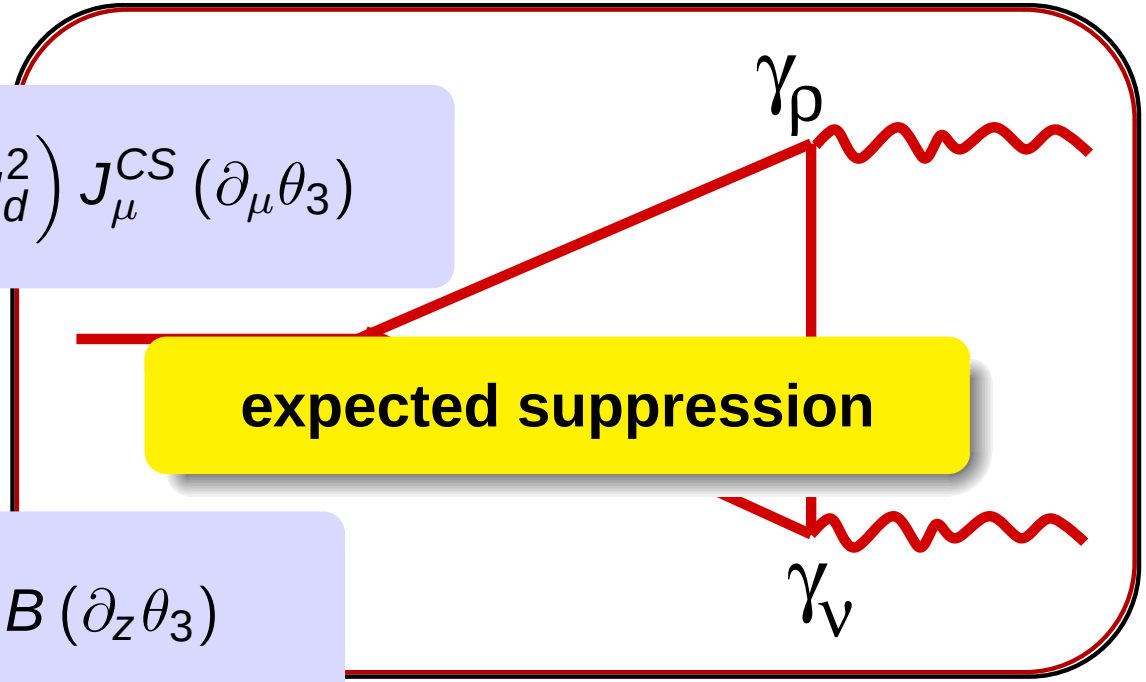


**expected suppression**

- **gluons do not couple to $J^{\mu 5,3}$**