Estimating thermal dilepton rates and electrical conductivity in Quenched QCD

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Based on work with

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arXiv:1012.4963

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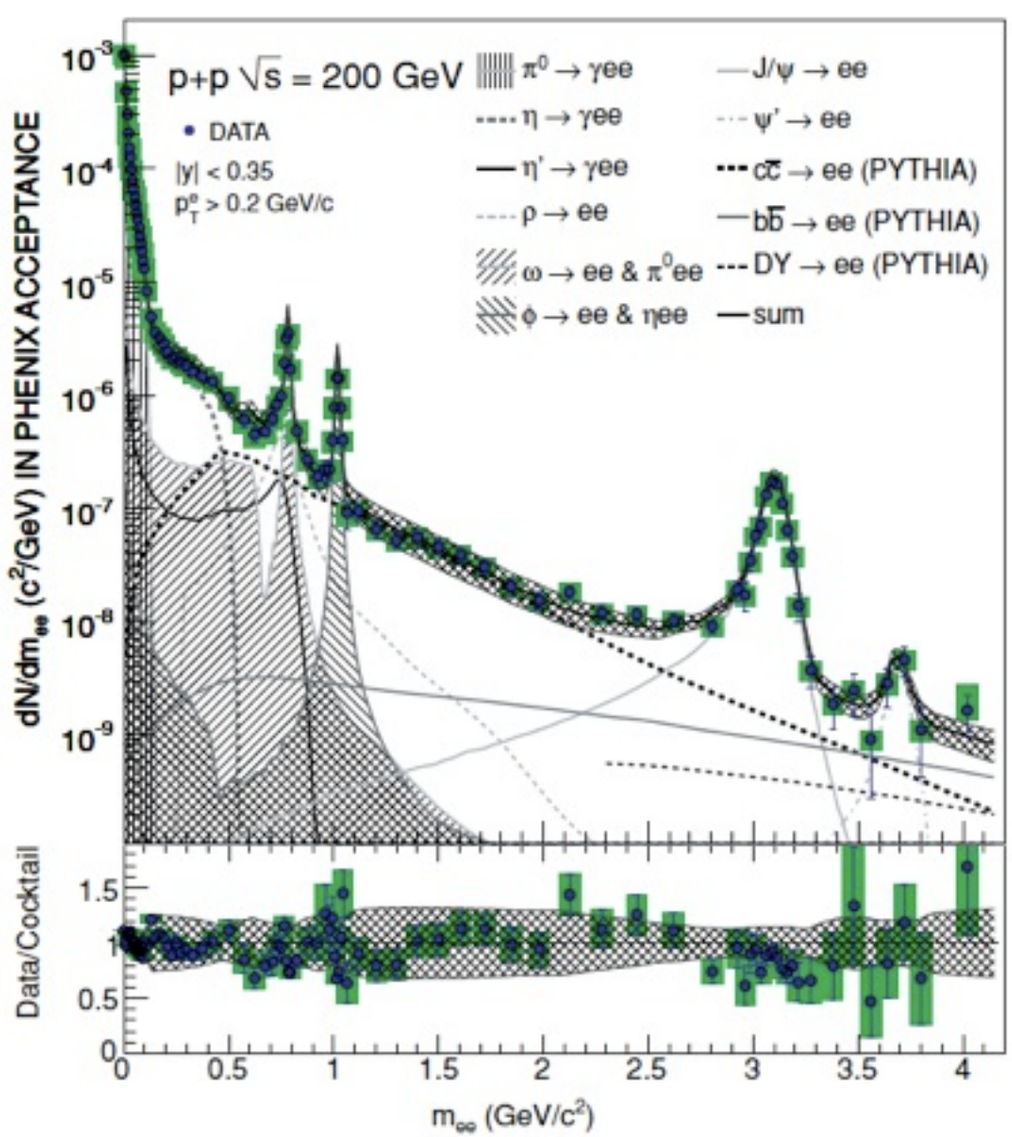
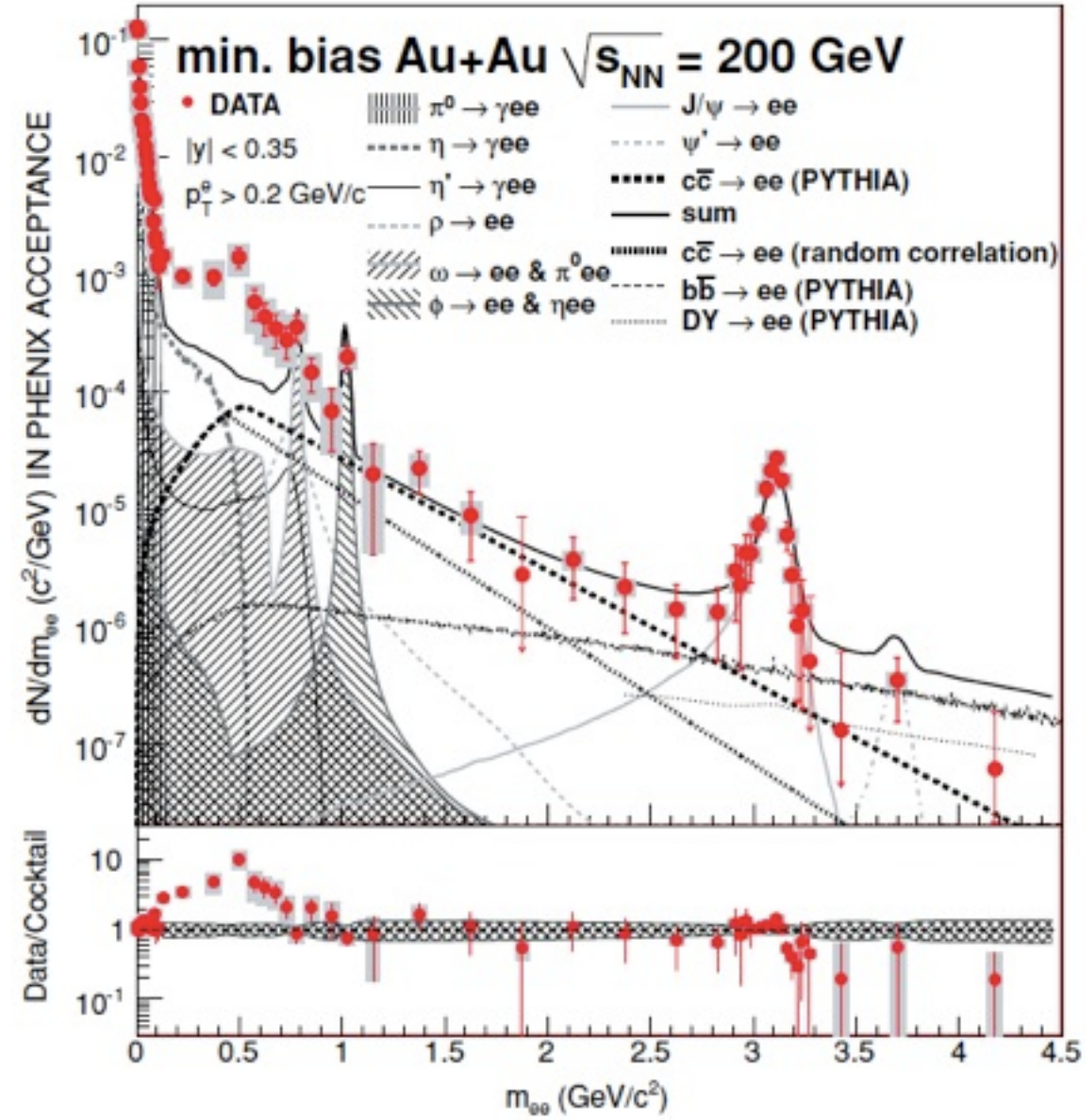
Outline

- Introduction & Motivation
  - thermal dilepton & photon emission rate, electrical conductivity
  - Euclidean correlation and spectral functions
- Vector correlation function on the lattices
  - finite volume & cut-off effects
  - thermal moments of spectral functions
  - continuum extrapolation
- Thermal dilepton rates and electric conductivity
  - chi-square fitting with Ansätz
  - Maximum Entropy Method analysis without Ansätz
- Conclusions

Dilepton rates

PHENIX: PRC **81(2010)034911**





- pp data well described by cocktails
- enhancement in the low mass region in AuAu data

Dilepton rates

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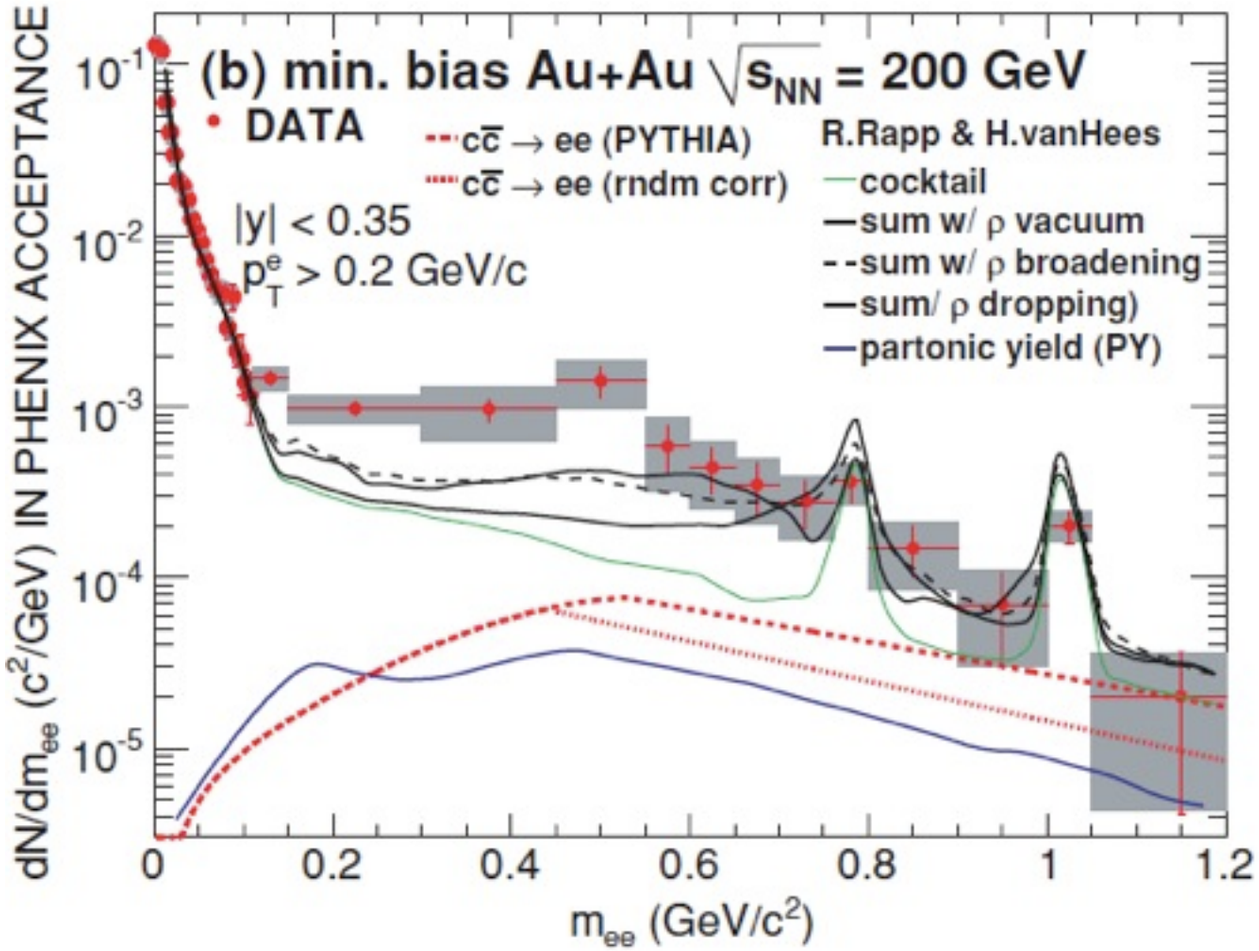
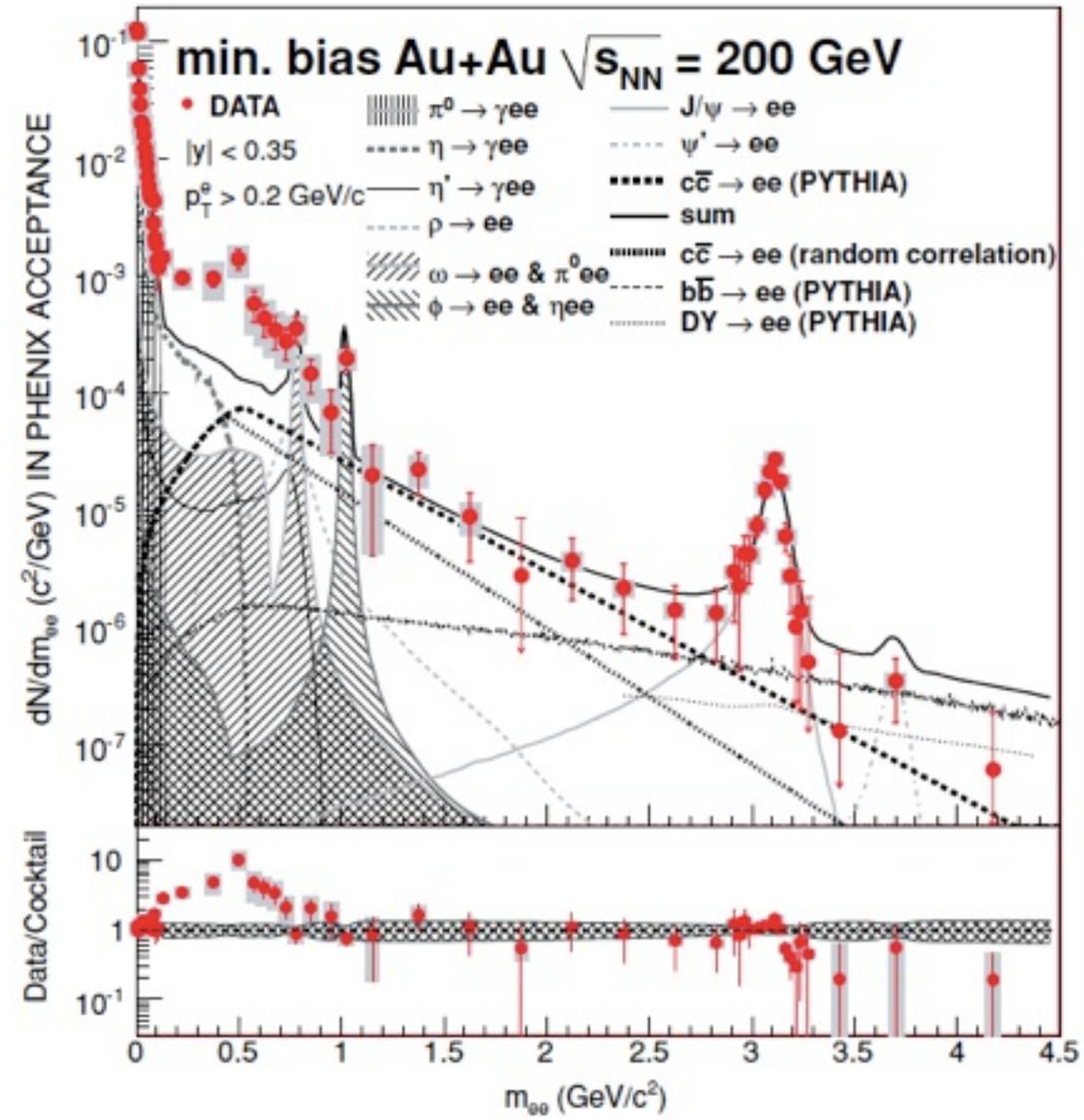
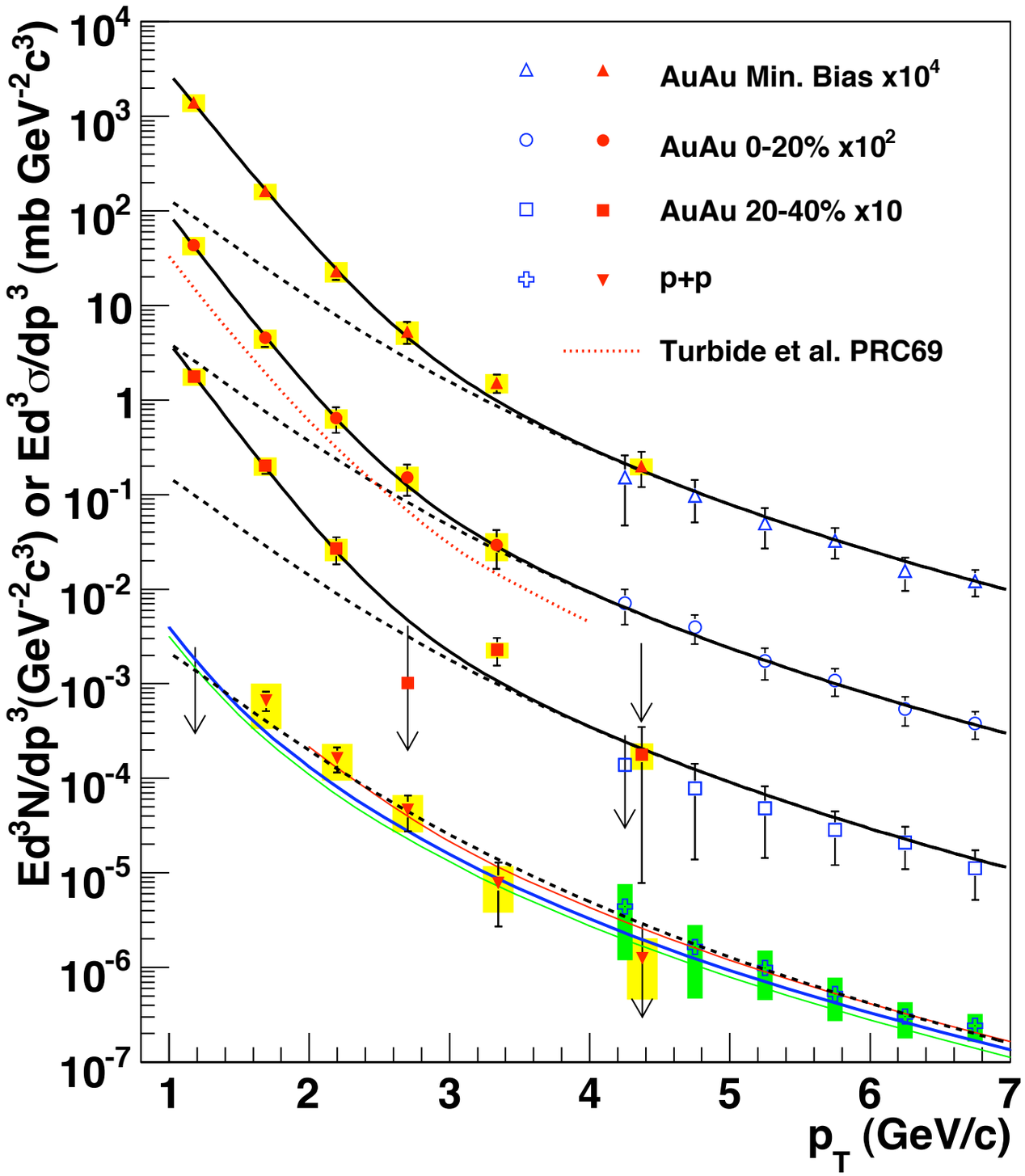




$$\frac{\mathrm{d}N_{l^+l^-}}{\mathrm{d}\omega \mathrm{d}^3p} = C_{em}\frac{\alpha_{em}^2}{6\pi^3}\frac{\rho_V(\omega,\vec{p},T)}{(\omega^2-\vec{p}^2)(\mathrm{e}^{\omega/T}-1)} \qquad C_{em} = e^2\sum_{f=1}^{n_f} Q_f^2$$

- Low mass region hadronic contribution not understood
- First principle calculation needed

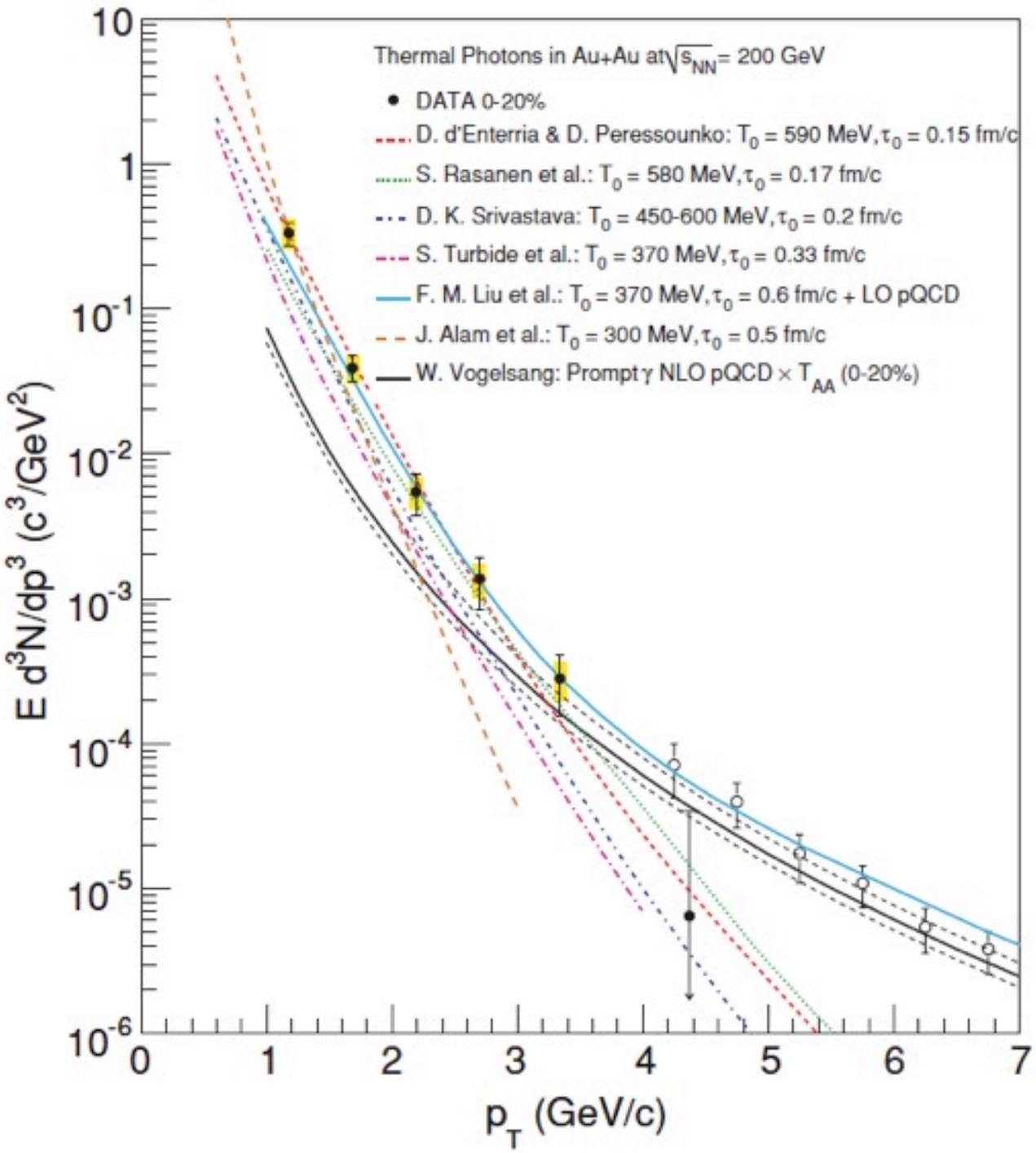
Direct photon production



- pp data well described by NLO pQCD at pt>2GeV/c
- Enhanced direct photon production in AuAu at pt <2.5 GeV/c

PHENIX: PRL 104(2010)132301

Direct photon production



- pp data well described by NLO pQCD at pt>2GeV/c
- Enhanced direct photon production in AuAu at pt <2.5 GeV/c
- Non-perturbative calculation of photon emission from QCP needed

$$\omega \frac{\mathrm{d}R_\gamma}{\mathrm{d}^3p} = C_{em} \frac{\alpha_{em}}{4\pi^2} \frac{\rho_V(\omega = |\vec{p}|, T)}{\mathrm{e}^{\omega/T} - 1}$$

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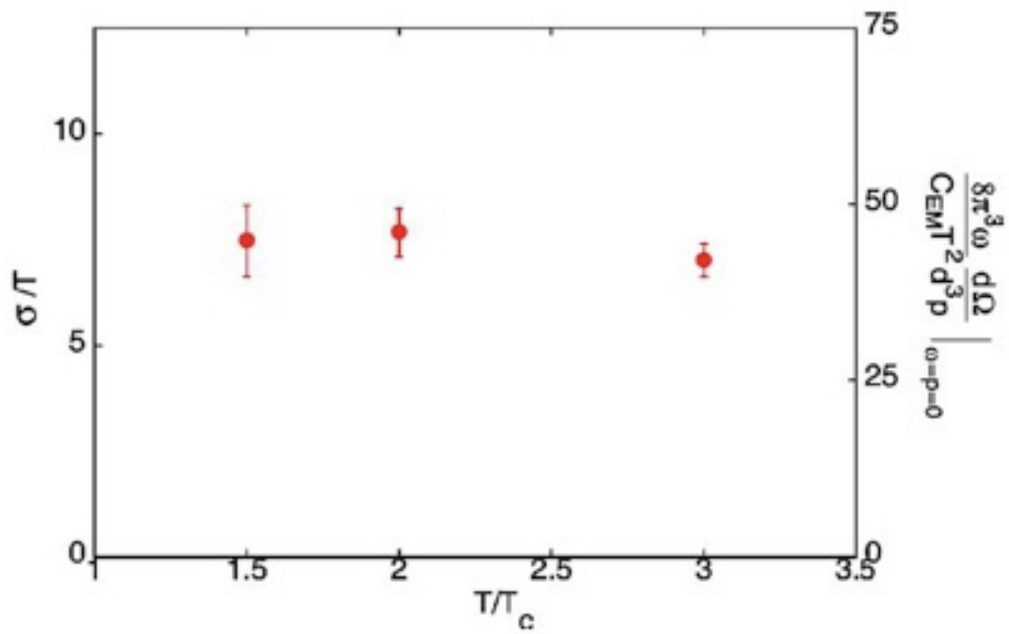
Electrical conductivity

Electrical conductivity: $$\frac{\sigma}{T} = \frac{C_{em}}{6} \lim_{\omega \to 0} \frac{\rho_{ii}(\omega)}{\omega T}$$

The emission rate of soft photons: $$\lim_{\omega \to 0} \omega \frac{\mathrm{d}R_\gamma}{\mathrm{d}^3 p} = \frac{3}{2\pi^2} \sigma(T) T \alpha_{em}$$

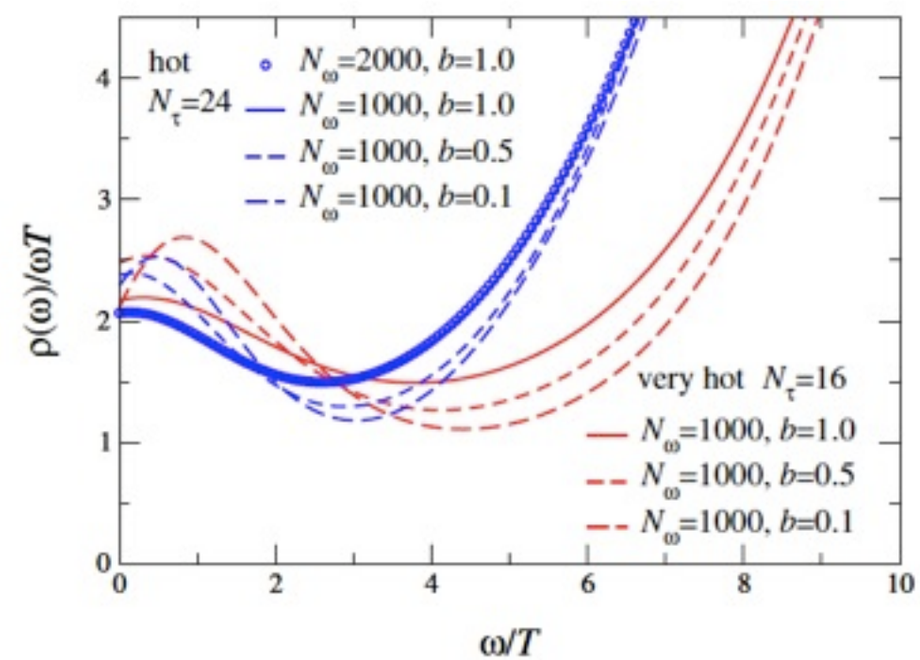
Quite different results from previous lattice calculations:

$$\sigma/T \simeq 7C_{em}$$



S. Gupta PLB 597(2004)57

$$\sigma/T \simeq (0.4 \pm 0.1) C_{em}$$



G. Aarts et al., PRL 99(2007) 022002

staggered fermions used

$N_\tau = 8 - 14,\ N_\sigma \leq 44$

$N_\tau = 16, 24,\ N_\sigma = 64$

Vector correlation & spectral functions

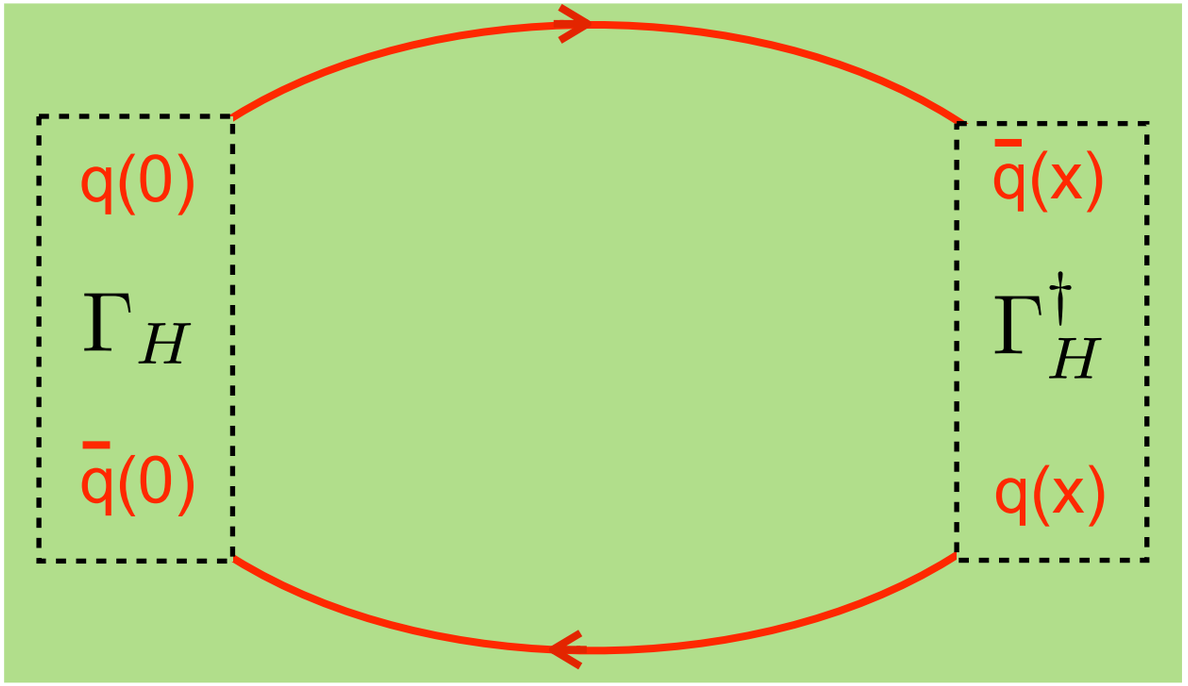
Spectral function

$$\rho(\omega,\vec{p}) = D^+(\omega,\vec{p}) - D^-(\omega,\vec{p}) = 2\,\mathrm{Im}D_R(\omega,\vec{p})$$

Euclidean correlation function

$$G_{\mu\nu}(\tau,\vec{p}) = \int \mathrm{d}^3x \langle J_\mu(\tau,\vec{x}) J_\nu^\dagger(0,\vec{0})\rangle\, e^{i\vec{p}\cdot\vec{x}}$$

$$J_\mu(\tau,\vec{x}) \equiv \bar{q}(\tau,\vec{x})\gamma_\mu q(\tau,\vec{x})$$



Spectral representation

$$G(\tau,\vec{p}) = \int \mathrm{d}^3x\, e^{-i\vec{p}\cdot\vec{x}}\, D^+(-i\tau,\vec{x})\,, \qquad D^+(t,\vec{x}) = D^-(t+i\beta,\vec{x})$$

$$G_H(\tau,\vec{p},T) = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi}\, \rho_H(\omega,\vec{p},T)\, \frac{\cosh(\omega(\tau-1/2T))}{\sinh(\omega/2T)} \quad , \quad H = 00,\ ii,\ V\ .$$

Vector correlation function

$$G_H(\tau,\vec{p},T) = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi}\,\rho_H(\omega,\vec{p},T)\,\frac{\cosh(\omega(\tau-1/2T))}{\sinh(\omega/2T)} \quad , \quad H = 00,\ ii,\ V\ .$$

p=0 in this work

time like correlator G_{00} and space like correlator G_{ii}

$$G_V(\tau,\vec{p},T) = G_{ii}(\tau,\vec{p},T) + G_{00}(\tau,\vec{p},T)$$

conserved current, J_0, gives τ-independent correlator G_{00}

$$G_{00}(T) \equiv -\chi_q T + \mathcal{O}(a^2)$$

the local, non-conserved current needs to be renormalized

$$J_\mu(\tau,\vec{x}) = 2\kappa Z_V \bar{\psi}(\tau,\vec{x})\gamma_\mu\psi(\tau,\vec{x})$$

avoid ambiguities of renormalization

$$R(\tau) \equiv \frac{G_V(\tau)}{G_{00}(\tau)} \quad ; \quad R(\tau) \equiv \frac{G_V(\tau)}{G_{00}(\tau)G_V^{free}(\tau T)}$$

Prior information on spectral functions

- free vector spectral function (in the infinite temperature limit)

$$\rho_{00}^{free}(\omega) = -2\pi T^2 \omega \delta(\omega)$$

$$\rho_{ii}^{free}(\omega) = 2\pi T^2 \omega \delta(\omega) + \frac{3}{2\pi}\omega^2 \tanh(\frac{\omega}{4T})$$

  - δ-functions cancel in $\rho_V(\omega) \equiv \rho_{00}(\omega) + \rho_{ii}(\omega)$

- vector spectral function at T<∞
  - δ-function in ρ_{00} is protected

$$\rho_{00}(\omega, T) = -2\pi \chi_q \omega \delta(\omega)$$

  - δ-function in ρ_{ii} is smeared out

possible form: Breit-Wigner (BW) form + modified continuum

$$\rho_{ii}(\omega, T) = \chi_q c_{BW} \frac{\omega\Gamma}{\omega^2 + (\Gamma/2)^2} + \frac{3}{2\pi}\left(1 + \frac{\alpha_s}{\pi}\right) \omega^2 \tanh(\frac{\omega}{4T})$$

3-4 parameters: $(\chi_q), c_{BW}, \Gamma, \alpha_s$

Basic ideas of Lattice QCD

Expectation values of the observable O in the path integral formalism

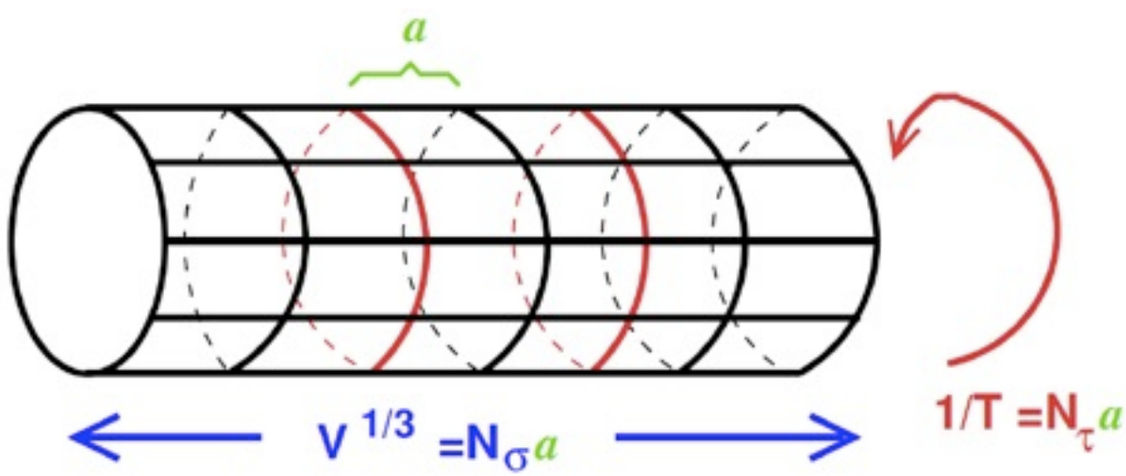
$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}\mathcal{U}\, \mathcal{D}\psi\, \mathcal{D}\bar{\psi}\, \mathcal{O}\, e^{-S_{lat}}$$

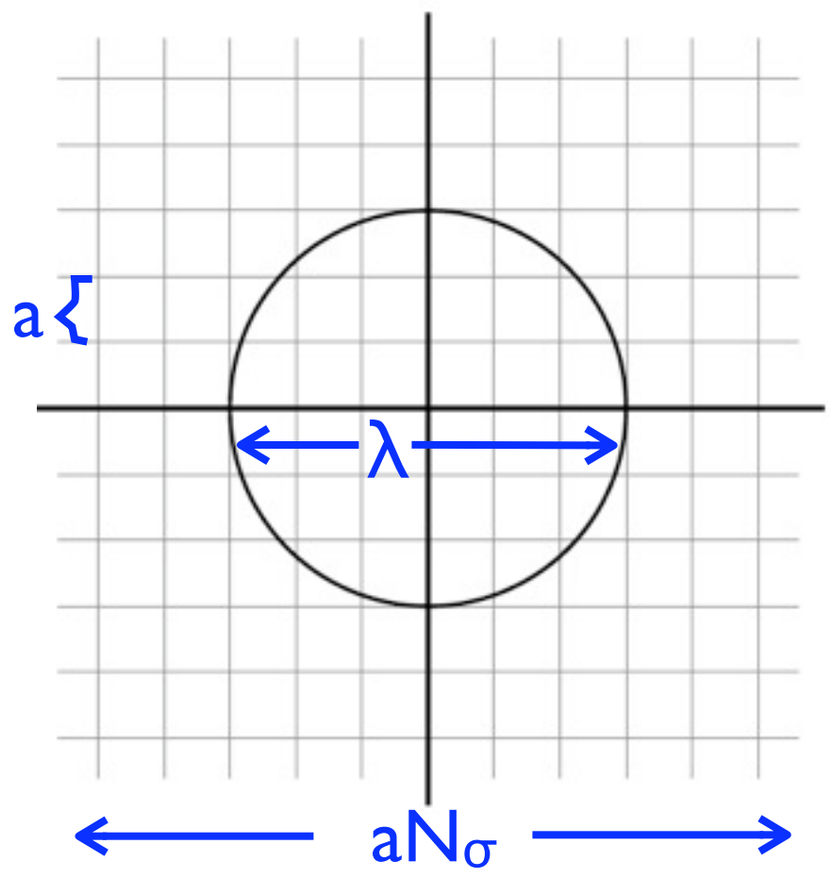
$$S_{lat} = S_g + S_f$$

$$Z = \int \mathcal{D}\mathcal{U}\, \mathcal{D}\psi\, \mathcal{D}\bar{\psi}\, e^{-S_{lat}} = \int \mathcal{D}\mathcal{U}\, e^{-S_g}\, \det M_f$$

- ★ discretize the gauge and fermion fields
- ★ Monte Carlo: generate an ensemble of the field configuration
- ★ calculate the observable on every field configuration of the ensemble
- ★ build ensemble average
- ★ to save computing time, set det M_f = constant: quenched approximation

Lattice QCD at finite temperature





- Four dim. Euclidean lattice $N_\sigma^3 \times N_\tau$
- Temperature $T = 1/(N_\tau a)$
- $a \ll \lambda \ll N_\sigma a$

★ Finite volume effects

★ Lattice cutoff effects

Input parameters

- lattice gauge coupling: $\beta(= 6/g^2)$
- lattice size: N_τ, N_σ
- quark masses
- ... ...

Vector correlation functions on large & fine lattices

- SU(3) gauge configurations at $T/T_c \simeq 1.45$
- lattice size $N_\sigma^3 \times N_\tau$ with N_σ = 32-128 & N_τ=16,24,32,48
- Non-perturbatively clover O(a) improved Wilson fermions
- Quark masses close to chiral limit $\kappa \simeq \kappa_c$

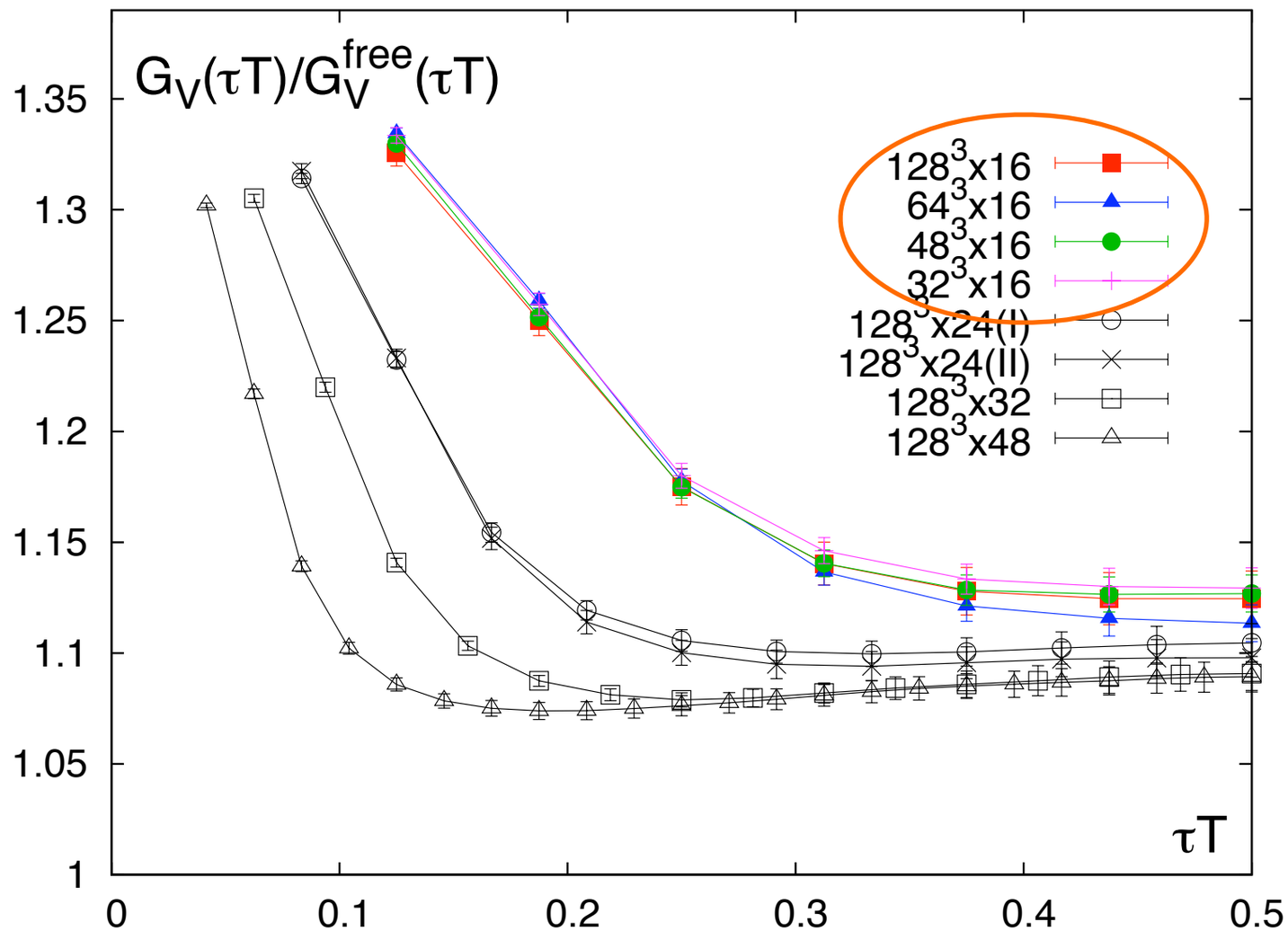
volume dependence

N_τ	N_σ	β	c_{SW}	κ	Z_V	a^{-1}[GeV]	a[fm]	#conf
16	32	6.872	1.4125	0.13495	0.829	6.43	0.031	251
16	48	6.872	1.4125	0.13495	0.829	6.43	0.031	229
16	64	6.872	1.4125	0.13495	0.829	6.43	0.031	191
16	128	6.872	1.4125	0.13495	0.829	6.43	0.031	191
24	128	7.192	1.3673	0.13431	0.842	9.65	0.020	340
	128	7.192	1.3673	0.13440	0.842	9.65	0.020	156
32	128	7.457	1.3389	0.13390	0.851	12.86	0.015	255
48	128	7.793	1.3104	0.13340	0.861	18.97	0.010	451

cut-off dep. & continuum extrapolation

close to continuum

Volume & cut-off dep. of vector corr. function



small volume dep.

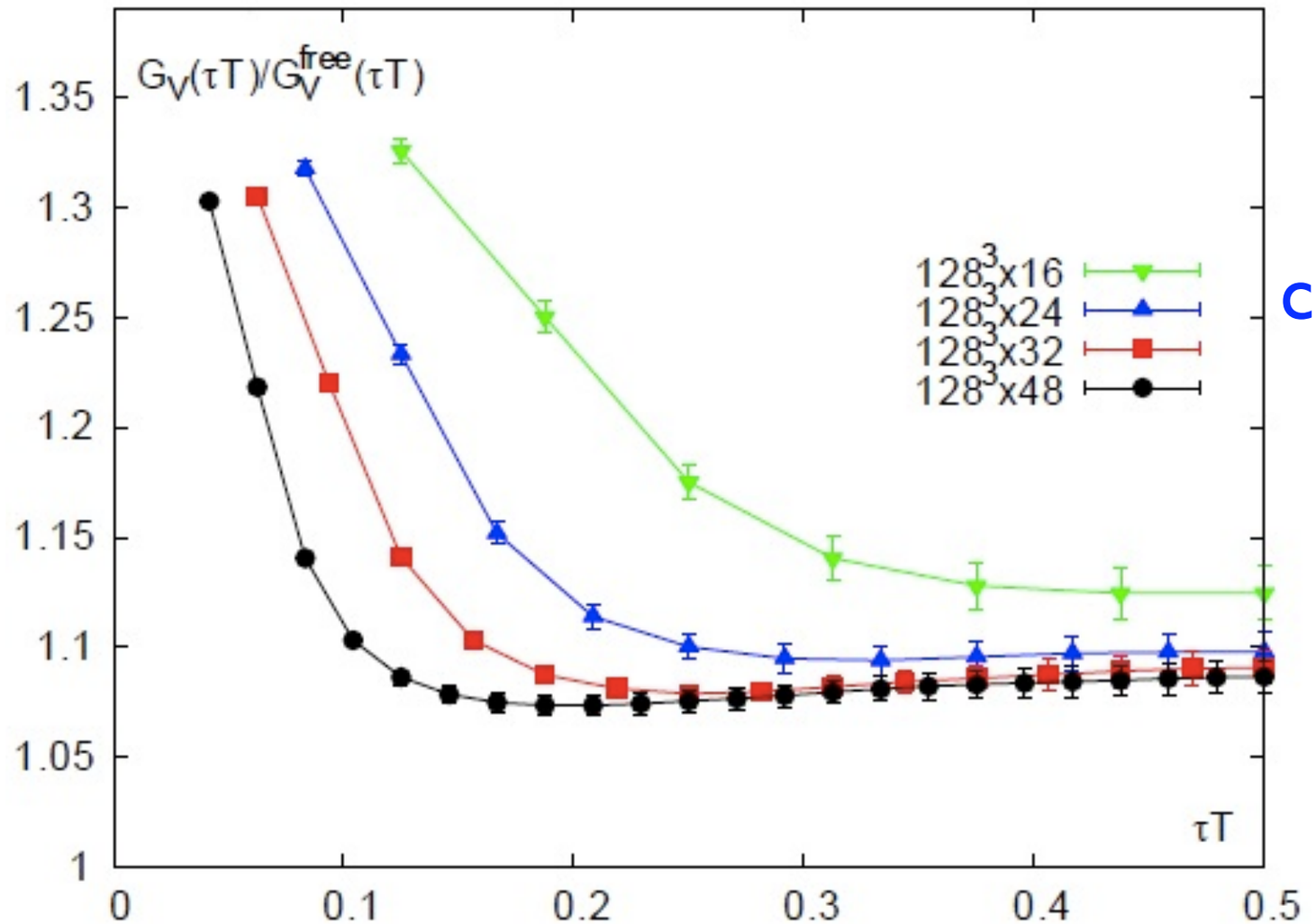
fix lattice volume $V=128^3$

vary N_τ at fixed $T \simeq 1.45T_c$

Normalized by free correlators in the continuum $G_V^{free}(\tau T)$

$$G_V^{free}(\tau T) = 6T^3 \left(\pi \, (1 - 2\tau T) \frac{1 + \cos^2(2\pi\tau T)}{\sin^3(2\pi\tau T)} + 2 \, \frac{\cos(2\pi\tau T)}{\sin^2(2\pi\tau T)} \right)$$

Volume & cut-off dep. of vector corr. function



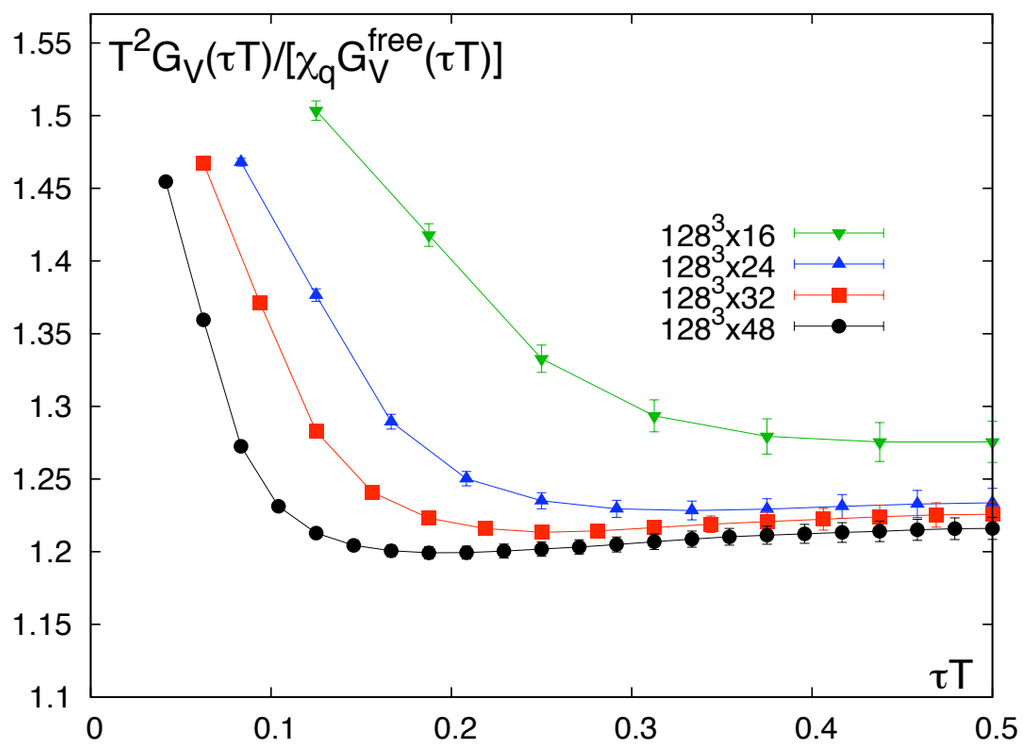
cut-off effects are more server than finite volume effects

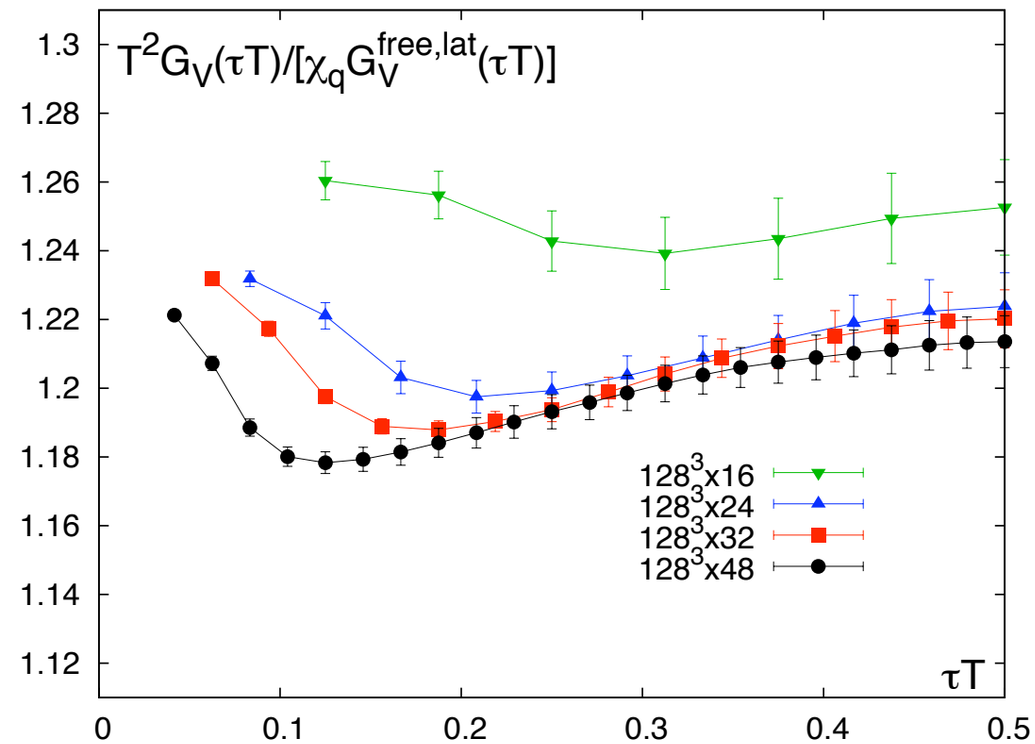
large N_T needed to perform continuum extrapolation

$G_V(\tau T)$ is close to the free case at large τT

incomplete cancelation between $G_{00}(\tau T)$ and BW-contribution to $G_{ii}(\tau T)$?

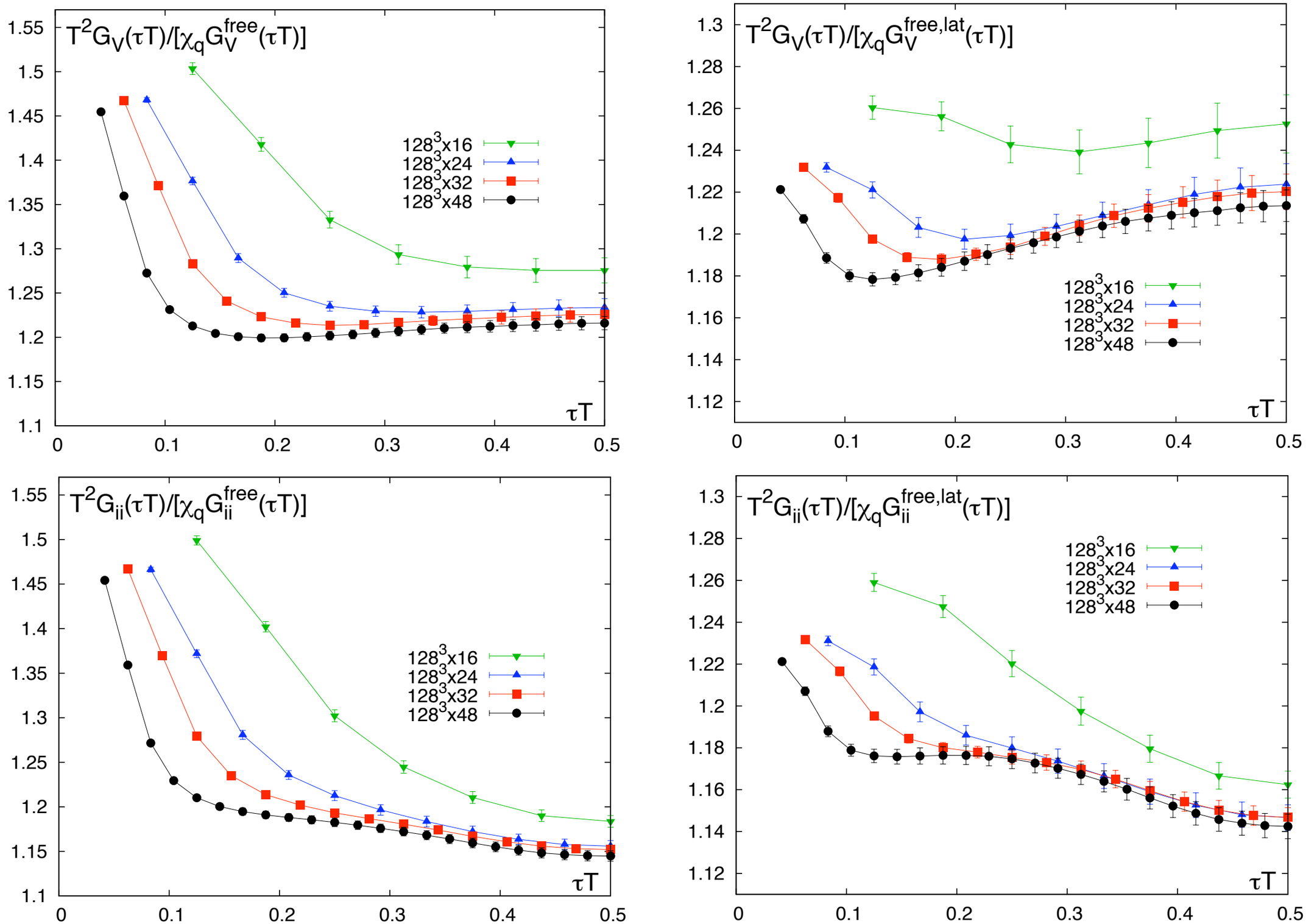
Cut-off effects of vector corr. function



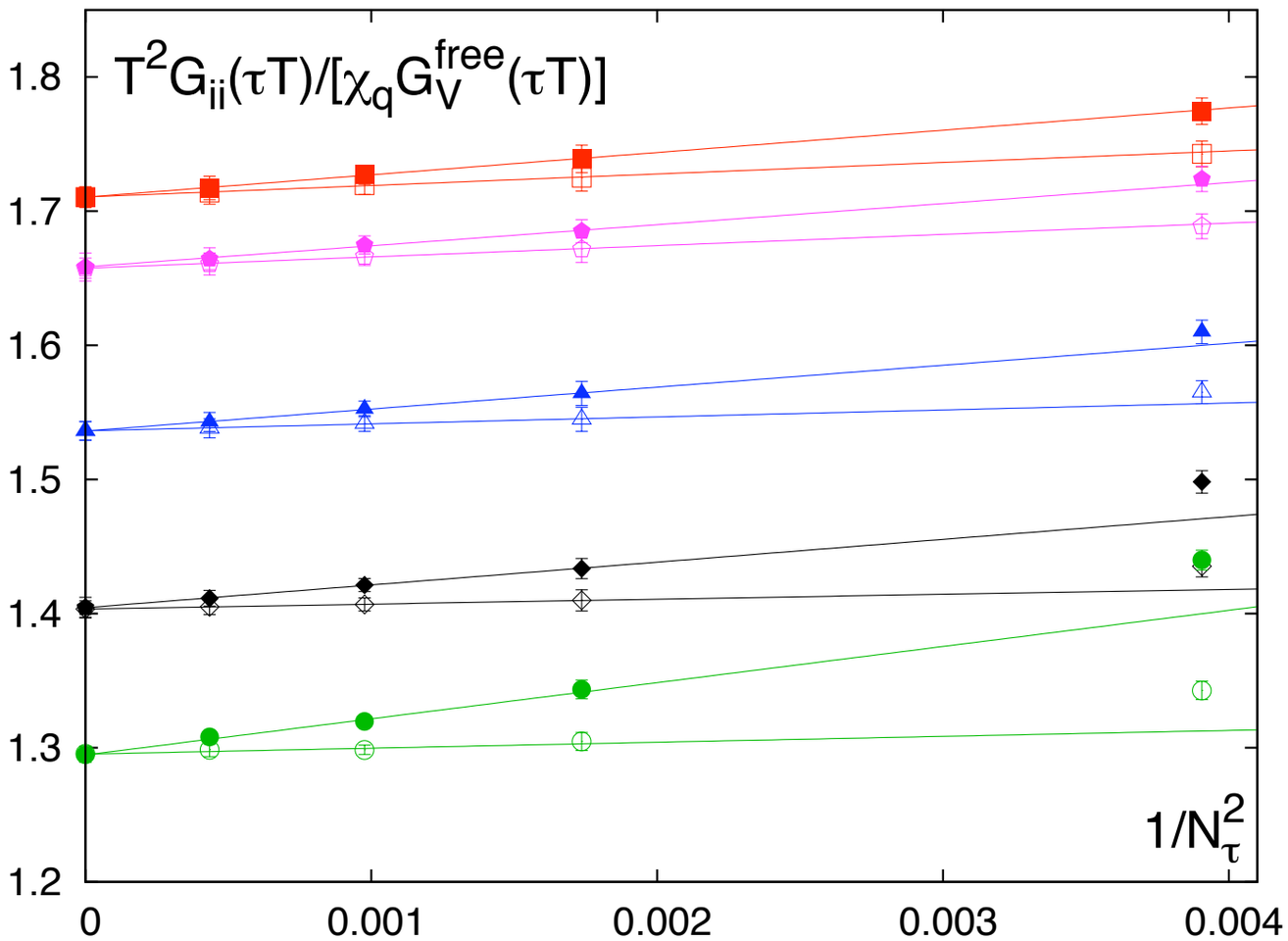


- ratios are free of renormalization ambiguities
- most of the cut-off effects are well described by discretization errors of non-interacting case

Cut-off effects of vector corr. function



Continuum extrapolation



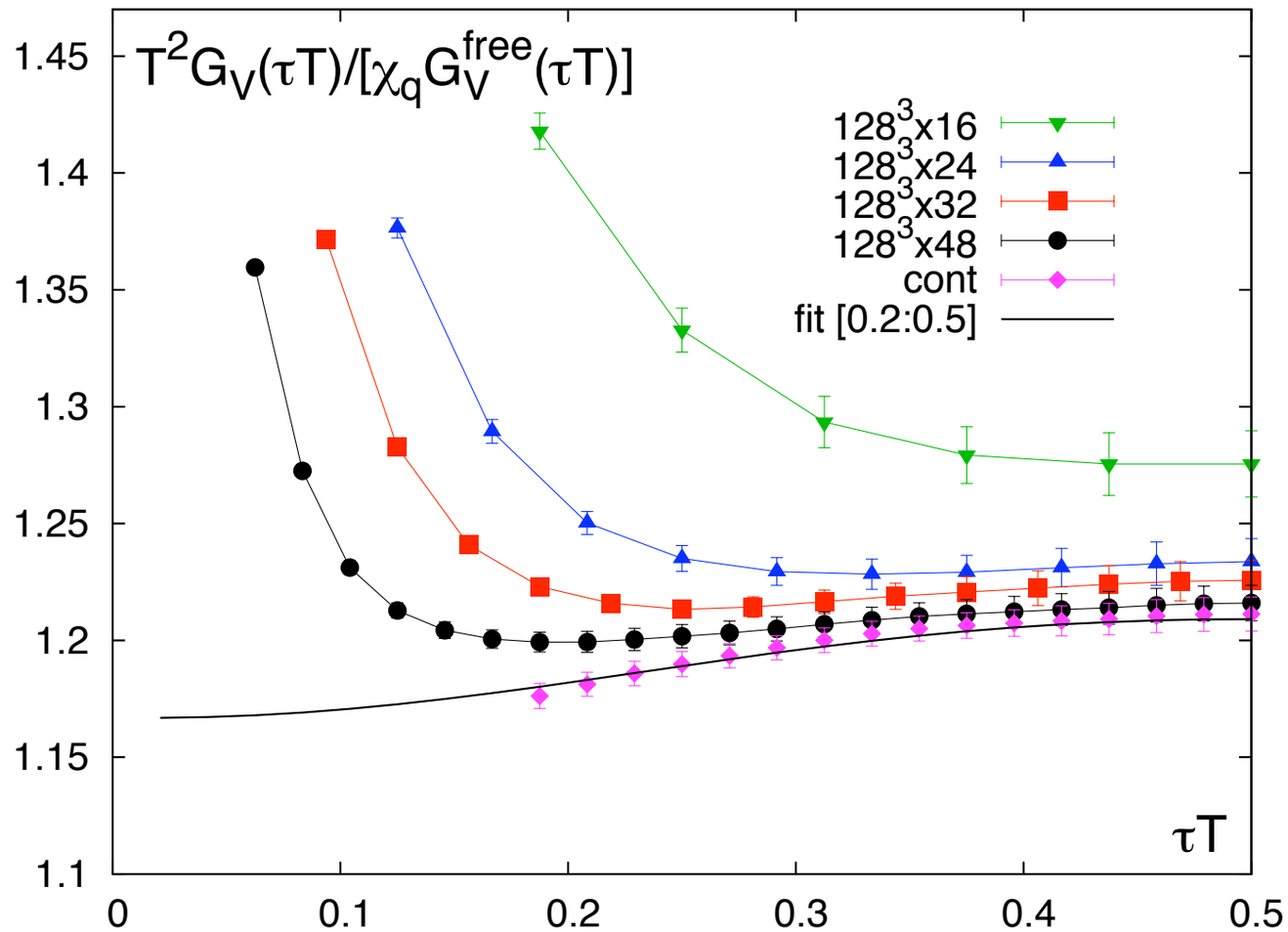
- extrapolation in $(aT)^2 = 1/N_\tau^2$

N_τ	∞
χ_q/T^2	0.897(3)
$G_V^{(2)}/(\widetilde{\chi}_q G_V^{(2),free})$	1.189(13)
$G_V(1/2)/(\widetilde{\chi}_q G_V^{free}(1/2))$	1.211(9)
$G_V(1/4)/(\widetilde{\chi}_q G_V^{free}(1/4))$	1.190(7)
$G_{ii}(1/2)/(\widetilde{\chi}_q G_{ii}^{free}(1/2))$	1.142(9)
$G_{ii}(1/4)/(\widetilde{\chi}_q G_{ii}^{free}(1/4))$	1.172(7)

- extrapolation at the other values of τT using spline interpolation on data at fixed cut-off

- extrapolation under control for $\tau T \gtrsim 0.2$

Continuum extrapolation



$$\frac{G_V(1/2)}{G_V^{free}(1/2)} = 1.086 \pm 0.008 \,,$$

$$\frac{G_V(1/4)}{G_V^{free}(1/4)} = (0.982 \pm 0.005)\frac{G_V(1/2)}{G_V^{free}(1/2)}$$

- Increase of $G_V(\tau T)/G_V^{free}(\tau T)$ with τT is obvious
- The rise with τT indicates that vector spectral function in the low frequency region is different from the free case
- Motivation for the Breit-Wigner type ansatz fitting

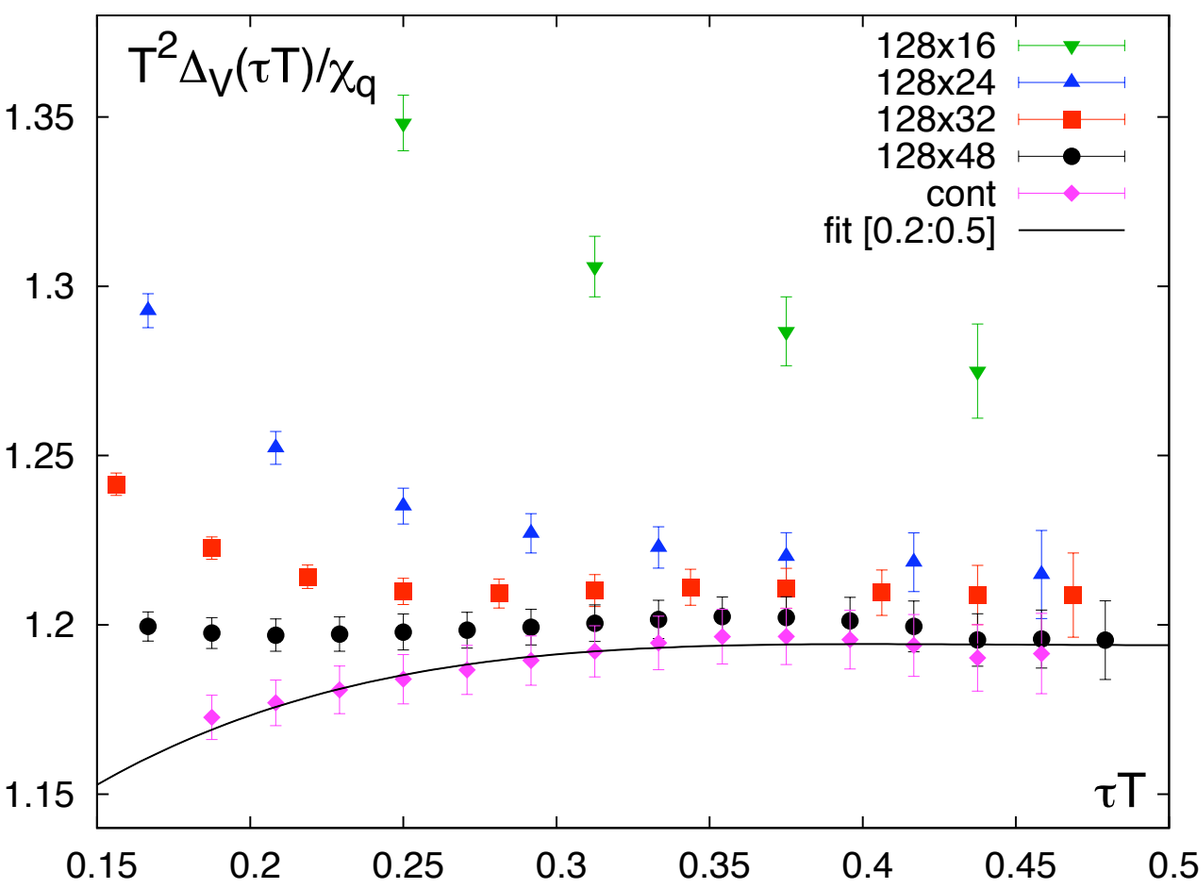
Curvature of vector correlator at τT=1/2

Taylor expansion around the mid-point τT=1/2

$$G_V(\tau T) = G_V^{(0)} \sum_{n=0}^{\infty} \frac{G_V^{(2n)}}{G_V^{(0)}} \left(\tau T - \frac{1}{2}\right)^{2n} = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi} \frac{\rho_V(\omega)}{\sinh(\omega/2T)} \left[1 + \frac{1}{2!}\left(\frac{\omega}{T}\right)^2 (\tau T - \frac{1}{2})^2 + \frac{1}{4!}\left(\frac{\omega}{T}\right)^4 (\tau T - \frac{1}{2})^4 + \cdots\right]$$

Thermal moments of vector spectral functions

$$G_V^{(n)} = \frac{1}{n!} \left.\frac{\mathrm{d}^n G_V(\tau, T)}{\mathrm{d}(\tau T)^n}\right|_{\tau T = 1/2} = \frac{1}{n!} \int_0^\infty \frac{\mathrm{d}\omega}{2\pi} \left(\frac{\omega}{T}\right)^n \frac{\rho_V(\omega)}{\sinh(\omega/2T)}, \quad G_V^{(0)} = G_V(\tau T = 1/2)$$



ratio of mid-point subtracted correlation functions

$$\Delta_V(\tau T) \equiv \frac{G_V(\tau T) - G_V^{(0)}}{G_V^{free}(\tau T) - G_V^{(0),free}} = \frac{G_V^{(2)}}{G_V^{(2),free}} \left(1 + \left(\frac{G_V^{(4)}}{G_V^{(2)}} - \frac{G_V^{(4),free}}{G_V^{(2),free}}\right)\left(\tau T - \frac{1}{2}\right)^2 + \cdots\right)$$

$$\frac{G_V^{(2)}}{G_V^{(0)}} = (0.982 \pm 0.012) \frac{G_V^{(2),free}}{G_V^{(0),free}},$$

$$\frac{G_{ii}^{(2)}}{G_{ii}^{(0)}} = (1.043 \pm 0.010) \frac{G_{ii}^{(2),free}}{G_{ii}^{(0),free}}.$$

Curvature of vector correlator at τT=1/2

Taylor expansion around the mid-point τT=1/2

$$G_V(\tau T) = G_V^{(0)} \sum_{n=0}^{\infty} \frac{G_V^{(2n)}}{G_V^{(0)}} \left(\tau T - \frac{1}{2}\right)^{2n} = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi} \frac{\rho_V(\omega)}{\sinh(\omega/2T)} \left[1 + \frac{1}{2!}\left(\frac{\omega}{T}\right)^2 (\tau T - \frac{1}{2})^2 + \frac{1}{4!}\left(\frac{\omega}{T}\right)^4 (\tau T - \frac{1}{2})^4 + \cdots\right]$$

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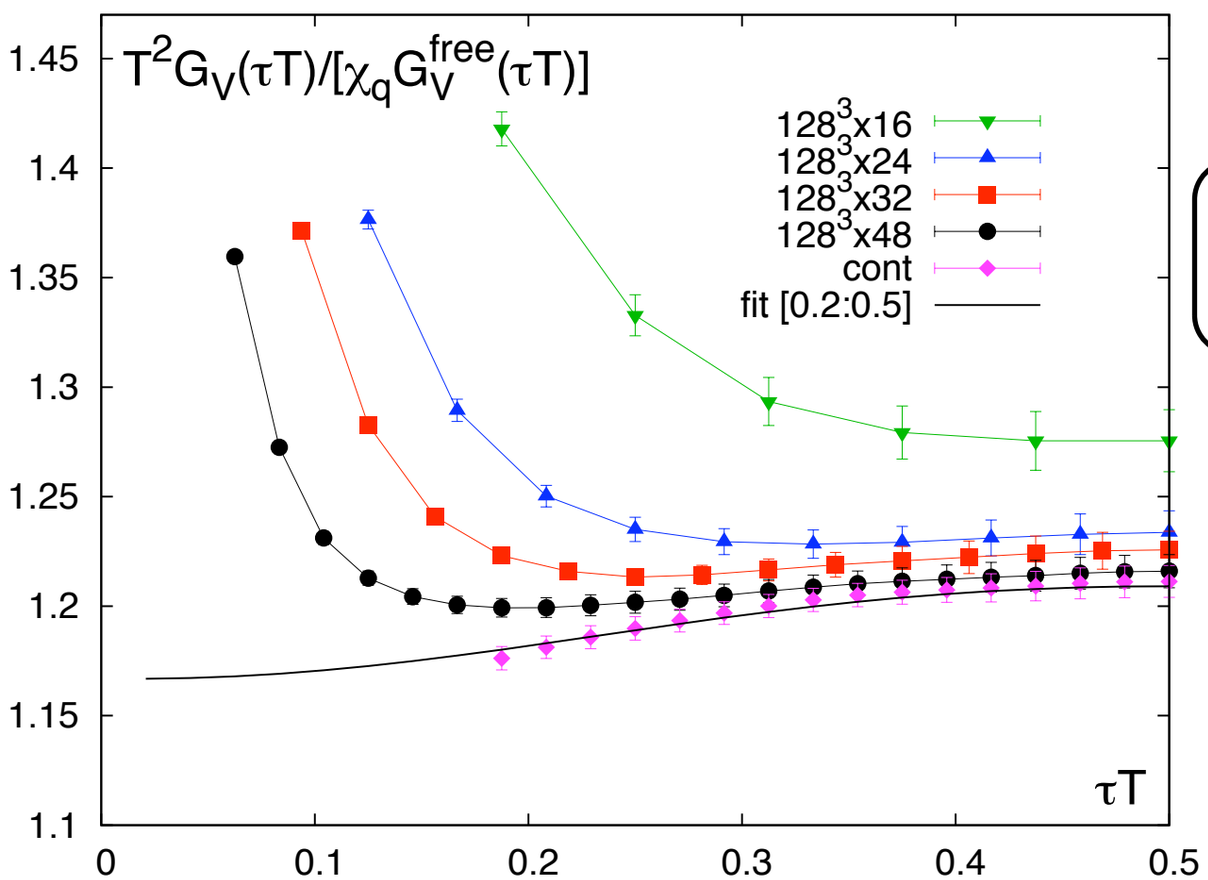

$$\frac{G_V(\tau T)}{G_V^{free}(\tau T)} = \frac{G_V^{(0)}}{G_V^{(0),free}} \left(1 + \left(\frac{G_V^{(2)}}{G_V^{(0)}} - \frac{G_V^{(2),free}}{G_V^{(0),free}}\right)\left(\tau T - \frac{1}{2}\right)^2 + \ldots\right)$$

$$\frac{G_V^{(2)}}{G_V^{(0)}} = (0.982 \pm 0.012) \frac{G_V^{(2),free}}{G_V^{(0),free}},$$

$$\frac{G_{ii}^{(2)}}{G_{ii}^{(0)}} = (1.043 \pm 0.010) \frac{G_{ii}^{(2),free}}{G_{ii}^{(0),free}}.$$

Breit-Wigner + continuum Ansatz

- The correlation function at τT=1/2 is about 2% larger than the corresponding free field value: $G_{ii}(1/2)/G_{ii}^{free}(1/2) = 1.024(8)$
- The second moment of the vector spectral function deviates from the free field value by about 7%: $G_V^{(2)}/G_V^{(2),free} = G_{ii}^{(2)}/G_{ii}^{(2),free} = 1.067(12)$
- The deviation from the free field value increase with increasing Euclidean time: $G_V(1/4)/G_V^{free}(1/4) = (0.982 \pm 0.005) G_V(1/2)/G_V^{free}(1/2)$

Fitting Ansatz

$$\rho_{00}(\omega, T) = -2\pi\chi_q \omega\delta(\omega)$$

$$\rho_{ii}(\omega, T) = \underbrace{2\chi_q\, c_{BW} \frac{\omega\Gamma/2}{\omega^2 + (\Gamma/2)^2}}_{\text{smeared delta}} + \underbrace{\frac{3}{2\pi}(1+k)\,\omega^2 \tanh(\frac{\omega}{4T})}_{\text{modified free spf}}$$

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Fitting Ansatz

$$\frac{\sigma}{T} = \frac{1}{6} \lim_{\omega \to 0} \frac{\rho_{ii}(\omega)}{\omega T} = \frac{C_{em}}{3} \frac{2 c_{BW} \widetilde{\chi}_q}{\widetilde{\Gamma}}$$

$$\rho_{ii}(\omega, T) = \underbrace{2\chi_q \, c_{BW} \frac{\omega\Gamma/2}{\omega^2 + (\Gamma/2)^2}}_{\text{smeared delta}} + \underbrace{\frac{3}{2\pi}(1+k)\,\omega^2 \tanh(\frac{\omega}{4T})}_{\text{modified free spf}}$$

Breit-Wigner + continuum Ansatz

- The correlation function at τT=1/2 is about 2% larger than the corresponding free field value: $G_{ii}(1/2)/G_{ii}^{free}(1/2) = 1.024(8)$
- The second moment of the vector spectral function deviates from the free field value by about 7%: $G_V^{(2)}/G_V^{(2),free} = G_{ii}^{(2)}/G_{ii}^{(2),free} = 1.067(12)$
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Fitting Ansatz

$$\frac{\sigma}{T} = \frac{1}{6} \lim_{\omega \to 0} \frac{\rho_{ii}(\omega)}{\omega T} = \frac{C_{em}}{3} \frac{2 c_{BW} \widetilde{\chi}_q}{\widetilde{\Gamma}}$$

$$\rho_{ii}(\omega, T) = 2\chi_q \, c_{BW} \frac{\omega\Gamma/2}{\omega^2 + (\Gamma/2)^2} + \frac{3}{2\pi}(1+k)\,\omega^2 \tanh(\frac{\omega}{4T})$$

$$\widetilde{\rho}_{ii}(\widetilde{\omega}) = \frac{2 c_{BW} \widetilde{\chi}_q}{\widetilde{\Gamma}} \frac{2\widetilde{\omega}(\widetilde{\Gamma}/2)^2}{\widetilde{\omega}^2 + (\widetilde{\Gamma}/2)^2} + \frac{3}{2\pi}(1+k)\ \widetilde{\omega}^2\ \tanh(\frac{\widetilde{\omega}}{4})$$

Fit to correlators & thermal moments

$$\widetilde{\rho}_{ii}(\widetilde{\omega}) = \frac{2c_{BW}\widetilde{\chi}_q}{\widetilde{\Gamma}} \frac{2\widetilde{\omega}(\widetilde{\Gamma}/2)^2}{\widetilde{\omega}^2 + (\widetilde{\Gamma}/2)^2} + \frac{3}{2\pi}(1+k)\ \widetilde{\omega}^2\ \tanh(\frac{\widetilde{\omega}}{4})$$

1. Correlator data extrapolated in the continuum

$$\widetilde{G}_{ii}(\tau T) = c_{BW}\widetilde{\chi}_q F_{BW}(\tau T, \widetilde{\Gamma}) \ + \ (1+k(T))\,\widetilde{G}_V^{free}(\tau T)$$

$$F_{BW}(\tau T, \widetilde{\Gamma}) = \frac{\widetilde{\Gamma}}{2\pi}\int_0^\infty \mathrm{d}\widetilde{\omega}\frac{\widetilde{\omega}}{(\widetilde{\Gamma}/2)^2+\widetilde{\omega}^2}\frac{\cosh(\widetilde{\omega}(\tau T - 1/2))}{\sinh(\widetilde{\omega}/2)}$$

2. The zeroth and second thermal moments

$$\widetilde{G}_{ii}^{(0)} \equiv \widetilde{G}_{ii}(1/2) = c_{BW}\widetilde{\chi}_q F_{BW}^{(0)}(\widetilde{\Gamma}) + 2\,(1+k(T))$$

$$\Delta_V(\tau T) = c_{BW}\widetilde{\chi}_q \frac{F_{BW}(\tau T,\widetilde{\Gamma}) - F_{BW}(1/2,\widetilde{\Gamma})}{\widetilde{G}_V^{free}(\tau T) - \widetilde{G}_V^{free}(1/2)} \ + \ 1 + k(T)$$

$$\frac{G_V^{(2)}}{G_V^{(2),free}} \equiv \Delta_V(1/2) = c_{BW}\widetilde{\chi}_q \frac{F_{BW}^{(2)}(\widetilde{\Gamma})}{\widetilde{G}_V^{(2),free}} \ + \ 1 + k(T)$$

$k = \alpha_s/\pi$

$g^2(T) = 4\pi\alpha_s \simeq 1.6$

$k = 0.0465(30)\ ,\ \widetilde{\Gamma} = 2.235(75)\ ,\ 2c_{BW}\widetilde{\chi}_q/\widetilde{\Gamma} = 1.098(27)$

$\chi^2/d.o.f. = 0.06,\ d.o.f. = 12$

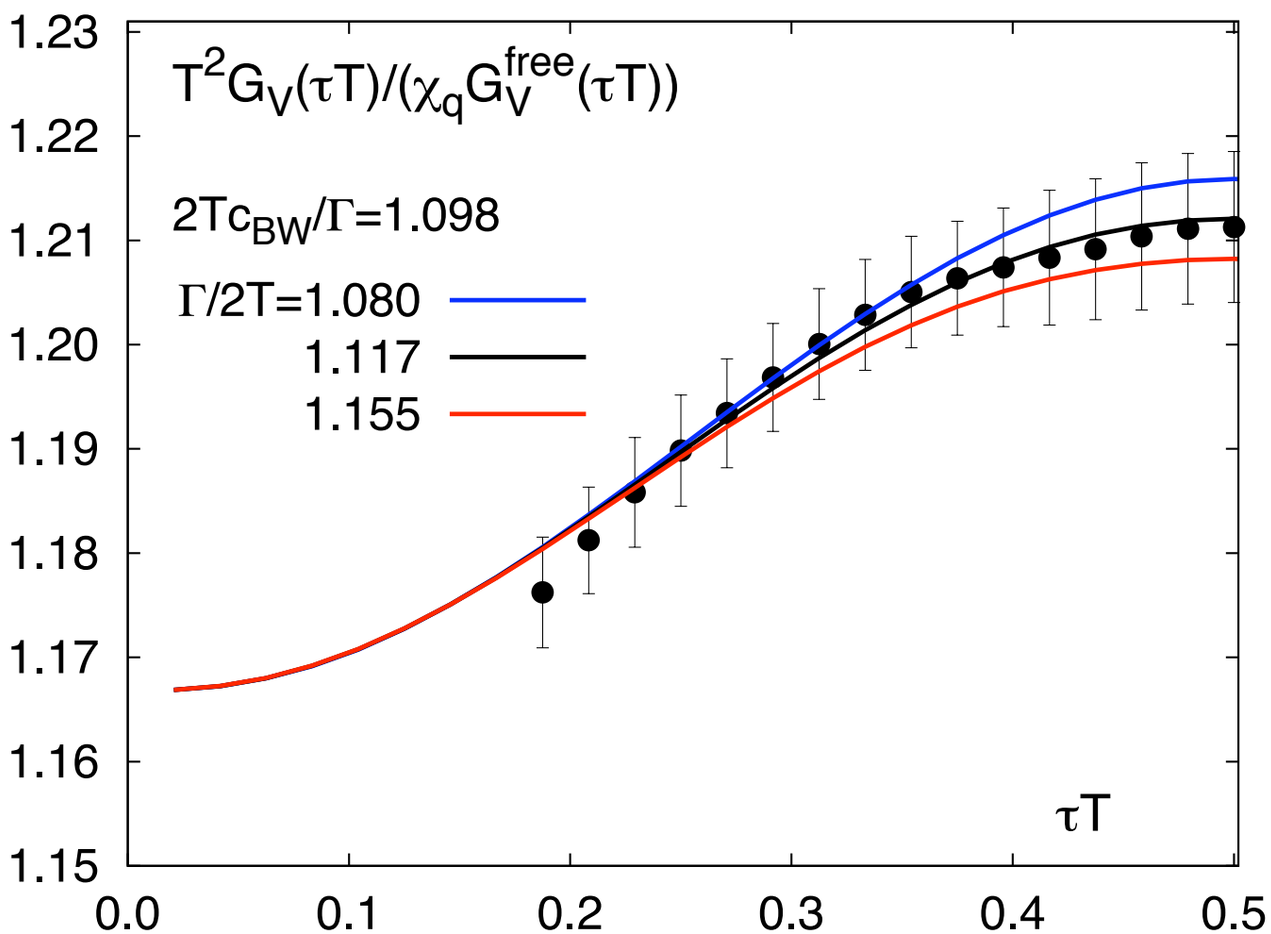
good agreement with
Kaczmarek et al., PRD70(2004)074505

Dependences on the width

$$\widetilde{\rho}_{ii}(\widetilde{\omega}) = \frac{2c_{BW}\widetilde{\chi}_q}{\widetilde{\Gamma}} \frac{2\widetilde{\omega}(\widetilde{\Gamma}/2)^2}{\widetilde{\omega}^2 + (\widetilde{\Gamma}/2)^2} + \frac{3}{2\pi}(1+k)\ \widetilde{\omega}^2\ \tanh(\frac{\widetilde{\omega}}{4})$$

$$k = 0.0465(30)\ ,\ \widetilde{\Gamma} = 2.235(75)\ ,\ 2c_{BW}\widetilde{\chi}_q/\widetilde{\Gamma} = 1.098(27)$$

→ vary width Γ with the other two parameters fixed



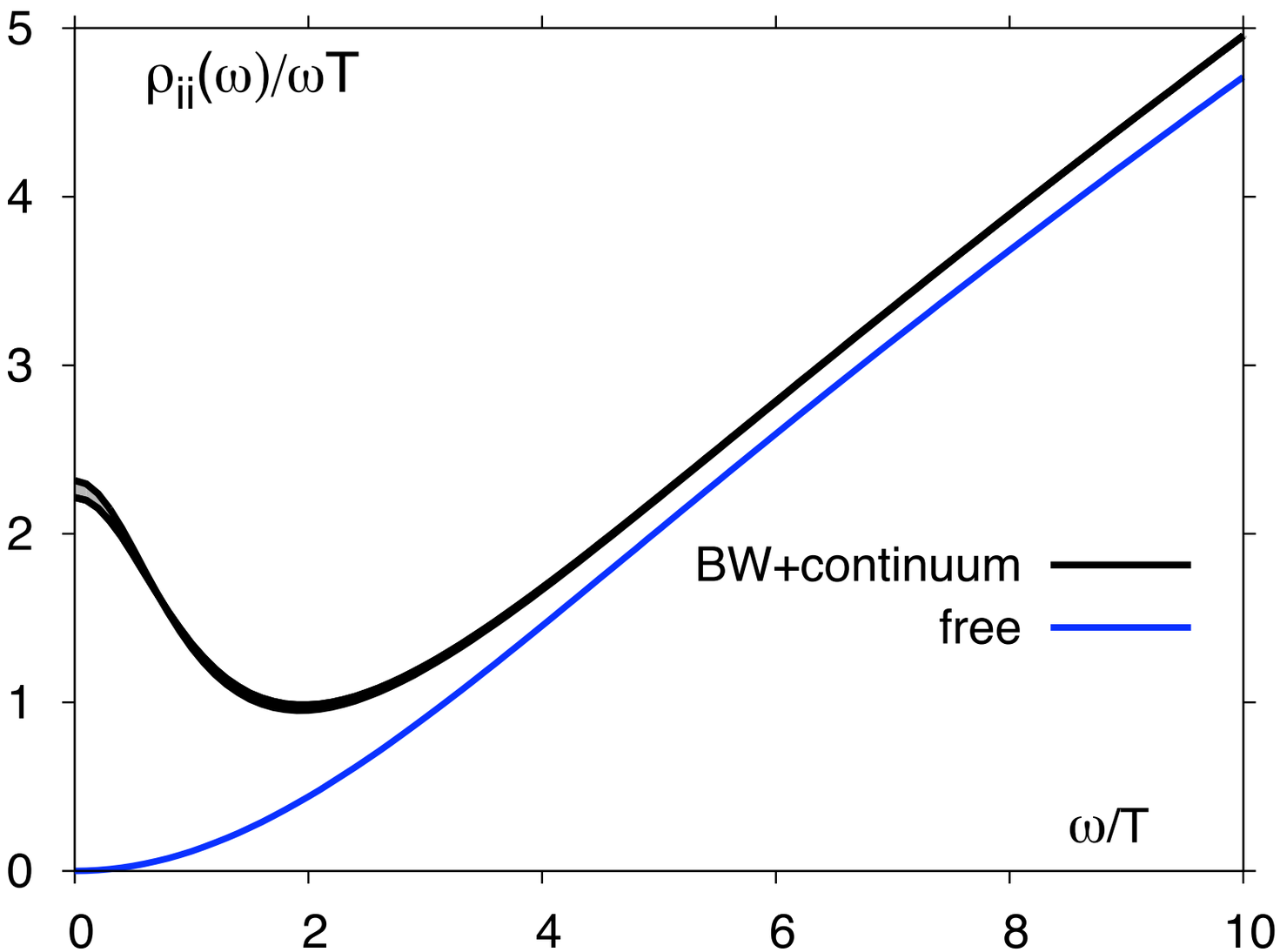
- vector correlation function is sensitive to the low energy, Breit-Wigner contribution only for distance $\tau T \gtrsim 0.25$

- fit parameters can be well constrained by the additional second thermal moment

Estimate of electrical conductivity

$$\widetilde{\rho}_{ii}(\widetilde{\omega}) = \frac{2c_{BW}\widetilde{\chi}_q}{\widetilde{\Gamma}} \frac{2\widetilde{\omega}(\widetilde{\Gamma}/2)^2}{\widetilde{\omega}^2 + (\widetilde{\Gamma}/2)^2} + \frac{3}{2\pi}(1+k)\ \widetilde{\omega}^2\ \tanh(\frac{\widetilde{\omega}}{4})$$

$k = 0.0465(30)$, $\widetilde{\Gamma} = 2.235(75)$, $2c_{BW}\widetilde{\chi}_q/\widetilde{\Gamma} = 1.098(27)$



$$\frac{\sigma}{T} = \frac{C_{em}}{6} \lim_{\omega\to 0} \frac{\rho_{ii}(\omega)}{\omega T}$$

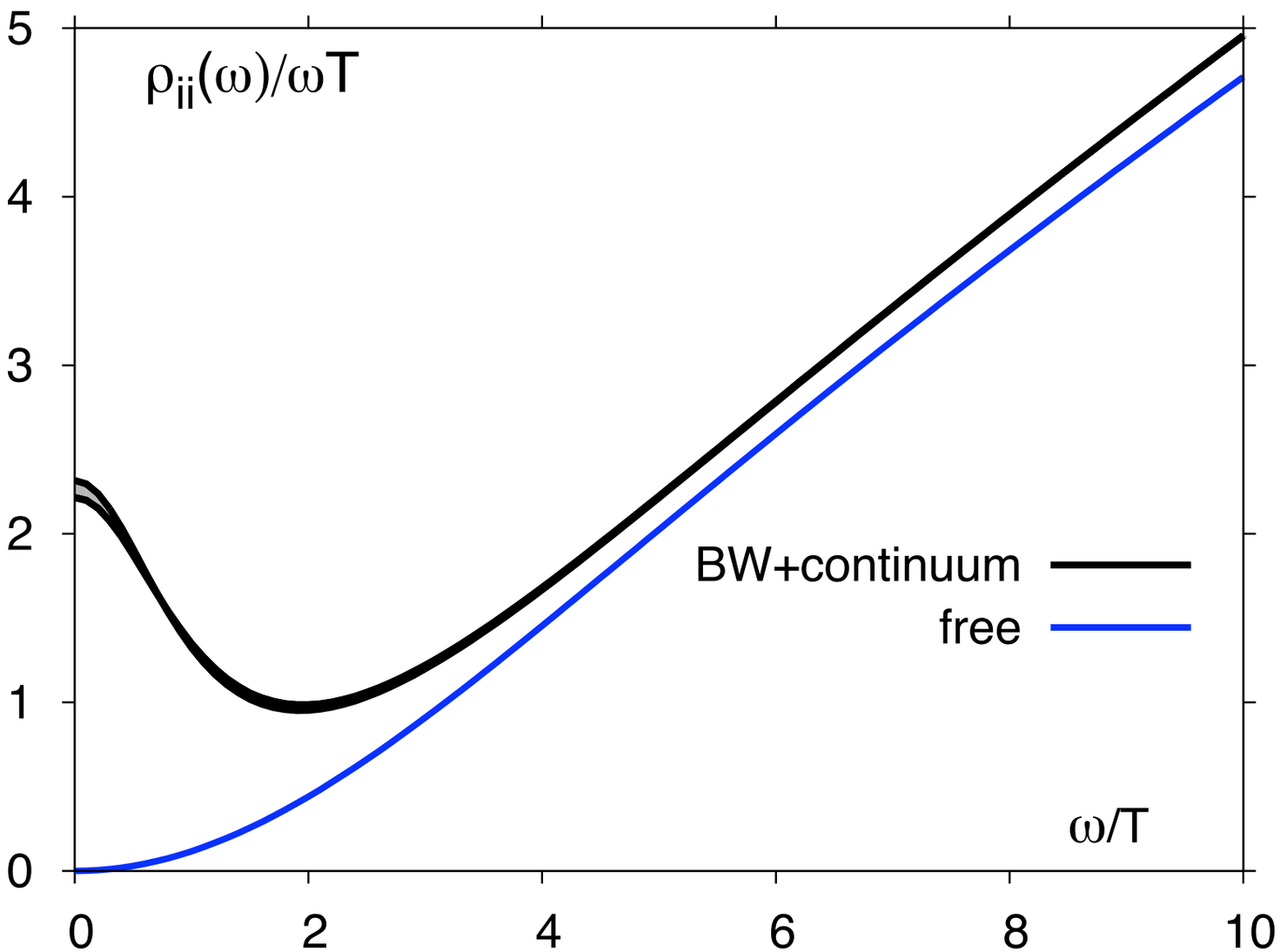
$$= \frac{C_{em}}{3} \frac{2c_{BW}\widetilde{\chi}_q}{\widetilde{\Gamma}}$$

$$= (0.37 \pm 0.01) C_{em}$$

Estimate of electrical conductivity

$$\widetilde{\rho}_{ii}(\widetilde{\omega}) = \frac{2c_{BW}\widetilde{\chi}_q}{\widetilde{\Gamma}} \frac{2\widetilde{\omega}(\widetilde{\Gamma}/2)^2}{\widetilde{\omega}^2 + (\widetilde{\Gamma}/2)^2} + \frac{3}{2\pi}(1+k)\ \widetilde{\omega}^2\ \tanh(\frac{\widetilde{\omega}}{4})$$

$k = 0.0465(30)$, $\widetilde{\Gamma} = 2.235(75)$, $2c_{BW}\widetilde{\chi}_q/\widetilde{\Gamma} = 1.098(27)$



$$\frac{\sigma}{T} = \frac{C_{em}}{6} \lim_{\omega\to 0} \frac{\rho_{ii}(\omega)}{\omega T}$$
$$= \frac{C_{em}}{3} \frac{2c_{BW}\widetilde{\chi}_q}{\widetilde{\Gamma}}$$
$$= (0.37 \pm 0.01) C_{em}$$

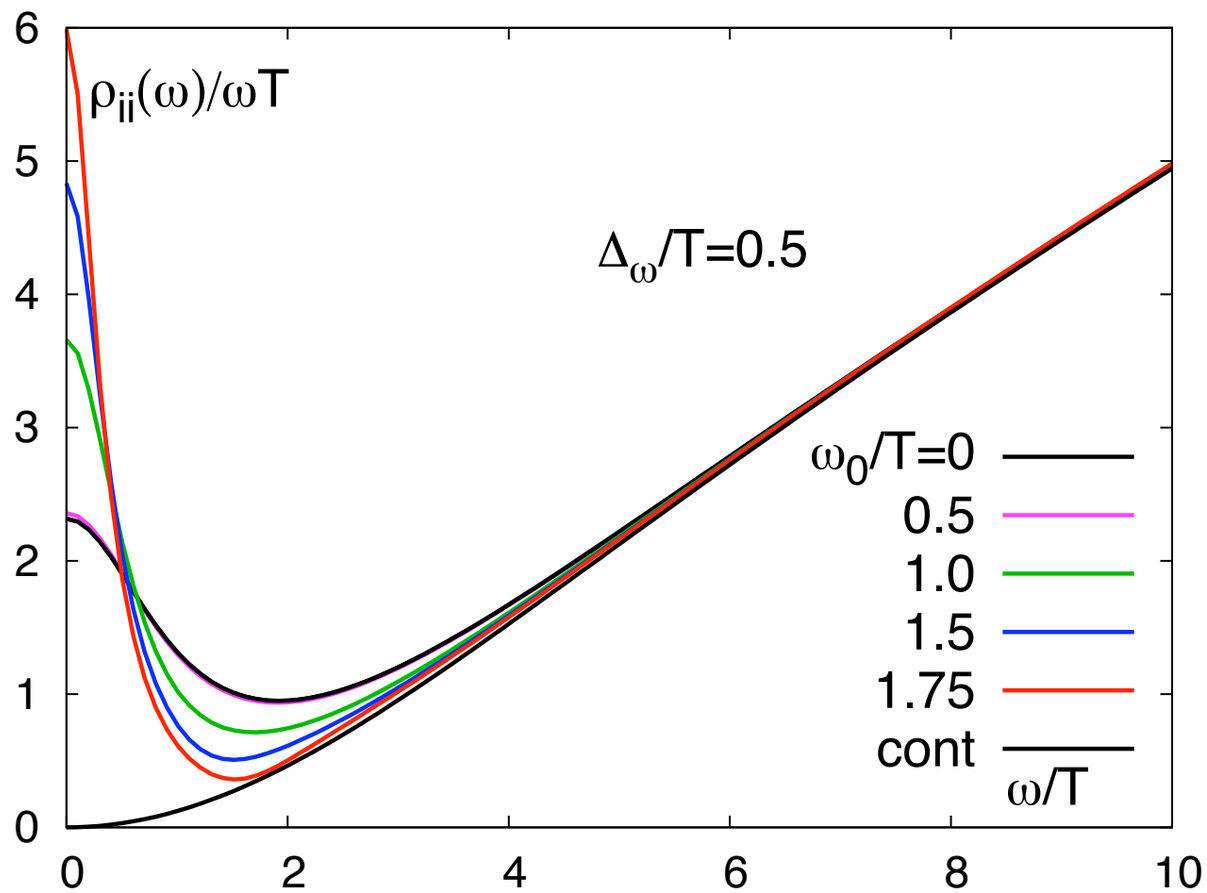
(accidentally) close to Aarts' result!

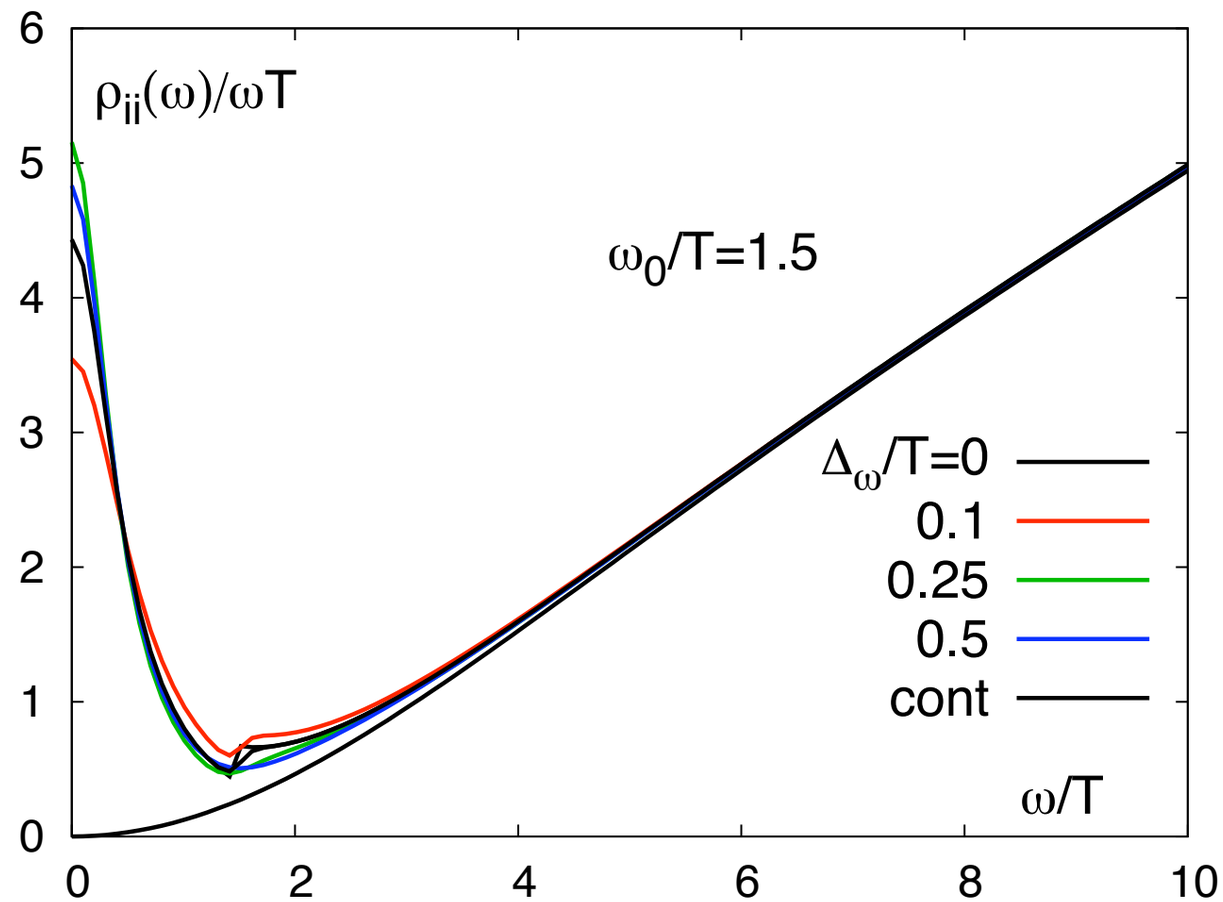
Breit-Wigner + truncated continuum Ansatz

$$\rho_{ii}(\omega) = 2\chi_q c_{BW} \frac{\omega\Gamma/2}{\omega^2 + (\Gamma/2)^2} + \frac{3}{2\pi}(1+k)\ \omega^2\ \tanh(\frac{\omega}{4T})\ \Theta(\omega_0, \Delta_\omega)$$

$$\Theta(\omega_0, \Delta_\omega) = \left(1 + \mathrm{e}^{(\omega_0^2 - \omega^2)/\omega\Delta_\omega}\right)^{-1}$$

delay the onset (ω_0) of the continuum part





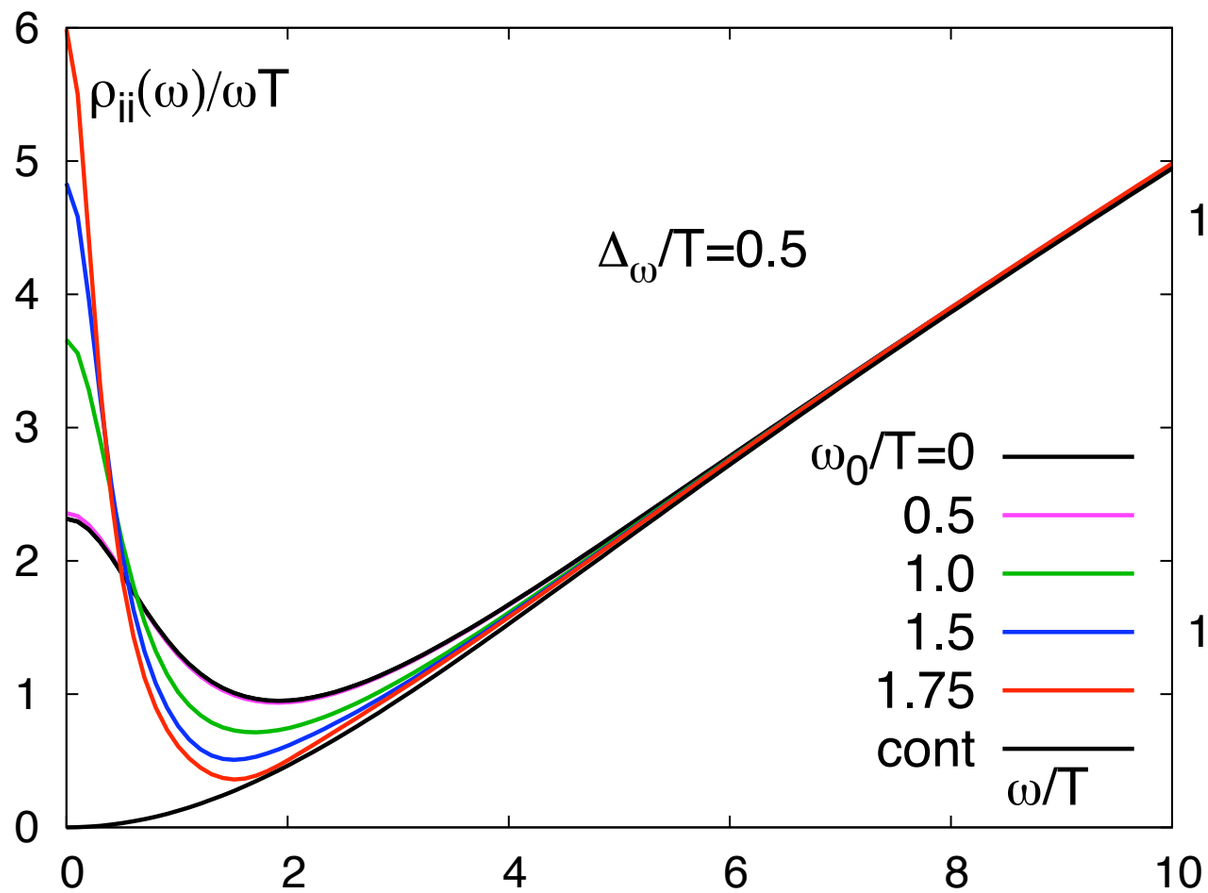
- Rise of BW peaks compensate for the cut from continuum parts
- Fits become worse with increasing ω_0 and/or increasing Δ_ω

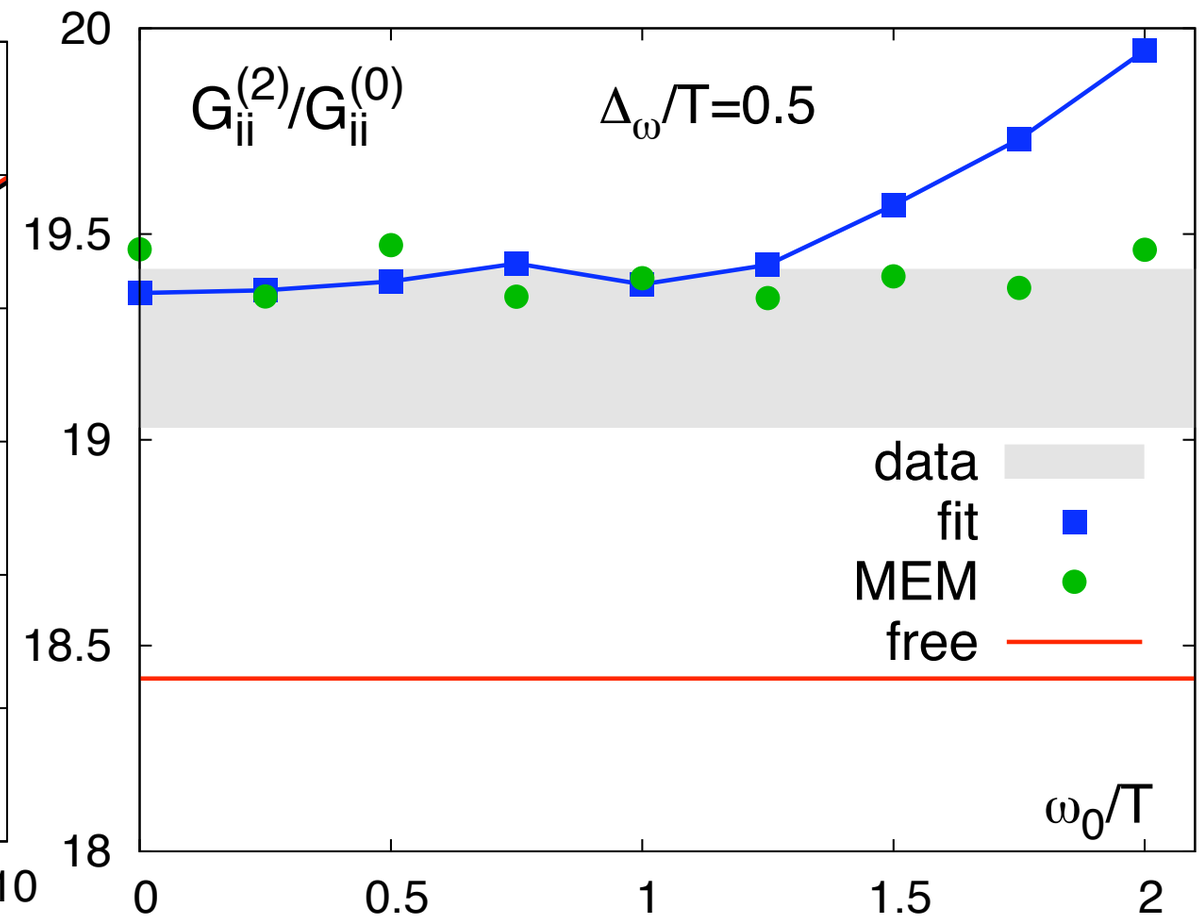
Breit-Wigner + truncated continuum Ansatz

$$\rho_{ii}(\omega) = 2\chi_q c_{BW} \frac{\omega\Gamma/2}{\omega^2 + (\Gamma/2)^2} + \frac{3}{2\pi}(1+k)\ \omega^2\ \tanh(\frac{\omega}{4T})\ \Theta(\omega_0, \Delta_\omega)$$

$$\Theta(\omega_0, \Delta_\omega) = \left(1 + \mathrm{e}^{(\omega_0^2 - \omega^2)/\omega\Delta_\omega}\right)^{-1}$$

delay the onset (ω_0) of the continuum part





- Ratio of thermal moments reacts sensitive to the truncation of the cont. part
- Spectral function should not deviate much from free behavior for $\omega/T \gtrsim (2-4)$

Maximum Entropy Method [Jarrell and Gubernatis, '96]

- Extract spectral function (spf) without ansatz

$$G(\tau, T) = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi} \frac{\cosh\left(\omega(\tau - 1/2T)\right)}{\sinh(\omega/2T)} \rho(\omega, T)$$

Discretized $\mathcal{O}(10)$

Continuous $\mathcal{O}(10^3)$

χ^2 fitting inconclusive

- Maximum Entropy Method (MEM)
  - successful in condensed matter physics, astrophysics, image processing...
  - A method to obtain the most probable image from insufficient data

MEM

Extract spf by Maximum Entropy Method [Asakawa et al., '01]

- Based on the Bayesian theorem

$$P[X|Y] = \frac{P[Y|X]P[X]}{P[Y]}, \quad P[X|Y]: \text{Probability of X given Y}$$

- Ingredients of MEM

$$P[\rho|GH] \propto P[G|\rho H]\ P[\rho|H]$$

ρ: spectral function
G: lattice data
H: prior information of ρ

$P[G|\rho H] \propto \exp(-\chi^2/2)$: likelihood function

$P[\rho|H] \propto \exp(\alpha S)$: prior probability

Shannon-Jaynes entropy: $S = \int_0^\infty \mathrm{d}\omega \left[\rho(\omega) - m(\omega) - \rho(\omega)\ \ln\left(\frac{\rho(\omega)}{m(\omega)}\right)\right]$

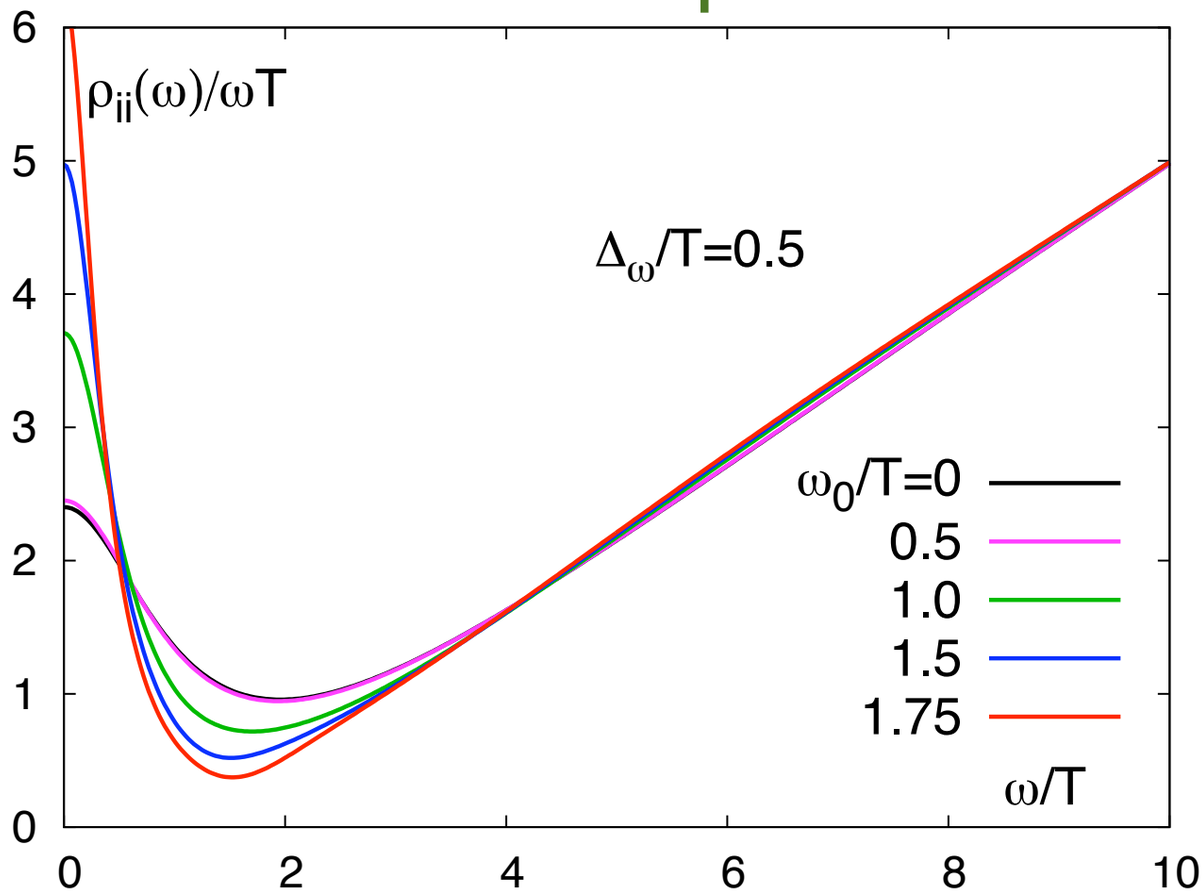
Default Model (DM): m(ω), includes the prior information of ρ, e.g. ρ is positive-definite

DM is the only input parameter in the MEM algorithm

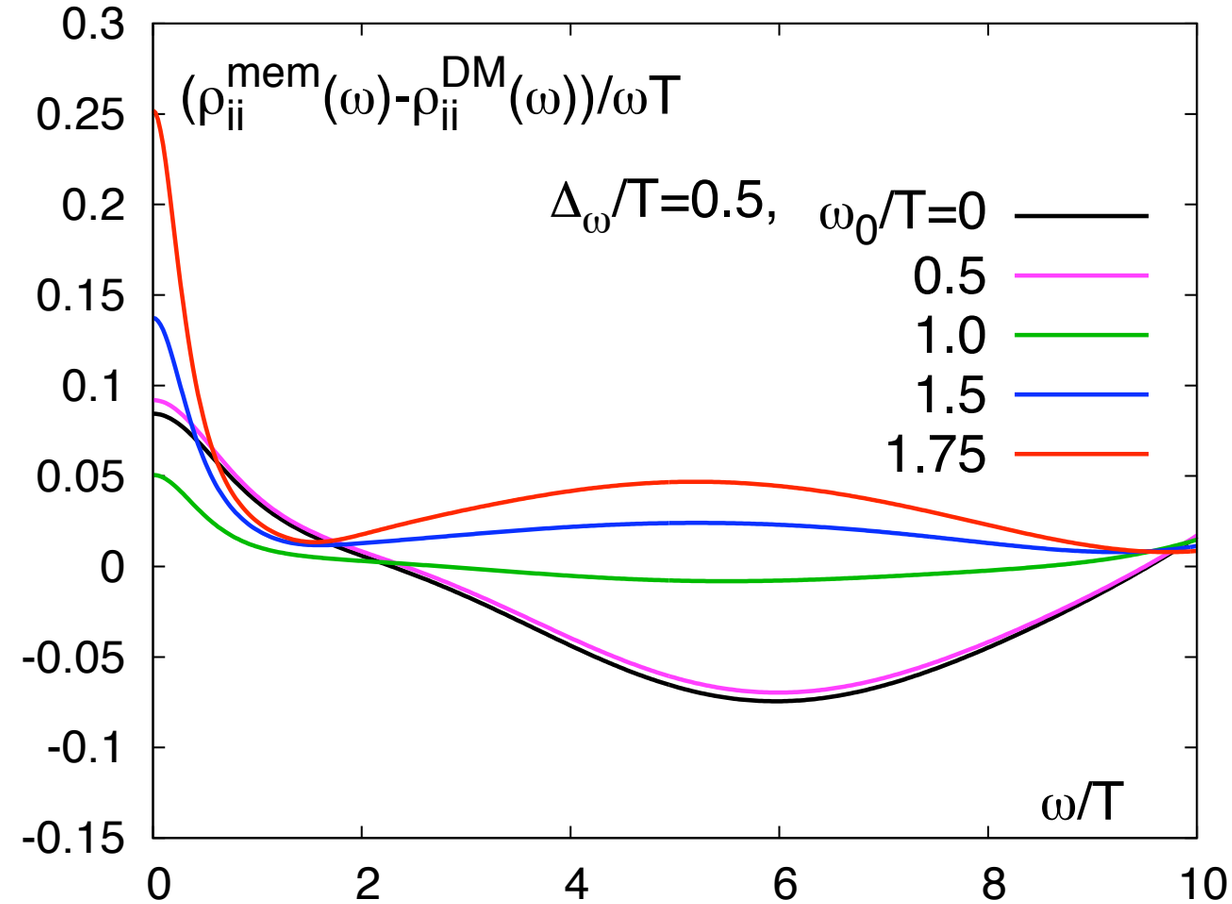
- Maximizing $P[\rho|GH]$ gives the most probable image

MEM analysis

MEM outputs

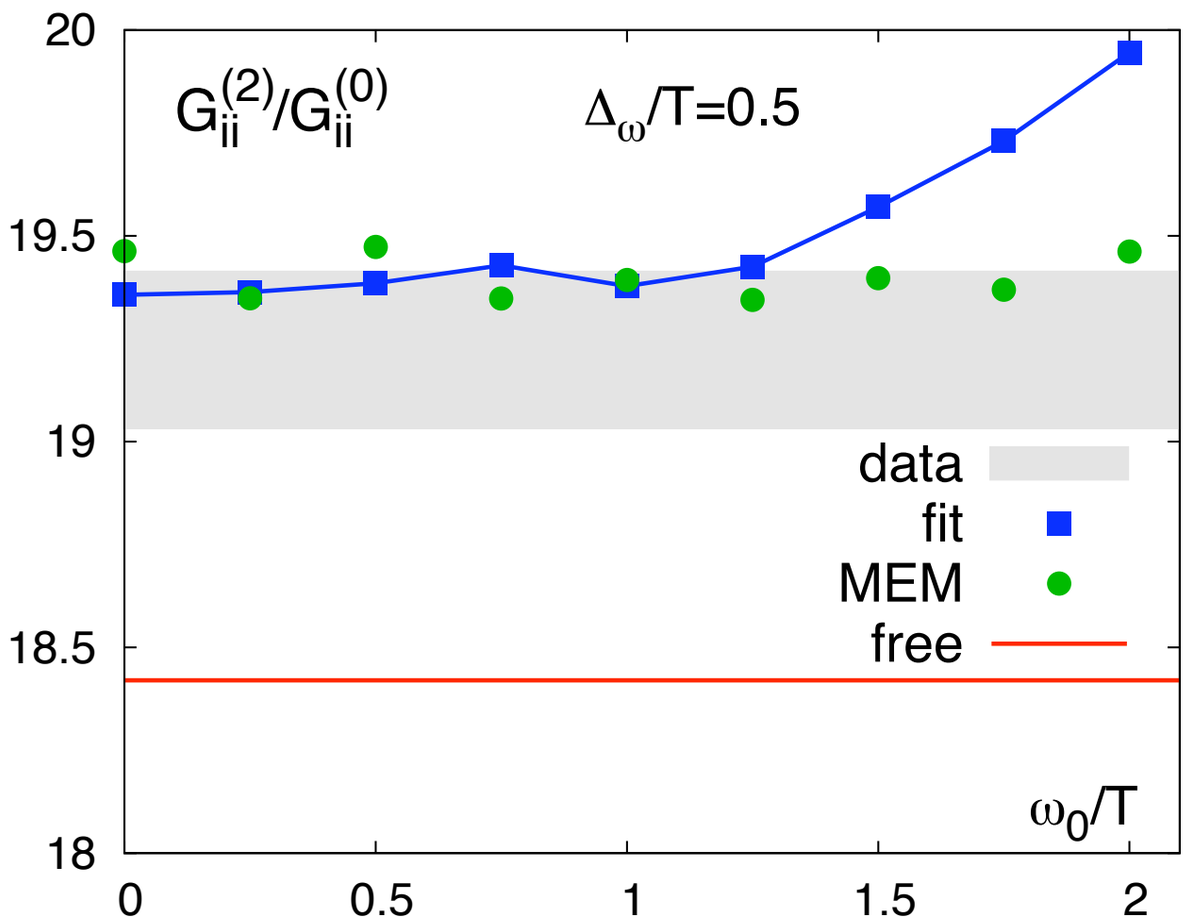


Differences

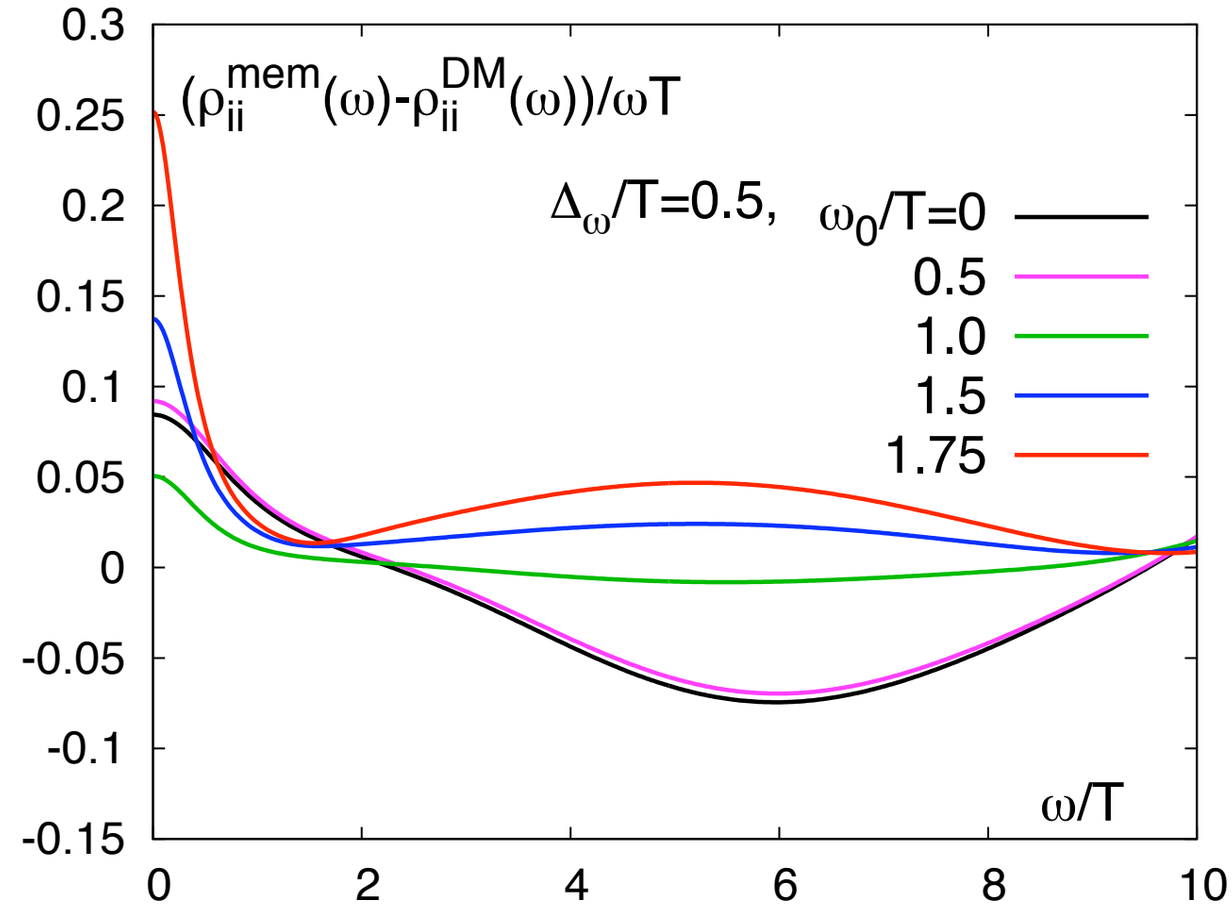


- Differences are smaller than 2% with smaller parameters and increase with worse default models

MEM analysis

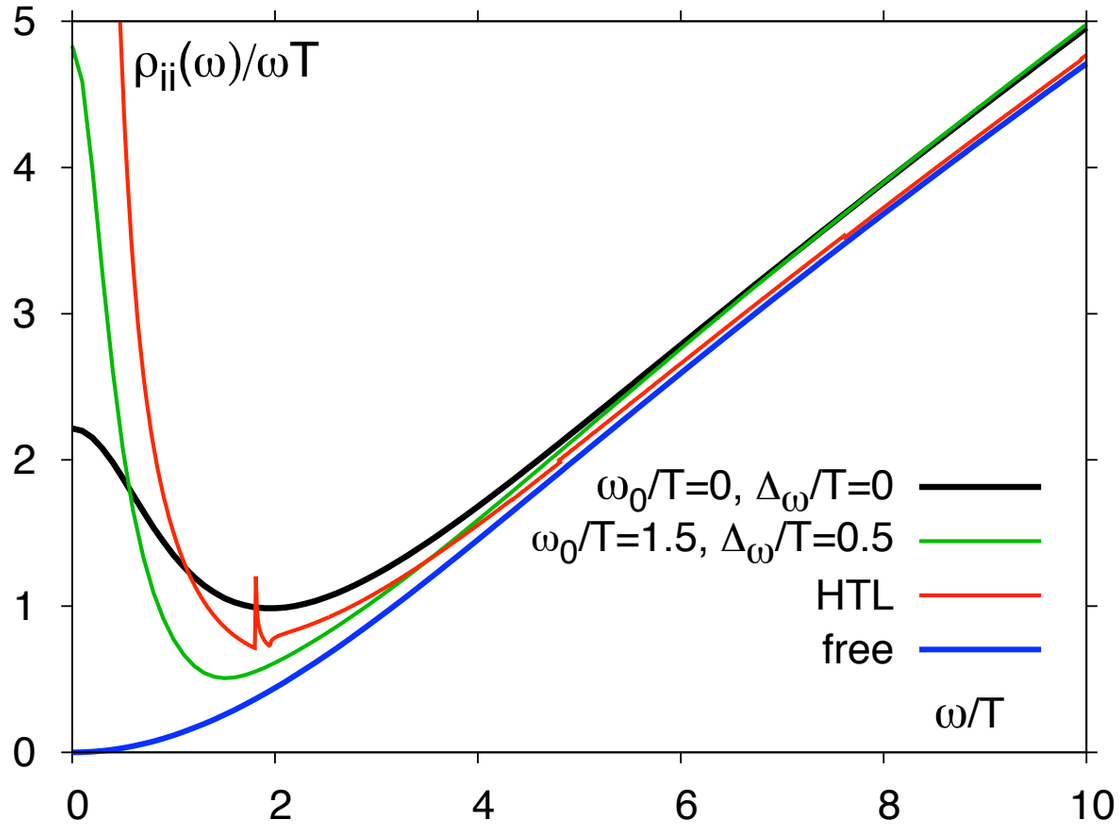


Differences

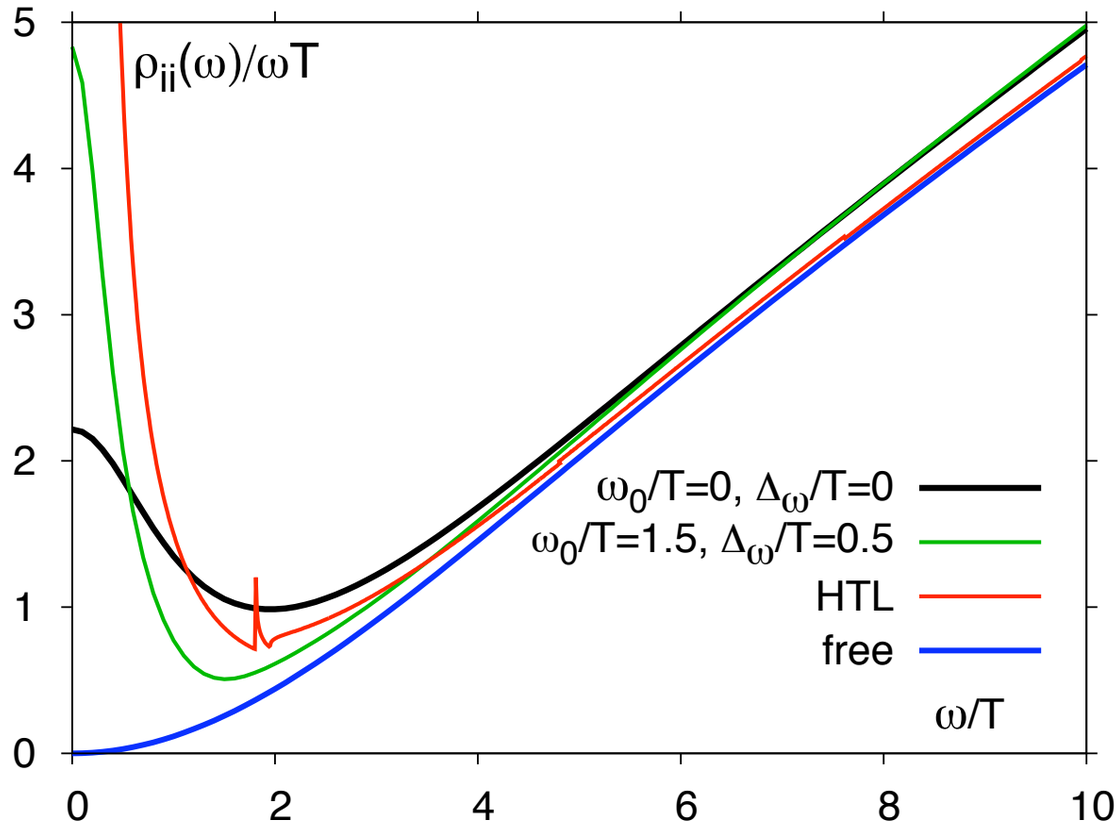


- Differences are smaller than 2% with smaller parameters and increase with worse default models
- MEM analysis reproduces the ratio of thermal moments even better
- Statistical errors are small

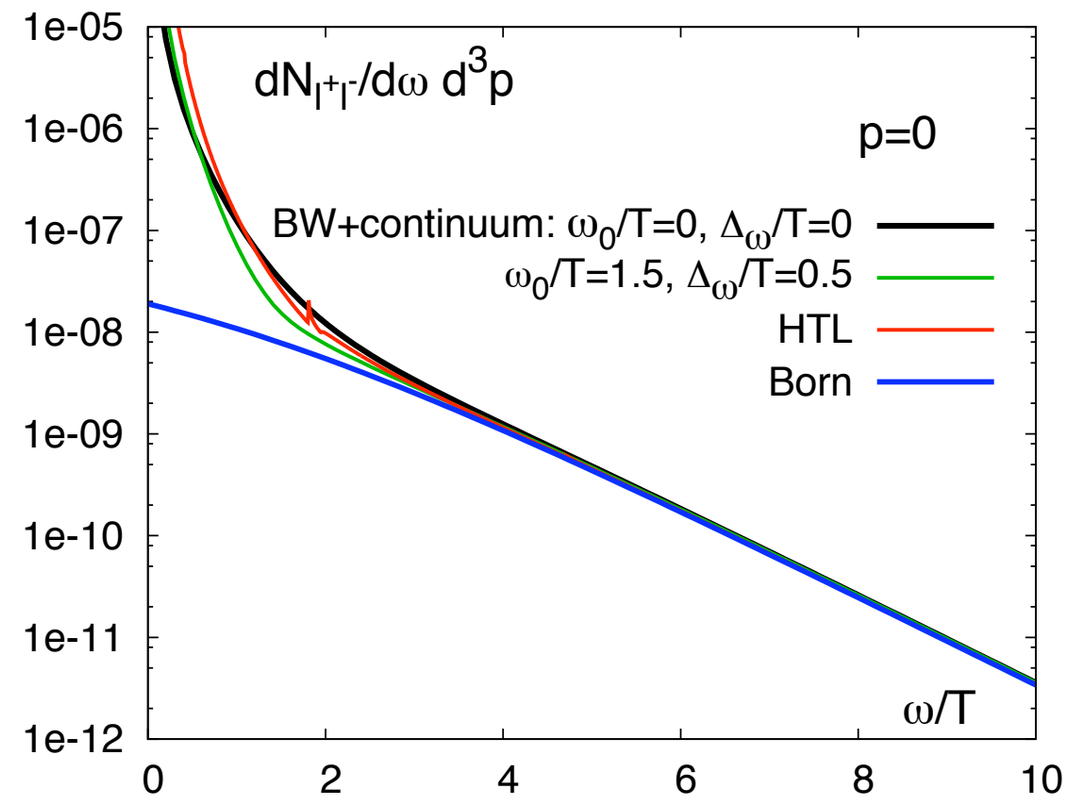
Thermal dilepton rate & electrical conductivity



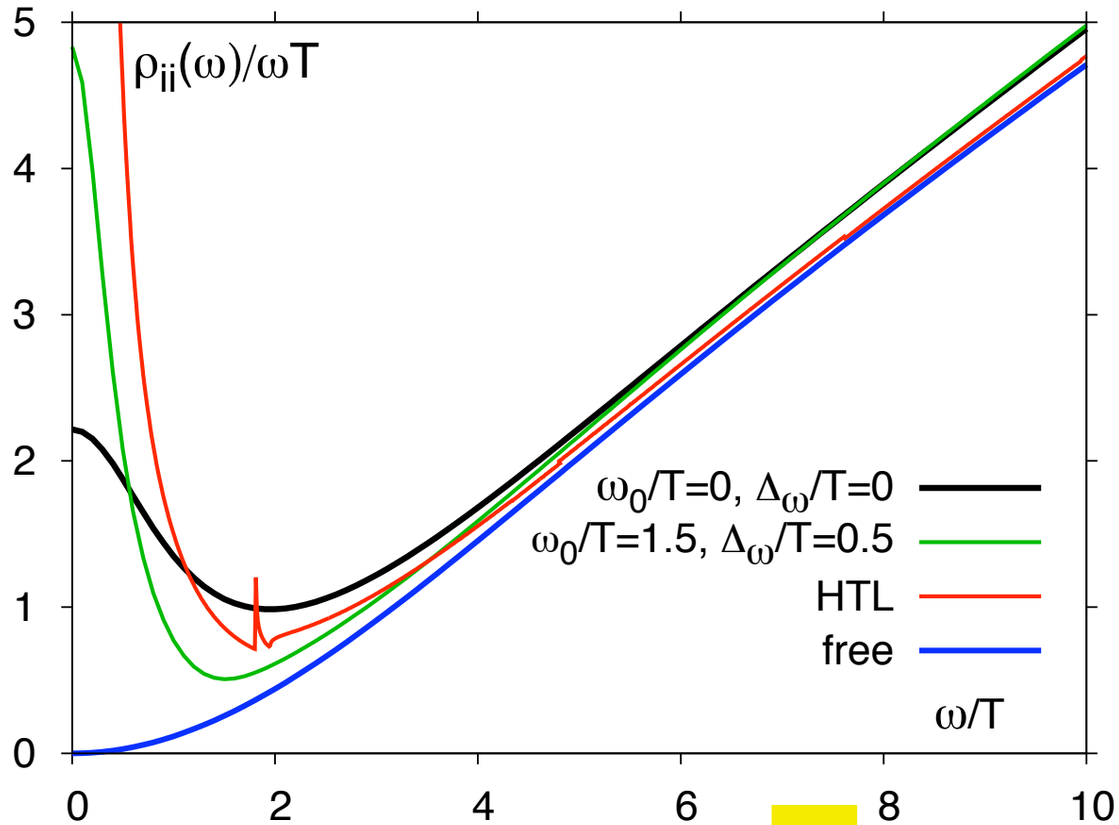
Thermal dilepton rate & electrical conductivity



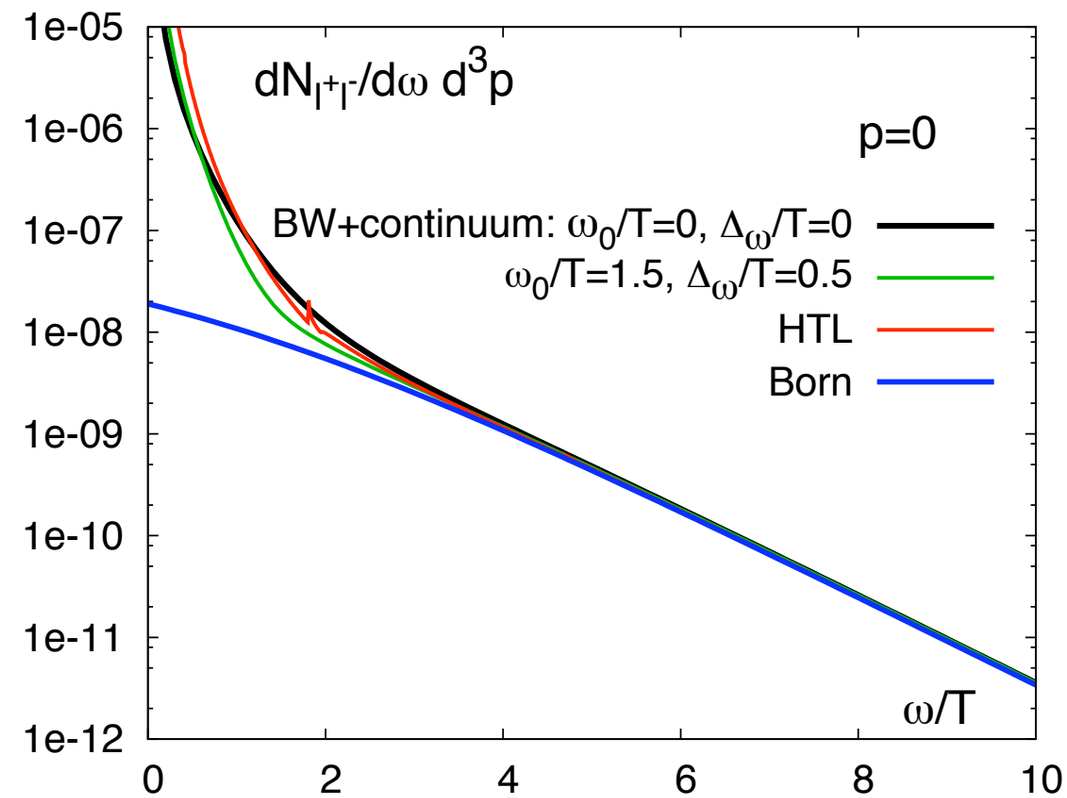
$$\frac{\mathrm{d}N_{l^+l^-}}{\mathrm{d}\omega \mathrm{d}^3p} = C_{em}\frac{\alpha_{em}^2}{6\pi^3}\frac{\rho_V(\omega,\vec{p},T)}{(\omega^2-\vec{p}^2)(\mathrm{e}^{\omega/T}-1)}$$



Thermal dilepton rate & electrical conductivity



$$\frac{\mathrm{d}N_{l^+l^-}}{\mathrm{d}\omega \mathrm{d}^3p} = C_{em}\frac{\alpha_{em}^2}{6\pi^3}\frac{\rho_V(\omega,\vec{p},T)}{(\omega^2-\vec{p}^2)(\mathrm{e}^{\omega/T}-1)}$$



$$1/3 \lesssim \frac{1}{C_{em}}\frac{\sigma}{T} \lesssim 1 \quad \text{at} \quad T \simeq 1.45\ T_c$$

$$\lim_{\omega\to 0}\omega\frac{\mathrm{d}R_\gamma}{\mathrm{d}^3p} = (0.0004\ -\ 0.0013)\,T_c^2 \simeq (1-3)\cdot 10^{-5}\ \mathrm{GeV}^2 \quad \text{at} \quad T \simeq 1.45\ T_c$$

Conclusion

- We calculated the vector correlation function at $T \simeq 1.45 T_c$ in quenched lattice QCD and performed a continuum extrapolation

- $G_V(\tau T)$ is well reproduced using a Breit-Wigner plus continuum ansatz for the vector spectral function

- Electrical conductivity $1/3 \lesssim \frac{1}{C_{em}} \frac{\sigma}{T} \lesssim 1 \quad \text{at} \quad T \simeq 1.45\, T_c$

- Dilepton rate approaches leading order Born rate at $\omega/T \gtrsim 4$