Chiral symmetry breaking in lattice QED mode with 2+1d defmion brane

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arXiv:1203.1091 [hep-lat] and progress

OUTLINE

- Introduction
- Lattice QED with 2+1d fermion brane
- Results
  - Chiral condensate and susceptivility
  - Low-lying mode distribution
  - Hadron-like spectrum
- Discussion and outlook

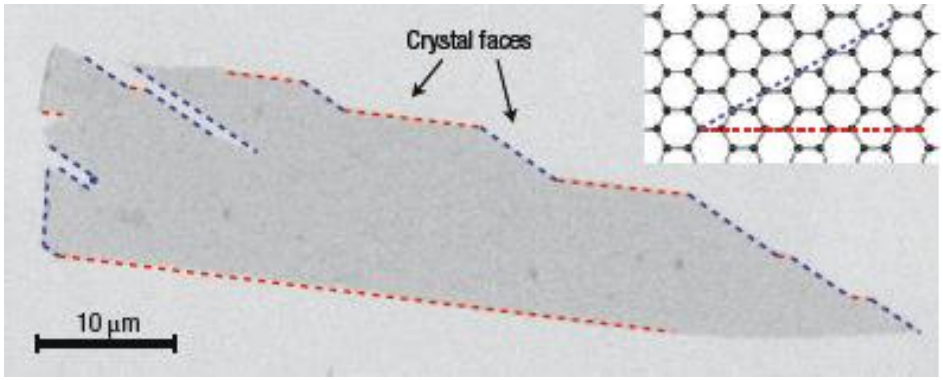
Introduction

GRAPHENE

- 2d honeycomb seat of Carbon atoms

  Geim and Novoselov, et al.

  (Nobel Prize in 2010)



Novoselov et al., (2004)

- Electromagnetic properties
  - High electron mobility

    (~100k cm^2/Vs > 77k cm^2/Vs (InSb)) at the room temperature,

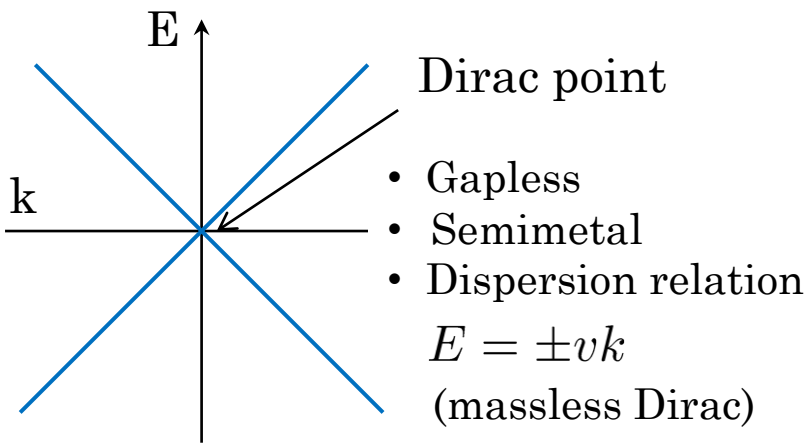
    and it means speed of electron is very fast !
  - Very thin (~ 4Å) and cheap (~ 1 ¢ /g) [Silicon: ~0.5mm, 50 ¢ /g]

    ⇒ next devise material
  - Many interesting topics

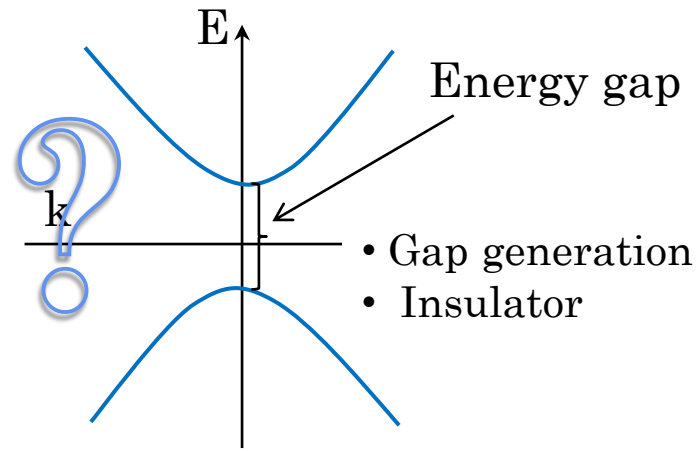
    Mott transition, anomalous Hall conductivity, …

Band structure and gap

- Suspended Graphene (near zero temp., free standing)
  - Model



Room temperature



$T \ll 20\ K$

  - Ideal situation: free-standing and pure
  - Condensation of electron-hole pair due to strong attraction ($\alpha_e \sim O(1)$)
  - Gap spontaneously appears → "insulator" at $T < T_c$
  - Experiment
    - Attraction becomes weak as $\alpha_e/\kappa \ll 1$ due to substrate effect
    - Gapless at $T \sim 20$ K → "semimetal"

Graphene lattice and Hamiltonian

- Tight binding approximation

$$H = t \sum_{x,\sigma=\pm} \sum_{i=1}^{3} \left[A_\sigma^\dagger(x) B_\sigma(x+b_i) + h.c. \right]$$

$$+ \; t' \sum_{x,\sigma=\pm} \sum_{i=1}^{6} \left[A_\sigma^\dagger(x) A_\sigma(x+a_i) + B_\sigma^\dagger(x) B_\sigma(x+a_i) \right]$$

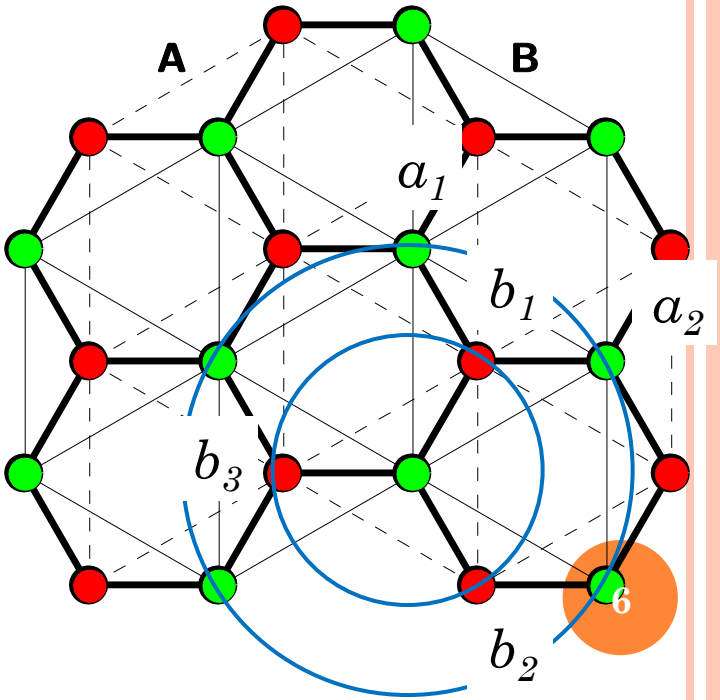
$$b_1 = \left(\frac{a_g}{2\sqrt{3}}, \frac{a_g}{2} \right), \quad b_2 = \left(\frac{a_g}{2\sqrt{3}}, -\frac{a_g}{2} \right),$$

$$b_3 = \left(-\frac{a_g}{\sqrt{3}}, 0 \right) \quad a_g = 0.142\,\mathrm{nm}$$

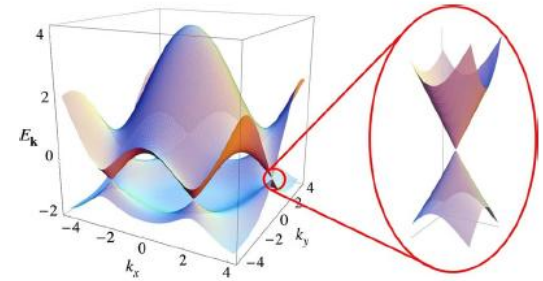
Experimental estimate :

$t = 2.8$ eV, $t' = 0.1$ eV Reich, et al., (2002)

- The second term is next-to-leading order
- Setting to $t' = 0$ is good approximation



Low energy approximation



- Expansion near Dirac point
  - Dirac (K) point : hexagonal points
  - Fourier transformation of H

$$H \simeq t \sum_{\sigma} \int_{\Omega_B} \frac{d^2k}{(2\pi)^2} \begin{pmatrix} \tilde{A}^\dagger_\sigma(k) & \tilde{B}^\dagger_\sigma(k) \end{pmatrix} \begin{pmatrix} 0 & E_k \\ E^\dagger_k & 0 \end{pmatrix} \begin{pmatrix} \tilde{A}_\sigma(k) \\ \tilde{B}_\sigma(k) \end{pmatrix}$$

with $E_k = \sum_{i=1}^{3} e^{-ikb_i}$

  - $|E_k| = 0 \Rightarrow k = \{K_1, K_2\},\ K_1 = \left(0, \pm\frac{4\pi}{3a}\right),\ K_2 = \left(\pm\frac{2\pi}{\sqrt{3}a}, \pm\frac{2\pi}{3a}\right)$

$$\begin{aligned} H_0 &= v_F \sum_{\sigma} \int_{DC} \frac{d^2k}{(2\pi)^2} \psi^\dagger_\sigma(k) \begin{pmatrix} 0 & k_x - ik_y & 0 & 0 \\ k_x + ik_y & 0 & 0 & 0 \\ 0 & 0 & 0 & -k_x + ik_y \\ 0 & 0 & -k_x - ik_y & 0 \end{pmatrix} \psi_\sigma(k) \\ &= \sum_{\sigma} \int_{DC} \bar{\psi}_\sigma(k) v_F k \cdot \gamma \psi_\sigma(k) \end{aligned}$$

Wallance (1947), G. W. Semenoff, (1984)

$$\psi_\sigma(k) = \begin{pmatrix} A_\sigma(k+K) \\ B_\sigma(k+K) \\ B_\sigma(k-K) \\ A_\sigma(k-K) \end{pmatrix},\ v_F = \frac{3ta_g}{2} \sim c/100,\ \gamma_i = \tilde{\tau}_i \otimes \tau_i$$

Dirac point

A, B sublattice

Low energy Graphene model

- Effective QED model
  - H_0 (tight-binding) ⇒ 2+1D QED with fermi velocity
  - Instantaneous interaction

$$\int d^2x dt\, \bar{\psi}\Big[i\gamma_0\partial_t + iv_F\gamma\cdot\partial\Big]\psi + \int d^2x dt d^2x' dt' \bar{\psi}\gamma_0\psi(t,x)\frac{e^2\delta(t-t')}{8\pi|x-x'|}\bar{\psi}\gamma_0\psi(x',t')$$

$$\to S_{\rm NR} = \frac{1}{2e^2}\int dt d^3x \vec{E}^2 + \int d^2x dt\, \bar{\psi}\Big[i\gamma_0(\partial_t+\phi) + iv_F\gamma\cdot\partial\Big]\psi$$

  - U(4) global symmetry

    In monolayer case, 16 generators $\underbrace{\{1,\gamma_5,i\gamma_3,[\gamma_5,\gamma_3]/2\}}_{\text{valley, sublattice}}\otimes\underbrace{\sigma_{i=0,1,2,3}}_{\text{spin}}$
  - Parity can be defined as $P=\gamma_0$
  - N_f = spin(=2) ×layer, N_f =2 : monolayer, N_f =4 : double layer
  - There were many studies with Schwinger-Dyson, large-N, Monte-Carlo

Gorbar (2001), Son (2005), Drut (2009)

<u>Concering issue</u>:

- Velocity free after rescaling $t\to t/v_F,\ \phi\to\phi v_F$
- Gauge variant action → difficult scaling study

Phase transition and chiral symmetry

- Mass gap in QED model
  - Low-energy strong QED dynamics: $\alpha_E = e^2/(4\pi v_F) \sim O(1),\ v_F \sim O(10^{-2})$
  - Definition of γ_5 "chiral" symmetry in 2+1 D

    $\psi' = e^{i\alpha\gamma_5}\psi, \quad \bar{\psi}' = \bar{\psi}e^{i\alpha\gamma_5},\ \gamma_5 = \sigma_3 \otimes I_2$

    Chirality corresponds to valley position.
    Metal-insulator phase, as a consequence of <u>mass gap in QED(3D, brane)</u>

    e.g. in large N analysis [Herbut (2006)] [Son (2007)]

  - Symmetry breaking in strong dynamics

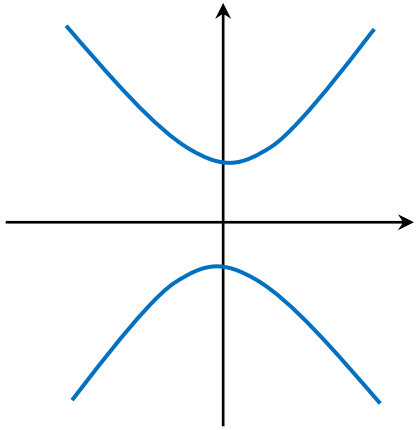
    Graphene: Valley sym. (flavor)

    $\boxed{U(2N_f) \to U_{a,b}(N_f)}$

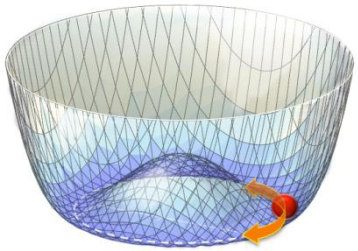
    Massless QED (3D): Chiral sym.

    $\boxed{U_L(N_f) \times U_R(N_f) \to U_V(N_f)}$

    [Coleman, Witten (1980)]
    [Jackiw, Templeton (1981)]
    [Appelquiest, Pisarski (1981)]



Mass gap ?



@wikipedia

Velocity effect

- Logarithmic scaling
  - Perturbatively velocity has logarithmic divergence

$$v(\mu) = v\left(1 + \frac{\alpha}{4}\ln\frac{\Lambda}{\mu}\right), \quad e^2(\mu) \sim \mathcal{O}(\alpha^2)$$

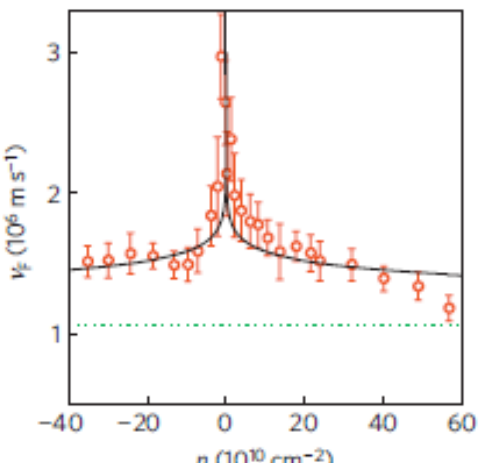
  - Effective coupling : $\alpha_E(\mu) = e^2(\mu)/(4\pi v_F(\mu))$ gives $\log^{-1}$ running behavior.
  - Experimental result is also agreement
    $\Rightarrow$ scaling of velocity is crucial !
  - UV fixed point in strong region

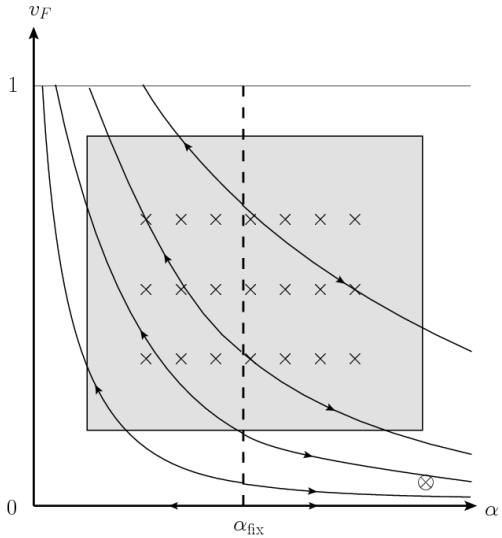
$$\frac{dv}{dl} = \frac{\alpha v}{4} - \left(\frac{1}{3} - \delta\right)\alpha^2 v = -C(\alpha - \alpha_{\rm fix}),$$

$\delta \simeq 0.03,\ C > 0,\ \alpha_{\rm fix} \simeq 0.8$

$\alpha > \alpha_{\rm FP}$: $v \to 0$, $\alpha < \alpha_{\rm FP}$: $v \to \infty$
$\Rightarrow$ unstable (UV) fixed point at $\alpha = \alpha_{\rm fix}$

Elias, et al (2007)





Lattice QED with 2+1d fermion brane

Strategy

- Monte Carlo study of relativistic model
  - Generalize to relativistic QED action with velocity

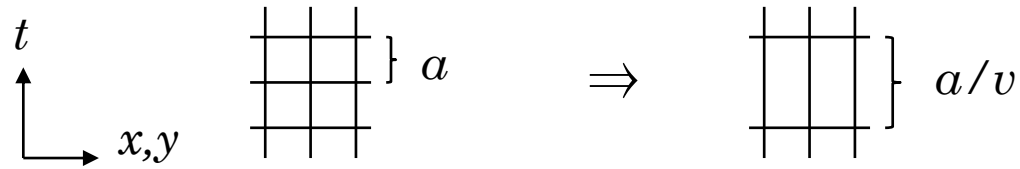
$$S_{QED_{2+1}} = \int d^2x dt \sum_{f=1}^{N_f} \bar{\psi}_f \Big[\gamma_0(\partial_t + A_0) + v_F \sum_{i=1,2} \gamma_i(\partial_i + A_i)\Big]\psi_f$$

    - Gauge invariance and including fermi velocity

      more realistic than non-relativistic approach (A_i=0, instantaneous field)
  - Rescaled

$$S = \int dx^2 dt \bar{\psi}\big[D_0(A_0)\gamma_0 + \vec{D}(A_i)\vec{\gamma}\big]\psi + \frac{1}{2e^2}\int d^3x dt\big[vF_{0i}^2 + \frac{1}{v}F_{ij}^2\big]$$

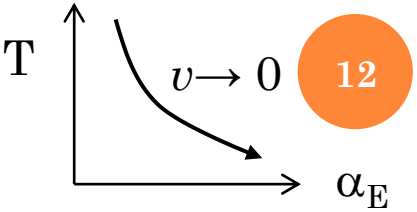
$$t \to tv,\ A_0 \to A_0/v$$



$$S = \sum_{t,x,y} \chi^\dagger\big[\eta_0 D_0(\theta_0) + \eta_i D_i(\theta_i) + m\big]\chi + \sum_{t,x,y,z} \frac{1}{2}\big[\beta v F_{0i}^2 + \frac{\beta}{v}F_{ij}^2\big]$$

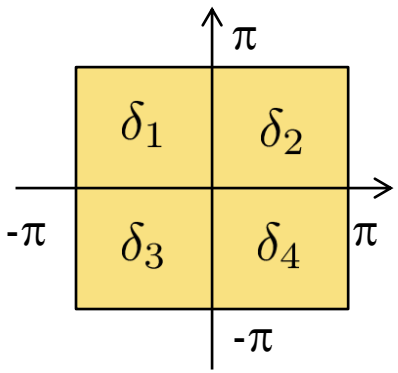
2+1 dimensional 3+1 dimensional

- "Strong" electric and "weak" magnetic field.
- Photon field at z direction is interaction free.
- Two bare parameters, (v. β)



Lattice QED model with fermion brane

- Staggered fermion in 2+1 dimension



$$\int_{-\pi}^{\pi} dp^3 M(p) \sim \sum_{f=1}^{8} \int_{-\pi/2}^{\pi/2} d^3p M(p+\delta_f)$$

$$\delta_f = \{(0,0,0)\,,(\pi,0,0),\,(0,\pi,0),\cdots,(\pi,\pi,\pi)\}$$

$$S = \sum_{n,m} \bar{\psi}(n)\Big[\frac{1}{2}\sum_{\mu}\Big\{\nabla_\mu(\gamma_\mu \otimes 1) + \nabla_\mu^2(\gamma_5 \otimes \tau_\mu\tau_3)\Big\} + m(1\otimes 1)\Big]\psi(m)$$

  - Chiral symmetry

    $\psi' = \exp[i\alpha(\gamma_5 \otimes \tau_3)]\psi,\ \bar{\psi}' = \bar{\psi}\exp[i\alpha(\gamma_5 \otimes \tau_3)]$

    which is rotation of spinor and flavor simultaneously.
  - Restore the global U(4) in the continuum limit, without root trick

- Non-Compact QED

$$S_g = \sum_{n=x,y,z,t} \frac{1}{2}\Big[\frac{\beta}{v}\sum_{i,,j}(\theta_i(n+\hat{j}) - \theta_i(n))^2 + \beta v \sum_{i}(\theta_i(n+\hat{4}) - \theta_i(n))^2\Big]$$

There were many lattice studies in 3D and 4D QED for chiral SB and scaling study.

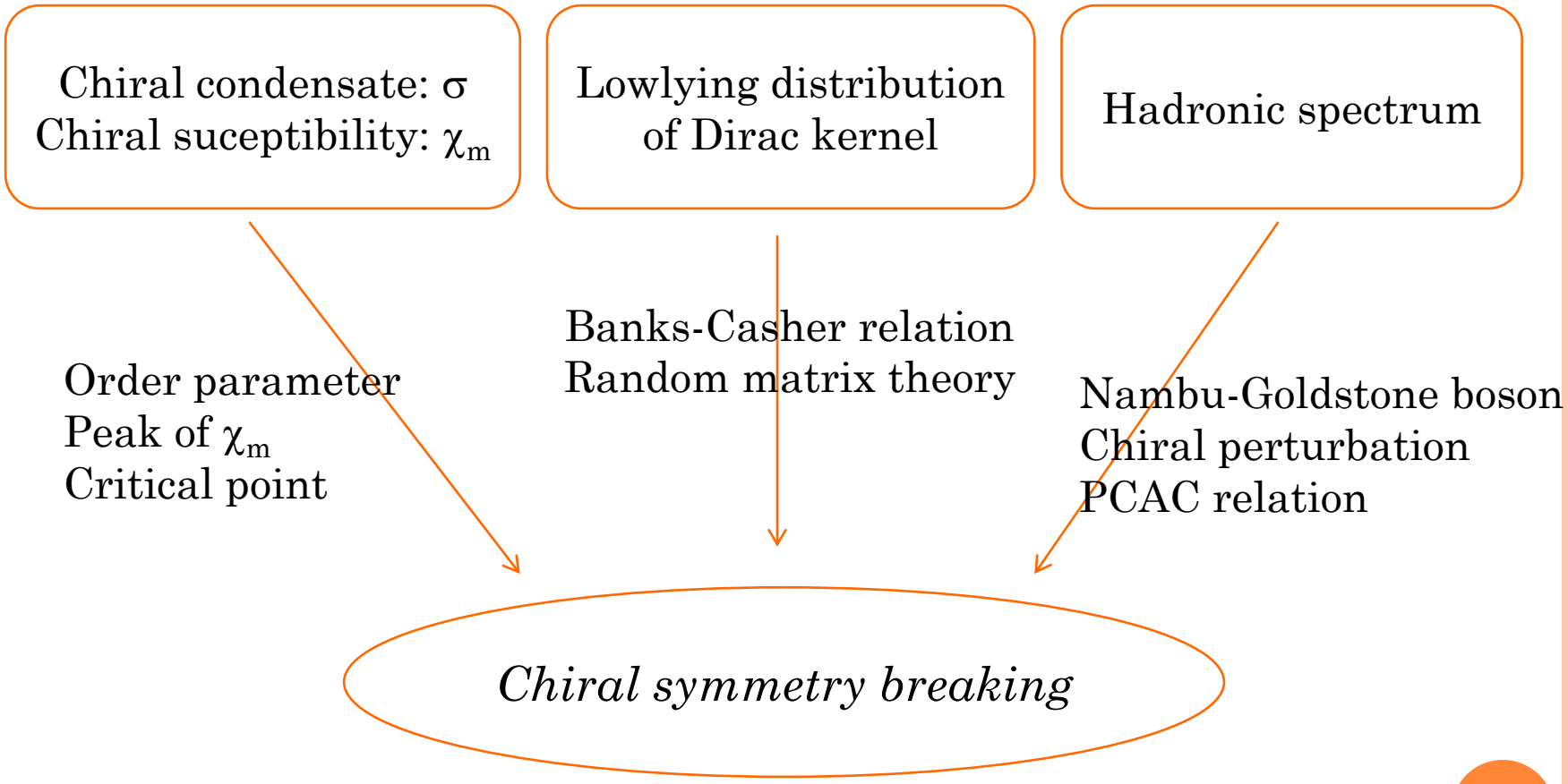
[Hands, Kogut, et al.(1989--2010)] [QCDSF (1990--1998)]
[Drut, Lahde (2009--2010)]

Lattice simulation

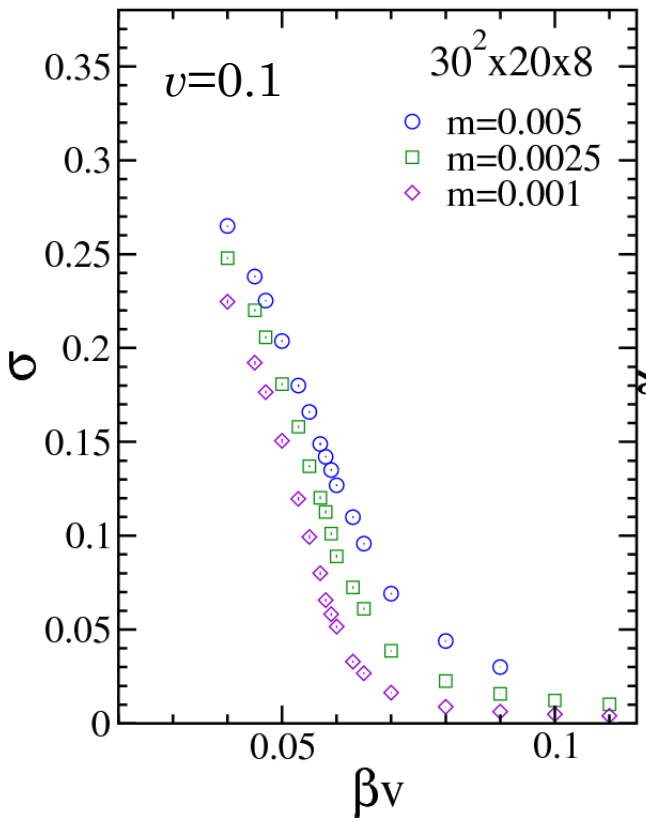
- Dynamical staggered fermion simulation
  - HMC simulation
  - Omelyan integrator
  - Hasenbush mass preconditioning, $m_h = 0.05$
  - Low-mode projection (if CG iteration is over than 10^4)
  - HMC time step $\tau \simeq 0.01 - 0.05$, HMC time $N_\tau = 1/\tau$
  - 10000 – 30000 HMC traj after 400 traj therm
  - 20 steps for use of statistics, O(100) total statistics
  - Jackknife error analysis, bin size = 10
  - Spatial periodic, temporal anti-periodic BC
  - Finite temperature, N_t=20 fixed.

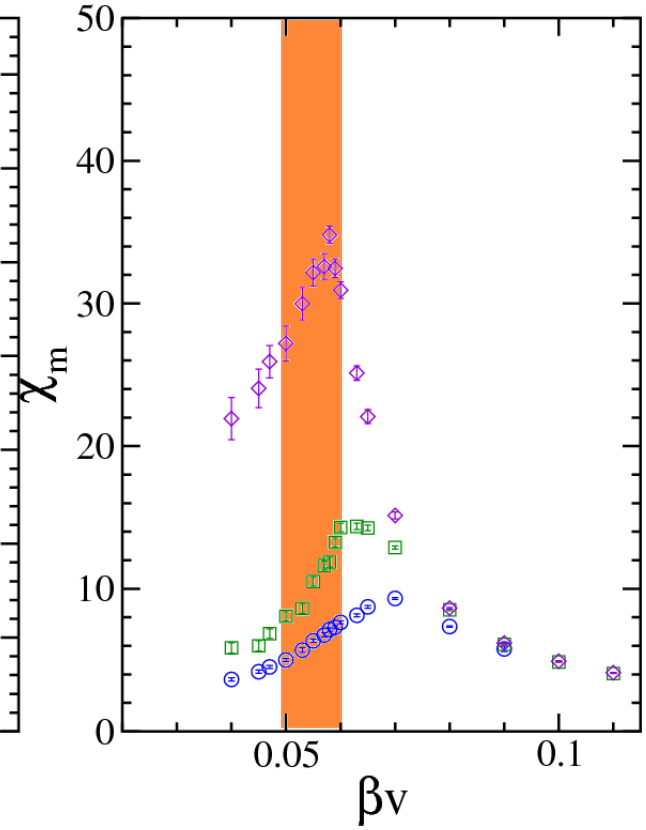
Results

Chiral symmetry breaking



Chiral condensate and susceptibility



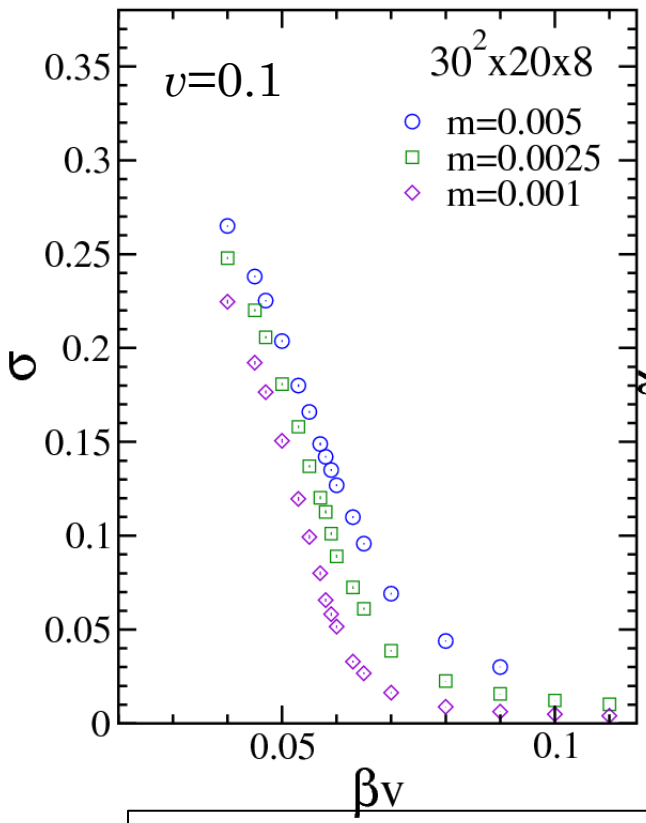


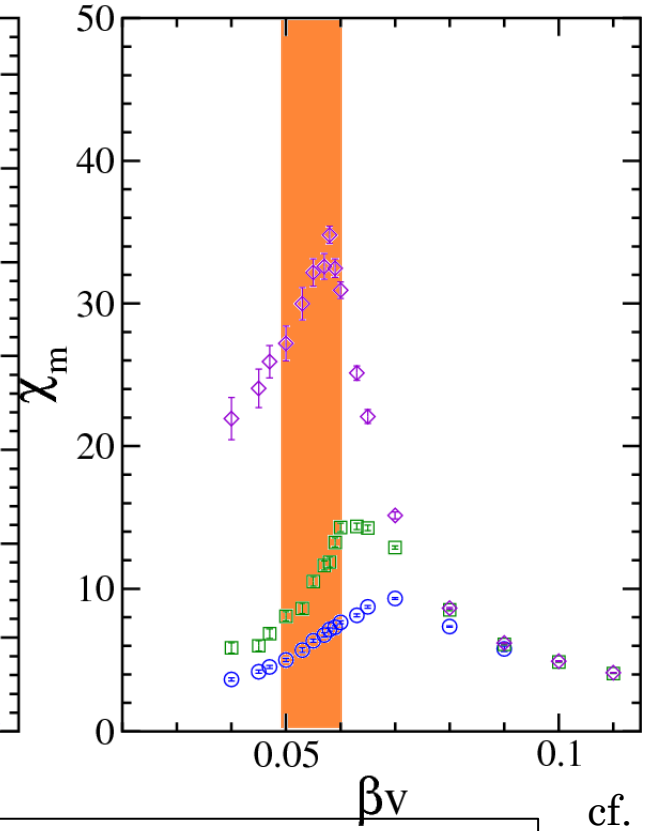
$$\sigma = \langle \mathrm{Tr} D_\chi^{-1} \rangle,$$

$$\chi_m = d\sigma/dm$$

$\alpha = 1/(4\pi\beta v)$

Chiral condensate and susceptibility





$$\sigma = \langle \mathrm{Tr} D_\chi^{-1} \rangle,$$
$$\chi_m = d\sigma/dm$$

$\alpha = 1/(4\pi\beta v)$

- Significant mass dependence
- Critical behavior of 2nd order transition
- Clear peak of χ_m
- Critical point:
  $(\beta v)_c \simeq 0.05 - 0.06$, $\alpha_c \simeq 1.3 = O(1)$.

cf. Schwinger-Dyson model:
$\alpha_c = 0.92 - 1.13$
Gamayun, et al. (2010), Khveshchenko (2009)

Non-relativistic MC
$\alpha_c = 1.11(6)$
Drut, Lahde (2009)

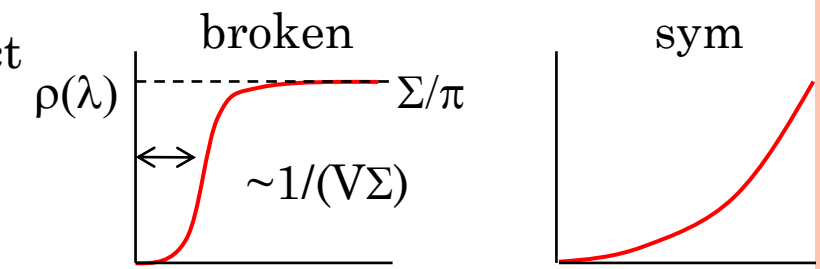
Lowmode distribution

- Spectral density
  - Lowmode distribution which is related to chiral symmetry breaking
  - If chiral symmetry breaking occurs, ρ(λ) is given by Banks-Casher relation:

$$\rho(\lambda) = \left\langle \frac{1}{V} \sum_n \delta(\lambda - \lambda_n) \right\rangle \qquad \lim_{m\to 0} \lim_{V\to\infty} \langle \bar{q}q \rangle = \Sigma = \pi\rho(0)$$

$\Rightarrow$ spectral density provides chiral condensate

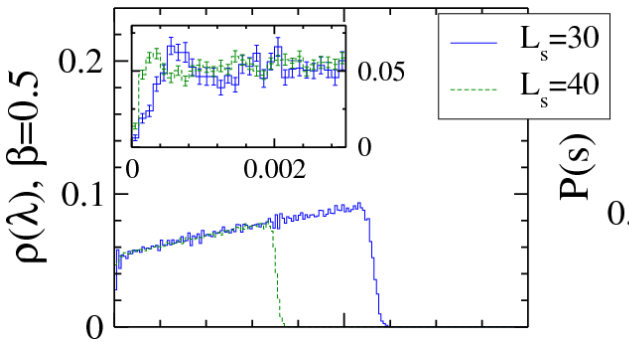
  - Gap appears due to finite size effect
  - Scale generation: 1/(VΣ)

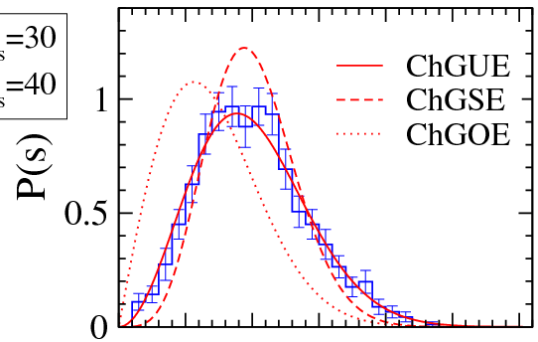


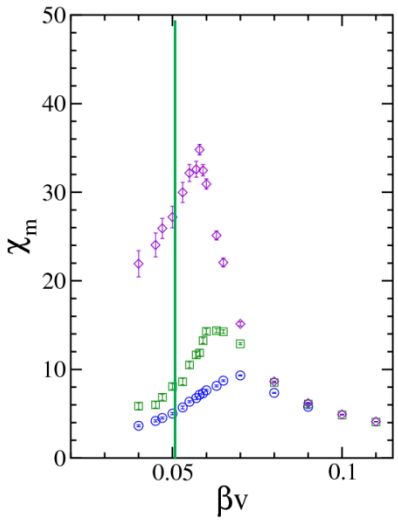
- Level spacing distribution
  - According to random matrix theory, we can distinguish universal distribution of level spacing: $s = (\lambda_2 - \lambda_1)/\langle \lambda_2 - \lambda_1 \rangle$

$$P(s) = \begin{cases} \frac{32}{\pi^2} s^2 e^{-\frac{4}{\pi}s^2} & \text{chGUE} \quad \leftarrow U(N_f)\times U(N_f)/U(N_f) \\ \frac{2^{18}}{3^6\pi^3} s^4 e^{-\frac{64}{9\pi}s^2} & \text{chGSE} \\ \pi s e^{-\frac{\pi}{2}s^2} & \text{chGOE} \end{cases}$$

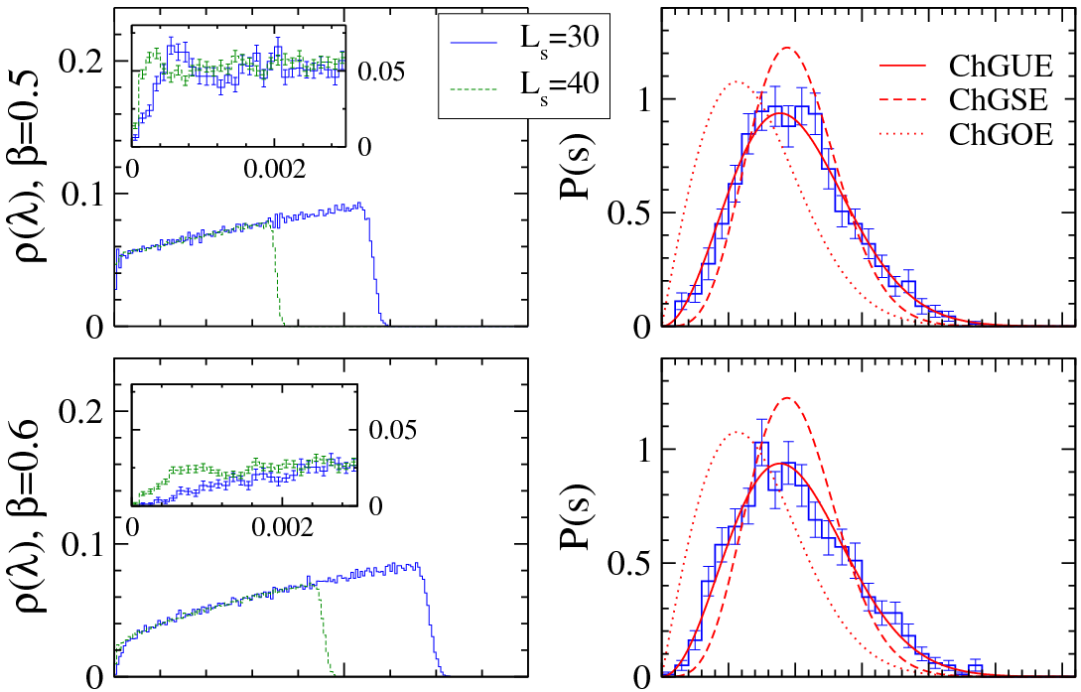
Lowmode distribution

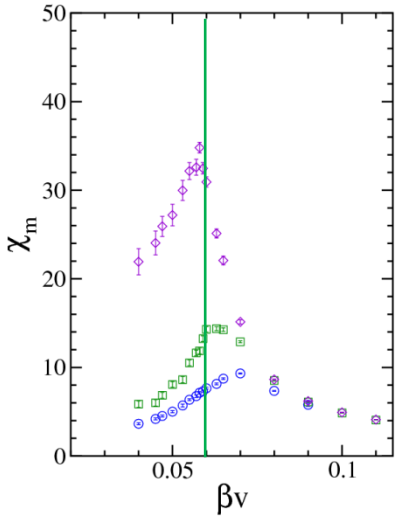




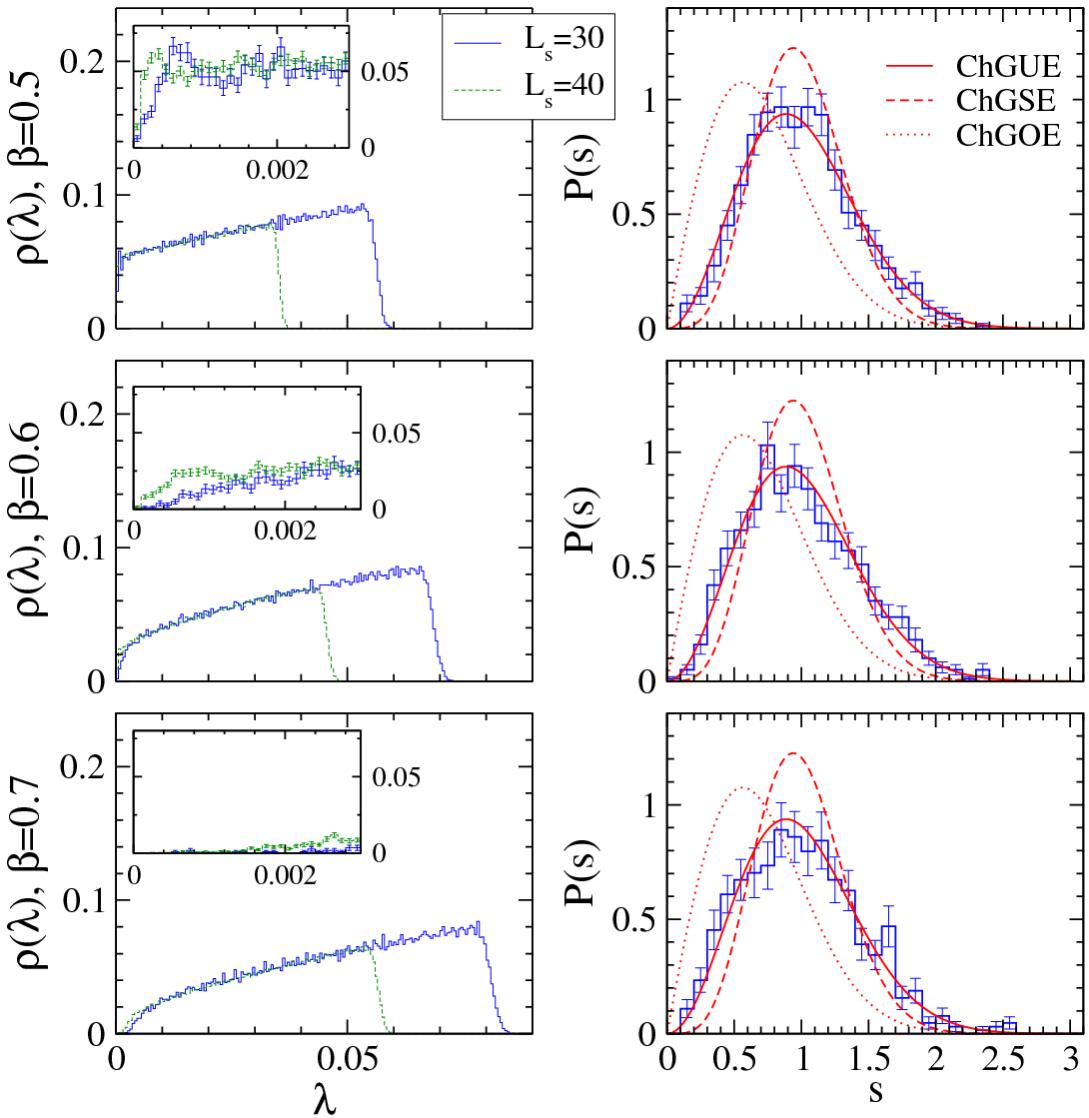


Lowmode distribution





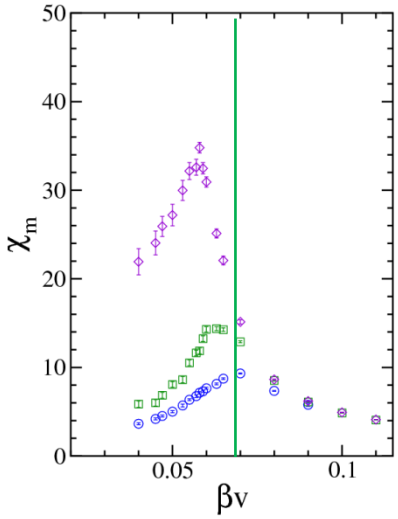
Lowmode distribution



- Correspondence between critical point and behavior of spectral density
- $\beta = 0.5$, "hard edge"
  $\beta > 0.6$, perturbative
  $$\rho(\lambda) \sim \lambda^d$$

cf. Damgaard, Heller, et al. (2000)

- Level spacing distribution is consistent with RMT



Hadronic spectrum

- Goldstone boson spectrum

Correlation function of NG boson: $P_{\rm GS} = \bar{\psi}(\gamma_5 \otimes \gamma_5)\psi$

$$G(x) = \sum_{y,t} \langle P_{GS}(x,y,t) P_{GS}(0) \rangle$$

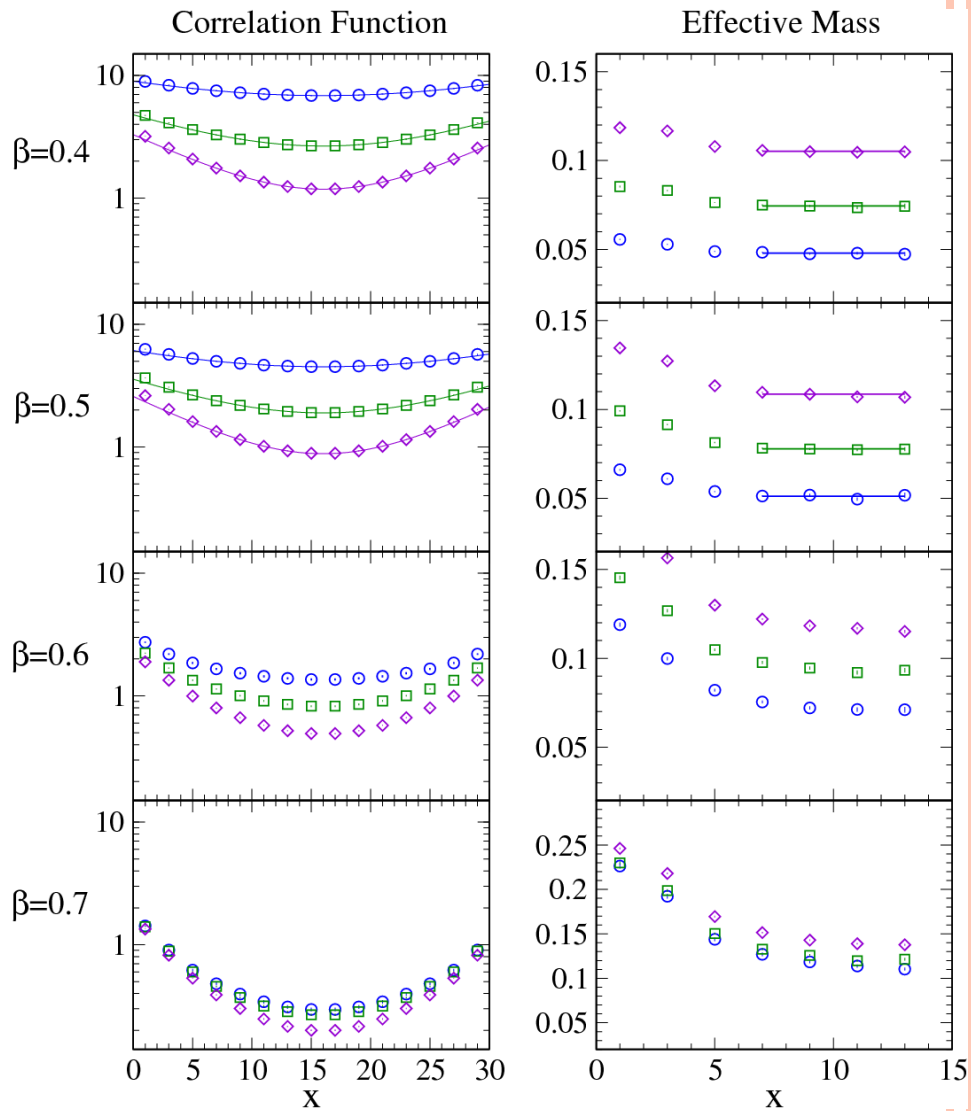
Effective (screening) mass m_π :

$$\lim_{x \gg 1} G(x) = Z\left[e^{-m_{\rm NG} x} + e^{-m_{\rm NG}(N_x - x)}\right]$$

<u>Existing asymptotic state of NG boson</u>

β= 0.4 – 0.5 : χ^2 fitting works well.
β= 0.6: seems to be plateau
β= 0.7: do not observe plateau.

NG boson state appear in strong broken phase



Hadronic spectrum

- Comparison with ChPT and PCAC

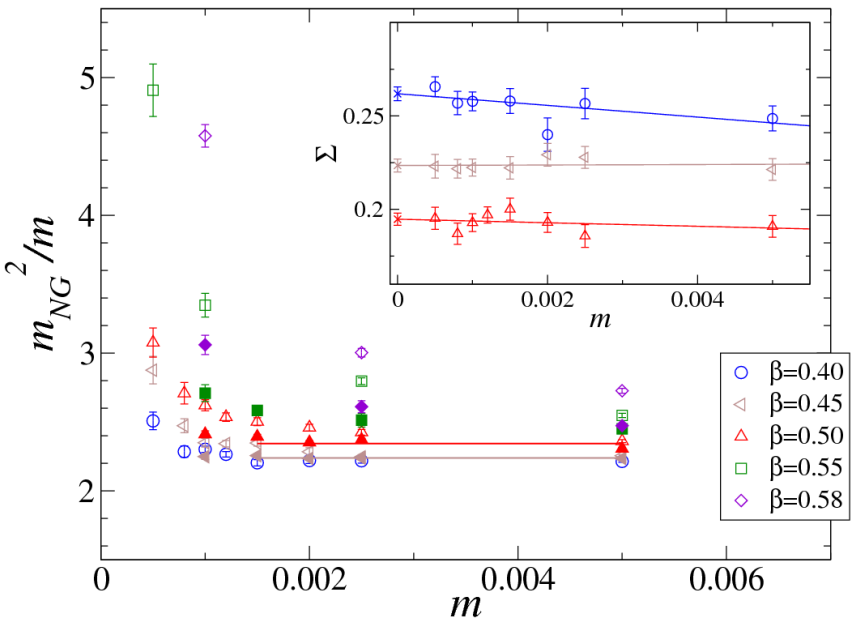
NG boson mass and decay constant have to be satisfied with ChPT and PCAC relation

$$m_{\mathrm{NG}}^2 = 2Bm$$

$$\langle 0|A_\mu|\pi\rangle = ip_\mu f_{\mathrm{NG}} \qquad f_{\mathrm{NG}} = \sqrt{\frac{\Sigma}{B}} : \text{ GMOR relation}$$

Hadronic spectrum

- Comparison with ChPT and PCAC

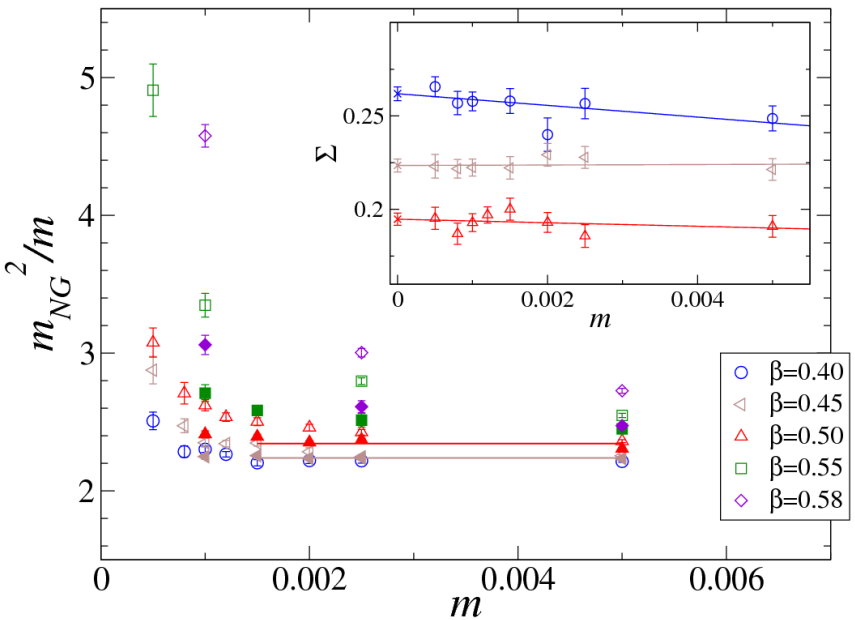


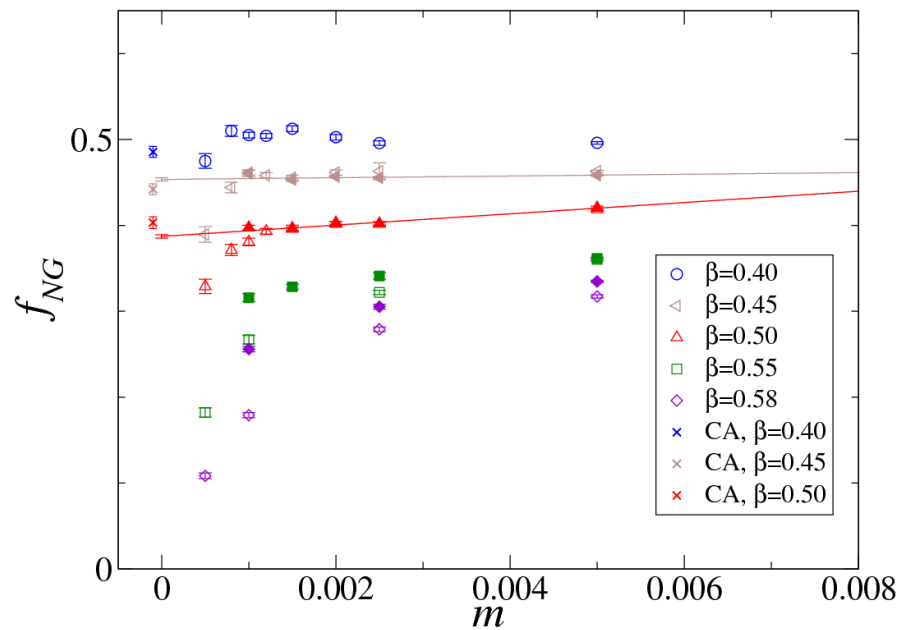
$$m_{\mathrm{NG}}^2 = 2Bm$$

- near critical point, following ChPT
- non-linear behavior at β > 0.55

Hadronic spectrum

- Comparison with ChPT and PCAC





$$m_{\mathrm{NG}}^2 = 2Bm$$

- near critical point, following ChPT
- non-linear behavior at β > 0.55

- GMOR relation:
  Σ: Banks-Casher relation
  B: constant fit
- β > 0.55, f_{NG} goes to zero.
- Consistent value with linear fit at β ≧0.5

SUMMARY

Lattice QED model with fermion brane

Relativistic definition:
2+1 D fermion + 4D gauge,
gauge invariance, velocity scaling

Chiral symmetry breaking

1st step

Continuum limit ?
Phase diagram ?
Renormalization ?

Graphene model

- Metal-insulator transition
- Hall effect

What is NG boson ?

PS operator:

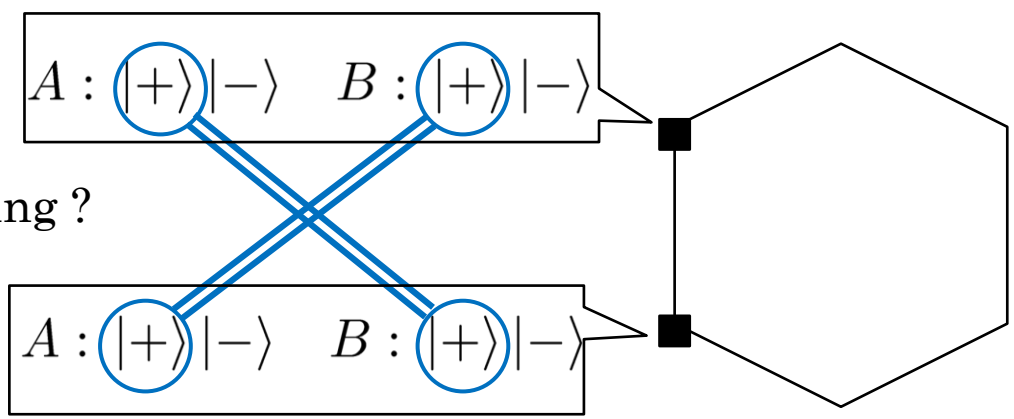
$$\bar{\psi}\gamma_5\tau\psi = \int \sum_{\sigma,\sigma'} \Big[- A^\dagger_\sigma(k+K)\tau_{\sigma\sigma'}B_{\sigma'}(k-K) - B^\dagger_\sigma(k+K)\tau_{\sigma\sigma'}A_{\sigma'}(k-K)$$
$$+B^\dagger_\sigma(k-K)\tau_{\sigma\sigma'}A_{\sigma'}(k+K) + A^\dagger_\sigma(k-K)\tau_{\sigma\sigma'}B_{\sigma'}(k+K)\Big]$$

NG boson: bound state of electron-hole with different valley and sublattice

Considering 4 NG boson states as spin degenerate

$$|++\rangle,\ |--\rangle,\ \frac{1}{2}(|+-\rangle + |-+\rangle),\ \frac{1}{2}(|+-\rangle - |-+\rangle)$$

Pairing ?

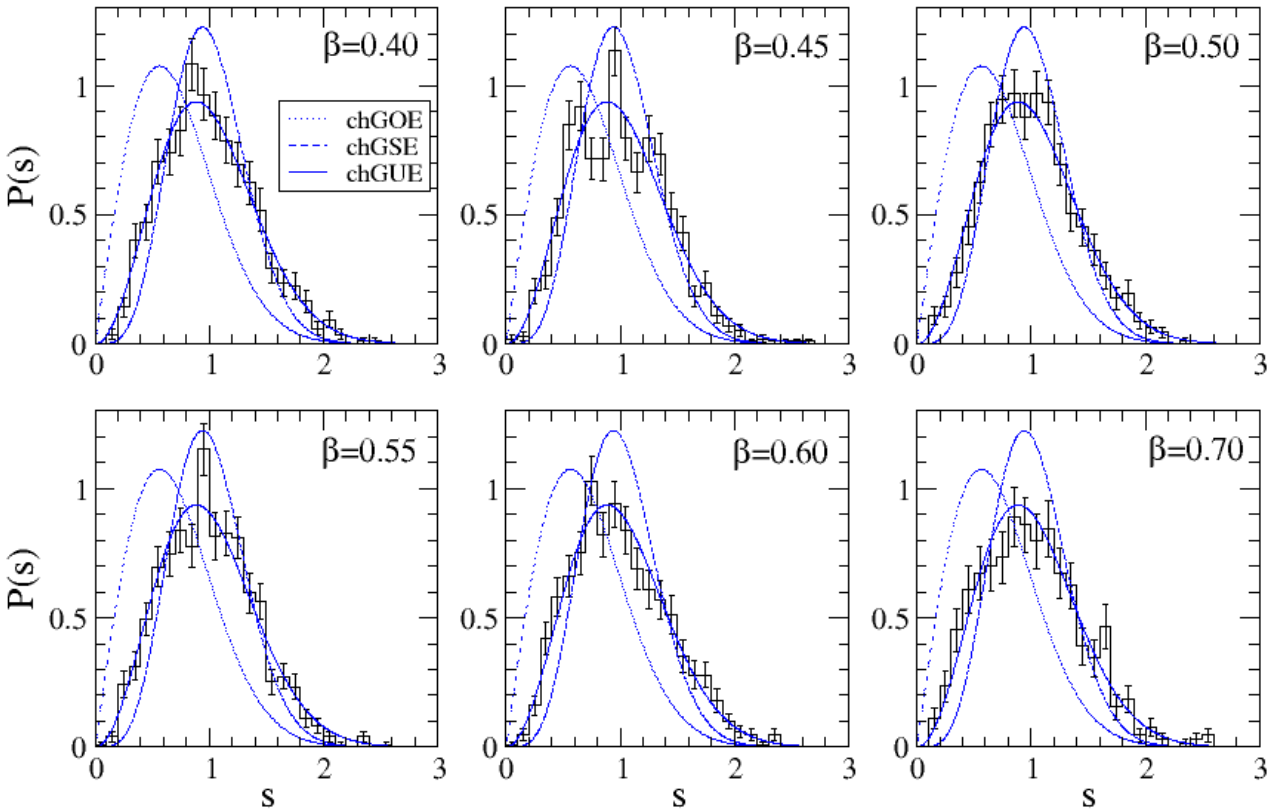


Backup slide

Comparison with others

	Our	**Drut and Lahde**
QED gauge field	Non-compact $\theta_i \neq 0,\ \ \theta_0 \neq 0$	Non-Compact $\theta_i = 0,\ \ \theta_0 \neq 0$
Gauge symmetry	○	×
Lattice size	$30^2 \times 20$, z = 8	$20^2 \times 20$, $28^2 \times 28$, z = 8
Fermi velocity	v = 0.05, 0.1	-
Gauge coupling	β_E , β_B ($\beta_E = \beta v$, $\beta_B = \beta / v$)	$\beta_E = \beta$
m	0.001, 0.0025, 0.005	0.0025 -- 0.02
Scaling	○	×

Low-lying mode distribution



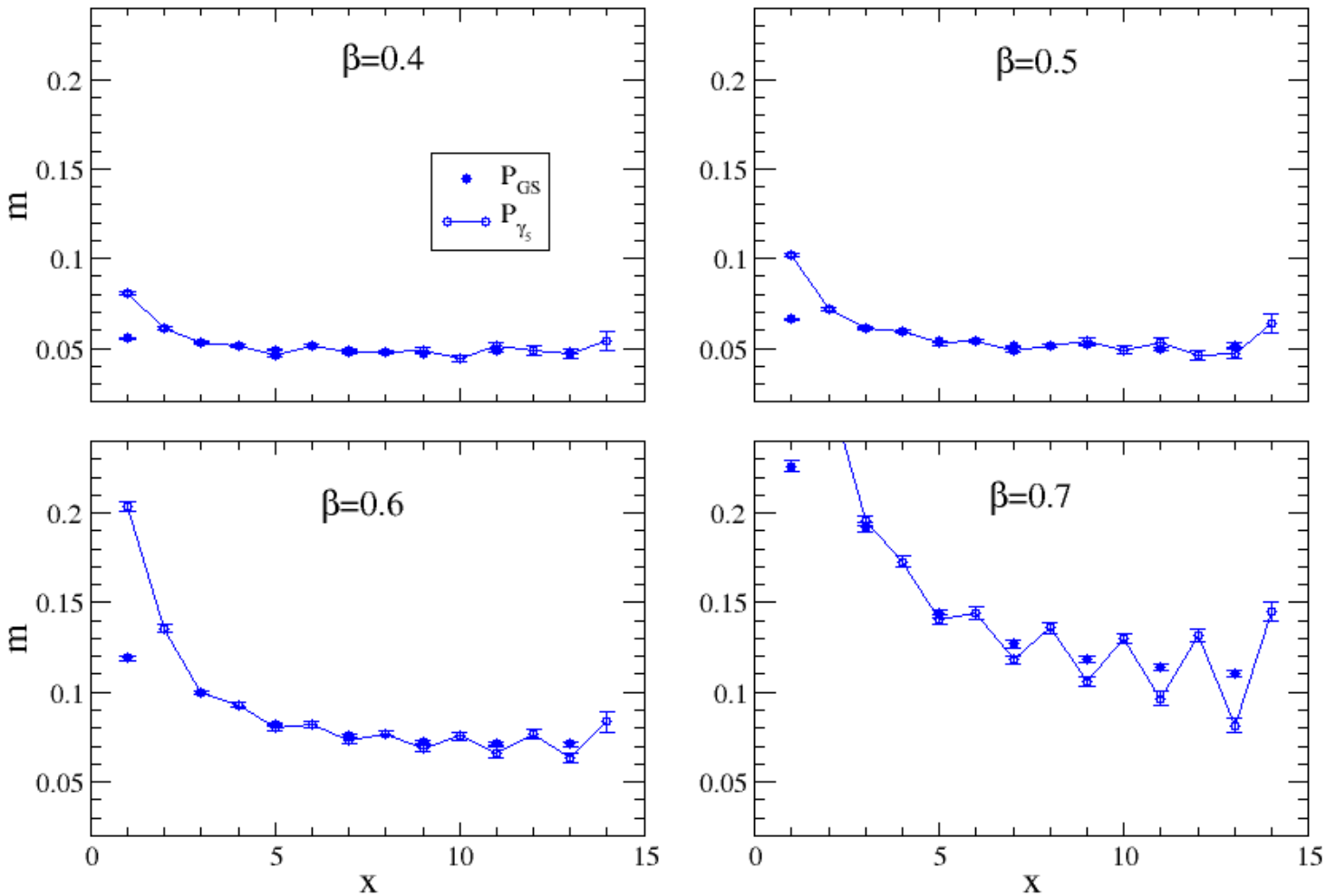
$$P(s) = \begin{cases} \boxed{\frac{32}{\pi^2} s^2 e^{-\frac{4}{\pi}s^2} \quad \text{chGUE}} \\ \frac{2^{18}}{3^6\pi^3} s^4 e^{-\frac{64}{9\pi}s^2} \quad \text{chGSE} \\ \pi s e^{-\frac{\pi}{2}s^2} \quad \text{chGOE} \end{cases}$$

$U(N_f) \times U(N_f)/U(N_f)$

$$s = (\lambda_2 - \lambda_1)/\langle \lambda_2 - \lambda_1 \rangle$$

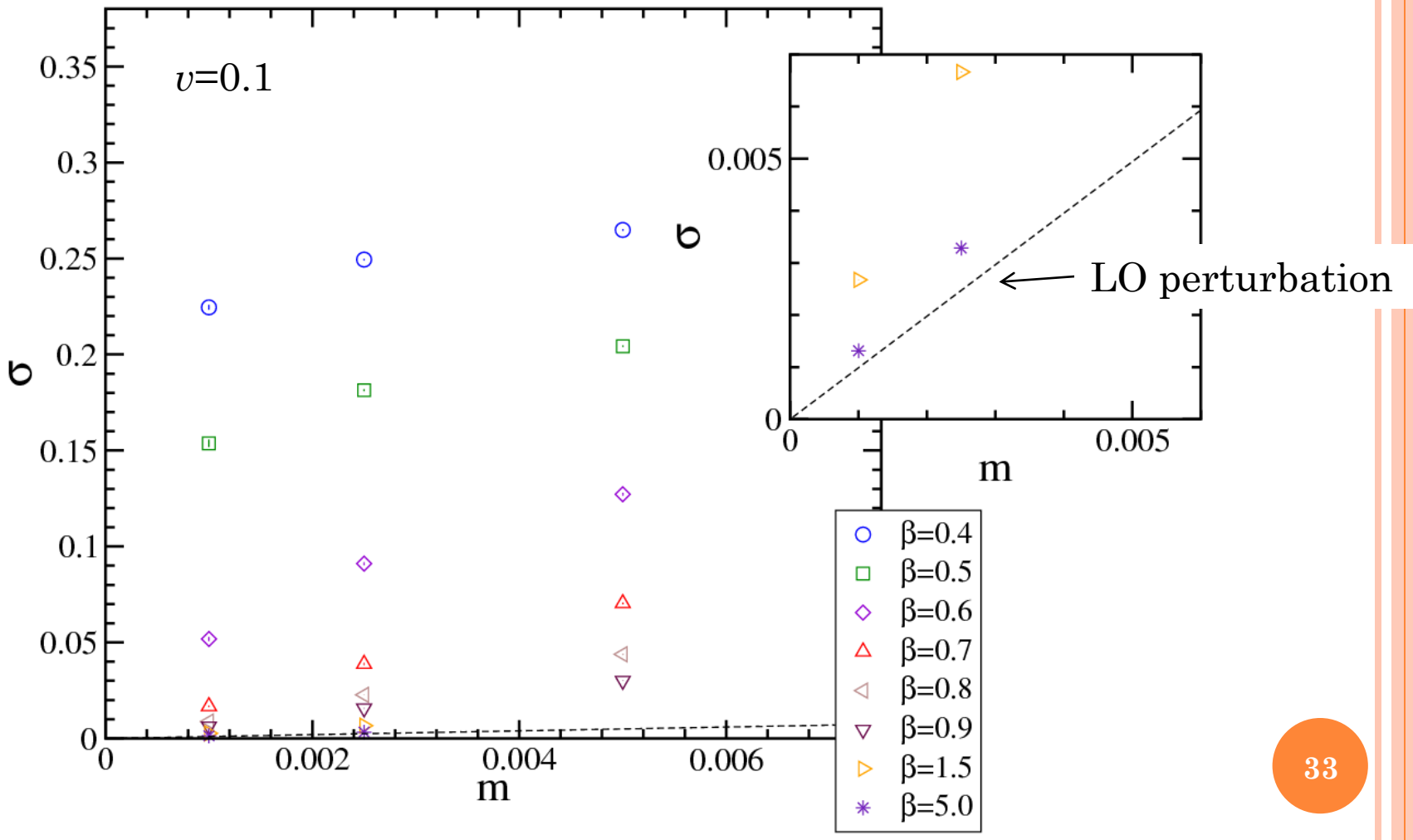
Hadron-like spectrum

- Comparison with $P_{\gamma 5}$, P_{GS}



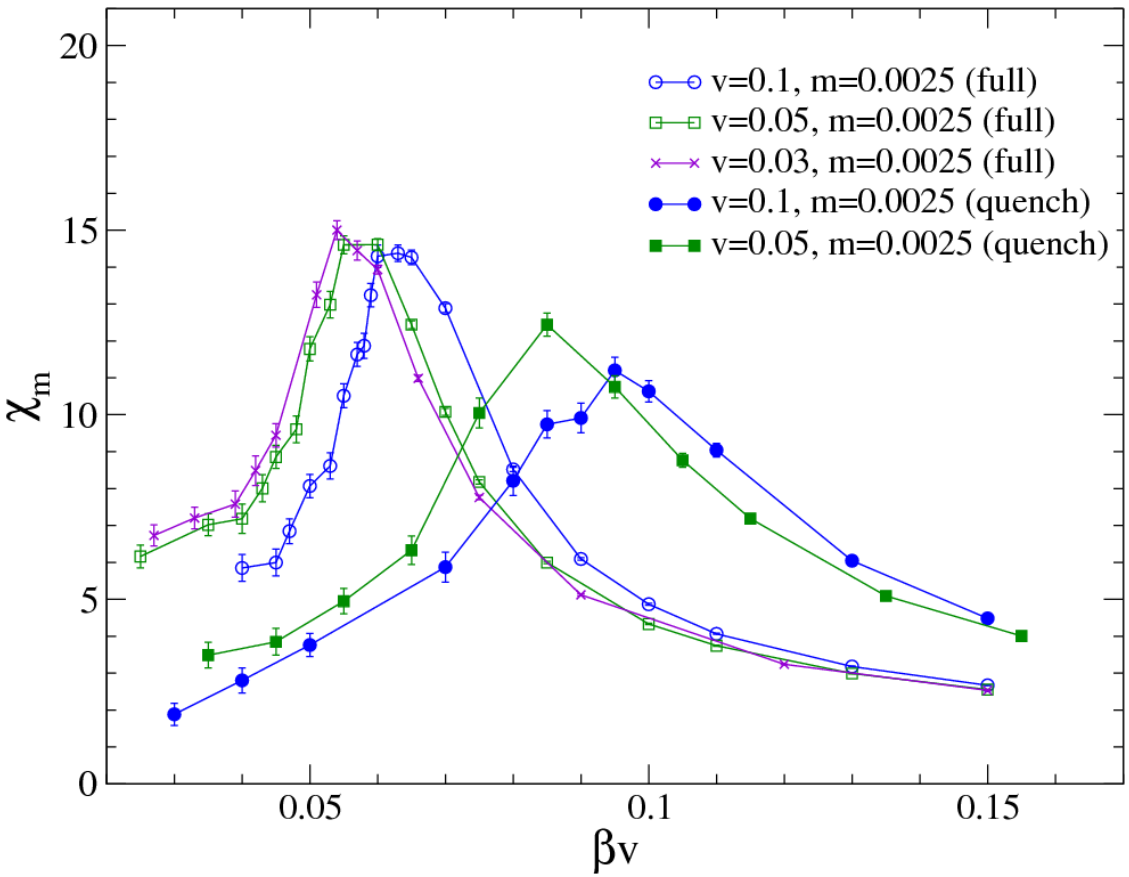
Mass dependence

- Chiral condensate



Vacuum polarization effect

- Chiral susceptibility



- Peak becomes sharp compared with quench results
- Dynamical effects
  - $v = 0.1$
    $(\beta v)_c \simeq 0.1$ (quench)
    $\rightarrow 0.062$ (full)
  - $v=0.05$
    $(\beta v)_c \simeq 0.09$ (quench)
    $\rightarrow 0.055$ (full)

  for α_c : 50--60% increase

cf. Gap eq. prediction
(quench) : $\alpha_c = 0.5$
(1-loop) : $\alpha_c = 0.92$
$\Rightarrow \simeq 50\%$ increase

Gamayun, et al. (2010)

[MC: $\alpha_c = 1.08$ in Drut (2009)]

Dynamical effect is significant !
(and also similar to gap equation.)