The study of decoupling of mirror fermions via an anomaly free model

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Dirac fermions and Weyl fermions

Dirac representation is reducible

$$R_{Dirac} : (\frac{1}{2}, 0) \bigoplus (0, \frac{1}{2})$$

Weyl Representation: $(\frac{1}{2}, 0)$ or $(0, \frac{1}{2})$

- Irreducible representation
- Characterize by the eigenvalue of γ_5:± 1
- Left handed & Right handed particles

Vector Theory vs. Chiral Theory

Vector Theory

- Left handed particles and right handed particles behave in the same way(same interactions).

Chiral Theory

- Left handed and right handed don't have to behave in the same way(they could have different form of interactions)

Chiral Gauge Theory

Weakly Coupled Chiral Gauge Theory

- Electroweak Theory $\rightarrow$ Perturbation expansion

Strongly Coupled Chiral Gauge Theory

- Potential Model for unknown Physics
  $\rightarrow$ No systematic way of solution
- Lattice?

Chiral fermion on lattice

- Replace Dirac operator with GW operator:

$$\{D, \gamma_5\} = aD\gamma_5 D$$

- Chiral symmetry of lattice action $S = \int \bar{\psi} D \psi$:

$$\hat{\gamma}_5 = (1 - aD)\gamma_5, \quad \bar{\psi} \to \bar{\psi} e^{i\alpha\hat{\gamma}_5}, \quad \psi \to e^{i\alpha\gamma_5}\psi$$
$$\bar{\psi}_{L/R} \equiv \bar{\psi}\frac{1 \mp \hat{\gamma}_5}{2}, \quad \psi_{L/R} \equiv \frac{1 \pm \gamma_5}{2}\psi$$

- D(therefore $\hat{\gamma}_5$) is gauge field dependent in the overlap formalism.

P. H. Ginsparg and K. G. Wilson (1982), H. Neuberger (1998)

Fermion measure problem for lattice chiral gauge theory

- Any fermion determinant of chiral theory is defined up to a complex phase.

$$\int D\bar{\psi}_L D\psi_L \exp(\bar{\psi}_L \mathrm{D}\psi_L)$$

$$= \begin{cases} \det D & \text{using base } \bar{\psi}_L(x) = \sum_i \bar{c}_i u_i(x) \\ \det D \cdot \det \mathrm{Q} & \text{using base } \{u'_i\} = \sum_l u_l(x) \mathrm{Q}^{-1}_{li} \end{cases}$$

- On lattice, the phase not only depends on which base you choose, but also depends on gauge field back ground.
- $U(1)$ arbitrariness for each gauge field configuration.
- How to determine the phase? $\rightarrow$ Fermion measure problem.

Decoupling the Mirror Fermions

The action of any vector theory can be written as:

$$S = S_{light} + S_{mirror}$$

Light Sector

- A chiral gauge theory we wish to obtain at the end

Mirror Sector

- contains all fermions with the "wrong" chirality

Decoupling the mirror fermions

- Adding interactions in the mirror sector and hopefully all fermions with the wrong chirality become heavy and disappear in the IR limit

1-0 Model

- Two dimension
- $U(1)$ lattice gauge theory
- Yukawa-Higgs-GW-fermion Model
- Study the mirror sector spectrum at fix gauge field background $A = 0$

1-0 Model

- Action

$$
\begin{aligned}
S &= S_{light} + S_{mirror} \\
S_{light} &= -(\bar{A}_+ \cdot D_1 \cdot A_+) - (\bar{X}_- \cdot D_0 \cdot X_-) \\
S_{mirror} &= S_\kappa - (\bar{A}_- \cdot D_1 \cdot A_-) - (\bar{X}_+ \cdot D_0 \cdot X_+) \\
&\quad + S_{Yuk.,Dirac} + S_{Yuk.,Maj}
\end{aligned}
$$

- Fields content of mirror sector:

Field	Q
A_-	1
X_+	0
ϕ	-1

Higgs field

- Unitary higgs Field:

$$\phi = e^{i\eta_x}, \ |\eta_x| \leq \pi$$

- Action:

$$S_\kappa = \frac{\kappa}{2} \sum_x \sum_{\hat{\mu}} [2 - (\phi_x^* U_{x,x+\hat{\mu}}^* \phi_{x+\hat{\mu}} + \mathrm{h.c.})]$$

$$U_{x,x+\hat{\mu}} = e^{iA_\mu(x)}$$

- Small κ phase $\rightarrow$ Higgs field correlated only over lattice spacing $\rightarrow$ decoupled from long distance physics

Interactions

Include all kinds of terms consistent with the to be gauged $U(1)$ symmetry:

- Dirac type Yukawa interaction:

$$S_{Yuk.,Dirac} = y[(\bar{A}_- \cdot \phi^* \cdot X_+) + (\bar{X}_+ \cdot \phi \cdot A_-)]$$

- Majorana type Yukawa interaction:

$$S_{Yuk.,Maj} = yh[(A_-^T \cdot \phi\gamma_2 \cdot X_+) - (\bar{X}_+ \cdot \gamma_2\phi^* \cdot \bar{A}_-^T)]$$

Splitting the vector-like partition function in the fixed gauge background

- $S = S_{light} + S_{mirror} \to Z = Z_{light} + Z_{mirror}$
- Using definite-chirality eigenvector of $\hat{\gamma}_5$ and γ_5

$$\hat{\gamma}_5 u_i = -u_i, \quad \hat{\gamma}_5 w_i = w_i$$

$$\gamma_5 v_i = v_i, \quad \gamma_5 t_i = -t_i$$

- General Dirac field Ψ_x can be decomposed into chiral components

$$\Psi_x = \sum_i \alpha_+^i v_i(x) + \alpha_-^i t_i(x), \; \bar{\Psi}_x = \sum_i \bar{\alpha}_+^i u_i^\dagger(x) + \bar{\alpha}_-^i w_i^\dagger(x)$$

Splitting the vector-like partition function

- Mirror sector

$$X_+ = \sum_i \beta^i_+ v_i,\ \bar{X}_+ = \sum_i \bar{\beta}^i_+ u_i^\dagger[0]$$
$$A_- = \sum_i \alpha^i_- t_i,\ \bar{A}_- = \sum_i \bar{\alpha}^i_- w_i^\dagger[A]$$

- Light sector

$$X_- = \sum_i \beta^i_- t_i,\ \bar{X}_- = \sum_i \bar{\beta}^i_- w_i^\dagger[0]$$
$$A_+ = \sum_i \alpha^i_+ v_i,\ \bar{A}_+ = \sum_i \bar{\alpha}^i_+ u_i^\dagger[A]$$

Splitting the vector-like partition function

- Splitting of partition function

$$Z[A; y, h] = Z_{light}[A] \times \frac{1}{J[A]} \times Z_{mirror}[A; y, h]$$

- Mirror partition function

$$Z_{mirror}[A; y, h] = \int d^2\alpha_- d^2\beta_+ d\phi e^{-S_{mirror}}$$

Probing the spectrum of mirror sector

- What to measure: Photon vacuum polarization operator.

$$\Pi^{mirror}_{\mu\nu}(x,y) \equiv \left.\frac{\delta^2 \ln Z_{mirror}[A]}{\delta A_\mu(x)\delta A_\nu(y)}\right|_{A=0}$$

- Mirror partition function in certain gauge background

$$Z_{mirror}[A;y,h] = \int d^2\alpha_- d^2\beta_+ d\phi e^{-S_{mirror}}$$

- Splitting theorem: An Important trick

$$\delta \ln Z_{mirror}[A] = \ln \frac{Z_{mirror}[A+\delta A]}{Z_{mirror}[A]} = \sum_i (\delta w_i^\dagger \cdot w_i) + \left\langle \frac{\delta S}{\delta O}\delta O \right\rangle$$

Polarization operator

$$
\begin{aligned}
\Pi^{mirror}_{\mu\nu} &= \delta_\nu j^w_\mu \\
&+ \langle \bar{\alpha}^i_- \alpha^j_- \rangle (w_i^\dagger \cdot (\delta_\mu \delta_\nu D + \delta_\nu \hat{P}_+ \delta_\mu D) \cdot t_j) \\
&+ \langle \bar{\alpha}^i_- \alpha^j_- \bar{\alpha}^k_- \alpha^l_- \rangle^C (w_i^\dagger \cdot \delta_\mu D \cdot t_j)(w_k^\dagger \cdot \delta_\nu D \cdot t_l) \\
&+ \frac{\kappa}{2} \langle (\phi^* \cdot \delta_\nu \delta_\mu U \cdot \phi) + \mathrm{h.c.} \rangle \\
&+ \frac{\kappa^2}{4} \langle (\phi^* \cdot \delta_\mu U \cdot \phi) + \mathrm{h.c.} \rangle \langle (\phi^* \cdot \delta_\nu U \cdot \phi) + \mathrm{h.c.} \rangle \\
&+ \frac{\kappa}{2} \left[\langle \bar{\alpha}^i_- \alpha^j_- ((\phi^* \cdot \delta_\nu \delta_\nu U \cdot \phi) + \mathrm{h.c.}) \rangle^C (w_i^\dagger \cdot \delta_\mu D \cdot t_j) \right. \\
&\quad \left. + (\mu \leftrightarrow \nu) \right] \\
&- y \langle \bar{\alpha}^i_- \beta^j_+ (w_i^\dagger \cdot \delta_\nu (\hat{P}_+ \delta_\mu \hat{P}_+) \cdot \phi^* v_j) \rangle
\end{aligned}
$$

Continued

$$
\begin{aligned}
- \quad & yh\langle\bar{\alpha}^i_-\bar{\beta}^j_+(u^\dagger_j\gamma_2\cdot\phi^*\cdot\delta_\nu(\hat{P}^T_+\cdot\hat{P}^T_+)\cdot w^*_i)\rangle \\
- \quad & y[(w^\dagger_i\cdot\delta_\mu D\cdot t_j)\langle\bar{\alpha}^i_-\alpha^j_-\bar{\alpha}^k_-(\beta^l_+(w^\dagger_k\delta_\mu\hat{P}\cdot\phi^* v_l) \\
& +h\bar{\beta}^l_+(u^\dagger_l\gamma 2\cdot\phi^*\cdot\delta_\nu\hat{P}^T_+\cdot w^*_k))\rangle+(\mu\leftrightarrow\nu)] \\
+ \quad & y^2\langle[\bar{\alpha}^i_-\beta^j_+(w^\dagger_i\cdot\delta_\mu\hat{P}_+\cdot\phi^* v_j)+h\bar{\alpha}^i_-\bar{\beta}^j_+(u^\dagger_j\gamma 2\cdot\phi^*\cdot\delta_\mu\hat{P}^T_+\cdot w^*_j)]\times \\
& [\bar{\alpha}^k_-\beta^l_+(w^\dagger_k\cdot\delta_\mu\hat{P}_+\cdot\phi^* v_l)+h\bar{\alpha}^k_-\bar{\beta}^l_+(u^\dagger_l\gamma 2\cdot\phi^*\cdot\delta_\mu\hat{P}^T_+\cdot w^*_k)]\rangle^C \\
- \quad & \frac{y\kappa}{2}\{\langle[(\phi^*\cdot\delta_\mu U\cdot\phi)+\mathrm{h.c.}][\bar{\alpha}^i_-\beta^j_+(w^\dagger_i\cdot\delta_\nu\hat{P}_+\cdot\phi^* v_j) \\
& +h\bar{\alpha}^i_-\bar{\beta}^j_+(u^\dagger_j\gamma 2\cdot\phi^*\cdot\delta_\mu\hat{P}^T_+\cdot w^*_j)]\rangle \\
& +(\mu\leftrightarrow\nu)\}
\end{aligned}
$$

E. Poppitz and Y. Shang, Int. J. Mod. Phys. A 25, 2761 (2010)

Small k behaviour of $\Pi_{\mu\nu}^{mirror}$ and massless particles

If there is massless particle:

- Singularity at $k = 0$:

$$\Pi_{\mu\nu}^{mirror} \xrightarrow{small\ k} 2C\frac{\delta_{\mu\nu}k^2 - k_\mu k_\nu}{k^2}$$

$$2C_{GS\ scalar} \simeq \kappa,\ 2C_{ch.ferm.} \simeq \frac{1}{2\pi} \simeq 0.159$$

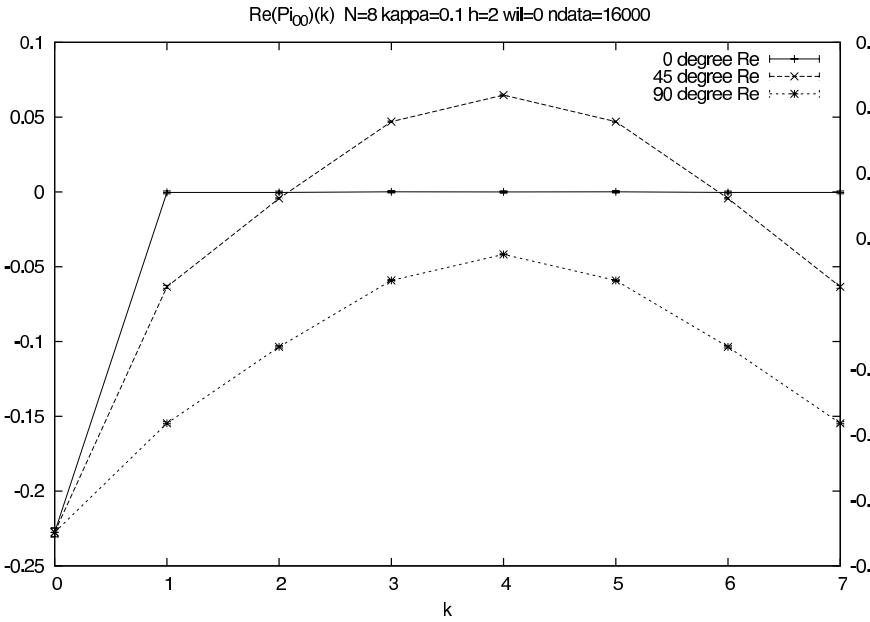
- Directional limit

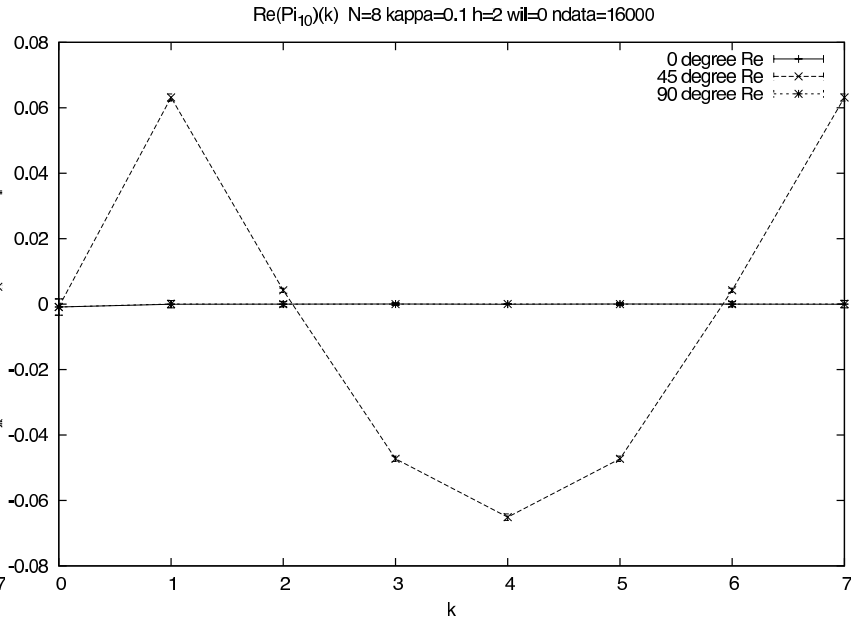
$$\tilde{\Pi}_{11}(\phi)\,|_{k\to 0} = C(1-\cos 2\phi),\quad \tilde{\Pi}_{21}(\phi)\,|_{k\to 0} = -C\sin 2\phi$$

No massless particles $\Rightarrow$ No singular behaviour

$$\tilde{\Pi}_{\mu\nu} \xrightarrow{small\ k} \frac{k^\mu k^\nu - k^2 g^{\mu\nu}}{m^2}$$

Massless particle in 1-0 model





Directional limit at $k \to 0 \Rightarrow$ Massless charged chiral fermions

$$\tilde{\Pi}_{11}(45^0) = -\tilde{\Pi}_{21}(45^0) = C$$
$$\tilde{\Pi}_{11}(90^0) = 2C$$

E. Poppitz and Y. Shang, Int. J. Mod. Phys. A 25, 2761 (2010)

Anomalies

Only depends on the representation of matter content

- Only massless particles can contribute to anomalies

Anomalous global symmetry

- Fine, sometimes can give information of the low energy spectrum

Anomalous gauge symmetry

- No good$\Rightarrow$ Unphysical degrees of freedom will not decouple

S. Weinberg, The quantum theory of fields. Vol.2: Modern applications

t'Hooft anomaly matching conditions

- Global chiral symmetry represented by by elementary fermions T_α
- Anomaly constant $\mathrm{Tr}\,[\{T_\alpha, T_\beta\}\,T_\gamma]$ is non-zero
- $\Rightarrow$ Same symmetry represented by low energy bound state $\mathcal{J}_\alpha$ with same anomaly constant

$$\mathrm{Tr}\,[\{\mathcal{J}_\alpha, \mathcal{J}_\beta\}\mathcal{J}_\gamma] = \mathrm{Tr}\,[\{T_\alpha, T_\beta\}\,T_\gamma]$$

S. Weinberg, The quantum theory of fields. Vol.2: Modern applications

t'Hooft anomaly matching conditions

Summarize

- High energy degrees of freedom are anomalous
  $\Rightarrow$ So will the low energy degrees of freedom
  $\Rightarrow$ Existence of massless bound states

Why massless particles in 1-0 Model

- Anomaly cancellation condition on lattice

$$\sum_L q^2 - \sum_R q^2 = 0$$

- Matter content of 1-0 Model(Anomalous)

Field	Q
A_-	1
X_+	0

$\Rightarrow$ Massless low energy bound states

3-4-5 Model

- Field content.

Field	Q
A_-	3
B_-	4
C_+	5
X_+	0
ϕ	-1

- Anomaly cancellation condition

$$\sum_L q^2 - \sum_R q^2 = 0$$

$\Rightarrow$ 3-4-5 model is anomaly free

Action

$$
\begin{aligned}
S &= S_{light} + S_{mirror} \\
S_{light} &= -(\bar{A}_+ \cdot D_3 \cdot A_+) - (\bar{B}_+ \cdot D_4 \cdot B_+) - (\bar{C}_- \cdot D_5 \cdot C_-) \\
&\quad -(\bar{X}_- \cdot D_0 \cdot X_-) \\
S_{mirror} &= S_\kappa - (\bar{A}_- \cdot D_3 \cdot A_-) - (\bar{B}_- \cdot D_4 \cdot B_-) - (\bar{C}_+ \cdot D_5 \cdot C_+) \\
&\quad -(\bar{X}_+ \cdot D_0 \cdot X_+) + S_{Yuk.,Dirac} + S_{Yuk.,Maj}
\end{aligned}
$$

Mirror sector interactions

- Break all mirror sector global symmetries explicitly except the to-be-gauged $U(1)$

$$
\begin{aligned}
S_{Yuk.,Dirac} &= y_{30}\bar{A}_- X_+ \phi^{-3} + y_{40}\bar{B}_- X_+ \phi^{-4} + y_{35}\bar{A}_- C_+ \phi^2 \\
&\quad + y_{45}\bar{B}_- C_+ \phi + y_{30}\bar{X}_+ A_- \phi^3 + y_{40}\bar{X}_+ B_- \phi^4 \\
&\quad + y_{35}\bar{C}_+ A_- \phi^{-2} + y_{45}\bar{C}_+ B_- \phi^{-1} \\
S_{Yuk.,Maj.} &= h_{30} A_-^T \gamma_2 X_+ \phi^3 + h_{40} B_-^T \gamma_2 X_+ \phi^4 + h_{35} A_-^T \gamma_2 C_+ \phi^8 \\
&\quad + h_{45} B_-^T \gamma_2 C_+ \phi^9 \\
&\quad - h_{30}\bar{X}_+ \gamma_2 \bar{A}_-^T \phi^{-3} - h_{40}\bar{X}_+ \gamma_2 \bar{B}_-^T \phi^{-4} \\
&\quad - h_{35}\bar{C}_+ \gamma_2 \bar{A}_-^T \phi^{-8} - h_{45}\bar{C}_+ \gamma_2 \bar{B}_-^T \phi^{-9}
\end{aligned}
$$

Polarization tensor of 3-4-5 model

$$
\begin{aligned}
\Pi_{\mu\nu} \;=\;& \delta_\nu(j_\mu^{wA} + j_\mu^{wB} + j_\mu^{uC}) \\
&+\langle \bar{\alpha}_-^i \alpha_-^j \rangle (w_{iA}^\dagger \cdot (\delta_\nu \delta_\mu D_3 + \delta_\nu \hat{P}_{+A} \cdot \delta_\mu D_3) \cdot t_j) \\
&+\langle \bar{\beta}_-^i \beta_-^j \rangle (w_{iB}^\dagger \cdot (\delta_\nu \delta_\mu D_4 + \delta_\nu \hat{P}_{+B} \cdot \delta_\mu D_4) \cdot t_j) \\
&+\langle \bar{\gamma}_+^i \gamma_+^j \rangle (u_{iC}^\dagger \cdot (\delta_\nu \delta_\mu D_5 + \delta_\nu \hat{P}_{-C} \cdot \delta_\mu D_5) \cdot v_j) \\
&+\Big\langle [\bar{\alpha}_-^i \alpha_-^j (w_{iA}^\dagger \cdot \delta_\mu D_3 \cdot t_j) + \bar{\beta}_-^i \beta_-^j (w_{iB}^\dagger \cdot \delta_\mu D_4 \cdot t_j) \\
&+\bar{\gamma}_+^i \gamma_+^j (u_{iC}^\dagger \cdot \delta_\mu D_5 \cdot v_j)][\bar{\alpha}_-^k \alpha_-^l (w_{kA}^\dagger \cdot \delta_\nu D_3 \cdot t_l) \\
&+\bar{\beta}_-^k \beta_-^l (w_{kB}^\dagger \cdot \delta_\nu D_4 \cdot t_l) + \bar{\gamma}_+^k \gamma_+^l (u_{kC}^\dagger \cdot \delta_\nu D_5 \cdot v_l)] \Big\rangle^C \\
&+\frac{\kappa}{2} \langle (\phi^* \cdot \delta_\nu \delta_\mu U^* \cdot \phi) + \text{h.c.} \rangle \\
&+\frac{\kappa^2}{4} \langle [(\phi^* \cdot \delta_\mu U^* \cdot \phi) + \text{h.c.}][(\phi^* \cdot \delta_\nu U^* \cdot \phi) + \text{h.c.}] \rangle^C \\
&+\frac{\kappa}{2} \Big\langle [(\phi^* \cdot \delta_\mu U^* \cdot \phi) + \text{h.c.}][\bar{\alpha}_-^i \alpha_-^j (w_{iA}^\dagger \cdot \delta_\mu D_3 \cdot t_j) + \bar{\beta}_-^i \beta_-^j (w_{iB}^\dagger \cdot \delta_\mu D_4 \cdot t_j) \\
&+\bar{\gamma}_+^i \gamma_+^j (u_{iC}^\dagger \cdot \delta_\mu D_5 \cdot v_j)] + (\mu \leftrightarrow \nu) \Big\rangle^C \\
&-y_{30} \langle \bar{\alpha}_-^i \chi_+^j \rangle (w_{iA}^\dagger \cdot \delta_\nu (\hat{P}_{+A} \cdot \delta_\mu \hat{P}_{+A}) \cdot \phi^{-3} \cdot v_j) - y_{40} \langle \bar{\beta}_-^i \chi_+^j \rangle (w_{iB}^\dagger \cdot \delta_\nu (\hat{P}_{+B} \cdot \delta_\mu \hat{P}_{+B}) \cdot \phi^{-4} \cdot v_j) \\
&-y_{35} \langle \bar{\alpha}_-^i \gamma_+^j \rangle (w_{iA}^\dagger \cdot \delta_\nu (\hat{P}_{+A} \cdot \delta_\mu \hat{P}_{+A}) \cdot \phi^2 \cdot v_j) - y_{45} \langle \bar{\beta}_-^i \gamma_+^j \rangle (w_{iB}^\dagger \cdot \delta_\nu (\hat{P}_{+B} \cdot \delta_\mu \hat{P}_{+B}) \cdot \phi \cdot v_j) \\
&-y_{35} \langle \bar{\gamma}_+^i \alpha_-^j \rangle (u_{iC}^\dagger \cdot \delta_\nu (\hat{P}_{-C} \cdot \delta_\mu \hat{P}_{-C}) \cdot \phi^{-2} \cdot t_j) - y_{45} \langle \bar{\gamma}_+^i \beta_-^j \rangle (u_{iC}^\dagger \cdot \delta_\nu (\hat{P}_{-C} \cdot \delta_\mu \hat{P}_{-C}) \cdot \phi^{-1} \cdot t_j) \\
&+h_{30} \langle \bar{\chi}_+^i \bar{\alpha}_-^j \rangle (u_{iX}^\dagger \cdot \phi^{-3} \cdot \gamma_2 \cdot \delta_\nu (\delta_\mu \hat{P}_{+A}^T \cdot \hat{P}_{+A}^T) \cdot w_{jA}^*) \\
&+h_{40} \langle \bar{\chi}_+^i \bar{\beta}_-^j \rangle (u_{iX}^\dagger \cdot \phi^{-4} \cdot \gamma_2 \cdot \delta_\nu (\delta_\mu \hat{P}_{+B}^T \cdot \hat{P}_{+B}^T) \cdot w_{jB}^*) \\
&+h_{35} \langle \bar{\gamma}_+^i \bar{\alpha}_-^j \rangle [(u_{iC}^\dagger \cdot \delta_\nu (\hat{P}_{-C} \cdot \delta_\mu \hat{P}_{-C}) \cdot \phi^{-8} \cdot \gamma_2 \cdot w_{jA}^*) \\
&+(u_{iC}^\dagger \cdot \phi^{-8} \cdot \gamma_2 \cdot \delta_\nu (\delta_\mu \hat{P}_{+A}^T \cdot \hat{P}_{+A}^T) \cdot w_{jA}^*)] \\
&+h_{35} \langle \bar{\gamma}_+^i \bar{\alpha}_-^j \rangle [(u_{iC}^\dagger \cdot \delta_\mu \hat{P}_{-C} \cdot \phi^{-8} \cdot \gamma_2 \cdot \delta_\nu \hat{P}_{+A}^T \cdot w_{jA}^*) + (\mu \leftrightarrow \nu)] \\
&+h_{45} \langle \bar{\gamma}_+^i \bar{\beta}_-^j \rangle [(u_{iC}^\dagger \cdot \delta_\mu \hat{P}_{-C} \cdot \phi^{-9} \cdot \gamma_2 \cdot \delta_\nu \hat{P}_{+B}^T \cdot w_{jB}^*) + (\mu \leftrightarrow \nu)] \\
&+h_{45} \langle \bar{\gamma}_+^i \bar{\beta}_-^j \rangle [(u_{iC}^\dagger \cdot \delta_\nu (\hat{P}_{-C} \cdot \delta_\mu \hat{P}_{-C}) \cdot \phi^{-9} \cdot \gamma_2 \cdot w_{jB}^*) \\
&+(u_{iC}^\dagger \cdot \phi^{-9} \cdot \gamma_2 \cdot \delta_\nu (\delta_\mu \hat{P}_{+B}^T \cdot \hat{P}_{+B}^T) \cdot w_{jB}^*)] \\
&\text{(continued)}
\end{aligned}
$$

Continues

$$
\begin{aligned}
&-\Big\langle \big[\bar{\alpha}^k_- \alpha^l_- (w^\dagger_{kA} \cdot \delta_\mu D_3 \cdot t_l) + \bar{\beta}^k_- \beta^l_- (w^\dagger_{kB} \cdot \delta_\mu D_4 \cdot t_l) \\
&+\bar{\gamma}^k_+ \gamma^l_+ (u^\dagger_{kC} \cdot \delta_\mu D_5 \cdot v_l)\big] \\
&\times \big[y_{30} \bar{\alpha}^i_- \chi^j_+ (w^\dagger_{iA} \cdot \delta_\nu \hat{P}_{+A} \cdot \phi^{-3} \cdot v_j) + y_{40} \bar{\beta}^i_- \chi^j_+ (w^\dagger_{iB} \cdot \delta_\nu \hat{P}_{+B} \cdot \phi^{-4} \cdot v_j) \\
&+y_{35} \bar{\alpha}^i_- \gamma^j_+ (w^\dagger_{iA} \cdot \delta_\nu \hat{P}_{+A} \cdot \phi^{2} \cdot v_j) + y_{45} \bar{\beta}^i_- \gamma^j_+ (w^\dagger_{iB} \cdot \delta_\nu \hat{P}_{+B} \cdot \phi \cdot v_j) \\
&+y_{35} \bar{\gamma}^i_+ \alpha^j_- (u^\dagger_{iC} \cdot \delta_\nu \hat{P}_{-C} \cdot \phi^{-2} \cdot t_j) + y_{45} \bar{\gamma}^i_+ \beta^j_- (u^\dagger_{iC} \cdot \delta_\nu \hat{P}_{-C} \cdot \phi^{-1} \cdot t_j) \\
&-h_{30} \bar{\chi}^i_+ \bar{\alpha}^j_- (u^\dagger_{iX} \cdot \phi^{-3} \cdot \gamma_2 \cdot \delta_\nu \hat{P}^T_{+A} \cdot w^*_{jA}) - h_{40} \bar{\chi}^i_+ \bar{\beta}^j_- (u^\dagger_{iX} \cdot \phi^{-4} \cdot \gamma_2 \cdot \delta_\nu \hat{P}^T_{+B} \cdot w^*_{jB}) \\
&-h_{35} \bar{\gamma}^i_+ \bar{\alpha}^j_- [(u^\dagger_{iC} \cdot \delta_\nu \hat{P}_{-C} \cdot \phi^{-8} \cdot \gamma_2 \cdot w^*_{jA}) + (u^\dagger_{iC} \cdot \phi^{-8} \cdot \gamma_2 \cdot \delta_\nu \hat{P}^T_{+A} \cdot w^*_{jA})] \\
&-h_{45} \bar{\gamma}^i_+ \bar{\beta}^j_- [(u^\dagger_{iC} \cdot \delta_\nu \hat{P}_{-C} \cdot \phi^{-9} \cdot \gamma_2 \cdot w^*_{jB}) \\
&+(u^\dagger_{iC} \cdot \phi^{-9} \cdot \gamma_2 \cdot \delta_\nu \hat{P}^T_{+B} \cdot w^*_{jB})]\big] + (\mu \leftrightarrow \nu)\Big\rangle^C \\
&+\Big\langle \big\{y_{30} \bar{\alpha}^i_- \chi^j_+ (w^\dagger_{iA} \cdot \delta_\mu \hat{P}_{+A} \cdot \phi^{-3} \cdot v_j) + y_{40} \bar{\beta}^i_- \chi^j_+ (w^\dagger_{iB} \cdot \delta_\mu \hat{P}_{+B} \cdot \phi^{-4} \cdot v_j) \\
&+y_{35} \bar{\alpha}^i_- \gamma^j_+ (w^\dagger_{iA} \cdot \delta_\mu \hat{P}_{+A} \cdot \phi^{2} \cdot v_j) + y_{45} \bar{\beta}^i_- \gamma^j_+ (w^\dagger_{iB} \cdot \delta_\mu \hat{P}_{+B} \cdot \phi \cdot v_j) \\
&+y_{35} \bar{\gamma}^i_+ \alpha^j_- (u^\dagger_{iC} \cdot \delta_\mu \hat{P}_{-C} \cdot \phi^{-2} \cdot t_j) + y_{45} \bar{\gamma}^i_+ \beta^j_- (u^\dagger_{iC} \cdot \delta_\mu \hat{P}_{-C} \cdot \phi^{-1} \cdot t_j) \\
&-h_{30} \bar{\chi}^i_+ \bar{\alpha}^j_- (u^\dagger_{iX} \cdot \phi^{-3} \cdot \gamma_2 \cdot \delta_\mu \hat{P}^T_{+A} \cdot w^*_{jA}) - h_{40} \bar{\chi}^i_+ \bar{\beta}^j_- (u^\dagger_{iX} \cdot \phi^{-4} \cdot \gamma_2 \cdot \delta_\mu \hat{P}^T_{+B} \cdot w^*_{jB}) \\
&-h_{35} \bar{\gamma}^i_+ \bar{\alpha}^j_- [(u^\dagger_{iC} \cdot \delta_\mu \hat{P}_{-C} \cdot \phi^{-8} \cdot \gamma_2 \cdot w^*_{jA}) + (u^\dagger_{iC} \cdot \phi^{-8} \cdot \gamma_2 \cdot \delta_\mu \hat{P}^T_{+A} \cdot w^*_{jA})] \\
&-h_{45} \bar{\gamma}^i_+ \bar{\beta}^j_- [(u^\dagger_{iC} \cdot \delta_\mu \hat{P}_{-C} \cdot \phi^{-9} \cdot \gamma_2 \cdot w^*_{jB}) + (u^\dagger_{iC} \cdot \phi^{-9} \cdot \gamma_2 \cdot \delta_\mu \hat{P}^T_{+B} \cdot w^*_{jB})]\big\} \\
&\times \big\{y_{30} \bar{\alpha}^k_- \chi^l_+ (w^\dagger_{kA} \cdot \delta_\nu \hat{P}_{+A} \cdot \phi^{-3} \cdot v_l) + y_{40} \bar{\beta}^k_- \chi^l_+ (w^\dagger_{kB} \cdot \delta_\nu \hat{P}_{+B} \cdot \phi^{-4} \cdot v_l) \\
&+y_{35} \bar{\alpha}^k_- \gamma^l_+ (w^\dagger_{kA} \cdot \delta_\nu \hat{P}_{+A} \cdot \phi^{2} \cdot v_l) + y_{45} \bar{\beta}^k_- \gamma^l_+ (w^\dagger_{kB} \cdot \delta_\nu \hat{P}_{+B} \cdot \phi \cdot v_l) \\
&+y_{35} \bar{\gamma}^k_+ \alpha^l_- (u^\dagger_{kC} \cdot \delta_\nu \hat{P}_{-C} \cdot \phi^{-2} \cdot t_l) + y_{45} \bar{\gamma}^k_+ \beta^l_- (u^\dagger_{kC} \cdot \delta_\nu \hat{P}_{-C} \cdot \phi^{-1} \cdot t_l) \\
&-h_{30} \bar{\chi}^k_+ \bar{\alpha}^l_- (u^\dagger_{kX} \cdot \phi^{-3} \cdot \gamma_2 \cdot \delta_\nu \hat{P}^T_{+A} \cdot w^*_{lA}) - h_{40} \bar{\chi}^k_+ \bar{\beta}^l_- (u^\dagger_{kX} \cdot \phi^{-4} \cdot \gamma_2 \cdot \delta_\nu \hat{P}^T_{+B} \cdot w^*_{lB}) \\
&-h_{35} \bar{\gamma}^k_+ \bar{\alpha}^l_- [(u^\dagger_{kC} \cdot \delta_\nu \hat{P}_{-C} \cdot \phi^{-8} \cdot \gamma_2 \cdot w^*_{lA}) + (u^\dagger_{kC} \cdot \phi^{-8} \cdot \gamma_2 \cdot \delta_\nu \hat{P}^T_{+A} \cdot w^*_{lA})] \\
&-h_{45} \bar{\gamma}^k_+ \bar{\beta}^l_- [(u^\dagger_{kC} \cdot \delta_\nu \hat{P}_{-C} \cdot \phi^{-9} \cdot \gamma_2 \cdot w^*_{lB}) + (u^\dagger_{kC} \cdot \phi^{-9} \cdot \gamma_2 \cdot \delta_\nu \hat{P}^T_{+B} \cdot w^*_{lB})]\big\}\Big\rangle^C
\end{aligned}
$$

(continued)

Continues again. $\sim$ 3000 lines of code

$$
\begin{aligned}
&-\frac{\kappa}{2}\Big\langle [(\phi^* \cdot \delta_\mu U^* \cdot \phi) + \text{h.c.}] \\
&\times \big\{ y_{30}\bar{\alpha}^i_- \chi^j_+ (w^\dagger_{iA} \cdot \delta_\nu \hat{P}_{+A} \cdot \phi^{-3} \cdot v_j) + y_{40}\bar{\beta}^i_- \chi^j_+ (w^\dagger_{iB} \cdot \delta_\nu \hat{P}_{+B} \cdot \phi^{-4} \cdot v_j) \\
&+ y_{35}\bar{\alpha}^i_- \gamma^j_+ (w^\dagger_{iA} \cdot \delta_\nu \hat{P}_{+A} \cdot \phi^{2} \cdot v_j) + y_{45}\bar{\beta}^i_- \gamma^j_+ (w^\dagger_{iB} \cdot \delta_\nu \hat{P}_{+B} \cdot \phi \cdot v_j) \\
&+ y_{35}\bar{\gamma}^i_+ \alpha^j_- (u^\dagger_{iC} \cdot \delta_\nu \hat{P}_{-C} \cdot \phi^{-2} \cdot t_j) + y_{45}\bar{\gamma}^i_+ \beta^j_- (u^\dagger_{iC} \cdot \delta_\nu \hat{P}_{-C} \cdot \phi^{-1} \cdot t_j) \\
&- h_{30}\bar{\chi}^i_+ \bar{\alpha}^j_- (u^\dagger_{iX} \cdot \phi^{-3} \cdot \gamma_2 \cdot \delta_\nu \hat{P}^T_{+A} \cdot w^*_{jA}) - h_{40}\bar{\chi}^i_+ \bar{\beta}^j_- (u^\dagger_{iX} \cdot \phi^{-4} \cdot \gamma_2 \cdot \delta_\nu \hat{P}^T_{+B} \cdot w^*_{jB}) \\
&- h_{35}\bar{\gamma}^i_+ \bar{\alpha}^j_- [(u^\dagger_{iC} \cdot \delta_\nu \hat{P}_{-C} \cdot \phi^{-8} \cdot \gamma_2 \cdot w^*_{jA}) + (u^\dagger_{iC} \cdot \phi^{-8} \cdot \gamma_2 \cdot \delta_\nu \hat{P}^T_{+A} \cdot w^*_{jA})] \\
&- h_{45}\bar{\gamma}^i_+ \bar{\beta}^j_- [(u^\dagger_{iC} \cdot \delta_\nu \hat{P}_{-C} \cdot \phi^{-9} \cdot \gamma_2 \cdot w^*_{jB}) \\
&+ (u^\dagger_{iC} \cdot \phi^{-9} \cdot \gamma_2 \cdot \delta_\nu \hat{P}^T_{+B} \cdot w^*_{jB})] \big\} + (\mu \leftrightarrow \nu) \Big\rangle^C
\end{aligned}
$$

How do we know we did something right?

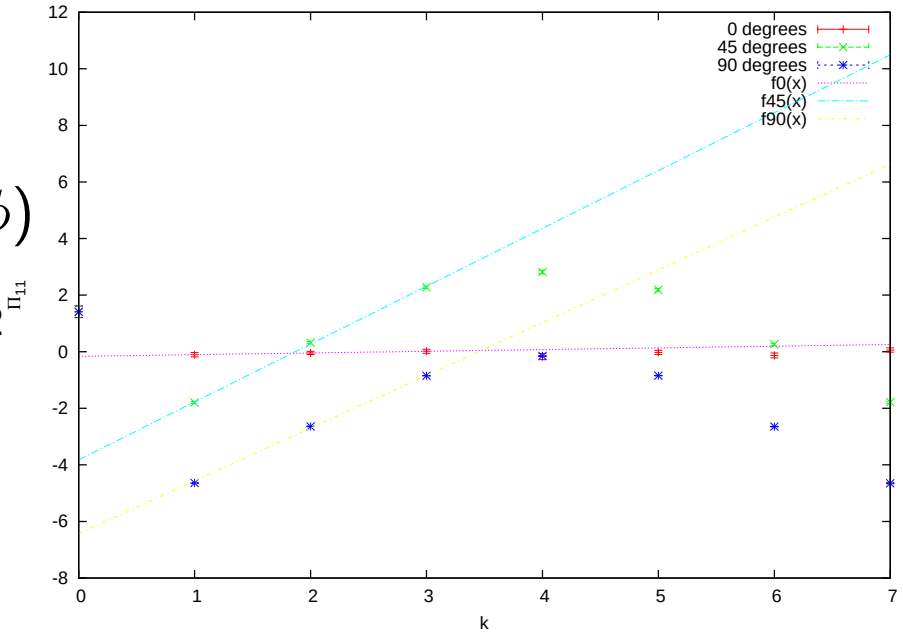
- Transversality of $\Pi_{\mu\nu}$
  1. Gauge invariance $\rightarrow \nabla \cdot \Pi_{\mu\nu} = 0$
  2. Lattice version in momentum space: $\sum_\mu (1 - e^{\frac{i2\pi}{N} k_\mu}) \tilde{\Pi}_{\mu\nu} = 0$
- Very strong condition

Massless particle?

- Massless particle

$$\tilde{\Pi}_{11}(\phi)\,|_{k\to 0} = C(1-\cos 2\phi)$$

$$2C_{ch.ferm.} \simeq q^2\frac{1}{2\pi} \simeq 0.159q^2$$

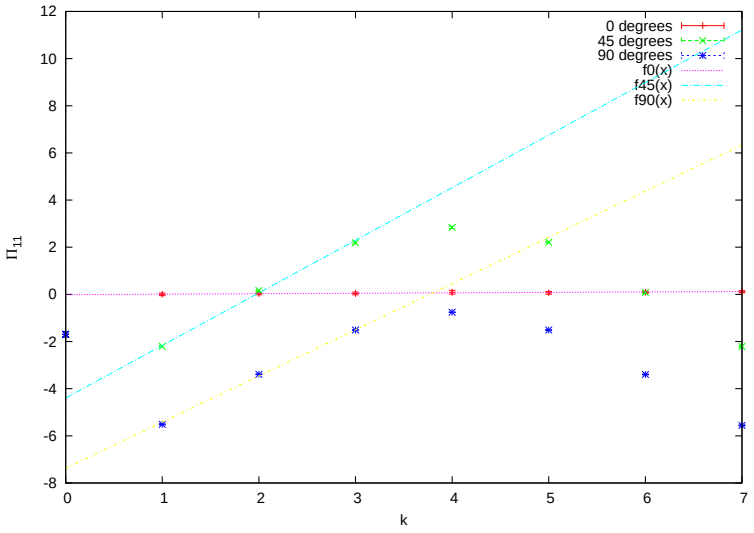


- No massless particle

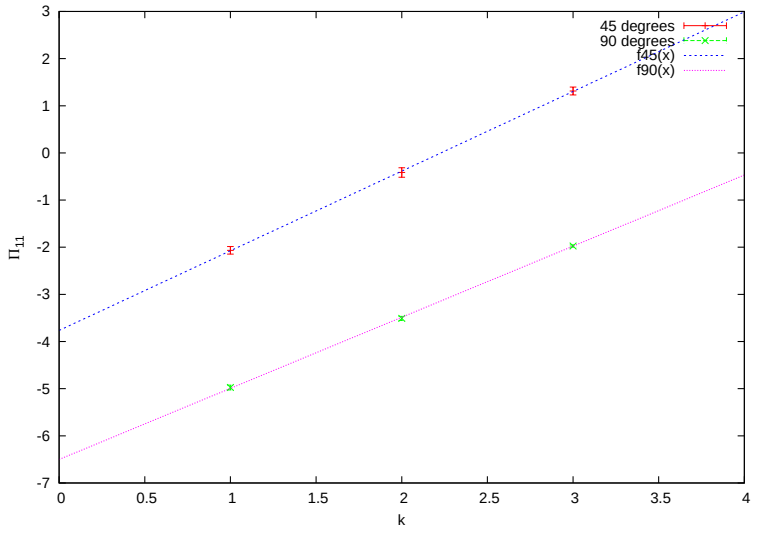
$$\tilde{\Pi}_{\mu\nu} \xrightarrow{small\ k} \frac{k^\mu k^\nu - k^2 g^{\mu\nu}}{m^2}$$

Another group of coupling constants for 3-4-5 model

8×8 lattice



10×10 lattice



Interpretation of the result

- Discontinuity $C \simeq 50 \times \frac{1}{4\pi} = (3^2 + 4^2 + 5^2) C_{ch.ferm.}$.
- Spectrum has to be anomaly free:
  1. Chiral 3_-, 4_-, 5_+ massless fermions.
  2. Vectorlike 5_-, 5_+ massless fermions (5_- could be a bound state of $\phi * 4_-$).

Summary and Conclusion

- An anomaly free model is studied to test the idea of decoupling the mirror fermions
- No argument can be made about the low energy spectrum of anomaly free model from symmetry point of view.
- Numerical simulation suggests non-analytic behavior of $\Pi_{\mu\nu}$ at $k = 0$,
- There is still massless particle in the mirror sector therefore it does not decouple from the light sector .