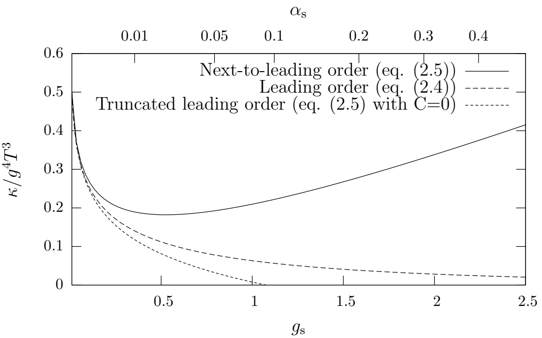
Toward NLO computation of thermal photon production rate

Aleksi Kurkela
with Jacopo Ghiglieri, Guy Moore, Derek Teaney, . . .

Motivation

A single NLO transport coefficient known:

- Heavy quark momentum diffusion coefficient: Caron-Huot, Moore 0801.2173



- Shows very poor perturbative convergence
  - Is this a generic feature of dynamical quantities?
    - Compare: pressure vs. quark number susceptibilities (no pure gluon diagrams in χ)
    - If is, is there anything that can be done? Resummations?

Motivation

To answer these questions, compute the next easiest NLO quantity:

- Thermal photon production rate
  - Phenomenologically interesting, improvement needed badly
  - If doesn't improve, at least get reliable errorbars

However: The calculation has not been done yet

Here: What technologies do we need for the calculation

Warning: Work under construction

Outline

- Motivation
- LO calculation
  - Hard region
  - Soft region
  - Collinear region
- NLO calculation
  - Collinear region
  - Soft region

LO calculation

We are interested in

$$\frac{d\Gamma_\gamma}{d^3k} = \frac{e^2}{(2\pi)^3 2k^0} D^>(K)$$

with electro-magnetic current-current Wightman correlator

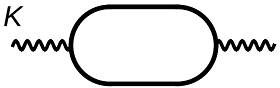
$$D^>(K) = \int d^4x\, e^{-iK\cdot x} \langle \underbrace{j_\mu(0)}_{\sum_{q=uds} e_q \bar{q}\gamma_\mu q} j^\mu(x)\rangle$$

Dynamical quantity ($k_0 \neq 0$):

- Euclidean methods + analytic continuation
- Real-time formalism

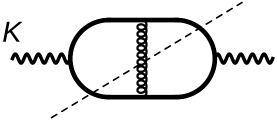
LO diagrams:

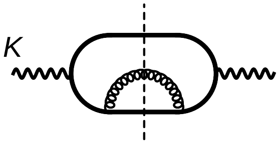
- 1 loop $\mathcal{O}(\alpha_{EM})$:



  - Kinematically disallowed for light-like K
    (both quarks can't be on-shell simultaneously)

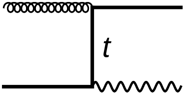
- 2 loops $\mathcal{O}(\alpha_{EM}\alpha_s)$:





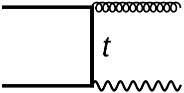
LO diagrams

- Cut diagrams correspond to:
  - Compton scattering:



$$\propto \int dq_\perp^2 \frac{d\sigma}{dq_\perp^2}$$

  - Pair annihilation:



$$\propto \int dq_\perp^2 \frac{d\sigma}{dq_\perp^2}$$

- Every time a scattering takes place, a quark can convert to a photon $\Rightarrow$ For $(K^2 = 0)$ $t \longrightarrow 0$, IR divergence:

$$D^> \propto \int_{\Lambda_{IR}^2} dq_\perp^2 \frac{d\sigma}{dq_\perp^2} \propto \ln\left(\frac{k_0 T}{\Lambda_{IR}^2}\right)$$

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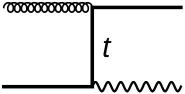

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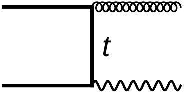

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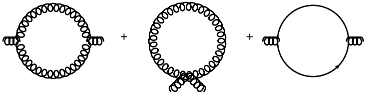
- IR divergences signal of missing physics!
  - For soft particles $p \sim gT$, large modifications to dispersion relation due to medium.

Hard Thermal Loops Braaten, Pisarski; Frenkel, Taylor; Blaizot-Iancu

Interaction with the medium generates a $\mathcal{O}(gT)$ correction to disp. relation

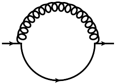
- Dominated by scattering with "Hard" particles at the scale T

$$\Pi_{gluon}(p \sim gT) =$$



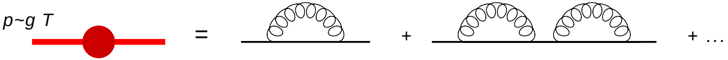
$$\sim g^2 T^2$$

$$\Sigma_{quark}(p \sim gT) =$$



$$\sim gT$$

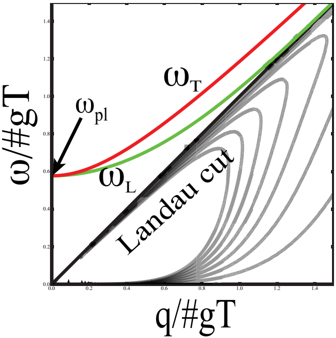
- Correction not small for "soft" $\mathcal{O}(gT)$ modes: need to resum
  $\Rightarrow$ HTL resummed perturbation theory



Expansion around quasi-particle excitations

In-medium dispersion relations

For momenta $\sim gT$, qualitatively different disp. rel.:

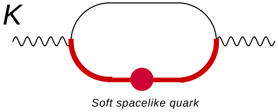


- Transverse and Longitudinal polarizations: ω_T, ω_L
- Minimum frequency: Plasma frequency: $\omega_{\rm pl}$
- Screening mass: m_D
- Non-zero spectral weight in spacelike region: Landau cut

- Similarly for quarks:
  - Effective thermal masses (m_∞), plasma frequencies and Landau cut
  - Positive and negative helicity/chirality modes: Plasminos

Leading order: Soft region Kapusta, Lichard, Seibert

Landau cut opens up phase space:

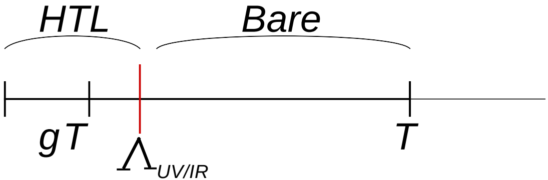


$$\propto \ln\left(\frac{\Lambda_{UV}}{m_\infty}\right) + \#$$

- Hard computation mangles the soft $\sim gT$ scale: IR divergence
- Soft computation mangles the hard $\sim T$ scale: UV divergence

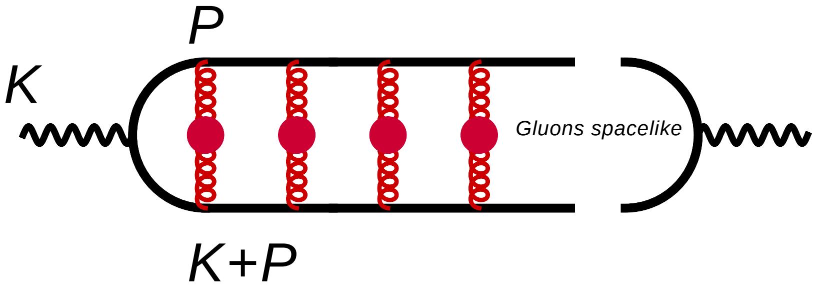
$\Rightarrow$ Dependence on cut off cancels in the sum

$$\underbrace{\ln\left(\frac{\Lambda_{UV}^2}{m_\infty^2}\right) + \#}_{\text{Soft: HTL}} + \underbrace{\ln\left(\frac{k_0 T}{\Lambda_{IR}^2}\right)}_{\text{Hard: Bare}} = \ln\left(\frac{k_0 T}{m_\infty^2}\right) + \#$$



Collinear region

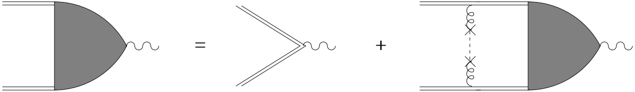
Introducing HTL gave rise to a completely new kinematic region: Aurenche et. al



- P and $K + P$ nearly collinear
- Gluons spacelike, in Landau cut
- Cutting graph gives bremsstrahlung and pair annihilation processes
- Guy Moore's talk!

Collinear region: AMY resummation Arnold, Moore, Yaffe

Splitting described by an integral equation:



$$2\mathbf{p}_\perp = i\delta E\,\mathbf{f}(\mathbf{p}_\perp; p, k) + \int \frac{d^2 q_\perp}{(2\pi)^2}\mathcal{C}(q_\perp)\Big[\mathbf{f}(\mathbf{p}_\perp) - \mathbf{f}(\mathbf{p}_\perp + \mathbf{q}_\perp)\Big]$$

$$\frac{dN_\gamma}{d^3\mathbf{k}d^4x} = \frac{2\alpha_{\mathrm{EM}}}{4\pi^2 k}\int_{-\infty}^{\infty}\frac{dp}{2\pi}\int\frac{d^2\mathbf{p}_\perp}{(2\pi)^2}\frac{n_f(k+p)[1-n_f(p)]}{2[p(p+k)]^2}$$
$$\times\left[p^2 + (p+k)^2\right]\mathrm{Re}\Big\{2\mathbf{p}_\perp \cdot \mathbf{f}(\mathbf{p}_\perp; p, k)\Big\}$$

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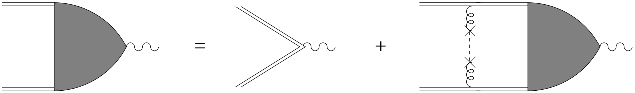


$$2\mathbf{p}_\perp \quad = \quad i\delta E\,\mathbf{f}(\mathbf{p}_\perp; p, k) + \int \frac{d^2 q_\perp}{(2\pi)^2}\mathcal{C}(q_\perp)\Big[\mathbf{f}(\mathbf{p}_\perp) - \mathbf{f}(\mathbf{p}_\perp + \mathbf{q}_\perp)\Big]$$

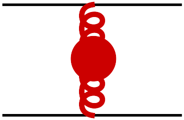
Takes two inputs:

1. Dispersion relations of splitter and splittee

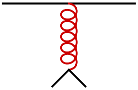
$$\delta E \approx k^0 - E(\mathbf{k}-\mathbf{p}) - E(\mathbf{p}) = -\left[\frac{k}{p(k+p)}\frac{\mathbf{p}_\perp^2 + m_\infty^2}{2}\right]$$

2. Rate of soft collisions: Collision kernel

$$\mathcal{C}(q_\perp) = \frac{d\Gamma}{dq_\perp^2} \sim g^2 T \frac{m_D^2}{q_\perp^2(q_\perp^2 + m_D^2)} \propto$$



$\Rightarrow$



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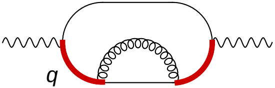
Why is NLO $\mathcal{O}(g)$:

- $\mathcal{O}(1)$ sensitivity to soft sector arose from IR divergent integrand:



$$\sim \int gT\,dq f(q), \qquad \text{with} \quad f(q \lesssim gT) \sim 1/q$$

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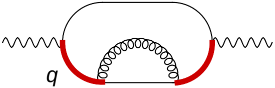
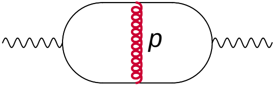


$$\sim \int gT\, dq f(q), \qquad \text{with} \quad f(q \lesssim gT) \sim 1/q$$

- When a generic loop momentum becomes soft, the integrand stays finite



$$\sim \int_{gT} dp f(p), \qquad f(p) \sim p^0$$

$\Rightarrow \mathcal{O}(g)$ sensitivity to gT to soft sector

Whenever a line gets soft, expect $\mathcal{O}(g)$ corrections

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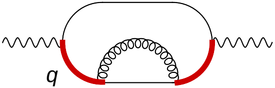

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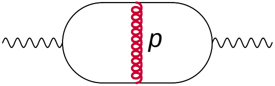

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Whenever a line gets soft, expect $\mathcal{O}(g)$ corrections

- Luckily NLO integrals not sensitive to ultra-soft scale $\sim g^2 T$.
  - NNLO probably prohibitively hard. Same story as g^6 pressure, but no technology to resum ultra-soft sector.

Collinear region

$$2\mathbf{p}_\perp = i\delta E\,\mathbf{f}(\mathbf{p}_\perp; p, k) + \int \frac{d^2 q_\perp}{(2\pi)^2}\mathcal{C}(q_\perp)\Big[\mathbf{f}(\mathbf{p}_\perp) - \mathbf{f}(\mathbf{p}_\perp + \mathbf{q}_\perp)\Big]$$

Order g corrections to collinear resummation from two sources:

1. Dispersion relations of splitter and splittee
2. Rate of soft collisions: NLO correction to collision kernel

Collinear region: 1. Dispersion relations

$$\delta E \approx -\left[\frac{k}{p(k+p)}\frac{\mathbf{p}_\perp^2 + m_\infty^2}{2}\right]$$

Order $\mathcal{O}(g)$ corrections to $m_\infty^2 \sim g^2 T^2 \big[1 + \# g + \ldots\big]$

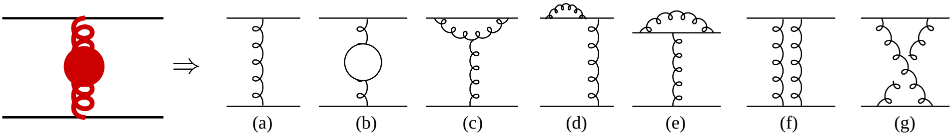
$$\begin{aligned} m_\infty^2 &\sim g^2 \int \frac{d^3p}{p} n_B(\omega_{\mathrm{HTL}}(p)) \sim g^2 \int \frac{d^3p}{p} n_B\left(\sqrt{m_\infty^2 + p^2}\right) \\ &\sim g^2 T^2 (1 - \# m_\infty / T) \end{aligned}$$

Note: actually m_∞^2 becomes negative for not so small g $\Rightarrow$ maybe better to solve m_∞ self-consistently

Collinear region: 2. Collision kernel

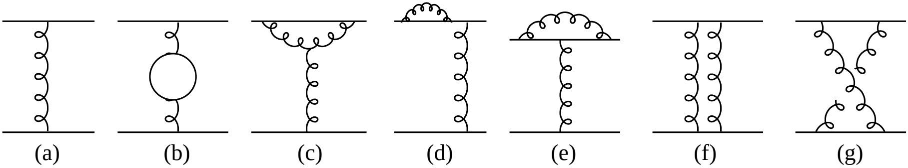
- Rate of soft collisions: NLO correction to collision kernel

$$\mathcal{C}(q_\perp \sim gT) \sim \mathcal{C}^{LO} + g\mathcal{C}^{NLO}$$



All propagators soft!! Looks almost prohibitively difficult!

Collinear region: NLO collision kernel Caron-Huot



However:

- Can be translated into EQCD (along the light cone) with a very clever coordinate transformation.
- EQCD is vastly simpler than HTL: just massive disp. rels.

$$
\begin{aligned}
\mathcal{C}^{NLO} \quad = \quad & (g^4 T^2 C_s C_A)\Big[\frac{7}{32 q_\perp^3} - \frac{m_D + 2\frac{q_\perp^2 - M2}{q_\perp}\arctan q_\perp/m_D}{4\pi(q_\perp^2+m_D^2)^2} \\
+ \quad & \frac{m_D - \frac{q_\perp^2+4m_D^2}{2q_\perp}\arctan(q_\perp/2m_D)}{8\pi q_\perp^4} - \frac{\arctan(q_\perp/m_D)}{2\pi q_\perp(q_\perp^2+m_D^2)} + \frac{\arctan(q_\perp/2m_D)}{2\pi q_\perp^3} \\
+ \quad & \frac{m_D}{4\pi(q_\perp^2+m_D^2)}\left(\frac{3}{q_\perp^2+4m_D^2} - \frac{2}{q_\perp^2+m_D^2} - \frac{1}{q_\perp^2}\right)\Big]
\end{aligned}
$$

Collinear region, Results

$$\frac{d\Gamma_\gamma}{d^3k} \supset \frac{\mathcal{A}(k)}{(2\pi)^3}\left[C_{\mathrm{brem}}(k) + C_{\mathrm{pair}}(k)\right]$$

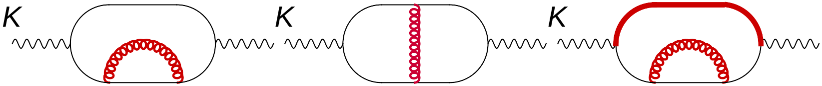
- $\sim 100\%$ corrections for realistic couplings
- Just one kinematic region, don't draw any conclusions!

VERY PRELIMINARY

"$2 \to 2$" NLO diagrams

Whenever any of the lines becomes soft, dispersion relation are $\mathcal{O}(g)$ mangled:

- Less scary diagrams



blobs suppressed from propagators

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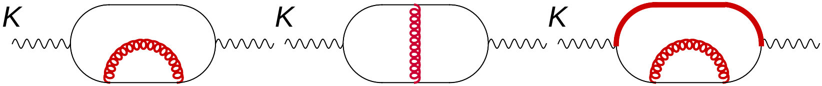
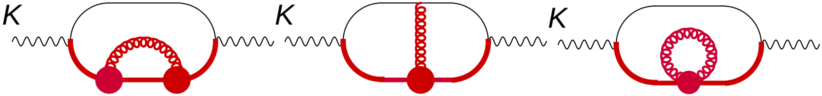


blobs suppressed from propagators

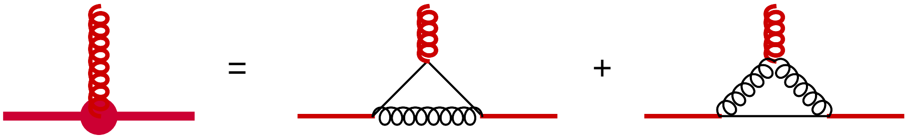
In addition, whenever vertex contains only soft momenta, it receives $\mathcal{O}(1)$ HTL correction:

- Frightening diagrams

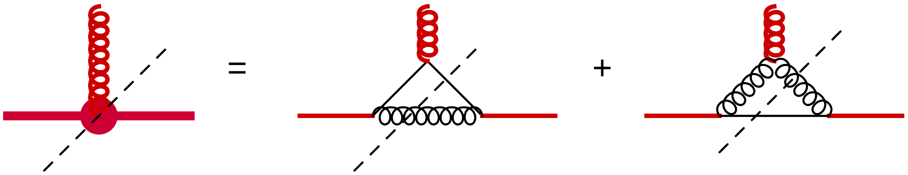


HTL's and real time

Compact notation may fool: Zoom in to the HTL vertex



Internal lines can be on-shell and can hence be cut:



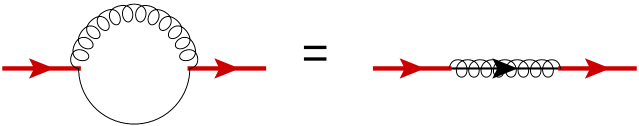
$\Rightarrow$ Need to understand HTL vertices in real time formalism

HTL's simplified by effective kinetic theory Wong; Blaizot,Iancu; Caron-Huot

Consider plasma of

- Soft quark and gluons with $p \sim gT$
- Hard (eikonal) particles: $\mathbf{v} = 1$, kinematics unaffected by softs

Example: Retarded self energy



- A soft quark couples to a heavy mode with: $\omega_0^2 i \not{v}$
- Hard particle propagates along light ray with velocity $\mathbf{v}$: $D_R^{eik}(\mathbf{x}, t) \sim \delta^3(\mathbf{v}t - \mathbf{x})$

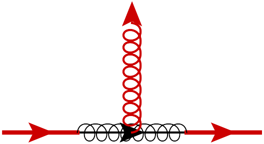
$$D_R^{eik}(\mathbf{k}, k_0) = \frac{-i}{-(k_0 + i\epsilon) + \mathbf{v} \cdot \mathbf{k}}$$

- Integrate over hard particle distribution (over directions): $\int \frac{d\Omega_{\mathbf{v}}}{4\pi}$

$$-i\Sigma_{\mathrm{HTL}} = \omega_0^2 \int \frac{d\Omega_{\mathbf{v}}}{4\pi} \frac{\not{v}}{-(k_0 + i\epsilon) + \mathbf{v} \cdot \mathbf{k}}$$

HTL vertices

The effective kinetic theory can be used to get all the other vertices
Example: 3-point retarded:



$$= \omega_0^2 \int \frac{d\Omega_{\mathbf{v}}}{4\pi} i\not{v} \frac{-i}{v \cdot Q^-} (igv_\mu t^a) \frac{-i}{v \cdot (Q+P)^-}$$

- Angular integrals trivial, always lead to familiar HTL arctan's

Conclusions?

Work is in process

- Collinear sector finished
- Easy diagrams under control
- fermionic HTLs understood
- Evaluation of the fully soft diagrams underway

Can we guess already now the answer?

- In the collinear region $\sim 100\%$ correction
- Fully soft sector complicated, different color structures. Sign?

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Winter is coming to Montreal, so it's good that we don't run out of things to do.