PHASE TRANSITIONS, TRANSPORT PROCESSES AND DYNAMIC CORRELATION IN STRONGLY COULOMB-COUPLED, ATOMIC PLASMAS

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Metal-Insulator Transition – Ionization

Freezing Transition – Wigner Crystallization

Ferromagnetic Transition – Magnetization

Correlation Functions, Static and Dynamic

Electronic Screening and Binding Energies

Cross Sections of Inelastic Scattering

Generalized Viscoelastic Navier-Stokes Formalism

Shear Viscosity and Elastic Constant

Glassy (or "Frozen") States – Quasi-elastic Peak

Crystalline Nucleation, Experimental

BASIC AND/OR DIMENSIONLESS PARAMETERS FOR ELECTRON LIQUIDS

$$\boxed{r_s \equiv \frac{a}{a_B} \equiv \left(\frac{3}{4\pi n}\right)^{1/3} \frac{m e^2}{\hbar^2}}$$

Wigner-Seitz radius

$$\boxed{\theta \equiv \frac{2 m k_B T}{\hbar^2 (3\pi^2 n)^{2/3}}}$$

Fermi-degeneracy parameter

$$\Gamma_e \equiv \frac{e^2}{a k_B T} = 2\left(\frac{4}{9\pi}\right)^{2/3} \frac{r_s}{\theta}$$

Coulomb-coupling parameter

$$\zeta \equiv \frac{n_\uparrow - n_\downarrow}{n_\uparrow + n_\downarrow}$$

Degree of spin polarization

$$\frac{1}{D_s} = \frac{2}{\pi} \int_0^\infty dk \left\{ 1 - \frac{1}{\varepsilon_e(k,0)} \right\}$$

Electronic screening length

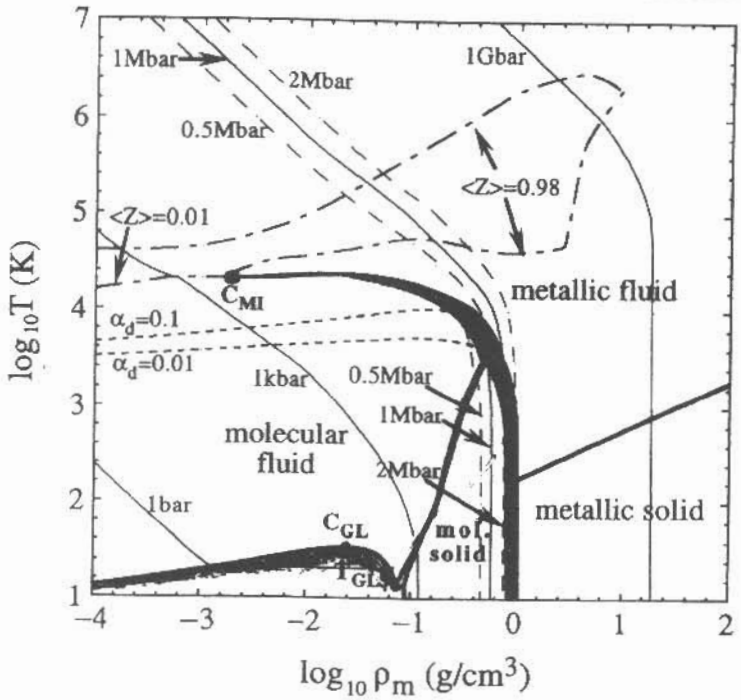
$$\frac{a}{D_s} = 1.239\, r_s^{\nu} \left(\tanh \frac{1.061}{\theta} \right)^{1/2} \qquad \nu = \frac{0.435 + 0.024\,\theta^{2.65}}{1 + 0.048\,\theta^{2.65}}$$

Coulomb-coupling parameter between protons with electronic screening

$$\Gamma_{pp}^{s} \equiv \frac{e^2}{a k_B T} \exp\left(-\frac{a}{D_s}\right)$$

Metal-Insulator Transitions in Hydrogen Matter

H. Kitamura and S. Ichimaru, J. Phys. Soc. Jpn 67, 950 (1998)



C_{MI}

$\rho_m = 0.0019$ g/cm^3

$T = 2.04\times10^4$ K

$P = 3.0$ kbar

Atomic ionization:

$$H \Rightarrow p + e^- - 13.6\ \text{eV}\ (= 1\ \text{Ry})$$

Molecular ionization:

$$H_2 \Rightarrow H_2^+ + e^- - 15.45\ \text{eV}\ (= 1.13\ \text{Ry})$$

Molecular dissociation:

$$H_2 \Rightarrow H + H - 4.447\ \text{eV}\ (= 0.33\ \text{Ry})$$

Molecular-ionic dissociation:

$$H_2^+ \Rightarrow H + p - 2.467\ \text{eV}\ (= 0.18\ \text{Ry})$$

Electronic Screening and Binding Energies

Effective radii, R_b, of bound electrons

Lowering of binding energies $\Leftrightarrow$ Increase in R_b

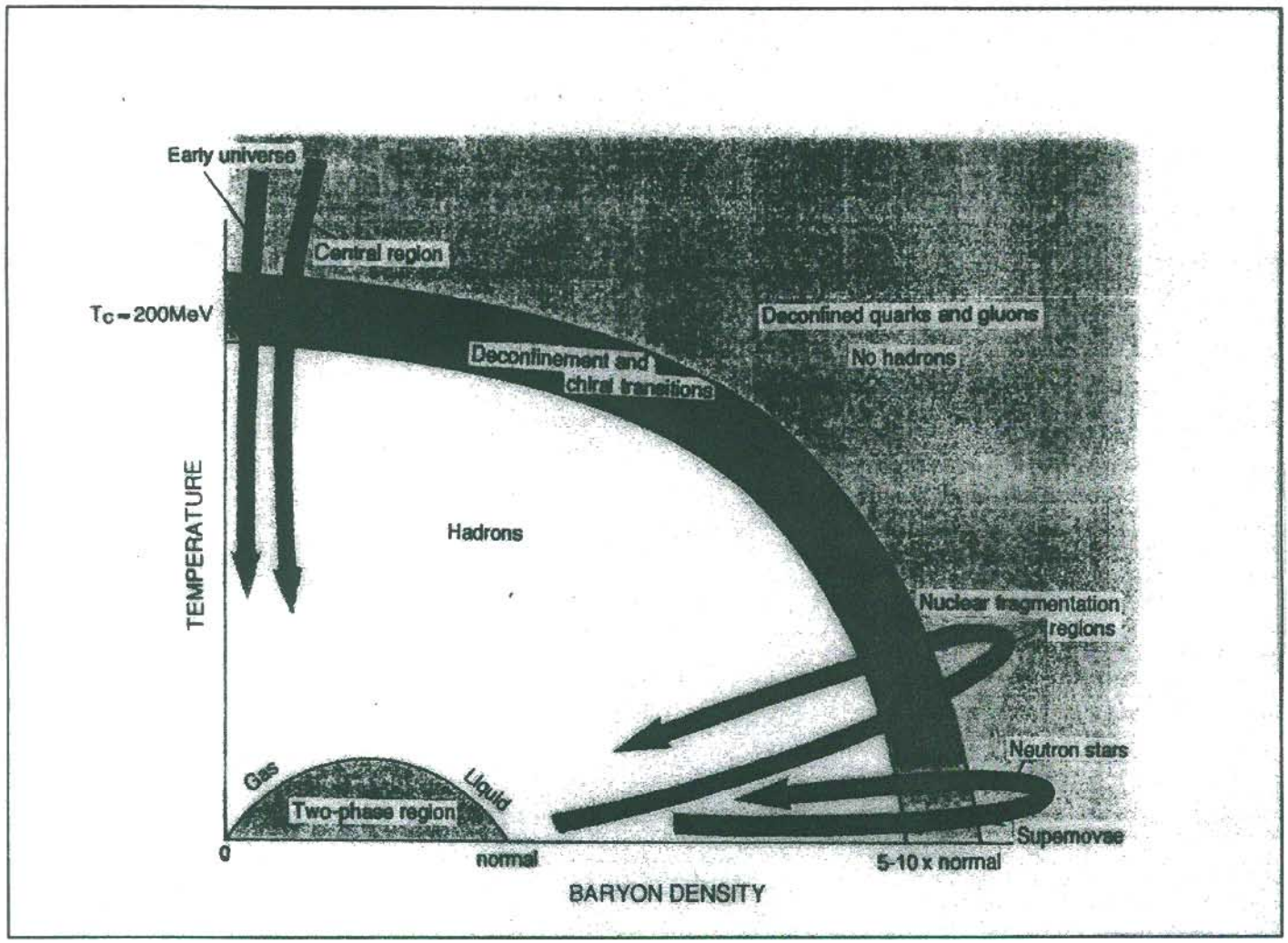
$$f(x) = 1 - 1.9585\,x + 1.2172\,x^2 - 0.24900\,x^3 + 0.012973\,x^4$$

$$f(0) = 1 \qquad f(1.17) = 0$$

$$a/R_b = r_s f(a_B/D_s) = 1, \quad \text{for "pressure ionization"}$$

PHASE DIAGRAM OF

EQUILIBRATED NUCLEAR MATTER



From G. Baym "The Quark Gluon Plasma" in *Elementary Processes in Dense Plasmas - The Proceedings of the Oji International Seminar,* edited by S. Ichimaru and S. Ogata (Addison-Wesley Pub. Co., Reading, Mass., 1995).

FUNDAMENTALS ON THE STATISTICAL THEORY OF STRONGLY COUPLED PLASMAS

S. Ichimaru, *STATISTICAL PLASMA PHYSICS*, I & II

(Westview, Boulder, CO, 2004)

DIELECTRIC FORMULATION

$$\Phi(\mathbf{k},\omega) = \frac{\Phi_{\mathrm{ext}}(\mathbf{k},\omega)}{\varepsilon(\mathbf{k},\omega)} \quad \underline{\mathbf{def}} \text{ Dielectric response function}$$

$$\delta\rho(\mathbf{k},\omega) = Z\,e\,\chi(\mathbf{k},\omega)\,\Phi_{\mathrm{ext}}(\mathbf{k},\omega) \quad \underline{\mathbf{def}} \text{ Polarizability}$$

$$\frac{1}{\varepsilon(\mathbf{k},\omega)} = 1 + Z^2 v(k)\,\chi(\mathbf{k},\omega)\,, \qquad v(k) = \frac{4\pi e^2}{k^2}$$

DYNAMIC STRUCTURE FACTOR

$$S(\mathbf{k},\omega) = \frac{1}{2\pi V}\int_{-\infty}^{\infty} \mathrm{d}t\,\langle \rho_{\mathbf{k}}(t'+t)\,\rho_{-\mathbf{k}}(t')\rangle_{t'}\,\exp(i\,\omega t)$$

FLUCTUATION-DISSIPATION THEOREM

$$S(\mathbf{k},\omega) = -\frac{\hbar N}{2\pi}\coth\left(\frac{\hbar\,\omega}{2\,k_{\mathrm{B}}T}\right)\mathrm{Im}\,\chi(\mathbf{k},\omega;[S(\mathbf{k},\omega)])$$

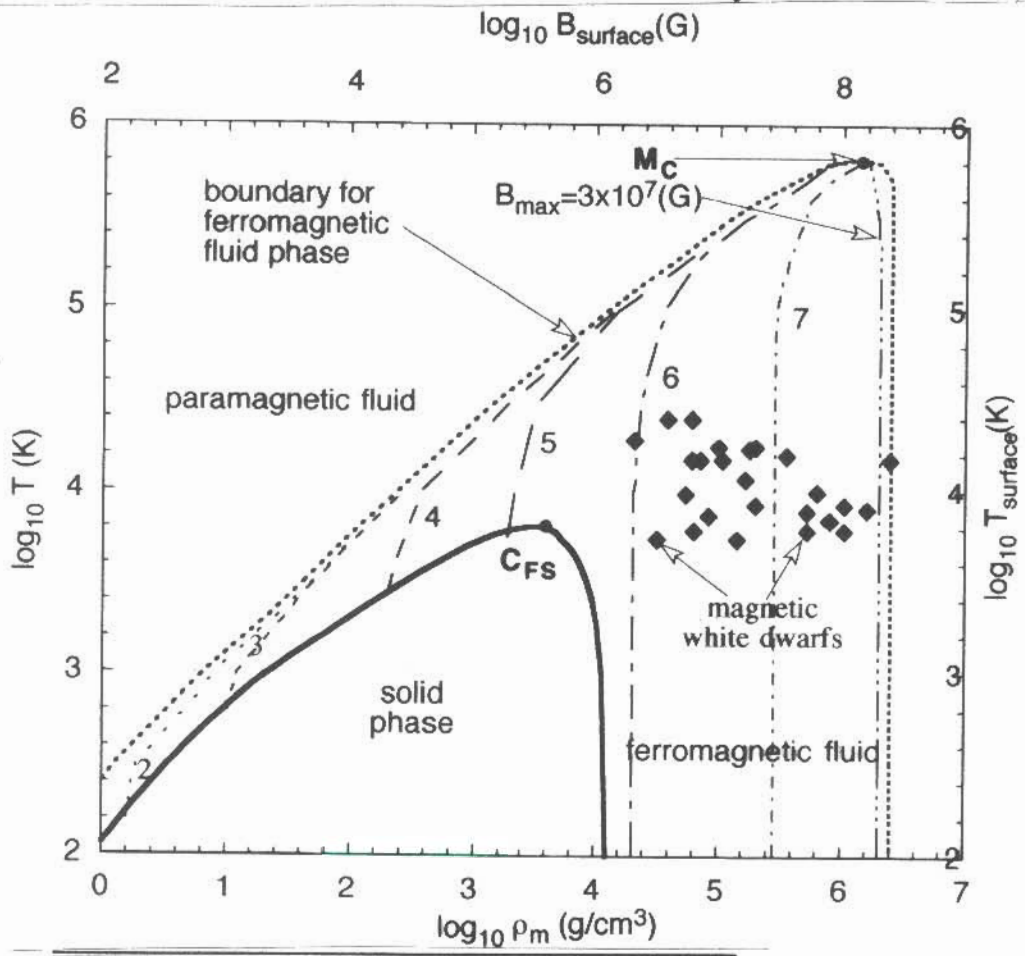
a self-consistent evaluation of $S(\mathbf{k},\omega)$

STATIC STRUCTURE FACTOR

$$S(k) = \frac{V}{N}\int_{-\infty}^{\infty} \mathrm{d}\omega\,S(\mathbf{k},\omega)$$

PHASE DIAGRAM OF METALLIC HYDROGEN AND MAGNETIC WHITE DWARFS

S. Ichimaru, Physics of Plasmas 8, 48 (2001)



	C_{FS}	M_C
ρ_m (g/cm^3)	4.0×10^3	1.4×10^6
R_s	161	22.4
T (K)	6.6×10^3	6.6×10^5
Θ	0.159	0.317
B_M (G)	2.6×10^5	8.0×10^6
Γ_s	517	34.4

strongly coupled degenerate

$$B_M = 4\pi\zeta\mu_p n$$

$$(\mu_p = 1.4\times10^{-23}\ \text{erg/G})$$

Numerals at the dashed and chain curves **denote the decimal exponents of B_M in gauss.** Diamond markers **plot observed surface-field strengths ($B_{surface}$) vs. surface temperatures ($T_{surface}$) for 25 magnetic white dwarfs.** [J. Weisheit, in *Elementary Processes in Dense Plasmas: Proc. Oji International Seminar* (eds. S. Ichimaru and S. Ogata) p. 61 (Addison-Wesley, Reading, MA, 1995)]

Is there a FERROMAGNETIC DOMAIN in the PHASE DIAGRAM of EQUILIBRATED NUCLEAR MATTER, that may account for the origin of NEUTRON-STAR MAGNETIC FIELDS ?

	ATOMIC PLASMAS	QCD MATTER
incoherent thermal limit at high temperatures	IONOZED GAS	Quark Gluon Plasma (QGP)
⇓	{increase in viscosity}	
lowering temperature and/or increase in density	FLUID METALLIC HYDROGEN	
{increase in coupling}	$\Gamma = e^2(4\pi n/3)^{1/3}/k_B T$	
⇓	{increase in viscoelasticity}	
⇓	{glassy state and crystalline nucleation}	
coherent limit at high densities (energies)	SOLID METALLIC HYDROGEN	Color Glass Condensate ? (CGC)

GENERALIZED VISCOELASTIC THEORY FOR GLASS TRANSITIONS IN STRONGLY COUPLED PLASMAS

S. Ichimaru and S. Tanaka, Phys. Rev. Lett. 56, 2815 (1986)

S. Tanaka and S. Ichimaru, Phys. Rev. A 35, 4743 (1987)

QUESTION: Can a "glassy plasma" be produced, when a one-component plasma (OCP) is supercooled below a Wigner-transition temperature, sufficiently fast to avoid homogeneous nucleation of crystals?

REMARKS: An OCP is probably a most difficult system to make a glass - - symmetric interaction; point charges; too "elusive" to lock themselves into a glassy state

PURPOSES OF THE STUDY

◊ **To develop a theory of dynamic correlation in the strongly coupled OCP through the generalized viscoelastic formalism, so that the MD simulation results for both $S(\mathbf{k},\omega)$ and η (shear viscosity) are well reproduced.**

◊ **To extend the theory to plasmas in a supercooled, metastable, fluid state ($\Gamma_m \approx 178 < \Gamma < \Gamma_c$ gt.t. $\sim 10^3$).**

◊ **To analyze a possibility of glass transition, revealed in the formation of a quasi-elastic peak in $S(\mathbf{k},\omega)$.**

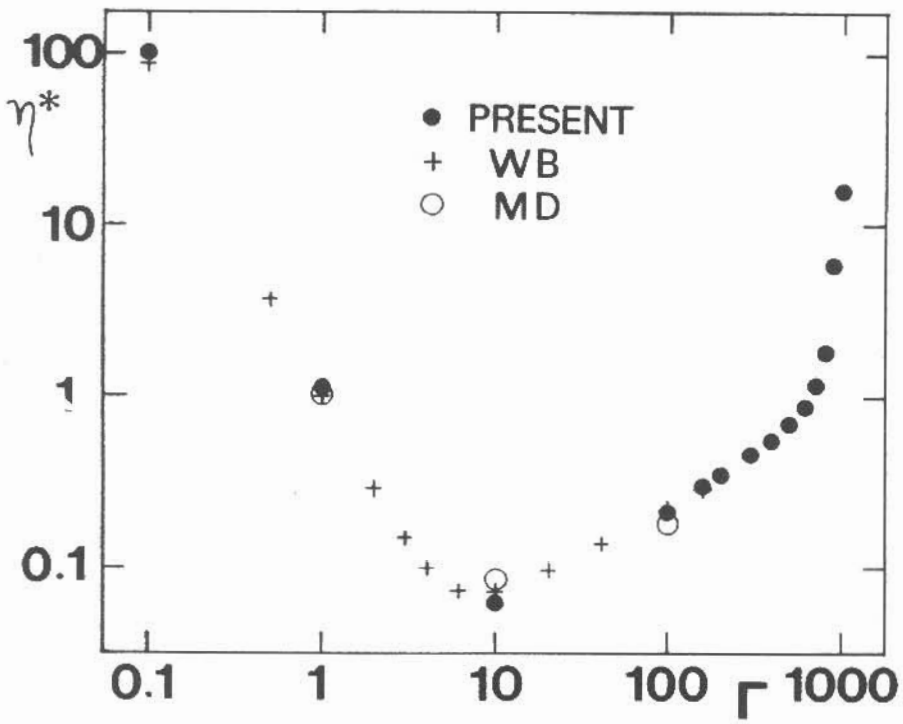
◊ To estimate the lifetime of the metastable fluid in terms of its self-diffusion and the probability of spontaneous nucleation of crystals - rate of "rapid quench" necessary to maintain the glassy state.

◊ To relate the theory with a possible experiment.

PRINCIPAL RESULTS

◊ Shear viscosity η in supercooled OCP fluids

$$\eta^* \equiv \eta / mn\omega_p a^2 \qquad \omega_p = \sqrt{4\pi n (Ze)^2 / m}$$



◊ If a Penning-trapped, non-neutral $^9Be^+$ plasma at $n \approx 10^{10}$ cm^{-3} is laser-cooled to $\Gamma = 900 - 1000$, within a time scale of $2 \times (10 - 10^5)$ sec, the resulting state will be a glass rather than a crystal.

DIELECTRIC RESPONSE IN VISCOELASTIC NAVIER-STOKES FORMALISM

$$\left[1+\tau_m \frac{\partial}{\partial t}\right]\left[\frac{\partial}{\partial \mathbf{r}}\Pi(\mathbf{r},t)-\frac{\partial}{\partial \mathbf{r}}P(\mathbf{r},t)+Zen\,\mathbf{E}(\mathbf{r},t)\right]$$

$$=-\eta\frac{\partial}{\partial \mathbf{r}}\cdot\frac{\partial}{\partial \mathbf{r}}\,\delta\mathbf{u}\,(\mathbf{r},t)-\left(\zeta+\frac{\eta}{3}\right)\frac{\partial}{\partial \mathbf{r}}\frac{\partial}{\partial \mathbf{r}}\cdot\delta\mathbf{u}\,(\mathbf{r},t)$$

δu: flow velocity P: pressure E: electric field

Π: momentum-flow tensor (isotropic part)

τ_m: viscoelastic relaxation time

η: shear viscosity ζ: bulk viscosity

$$\epsilon(k,\omega) = 1 - \frac{v(k)\,\chi_0(k,\omega)}{1+v(k)\,G(k,\omega)\,\chi_0(k,\omega)}$$

$$v(k) = 4\pi(Ze)^2/k^2$$

$$\chi_0(k,\omega) = \frac{(n/m)\,k^2}{\omega^2-(k_BT/m)\,k^2}$$

$$G(k,\omega) = \frac{k^2}{k_D^2}\left\{1-\frac{1}{k_BT}\left(\frac{\partial P}{\partial n}\right)_T+\frac{i\omega}{nk_BT}\frac{\eta_\ell}{1-i\omega\tau_m}\right\}$$

$$\eta_\ell = \frac{4}{3}\eta+\zeta \qquad \text{longitudinal viscosity}$$

We shall henceforth neglect ζ. $k_D^2 = 4\pi(Ze)^2 n/k_BT$

- Vieillefosse and Hansen (1975) have found ζ to be negligible compared to η in the OCP.

GENERALIZATION: hydrodynamics $\Rightarrow$ large k & ω

◊ $\chi_0(\mathbf{k},\omega) \to -\int d\mathbf{p}\, \frac{1}{\omega - \mathbf{k}\cdot\mathbf{v} + i0}\, \mathbf{k}\cdot \frac{\partial \mathbf{f}_0(\mathbf{p})}{\partial \mathbf{p}}$: Vlasov polarizability

◊ $G(k) = \frac{k^2}{k_D^2}\left[1 - \frac{1}{k_B T}\left(\frac{\partial P}{\partial n}\right)_T\right]$: compressibility sum rule

$$\Rightarrow\ G(k) = 1 + \frac{k_B T}{n\, v(k)}\left[1 - \frac{1}{S(k)}\right]$$

◊ $\lim_{\omega\to\infty} G(\mathbf{k},\omega) = I(k)$: third frequency-moment sum rule

$$I(k) = \frac{1}{n}\int \frac{d\mathbf{q}}{(2\pi)^3}\left[\frac{\mathbf{k}\cdot\mathbf{q}}{q^2} + \frac{\mathbf{k}\cdot(\mathbf{k}-\mathbf{q})}{|\mathbf{k}-\mathbf{q}|^2}\right]\frac{\mathbf{k}\cdot\mathbf{q}}{q^2}\left[1 - S(|\mathbf{k}-\mathbf{q}|)\right]$$

$$= -\frac{4}{15}\frac{U_{\text{int}}}{N k_B T}\frac{k^2}{k_D^2} + \cdots$$

$$\frac{\eta_\ell}{\tau_m} \to \frac{\eta_\ell(k)}{\tau_m(k)} = n k_B T \frac{k_D^2}{k^2}\left[G(k) - I(k)\right]$$

$$\boxed{G(\mathbf{k},\omega) = \frac{G(k) - i\omega\tau_m(k) I(k)}{1 - i\omega\tau_m(k)}}$$

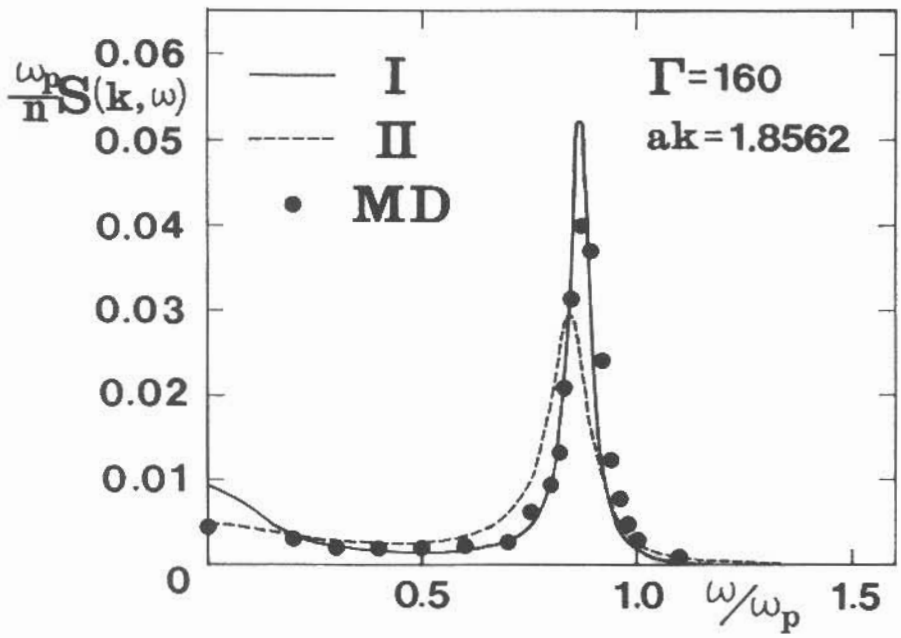
QUASI-ELASTIC PEAK

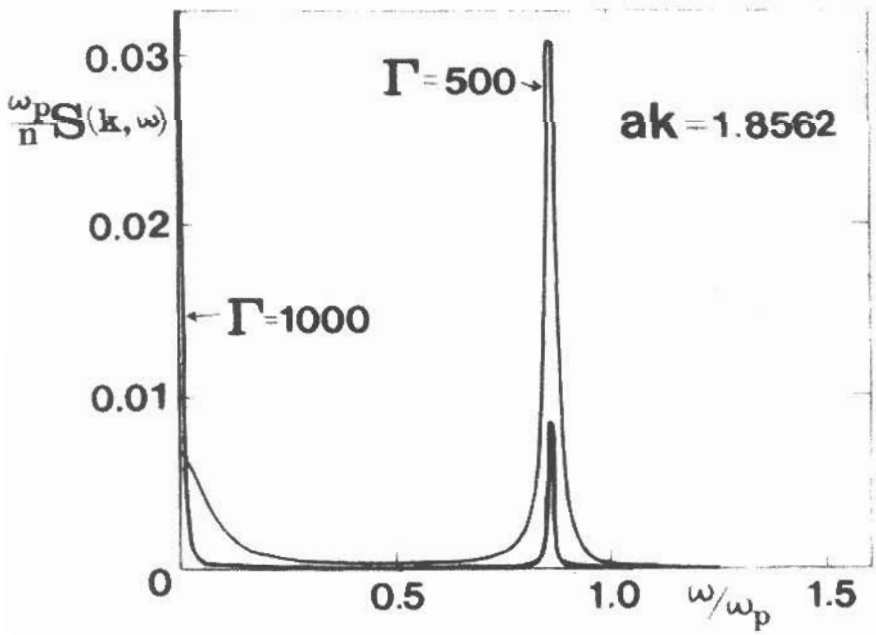
$$S(\mathbf{k},\omega) \underset{\text{low } \omega \,\&\, \mathbf{k}}{\to} \left(\frac{k}{k_D}\right)^4 \frac{\eta_\ell}{\pi k_B T\left[1 + (\tau_m\omega)^2\right]} \underset{\eta_\ell,\,\tau_m\to\infty\ (D\to 0)}{\to} n S(k)\,\delta(\omega)$$

$$\frac{\eta_\ell}{\tau_m} = n k_B T\left[1 - \frac{1}{k_B T}\left(\frac{\partial P}{\partial n}\right)_T + \frac{4}{15}\frac{U_{\text{int}}}{N k_B T}\right]$$

A "frozen state" $\langle \delta n(0,0)\, \delta n(\mathbf{r},t)\rangle$ may persist for $t \to \infty$.

DYNAMIC STRUCTURE FACTORS OF STRONGLY COUPLED OCP

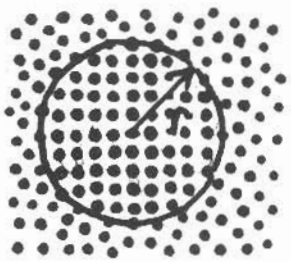




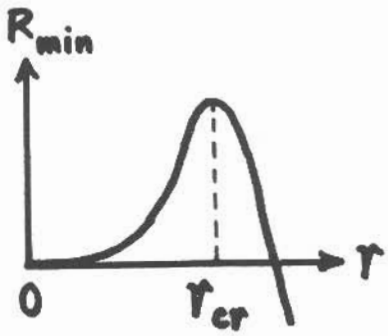
DIFFUSION TIME AND NUCLEATION TIME

PROBABILITY OF NUCLEATION

$$P(r) \propto \exp\left[-\frac{R_{min}(r)}{k_B T}\right]$$



$$\frac{R_{min}(r)}{k_B T} = \frac{4\pi\alpha}{k_B T} r^2 - \frac{\mu_F - \mu_c}{k_B T}\left(\frac{r}{a}\right)^3$$



$$4\pi\alpha a^2 = \gamma k_B T_m$$

$$\gamma \simeq 2$$

Turnbull (1950)

- critical radius r_{cr} of nucleation

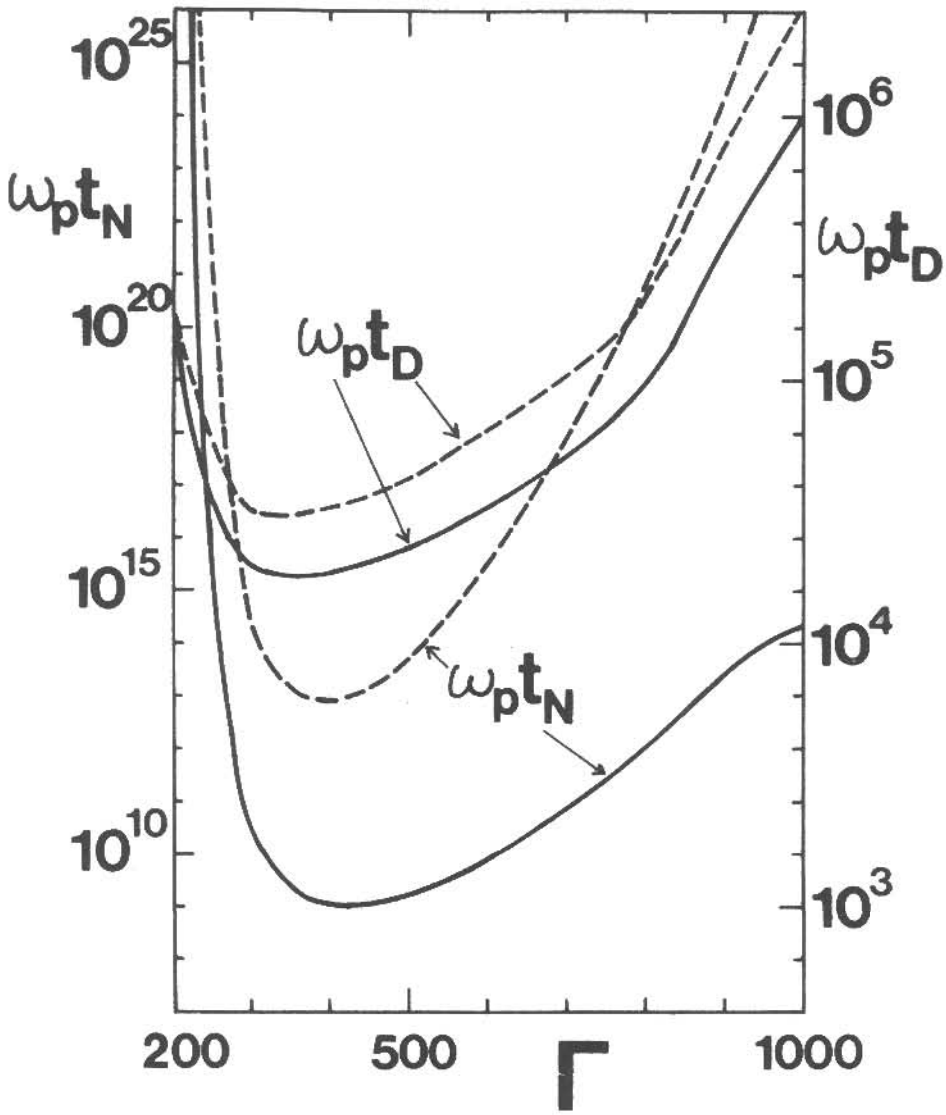
$$\left.\frac{d}{dr} R_{min}(r)\right|_{r=r_{cr}} = 0 \Rightarrow \frac{r_{cr}}{a} = \frac{4}{3}\frac{k_B T}{\mu_F - \mu_c}\frac{\Gamma}{\Gamma_m}$$

- diffusion time t_D

$$D^* \left(\equiv \frac{D}{a^2 \omega_p}\right) = \frac{2}{27(R/a)\Gamma\eta^*}$$

$$\omega_p t_D \equiv \omega_p\left(\frac{r_{cr}^2}{D}\right) = \left(\frac{r_{cr}}{a}\right)^2 \frac{1}{D^*}$$

- nucleation time t_N

$$\omega_p t_N \equiv \omega_p t_D \exp\left[\frac{R_{min}(r_{cr})}{k_B T}\right]$$

TIMES OF NUCLEATION AND DIFFUSION