Long Baseline Neutrino Physics

Manfred Lindner
Max-Planck-Institute, Heidelberg

TASI 2006

Theoretical Advanced Study Institute
In Elementary Particle Physics
University of Colorado, June 4-30, 2006

1. Introduction

The Birth of the Neutrino

W. Pauli

Before 1930: neutron → proton $+e^-$

2-body decay → monoenergetic spectrum expected

experiment: continuous β-decay spectrum

Pauli: energy-momentum conservation

→ postulate new particle

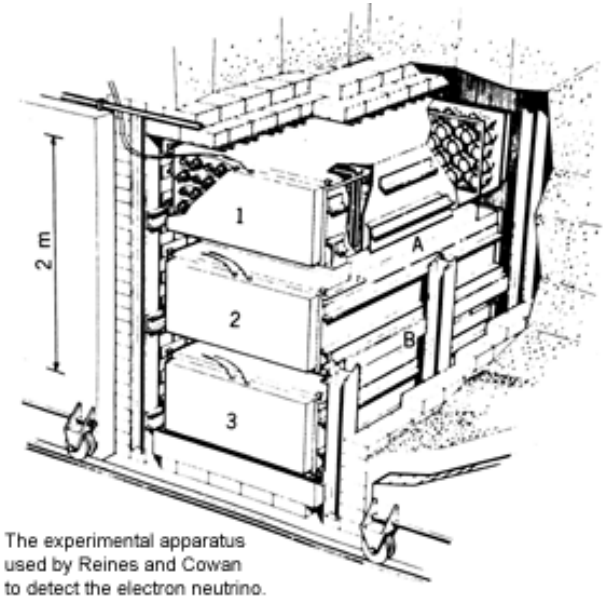
→ invisible, since Q=0

→ spin ½, ...

Letter to Tübingen Dec. 1930 …

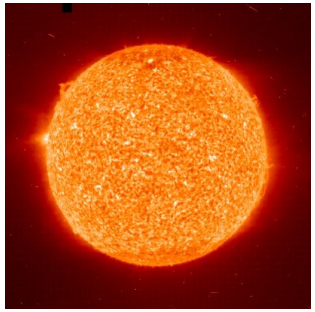
… will never be detected

- Cowen & Reines 1954-56 project ``poltergeist''
  → detection of reactor neutrinos
  → Nobel price for F. Reines 1995

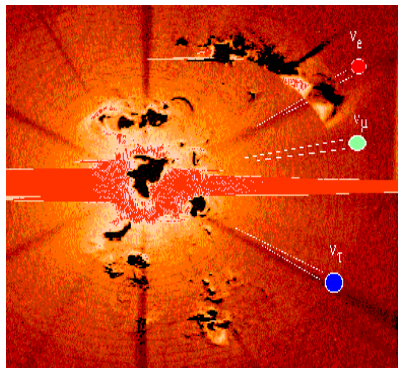


The experimental apparatus used by Reines and Cowan to detect the electron neutrino.

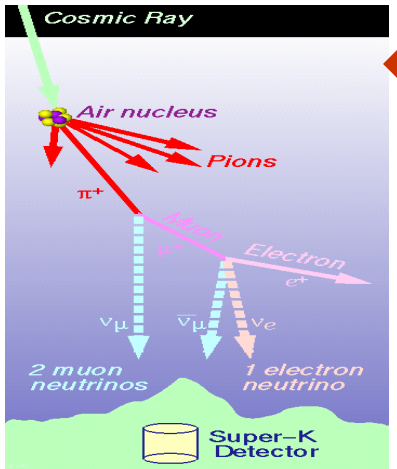
Neutrino Sources & New Physics



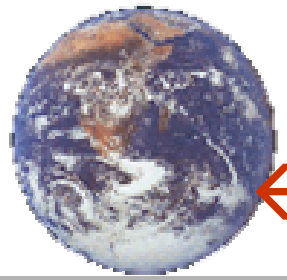
←Sun



←Cosmology

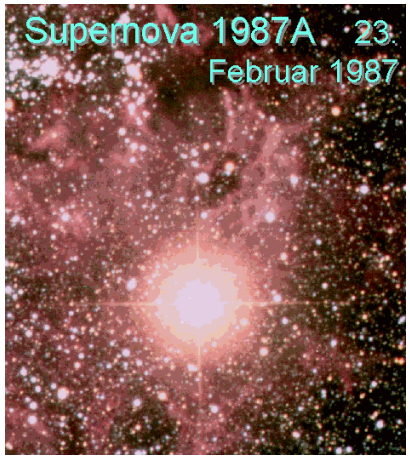


←Atmosphere



←Earth

Astronomy: →
Supernovae
UHE ν's
...



Reactors→

Accelerators→
Laboratories

Physics Beyond the Standard Model

Theoretical arguments:

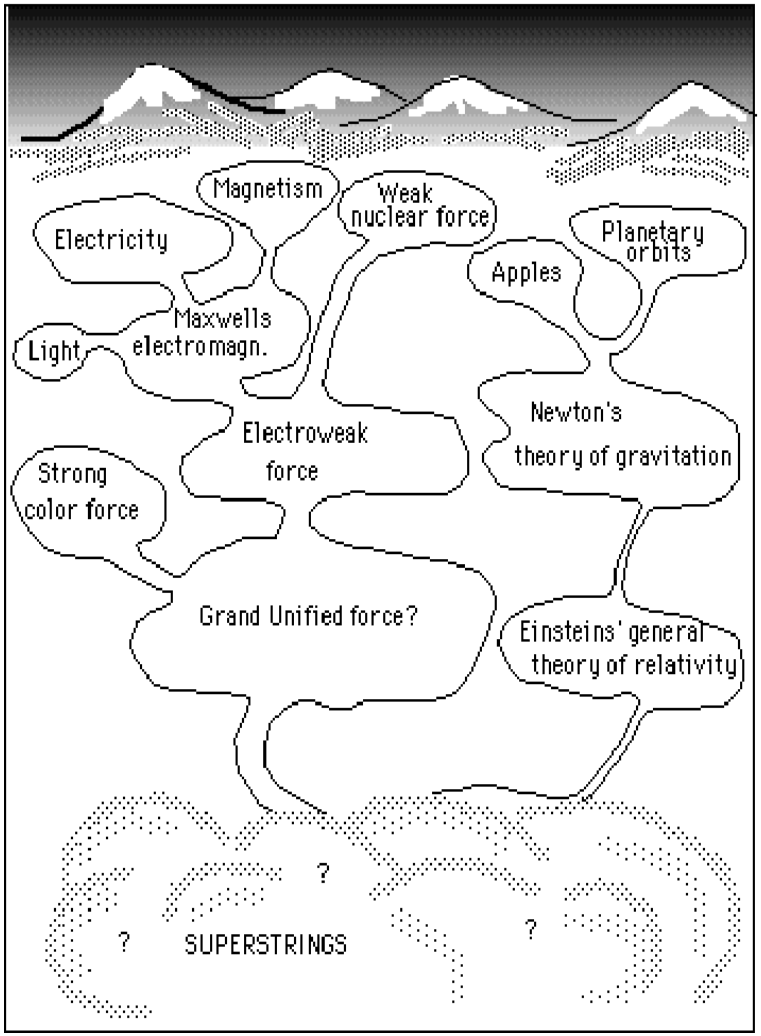
SM does not exist without cutoff
Higgs-doublett = only simplest extension
Gauge hierarchy problem
Why: 3 generations , fermion representations
Many parameters (9+? Masses, 4+? Mixings)
Charge quantisation, unification: GUTs, ...,
Gravitation, ...

2 directions: Sym. breaking & Flavour

Experimental facts:

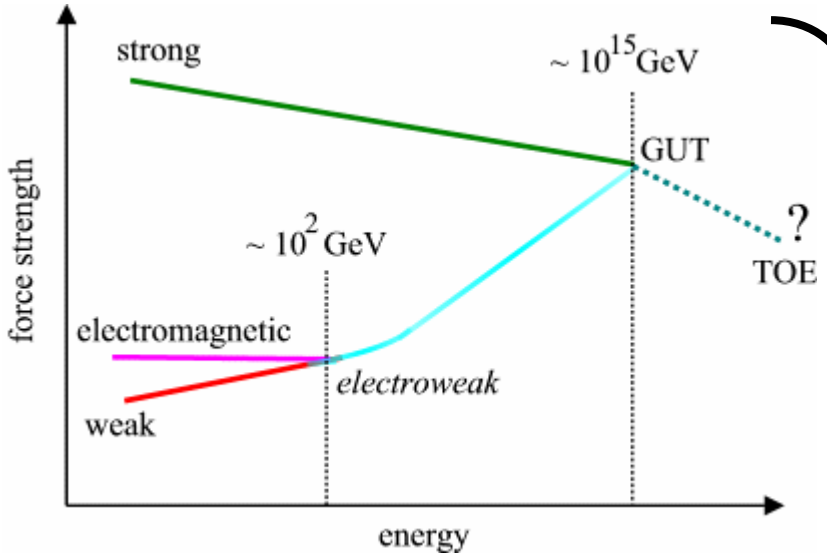
- Dark Matter & Dark Energy exist!
- Neutrino masses have been detected!
- Baryon asymetry of the universe ←→ $m_\nu > 0$

→ physics beyond the standard model
→ results ←→ implications for theory



New Physics Beyond the SM

gauge bosons →



experimental facts:

Dark Matter
Dark Energy
baryon asymetry ←→ $m_\nu > 0$
neutrino masses & mixings
→ precision

Higgs →

gauge hierarchy problem:

$\delta m_H^2 \sim \Lambda^2$

SUSY ~TeV

quarks leptons →

flavour problem: 3 generations
many parameters (m_i ,mixings)
unification into GUTs

$m_\nu = (m^D)^T M_R^{-1} m_D$

$\sim\Lambda_{GUT}$ +seesaw

astrophysics & cosmology

2. Neutrino Masses

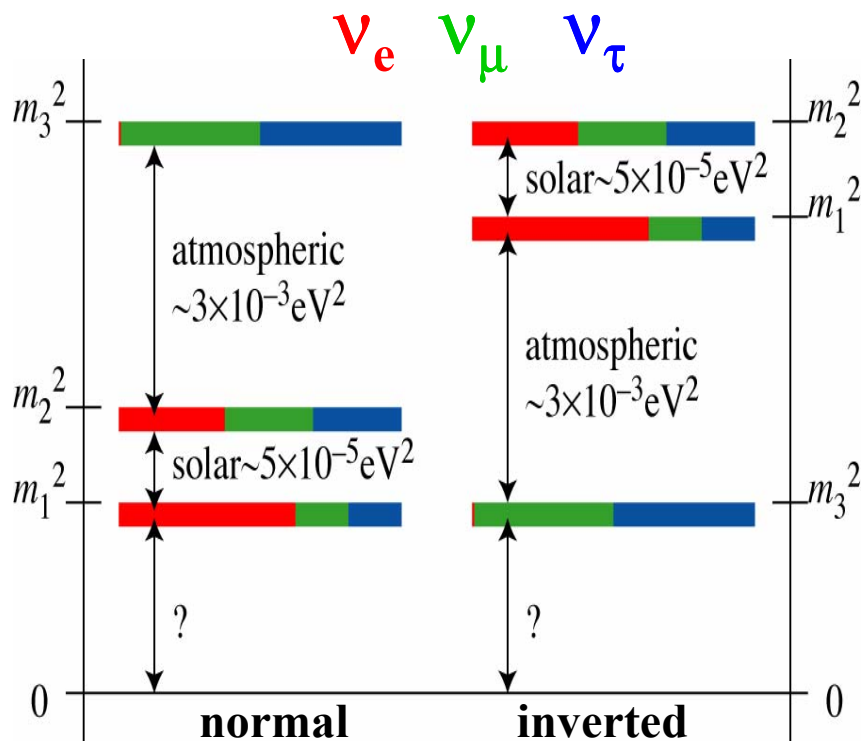
Parameters for 3 Light Neutrinos

mass & mixing parameters: m_1 , Δm^2_{21}, $|\Delta m^2_{31}|$, $\mathrm{sign}(\Delta m^2_{31})$

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \mathrm{diag}(e^{i\alpha}, e^{i\beta}, 1)$$

questions:

- ➔ Dirac or Majorana
- ➔ absolute mass scale: m_1
- ➔ mass ordering: $\mathrm{sgn}(\Delta m^2_{31})$
- ➔ how small is θ_{13}, θ_{23} maximal?
- ➔ leptonic CP violation
- ➔ LSND ←➔ sterile neutrino(s)
- ➔ L/E pattern of oscillations



4 Ways to measure Masses & Mixings

<u>4 different methods:</u>

- **kinematical**
- **lepton number violation ←→ Majorana**
- **astrophysics & cosmology**
- **oscillations**

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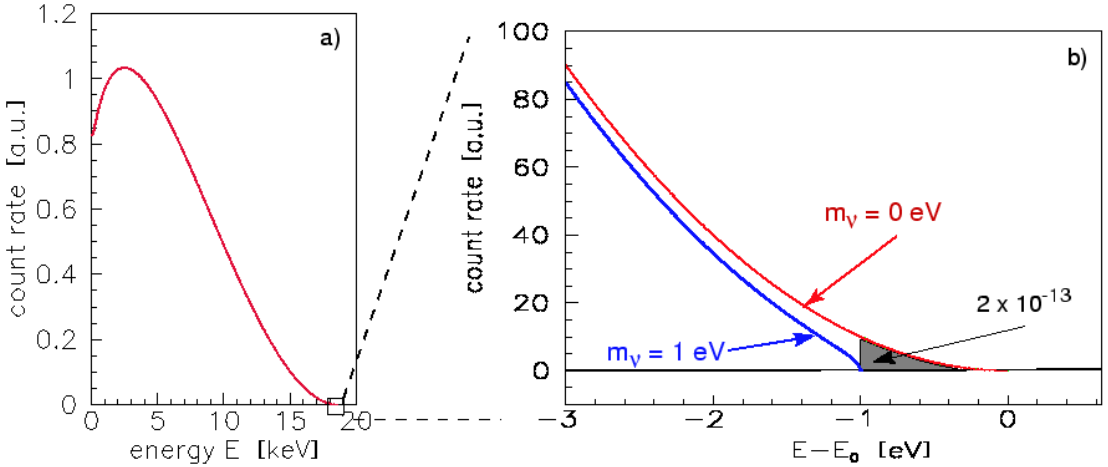
Summary:Kinematical Mass Determination

Relativistic kinematics:

$$E^2 = p^2 + m^2; \quad \sum p_i^\mu = \sum p_f^\mu$$

Endpoint of decays:

Tritium $\rightarrow He^3 + e^- + \overline{\nu}_e$



Bounds:

"Elektron-Neutrino":	$m < 2.2$ eV	(Mainz, Troitsk)
"Muon-Neutrino":	$m < 170$ keV	
"Tau-Neutrino":	$m < 15.5$ MeV	

Sensitivity $\Leftrightarrow$ **degenerate ν-spectrum**

$\Rightarrow$ **Oscillations:** $\Delta m_{ij}^2 \ll m_i^2 \Rightarrow$ $\boxed{\sum m_i^2 |U_{ei}|^2 < (2.2 \text{ eV})^2}$

Future: **KATRIN:** $\longrightarrow 0.25$ **eV** $\Leftrightarrow$ cosmological bound (WMAP; 2df galaxy survey, Lyman α, ...)

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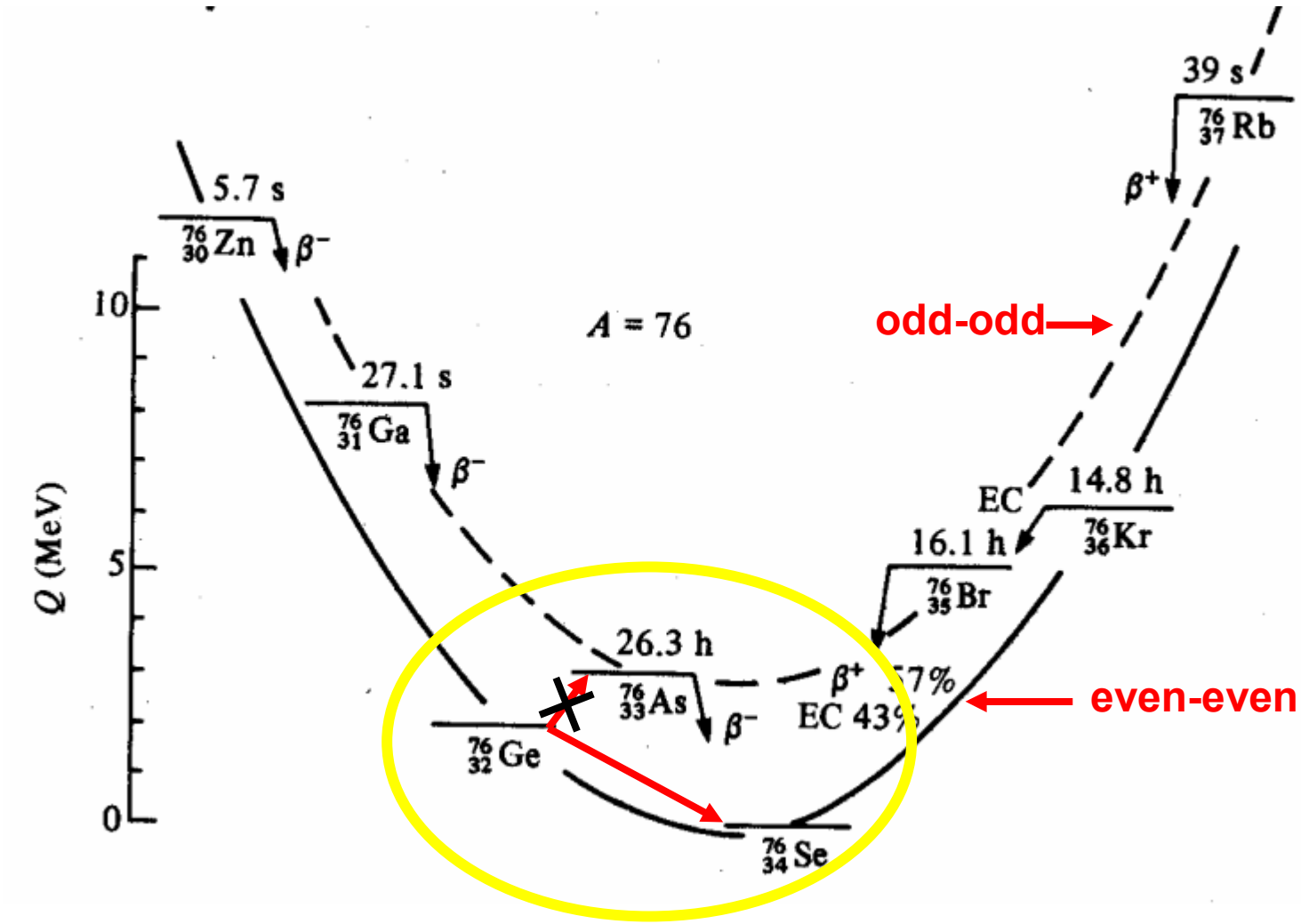
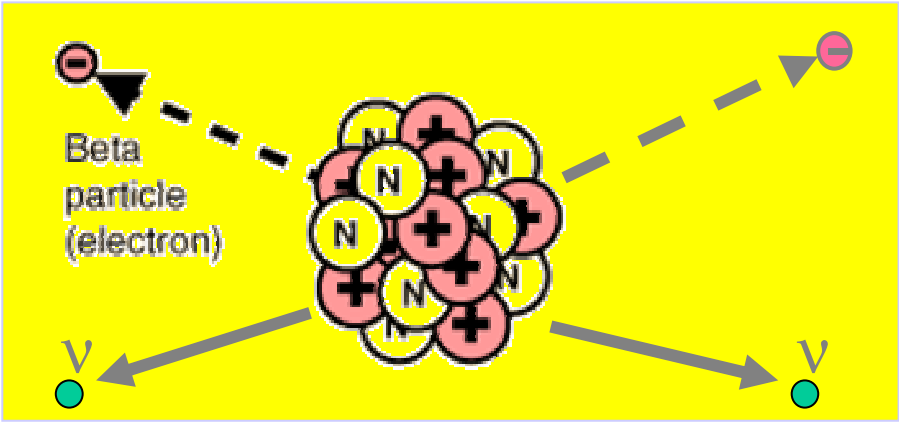
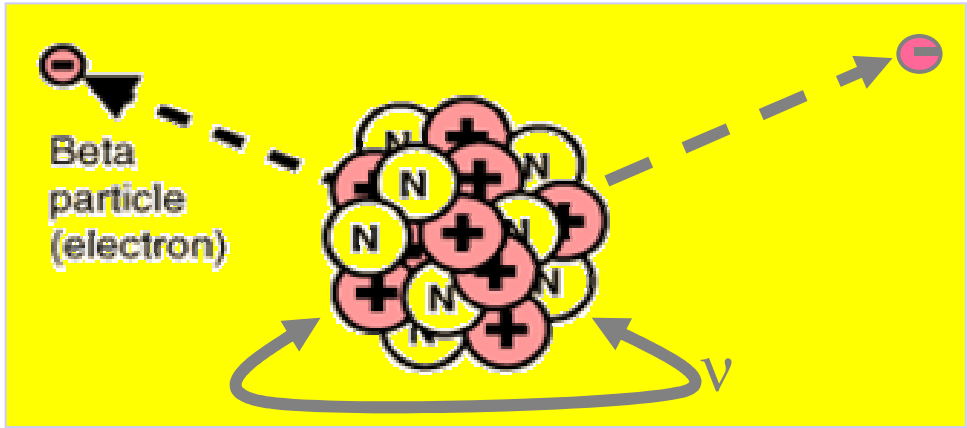
Mass Parabolas



Ground states of even-even nuclei: 0^+

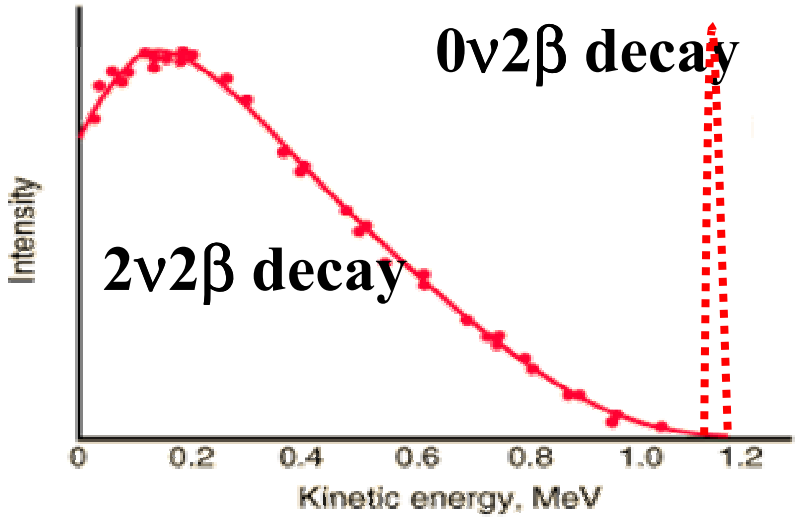
Lepton Number Violation & 0ν2β Decay





2ν2β decay **^{76}Ge: τ=1.5 × 10^{21} y**

Majorana ν ➔ 0ν2β decay



0ν2β

⇕

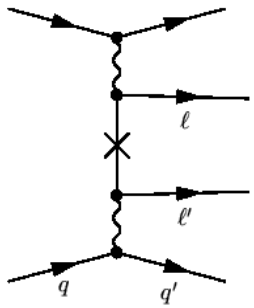
Majorana neutrinos

⇕

lepton number violation

Neutrino-less Double β-Decay

$0\nu 2\beta$-decay $\propto$

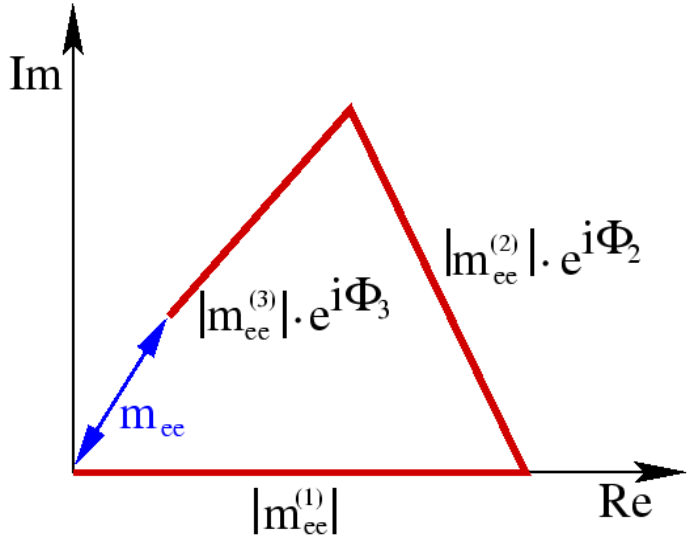


$\propto |\langle m_{ee}\rangle| = |\sum m_i U_{ei}^2|$ **$\leq$ 0.35 eV ?**

Heidelberg-Moscow experiment

$$m_{ee} = |m_{ee}^{(1)}| + |m_{ee}^{(2)}| \cdot e^{i\Phi_2} + |m_{ee}^{(3)}| \cdot e^{i\Phi_3}$$

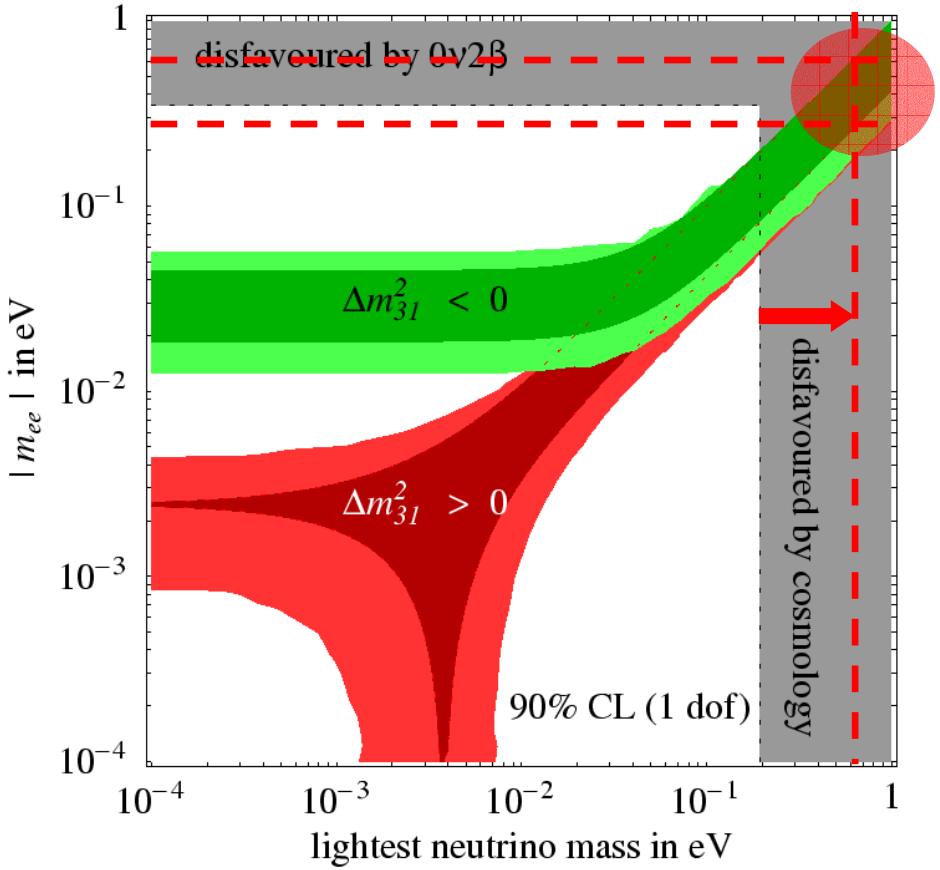
$$\begin{aligned} |m_{ee}^{(1)}| &= |U_{e1}|^2 m_1 \\ |m_{ee}^{(2)}| &= |U_{e2}|^2 \sqrt{m_1^2 + \Delta m_{21}^2} \\ |m_{ee}^{(3)}| &= |U_{e3}|^2 \sqrt{m_1^2 + \Delta m_{31}^2} \end{aligned}$$



solar $\Rightarrow$ $|U_{e1}|^2, |U_{e2}|^2, \Delta m_{21}^2$ atmosph. $\Rightarrow$ $|\Delta m_{31}^2|$ CHOOZ $\Rightarrow$ $|U_{e3}|^2 < 0.05$

➔ **free parameters: m_1 , sign(Δm^2_{31}) , CP-phases Φ_1, Φ_2**

- sign of Δm^2_{31}
- partial compensations:
  variation of Φ_i, $|U_{e3}|^2$, ...
- model dependent:
  LR, SUSY R-partity violation, ...
- nuclear matrix elements ...
- $|m_{ee}| \Rightarrow$ one CP-phase comb.
  $\Rightarrow$ 2nd phase?



➔better experiments are important ⬅➡ L-violating effects

<u>sensitivity aim:</u> **|<m>| ≥ 10^{-2} eV ⬅➡ inverted hirarchy &** $\sqrt{|\Delta m^2_{31}|} \approx 0.05\text{eV}$

New Experiments

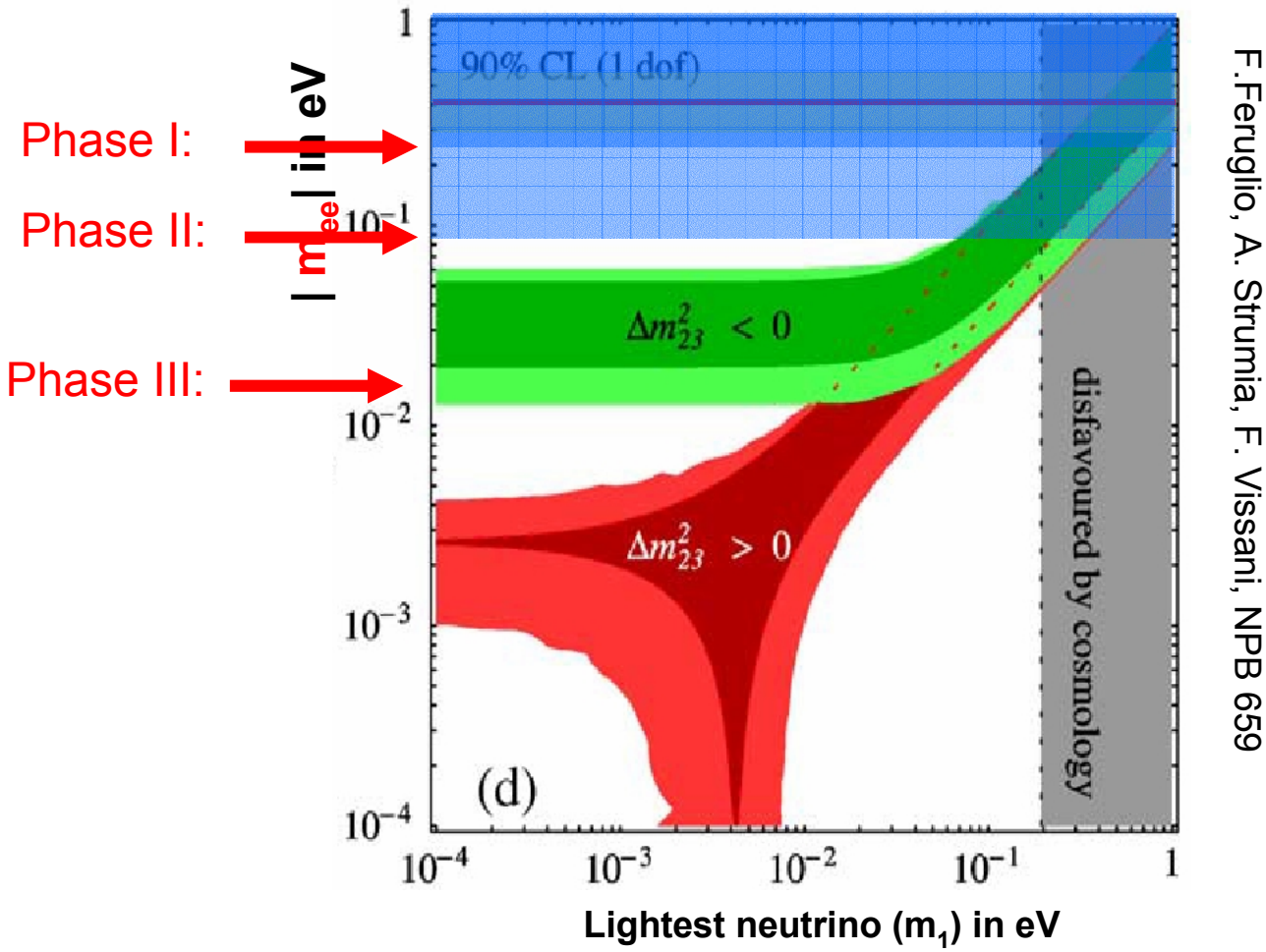
Approved:

- CUORE (Te-130)
- GERDA (Ge-76)

Planned:

- Majorana
- EXO
- MOON
- Super-NEMO
- ...

Phases and Physics reach of GERDA



F.Feruglio, A. Strumia, F. Vissani, NPB 659

ME from Faessler➔ P-I: 0.31 eV, P-II: 0.12 eV; P-III: 0.02 eV

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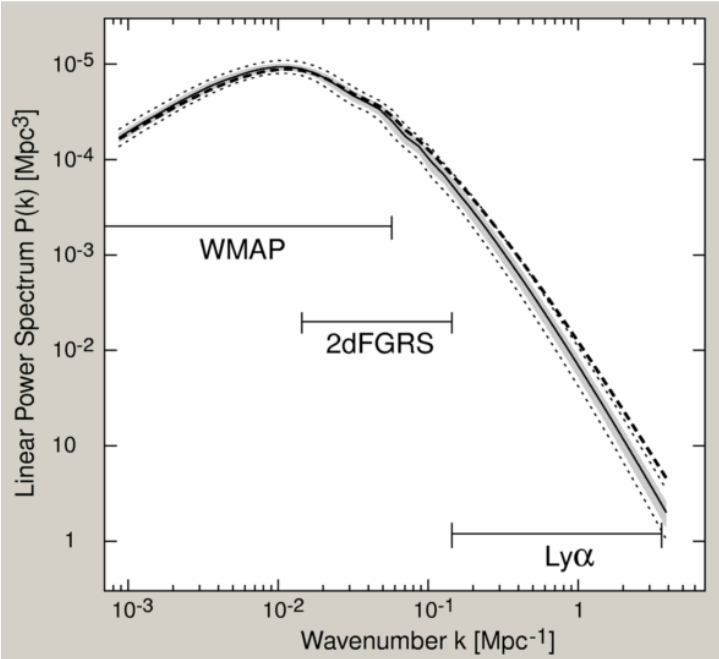
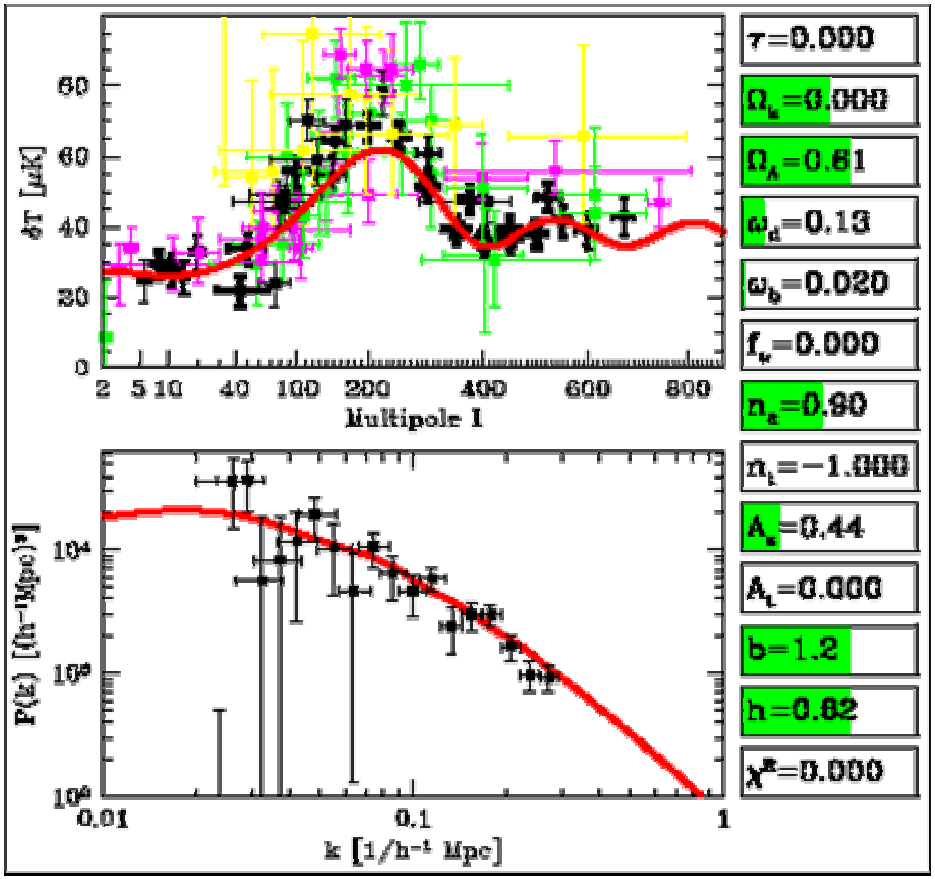
- **astrophysics & cosmology → see next section**
- **oscillations**

Cosmology and Neutrino Mass

- **ν's are hot dark matter ➔ smears structure formation on small scales**





- **WMAP+2dFGRS + Lyα**

➔ **mass bound: $\Sigma m_\nu < 0.7 \ldots 1.2$ eV**

- **3 degenerate neutrinos**

➔ **$m_\nu < 0.4$ eV future improvements: ~factor 5...**

- **comparison with KATRIN, 0ν2β, LSND**

$f_\nu = \Omega_\nu / \Omega_{matter}$

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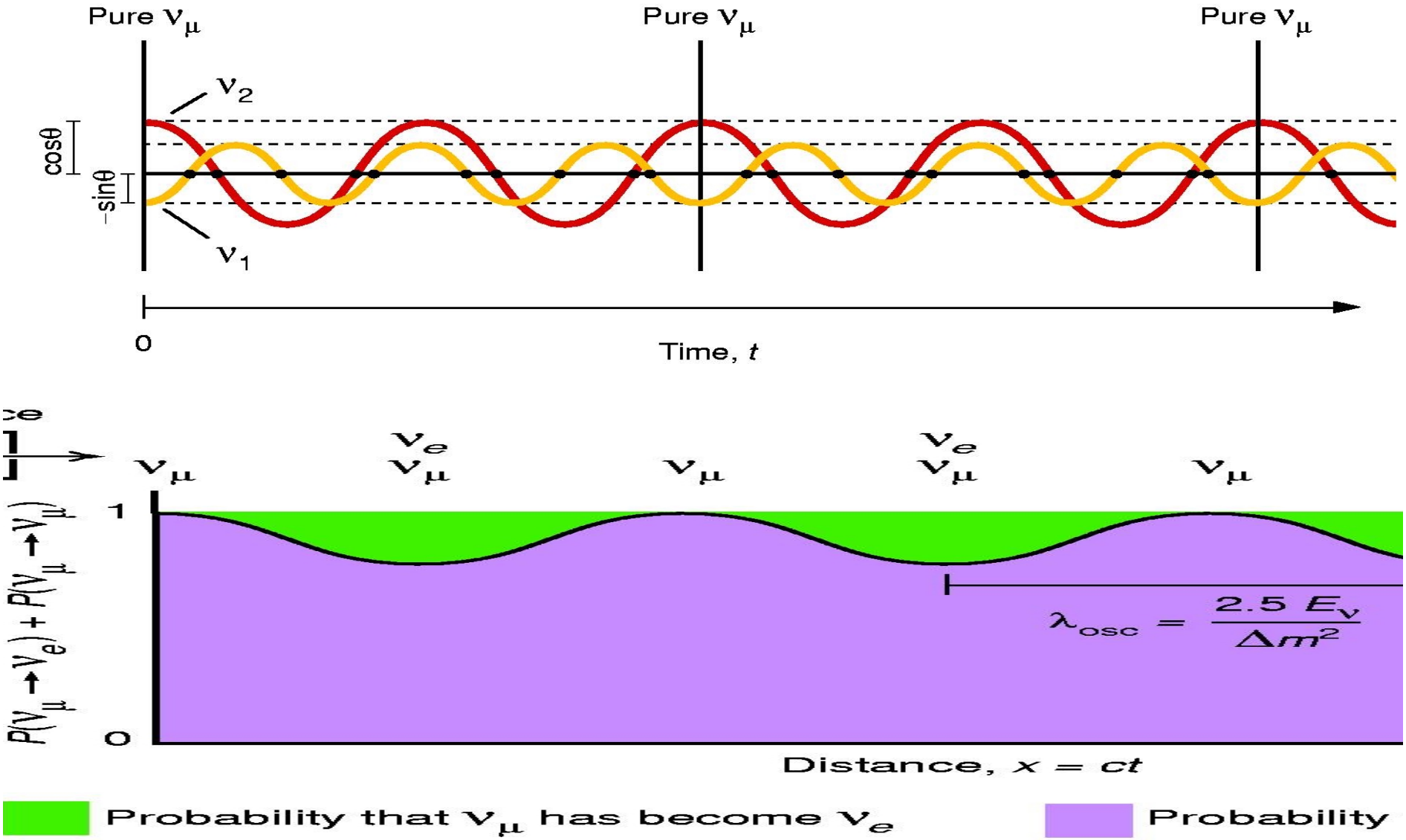
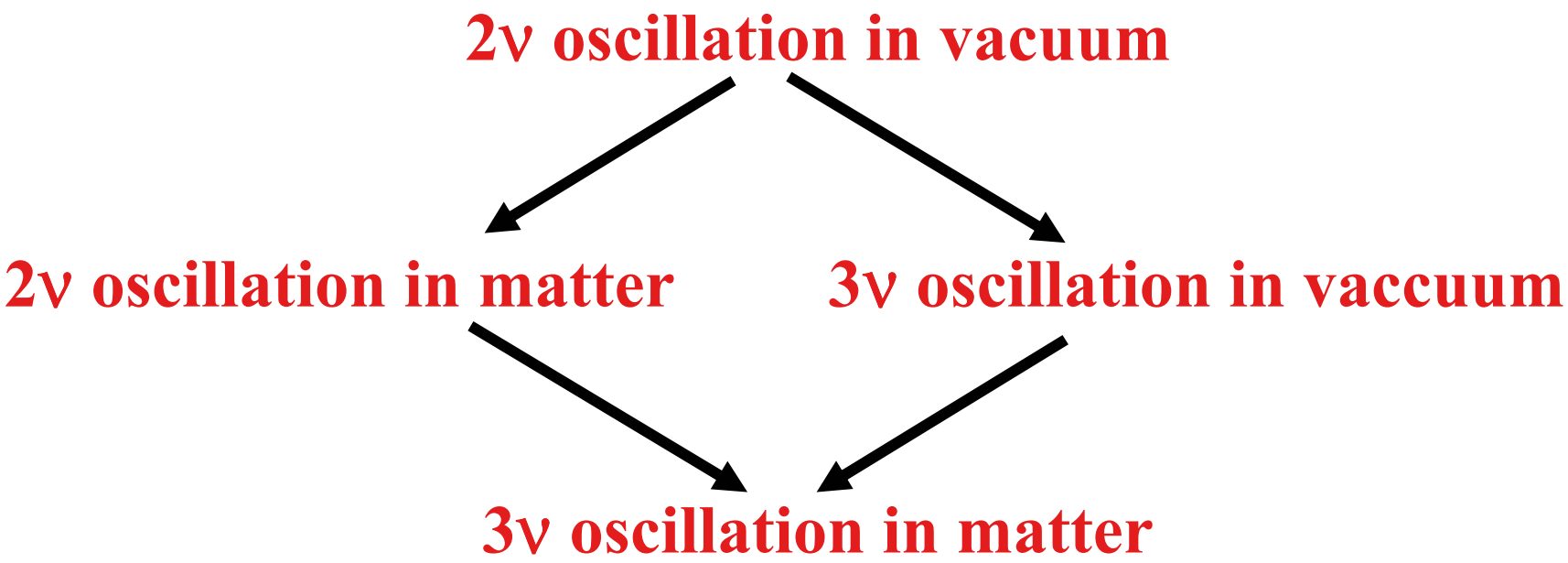
Neutrino Oscillations



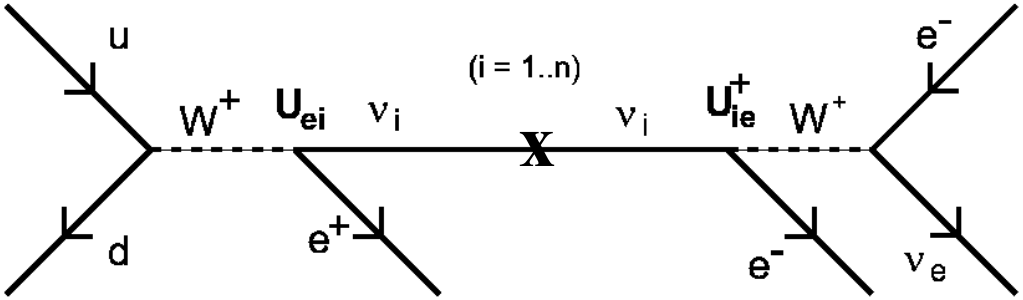
Different Levels of Complexity

Oscillations can involve 2 or 3 neutrinos in vacuum or matter
➔4 possibilities:



Neutrino Oscillations for N=2

Production
Propagation
Detection



$p + p \rightarrow d + \nu_e + e^+$ electron scattering

2 Flavours ν_e, ν_μ:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \cdot \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

flavour states — mixing matrix · mass eigenstates

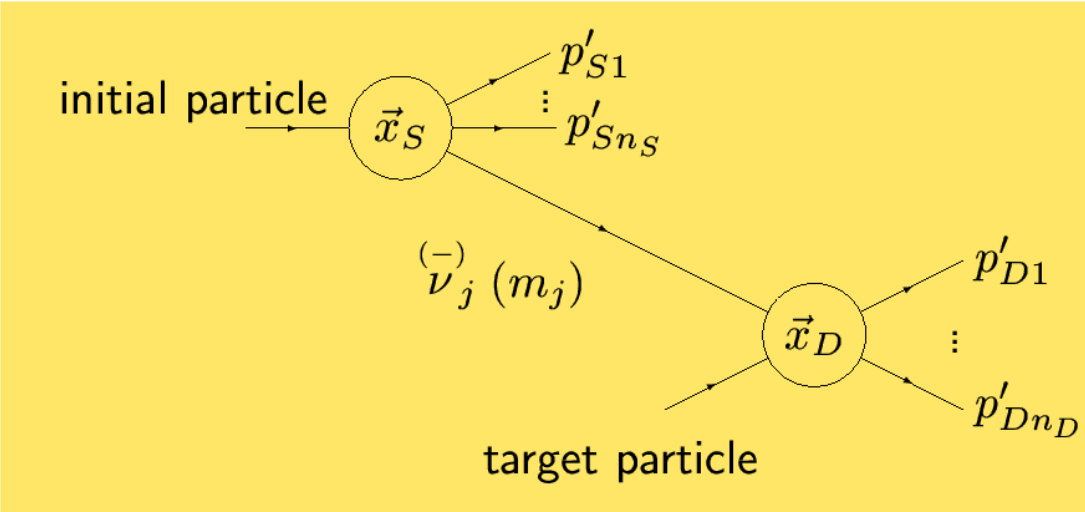
- Production as flavour eigenstate from W-exchange
- Detection via W-exchange $\equiv$ projection on flavour state
- Propagation as mass eigenstate $\Rightarrow$ use mixing matrix at vertex
- Is a simple QM treatment justified?

Neutrino Oscillations in QFT

QFT description of a neutrino produced in a decay at rest:

- localized source and detector
- $L = |\vec{x}_D - \vec{x}_S|$
- initial particle at rest
- target particle at rest

... DIF similar



Transition probability from Feynman diagram:

$$\left\langle P_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta} \right\rangle_{\mathcal{P}} \propto \int dP_S \int_{\mathcal{P}} \frac{d^3 p_{D1}}{2E_{D1}} \cdots \frac{d^3 p_{Dn_D}}{2E_{Dn_D}} \left| \mathcal{A}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta} \right|^2$$

Neutrino propagator in ultra-relativistic limit ➔ e^{ipx}

QFT derivation: Avoids confusion (factors of two…, etc.)

Kinematics: Equal Energy or equal Momenta?

- Consider e.g. pion decay at rest: $\pi^+ \to \mu^+ + \nu_\mu$
- Neutrino energy and momentum determined by energy-momentum conservation

$$p_k^2 = \frac{m_\pi^2}{4}\left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2 - \frac{m_k^2}{2}\left(1 + \frac{m_\mu^2}{m_\pi^2}\right) + \frac{m_k^4}{4\,m_\pi^2}$$

$$E_k^2 = \frac{m_\pi^2}{4}\left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2 + \frac{m_k^2}{2}\left(1 - \frac{m_\mu^2}{m_\pi^2}\right) + \frac{m_k^4}{4\,m_\pi^2}$$

- **For $E \gg m$:** $\boxed{p_k \simeq E - \xi\,\frac{m_k^2}{2E}\,, \qquad E_k \simeq E + (1-\xi)\,\frac{m_k^2}{2E}}$

with $E = \frac{m_\pi}{2}\left(1 - \frac{m_\mu^2}{m_\pi^2}\right) \simeq 30\,\mathrm{MeV}\,, \qquad \xi = \frac{1}{2}\left(1 + \frac{m_\mu^2}{m_\pi^2}\right) \simeq 0.8$

$\Rightarrow$ **neither equal energy nor equal momentum!**

$$e^{ipx} \Rightarrow \quad \boxed{p_\mu \cdot x^\mu = p_k L - E_k T = -\frac{m_k^2 L}{2E}} \text{ for } L = T$$

$\Rightarrow$ ξ drops out of the oscillation formulae $\Leftrightarrow$ naive treatment correct

- Shown for π-decay, but valid in general (DIF, N-body, ..., different ξ)

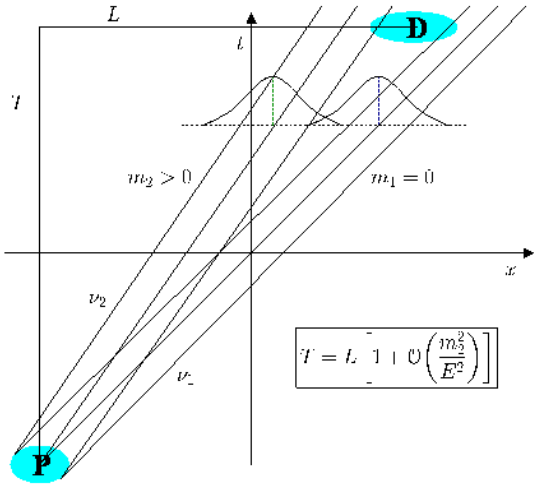
Localized Source and Detector:

- Feynman rules for particles of given momentum ($\simeq$ on-shell)
  $\Rightarrow$ this corresponds to an infinitely extended (non-localized) plane wave

- Localized source (wave packet) and detector in space-time $(\Delta x_S, \Delta t_S)$, $(\Delta x_D, \Delta t_D)$:

  $\Rightarrow$ Source: Fourier superposition of momenta with $\sigma_S^2 \simeq min(\Delta x_S^2, \Delta t_S^2)$
  $\Rightarrow$ Detector: projection on a superposition of momenta with $\sigma_D^2 \simeq min(\Delta x_D^2, \Delta t_D^2)$

- Different masses and momenta $\Rightarrow$ dispersion $\Rightarrow$ loss of coherence



- separation of wave packets: $\boxed{\Delta x_{ij} = \frac{|\Delta m_{ij}^2| L}{2E^2}}$

- coherence condition: $\Delta x_{ij} < \sigma := \sqrt{\sigma_S^2 + \sigma_D^2}$

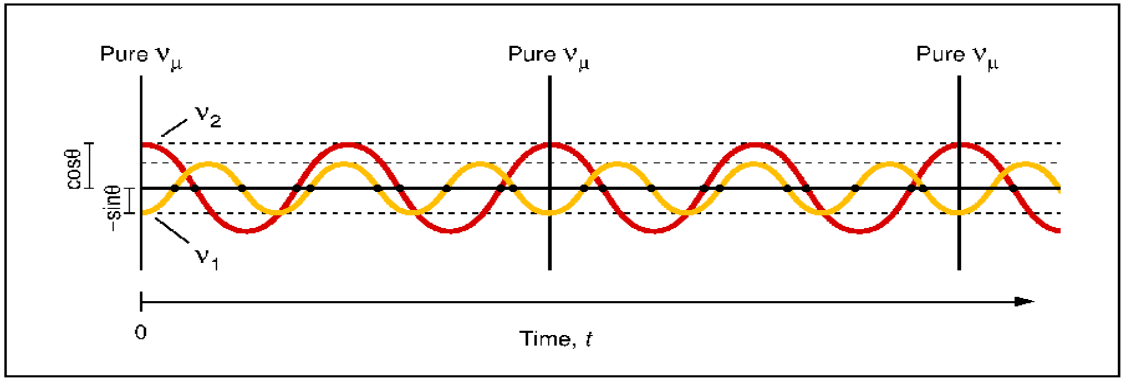
- coherence length: $L < \frac{2E^2 \sigma}{|\Delta m_{ij}^2|}$

- Oscillations from QFT $\Rightarrow$ $P_{\nu_\alpha \to \nu_\beta}(L,T) = \left| \sum_k U_{\alpha k}^* \, e^{i p_k L - i E_k T} \, U_{\beta k} \right|^2$
- Very interesting QM effects (σ, decay)

Two Neutrino Oscillations

2 Neutrinos: ν_e, ν_μ

$$|\nu_e(0)\rangle = \cos\theta\,|\nu_1\rangle + \sin\theta\,|\nu_2\rangle$$
$$|\nu_\mu(0)\rangle = -\sin\theta\,|\nu_1\rangle + \cos\theta\,|\nu_2\rangle$$



$$|\nu_\mu(t)\rangle = -\sin\theta\ \exp[-\frac{iE_1 t}{\hbar}]\ |\nu_1\rangle + \cos\theta\ \exp[-\frac{iE_2 t}{\hbar}]\ |\nu_2\rangle$$

$$E_i = \sqrt{p_i^2 + m_i^2} \quad \xrightarrow{p_i = p \gg m_i} \quad \simeq p + \frac{m_i^2}{2p} \quad \simeq p + \frac{m_i^2}{2E}$$

$$L = c \cdot t \qquad \Delta m^2 = m_2^2 - m_1^2 \Rightarrow \quad E_2 - E_1 = \frac{\Delta m^2}{2E}$$

2ν-transition-probability:

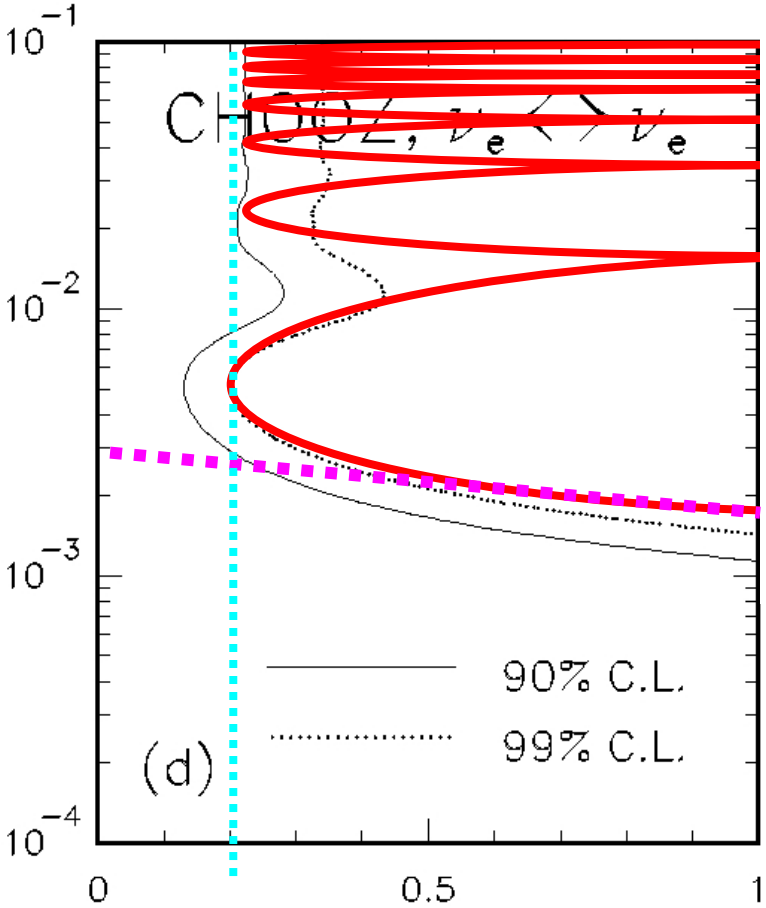
$$P(\nu_\mu \to \nu_e) = |\langle \nu_\mu(t)|\nu_e(0)\rangle|^2 = \sin^2 2\theta \cdot \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

ν_e, ν_μ, ν_τ ➔ 9 oscillation channels for neutrinos

ν_e, ν_μ, ν_τ ➔ 9 channels for anti-neutrinos (assuming 3ν !)

2 Flavour Exclusion Limits

$$P(\nu_\mu \to \nu_e) = \sin^2(2\theta) \cdot \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$



small Δm^2:

$$P \simeq \sin^2(2\theta)\left(\frac{\Delta m^2 L}{4E}\right)^2 < P_{exp.}$$

$$\Rightarrow \sin(2\theta) \cdot \Delta m^2 < \frac{\sqrt{P}4E}{L}$$

$\sin(\Delta m^2 L/4E) = 1$:

$$P \simeq \sin^2(2\theta) < P_{exp.}$$

$$\sin(2\theta) < \sqrt{P}$$

Inbetween: Oscillation + energy smearing

Other Oscillation Phenomena

Possibilities:	**Requirements:**
• different neutrino flavours	✓
• $B_0 \leftrightarrow \overline{B_0}$ $K_0 \leftrightarrow \overline{K_0}$	✓
• neutrinos $\leftrightarrow$ anti-neutrinos	✓ new physics
• neutron $\leftrightarrow$ anti-neutron	✓ new physics
• different charged lepton flavours	✓ new basis…
• within up and down quark flavours	✓ distance…

Crutial:

- identical quantum numbers
- mixing

N ≥ 2 with CP Violation and Matter Effects

Precision: $N = 2$ description insufficient $\Rightarrow$ modifications

- $2 \to 3$ **neutrino framework** $\Rightarrow$ **more parameters & CP effects**
- **MSW: parameter mapping in matter**

Quantum mechanical treatment for N ultra-relativistic neutrinos:

$$P_{\nu_{e_l} \to \nu_{e_m}}(L/E) = \left| \sum_j U_{mj} U_{lj}^* \exp\left(\frac{-i\ m_j^2 L}{2E} \right) \right|^2$$

- masses m_j associated to mass eigenfields ν_j
- flavour eigenstates ν_{e_l}, ν_{e_m}, ...
- neutrino energy E
- source and detector distance L
- unitary mixing matrix U

General 3x3 neutrino mixing matrix:

has (up to) 3 angles + 1 Dirac-phase +2 Majorana-phases: θ_{12}, θ_{23}, θ_{13}, δ, Φ_1, Φ_2

$$U_{MNS} = U \cdot \mathbf{diag}\left(\exp[\mathbf{i\Phi_1}], \exp[\mathbf{i\Phi_2}], \mathbf{1}\right)$$

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- Only U enters in neutrino oscillations: $J_{ij}^{e_l e_m} := U_{li}U_{lj}^* U_{mi}^* U_{mj}$
- All oscillation frequencies show up: $\Delta_{ij} := \frac{\Delta m_{ij}^2 L}{4E} = \frac{(m_i^2 - m_j^2)L}{4E}$

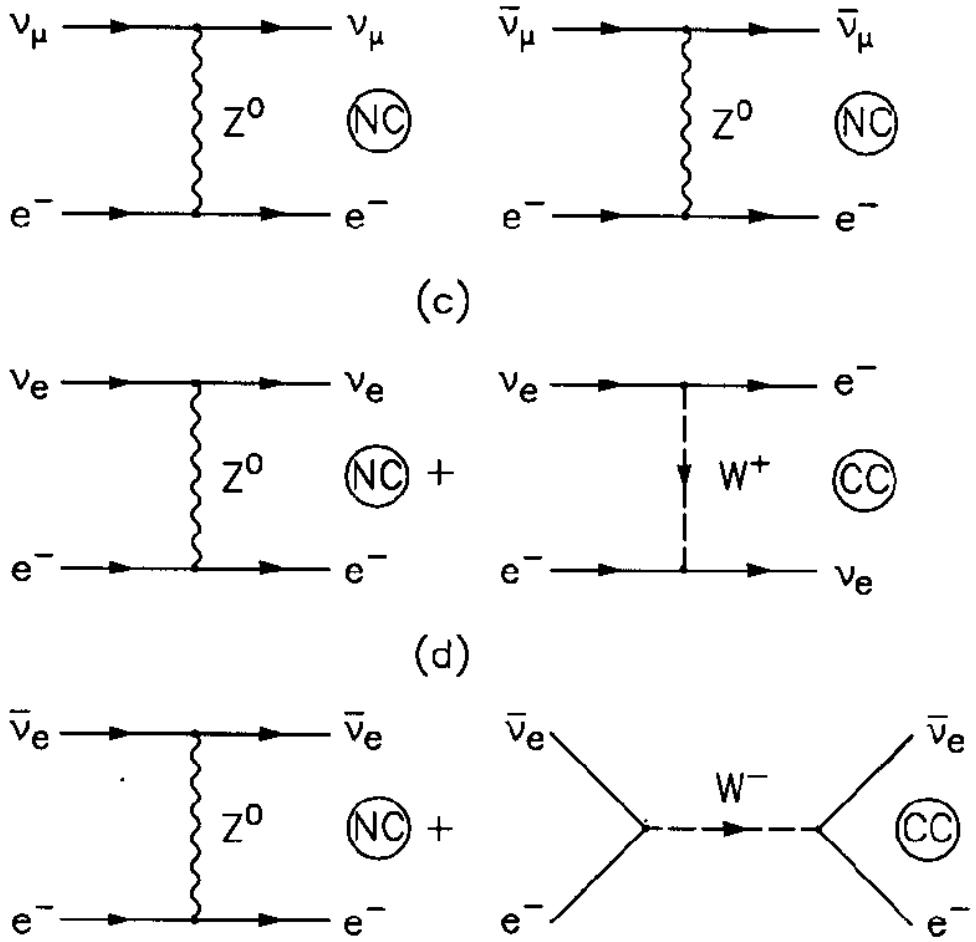
$$P(\nu_{e_l} \to \nu_{e_m}) = \underbrace{\delta_{lm} - 4\sum_{i>j} \mathrm{Re} J_{ij}^{e_l e_m} \sin^2 \Delta_{ij}}_{P_{CP}} \underbrace{-2\sum_{i>j} \mathrm{Im} J_{ij}^{e_l e_m} \sin 2\Delta_{ij}}_{P_{\not{C}P}}$$

$\Rightarrow$ **Leptonic CP violation, genuine 3 flavour and matter effects**

➔ **lengthy 3 flavour oscillation formulae**

Matter Effects and MSW Resonance

Mikheyev-Smirnov-Wolfenstein: coherent forward scattering



$\mathcal{L}_{NC}$ = flavour universal

$\mathcal{L}_{CC} = \sqrt{2} G_F n_e \quad \Leftrightarrow \quad$ only ν_e

MSW-resonance energy (Δm^2_{31})

Earth: $\mathbf{E_{res}} \simeq \mathbf{10}$ GeV

for beams
dominated by average density

$$\rho = \rho_{\text{average}} + \delta\rho$$

$$\mathcal{L}_{CC} = -\frac{G_F}{\sqrt{2}} \left[\bar{e}\gamma_\mu(1-\gamma_5)\nu_e\right] \left[\bar{\nu}_e\gamma^\mu(1-\gamma_5)e\right]$$

Fierz Transf.

$$= -\frac{G_F}{\sqrt{2}} \left[\bar{e}\gamma_\mu(1-\gamma_5)e\right] \left[\bar{\nu}_e\gamma^\mu(1-\gamma_5)\nu_e\right]$$

$$\boxed{H_{eff}(\nu) = \langle \mathcal{L}_{eff} \rangle_e = \bar{\nu}_e \; V_{eff} \; \nu_e}$$

$$\langle \bar{e}\gamma_o e \rangle = \langle e^+ e \rangle = N_e$$

$$\langle \bar{e}\gamma_i e \rangle ; \langle \bar{e}\gamma_o\gamma_5 e \rangle ; \langle \bar{e}\gamma_o\gamma_5 e \rangle \simeq 0$$

$$\boxed{V_{eff}(\nu_e) = \sqrt{2} G_F N_e}$$

Hamiltonian for 3 Neutrino Oscillations in Flavour Basis:

$$\mathbf{H} = H_0 + \delta\mathbf{H}_{CC} + \delta\mathbf{H}_{NC} = \frac{1}{2E}\mathbf{U}\begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix}\mathbf{U}^{\mathbf{T}} + \frac{1}{2E}\begin{pmatrix} \mathbf{A}+\mathbf{A}' & 0 & 0 \\ 0 & \mathbf{A}' & 0 \\ 0 & 0 & \mathbf{A}' \end{pmatrix}$$

- $\mathbf{A} = \pm\frac{\mathbf{2\sqrt{2}G_F Y \rho E}}{\mathbf{m_n}} = 2V \cdot E$ $\quad \nu \oplus \text{matter}$ and $\overline{\nu} \oplus \text{anti} - \text{matter} \Rightarrow$ "+"
- $Y = e^-$/nucleon ρ =matter density m_n =nucleon mass
- Overall phases drop out: $m_i \to m_i - m_1 \Rightarrow$ m_1 and A' can be eliminated

$$\mathbf{H}' = \frac{1}{2E}\mathbf{U}\begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m_{21}^2 & 0 \\ 0 & 0 & \Delta m_{31}^2 \end{pmatrix}\mathbf{U}^{\mathbf{T}} + \frac{1}{2E}\begin{pmatrix} \mathbf{A} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Note: A' can not be eliminated if sterile neutrinos are present

- In good approximation $\Delta m^2_{12} \simeq 0$
- $\mathbf{U}$ can be written as a sequence of rotations: $\mathbf{U} = R_{23} R_{13} R_{12}$

$$\mathbf{H}'' = \tfrac{1}{2E}\mathbf{R_{23}}\mathbf{R_{13}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \Delta m^2_{31} \end{pmatrix} \mathbf{R_{13}^{-1}}\mathbf{R_{23}^{-1}} + \tfrac{1}{2E}\mathbf{R_{23}} \begin{pmatrix} \mathbf{A} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \mathbf{R_{23}^{-1}}$$

$$= \tfrac{1}{2E}\mathbf{R_{23}} \left[\mathbf{R_{13}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \Delta m^2_{31} \end{pmatrix} \mathbf{R_{13}^{-1}} + \begin{pmatrix} \mathbf{A} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right] \mathbf{R_{23}^{-1}}$$

$$= \tfrac{1}{2E}\mathbf{R_{23}} \left[\begin{pmatrix} \bullet & 0 & \bullet \\ 0 & 0 & 0 \\ \bullet & 0 & \bullet \end{pmatrix} + \begin{pmatrix} \mathbf{A} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right] \mathbf{R_{23}^{-1}}$$

$$= \tfrac{1}{2E}\mathbf{R_{23}} \left[\mathbf{R'_{13}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \Delta(m^2_{31})' \end{pmatrix} (\mathbf{R'_{13}})^{-1} \right] \mathbf{R_{23}^{-1}}$$

$\Rightarrow$ re-insert R_{12} $\Rightarrow$ U' $\Rightarrow$ parameter mapping in 1-3 subspace

- **Different** mappings for **neutrinos** and **antineutrinos**
- 1-3 sub-space mapping like in 2 neutrino case
- Relevant quantitiy $\boxed{C_\pm^2 = \left(\frac{A}{\Delta m_{31}^2} - \cos 2\theta_{13}\right)^2 + \sin^2 2\theta_{13}}$
- MSW resonance condition for $\theta_{13} \simeq 0$: $\Delta m_{31}^2 = A = 2VE = \pm \frac{2\sqrt{2} G_F Y \rho E}{m_n}$
- Effective parameters in matter:

$$\begin{aligned}
\sin^2 2\theta'_{13} &= \frac{\sin^2 2\theta_{13}}{C_\pm^2} \\
\Delta m_{31,m}^2 &= \Delta m^2 C_\pm \\
\Delta m_{32,m}^2 &= \frac{\Delta m^2 \ (C_\pm + 1) + A}{2} \\
\Delta m_{21,m}^2 &= \frac{\Delta m^2 \ (C_\pm - 1) - A}{2}
\end{aligned}$$

- Corrections due to
  - $\Delta m_{12}^2 \neq 0$
  - non-constant matter profiles $\Rightarrow$ solve Schrödinger equation

Analytic Approximations

- $\Delta = \Delta m^2_{31} L/4E$
- qualitative understanding $\Rightarrow$ expand in $\alpha = \Delta m^2_{21}/\Delta m^2_{31}$ and $\sin^2 2\theta_{13}$
- matter effects $\hat{A} = A/\Delta m^2_{31} = 2VE/\Delta m^2_{31}$; $V = \sqrt{2} G_F n_e$

$$P(\nu_\mu \to \nu_\mu) \approx 1 - \cos^2\theta_{13} \sin^2 2\theta_{23} \sin^2\Delta + 2\,\alpha\, \cos^2\theta_{13} \cos^2\theta_{12} \sin^2 2\theta_{23} \Delta \cos\Delta$$

$$\begin{aligned} P(\nu_e \to \nu_\mu) \approx\ & \sin^2 2\theta_{13} \sin^2\theta_{23} \frac{\sin^2((1-\hat{A})\Delta)}{(1-\hat{A})^2} \\ \pm\ & \sin\delta_{\rm CP}\ \alpha \sin 2\theta_{12} \cos\theta_{13} \sin 2\theta_{13} \sin 2\theta_{23} \sin(\Delta) \frac{\sin(\hat{A}\Delta)\sin((1-\hat{A})\Delta)}{\hat{A}(1-\hat{A})} \\ +\ & \cos\delta_{\rm CP}\alpha \sin 2\theta_{12} \cos\theta_{13} \sin 2\theta_{13} \sin 2\theta_{23} \cos(\Delta) \frac{\sin(\hat{A}\Delta)\sin((1-\hat{A})\Delta)}{\hat{A}(1-\hat{A})} \\ +\ & \alpha^2 \sin^2 2\theta_{12} \cos^2\theta_{23} \frac{\sin^2(\hat{A}\Delta)}{\hat{A}^2} \end{aligned}$$

➔ analytic discussion / full simulations
➔ degeneracies, correlations ➔ $(\sin^2 2\theta_{13})_{\rm eff}$

Cervera et al.
Freund, Huber, ML
Akhmedov, Johansson , ML, Ohlsson, Schwetz

Degeneracies, Correlations, ...

Fixed L/E ➔ probabilities invarinat under transformations:

- $\theta_{23} \rightarrow \pi/2 - \theta_{23}$ Fogli, Lisi

$P(\nu_e \rightarrow \nu_\mu)$ not really invariant ➔ compensation by small parameter off-sets

- $\Delta m^2 \rightarrow -\Delta m^2$ **compensated by offset in δ** Minakata, Nunokawa

- **$P(\nu_e \rightarrow \nu_\mu)$=const. ➔ $\delta - \theta_{13}$ manifolds** Koike, Ota, Sato & Burguet-Castell et al.

- **➔ 8-fold degeneracy** Barger, Marfatia, Whisnant

- **parameter extraction suffers from correlations & degeneracies**
- **how to break degeneracies & correlations?**

Baseline & MSW Matter Effect

Beams in Earth Matter:

$\Rightarrow$ electron density profile as function of radius **Stacy**

density errors **Geller & Hara**

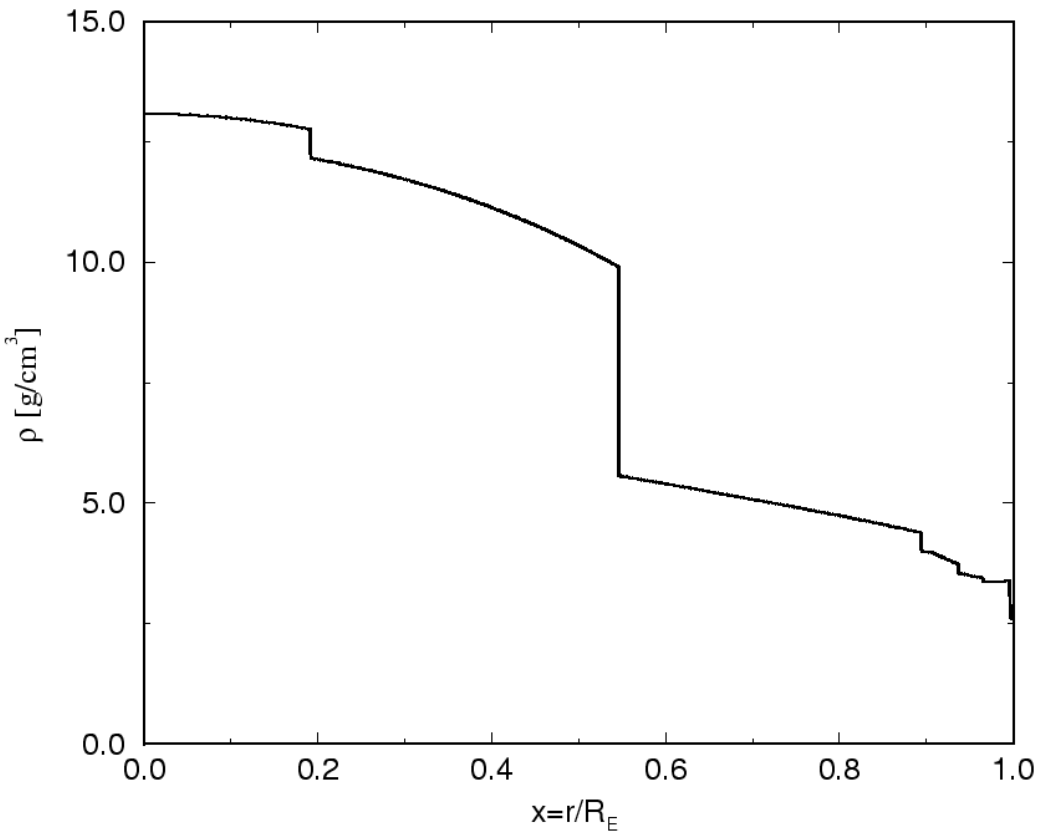
Large $L \Rightarrow$ steep angles

$L = 2800$ km $\Leftrightarrow 13^o$

$L = 7300$ km $\Leftrightarrow 35^o$

$L = 12750$ km $\Leftrightarrow 90^o$

$L \leq \mathcal{O}(10\ 000\ \mathrm{km})$ $\Leftrightarrow$ **mantle**



- $E_{resonance} \simeq 10 - 15$ GeV, matter effects grow with distance L
- Average density profile uncertainties decrease with L $\Rightarrow$ $\simeq 5\%$ error

The Magic Baseline

$$
\begin{aligned}
P(\nu_e \to \nu_\mu) \quad \approx \quad & \sin^2 2\theta_{13} \sin^2\theta_{23} \; \frac{\sin^2((1-\hat{A})\Delta)}{(1-\hat{A})^2} \\
\pm \quad & \sin\delta_{\rm CP} \; \alpha \sin 2\theta_{12} \cos\theta_{13} \sin 2\theta_{13} \sin 2\theta_{23} \sin(\Delta) \frac{\sin(\hat{A}\Delta)\sin((1-\hat{A})\Delta)}{\hat{A}(1-\hat{A})} \\
+ \quad & \cos\delta_{\rm CP} \alpha \sin 2\theta_{12} \cos\theta_{13} \sin 2\theta_{13} \sin 2\theta_{23} \cos(\Delta) \frac{\sin(\hat{A}\Delta)\sin((1-\hat{A})\Delta)}{\hat{A}(1-\hat{A})} \\
+ \quad & \alpha^2 \sin^2 2\theta_{12} \cos^2\theta_{23} \frac{\sin^2(\hat{A}\Delta)}{\hat{A}^2}
\end{aligned}
$$

- All terms besides the first vanish for $\sin(\hat{A}\Delta) = 0$
- Condition for uncorrelated sensitivity to θ_{13} $\boxed{\hat{A}\Delta = \pi}$

$\Rightarrow$ inserting $\hat{A} = A/\Delta m^2_{31}$, $A = 2VE$, $\Delta = \Delta m^2_{31} L/4E$ one finds

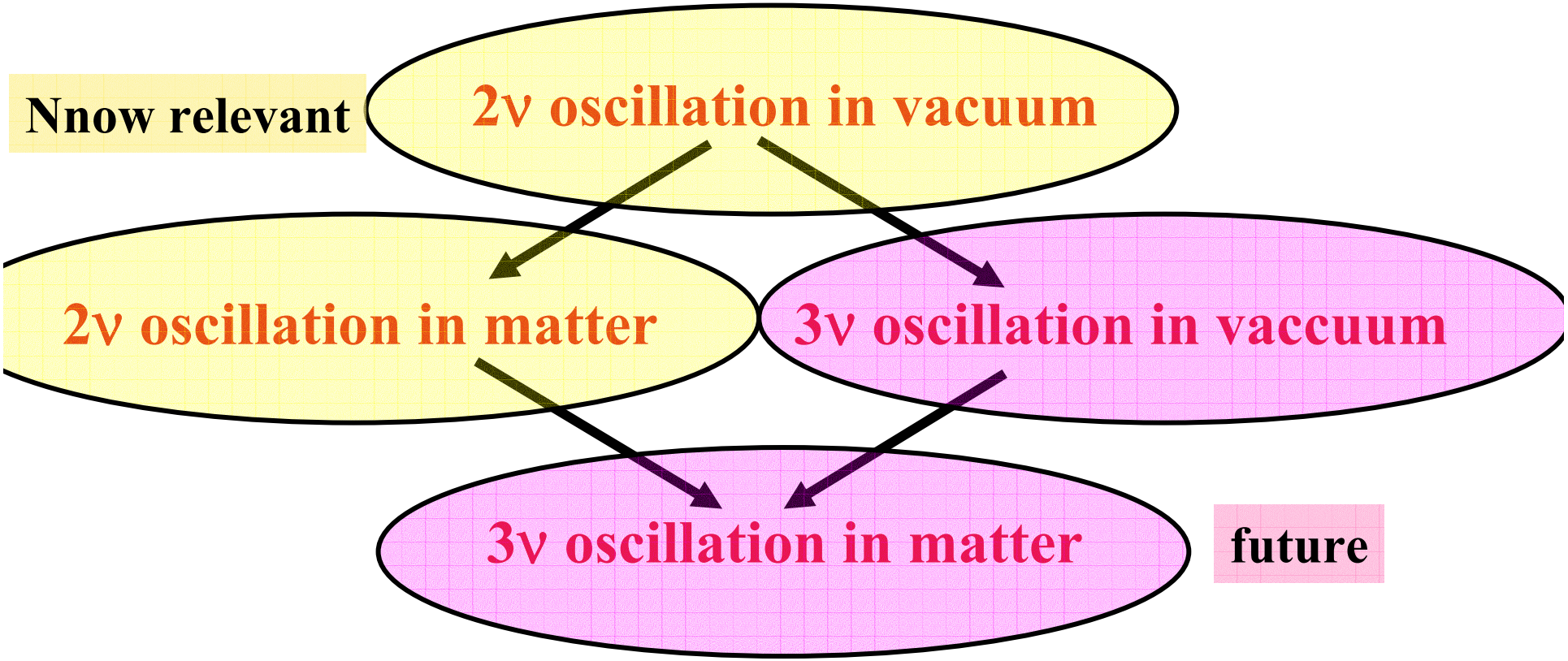
$$L_{magic} = \frac{2\pi}{\sqrt{2} G_F n_e} = 7630 \text{ km} \cdot \frac{\rho}{4.3 g/cm^3}$$

Huber, Winter

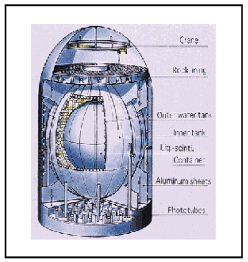
- Note that this is not the MSW resonance condition

Different Levels of Complexity

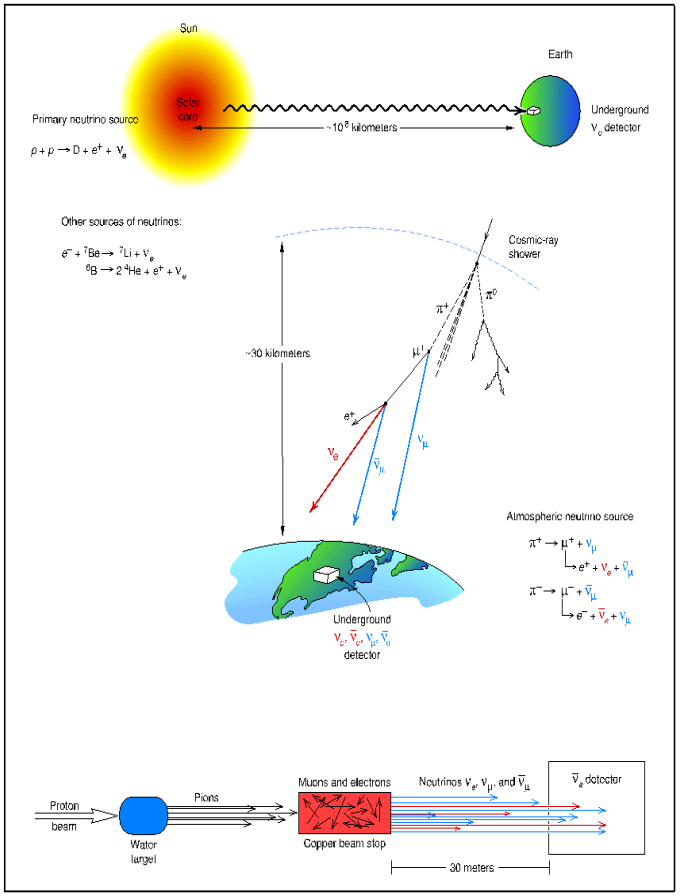
Oscillations can involve 2 or 3 neutrinos in vacuum or matter
➔4 possibilities:



Neutrino Oscillation Results: Overview



⇔



KamLAND **solar**
CHOOZ

atmospheric
+
K2K

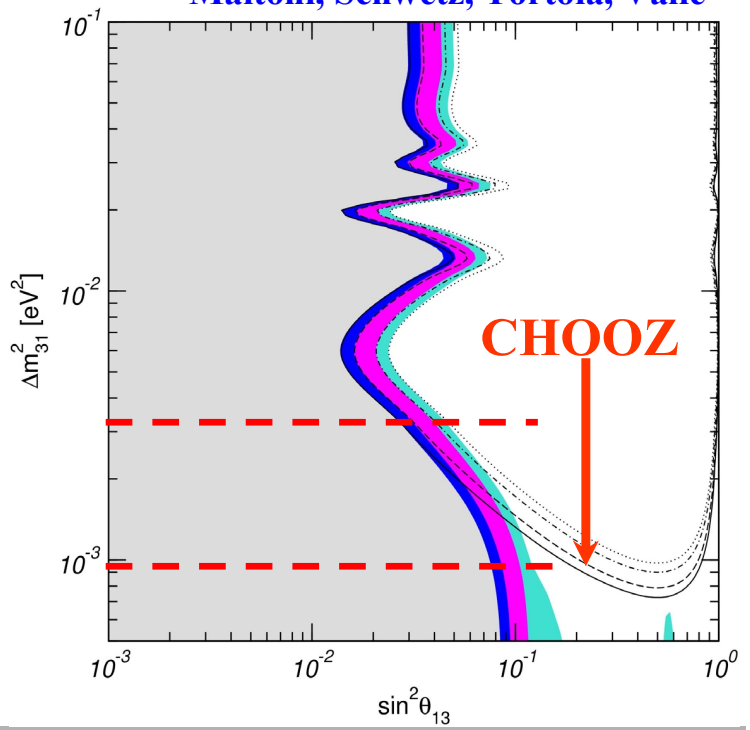
neutrino oscillation signals

LSND?

$\Delta m^2_{12} = 8.2 \pm 0.3 * 10^{-5} eV^2$
$\tan^2\theta_{12} = 0.39 \pm 0.05$
$\Delta m^2_{23} = 2.2 \pm 0.6 * 10^{-3} eV^2$
$\tan^2\theta_{23} = 1.0 \pm 0.3$

$\sin^2\theta_{13} \leq 0.041$ @ 3σ

Maltoni, Schwetz, Tortola, Valle



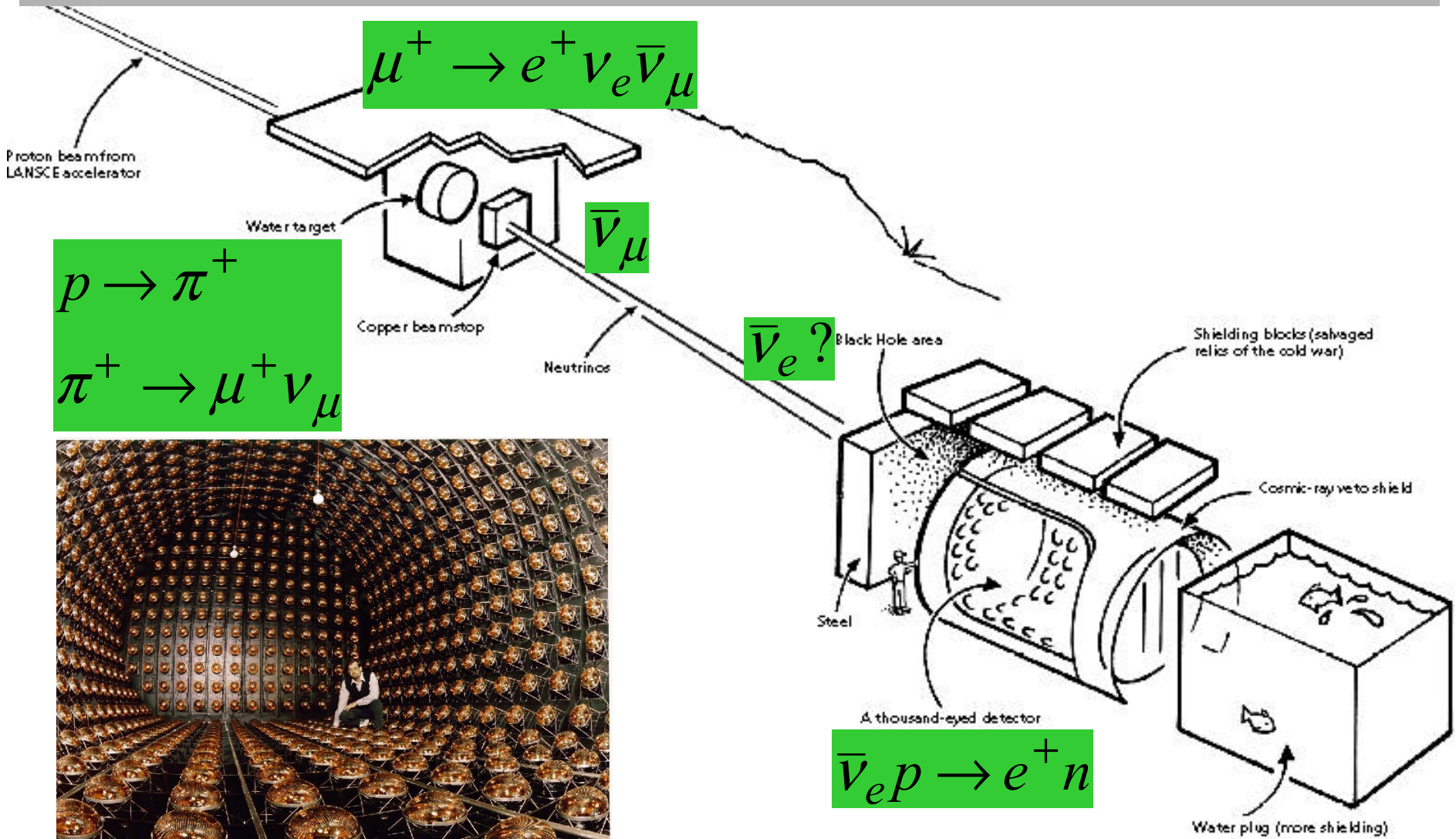
➔ **approximately two 2x2 oscillations**

LSND

$$\mu^+ \to e^+ \nu_e \bar{\nu}_\mu$$

$$p \to \pi^+$$

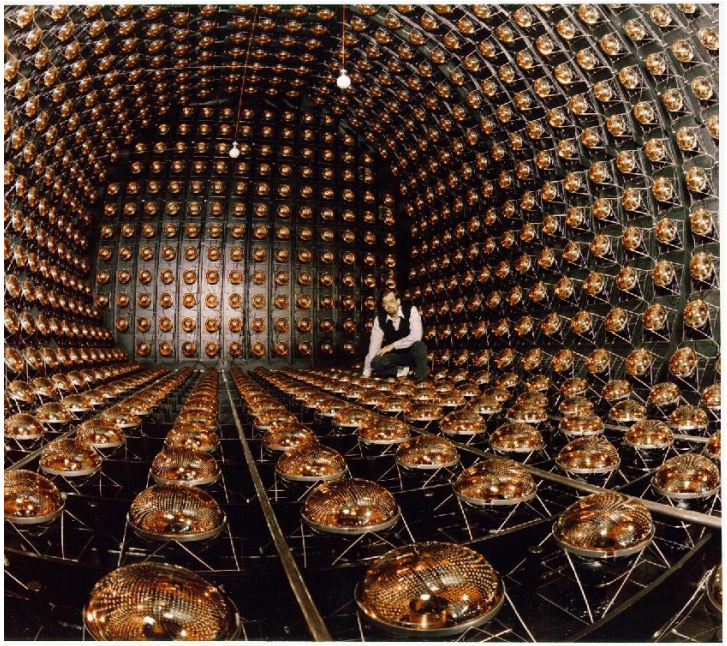
$$\pi^+ \to \mu^+ \nu_\mu$$



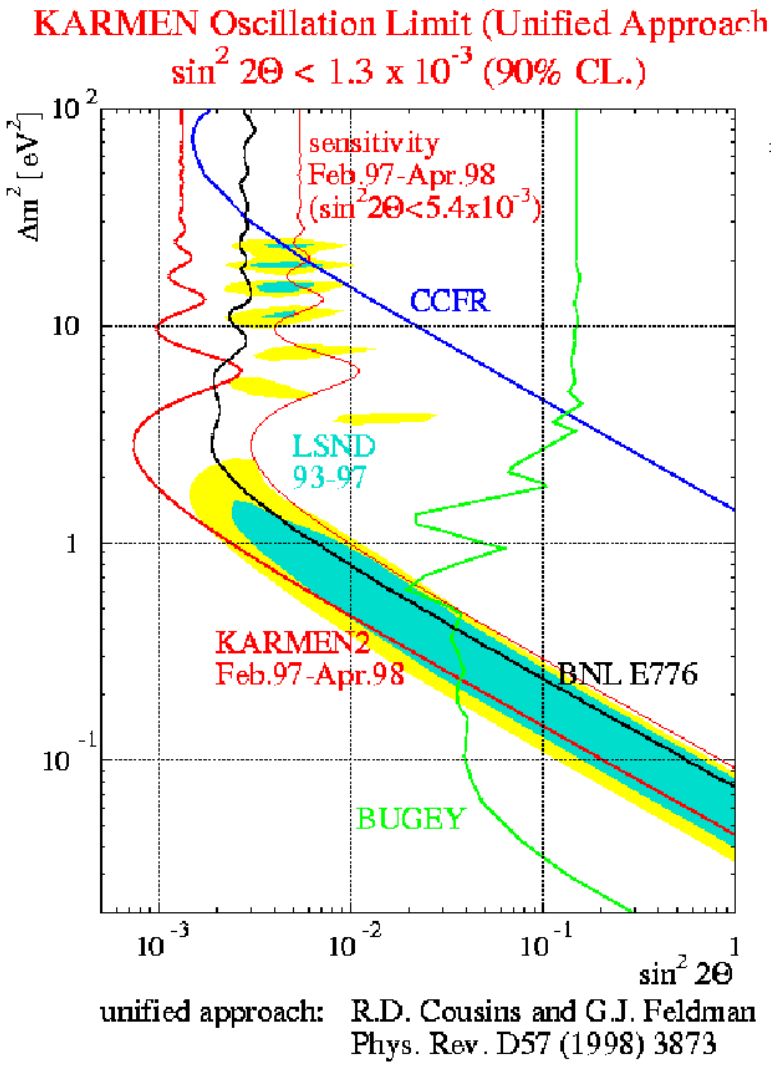
$\bar{\nu}_\mu$

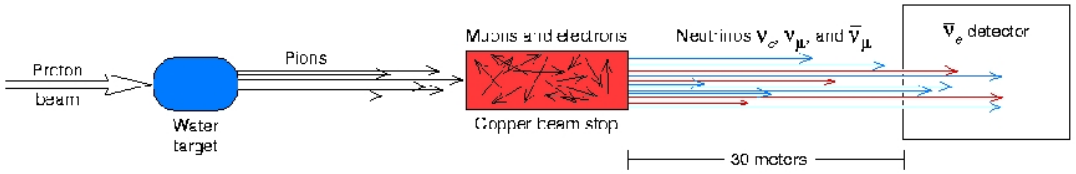
$\bar{\nu}_e$?

$$\bar{\nu}_e p \to e^+ n$$



LSND claims Oscillations





- LSND claims $\overline{\nu}_\mu \to \overline{\nu}_e$ and $\nu_\mu \to \nu_e$ osc.

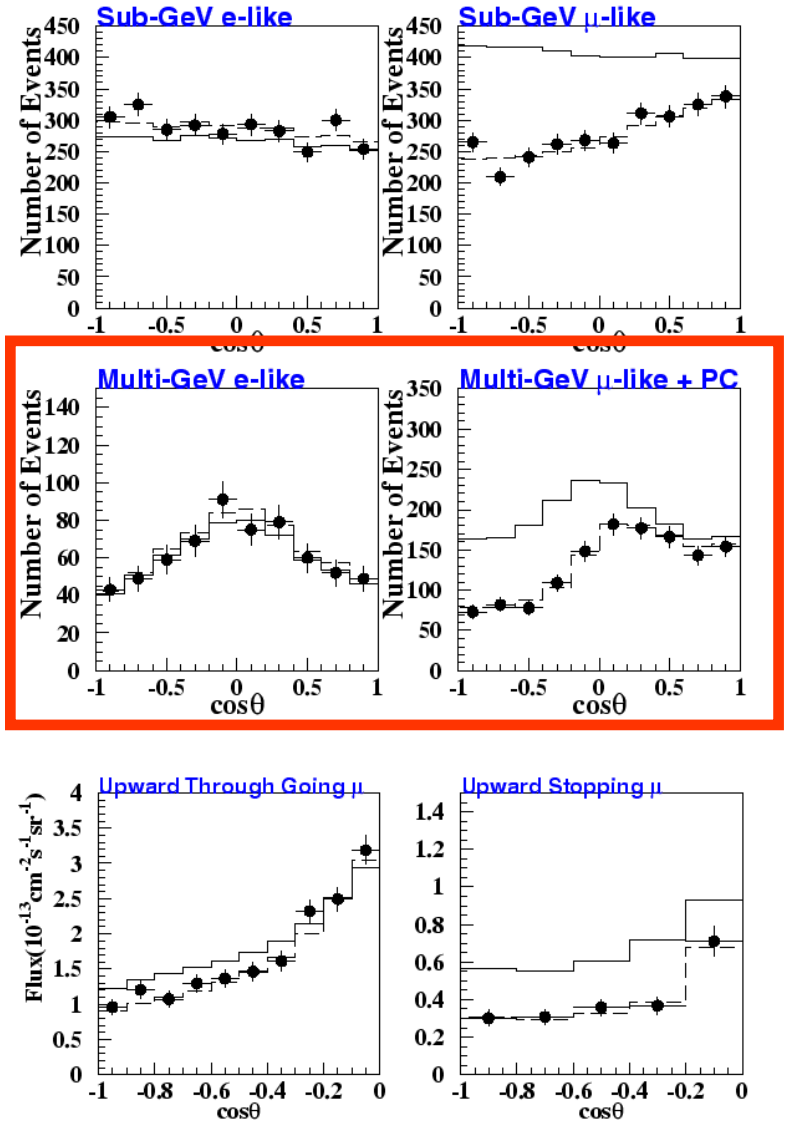
  1) Third $\Delta m^2 \simeq 1$ eV$\Rightarrow$ 4 light neutrinos?
  $\Leftrightarrow$ Z line-shape

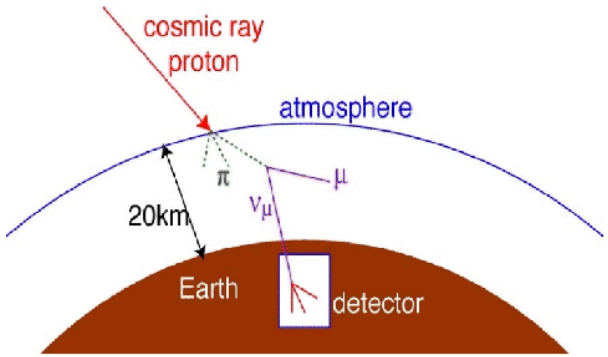
  2) CPT violation = different param. for ν and $\overline{\nu}$
  $\Leftrightarrow$ local QFT!?

- Partly ruled out by KARMEN
- Partly ruled out by $\to$cosmology
- Will be testet soon by $\Rightarrow$ **MiniBooNE**

3+1 ➔ tension in global fits
3+2 OK, 3+N ➔➔cosmology

Atmospheric Oscillations @SuperK





Oscillation fit (including K2K):

$$|\Delta m^2_{31}| = (2.0 \pm 1.0) \cdot 10^{-3}\mathrm{eV}^2$$
$$\sin^2 2\theta_{23} \geq 0.85$$

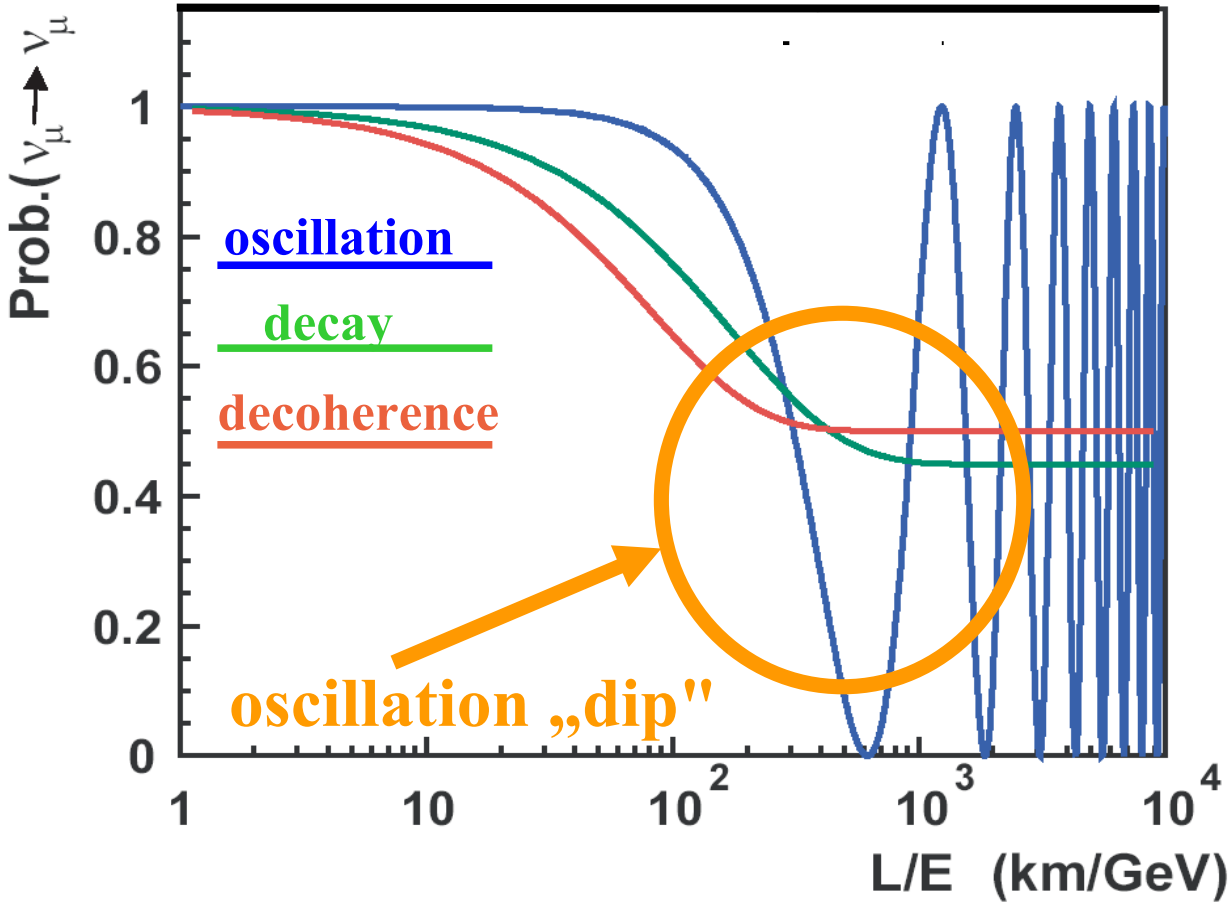
- 8σ signal for ν_μ disappearance
- NOT $\nu_\mu \to \nu_e$ (consistent with Chooz)
- $\Rightarrow$ $\nu_\mu \to \nu_\tau$ (some $\tau's$ seen)
- NOT $\nu_\mu \to \nu_s$ from NC/CC comparison
- L/E confirmed by K2K ($\simeq 2\sigma$)
- sensitivity for L/E-dependence of oscillations

L/E Dependence: Atmospheric Oscillations

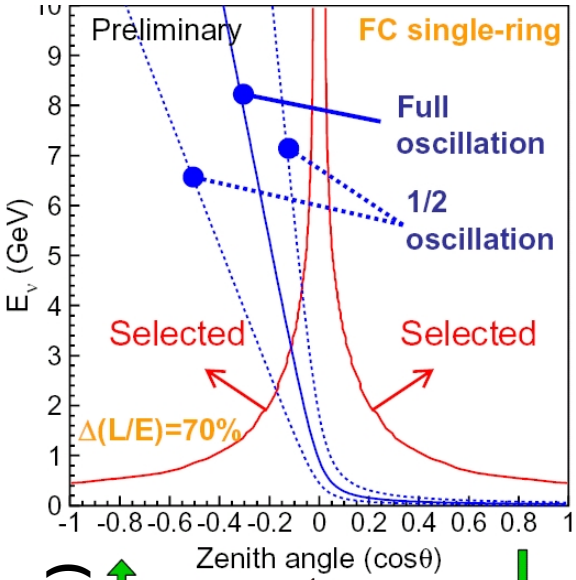
Neutrino oscillation : $P_{\mu\mu} = 1 - \sin^2 2\theta \sin^2(1.27 \frac{\Delta m^2 L}{E})$

Neutrino decay : $P_{\mu\mu} = (\cos^2\theta + \sin^2\theta \times \exp(-\frac{m}{2\tau}\frac{L}{E}))^2$

Neutrino decoherence : $P_{\mu\mu} = 1 - \frac{1}{2}\sin^2 2\theta \times (1 - \exp(-\gamma_0 \frac{L}{E}))$



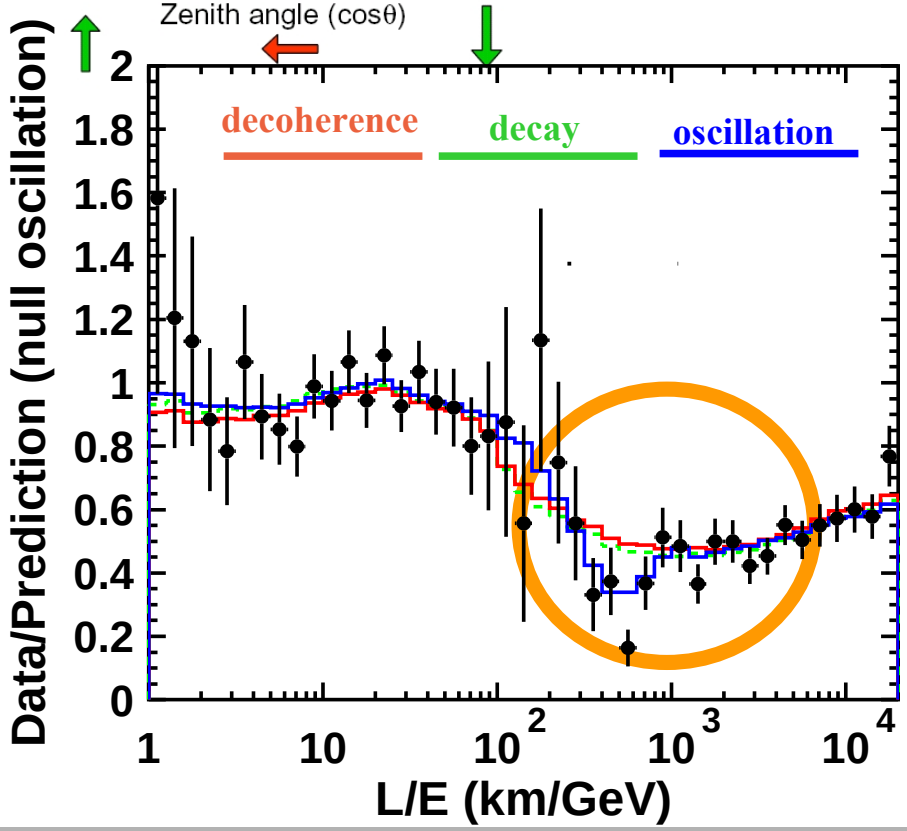
L/E dependence smeared out!



Bad L/E resolution for:

- horizontal events (dL/dcosθ is big!)
- events with small energy

←cuts in the E-cosθ plane



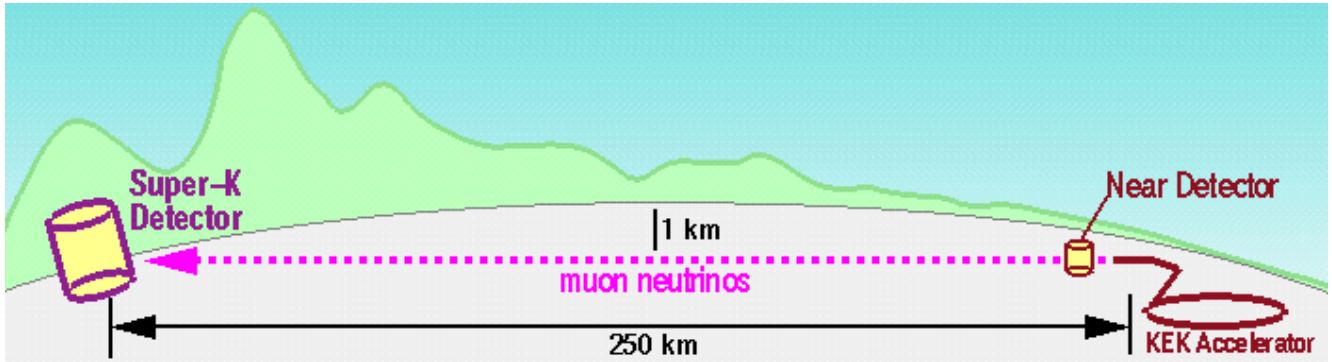
SK I + II (preliminary)

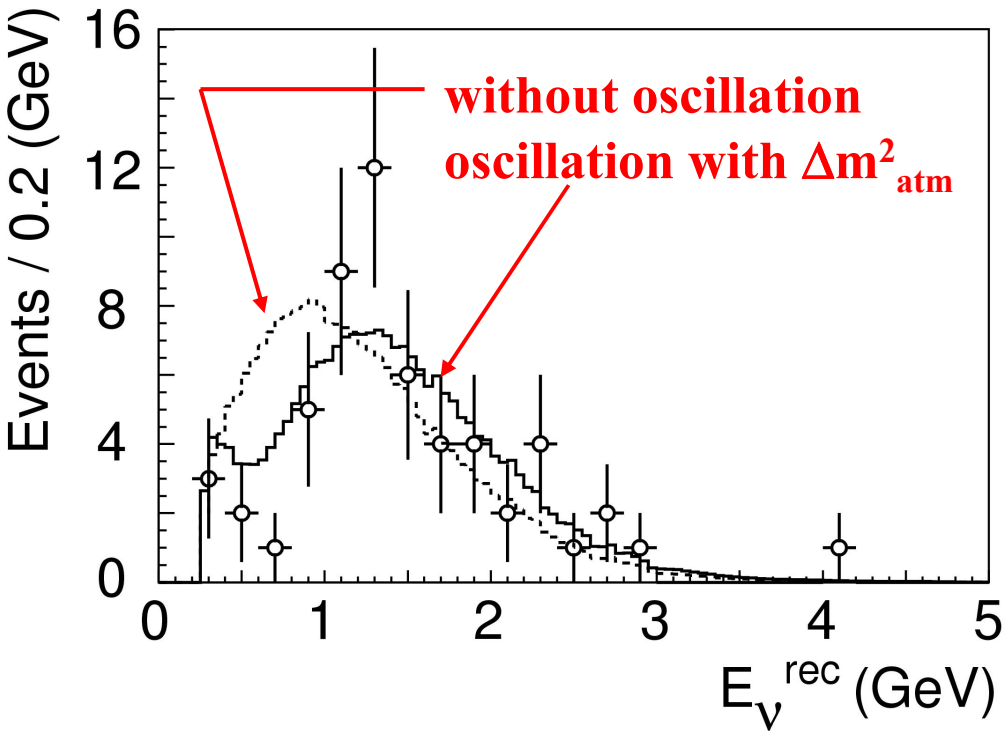
Result:

- 4.4σ for decay
- 4.8σ for de-coherence
- $\Delta m^2 = 2.4\ 10^{-3} eV^2$

←→long baseline exp.

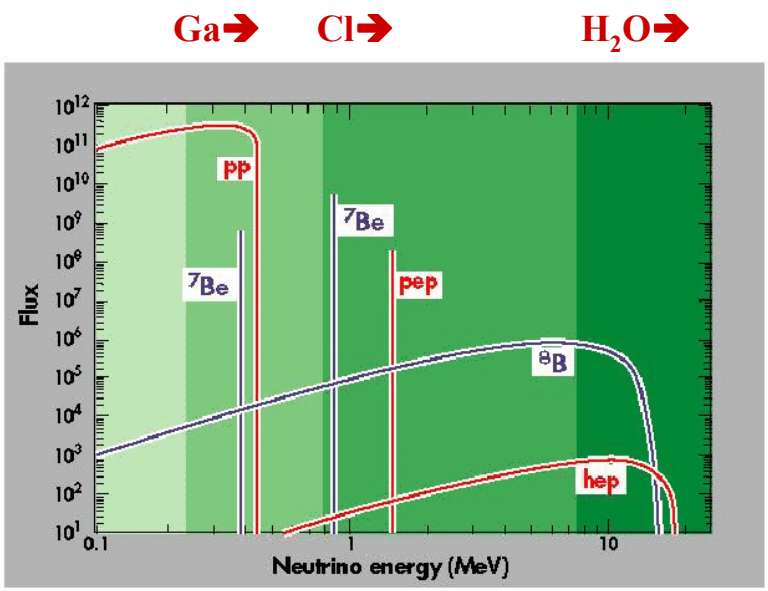
K2K confirms atmospheric Δm^2

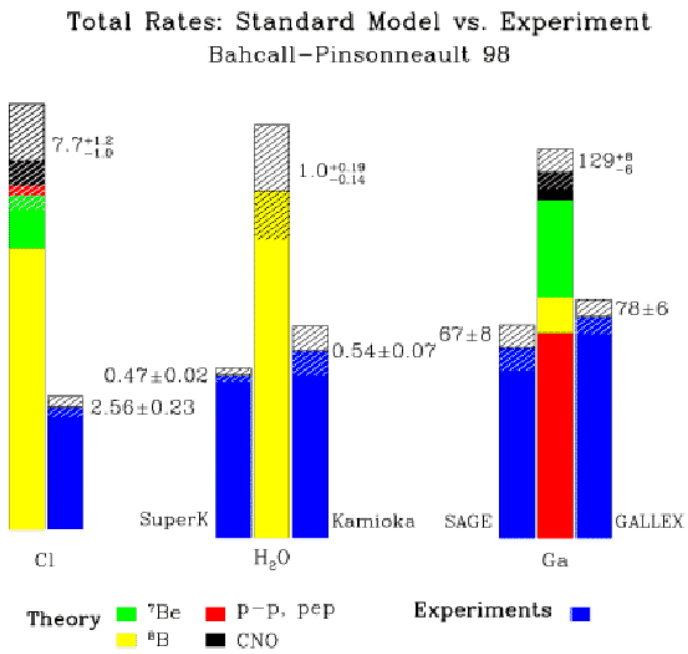




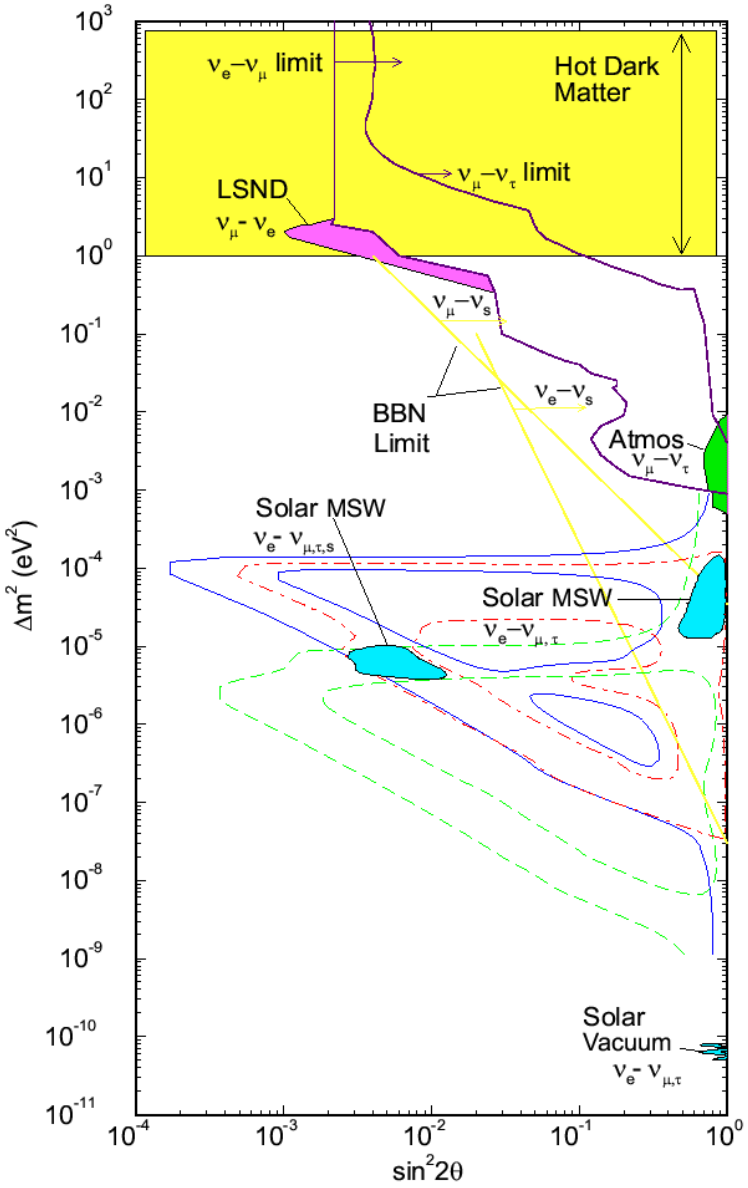
First Indications from Solar Neutrinos

Before ~2000

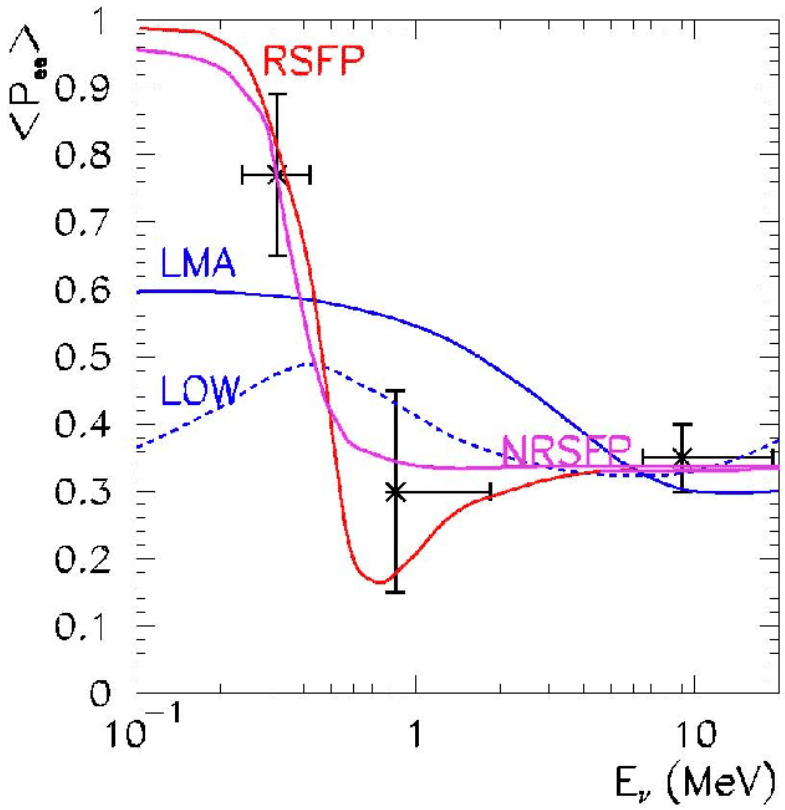




- Consistent deficit of total CC neutrino rates
- Combining $Cl/H_2O/Ga/D_2O \Rightarrow$ some spectral information
- Depends on solar modelling, but should be robust
- Can be explained by oscillation $\Rightarrow$ different regimes $\Rightarrow$

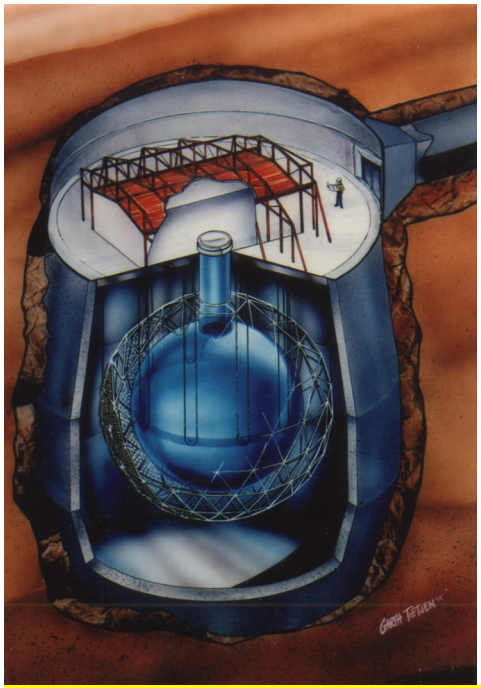


oscillation fits (≃3 years ago)

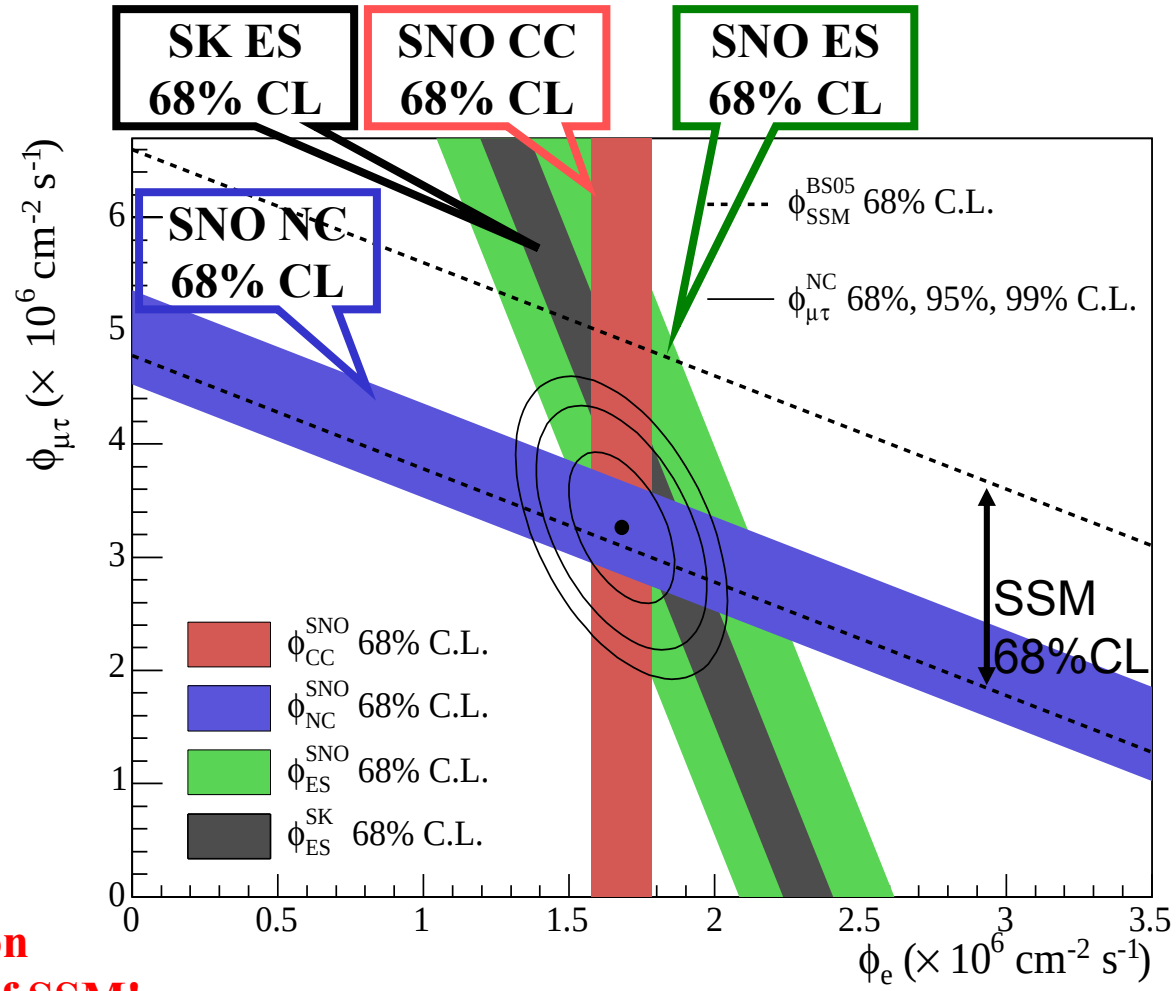


- **Which oscillation solution?**
  - SNO day/night ⇒ LMA best
- **RSFP worked even better!**
  - Reqires large magnetic moments
  - ... and large magnetic fields

Solar ν´s: NC, CC, ES Rates from SNO



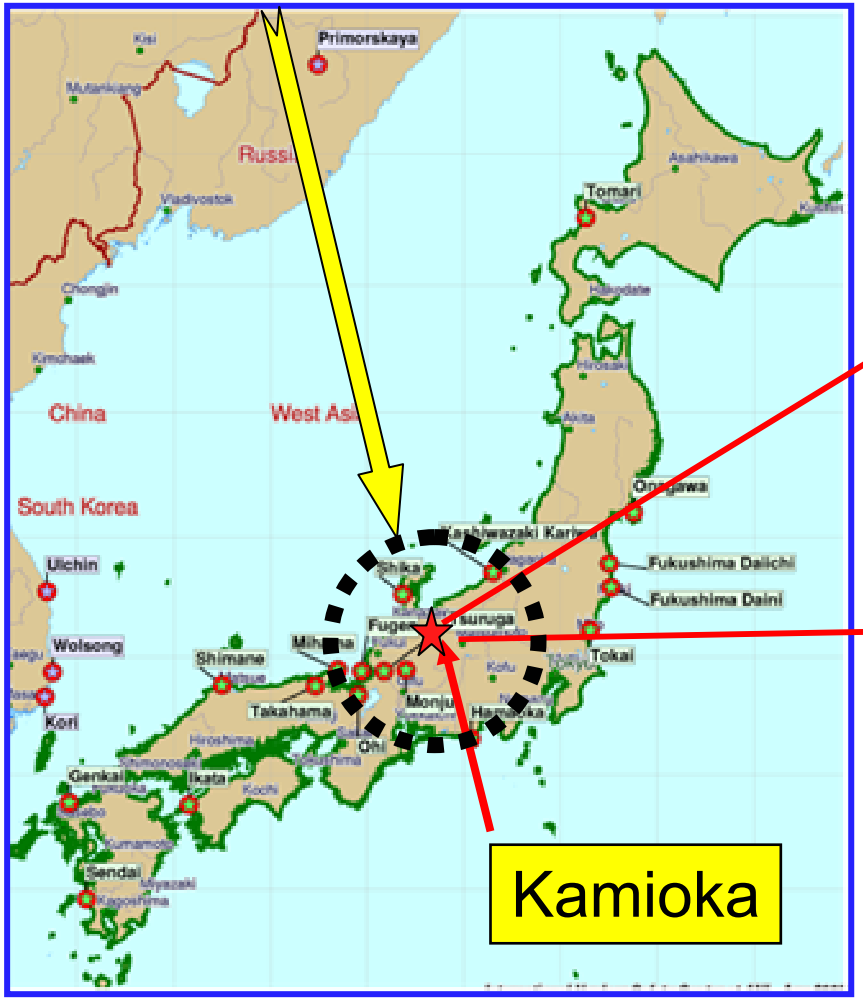
$\Phi_{CC} = \Phi_e$
$\Phi_{ES} = \Phi_e + 1/6\ \Phi_{\mu\tau}$
$\Phi_{NC} = \Phi_e + \Phi_\mu + \Phi_\tau$



Clear proof of flavour conversion
Independent of SSM ←→ test of SSM!
No conversion and conversion to sterile ν's excluded by many sigma
≃ no L/E dependence

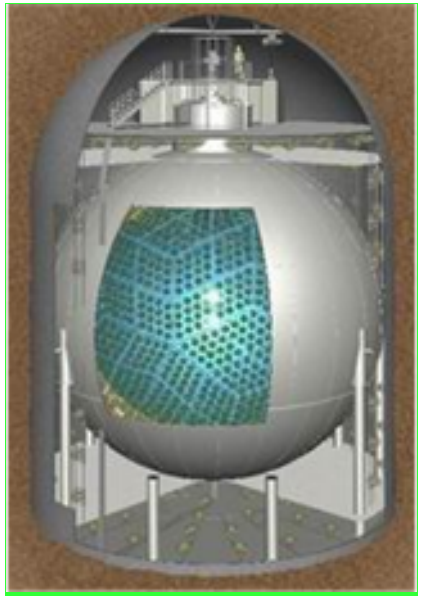
KamLAND tests solar Oscillations

26 power reactors are located in narrow band at L~180km from KamLAND, producing 80GW$_{th}$, 7% of World reactor power !!



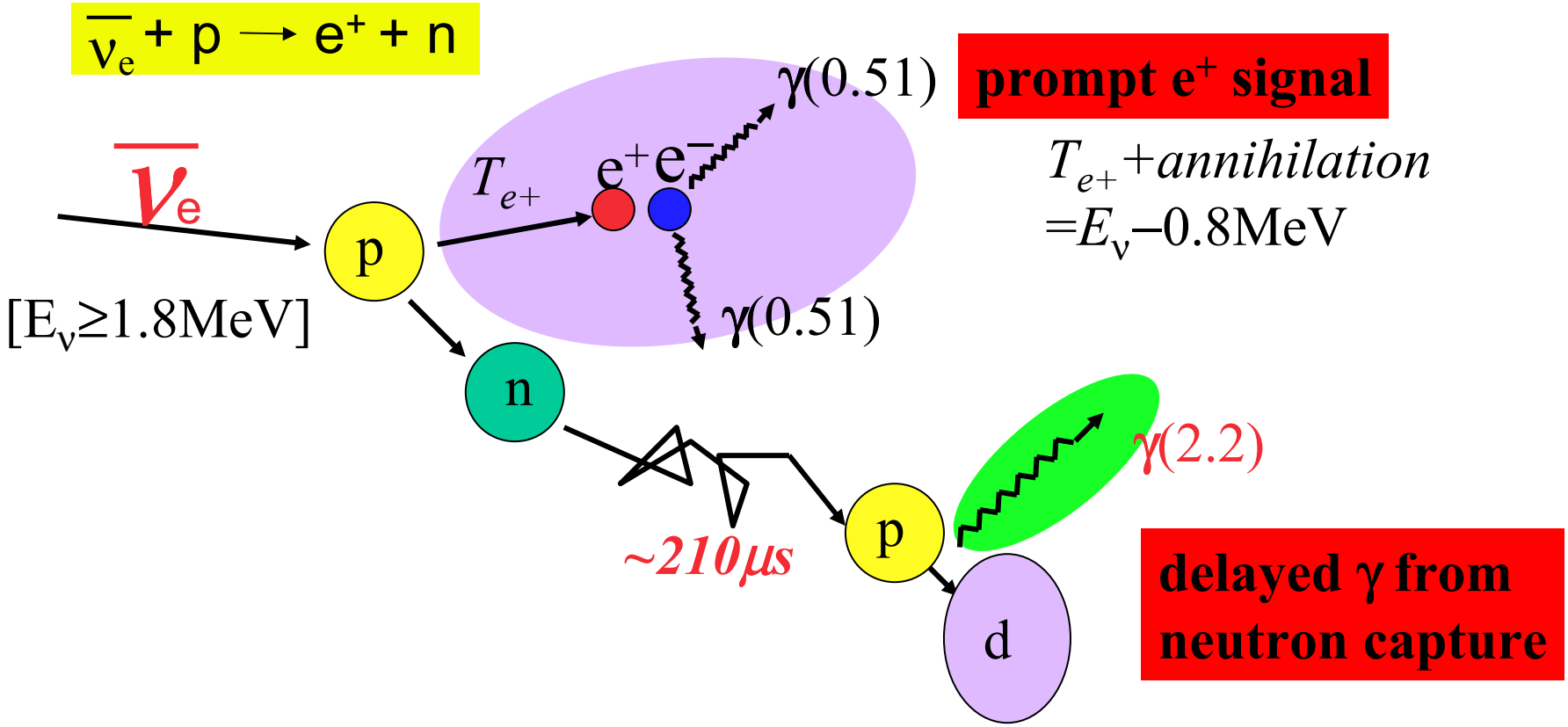
<L>~180km

KamLAND



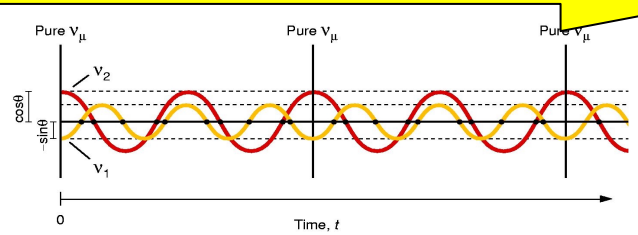
Contribution from overseas
Korea 2.46%
Other countries 0.7%

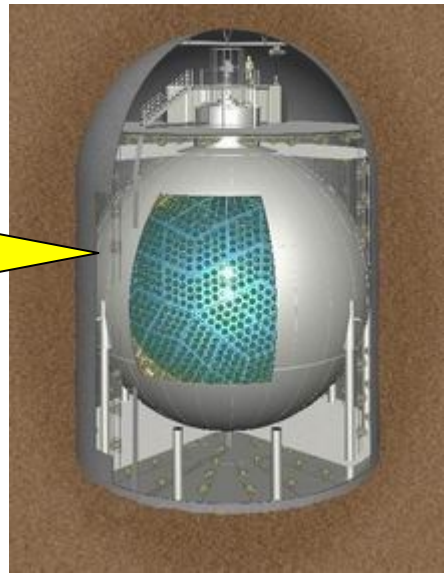
Neutrino Detection at KamLAND

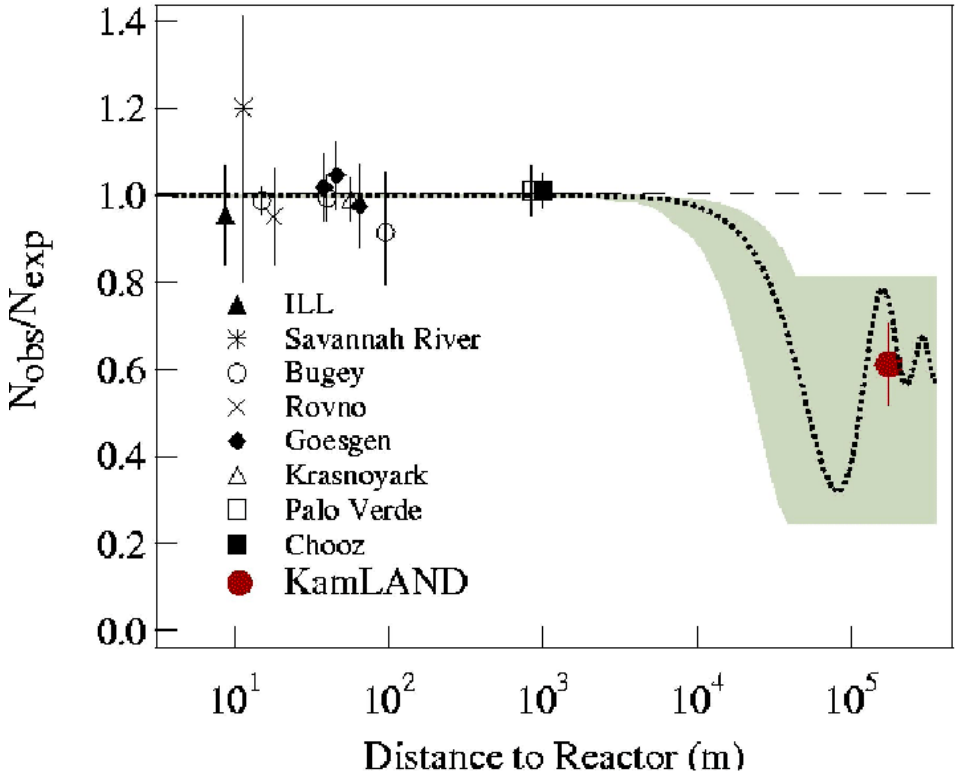


position & time correlation
and delayed energy information
➔ enormous background reduction!

flux ~1/L² + oscillations?







Compare to solar oscillations:

- **rate consistent**

➔tests solar oscillations with reactor anti-ν's

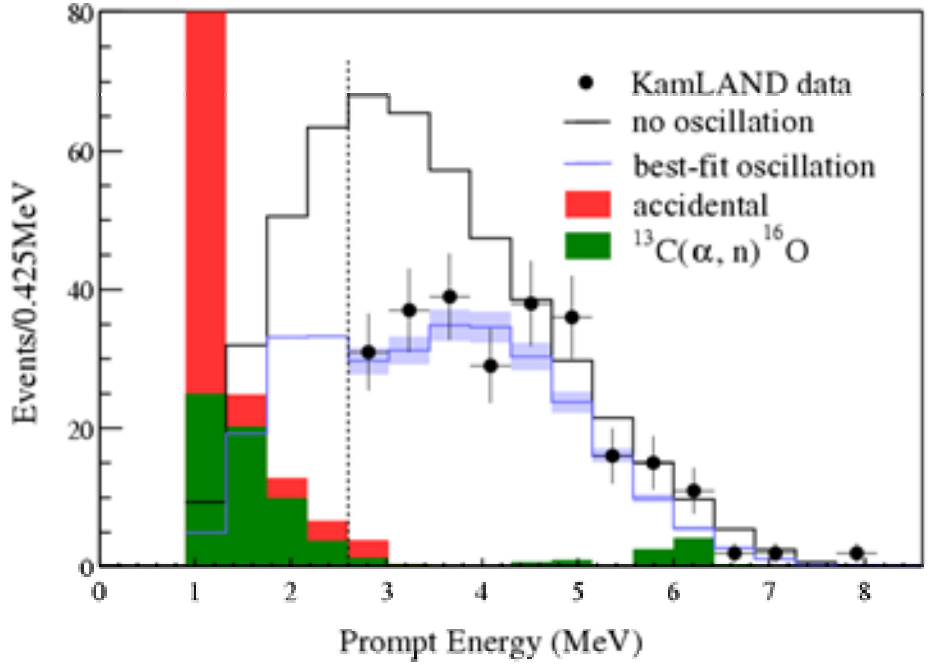
- **excludes RSFP (no B fields)**
- **≃ excludes CPT-violation ←➔ LSND**
- **rate effect & some L/E sensitivity**

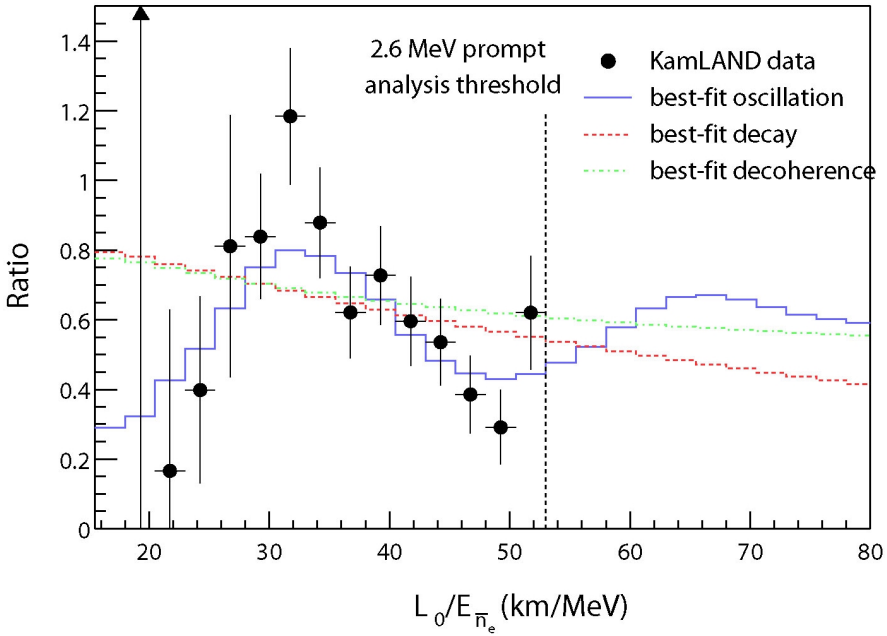
KamLAND Results

Best fit to latest results (2005):

$\Delta m^2 = 7.9^{+0.6}_{-0.5} \times 10^{-5}\ eV^2$

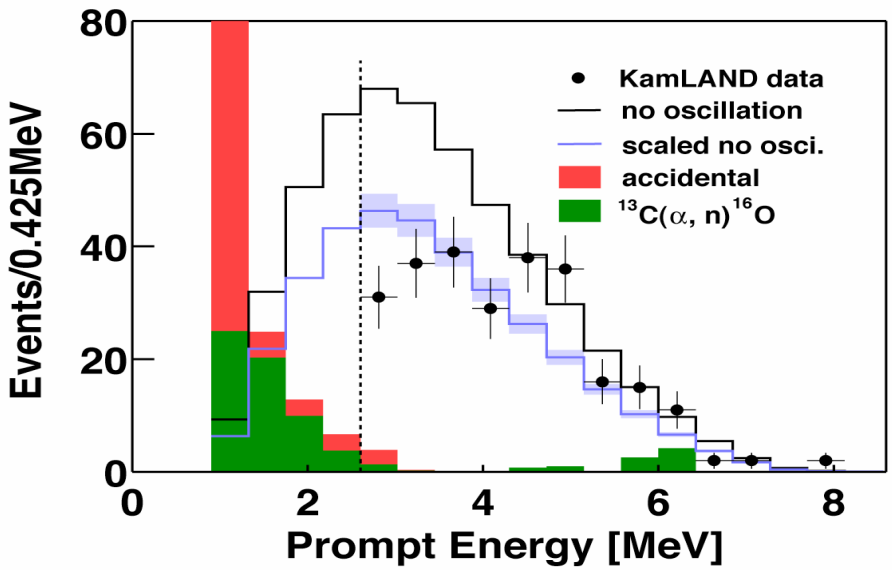
$\tan^2\theta = 0.46$





some L/E dependence

Testing Solar L/E with KamLAND

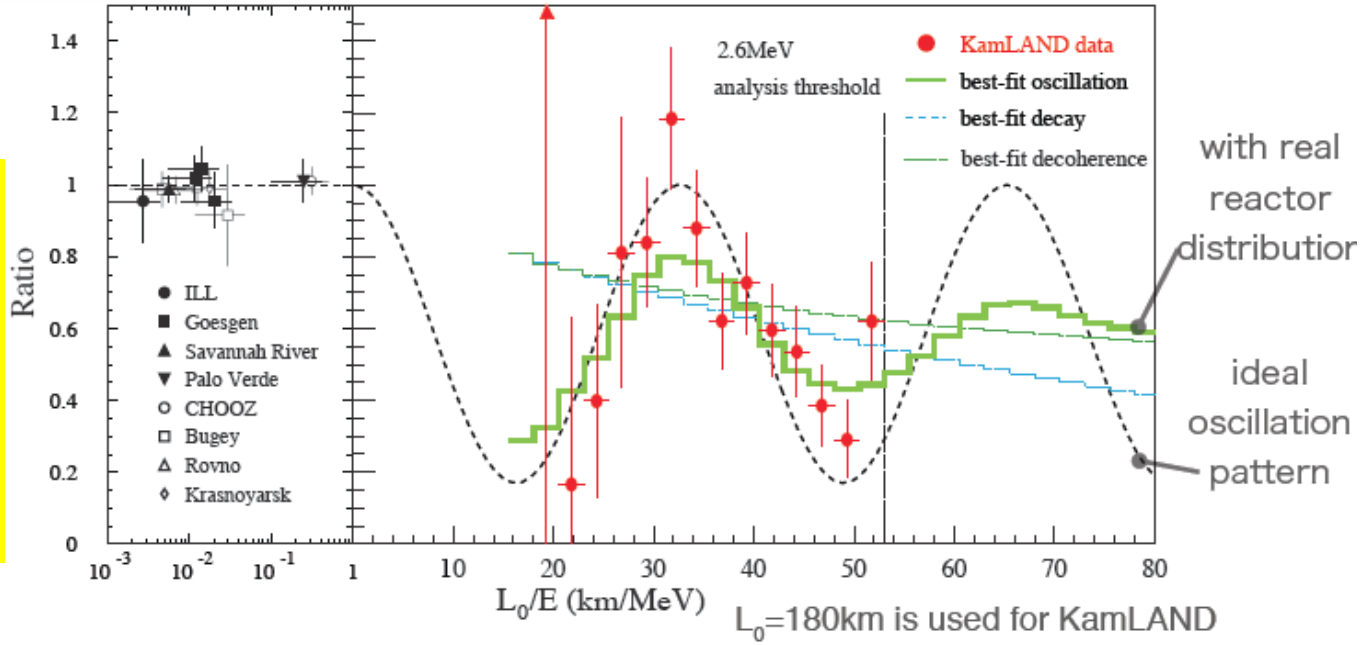


rate plus shape ➔ oscillations at 99.999995% CL

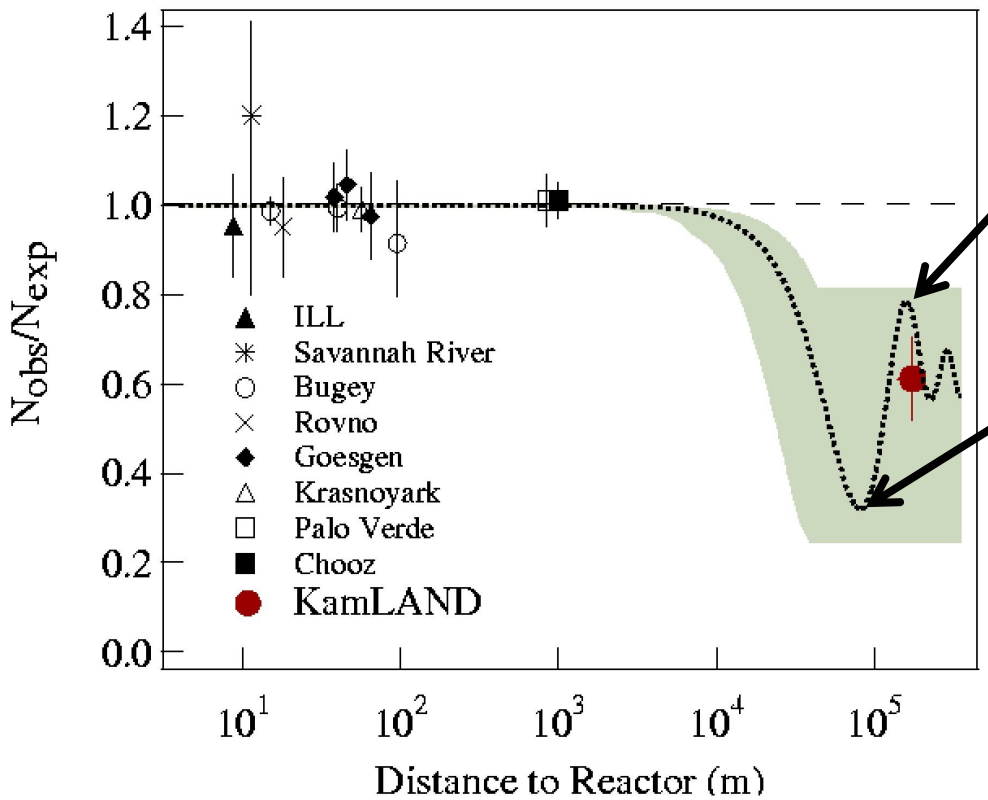
<u>Best fit:</u> $\Delta m^2 = 7.9^{+0.6}_{-0.5} \times 10^{-5} eV^2$
$\tan^2\theta = 0.46$

<u>improved tests of L/E:</u>

- **Super Kamiokande**
- **KamLAND**
- **MINOS**
- **…**



KamLAND Implications



poor θ_{12} sensitivity

good θ_{12} sensitivity

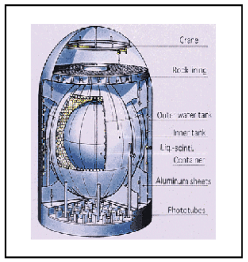
➔KamLAND is not in the ideal place!

➔ optimal ~ 80 km

➔New reactor experiments at ideal location(s)

- **better θ_{12} sensitivity**
- **better Δm^2 sensitivity**

Neutrino Oscillations Status Summary



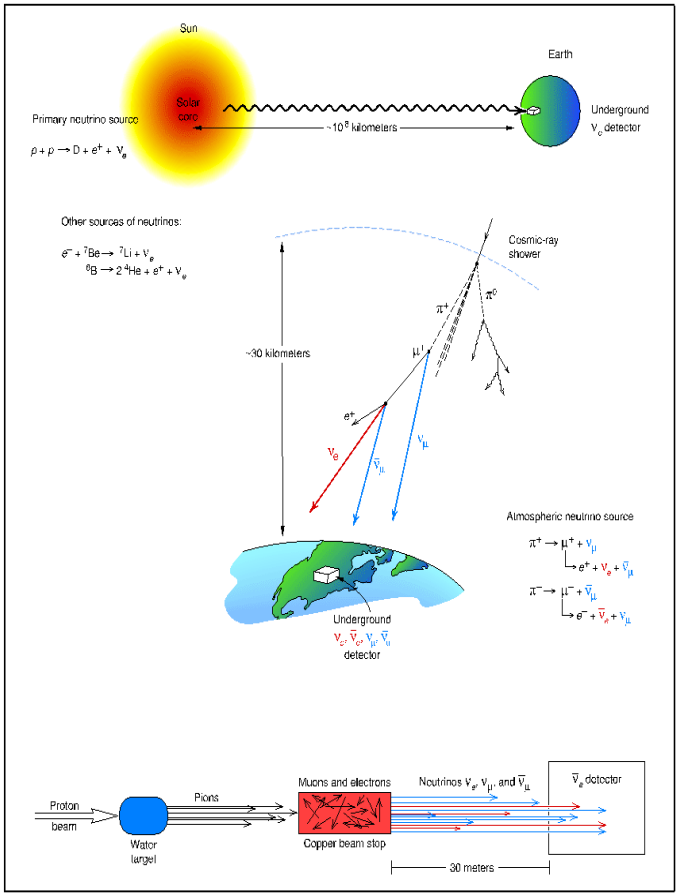
⇔

KamLAND solar
CHOOZ

atmospheric
+
K2K

neutrino oscillation signals

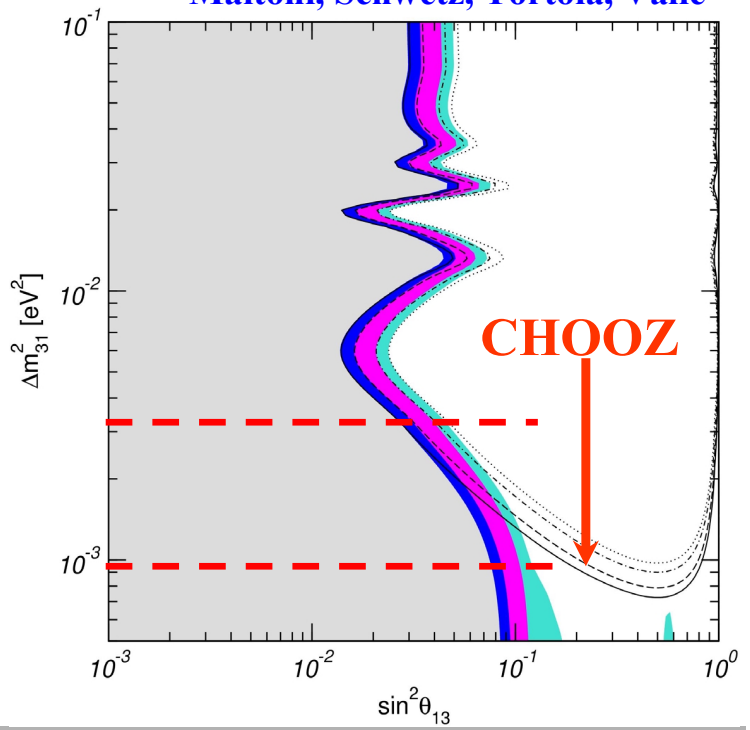
LSND?



$\Delta m^2_{12} = 8.2 \pm 0.3 * 10^{-5} eV^2$
$\tan^2\theta_{12} = 0.39 \pm 0.05$
$\Delta m^2_{23} = 2.5 \pm 0.6 * 10^{-3} eV^2$
$\tan^2\theta_{23} = 1.0 \pm 0.3$

$\sin^2\theta_{13} \leq 0.041$ @ 3σ

Maltoni, Schwetz, Tortola, Valle



➔ **approximately two 2x2 oscillations**