Sally Dawson, BNL
Introduction to the Standard Model
TASI, 2006

- Introduction to the Standard Model
  - Review of the SU(2) x U(1) Electroweak theory
  - Experimental status of the EW theory
  - Constraints from Precision Measurements
- Theoretical problems with the Standard Model
- Beyond the SM
  - Why are we sure there is physics BSM?
  - What will the LHC and Tevatron tell us?

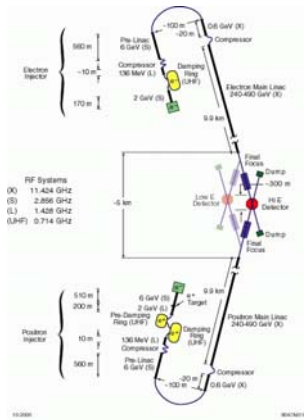
Lecture 1

- Introduction to the Standard Model
  - Just the SU(2) x U(1) part of it….
- Some good references:
  - Chris Quigg, *Gauge Theories of the Strong, Weak, and Electromagnetic Interactions*
  - Michael Peskin, *An Introduction to Quantum Field Theory*
  - Sally Dawson, *Trieste lectures*, hep-ph/9901280

Science Timeline

Tevatron LHC LHC Upgrade ILC

2006 2007 2012



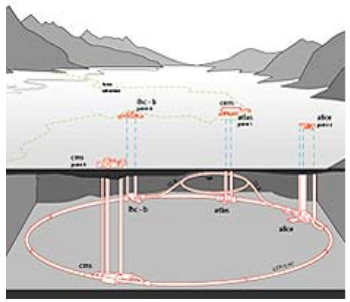
Large Hadron Collider (LHC)

- proton-proton collider at CERN (2007)
- 14 TeV energy
  - 7 mph slower than the speed of light
  - *cf*. 2TeV @ Fermilab

  (307 mph slower than the speed of light)
- Typical energy of quarks and gluons 1-2 TeV

LHC is Big….

- ATLAS is 100 meters underground, as deep as Big Ben is tall



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- The accelerator circumscribes 58 square kilometers, as large as the island of Manhattan

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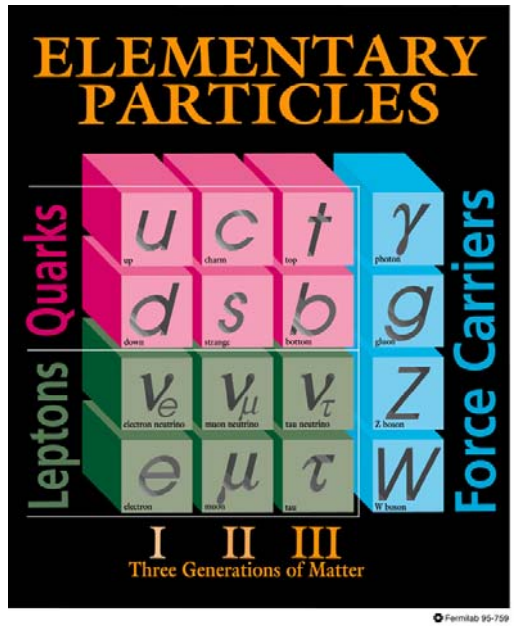
LHC Will Require Detectors of Unprecedented Scale

- CMS is 12,000 tons (2 x's ATLAS)
- ATLAS has 8 times the volume of CMS

What we know

- The photon and gluon appear to be massless
- The W and Z gauge bosons are heavy
  - M_W=80.404 ± 0.030 GeV
  - M_Z =91.1875 ± 0.0021 GeV
- There are 6 quarks
  - M_t=172.5 ±2.3 GeV
  - M_t >> all the other quark masses
- There appear to be 3 distinct neutrinos with small but non-zero masses
- The pattern of fermions appears to replicate itself 3 times
  - Why not more?



Abelian Higgs Model

- Why are the W and Z boson masses non-zero?
- U(1) gauge theory with single spin-1gauge field, A_μ

$$L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$F_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu$$

- U(1) local gauge invariance:

$$A_\mu(x) \to A_\mu(x) - \partial_\mu \eta(x)$$

- Mass term for A would look like:

$$L = -\frac{1}{4} F_{\mu\nu} \; F^{\mu\nu} + \frac{1}{2} m^2 A_\mu A^\mu$$

- Mass term violates local gauge invariance
- We understand why $M_A = 0$

Gauge invariance is guiding principle

Abelian Higgs Model, 2

- Add complex scalar field, φ, with charge –e:

$$L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \left| D_\mu \phi \right|^2 - V(\phi)$$

- Where

$$D_\mu = \partial_\mu - ieA_\mu \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$V(\phi) = \mu^2 |\phi|^2 + \lambda \left(|\phi|^2 \right)^2$$

- L is invariant under local U(1) transformations:

$$\boxed{\begin{aligned} A_\mu(x) &\to A_\mu(x) - \partial_\mu \eta(x) \\ \phi(x) &\to e^{-ie\eta(x)} \phi(x) \end{aligned}}$$

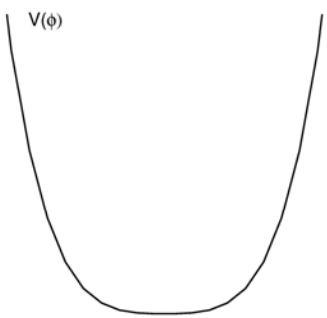
Abelian Higgs Model, 3

- Case 1: $\mu^2 > 0$
  - QED with $M_A=0$ and $m_\varphi=\mu$
  - Unique minimum at $\varphi=0$

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \left|D_\mu \phi\right|^2 - V(\phi)$$

$$D_\mu \quad = \partial_\mu - ieA_\mu$$

$$V(\phi) = \mu^2\left|\phi\right|^2 + \lambda\left(\left|\phi\right|^2\right)^2$$

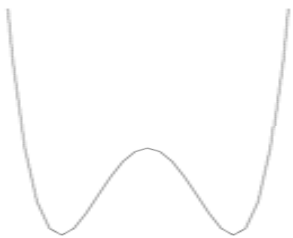


By convention, $\lambda > 0$

Abelian Higgs Model, 4

- Case 2: $\mu^2 < 0$

$$V(\phi) = -\left|\mu^2\right|\left|\phi\right|^2 + \lambda\left(\left|\phi\right|^2\right)^2$$



- Minimum energy state at:

$$<\phi> = \sqrt{-\frac{\mu^2}{2\lambda}} \equiv \frac{v}{\sqrt{2}}$$

Vacuum breaks U(1) symmetry

Aside: What fixes sign (μ^2)?

Abelian Higgs Model, 5

- Rewrite $\phi \equiv \frac{1}{\sqrt{2}} e^{i\frac{\chi}{v}} (v+h)$

  χ and h are the 2 degrees of freedom of the complex Higgs field

- L becomes:

$$L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - evA_\mu \partial^\mu \chi + \frac{e^2 v^2}{2} A^\mu A_\mu + \frac{1}{2}\left(\partial_\mu h \partial^\mu h + 2\mu^2 h^2\right)$$
$$+ \frac{1}{2} \partial_\mu \chi \partial^\mu \chi + (h, \chi \text{ interactions})$$

- Theory now has:
  - Photon of mass $M_A = ev$
  - Scalar field h with mass-squared $-2\mu^2 > 0$
  - Massless scalar field χ *(Goldstone Boson)*

Abelian Higgs Model, 6

- What about mixed χ-A propagator?
  - Remove by gauge transformation

$$A'_\mu \equiv A_\mu - \frac{1}{ev}\partial_\mu \chi$$

- χ field disappears
  - We say that it has been *eaten* to give the photon mass
  - χ field called Goldstone boson
  - *This is Abelian Higgs Mechanism*
  - This gauge (unitary) contains only physical particles

$$\boxed{L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{e^2v^2}{2}A'^{\mu}A'_\mu + \frac{1}{2}\left(\partial_\mu h \partial^\mu h\right) - V(h)}$$

Higgs Mechanism summarized

Spontaneous breaking of a gauge theory by a non-zero VEV of a scalar field results in the disappearance of a Goldstone boson and its transformation into the longitudinal component of a massive gauge boson

Gauge Invariance

What about gauge invariance? Choice above called unitarity gauge

- No χ field
- Bad high energy behavior of A propagator

$$\Delta_{\mu\nu}(k) = -\frac{i}{k^2 - M_A{}^2}\left(g_{\mu\nu} - \frac{k^\mu k^\nu}{M_A{}^2}\right)$$

- R_ξ gauges more convenient:
- $L_{GF} = (1/2\xi)(\partial_\mu A^\mu + \xi e v \chi)^2$

$$L_2 = -\frac{1}{2}A_\mu\left(-g^{\mu\nu}\partial^2 + \left(1-\frac{1}{\xi}\right)\partial^\mu\partial^\nu - e^2v^2\right)A_\mu + \frac{1}{2}\left(\partial_\mu h\partial^\mu h + 2\mu^2h^2\right)$$
$$+\frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \frac{\xi}{2}e^2v^2\chi^2$$

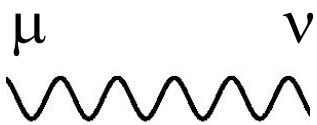
More on R_ξ gauges

Mass of Goldstone boson χ depends on ξ

$\xi=1$: Feynman gauge with massive χ

$\xi=0$: Landau gauge

$\xi\to\infty$: Unitarity gauge

Gauge Boson, A 

$$\frac{-i}{k^2-M_A^2}\left(g^{\mu\nu}-\frac{k^\mu k^\nu}{k^2-\xi M_A^2}(1-\xi)\right)$$

Higgs, h

$$\frac{i}{k^2-M_h^2}$$

Goldstone Boson, χ, or Faddeev-Popov ghost

$$\frac{i}{k^2-\xi M_A^2}$$

Non-Abelian Higgs Mechanism

- Vector fields $A^a_\mu(x)$ and scalar fields $\varphi_i(x)$ of SU(N) group

$$\Phi = \begin{pmatrix} \phi_1 \\ \cdot \\ \cdot \\ \cdot \\ \phi_N \end{pmatrix}$$

$$L_\Phi = (D^\mu \Phi^+)(D^\mu \Phi) - V(\Phi),$$

$$V(\Phi) = \mu^2 \Phi^+ \Phi + \lambda (\Phi^+ \Phi)^2$$

- L is invariant under the non-Abelian symmetry:

$$\phi_i \to (1 - i\eta^a \tau^a)_{ij} \phi_j$$

$$D_\mu \phi = \left(\partial_\mu - ig\tau^a A^a_\mu\right)\phi$$

- τ_a are group generators, a=1…N^2-1 for SU(N)

For SU(2): $\tau^a = \sigma^a/2$

Non-Abelian Higgs Mechanism, 2

- In exact analogy to the Abelian case

$$(D^\mu \Phi^+)(D^\mu \Phi) \quad \rightarrow \quad \ldots + g^2 (\tau^a \phi^+)_i (\tau^b \phi)_i A^a_\mu A^{b\mu} + \ldots$$

$$\rightarrow^{\phi \rightarrow \phi_0} \ldots + g^2 (\tau^a \phi_0{}^+)_i (\tau^b \phi_0)_i A^a_\mu A^{b\mu} + \ldots$$

- $\tau^a \varphi_0 \neq 0$ $\Rightarrow$ Massive vector boson + Goldstone boson
- $\tau^a \varphi_0 = 0$ $\Rightarrow$ Massless vector boson + massive scalar field

Non-Abelian Higgs Mechanism, 3

- Consider SU(2) example $$D_\mu \phi = \left(\partial_\mu - ig \frac{\sigma^a}{2} A_\mu^a \right) \phi$$
- Suppose φ gets a VEV: $$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$
- Gauge boson mass term $$\left| D_\mu \phi \right|^2 = \frac{1}{2} g^2 (0, v) \tau^a \tau^b \begin{pmatrix} 0 \\ v \end{pmatrix} A_\mu^a A^{b\mu}$$
- Using the property of group generators, $\{\tau^a, \tau^b\} = \delta^{ab}/2$
- Mass term for gauge bosons:

$$\boxed{L_{mass} = \frac{g^2 v^2}{8} A_\mu^a A^{a\mu}}$$

Standard Model Synopsis

- Group: $\underbrace{SU(3)}_{\text{QCD}} \times \underbrace{SU(2) \times U(1)}_{\text{Electroweak}}$

- Gauge bosons:
  - SU(3): G_μ^i, i=1…8
  - SU(2): W_μ^i, i=1,2,3
  - U(1): B_μ
- Gauge couplings: g_s, g, g'
- SU(2) Higgs doublet: Φ

SM Higgs Mechanism

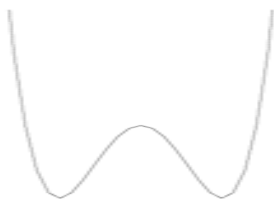
- Standard Model includes complex Higgs SU(2) doublet

$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix}\phi_1 + i\phi_2 \\ \phi_3 + i\phi_4\end{pmatrix} = \begin{pmatrix}\varphi^+ \\ \varphi^0\end{pmatrix}$$

- With SU(2) x U(1) invariant scalar potential

$$V = \mu^2\Phi^+\Phi + \lambda(\Phi^+\Phi)^2$$ Invariant under $\Phi \rightarrow -\Phi$

- If $\mu^2 < 0$, then spontaneous symmetry breaking
- Minimum of potential at: $\langle\Phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0 \\ v\end{pmatrix}$
  - Choice of minimum breaks gauge symmetry
  - Why is $\mu^2 < 0$?



More on SM Higgs Mechanism

- Couple Φ to SU(2) x U(1) gauge bosons (W_i^μ, i=1,2,3; B^μ)

$$L_S = (D^\mu \Phi)^+ (D^\mu \Phi) - V(\Phi)$$

$$D_\mu = \partial_\mu - i\frac{g}{2}\sigma^i W^i{}_\mu - i\frac{g'}{2}Y_\Phi B_\mu$$

Justify later: $Y_\Phi=1$

- Gauge boson mass terms from:

$$(D_\mu \Phi)^+ D^\mu \Phi \to \ldots + \frac{1}{8}(0,v)(gW_\mu^a \sigma^a + g'B_\mu)(gW^{b\mu}\sigma^b + g'B^\mu)\begin{pmatrix}0\\ v\end{pmatrix} + \ldots$$

$$\to \ldots + \frac{v^2}{8}\left(g^2(W_\mu^1)^2 + g^2(W_\mu^2)^2 + (-gW_\mu^3 + g'B_\mu)^2\right) + \ldots$$

More on SM Higgs Mechanism

- With massive gauge bosons:

$$W_\mu^\pm = (W_\mu^1 \mp W_\mu^2)/\sqrt{2}$$
$$Z_\mu^0 = (g\, W_\mu^3 - g' B_\mu)/\sqrt{(g^2+g'^2)}$$

$$M_W = gv/2$$
$$M_Z = \sqrt{(g^2+g'^2)}\,v/2$$

- Orthogonal combination to Z is massless photon

$$A_\mu^0 = (g'\, W_\mu^3 + gB_\mu)/\sqrt{(g^2+g'^2)}$$

$$M_W = M_Z \cos\theta_W$$

- Weak mixing angle defined

$$\cos\theta_W = \frac{g}{\sqrt{g^2+g'^2}} \qquad \sin\theta_W = \frac{g'}{\sqrt{g^2+g'^2}}$$

- $Z = -\sin\theta_W B + \cos\theta_W W^3$
- $A = \cos\theta_W B + \sin\theta_W W^3$

More on SM Higgs Mechanism

- Generate mass for W,Z using Higgs mechanism
  - Higgs VEV breaks SU(2) x U(1)→$U(1)_{em}$
  - Single Higgs doublet is minimal case
- Just like Abelian Higgs model
  - Goldstone Bosons

$$\Phi' = \frac{1}{\sqrt{2}} e^{-i\frac{\vec{\omega}_i \vec{\sigma}_i}{v}}$$

$$\omega_i \rightarrow \omega^{\pm},\ z$$

- Before spontaneous symmetry breaking:
  - Massless W_i, B, Complex Φ
- After spontaneous symmetry breaking:
  - Massive $W^{\pm}$,Z; massless γ; physical Higgs boson h

Fermi Model

- Current-current interaction of 4 fermions

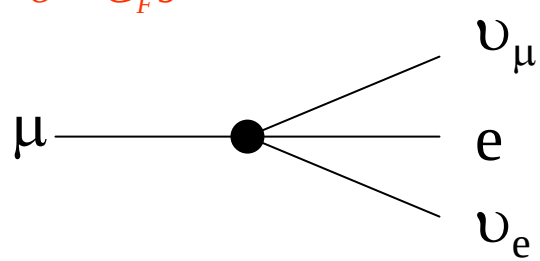
$$L_{FERMI} = -2\sqrt{2}G_F J_\rho^+ J^\rho$$

- Consider just leptonic current

$$J_\rho^{lept} = \bar{\nu}_e \gamma_\rho \left(\frac{1-\gamma_5}{2}\right) e + \bar{\nu}_\mu \gamma_\rho \left(\frac{1-\gamma_5}{2}\right) \mu + hc$$

- Only left-handed fermions feel charged current weak interactions (maximal P violation)
- This induces muon decay

$$\sigma \approx G_F^2 s$$



$G_F = 1.16637 \times 10^{-5}\ \mathrm{GeV}^{-2}$

This structure known since Fermi

Fermion Multiplet Structure

- Ψ_L couples to $W^{\pm}$ (cf Fermi theory)
  - Put in SU(2) doublets with weak isospin $I_3=\pm 1$
- Ψ_R doesn't couple to $W^{\pm}$
  - Put in SU(2) singlets with weak isospin $I_3=0$
- Fix weak hypercharge to get correct coupling to photon

Fermions in U(1) Theory

- Lagrangian is invariant under global U(1) symmetry

$$L = \overline{\psi}_f (i\partial - m_f)\psi$$

- Gauge the symmetry by requiring a local symmetry, $\eta \rightarrow \eta(x)$

$$\psi \rightarrow e^{iQ_f e\eta}\psi$$

- Local symmetry requires minimal substitution

$$\partial_\mu \rightarrow D_\mu + iQ_f eA_\mu(x)$$

Coupling Fermions to SU(2) x U(1) Gauge Fields

$$D_\mu = \partial_\mu - ig\frac{\sigma^a}{2}W_\mu^a - ig'\frac{Y}{2}B_\mu$$

- In terms of mass eigenstates

$$D_\mu = \partial_\mu - i\frac{g}{2\sqrt{2}}\left(W_\mu^+\sigma^+ + W_\mu^-\sigma^-\right) - i\frac{1}{2\sqrt{g^2+g'^2}}Z_\mu\left(g^2\sigma^3 - g'^2 Y\right) - i\frac{gg'}{2\sqrt{g^2+g'^2}}A_\mu\left(\sigma^3 + Y\right)$$

- Re-arrange couplings

$$Q_{em} = \frac{Y + I_3}{2} \qquad e = \frac{gg'}{\sqrt{g^2+g'^2}} \qquad g = \frac{e}{\sin\theta_W}$$

$$\boxed{D_\mu = \partial_\mu - i\frac{g}{2\sqrt{2}}\left(W_\mu^+\sigma^+ + W_\mu^-\sigma^-\right) - i\frac{g}{2\cos\theta_W}Z_\mu\left(I_3 - 2\sin^2\theta_W\right) - ieQ_{em}A_\mu}$$

Now include Leptons

- Simplest case, include an SU(2) doublet of left-handed leptons

$$\Psi_L = \begin{pmatrix} \nu_L = \frac{1}{2}(1-\gamma_5)\nu \\ e_L = \frac{1}{2}(1-\gamma_5)e \end{pmatrix}$$

- Only right-handed electron, $e_R=(1+\gamma_5)e/2$

$I_3=\pm 1$

  - No right-handed neutrino
- Define weak hypercharge, Y, such that $Q_{em}=(I_3+Y)/2$
  - $Y^e_L=-1$
  - $Y^e_R=-2$

To make charge come out right

**Standard Model has massless neutrinos—discovery of non-zero neutrino mass evidence for physics beyond the SM*

Leptons, 2

- By construction Isospin, I_3, commutes with weak hypercharge $[I_3,Y]=0$
- Couple gauge fields to leptons

$$\psi_L = \frac{1-\gamma_5}{2}\psi \qquad \psi_R = \frac{1+\gamma_5}{2}\psi$$

$$L_{leptons} = \bar{e}_R i\gamma^\mu\left(\partial_\mu - i\frac{g'}{2}YB_\mu\right)e_R$$

$$+\overline{\Psi}_L i\gamma^\mu\left(\partial_\mu - i\frac{g'}{2}YB_\mu - i\frac{g}{2}\sigma_i W_i^\mu\right)\Psi_L$$

- Write in terms of charged and neutral currents

$$L_{leptons} = (kinetic) + g\left(W_\mu^+ J^{\mu+} + W_\mu^- J^{\mu-} + Z_\mu J_Z^\mu\right) + eA_\mu J_{em}^\mu$$

$$J_{em}^\mu = -Q_{em}\bar{e}\gamma^\mu e$$

$$J^{+\mu} = -\frac{1}{\sqrt{2}}\left(\bar{e}_L\gamma^\mu\nu_L\right)$$

$$J_Z^\mu = -\frac{1}{2\cos\theta}\left(\bar{\nu}_L\gamma^\mu\nu_L + \bar{e}_L\gamma^\mu\left[-1+2\sin^2\theta_W\right]e_L + \bar{e}_R\gamma^\mu\left[2\sin^2\theta_W\right]e_R\right)$$

After spontaneous symmetry breaking….

- Couplings to leptons fixed:

$$e^+e^-\gamma^\mu: \quad +ie\gamma^\mu = -ieQ_e\gamma^\mu$$

$$\bar{\nu}eW^{+\mu}: \quad -i\frac{g}{2\sqrt{2}}\gamma^\mu(1-\gamma_5)$$

$$\bar{\nu}\nu Z^\mu: \quad -i\frac{g}{4}\gamma^\mu(1-\gamma_5)$$

$$\bar{e}eZ^\mu: \quad -i\frac{g}{4\cos\theta_W}\gamma^\mu\left[R_e(1+\gamma_5)+L_e(1-\gamma_5)\right]$$

$L_e = I_3 - 2Q_{em}\sin^2\theta_W$

$R_e = -2Q_{em}\sin^2\theta_W$

Higgs VEV Conserves Electric Charge

$$Q_{em} = \frac{Y + I_3}{2}$$

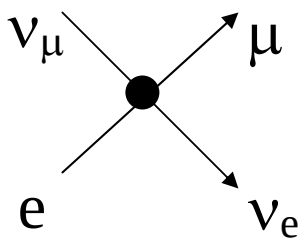
$$Y_\Phi = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad I_{3\Phi} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$\boxed{Q_{em} < \Phi > = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} = 0}$$

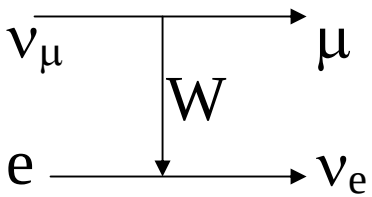
Muon decay

- Consider $\nu_\mu e \to \mu \nu_e$
- Fermi Theory:



$$-i2\sqrt{2}G_F g_{\mu\nu}\bar{u}_\mu\gamma^\mu\left(\frac{1-\gamma_5}{2}\right)u_{\nu_\mu}\bar{u}_{\nu_e}\gamma^\nu\left(\frac{1-\gamma_5}{2}\right)u_e$$

- EW Theory:



$$\frac{ig^2}{2}\frac{1}{k^2-M_W^2}g_{\mu\nu}\bar{u}_\mu\gamma^\mu\left(\frac{1-\gamma_5}{2}\right)u_{\nu_\mu}\bar{u}_{\nu_e}\gamma^\nu\left(\frac{1-\gamma_5}{2}\right)u_e$$

For $|k| << M_W$, $2\sqrt{2}G_F = g^2/2M_W^2$

For $|k| >> M_W$, $\sigma \sim 1/E^2$

Higgs Parameters

- G_F measured precisely

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W{}^2} = \frac{1}{2v^2} \qquad \boxed{v^2 = (\sqrt{2}G_F)^{-1} = (246GeV)^2}$$

- Higgs potential has 2 free parameters, μ^2, λ

$$V = \mu^2 \Phi^+ \Phi + \lambda (\Phi^+ \Phi)^2$$

- Trade μ^2, λ for v^2, $M_h{}^2$

$$V = \frac{M_h{}^2}{2} h^2 + \frac{M_h{}^2}{2v} h^3 + \frac{M_h{}^2}{8v^2} h^4 \qquad \boxed{\begin{aligned} v^2 &= -\frac{\mu^2}{2\lambda} \\ M_h{}^2 &= 2v^2\lambda \end{aligned}}$$

  - Large M_h $\rightarrow$strong Higgs self-coupling
  - A priori, Higgs mass can be anything

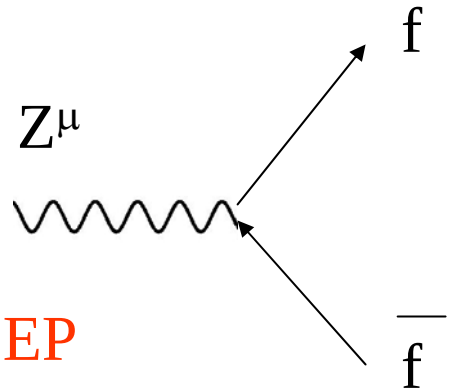
Parameters of SU(2) x U(1) Sector

- $g, g', v, M_h \Rightarrow$ Trade for:
  - α=1/137.03599911(46) from $(g-2)_e$ and quantum Hall effect
  - G_F=1.16637(1) x 10^{-5} GeV^{-2} from muon lifetime
  - M_Z=91.1875±0.0021 GeV
  - Plus masses

Decay widths trivial

- Calculate decay widths from:

$$\boxed{\Gamma(V \to f\bar{f}) = \frac{1}{2M_V}|A|^2 \frac{1}{8\pi}}$$



  – Z decays measured precisely at LEP

$$\Gamma(Z \to \nu\bar{\nu}) = \frac{G_F M_Z^3}{12\pi\sqrt{2}}$$

$$\Gamma(Z \to e^+e^-) = \frac{G_F M_Z^3}{12\pi\sqrt{2}}\left(R_e{}^2 + L_e^2\right)$$

Now Add Quarks to Standard Model

- Include color triplet quark doublet $Q_L = \begin{pmatrix} u_{iL} \\ d_{iL} \end{pmatrix}$ i=1,2,3 for color
  - Right handed quarks are SU(2) singlets, $u_R=(1+\gamma_5)u$, $d_R=(1+\gamma_5)d$
- With weak hypercharge
  - $Y_{uR}=4/3$, $Y_{dR}=-2/3$, $Y_{QL}=1/3$

$Q_{em}=(I_3+Y)/2$

- Couplings of charged current to W and Z's take the form:

$$L_{Zqq} = -\frac{g}{4\cos\theta_W}\bar{q}\gamma^\mu\left[L_q(1-\gamma_5)+R_q(1+\gamma_5)\right]qZ_\mu$$

$$L_{Wqq} = -\frac{g}{2\sqrt{2}}\left(\bar{u}\gamma^\mu(1-\gamma_5)dW_\mu^+ + \bar{d}\gamma^\mu(1-\gamma_5)uW_\mu^-\right)$$

$$L_q = I_3 + 2Q_{em}\sin^2\theta_W$$

$$R_q = 2Q_{em}\sin^2\theta_W$$

Fermions come in generations

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad u_R, \quad d_{R,} \begin{pmatrix} \nu \\ e \end{pmatrix}_L, \quad e_R$$

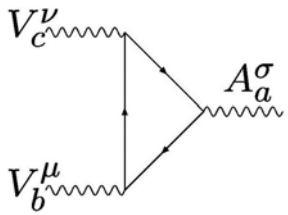
$$\begin{pmatrix} c \\ s \end{pmatrix}_L \quad c_R, \quad s_{R,} \begin{pmatrix} \nu \\ \mu \end{pmatrix}_L, \quad \mu_R$$

$$\begin{pmatrix} t \\ b \end{pmatrix}_L \quad t_R, \quad b_{R,} \begin{pmatrix} \nu \\ \tau \end{pmatrix}_L, \quad \tau_R$$

Except for masses, the generations look identical

Need Complete Generations

- Complete generations needed for ***anomaly cancellation***
- Triangle diverges; grows with energy



$$T^{\mu\nu\sigma} \approx \int \frac{d^n k}{(2\pi)^n} \frac{1}{k^3}$$

- Anomaly independent of mass; depends only on gauge properties

$$T^{\mu\nu\sigma} \approx Tr\left[\eta_i t^a, \left\{t^b, t^c\right\}\right]$$

$$\eta_i = \pm 1 \;\; for \;\; \psi_{L,R}$$

Sensible theories can't have anomalies

Anomalies, 2

- Standard Model Particles:

Particle	SU(3)	$SU(2)_L$	$U(1)_Y$
(u_L,d_l)	3	2	1/3
u_R	3	1	-1/3
d_R	3	1	1/3
(ν_L,e_L)	1	2	-1
e_R	1	1	2

- V=B, A=W_a (SU(2) gauge bosons)

$$\sum T_3 Q_{em}{}^2 = N_c N_g (1)\left(\frac{2}{3}\right)^2 + N_c N_g (-1)\left(-\frac{1}{3}\right)^2 + N_g (-1)(1)^2 = 0$$

- N_c=3 is number of colors; N_g is number of generations
- Anomaly cancellation requires complete generation

Gauge boson self-interactions

- Yang-Mills for gauge fields:

$$L_{YM} = -\frac{1}{4} W_{\mu\nu}{}^{a} W^{a,\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

$$W^{a}_{\mu\nu} = \partial_\mu W^{a}_{\nu} - \partial_\nu W^{a}_{\mu} + g\varepsilon^{abc} W^{b}_{\mu} W^{c}_{\nu}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

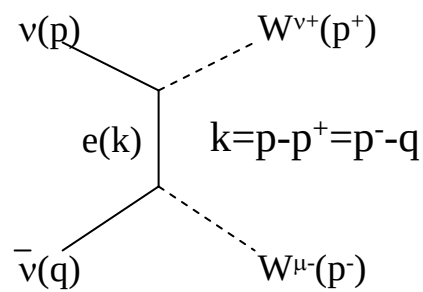
- In terms of physical fields:

$$\begin{aligned} L_{YM} = & -\frac{1}{4}\left|\partial_\mu A_\nu - \partial_\nu A_\mu - ie(W^-_\mu W^+_\nu - W^-_\nu W^+_\mu)\right|^2 \\ & -\frac{1}{4}\left|\partial_\mu Z_\nu - \partial_\nu Z_\mu + ie\frac{c_W}{s_W}(W^-_\mu W^+_\nu - W^-_\nu W^+_\mu)\right|^2 \\ & -\frac{1}{2}\left|\partial_\mu W_\nu{}^{+} - \partial_\nu W_\mu{}^{+} - ie(W^+_\mu A_\nu - W^+_\nu A_\mu) + ie\frac{c_W}{s_W}(W^+_\mu Z_\nu - W^+_\nu Z_\mu)\right|^2 \end{aligned}$$

- Triple and quartic gauge boson couplings fundamental prediction of model $\Rightarrow$ *Related to each other*

Consider W^+W^- pair production

Example: $\nu\bar{\nu}\rightarrow W^+W^-$



- t-channel amplitude:

$$\boxed{A_t(\nu\bar{\nu}\rightarrow W^+W^-) = -i\frac{g^2}{8}\bar{v}(q)\gamma^\mu(1-\gamma_5)\frac{k}{k^2}\gamma^\nu(1-\gamma_5)u(p)\varepsilon_\mu(p^-)\varepsilon_\nu(p^+)}$$

- In center-of-mass frame:

$$p = \frac{\sqrt{s}}{2}(1,0,0,1)$$

$$q = \frac{\sqrt{s}}{2}(1,0,0,-1)$$

$$p^+ = \frac{\sqrt{s}}{2}(1,0,\beta_W\sin\theta,\beta_W\cos\theta)$$

$$p^- = \frac{\sqrt{s}}{2}(1,0,-\beta_W\sin\theta,-\beta_W\cos\theta)$$

$$s = (p+q)^2$$

$$t = k^2 = (p-p')^2$$

$$\beta_W = \sqrt{1-4M_W^2/s}$$

W^+W^- pair production, 2

- Interesting physics is in the longitudinal W sector:

$$A_t(\nu\bar{\nu} \to W_L^{\ +} W_L^{\ -}) = i\frac{g^2}{4M_W^2}\bar{v}(q)k(1+\gamma_5)u(p)$$

$$\varepsilon^+ \to \frac{p^+}{M_W} + O\left(\frac{M_W^2}{s}\right)$$

- Use Dirac Equation: $\not{p}u(p)=0$

$$\longrightarrow \quad \left|A_t(\nu\bar{\nu} \to W_L^{\ +} W_L^{\ -})\right|^2 = 2G_F^2 s^2 \sin^2\theta + O\left(\frac{M_W^2}{s}\right)$$

Grows with energy

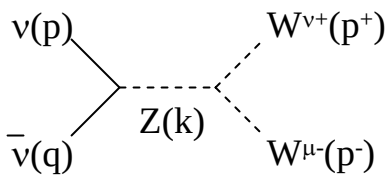
W^+W^- pair production, 3

- SM has additional contribution from s-channel Z exchange

$$A_s(\nu\bar{\nu} \to W^+W^-) = -i\frac{g^2}{4(s-M_Z^2)}\bar{v}(q)\gamma_\mu(1-\gamma_5)u(p)\left(g^{\mu\nu} - \frac{k^\mu k^\nu}{M_Z^2}\right)\left[g_{\lambda\rho}(p^- - p^+)_\nu + g_{\lambda\nu}(p^+ + k)_\rho - g_{\rho\nu}(p^- + k)_\lambda\right]\varepsilon^\lambda(p^+)\varepsilon^\rho(p^-)$$

- For longitudinal W's

$$A_s(\nu\bar{\nu} \to W_L^+W_L^-) = i\frac{g^2}{4M_W^2}\bar{v}(q)(p^+ - p^-)(1-\gamma_5)u(p)$$



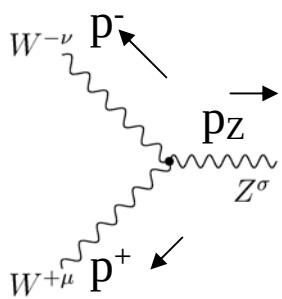
$$A_s(\nu\bar{\nu} \to W_L^+W_L^-) = -i\frac{g^2}{4M_W^2}\bar{v}(q)k(1+\gamma_5)u(p)$$

Contributions which grow with energy cancel between t- and s-channel diagrams

⇨

Depends on special form of 3-gauge boson couplings

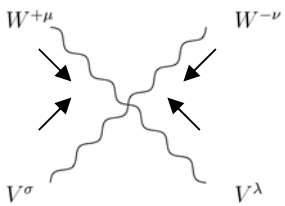
Feynman Rules for Gauge Boson Vertices



$$ig_{WWV}\, V^{\mu\nu\sigma}(p^+, p^-, p_Z)$$

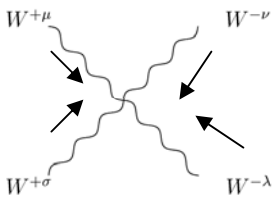
$$\boxed{\begin{aligned} g_{WW\gamma} &= e \\ g_{WWZ} &= e\cot\theta_W \end{aligned}}$$

$$V^{\mu\nu\sigma}(p^+, p^-, p_Z) = (p^+ - p^-)^{\sigma} g^{\mu\nu} + (p^- - p_Z)^{\mu} g^{\sigma\nu} + (p_Z - p^+)^{\nu} g^{\mu\sigma}$$



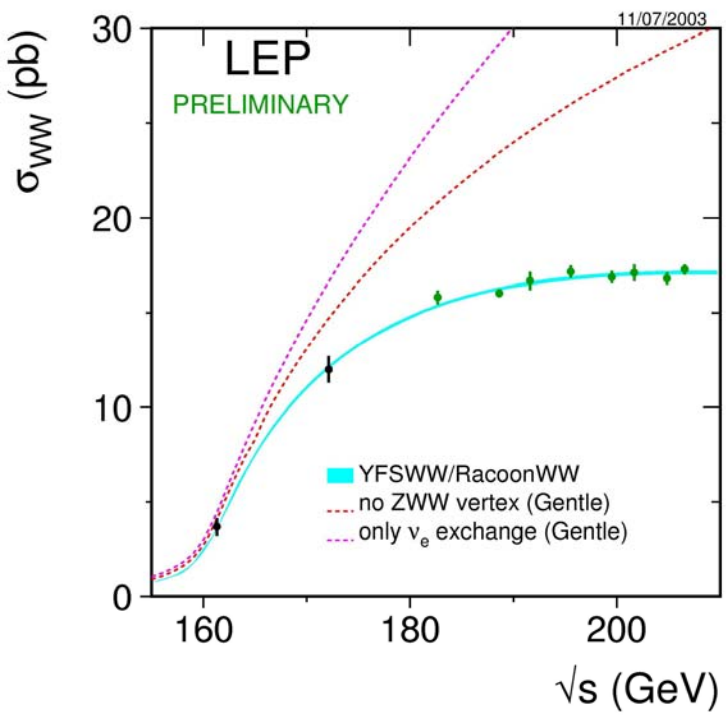
$$ig_{WWVV}\left(2g^{\sigma\lambda}g^{\mu\nu} - g^{\sigma\mu}g^{\lambda\nu} - g^{\lambda\mu}g^{\sigma\nu}\right)$$

$$\boxed{\begin{aligned} g_{WW\gamma\gamma} &= e^2 \\ g_{WW\gamma Z} &= e^2\cot\theta_W \\ g_{WWZZ} &= e^2\cot^2\theta_W \end{aligned}}$$



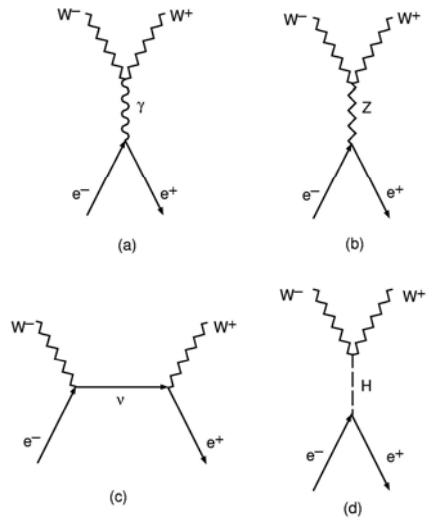
$$ig^2\left(2g^{\sigma\mu}g^{\lambda\nu} - g^{\mu\nu}g^{\lambda\sigma} - g^{\lambda\mu}g^{\sigma\nu}\right)$$

No deviations from SM at LEP2



LEP EWWG, hep-ex/0312023

No evidence for Non-SM 3 gauge boson vertices



Contribution which grows like $m_e^2 s$ cancels between Higgs diagram and others

What about fermion masses?

- Fermion mass term:

$$L = m\overline{\Psi}\Psi = m\left(\overline{\Psi}_L \Psi_R + \overline{\Psi}_R \Psi_L\right)$$

← Forbidden by SU(2)xU(1) gauge invariance

- Left-handed fermions are SU(2) doublets

$$Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L$$

- Scalar couplings to fermions:

$$L_d = -\lambda_d \overline{Q}_L \Phi d_R + h.c.$$

- Effective Higgs-fermion coupling

$$L_d = -\lambda_d \frac{1}{\sqrt{2}} (\overline{u}_L, \overline{d}_L) \begin{pmatrix} 0 \\ v+h \end{pmatrix} d_R + h.c.$$

- Mass term for down quark:

$$\boxed{\lambda_d = -\frac{M_d \sqrt{2}}{v}}$$

Fermion Masses, 2

- M_u from $\Phi_c = i\sigma_2\Phi^*$ (not allowed in SUSY)

$$\Phi_c = \begin{pmatrix} \bar{\phi}^0 \\ -\phi^- \end{pmatrix}$$

$$\boxed{\lambda_u = -\frac{M_u\sqrt{2}}{v}}$$

$$L = -\lambda_u \overline{Q}_L \Phi_c u_R + hc$$

- For 3 generations, α, β=1,2,3 (flavor indices)

$$\boxed{L_Y = -\frac{(v+h)}{\sqrt{2}} \sum_{\alpha,\beta} \left(\lambda_u^{\alpha\beta} \bar{u}_L^{\alpha} u_R^{\beta} + \lambda_d^{\alpha\beta} \bar{d}_L^{\alpha} d_R^{\beta} \right) + h.c.}$$

Fermion masses, 3

- Unitary matrices diagonalize mass matrices

$$\boxed{\begin{aligned} u_L^\alpha &= U_u^{\alpha\beta} u_L^{m\beta} \qquad & d_L^\alpha &= U_d^{\alpha\beta} d_L^{m\beta} \\ u_R^\alpha &= V_u^{\alpha\beta} u_R^{m\beta} \qquad & d_R^\alpha &= V_d^{\alpha\beta} d_R^{m\beta} \end{aligned}}$$

  - Yukawa couplings are ***diagonal*** in mass basis
  - Neutral currents remain flavor diagonal
  - Not necessarily true in models with extended Higgs sectors
- Charged current:

$$\boxed{J^{+\mu} = \frac{1}{\sqrt{2}} \bar{u}_L^\alpha \gamma^\mu d_L^\alpha = \frac{1}{\sqrt{2}} \bar{u}_L^{m\alpha} \gamma^\mu (U_u^+ V_d)_{\alpha\beta} d_L^{\beta m}}$$

CKM matrix

Review of Higgs Couplings

- Higgs couples to fermion mass
  - Largest coupling is to heaviest fermion

$$L = -\frac{m_f}{v}\bar{f}fh = -\frac{m_f}{v}\left(\bar{f}_L f_R + \bar{f}_R f_L\right)h$$

v=246 GeV

  - Top-Higgs coupling plays special role?
  - No Higgs coupling to neutrinos
- Higgs couples to gauge boson masses

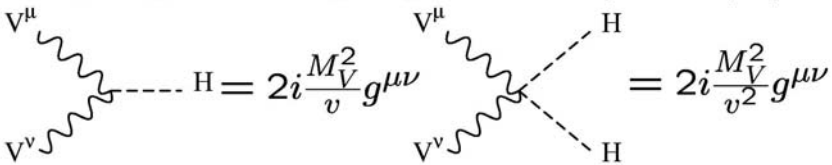
$$L = gM_W W^{+\mu}W_\mu^{-}h + \frac{gM_Z}{\cos\theta_W}Z^\mu Z_\mu h + \ldots.$$

$$g^2 = \frac{G_F}{\sqrt{2}}8M_W^2 = \frac{e^2}{\sin^2\theta_W} = \frac{4\pi\alpha}{\sin^2\theta_W}$$

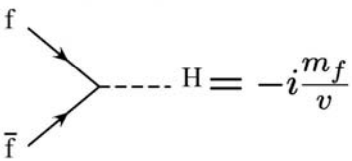
- Only free parameter is Higgs mass!
- Everything is calculable….***testable theory***

Review of Higgs Boson Feynman Rules

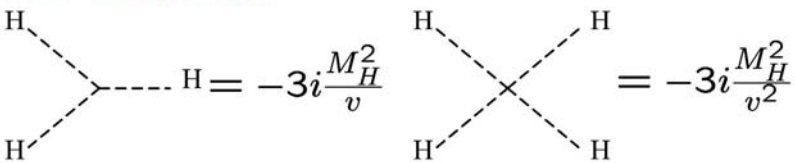
- Couplings to EW gauge bosons ($V = W, Z$):



- Couplings to fermions ($f = l, q$):



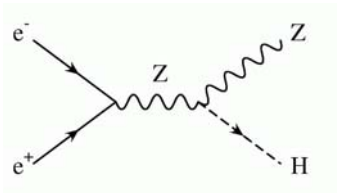
- Self-couplings:



- Higgs couples to heavy particles
- ***No tree level coupling to photons (γ) or gluons (g)***
- $M_h^2 = 2v^2\lambda \Rightarrow$ large M_h is strong coupling regime
  - M_h is parameter which separates perturbative/non-perturbative regimes

Higgs Searches at LEP2

- LEP2 searched for $e^+e^- \rightarrow Zh$
- Rate turns on rapidly after threshold, peaks just above threshold, $\sigma \sim \beta^3/s$
- Measure recoil mass of Higgs; ***result independent of Higgs decay pattern***
  - $P_{e-} = \sqrt{s}/2(1,0,0,1)$
  - $P_{e+} = \sqrt{s}/2(1,0,0,-1)$
  - $P_Z = (E_Z, p_Z)$
- Momentum conservation:
  - $(P_{e-} + P_{e+} - P_Z)^2 = P_h^2 = M_h^2$
  - $s - 2\sqrt{s}\, E_Z + M_Z^2 = M_h^2$
- LEP2 limit, ***M_h > 114.1 GeV***



Conclusion

- Standard Model consistent theory
  - Well tested (see lectures 2&3)
  - Theoretical issues (see lectures 4&5)