Fall, 2004

Motivation of Supersymmetry and its Nature

(1) Why SUSY ?
(gauge hierachy problem)

(2) Poincaré symmetry
(spacetime sym of QFT)

(3) Extended Poincaré symmetry
(Supersymmetry)

(4) Properties of SUSY multiplets

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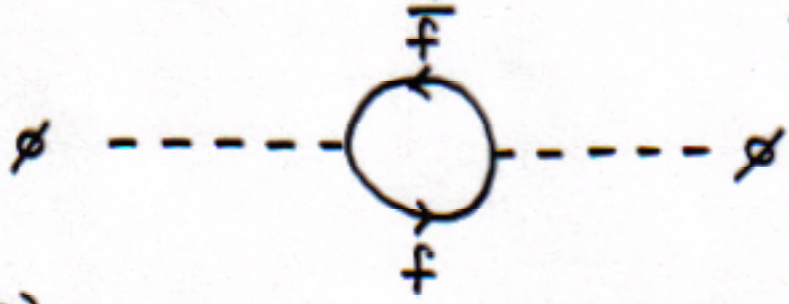
(1) Why SUSY?

* Higgs (Scalar field) self-energy correction:



$$\Pi^{f}_{\phi\phi}(p=0)$$

$$= -N(f)\int\frac{d^4k}{(2\pi)^4}\left(i\frac{\lambda_f}{\sqrt{2}}\right)\frac{i}{\not{k}-m_f}\left(i\frac{\lambda_f}{\sqrt{2}}\right)\frac{i}{\not{k}-m_f}$$

$$= -2N(f)\,\lambda_f^2\int\frac{d^4k}{(2\pi)^4}\Big[\underbrace{\frac{1}{k^2-m_f^2}}_{\mathcal{O}(\Lambda^2)}+\frac{2m_f^2}{(k^2-m_f^2)^2}\Big]$$

multiplicative factor $\left(\begin{matrix} q:3 \\ \ell:1 \end{matrix}\right)$

Λ : E scale at which SM looses effectiveness (cut-off of integral). New Physics scale

* Is $\Lambda \approx M_{Pl}$ (10^{19} GeV)?

$$\underline{m_H^2} = m_0^2 + \delta m_H^2 \sim \underline{m_0^2} - \Lambda^2$$

$(\lesssim 10^3\,\text{GeV})^2$ (Unitarity)

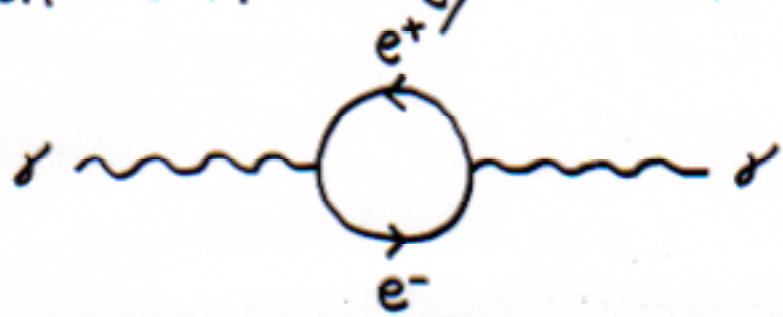
$\approx \Lambda^2 \approx (10^{19}\,\text{GeV})^2$ (bare scalar mass)

Fine tuning (possible, but NOT natural)

Gauge Hierarchy Problem

($\Lambda \approx 10^3$ GeV cures!)

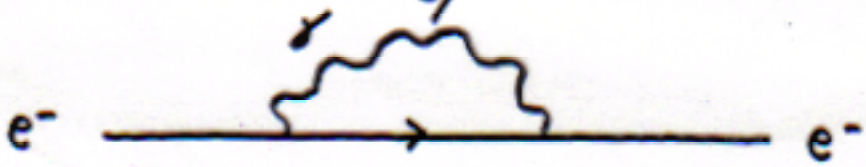
* Photon self-energy correction :



$$\Pi^{\mu\nu}_{\gamma\gamma}(p=0)$$
$$= -\int \frac{d^4k}{(2\pi)^4} (-ie\gamma^\mu) \frac{i}{\not k - m_e} (-ie\gamma^\nu) \frac{i}{\not k - m_e}$$
$$= 0$$

U(1) Gauge-invariance (Sym)
protects γ to get mass
from higher order correction.

* Electron self-energy correction :



$$\Pi_{ee}(p=0)$$
$$= \int \frac{d^4k}{(2\pi)^4} (-ie\gamma^\mu) \frac{i}{\not k - m_e} (-ie\gamma^\nu) \frac{-i g_{\mu\nu}}{k^2}$$
$$= -4e^2 m_e \underbrace{\int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2(k^2 - m_e^2)}}_{\mathcal{O}(\log \frac{\Lambda}{m_e})} \rightarrow \delta m_e \simeq 0.2\, m_e$$

for $\Lambda \approx M_{Pl}$

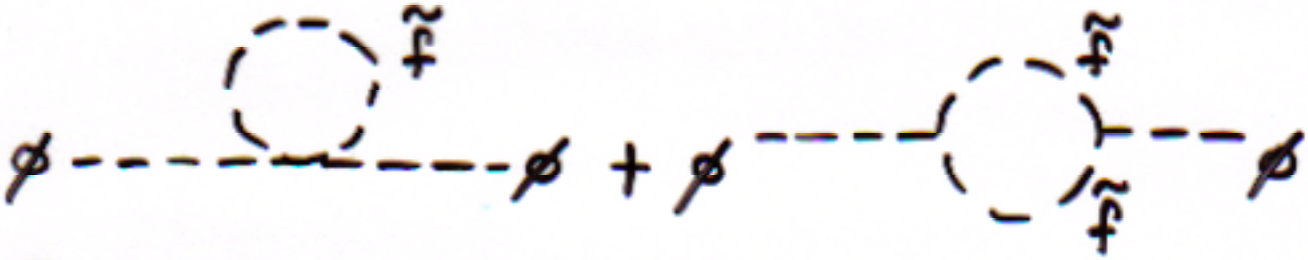
Chiral symmetry (if $m_e \to 0$)
protects e^- to get large mass correction.

* Higgs Λ^2 cancellation

Assume 2 scalar fields $(\tilde{f}_L, \tilde{f}_R)$

one counterpartner for each chiral fermion (f_L, f_R)

with the same $N(\tilde{f}) = N(f)$



$$\Pi^{\tilde{f}}_{\phi\phi}(p=0) = N(\tilde{f})\,\tilde{\lambda}_f \underbrace{\int \frac{d^4k}{(2\pi)^4}\left[\frac{1}{k^2 - m_{\tilde{f}_L}^2} + \frac{1}{k^2 - m_{\tilde{f}_R}^2}\right]}_{\mathcal{O}(\Lambda^2)\ :\ \text{another } \Lambda^2} + \mathcal{O}(\log\Lambda)$$

If $\tilde{\lambda}_f = \lambda_f^2$, (basically same coupling)

$$\Delta m_\phi^2\big|_{\text{total}} = \Pi^{f}_{\phi\phi} + \Pi^{\tilde{f}}_{\phi\phi}$$

$$= i\,\frac{3N(f)}{8\pi^2}\lambda_f^2\left[m_f^2 \log\frac{m_f^2}{\mu^2} - m_{\tilde{f}}^2 \log\frac{m_{\tilde{f}}^2}{\mu^2}\right]$$

(μ : renormalization scale
$m_{\tilde{f}_L} = m_{\tilde{f}_R}$ assumed for simplicity)

$\rightarrow$ Λ^2 cancelled !

(if $m_{\tilde{f}} = m_f \longrightarrow$ entire loop correction $= 0$)

If there exists New Physics (or new symmetry)

that can provide

a scalar counterpartner for each chiral fermion
$(s=0)$ $(s=\frac{1}{2})$

with the same coupling
$(\tilde{\lambda}_f = \lambda_f^2)$

(and the same mass) ,
$(m_{\tilde{f}} = m_f)$

we can remove Λ^2-divergence.

(entire loop correction)

(2) Poincaré Symmetry

(spacetime sym of QFT)

- <u>Poincaré algebra</u>

$$[P^\mu, P^\nu] = 0$$

$$[J^{\mu\nu}, P^\lambda] = i(g^{\nu\lambda}P^\mu - g^{\mu\lambda}P^\nu)$$

$$[J^{\mu\nu}, J^{\lambda\rho}] = i(g^{\nu\lambda}J^{\mu\rho} - g^{\mu\lambda}J^{\nu\rho} + g^{\mu\rho}J^{\nu\lambda} - g^{\nu\rho}J^{\mu\lambda})$$

P^μ : spacetime translation

$J^{\mu\nu}$: spacetime rotation (3D rotation + boost)

($J^{\mu\nu}$ satisfies $SO(3,1) \simeq SL(2,\mathbb{C})$ Lorentz algebra)

· Spinor representation of Lorentz group

A field transforms under Lorentz sym as

$$\psi_\alpha(x) \longrightarrow \left[e^{-\frac{i}{2}\theta^{\mu\nu}S_{\mu\nu}}\right]_\alpha{}^\beta \psi_\beta(x)$$

($S_{\mu\nu}$ has the same commutation relation as $J_{\mu\nu}$.)

Define
$$\left[\begin{array}{l} J^i \equiv \frac{1}{2}\epsilon^{ijk}S_{jk} \\ K^i \equiv S^{0i} \end{array}\right.$$

$[J^i, J^j] = i\epsilon^{ijk}J_k$
$[J^i, K^j] = i\epsilon^{ijk}K_k$
$[K^i, K^j] = -i\epsilon^{ijk}J_k$

and $\vec{J}_\pm = \frac{1}{2}(\vec{J} \pm i\vec{K})$

($SO(3,1) \simeq SL(2,\mathbb{C})$)

→ Irreducible rep. of Lorentz group $= (j_+, j_-)$

($\vec{J}_\pm^{\,2} = j_\pm(j_\pm + 1)$)

(ex) scalar : $(0,0)$

spin $\frac{1}{2}$ fermion : $(\frac{1}{2},0)$ or $(0,\frac{1}{2})$

- $(\frac{1}{2}, 0)$ representation

  : for left-handed 2-comp. Weyl spinor

  $\psi_\alpha \quad (\alpha = 1, 2)$

- $(0, \frac{1}{2})$ representation

  : for right-handed 2-comp. Weyl spinor

  $\bar{\psi}^{\dot{\alpha}} \quad (\dot{\alpha} = 1, 2)$

- 4-comp Dirac spinor

  : $\Psi_D = \begin{pmatrix} \xi_\alpha \\ \bar{\eta}^{\dot{\alpha}} \end{pmatrix}$

- $\psi_\alpha \to M_\alpha{}^\beta \psi_\beta \quad , \quad \bar{\psi}_{\dot{\alpha}} \to M^*{}_{\dot{\alpha}}{}^{\dot{\beta}} \psi_{\dot{\beta}}$

  where $M = e^{i \frac{\vec{\sigma}}{2}(\vec{\theta} - i\vec{\varphi})}$

  ($\vec{\theta}$: 3 rotation angle , $\vec{\varphi}$: boost parameter)

- $\bar{\psi}_{\dot{\alpha}}$ and $\psi_\alpha{}^*$ transforms in the same way.

  $\to \quad \bar{\psi}_{\dot{\alpha}} = \psi_\alpha{}^* \quad , \quad \bar{\psi}^{\dot{\alpha}} \equiv \psi^{\alpha *}$

(3) Extended Poincaré Symmetry
(= Supersymmetry)

(spacetime sym of supersymmetric QFT)

- In 1975, Haag, Lopuszanski, Sohnius proved the only possible extension of Poincaré group is by fermionic generators of either $(\frac{1}{2},0)$ or $(0,\frac{1}{2})$ representation.

  (ie, spin $\frac{1}{2}$ generators)

→ This new symmetry
- relates boson to fermion
- is the unique extension of spacetime sym

(aesthetic motivation to believe in SUSY)

- Introduce new fermionic (anti-commuting) operators

Q_α : $(\frac{1}{2},0)$ representation

$\bar{Q}^{\dot{\alpha}}$: $(0,\frac{1}{2})$ representation

$$\left(Q_\alpha |B\rangle = |F\rangle \ , \quad Q_\alpha |F\rangle = |B\rangle \right)$$

Supersymmetry algebra

$$[P^\mu, P^\nu] = 0$$

$$[J^{\mu\nu}, P^\lambda] = i(g^{\nu\lambda} P^\mu - g^{\mu\lambda} P^\nu)$$

$$[J^{\mu\nu}, J^{\lambda\rho}] = i(g^{\nu\lambda} J^{\mu\rho} - g^{\mu\lambda} J^{\nu\rho} + g^{\mu\rho} J^{\nu\lambda} - g^{\nu\rho} J^{\mu\lambda})$$

$$[Q_\alpha, P^\mu] = 0 \qquad [\bar{Q}^{\dot\alpha}, P^\mu] = 0$$

$$[Q_\alpha, J^{\mu\nu}] = i(\sigma^{\mu\nu})_\alpha{}^\beta Q_\beta \qquad [\bar{Q}^{\dot\alpha}, J^{\mu\nu}] = i(\bar{\sigma}^{\mu\nu})^{\dot\alpha}{}_{\dot\beta} \bar{Q}^{\dot\beta}$$

$$\{Q_\alpha, \bar{Q}_{\dot\beta}\} = 2(\sigma^\mu)_{\alpha\dot\beta} P_\mu$$

$$\{Q_\alpha, Q_\beta\} = 0 \qquad \{\bar{Q}_{\dot\alpha}, \bar{Q}_{\dot\beta}\} = 0$$

$$\left(\begin{aligned} \sigma^\mu &\equiv (1, \vec{\sigma}) \\ \sigma^{\mu\nu} &\equiv \tfrac{1}{4}(\sigma^\mu \bar{\sigma}^\nu - \sigma^\nu \bar{\sigma}^\mu) \\ \bar{\sigma}^{\mu\nu} &\equiv \tfrac{1}{4}(\bar{\sigma}^\mu \sigma^\nu - \bar{\sigma}^\nu \sigma^\mu) \end{aligned} \right)$$

Poincaré group $\subset$ Supersymmetry group

(Lie algebra) (graded Lie algebra)

[ ] only [ ], { }

(4) Properties of SUSY multiplet

· SUSY multiplet = {particle & its superpartners}

supersymmetry representation of an irreducible state

(best described by superfield)

(ex) $\{ |F\rangle, |B\rangle = Q_\alpha |F\rangle \}$

(i) <u>Same mass ($m_F = m_B$)</u>

$$P^2 |F\rangle = m_F^2 |F\rangle$$

$$P^2 |B\rangle = P^2 Q_\alpha |F\rangle = Q_\alpha P^2 |F\rangle$$

$$= Q_\alpha m_F^2 |F\rangle = m_F^2 |B\rangle \qquad ([Q_\alpha, P^2] = 0)$$

(ii) <u>Equal number of F and B states ($n_F = n_B$)</u>

· Define an operator $(-1)^F$

$$(-1)^F |B\rangle = +|B\rangle, \quad (-1)^F |F\rangle = -|F\rangle$$

$(\rightarrow Q_\alpha (-1)^F = -(-1)^F Q_\alpha)$ immediate consequence

· Use $\{Q_\alpha, \bar{Q}_{\dot\beta}\} = 2\sigma^\mu{}_{\alpha\dot\beta} P_\mu$

$$\mathrm{Tr}\left[(-1)^F \{Q_\alpha, \bar{Q}_{\dot\beta}\}\right]$$

$$= \mathrm{Tr}\left[(-1)^F Q_\alpha \bar{Q}_{\dot\beta} + (-1)^F \bar{Q}_{\dot\beta} Q_\alpha\right]$$

$$= \mathrm{Tr}\left[\underbrace{-Q_\alpha (-1)^F \bar{Q}_{\dot\beta}}_{\text{'last page consequence'}} + \underbrace{Q_\alpha (-1)^F \bar{Q}_{\dot\beta}}_{\text{'cyclic property of Trace'}}\right]$$

$$= 0$$

$$\rightarrow \mathrm{Tr}\left[(-1)^F 2\sigma^\mu_{\alpha\dot\beta} P_\mu\right]$$

$$= 2\sigma^\mu_{\alpha\dot\beta}\, \mathrm{Tr}\left[(-1)^F P_\mu\right] = 0$$

$$\rightarrow \mathrm{Tr}\left[(-1)^F\right] = 0$$ for fixed non-zero P_μ

$\rightarrow$ SUSY representation contains equal number of bosonic & fermionic states

(iii) <u>"massless" SUSY multiplet $= \{|\lambda\rangle, |\lambda+\tfrac{1}{2}\rangle\}$</u>

only two states different by spin $\frac{1}{2}$

Non-zero masses of particles are generated by SUSY-breaking effects.

(EWSB triggered by SUSY-breaking)

$\rightarrow$ Consider only massless states.

· A massless state : $|p, \lambda\rangle$

In reference frame of $P_\mu = (E, 0, 0, E)$

$$\{Q_\alpha, \bar{Q}_{\dot\beta}\} = 2\sigma^\mu{}_{\alpha\dot\beta} P_\mu$$
$$= 2(\sigma^0 E - \sigma^3 E)_{\alpha\dot\beta}$$
$$= 4E \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}_{\alpha\dot\beta}$$

$\rightarrow$ $\{Q_1, \bar{Q}_{\dot 1}\} = 0 \quad \rightarrow Q_1 = 0, \ \bar{Q}_{\dot 1} = 0$

$$\left(\{Q_1, \bar{Q}_{\dot 1}\} = Q_1 \bar{Q}_{\dot 1} + \bar{Q}_{\dot 1} Q_1 = \underbrace{Q_1 Q_1^*}_{\geq 0} + \underbrace{Q_1^* Q_1}_{\geq 0} \right)$$

$\{Q_2, \bar{Q}_{\dot 2}\} = 4E \rightarrow$ Define

$$a \equiv \frac{Q_2}{\sqrt{4E}}, \quad a^\dagger \equiv \frac{\bar{Q}_{\dot 2}}{\sqrt{4E}}$$

$a, a^\dagger$ satisfy algebra of fermionic annihilation, creation operators

$$\{a, a^\dagger\} = 1, \quad \{a, a\} = 0, \quad \{a^\dagger, a^\dagger\} = 0$$

· Start with the lowest helicity λ_0 $(a|\lambda_0\rangle = 0)$

$\rightarrow$ $a^\dagger|\lambda_0\rangle = |\lambda_0 + \frac{1}{2}\rangle$

and no more. $(a^\dagger a^\dagger|\lambda_0\rangle = 0)$

$\rightarrow$ Only $|\lambda_0\rangle$ and $|\lambda_0 + \frac{1}{2}\rangle$ states survive.

$\{|0\rangle, |\frac{1}{2}\rangle\}$, $\{|\frac{1}{2}\rangle, |1\rangle\}$, $\{|1\rangle, |\frac{3}{2}\rangle\}$, ...

are allowed SUSY multiplets.

(iv) <u>same charge</u>

Since SUSY is the only possible extension of Poincaré symmetry, it commutes with all other (internal) symmetry.

(ex) $\{\underbrace{|F\rangle}_{SU(2)_L \text{ doublet}}, \underbrace{|B\rangle = Q_a|F\rangle}_{\uparrow}\}$

If there exists New Physics (or new symmetry)

that can provide

a scalar counterpartner for each chiral fermion
$(s=0)$ $(s=\frac{1}{2})$

with the same coupling
$(\tilde{\lambda}_f = \lambda_f^2)$

(and the same mass),
$(m_{\tilde{f}} = m_f)$

we can remove Λ^2-divergence.
(entire loop correction)

→ Supersymmetry does it!

(Plus, it naturally arises as an extension of spacetime symmetry.)

* Summary

- Higgs (scalar) mass stabilization addresses a hierarchy problem (fine-tuning).

- SUSY is a good candidate to resolve this problem. (Λ^2-divergence cancellation)

- SUSY is a unique extension of Poincaré (spacetime) symmetry of QFT.

- For a given particle, there exists a superpartner of the same mass, charge and only the spin is different by $\frac{1}{2}$.
  (doubles the particle spectrum)