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Construction of Supersymmetric Lagrangian

(1) SUSY transformation & Superfields

(2) Chiral superfield & Vector superfield

(3) Supersymmetric Lagrangian

(4) Soft-breaking of SUSY

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(1) SUSY transf. & Superfields

· Supermultiplet = { particle, superpartner }

{ $s=0$, $s=\frac{1}{2}$ } : chiral superfield

"squark" quark MATTER

"slepton" lepton

higgs "higgsino"

{ $s=\frac{1}{2}$, $s=1$ } : vector superfield

"gaugino" gauge boson FORCE

(Gauge Int.)

{ $s=\frac{3}{2}$, $s=2$ } : gravity supermultiplet

"gravitino" graviton GRAVITY

· A compact way to describe supermultiplet

: superfield

contains boson & fermion in the same field

Superspace and Superfield

- Introduce $\underbrace{\theta^\alpha, \bar{\theta}^{\dot{\alpha}}}_{\text{fermionic coordinates (superspace)}}$ for fermionic generators $Q_\alpha, \bar{Q}_{\dot{\alpha}}$. $(\alpha, \dot{\alpha} = 1, 2)$

  (analogy : x^μ for P_μ)

- Spacetime is extended with SUSY.

Point in Minkowski space $\rightarrow$	in Superspace
x^μ	$x^\mu, \theta^\alpha, \bar{\theta}^{\dot{\alpha}}$
Field in Minkowski space $\rightarrow$	in Superspace
$\phi(x^\mu)$	$\phi(x^\mu, \theta^\alpha, \bar{\theta}^{\dot{\alpha}})$ (superfield)

$\theta, \bar{\theta}$: Grassman variables (fermionic)

$$\{\theta_\alpha, \theta_\beta\} = \{\theta_\alpha, \bar{\theta}_{\dot{\beta}}\} = \{\bar{\theta}_{\dot{\alpha}}, \bar{\theta}_{\dot{\beta}}\} = 0 \rightarrow \theta_1\theta_1 = 0$$

$$\int d\theta_\alpha = \int d\bar{\theta}_{\dot{\alpha}} = 0$$

$$\int d\theta_\alpha \theta_\alpha = \int d\bar{\theta}_{\dot{\alpha}} \bar{\theta}_{\dot{\alpha}} = 1$$

$$\phi(\theta) = c_0 + c_1 \underbrace{\theta f}_{\theta^\alpha f_\alpha} + c_2 \underbrace{\theta\theta}_{\theta^\alpha\theta_\alpha = \theta^1\theta_1 + \theta^2\theta_2 = \theta^1\theta^2 - \theta^2\theta^1} + (\text{no more})$$

· We can express SUSY algebra in terms of commutators (or Lie algebra) with $\theta, \bar{\theta}$.

$$\begin{cases} \{Q_\alpha, \bar{Q}_{\dot\beta}\} = 2\sigma^\mu{}_{\alpha\dot\beta} P_\mu \\ \{Q_\alpha, Q_\beta\} = \{\bar{Q}_{\dot\alpha}, \bar{Q}_{\dot\beta}\} = 0 \\ [P^\mu, Q_\alpha] = [P^\mu, \bar{Q}_{\dot\beta}] = 0 \end{cases}$$

$$\begin{cases} [\theta Q, \bar{\theta}\bar{Q}] = 2\theta\sigma^\mu\bar{\theta} P_\mu \\ [\theta Q, \theta Q] = [\bar{\theta}\bar{Q}, \bar{\theta}\bar{Q}] = 0 \\ [P^\mu, \theta Q] = [P^\mu, \bar{\theta}\bar{Q}] = 0 \end{cases}$$

· SUSY transformation

· Group element of SUSY transf.

$$G(x,\theta,\bar\theta) = e^{i[\theta Q + \bar\theta \bar Q - x\cdot P]}$$

(in analogy of group element of Lie algebra)

· $G(0,\xi,\bar\xi)\, G(x,\theta,\bar\theta)$

$$= e^{i[\xi Q + \bar\xi \bar Q]}\, e^{i[\theta Q + \bar\theta \bar Q - x\cdot P]}$$

$$= e^{i[(\theta+\xi)Q + (\bar\theta+\bar\xi)\bar Q - (x + i\theta\sigma\bar\xi - i\xi\sigma\bar\theta)\cdot P]}$$

(by Baker–Campbell–Hausdorff formula & SUSY algebra
$e^A e^B = e^{A+B+\frac{1}{2}[A,B]+\cdots}$)

$$= G(x + i\theta\sigma\bar\xi - i\xi\sigma\bar\theta,\ \theta+\xi,\ \bar\theta+\bar\xi)$$

· Multiplication of group element induces a motion in parameter space.

$$\phi(x,\theta,\bar\theta) \xrightarrow{\xi,\bar\xi} \phi(x+i\theta\sigma\bar\xi - i\xi\sigma\bar\theta,\ \theta+\xi,\ \bar\theta+\bar\xi)$$

$$= \phi(x,\theta,\bar\theta) + \delta_S\phi(x,\theta,\bar\theta)$$

$$\left(\begin{aligned}\delta_S\phi &= e^{i[\xi Q+\bar\xi\bar Q]}\phi - \phi \\ &= i[\xi Q + \bar\xi \bar Q]\phi\end{aligned}\right)$$ infinitesimal SUSY transf.

· This motion can be generated by the differential operator form.

$$\delta_s \phi(x,\theta,\bar\theta)$$

$$= i[\xi Q + \bar\xi \bar Q]\,\phi(x,\theta,\bar\theta)$$

$$= i\,[\,\xi^\alpha \underbrace{(-i\partial_\alpha + \sigma^\mu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_\mu)}_{=Q_\alpha} + \bar\xi^{\dot\alpha}\underbrace{(\bar\partial_{\dot\alpha} - \theta^\alpha\sigma^\mu_{\alpha\dot\alpha}\partial_\mu)}_{=\bar Q_{\dot\alpha}}]\,\phi$$

in differential operator forms

(analogy : $P_\mu = i\partial_\mu$)

$$\left(\partial_\alpha \equiv \frac{\partial}{\partial\theta^\alpha}\quad,\quad \bar\partial_{\dot\alpha} \equiv \frac{\partial}{\partial\bar\theta^{\dot\alpha}}\right)$$

Superfield in components

- Most general form of superfield $\phi(x,\theta,\bar\theta)$ in power series of $\theta, \bar\theta$:

$$\phi(x,\theta,\bar\theta) = \varphi(x) + \theta\psi(x) + \bar\theta\bar\chi(x)$$
$$+ \theta\theta F(x) + \bar\theta\bar\theta H(x) + \theta\sigma^\mu\bar\theta A_\mu(x)$$
$$+ \theta\theta\bar\theta\bar\lambda(x) + \bar\theta\bar\theta\theta\zeta(x) + \theta\theta\bar\theta\bar\theta D(x)$$

Components
- φ, F, H, D : scalars $(s=0)$
- $\psi, \bar\chi, \bar\lambda, \zeta$: Weyl spinors $(s=1/2)$
- A_μ : vector boson $(s=1)$

(Cf. graviton $(s=2)$, gravitino $(s=3/2)$ emerge only in general coordinate transf. invariance (General Relativity).

$$\underset{(s=2)}{h_{\mu\nu}} \longrightarrow h_{\mu\nu} + c\,\bar\xi(\gamma_\mu \underset{(s=3/2)}{\xi_\nu} + \gamma_\nu \xi_\mu)$$
)

$\phi = \{ \varphi, F, H, D, \psi, \bar{\chi}, \bar{\lambda}, J, A_\mu \}$

is a redundant representation.

(irreducible SUSY multiplet
$= \{ |\lambda\rangle, |\lambda + \frac{1}{2}\rangle \}$)

→ Give some conditions to most general ϕ to find some irreducible superfields.

$\bar{D}_{\dot{\alpha}} \phi = 0$ → turns out ϕ = LH chiral superfield

$D_\alpha \phi = 0$ → ϕ = RH chiral superfield

$\phi = \phi^\dagger$ → ϕ = vector superfield

($D_\alpha, \bar{D}_{\dot{\alpha}}$ = SUSY covariant derivatives)

· SUSY covariant derivatives

· Infinitesimal U(1) sym transf. (analogy)

$$\phi \to \phi + \underbrace{i\Lambda\phi}_{\delta\phi = i\Lambda\phi}$$

Try ∂_μ :

$$\partial_\mu\phi \to \partial_\mu\phi + \underbrace{i(\partial_\mu\Lambda)\phi + i\Lambda(\partial_\mu\phi)}_{\delta(\partial_\mu\phi) \neq i\Lambda(\partial_\mu\phi)}$$

Not covariant

Define $D_\mu \equiv \partial_\mu - ieA_\mu$: U(1) covariant derivative

$$D_\mu\phi \to D_\mu\phi + \underbrace{i\Lambda(D_\mu\phi)}_{\delta(D_\mu\phi) = i\Lambda(D_\mu\phi)}$$

Covariant (transf. as ϕ)

· Similarly for SUSY transf.

$$\phi \to \phi + \underbrace{i(\xi Q + \bar{\xi}\bar{Q})\phi}_{\delta_s\phi = i(\xi Q + \bar{\xi}\bar{Q})\phi}$$

$$\left[\begin{array}{l} D_\alpha \equiv \partial_\alpha + i\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu \\ \bar{D}_{\dot{\alpha}} \equiv -\bar{\partial}_{\dot{\alpha}} - i\theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu \end{array}\right]$$

SUSY covariant derivatives.

(2) Chiral and Vector Superfields

· Left-Handed Chiral Superfield

$$\bar{D}_{\dot{\alpha}}\,\phi(x,\theta,\bar{\theta}) = 0$$

$$\phi = \Phi = \varphi(x) + \sqrt{2}\,\theta\psi(x) - i\theta\sigma^{\mu}\bar{\theta}\,\partial_{\mu}\varphi(x)$$
$$+ \frac{i}{\sqrt{2}}\theta\theta\,\partial_{\mu}\psi(x)\sigma^{\mu}\bar{\theta} - \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\,\partial^{\mu}\partial_{\mu}\varphi(x) - \theta\theta F(x)$$

φ : scalar $(s=0)$

ψ : Weyl spinor $(s=1/2)$

F : Auxiliary (unphysical) field
(replaced by others after EOM is taken)

→ No $\bar{\chi}(x)$ or $A_{\mu}(x)$.
(RH chiral spinor) (vector boson)

→ Φ is the Left-Handed chiral superfield.
$\{|0\rangle, |\tfrac{1}{2}\rangle\}$

· infinitesimal SUSY transf of component fields :

$$\delta_s\varphi = \sqrt{2}\,\xi\psi \qquad (B\to F)$$
$$\delta_s\psi_{\alpha} = -\sqrt{2}F\xi_{\alpha} - i\sqrt{2}\,\sigma^{\mu}_{\alpha\dot{\alpha}}\partial_{\mu}\varphi \qquad (F\to B)$$
$$\delta_s F = \partial_{\mu}(-i\sqrt{2}\,\psi\sigma^{\mu}\xi) \qquad (\text{aux.}\to\text{total derivative})$$

obtain by equating
$$\left(\begin{aligned} \delta_s\phi &= \delta_s\varphi + \sqrt{2}\,\theta\delta_s\psi + \cdots \\ \delta_s\phi &= i[\xi^{\alpha}(-i\partial_{\alpha} + \sigma^{\mu}_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_{\mu}) + \cdots]\phi \end{aligned}\right)$$

· Right-Handed Chiral Superfied

$$D_\alpha \phi(x,\theta,\bar\theta) = 0$$

$\phi = \Phi^\dagger$ (conjugate of LH chiral superfield)

$$= \varphi^*(x) + \sqrt{2}\,\bar\theta\bar\psi(x) + i\theta\sigma^\mu\bar\theta\,\partial_\mu\varphi^*(x)$$

$$- \frac{i}{\sqrt{2}}\bar\theta\bar\theta\,\theta\sigma^\mu\partial_\mu\bar\psi(x) - \frac{1}{4}\theta\theta\bar\theta\bar\theta\,\partial^\mu\partial_\mu\varphi^*(x) - \bar\theta\bar\theta\,F^*(x)$$

· Vector Superfield

$$\phi(x,\theta,\bar\theta) = \phi^\dagger(x,\theta,\bar\theta)$$

In Wess-Zumino gauge (eliminates many unphysical deg. of freedom)

$$\phi = V = \theta\sigma^\mu\bar\theta V_\mu + i\theta\theta\bar\theta\bar\lambda - i\bar\theta\bar\theta\theta\lambda + \frac{1}{2}\theta\theta\bar\theta\bar\theta D$$

V_μ : Vector boson (gauge) $(s=1)$

λ_α : Weyl spinor (gaugino) $(s=1/2)$

D : auxiliary (unphysical) field

→ V is the vector superfield. $\{|\frac{1}{2}\rangle, |1\rangle\}$

(transforms as adjoint representation

$$e^V \to e^{-i\Lambda^\dagger} e^V e^{i\Lambda}$$
)

· infinitesimal SUSY transf of component fields

$$\delta\lambda_\alpha = -iD\xi_\alpha - \frac{1}{2}(\sigma^\mu\bar\sigma^\nu)_\alpha{}^\beta\xi_\beta(\partial_\mu V_\nu - \partial_\nu V_\mu) \quad (F\to B)$$

$$\delta V^\mu = i(\xi\sigma^\mu\bar\lambda - \lambda\sigma^\mu\bar\xi) - \partial^\mu(\xi\chi + \bar\xi\bar\chi) \quad (B\to F)$$

$$\delta D = \partial_\mu(-\xi\sigma^\mu\bar\lambda + \lambda\sigma^\mu\bar\xi) \quad (\text{aux.} \to \text{tot. derivative})$$

* Renormalizable SUSY $\mathcal{L}$

(before Gauge theory)

$$\mathcal{L} = \int d^2\theta\, d^2\bar{\theta}\, \Phi_i^\dagger \Phi_i + \left(\int d^2\theta\, f(\Phi) + \text{H.C.} \right)$$

Superpotential : $f(\Phi) = \frac{1}{2} m_{ij} \Phi_i \Phi_j + \frac{1}{3} \lambda_{ijk} \Phi_i \Phi_j \Phi_k$

Higher terms are NOT renormalizable

Euler-Lagrange Eq. $\rightarrow$ $F_i^\dagger = -\frac{\partial f(\varphi)}{\partial \varphi} = -m_{ij}\varphi_j - \lambda_{ijk}\varphi_j\varphi_k$

auxiliary scalar

$$\mathcal{L} = \partial_\mu \varphi_i^\dagger \partial^\mu \varphi_i + i \bar{\psi}_i \bar{\sigma}^\mu \partial_\mu \psi_i$$

K.E. of φ, ψ (derivative)

$$- |m_{ij}\varphi_j + \lambda_{ijk}\varphi_j\varphi_k|^2 - \frac{1}{2} m_{ij}\psi_i\psi_j$$

scalar int. & mass of φ, ψ (equal)

$$- \lambda_{ijk}\psi_i\psi_j\varphi_k$$

Yukawa int.

* Gauge Interaction (thru Minimal Coupling)

Vector Superfield

$$\int d^2\theta\, d^2\bar\theta\, \Phi^\dagger \Phi \longrightarrow \int d^2\theta\, d^2\bar\theta\, \Phi^\dagger e^{2gV} \Phi$$

$$= |D_\mu \varphi|^2 + i\bar\psi \bar\sigma^\mu D_\mu \psi + ig\sqrt{2}\,(\varphi^* \lambda \psi - \bar\lambda \bar\psi \varphi)$$

gauge & matter int. gauge-strength Yukawa among φ, ψ, λ

$$+ g\varphi^* D \varphi + |F|^2$$

scalar int.

$$\left(\begin{array}{l} D_\mu \equiv \partial_\mu + igV_\mu^a T^a \\ \text{Euler-Lagrange eq.} \rightarrow \quad D_a = -g \sum_{ij} \varphi_i^\dagger T_a^{ij} \varphi_j \end{array} \right)$$

Additional $\quad W^\alpha \equiv \bar D^2 e^{-V} D^\alpha e^V$ (field-strength superfield)

$$\int d^2\theta\, \frac{1}{32g^2} W^\alpha W_\alpha + \text{H.C.}$$

$$= \underbrace{-\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}}_{\text{K.E. of gauge}} \underbrace{- \frac{i}{2} \lambda^a \sigma_\mu \partial^\mu \bar\lambda^a}_{\text{K.E. of gaugino}}$$

$$\underbrace{+ \frac{1}{2} D^a D^a}_{\text{scalar int.}} \underbrace{+ \frac{1}{2} g f^{abc} \lambda^a \sigma_\mu V^{\mu b} \bar\lambda^c}_{\text{gauge \& gaugino int.}} + \text{H.C.}$$

* SUSYic Gauge Theory $\mathcal{L}$

$$\mathcal{L}_{SUSY} = \int d^2\theta d^2\bar{\theta}\ \Phi^\dagger_i (e^{2gV})_{ij} \Phi_j$$
$$+ \int d^2\theta\, f(\Phi) + \int d^2\bar{\theta}\, f(\Phi^\dagger)$$
$$+ \int d^2\theta \frac{1}{32g^2} W^\alpha W_\alpha + \int d^2\bar{\theta} \frac{1}{32g^2} \bar{W}_{\dot\alpha} \bar{W}^{\dot\alpha}$$

$$\left(\begin{array}{l} V \equiv V^a T^a \\ f(\Phi) \equiv \frac{1}{2} m_{ij} \Phi_i \Phi_j + \frac{1}{3} \lambda_{ijk} \Phi_i \Phi_j \Phi_k \\ W_\alpha \equiv \bar{D}^2 [\, e^{-V} D_\alpha e^V \,] \end{array} \right)$$

Includes ingredients of SM :

kinetic term of φ, ψ, V_μ

Yukawa int. of $\psi\psi\varphi$

scalar int. of φ

gauge-matter int. V_μ & ψ, V_μ & φ

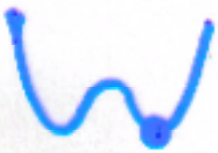
We're almost ready for SUSYic SM!

(3) Soft Breaking of SUSY

- SUSY, if it exists, must be broken.

  ($\tilde{e} > 99$ GeV while $e = 0.5$ GeV [LEP2])

- Spontaneous sym breaking is fine.



The vacuum is not symmetric but Lagrangian is.

- Is SUSY broken spontaneously?

  Not successful to make realistic models.

(If SUSY is at Lagrangian level,

$$\mathrm{Str}\, M^2 \equiv \sum_J (-1)^{2J} (2J+1)\, m_J^2 = 0$$

$$\sum m_0^2 - 2\sum m_{1/2}^2 + 3\sum m_1^2 = 0$$

: for each set of quantum numbers.

For $e = -\frac{1}{3}$, color = Red

$$\sum m_0^2 - \underbrace{2(m_d^2 + m_s^2 + m_b^2)}_{\sim 40\ \mathrm{GeV}^2} = 0$$

$\rightarrow$ heaviest $m_0 \lesssim 7$ GeV (must have been detected.))

- SUSY is broken explicitly.

  But NOT arbitrarily.

  if we still want SUSY as a solution of gauge hierarchy problem (Λ^2-div cancellation)

- Only 4 terms can break SUSY explicitly while keeping Λ^2-div cancellation.

  (soft breaking terms)

  scalar mass : $-m_i^2 |\varphi_i|^2$

  gaugino mass : $-\frac{1}{2} m_i \bar{\lambda}_i \lambda_i$

  trilinear scalar int : $-A_{ijk} \varphi_i \varphi_j \varphi_k$

  bilinear scalar int : $-B_{ij} \varphi_i \varphi_j$

  fermion mass, quartic scalar int, ...
  are NOT allowed as soft breaking terms.

- To avoid creating another fine-tuning, soft breaking terms should be near EW scale.

  $\Rightarrow$ Superpartner mass $\lesssim \mathcal{O}(1\,\text{TeV})$

  LHC may find some of them.

Q. How to understand

SUSY explicitly broken (soft breaking)?

Not Symmetry at all!

A. As a result of Spontaneous SUSY breaking in a fundamental (high E) theory.

High E — Supergravity (local SUSY)

$\exp[i(\xi(x)Q + \bar{\xi}(x)\bar{Q} - a^{\mu}(x)P_{\mu})]$

gravity multiplet

Non-renormalizable

Spontaneous Breaking

$\downarrow$

Low E — low-E global SUSY

$\exp[i(\xi Q + \bar{\xi}\bar{Q} - a^{\mu}P_{\mu})]$

No gravity multiplet

Renormalizable

Soft Breaking

* Summary

1. Spacetime is extended.

$$x^\mu \to x^\mu, \theta^\alpha, \bar{\theta}^{\dot{\alpha}} \quad \text{(superspace)}$$
$$\phi(x) \to \phi(x, \theta, \bar{\theta}) \quad \text{(superfield)}$$

2. Two important superfields to construct low-E SUSYic Lagrangian:

$\{s=0, s=\frac{1}{2}\}$: chiral superfield $(\bar{D}_{\dot{\alpha}}\Phi = 0)$

$\{s=\frac{1}{2}, s=1\}$: vector superfield $(V^\dagger = V)$

3. SUSYic Lagrangian can provide all SM ingredients.

We will build SUSYic SM next time.

4. SUSY is broken explicitly by soft breaking terms at TeV-scale.

probable at LHC or NLC