Fall, 2004

Brief Review of Minimal Supersymmetric Standard Model

(1) Definition of MSSM

(2) EWSB in MSSM

(3) Superparticles Mixing

(4) Feynman Rules

(1) Definition of MSSM

* Ingredients of MSSM

"Minimal" : Minimal number of Superfields and Interactions

- Minimal gauge group

  SM $SU(3)_c \times \underbrace{SU(2)_L \times U(1)_Y}_{\text{SSB to } U(1)_{QED}}$

- Minimal particle contents

  all SM ptls + <u>1 extra Higgs</u>
  + superparticles of these

- Soft breaking terms

  no superpartner w/ same mass

- An exact <u>R-parity</u>

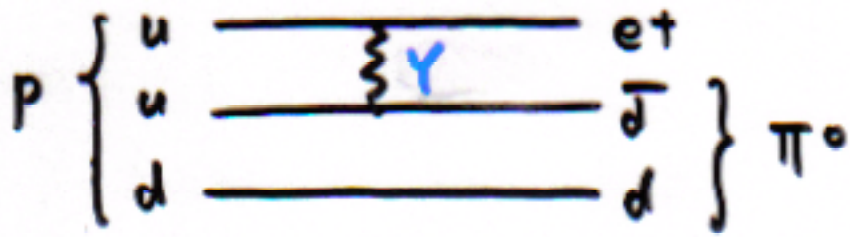
  to avoid Baryon, Lepton # violation

Proton decay

Many new physics predicts proton decay.

$\tau_p \gtrsim 10^{33}$ yrs constrains models.

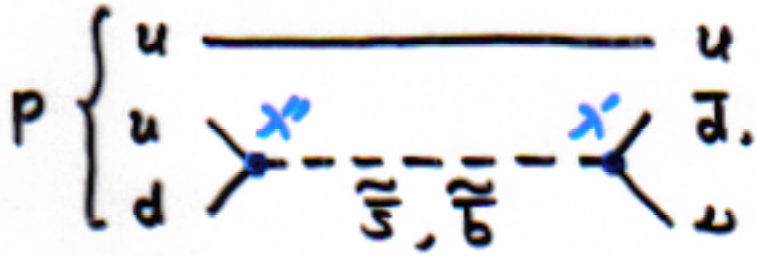
- In SU(5) GUT



: $p \to e^+ \pi^0$

$\tau_p \sim 10^{31}$ yrs

- In MSSM without R-parity



: $p \to \nu \pi^+$ (νK^+)

To satisfy exp. constraint ($\tau_p \gtrsim 10^{33}$ yrs),

$$|\lambda' \lambda''| \lesssim 10^{-11}$$

R-parity $\longrightarrow$ $\lambda' = 0$, $\lambda'' = 0$

(no problem with τ_p constraint)

* R-parity

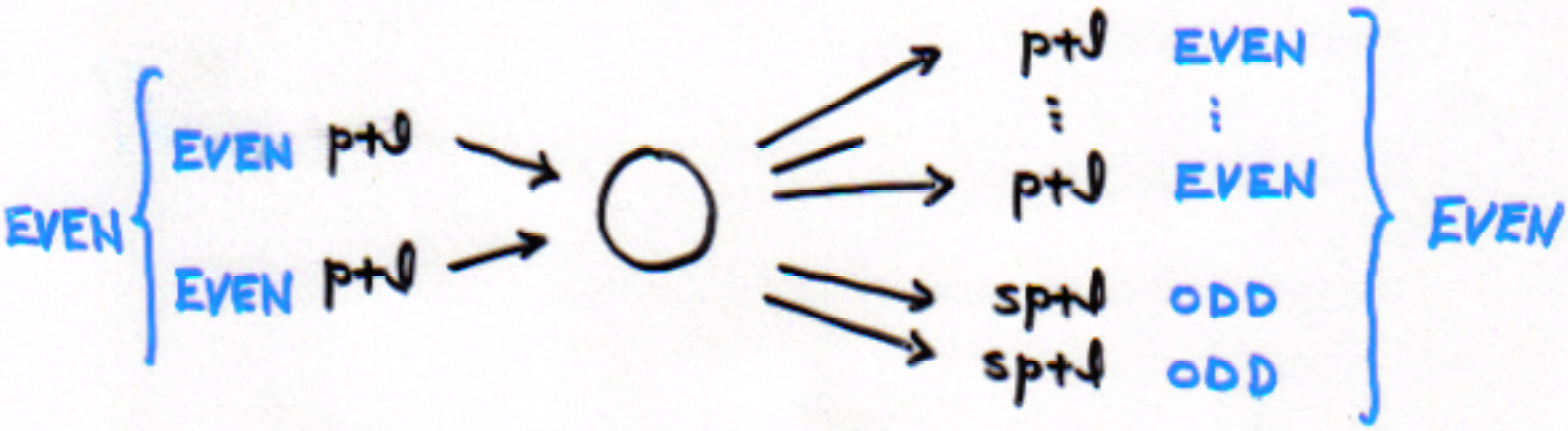
Without this → L#, B# violation

→ unacceptable fast nucleon decay

$$R = (-1)^{2J+3B+L}$$

$$= \begin{cases} \underset{\text{EVEN}}{+1} : \text{for all particles} \\ \qquad\qquad \text{including extra Higgs} \\ \underset{\text{ODD}}{-1} : \text{for all sparticles} \end{cases}$$

• Sparticles creation :

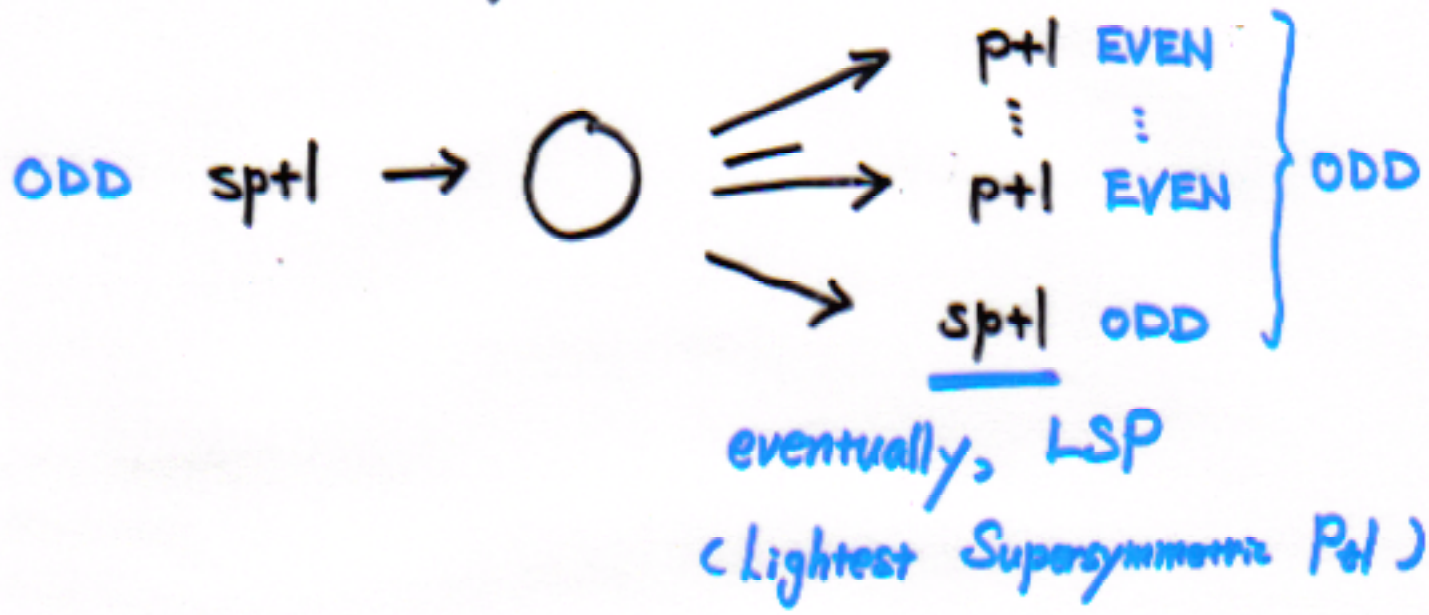


All sparticles should be created in pair.

Not good news : reduced mass reach in collider.

$$\left(\sqrt{s} > 2\, m_{LSP} \right)$$

· Sparticle decay :



LSP cannot decay into other ptls or sptls.
(absolutely stable)

It should be also Neutral for cosmological reason.
(no exotic isotopes)

Stable, Neutral LSP → Not detected at detector
→ missing E

· Immediate Consequence of R-parity :

- L#, B# violation prevented
- Sptls created in pair
- Stable, (Neutral) LSP

* MSSM Fields Content

in Weak-int. eigenstates

<Chiral Supermultiplet>

fermions	bosons	
left-chiral spinor		
$L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L$	$\begin{pmatrix} \tilde{\nu} \\ \tilde{e} \end{pmatrix}_L$	slepton
$N^c = \nu_L^c \ (= \nu_R^*)$	$\tilde{\nu}_L^c$	sneutrino
$E^c = e_L^c$	$\tilde{e}_L^c$	selectron
$Q = \begin{pmatrix} u \\ d \end{pmatrix}_L$	$\begin{pmatrix} \tilde{u} \\ \tilde{d} \end{pmatrix}_L$	squark
$U^c = u_L^c$	$\tilde{u}_L^c$	sup
$D^c = d_L^c$	$\tilde{d}_L^c$	sdown
$\begin{pmatrix} \tilde{\bar{H}}^+ \\ \tilde{\bar{H}}^0 \end{pmatrix}_L$ higgsino	$\bar{H} = \begin{pmatrix} \bar{H}^+ \\ \bar{H}^0 \end{pmatrix}_L$	
$\begin{pmatrix} \tilde{H}^0 \\ \tilde{H}^- \end{pmatrix}_L$ higgsino	$H = \begin{pmatrix} H^0 \\ H^- \end{pmatrix}_L$	

(Superfields notation ($\wedge$) : $\hat{L}$, $\hat{N}^c$, $\hat{E}^c$, ...)

⟨ Gauge Supermultiplet ⟩

gauge boson	gaugino	
g_i	$\tilde{g}_i$	gluino
$W^{\pm}$	$\tilde{W}^{\pm}$	wino
Z^0	$\tilde{Z}^0$	zino
γ	$\tilde{\gamma}$	photino
W^3	$\tilde{W}^3$	wino
B	$\tilde{B}$	bino

* Superpotential for MSSM

(one generation only)

(^ : superfield)

$$f_{MSSM} = \lambda_E \hat{L}^T (i\sigma^2) \hat{H} \hat{E}^c + \lambda_D \hat{Q}^T (i\sigma^2) \hat{H} \hat{D}^c$$

e mass d mass

$$+ \lambda_u \hat{Q}^T (i\sigma^2) \hat{\bar{H}} \hat{U}^c + \mu \hat{H}^T (i\sigma^2) \hat{\bar{H}}$$

u mass Higgs mixing (to avoid PQ sym)

$\left(i\sigma^2 = \begin{pmatrix} & 1 \\ -1 & \end{pmatrix} \right.$: to construct SU(2) singlets from SU(2) doublets $\left. \right)$

$$\lambda_E \hat{L}^T (i\sigma^2) \hat{H} \hat{E}^c = \lambda_E (-\hat{e}_L \hat{H}^0 \hat{e}_L^c + \hat{\nu}_L \hat{H}^- \hat{e}_L^c)$$

hypercharge	Y:	$-1/2$	$-1/2$	1	$-1/2$	$-1/2$	1
weak isospin	T_3:	$-1/2$	$1/2$	0	$1/2$	$-1/2$	0
electric char.	Q:	-1	0	1	0	-1	1

$(Q = Y + T_3)$

$$\lambda_u \hat{Q}^T (i\sigma^2) \hat{\bar{H}} \hat{U}^c = \lambda_u (-\hat{d}_L \hat{\bar{H}}^+ \hat{u}_L^c + \hat{u}_L \hat{\bar{H}}^0 \hat{u}_L^c)$$

Y:	$1/6$	$1/2$	$-2/3$	$1/6$	$1/2$	$-2/3$
T_3:	$-1/2$	$1/2$	0	$1/2$	$-1/2$	0
Q:	$-1/3$	1	$-2/3$	$2/3$	0	$-2/3$

∴ We need 2 Higgs doublets w/ different hypercharge.

$$\hat{H} = \begin{pmatrix} \hat{H}^0 \\ \hat{H}^- \end{pmatrix}_{Y=-\frac{1}{2}} \qquad \hat{\bar{H}} = \begin{pmatrix} \hat{\bar{H}}^+ \\ \hat{\bar{H}}^0 \end{pmatrix}_{Y=\frac{1}{2}}$$

* Illustration of Higgs mechanism

$$\mathcal{L}_{\text{superpotential}} = \int d^2\theta \left(m_{ij} \hat{\Phi}_i \hat{\Phi}_j + \lambda_{ijk} \hat{\Phi}_i \hat{\Phi}_j \hat{\Phi}_k \right) + h.c.$$

$$= -\lambda_{ijk} \psi_i \psi_j \phi_k + h.c. + \cdots$$

$$\lambda_E \underset{\psi}{e_L} \underset{\psi}{e_L^c} \underset{\phi}{H^0} + h.c. \quad \leftarrow \lambda_E \hat{e}_L \hat{H}^0 \hat{e}_L^c$$

$$= -\lambda_E H^0 \bar{e}_R e_L + h.c.$$

$$= -\lambda_E v \left(\bar{e}_R e_L + \bar{e}_L e_R \right) \quad \leftarrow \text{Higgs mech.}$$

$$= -\lambda_E v \left(\bar{e} P_L e + \bar{e} P_R e \right)$$

$$= -\underbrace{\lambda_E v}_{m_e} \bar{e} e$$

mass term expressed with Dirac spinor

$$\left[\underset{\text{Dirac spinor}}{e} = \begin{pmatrix} e_L \\ e_R \end{pmatrix} \begin{matrix} \leftarrow \text{Left Weyl} \\ \leftarrow \text{Right Weyl} \end{matrix} \right]$$

(Higgs mech. is possible only w/ Spont. EWS breaking)

* Soft-breaking terms

$$-\mathcal{L}_{soft}$$

$$= m_{\tilde{q}}^2 |\tilde{q}_L|^2 + m_{\tilde{u}}^2 |\tilde{u}_L^c|^2 + m_{\tilde{d}}^2 |\tilde{d}_L^c|^2$$

squark mass

$$+ m_{\tilde{l}}^2 |\tilde{l}_L|^2 + m_{\tilde{e}}^2 |\tilde{e}_L^c|^2$$

slepton mass

$$+ \Big(\lambda_E A_E H \tilde{l}_L \tilde{e}_L^c + \lambda_D A_D H \tilde{q}_L \tilde{d}_L^c + \lambda_u A_u \bar{H} \tilde{q}_L \tilde{u}_L^c$$

trilinear scalar int.

$$+ B\mu H \bar{H} + h.c. \Big)$$

bilinear scalar

$$+ m_H^2 |H|^2 + m_{\bar{H}}^2 |\bar{H}|^2$$

Higgs scalar mass

$$+ \frac{1}{2} M_1 \tilde{B}\tilde{B} + \frac{1}{2} M_2 \tilde{W}\tilde{W} + \frac{1}{2} M_3 \tilde{g}\tilde{g}$$

gaugino mass

(Soft-breaking terms
: To reflect no sparticles w/ same mass
& still cancel quadratic-div.)

(2) EW Sym. Breaking in MSSM

* Scalar Higgs Potential

· $V_{\text{Higgs scalar}}$

$$= \underbrace{(\text{F-term}) + (\text{D-term})}_{\text{SUSYic}} + \underbrace{(\text{Soft-breaking term})}_{\cancel{\text{SUSY}}}$$

$$\left(\begin{array}{c} F_i = -\dfrac{\partial f}{\partial \varphi_i}, \quad D^a = -g \sum_i \varphi_i^\dagger T_a \varphi_i \\ \text{after solving } \mathcal{E}\text{-}\mathcal{L} \text{ eqn.} \\ V = \underbrace{\sum_i |F_i|^2}_{\text{(F-term)}} + \underbrace{\frac{1}{2}\sum_a |D^a|^2}_{\text{(D-term)}} \end{array} \right)$$

$$= \underbrace{m_1^2 |H^0|^2 + m_2^2 |\bar{H}^0|^2 + (m_3^2 H^0 \bar{H}^0 + h.c.)}_{\text{quadratic}}$$

$$+ \underbrace{\frac{g_1^2 + g_2^2}{8} \left(|H^0|^2 - |\bar{H}^0|^2 \right)^2}_{\text{quartic}} + \underbrace{(\bar{H}^+, H^- \text{ term})}_{\text{no contribution to potential minimiz}}$$

$$\left(\begin{array}{l} m_1^2 \equiv \underbrace{m_H^2}_{} + \mu^2 \\ m_2^2 \equiv \underbrace{m_{\bar{H}}^2}_{} + \mu^2 \\ m_3^2 \equiv \underbrace{B\mu}_{} \end{array} \right)$$

$\sim$: soft-breaking

$m_1^2 = m_2^2$ if unbroken SUSY

* Conditions for Spontaneous EWS Breaking

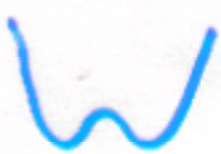
(1) Potential (V_{Higgs}) bounded (to be able to define vacuum)

SM	MSSM
$\lambda > 0$	$m_1^2 + m_2^2 \geq 2\|m_3\|^2$

In direction of $|H^0| = |\bar{H}^0|$, quartic term vanishes.

(2) VEV $\neq 0$

SM	MSSM
$\mu^2 < 0$	$m_1^2 m_2^2 < m_3^4$



Determinant of 2nd derivative of V_{Higgs} at origin < 0

Above 2 conditions for EWSB cannot be satisfied simultaneously if $m_1^2 = m_2^2$ (ie, unless soft-breaking terms)

EWSB is Not possible
Unless SUSY is broken!

(3) Superparticles Mixing

After EWSB, 2 or more fields can mix if they share $SU(3)_c \times U(1)_{QED}$ quantum numbers.

* Slepton (or Squark) mixing

$$\Psi_{Dirac} = \begin{pmatrix} \ell_L \\ \ell_R \end{pmatrix} \begin{matrix} \cdots \tilde{\ell}_L \\ \cdots \tilde{\ell}_R \end{matrix} \Bigg\} \text{ sleptons}$$

Weyl spinors

$$\mathcal{L}_{soft} = \cdots - \lambda H \tilde{\ell}_L \tilde{\ell}_R^* + h.c.$$

v (off-diagonal term)

mass eigenstates : $\tilde{\ell}_1$, $\tilde{\ell}_2$

(inter-generation mixing of sleptons (squarks)
: very model-dependent
It can cause unacceptably large FCNC)

* Gaugino – Higgsino mixing

$$\left\langle \begin{array}{lllll} \text{Gauginos} : & \lambda^+ & \lambda^- & \lambda_Z & \lambda_\gamma \\ & (\tilde{W}^+ & \tilde{W}^- & \tilde{Z}^0 & \tilde{\gamma}) \\ \text{Higgsinos} : & \tilde{H}^+ & \tilde{H}^- & \tilde{H}^0 & \tilde{H}^0 \end{array} \right.$$

(off-diagonals)

$$\int d^2\theta\, d^2\bar{\theta}\, \Phi^\dagger e^{2gV} \Phi = \cdots + ig\sqrt{2}\left(\phi^* \lambda \psi - \bar{\lambda}\bar{\psi}\phi\right)$$

min. coupling of gauge int.

Higgs (v) — Higgsino, Gaugino

* Chargino (Mix of $\lambda^\pm, \tilde{H}^+, \tilde{H}^-$)

mass eigenstates : $\tilde{\chi}_1^+$, $\tilde{\chi}_2^+$

Dirac spinor $\Psi_D = \begin{pmatrix} \psi \\ \bar{\chi} \end{pmatrix}$

* Neutralino (Mix of $\lambda_Z, \lambda_\gamma, \tilde{H}^0, \tilde{H}^0$)

mass eigenstates : $\tilde{\chi}_1^0, \tilde{\chi}_2^0, \tilde{\chi}_3^0, \tilde{\chi}_4^0$

Majorana spinor $\Psi_M = \begin{pmatrix} \psi \\ \bar{\psi} \end{pmatrix}$

$\Psi_M^c \equiv C\bar{\Psi}_M^T = \Psi_M$: self charge-conjug.

* Higgs Scalar Mixing (not sparticles)

4 complex scalars $(\bar{H}^+, H^-, \bar{H}^0, H^0)$

: 8 degrees of freedom

3 dof : absorbed by $W^\pm$, Z

Higgs mechanism

5 dof : Physical Higgs bosons

A	pseudo scalar ($P=-1$)
h, H	scalars
$H^\pm$	charged scalars

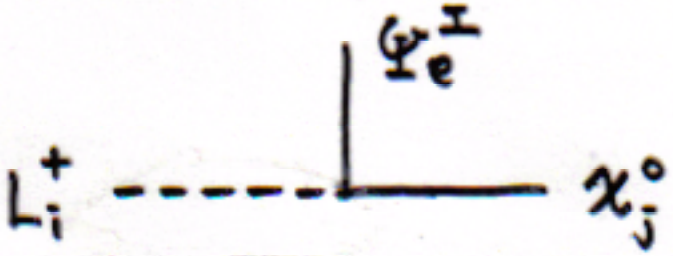
(4) Feynman Rules

Masses & Mixing angles of mixing fields depend on many unknown parameters.

(complicated mixing pattern)

→ Complicated Feynman rules for MSSM

ex) electron - selectron - neutralino



$$i\Big[\Big\{\frac{e}{\sqrt{2}\sin\theta\cos\theta} Z_L^{Ii}\left(Z_N^{1j}\sin\theta + Z_N^{2j}\cos\theta\right) + g^I Z_L^{(I+3)i} Z_N^{3j}\Big\}\frac{1-\gamma_5}{2} + \Big\{\frac{-e\sqrt{2}}{\cos\theta} Z_L^{(I+3)i} Z_N^{1j*} + g^I Z_L^{Ii} Z_N^{3j*}\Big\}\frac{1+\gamma_5}{2}\Big]$$

(g^I : Yukawa coupling ($m_e^I = -\frac{v}{\sqrt{2}} g^I$)
Z_L : Transf. Matrix (selectrons → mass eigenst.)
Z_N : Transf. Matrix (neutralinos → mass eigenst.))

For complete list, See Rosiek (PRD V41, 11, 1990)

* Summary

- MSSM needs an extra Higgs doublet
  for proper fermion mass
  and R-parity.
  to avoid L#, B# violation

- R-parity tells us superparticles are produced in pair and the LSP is stable.

- EWSB is deeply related w/ ~~SUSY~~ in MSSM.
  Former is NOT possible w/o latter.

- There are many mixings in MSSM.
  slepton, squark, chargino, neutralino, higgses

- Complicated mixing pattern makes complicated Feynman rules.