Nucleon Structure from Lattice QCD

J.W. Negele

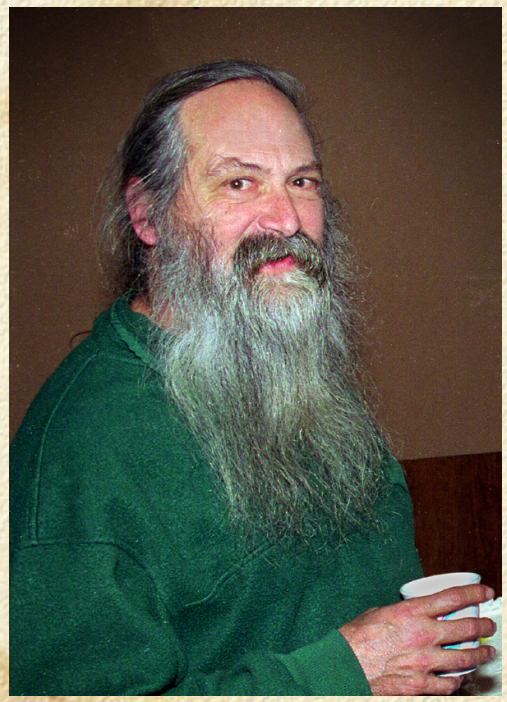
Celebration of 25 Years of Lattice QCD and QCDOC Dedication

BNL September 30, 2005

Outline

- Introduction
  - Celebrating two milestones
- Highlights from hadron structure
  - Deep inelastic scattering
  - Lattice calculations
  - Axial charge
  - Quark momentum fraction
  - Form factors and generalized form factors
  - Transverse structure
  - Origin of nucleon spin
  - Baryon shapes
- Challenges in the SciDAC/QCDOC era

"25" Years of Lattice QCD



PHYSICAL REVIEW D VOLUME 15, NUMBER 4 15 FEBRUARY 1977

Gauge fixing, the transfer matrix, and confinement on a lattice*

Michael Creutz
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(Received 29 October 1976)

We use the transfer-matrix formalism of statistical mechanics to relate Wilson's Lagrangian approach and the Kogut-Susskind Hamiltonian approach to gauge theories on a lattice. As a preliminary we discuss gauge fixing in Wilson's theory. This process leaves invariant Green's functions of gauge singlet operators. Taking the timelike lattice spacing to zero, we extract the Kogut-Susskind Hamiltonian from the transfer matrix in the gauge $A_0 = 0$.

VOLUME 42, NUMBER 21 PHYSICAL REVIEW LETTERS 21 MAY 1979

Experiments with a Gauge-Invariant Ising System

Michael Creutz, Laurence Jacobs, and Claudio Rebbi
Physics Department, Brookhaven National Laboratory, Upton, New York 11973
(Received 19 March 1979)

Using Monte Carlo techniques, we evaluate the path integral for the four-dimensional lattice gauge theory with a Z_2 gauge group. The system exhibits a first-order transition. This is contrary to the implications of the approximate Migdal recursion relations but consistant with mean-field-theory arguments. Our "data" agree well with a low-temperature expansion and the exact duality between the high- and low-temperature phases.

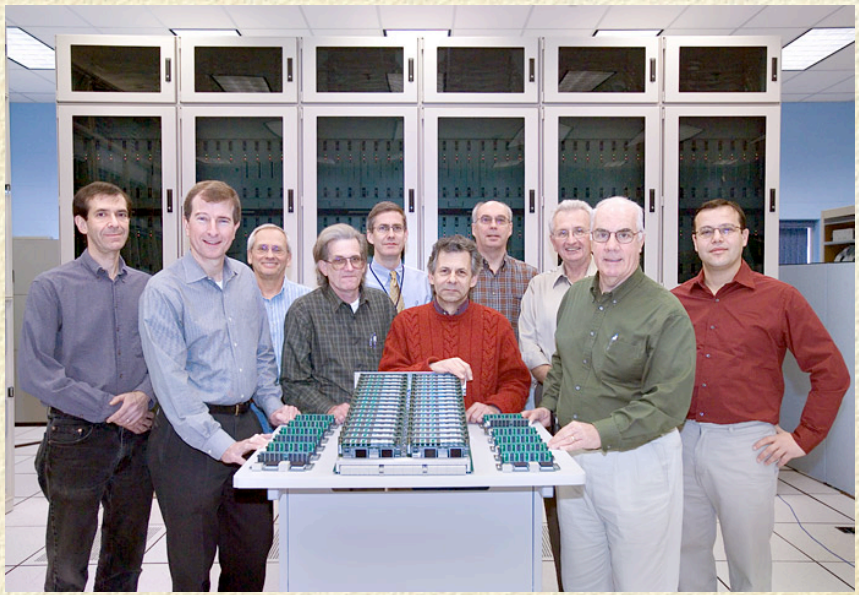
PHYSICAL REVIEW D VOLUME 20, NUMBER 8 15 OCTOBER 1979

Monte Carlo study of Abelian lattice gauge theories

Michael Creutz, Laurence Jacobs,* and Claudio Rebbi
Physics Department, Brookhaven National Laboratory, Upton, New York 11973
(Received 21 June 1979)

Using Monte Carlo techniques, we study the thermodynamics of four-dimensional Euclidean lattice gauge theories, with gauge groups Z_N and U(1). For $N \leq 4$ the models exhibit a single first-order phase transition, while for $N \geq 5$ we observe two transitions of higher order. As N increases, one of these transitions moves toward zero temperature, whereas the other remains at finite temperature and survives in the U(1) limit. The behavior of the Wilson loop factor is also analyzed for the Z_2 and Z_6 models.

Dedication of the US QCDOC Computer



Introduction

- How do hadrons arise from QCD?
- Lagrangian constrained by Lorentz invariance, gauge invariance and renormalizability:

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{4}F_{\mu\nu}^2$$

$$D_\mu = \partial_\mu - igA_\mu \qquad F_{\mu\nu} = \frac{i}{g}[D_\mu, D_\nu]$$

- Deceptively simple Lagrangian produces amazingly rich and complex structure of strongly interacting matter in our universe

Goals

- Quantitative calculation of hadron observables from first principles
  - Agreement with experiment
  - Credibility for predictions and guiding experiment
- Insight into how QCD works
  - Mechanisms
    - Diquark correlations
    - Paths that dominate action - instantons
    - Variational wave functions
  - Dependence on parameters
    - m_q - role of chiral symmetry
    - N_c, N_f, gauge group

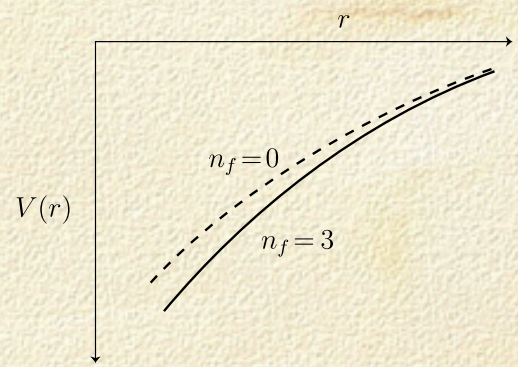
Computational Issues

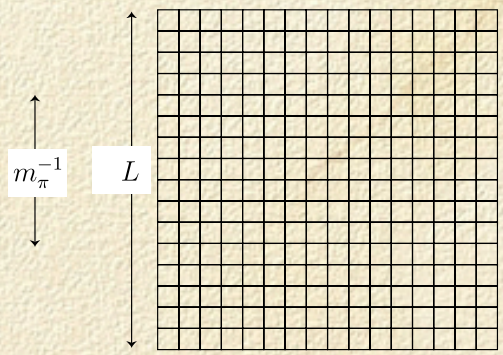
- Fermion determinant - Full QCD
- Small lattice spacing
- Small quark mass
- Large lattice volume

$$\frac{1}{m_\pi} \leq \frac{L}{4}$$

L(fm)	m_π (Mev)
1.6	500
4.0	200
5.7	140

- Cost ~ $(m_q)^{-2.5}(m_\pi)^{-4} \sim (m_\pi)^{-9}$





Hadron structure revealed by high energy scattering

- High energy scattering measures correlation functions along light cone
  - Asymptotic freedom: reaction theory perturbative
  - Unambiguous measurement of operators in light cone frame
  - Must think about physics on light cone
- Parton distribution q(x) gives longitudinal momentum distribution of light-cone wave function
- Generalized parton distribution gives transverse spatial structure of light-cone wave function $q(x, r_\perp)$

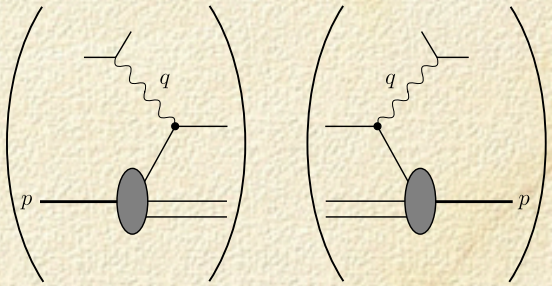
Parton and generalized parton distributions

High energy scattering: light-cone correlation function $(\lambda = p^+ x^-)$

$$\mathcal{O}(x) = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \bar{\psi}(-\frac{\lambda}{2}n)\, \not{n} \mathcal{P} e^{-ig\int_{-\lambda/2}^{\lambda/2} d\alpha n\cdot A(\alpha n)} \psi(\frac{\lambda}{2}n)$$

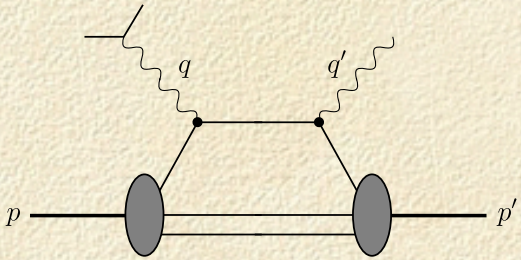
Deep inelastic scattering: diagonal matrix element

$$\langle P|\mathcal{O}(x)|P\rangle = q(x)$$



Deeply virtual Compton scattering: off-diagonal matrix element

$$\langle P'|\mathcal{O}(x)|P\rangle = \langle\gamma\rangle H(x,\xi,t) + \tfrac{i\Delta}{2m}\langle\sigma\rangle E(x,\xi,t)$$
$$\Delta = P' - P, \quad t = \Delta^2, \quad \xi = -n\cdot\Delta/2$$



Moments of parton distributions

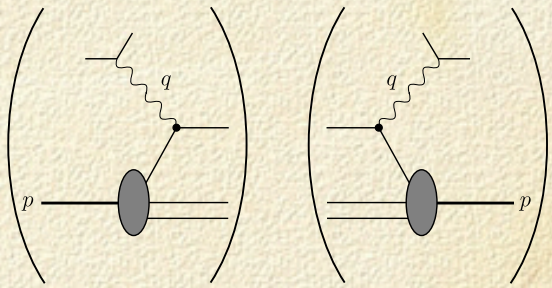
Expansion of $\mathcal{O}(x) = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \bar{\psi}(-\frac{\lambda}{2}n)\ \not{n}\mathcal{P} e^{-ig\int_{-\lambda/2}^{\lambda/2} d\alpha n\cdot A(\alpha n)} \psi(\frac{\lambda}{2}n)$

Generates tower of twist-2 operators

$$\mathcal{O}_q^{\{\mu_1\mu_2\ldots\mu_n\}} = \overline{\psi}_q \gamma^{\{\mu_1} iD^{\mu_2} \ldots iD^{\mu_n\}} \psi_q$$

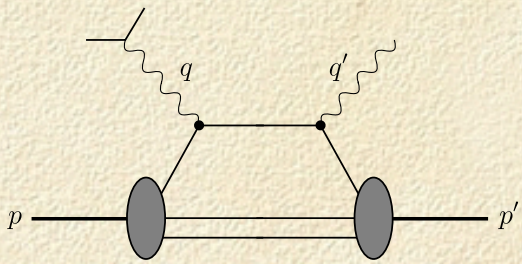
Diagonal matrix element

$$\langle P|\mathcal{O}_q^{\{\mu_1\mu_2\ldots\mu_n\}}|P\rangle \sim \int dx\, x^{n-1} q(x)$$

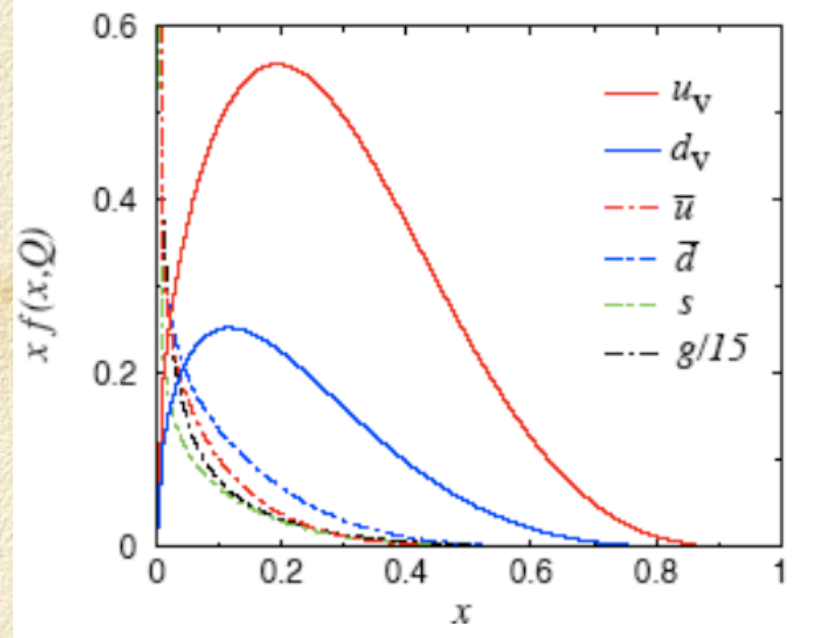


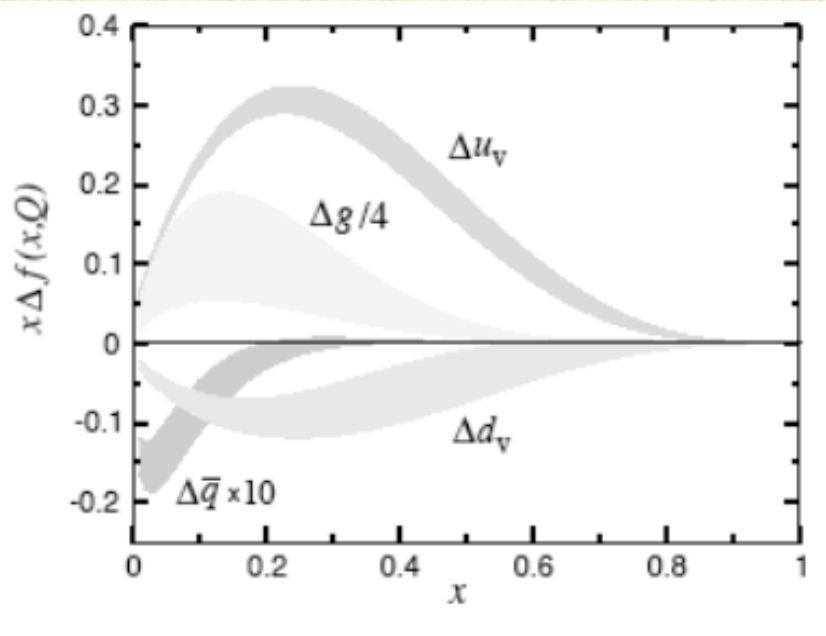
Off-diagonal matrix element

$$\langle P'|\mathcal{O}_q^{\{\mu_1\mu_2\ldots\mu_n\}}|P\rangle$$
$$\sim \int dx\, x^{n-1} [H(x,\xi,t), E(x,\xi,t)]$$
$$\rightarrow A_{ni}(t), B_{ni}(t), C_n(t)$$



Moments of parton distributions





$$\langle p|\bar{\psi}\gamma_\mu D_{\mu_1}\cdots D_{\mu_n}\psi|p\rangle \rightarrow \quad \langle x^n\rangle_q \quad = \int_0^1 dx x^n[q(x)+(-1)^{(n+1)}\bar{q}(x)]$$

$$\langle p|\bar{\psi}\gamma_5\gamma_\mu D_{\mu_1}\cdots D_{\mu_n}\psi|p\rangle \rightarrow \quad \langle x^n\rangle_{\Delta q} \quad = \int_0^1 dx x^n[\Delta q(x)+(-1)^{(n)}\Delta\bar{q}(x)]$$

$$\langle p|\bar{\psi}\gamma_5\sigma_{\mu\nu} D_{\mu_1}\cdots D_{\mu_n}\psi|p\rangle \rightarrow \quad \langle x^n\rangle_{\delta q} \quad = \int_0^1 dx x^n[\delta q(x)+(-1)^{(n+1)}\delta\bar{q}(x)]$$

where $q = q_\uparrow + q_\downarrow, \quad \Delta q = q_\uparrow - q_\downarrow, \quad \delta q = q_\top + q_\perp,$

Full QCD Calculations

Collaboration	m_π (MeV)	Gluon action	Quark action
LHPC / SESAM	> 650	Wilson	Wilson
QCDSF / UKQCD	> 550	Wilson	Clover improved Wilson
RBCK	> 500	DBW2	Domain wall
LHPC / MILC	> 350	Asqtad	Staggered sea HYP Domain wall valence

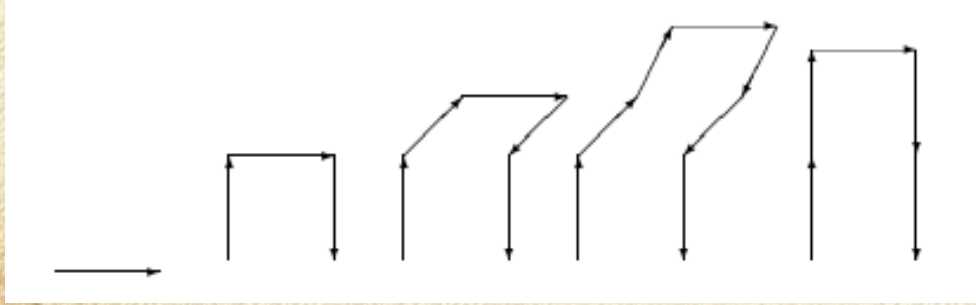
Chiral regime

- Hybrid: domain wall valence/staggered sea
- MILC sea quark configurations
  - $N_F = 3$, staggered sea quarks
  - $a = 0.125$ fm, $20^3 \times 32$ ($28^3 \times 32$) lattices, $L = 2.6$ (3.5) fm
  - m_π = 359, 498, 605, 696, 775 MeV
- HYP smeared domain wall valence quarks (same pion mass)
- Collaborators:

R. Brower	J. Osborn	C. Alexandrou	B. Bistrovic
R. Edwards	A. Pochinksy	Ph. Haegler	J. Bratt
G. Fleming	D. Renner	W. Schroers	D. Sigaev
O. Jahn	D. Richards	A. Tsapalis	P. Varilly
K. Orginos			

Asqtad Action: $O(a^2)$ perturbatively improved

- Symansik improved glue
  - $S_g(U) = C_0 W^{1x1} + C_1 W^{1x2} + C_2 W^{cube}$
- Smeared staggered fermions $S_f(V,U)$
  - Fat links remove taste changing gluons
  - Tadpole improved



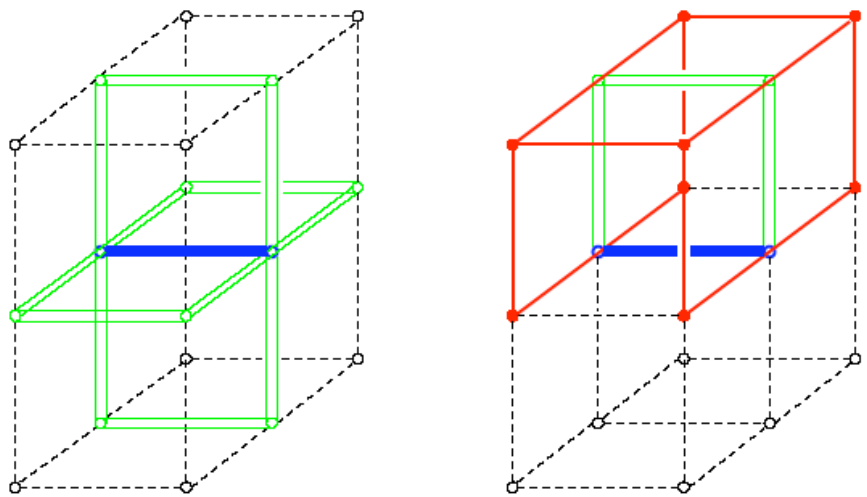
HYP Smearing

- Three levels of SU(3) projected blocking within hypercube
- Minimize dislocations - important for DW fermions

$$V_{i,\mu} = Proj_{SU(3)}[(1-\alpha_1)U_{i,\mu} + \frac{\alpha_1}{6}\sum_{\pm\nu\neq\mu}\tilde{V}_{i,\nu;\mu}\tilde{V}_{i+\hat{\nu},\mu;\nu}\tilde{V}^{\dagger}_{i+\hat{\mu},\nu;\mu}],$$

$$\tilde{V}_{i,\mu;\nu} = Proj_{SU(3)}[(1-\alpha_2)U_{i,\mu} + \frac{\alpha_2}{4}\sum_{\pm\rho\neq\nu,\mu}\bar{V}_{i,\rho;\nu\mu}\bar{V}_{i+\hat{\rho},\mu;\rho\nu}\bar{V}^{\dagger}_{i+\hat{\mu},\rho;\nu\mu}],$$

$$\bar{V}_{i,\mu;\nu\rho} = Proj_{SU(3)}[(1-\alpha_3)U_{i,\mu} + \frac{\alpha_3}{2}\sum_{\pm\eta\neq\rho,\nu,\mu}U_{i,\eta}U_{i+\hat{\eta},\mu}U^{\dagger}_{i+\hat{\mu},\eta}].$$

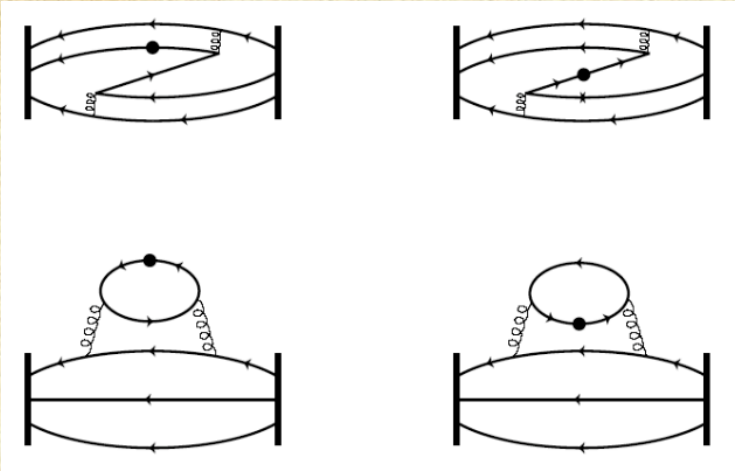


Perturbative renormalization

HYP smeared domain wall fermions - B. Bistrovic

operator	$H(4)$	NOS	HYP	APE
$\bar{q}[\gamma_5]q$	$1_1^{\pm}$	0.792	0.981	1.046
$\bar{q}[\gamma_5]\gamma_\mu q$	$4_4^{\mp}$	0.847	0.976	0.994
$\bar{q}[\gamma_5]\sigma_{\mu\nu}q$	$6_1^{\mp}$	0.883	0.992	0.993
$\bar{q}[\gamma_5]\gamma_{\{\mu}D_{\nu\}}q$	$6_3^{\pm}$	0.991	0.979	0.954
$\bar{q}[\gamma_5]\gamma_{\{\mu}D_{\nu\}}q$	$3_1^{\pm}$	0.982	0.975	0.951
$\bar{q}[\gamma_5]\gamma_{\{\mu}D_\nu D_{\alpha\}}q$	$8_1^{\mp}$	1.134	0.988	0.934
$\bar{q}[\gamma_5]\gamma_{\{\mu}D_\nu D_{\alpha\}}q$	mixing	5.71×10^{-3}	1.88×10^{-3}	8.21×10^{-4}
$\bar{q}[\gamma_5]\gamma_{\{\mu}D_\nu D_{\alpha\}}q$	$4_2^{\mp}$	1.124	0.987	0.934
$\bar{q}[\gamma_5]\gamma_{\{\mu}D_\nu D_\alpha D_{\beta\}}q$	$2_1^{\pm}$	1.244	0.993	0.919
$\bar{q}[\gamma_5]\sigma_{\mu\{\nu}D_{\alpha\}}q$	$8_1^{\pm}$	1.011	0.994	0.964
$\bar{q}[\gamma_5]\gamma_{[\mu}D_{\nu]}q$	$6_1^{\mp}$	0.979	0.982	0.989
$\bar{q}[\gamma_5]\gamma_{[\mu}D_{\{\nu]}D_{\alpha\}}q$	$8_1^{\pm}$	0.955	0.959	0.965

Hadron matrix elements on the lattice



- Measure $\langle \mathcal{O} \rangle$ for m_q, a, L
- Connected diagrams
- Disconnected diagrams (cancel for $\langle \mathcal{O} \rangle_u - \langle \mathcal{O} \rangle_d$)
- Extrapolate $m_q : m_\pi \to 140 \text{ MeV}$
  $a \to \sim 0.05 \text{ fm}$
  $L \to \sim 5 \text{ fm}$

Nucleon axial charge in full lattice QCD

- Why g_A?
- Matrix element of axial current $A_\mu = \bar{q}\gamma_\mu\gamma_5\frac{\vec{\tau}}{2}q$

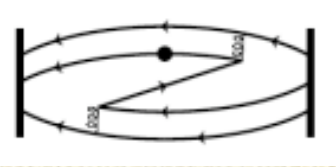
$$\langle N(p+q)|A_\mu|N(p)\rangle = \bar{u}(p+q)\frac{\vec{\tau}}{2}\left[g_A(q^2)\gamma_\mu\gamma_5 + g_P(q^2)q_\mu\gamma_5\right]u(p)$$

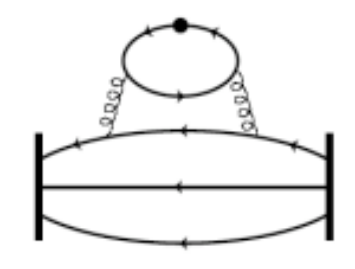
$$g_A(0) = 1.2695 \pm 0.0029$$

- Adler Weisberger $g_A^2 - 1 \sim \int(\sigma_{\pi^+p} - \sigma_{\pi^-p})$
- Goldberger Treiman $g_A \to f_\pi g_{\pi NN}/M_N$
- Spin content $\langle 1\rangle_{\Delta q} = \int_0^1 dx[\Delta q(x) + \Delta\bar{q}(x)]$

$$g_A = \langle 1\rangle_{\Delta u} - \langle 1\rangle_{\Delta d} \qquad \Sigma = \langle 1\rangle_{\Delta u} + \langle 1\rangle_{\Delta d} + \langle 1\rangle_{\Delta s}$$

Nucleon axial charge



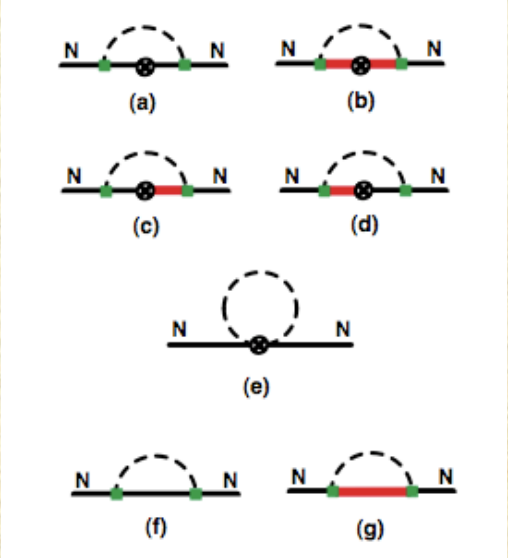


- Gold-Plated observable
  - Accurately measured
  - No disconnected diagrams
  - Chiral perturbation theory for $g_A(m_\pi^2, V)$
  - Renormalization - 5-d conserved current
- First calculation in chiral regime: hep-lat/0510062

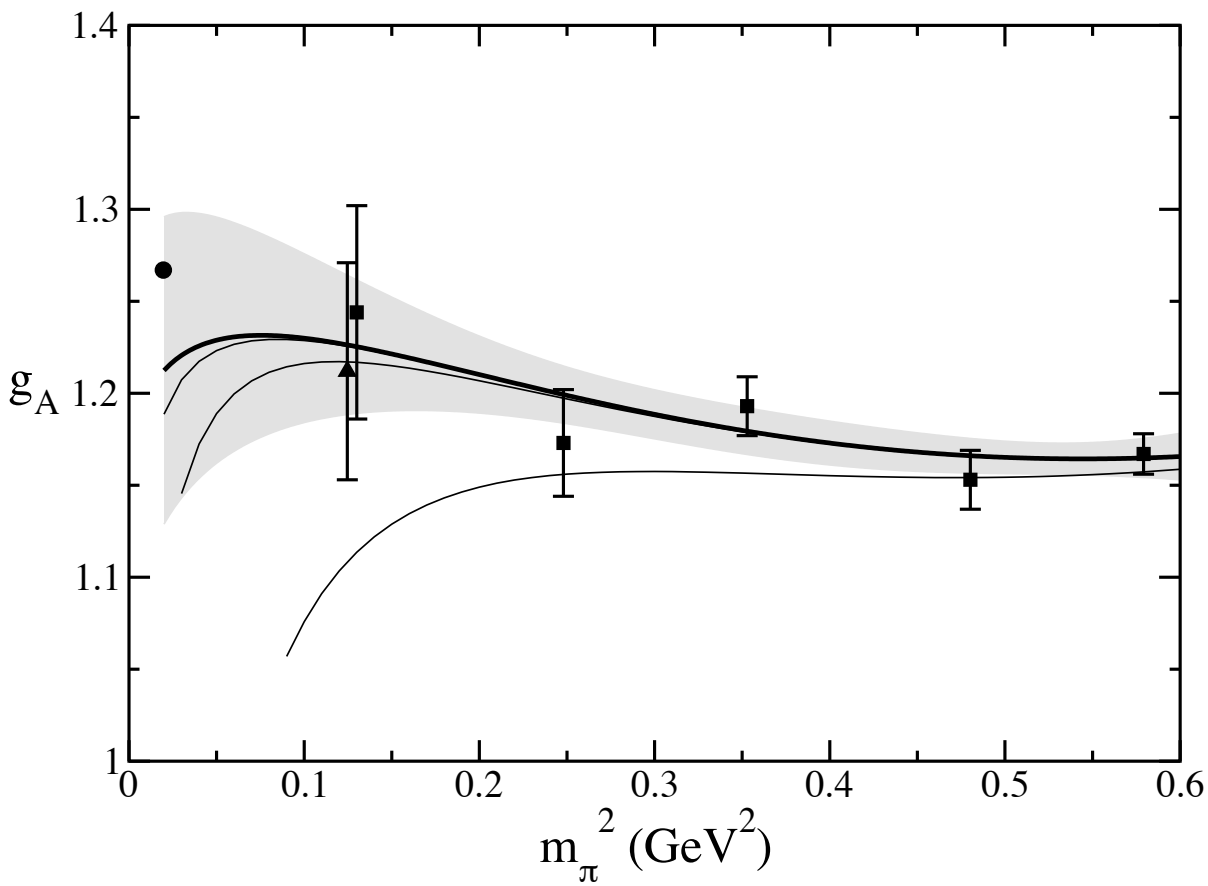
Nucleon Axial Charge

- Chiral perturbation theory $g_A(m_\pi^2, V)$
  - Beane and Savage hep-ph/0404131
  - Detmold and Lin hep-lat/0501007
- 1-loop theory has 6 parameters
  - Fix f_π, $m_\Delta - m_N$, $g_{\Delta N}$ (0.3% error)
  - Fit g_A, $g_{\Delta\Delta}$, C
  - Result $g_A(m_\pi = 140) = 1.212 \pm 0.084$

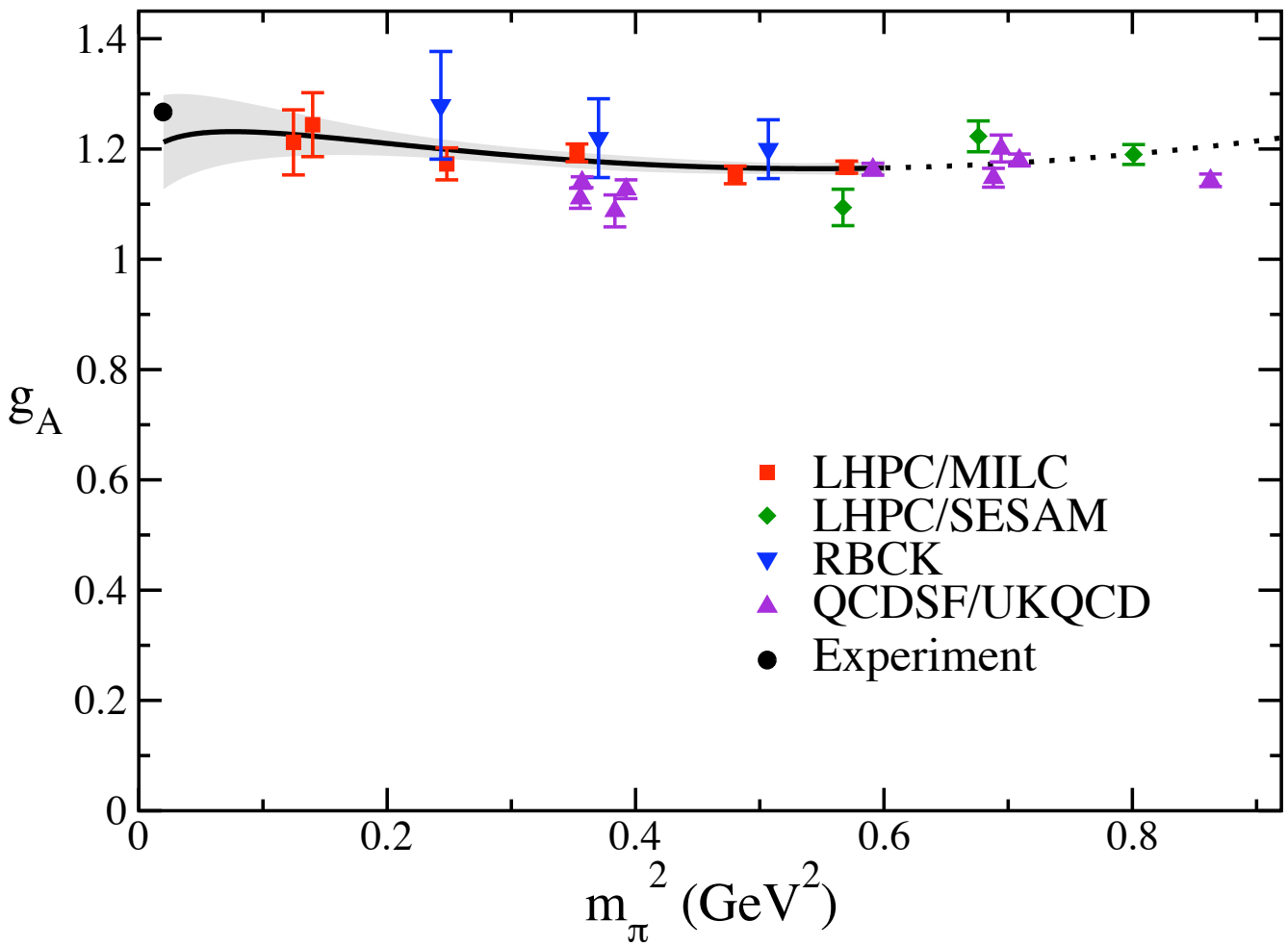
    $\text{Expt.} = 1.2695 \pm 0.0029$



Nucleon axial charge g_A $\langle 1 \rangle_{\Delta q}^{u-d}$

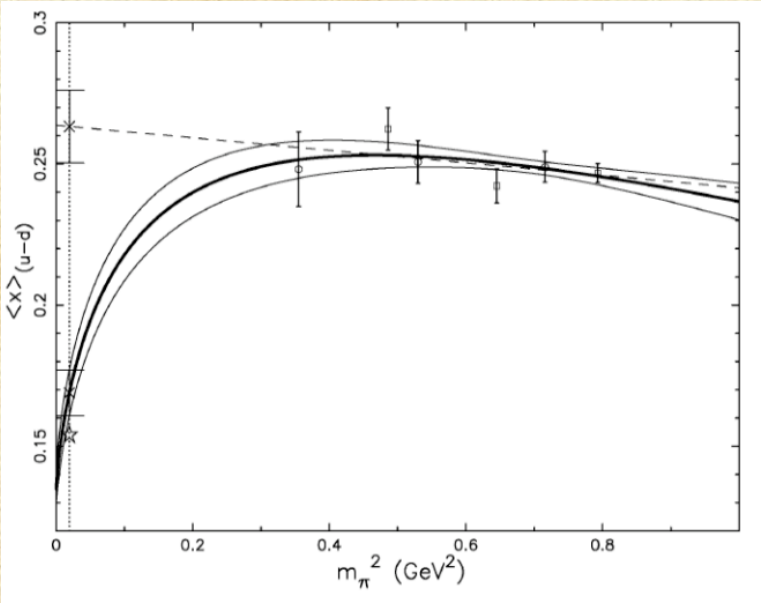


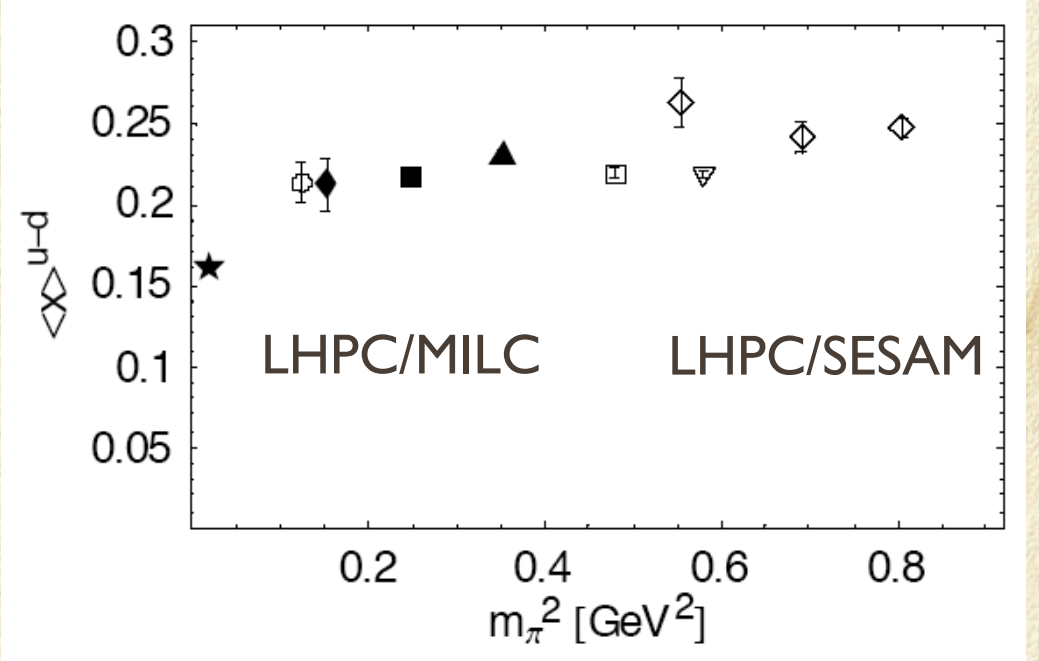
Nucleon axial charge g_A $\langle 1 \rangle_{\Delta q}^{u-d}$



Quark momentum fraction $\langle x \rangle_q^{u-d}$

Full QCD LHPC hep-lat/0209160



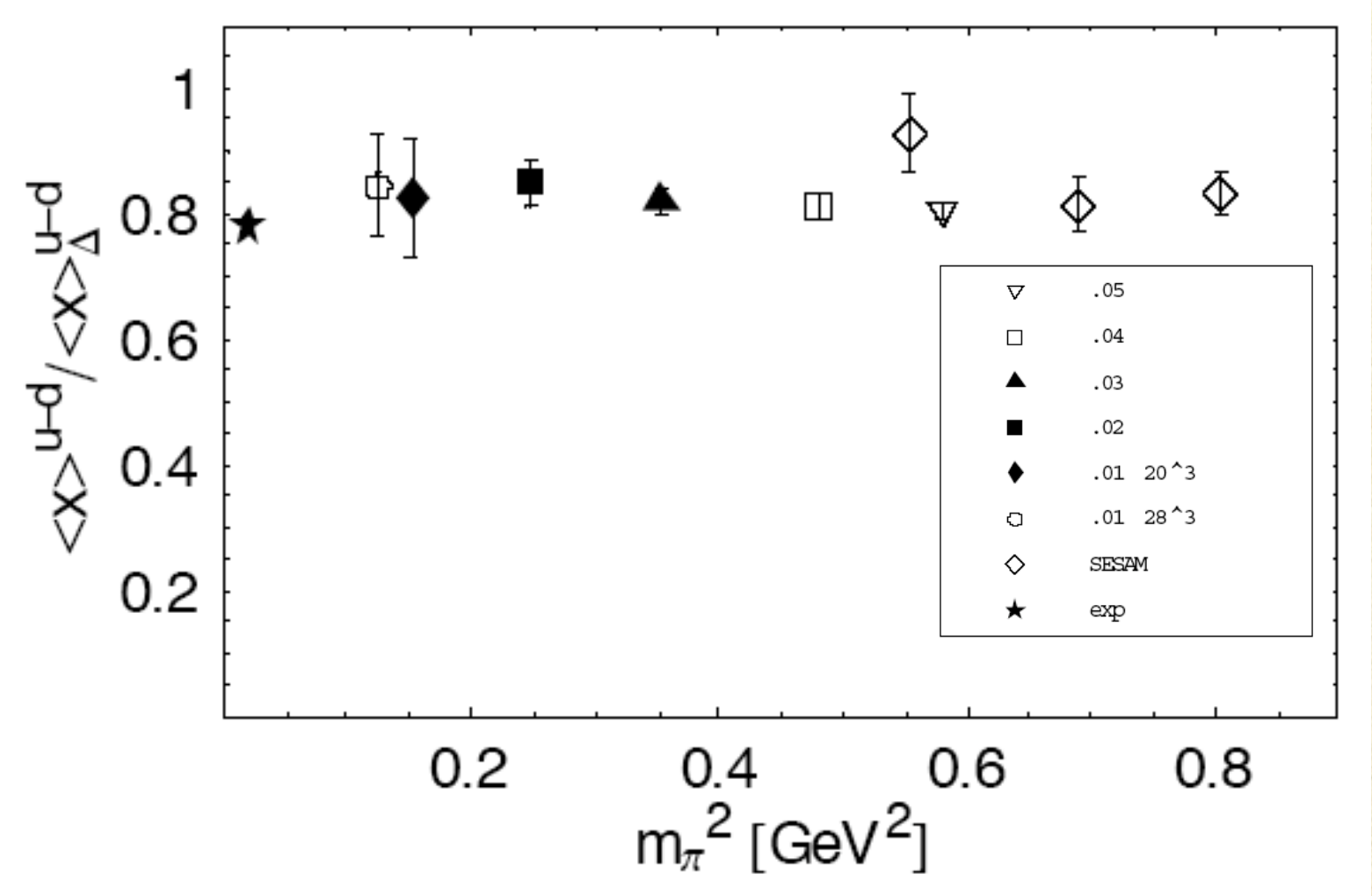


Physical chiral extrapolation formula hep-lat/0103006

$$\langle x^n \rangle_u - \langle x^n \rangle_d \sim a_n \left[1 - \frac{(3{g_A}^2 + 1) m_\pi^2}{(4\pi f_\pi)^2} \ln\left(\frac{m_\pi^2}{m_\pi^2 + \mu^2} \right) \right] + b_n m_\pi^2$$

Quark momentum fraction ratio $\langle x \rangle_q^{u-d} / \langle x \rangle_{\Delta q}^{u-d}$

LHPC/MILC LHPC/SESAM



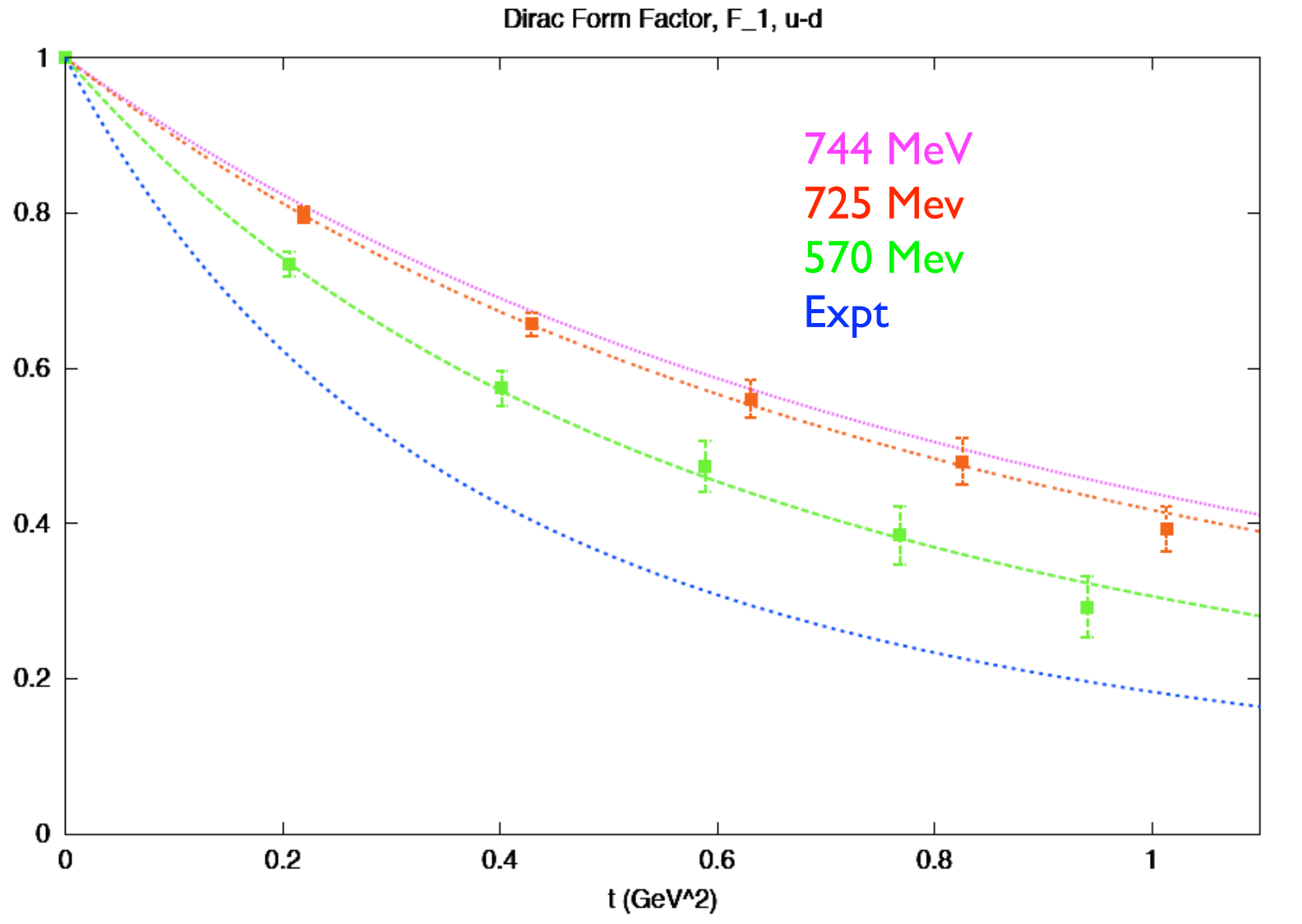
Electromagnetic form factors

- Simplest off-diagonal matrix element

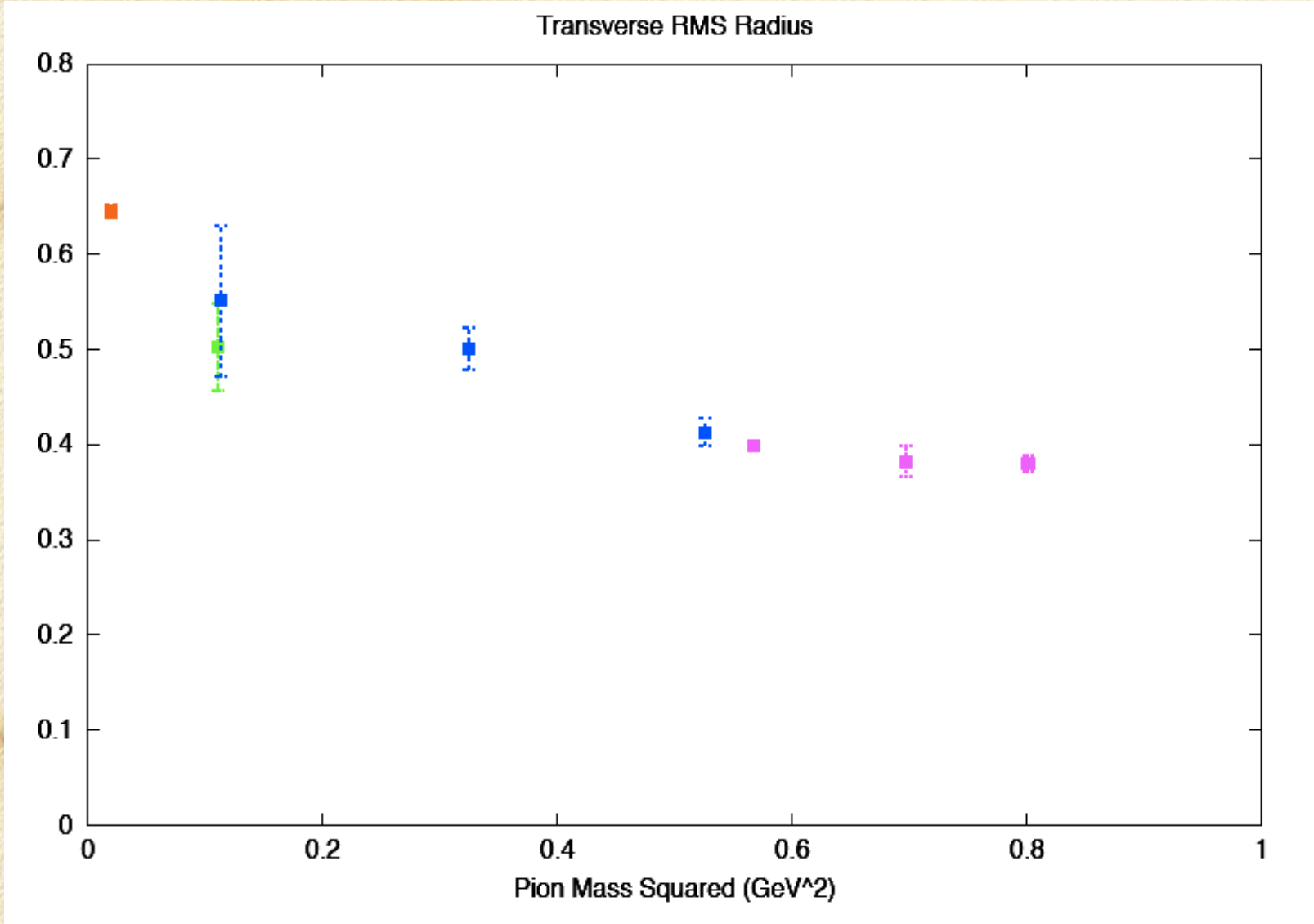
$$\langle p|\bar{\psi}\gamma^\mu\psi|p'\rangle = \bar{u}(p)[F_1(q^2)\gamma^\mu + F_2(q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2m}]u(p')$$

- Fourier transform of charge density if $\mathrm{L_{system}} \gg \mathrm{L_{wavepacket}} \gg \frac{1}{m}$
  - Pb: 5 fm >> 10^{-5} fm, Proton: 0.8 fm ~ 0.2 fm: marginal
  - For transverse Fourier transform of light cone w. f., m → $p_+ \sim \infty$
- Large q^2: ability of one quark to share q^2 with other constituents to remain in ground state - q^2 counting rules

F_1 Form factor



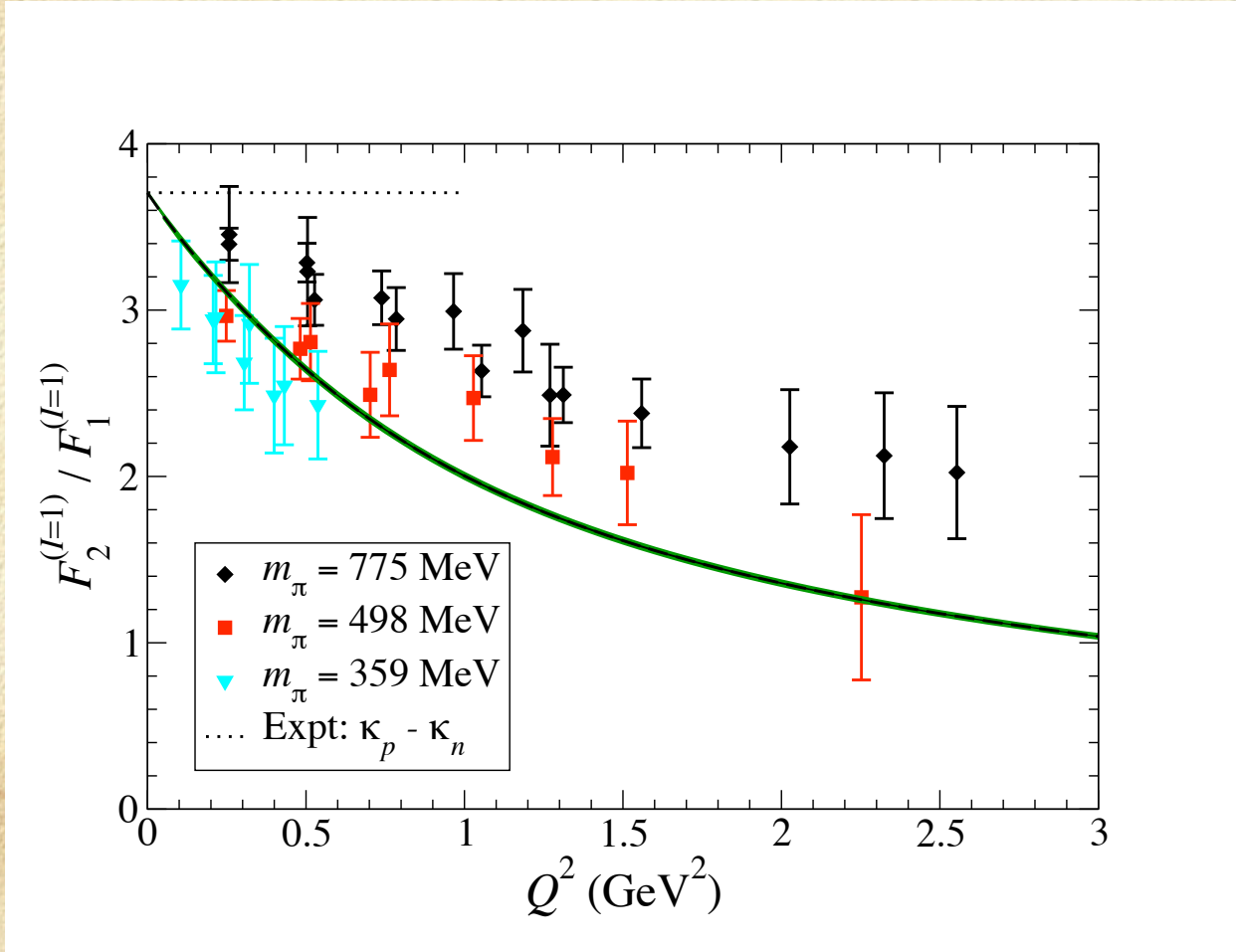
Transverse RMS radius - slope of F_1



Form factor ratio: F_2 / F_1

$$\frac{Q^2 F_2(Q^2)}{\log^2(Q^2/\Lambda^2) F_1(Q^2)} \sim \text{const.}$$

Balitsky, Ji, Yuan hep-ph/0212351



Generalized form factors

$$\mathcal{O}_q^{\{\mu_1\mu_2\ldots\mu_n\}} = \overline{\psi}_q \gamma^{\{\mu_1} iD^{\mu_2} \ldots iD^{\mu_n\}} \psi_q$$

$$\bar{P} = \tfrac{1}{2}(P' + P)$$

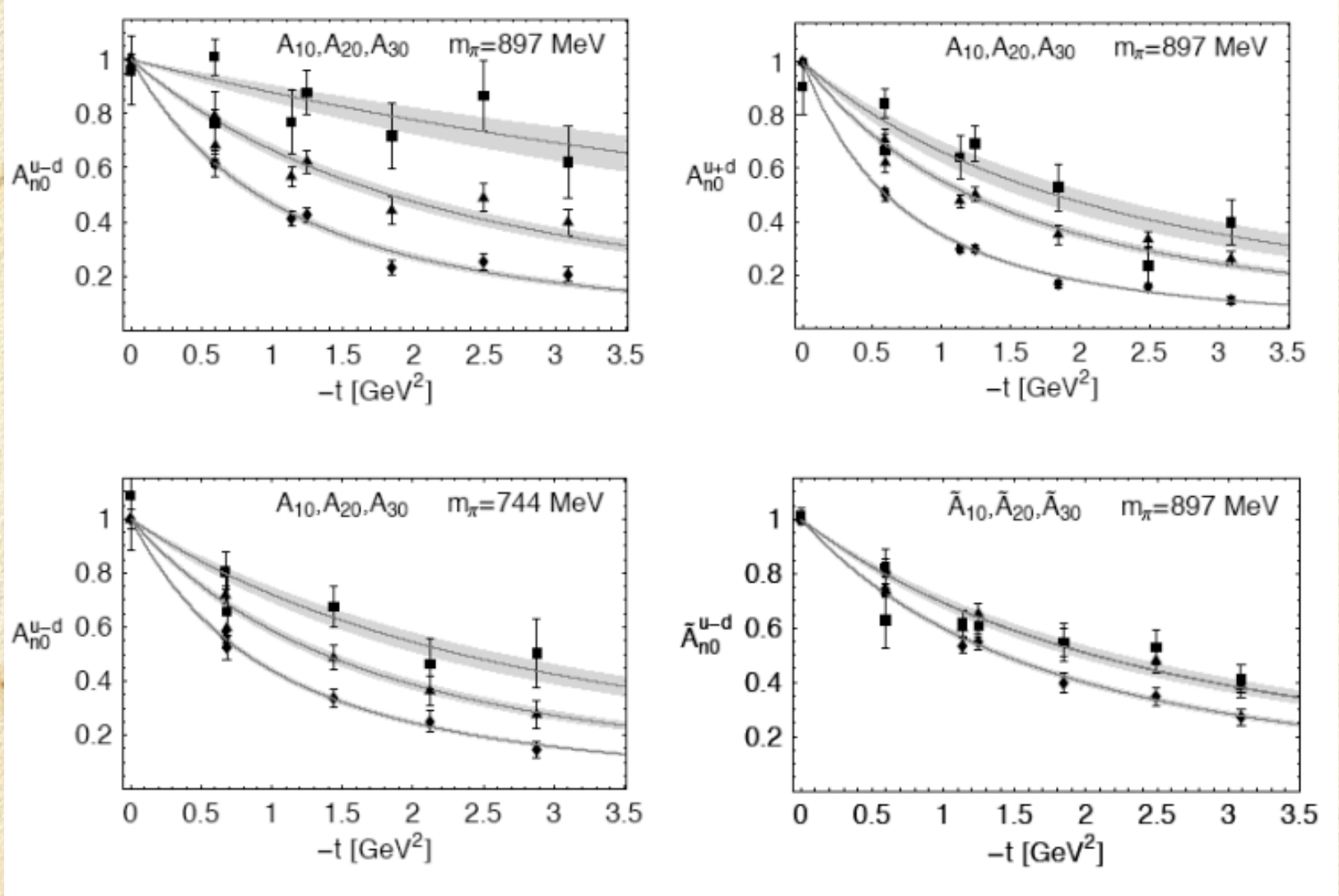
$$\Delta = P' - P$$

$$t = \Delta^2$$

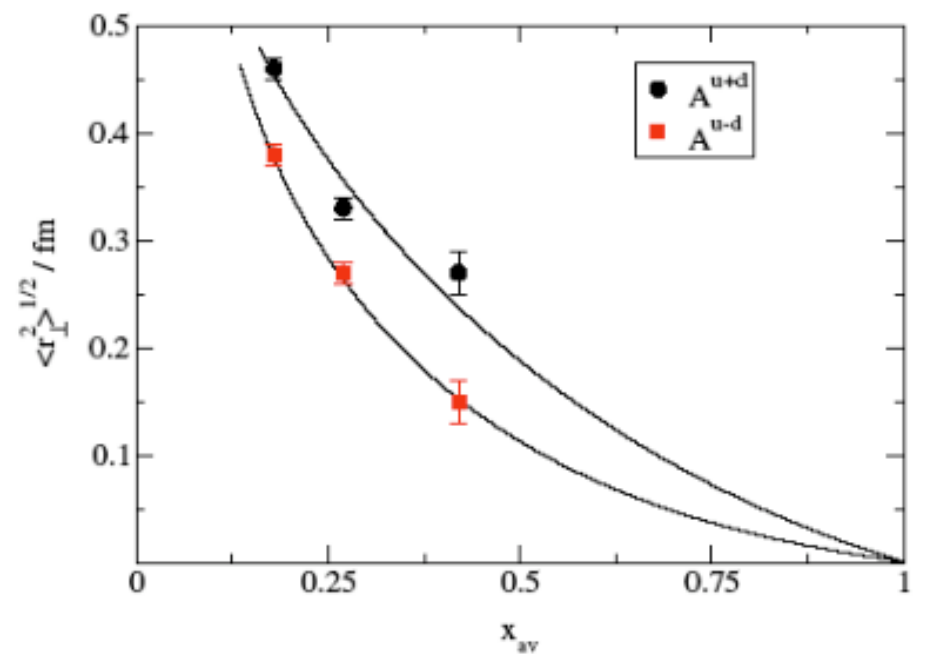
$$\begin{aligned}
\langle P'|\mathcal{O}^{\mu_1}|P\rangle &= \langle\!\langle \gamma^{\mu_1} \rangle\!\rangle A_{10}(t) \\
&+ \frac{i}{2m} \langle\!\langle \sigma^{\mu_1\alpha} \rangle\!\rangle \Delta_\alpha B_{10}(t)\,, \\
\langle P'|\mathcal{O}^{\{\mu_1\mu_2\}}|P\rangle &= \bar{P}^{\{\mu_1} \langle\!\langle \gamma^{\mu_2\}} \rangle\!\rangle A_{20}(t) \\
&+ \frac{i}{2m} \bar{P}^{\{\mu_1} \langle\!\langle \sigma^{\mu_2\}\alpha} \rangle\!\rangle \Delta_\alpha B_{20}(t) \\
&+ \frac{1}{m} \Delta^{\{\mu_1} \Delta^{\mu_2\}} C_2(t)\,, \\
\langle P'|\mathcal{O}^{\{\mu_1\mu_2\mu_3\}}|P\rangle &= \bar{P}^{\{\mu_1} \bar{P}^{\mu_2} \langle\!\langle \gamma^{\mu_3\}} \rangle\!\rangle A_{30}(t) \\
&+ \frac{i}{2m} \bar{P}^{\{\mu_1} \bar{P}^{\mu_2} \langle\!\langle \sigma^{\mu_3\}\alpha} \rangle\!\rangle \Delta_\alpha B_{30}(t) \\
&+ \Delta^{\{\mu_1} \Delta^{\mu_2} \langle\!\langle \gamma^{\mu_3\}} \rangle\!\rangle A_{32}(t) \\
&+ \frac{i}{2m} \Delta^{\{\mu_1} \Delta^{\mu_2} \langle\!\langle \sigma^{\mu_3\}\alpha} \rangle\!\rangle \Delta_\alpha B_{32}(t),
\end{aligned}$$

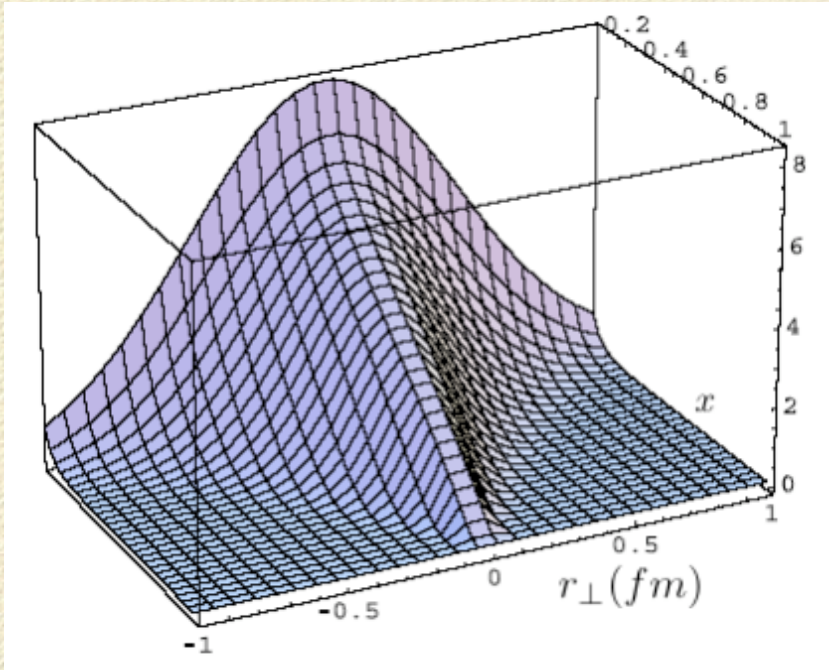
First three moments: $A_{10}, A_{20}, A_{30}, \tilde{A}_{10}, \tilde{A}_{20}, \tilde{A}_{30}$

LHPC hep-lat/0304018



Transverse size of light-cone wave function





$$x_{\rm av}^n = \frac{\int d^2 r_\perp \int dx\, x \cdot x^{n-1} q(x, \vec{r}_\perp)}{\int d^2 r_\perp \int dx x^{n-1} q(x, \vec{r}_\perp)}$$

$q(x, \vec{r}_\perp)$ **model** (Burkardt hep-ph/0207047)

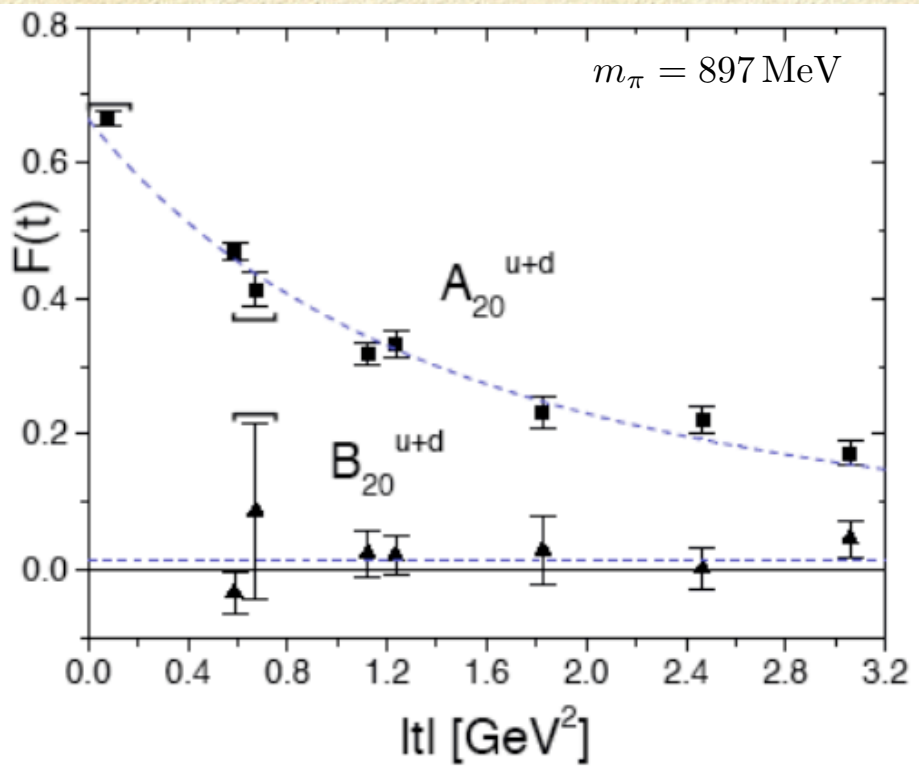
Origin of nucleon spin

"Spin crisis" - only ~ 30% arises from quark spins

quark spin contribution $\frac{1}{2}\Delta\Sigma = \frac{1}{2}\langle 1\rangle_{\Delta u+\Delta d}$ $\sim \frac{1}{2}0.682(18)$

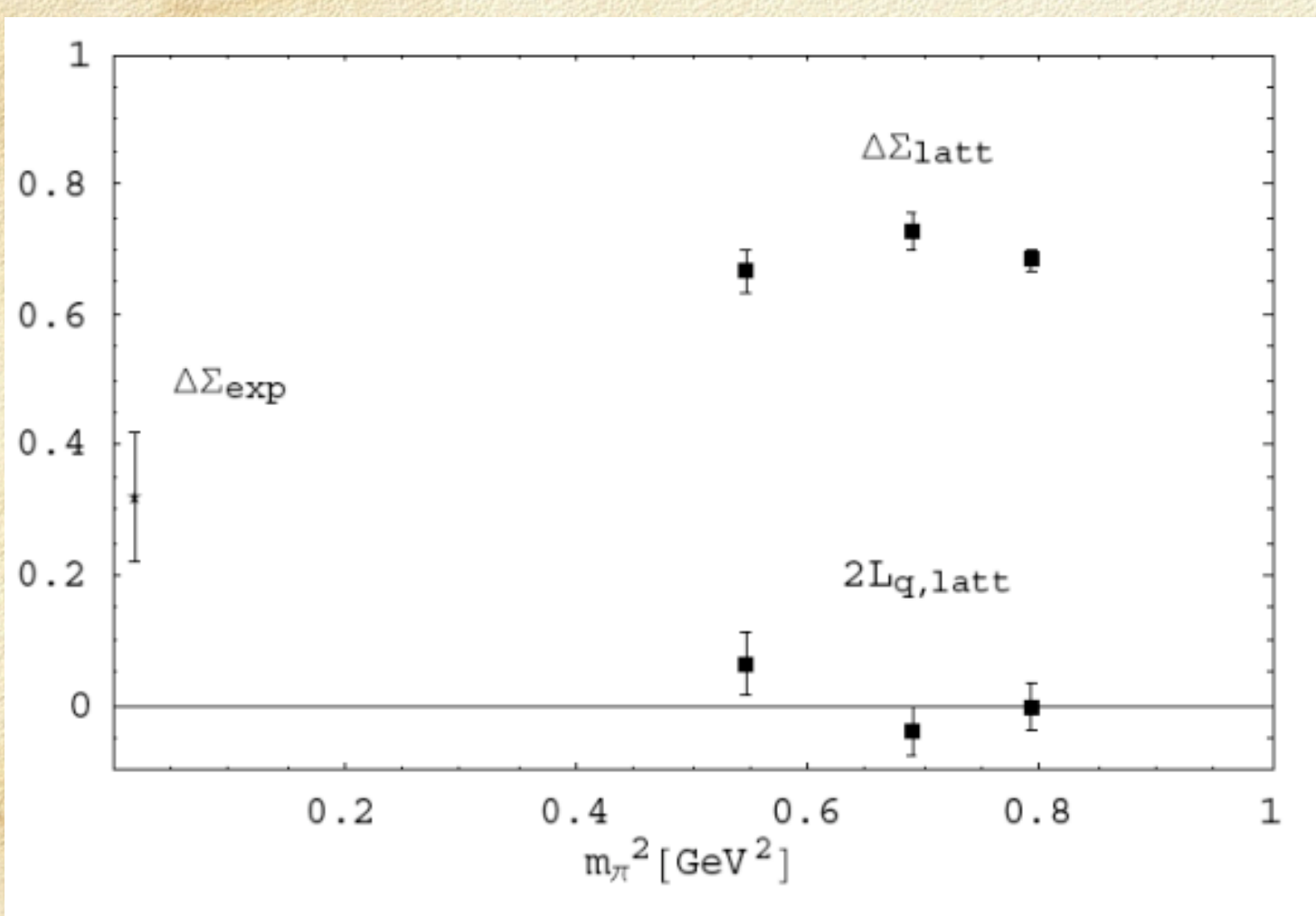
total quark contribution (spin plus orbital) Ji hep-ph/9603249

$$J_q = \frac{1}{2}[A_{20}^{u+d}(0) + B_{20}^{u+d}(0)] = \frac{1}{2}[\langle x\rangle_{u+d} + B_{20}^{u+d}(0)] \quad \sim \frac{1}{2}0.675(7)$$



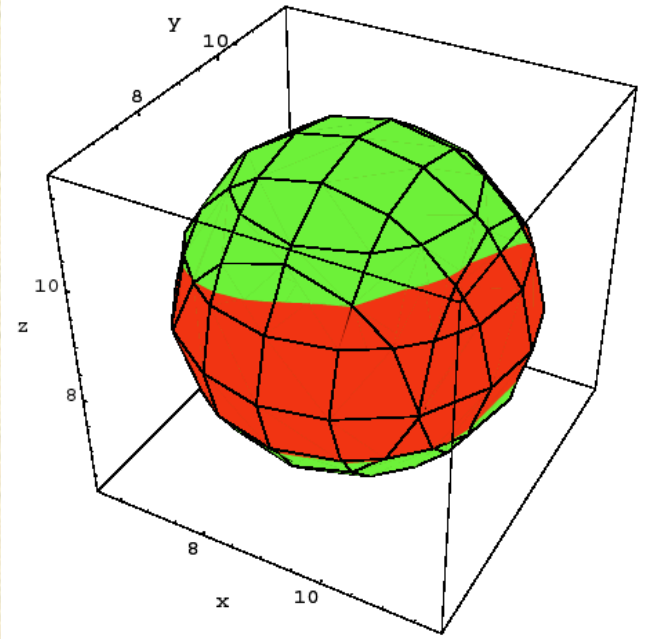
Spin Inventory
68% quark spin
0% quark orbital
32% gluons

Nucleon spin decomposition



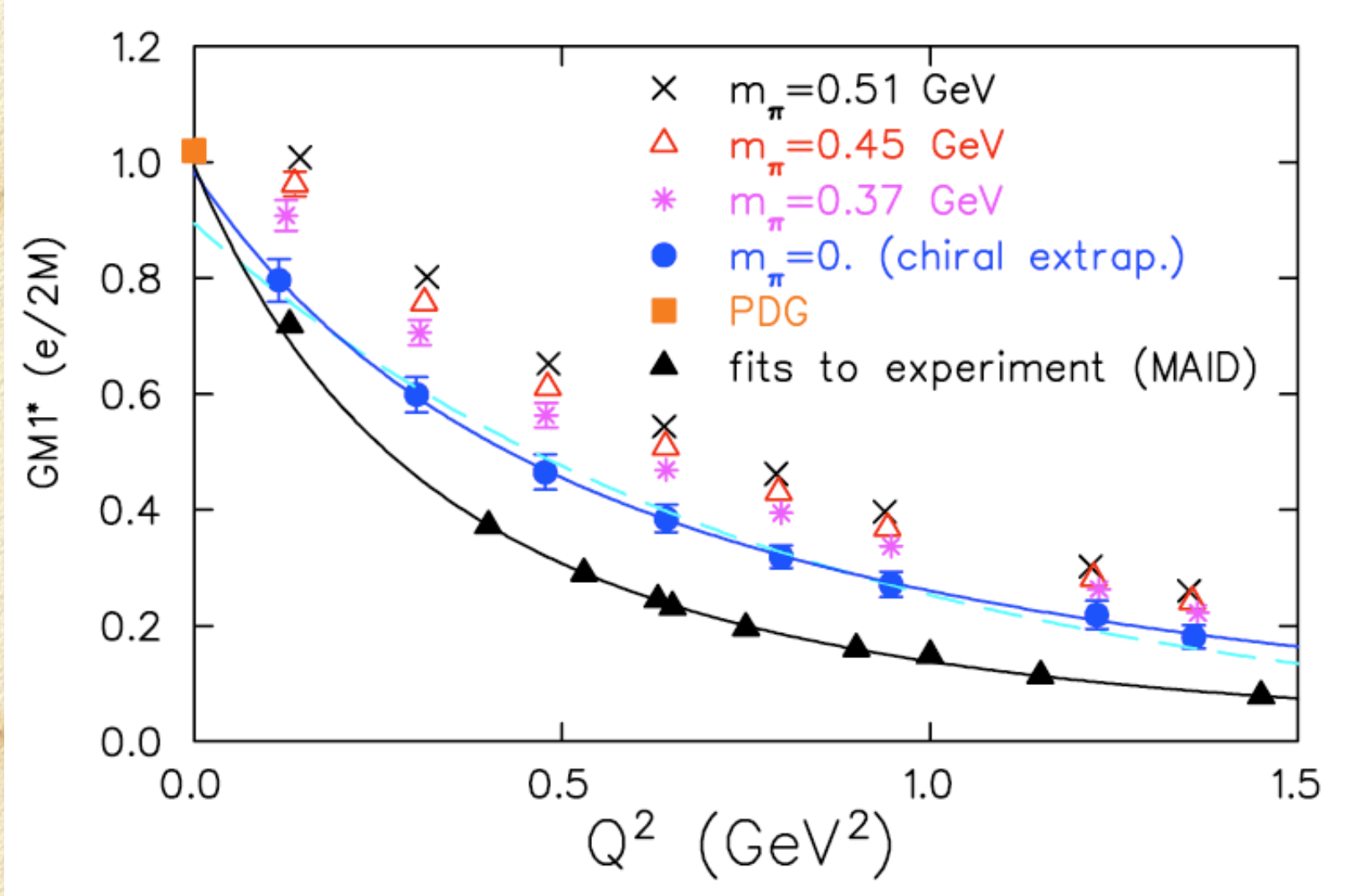
Baryon shapes

- Observe oblate deformation of spin 3/2 Δ directly on lattice from density-density correlation function (Alexandrou, nucl-th/0311007)

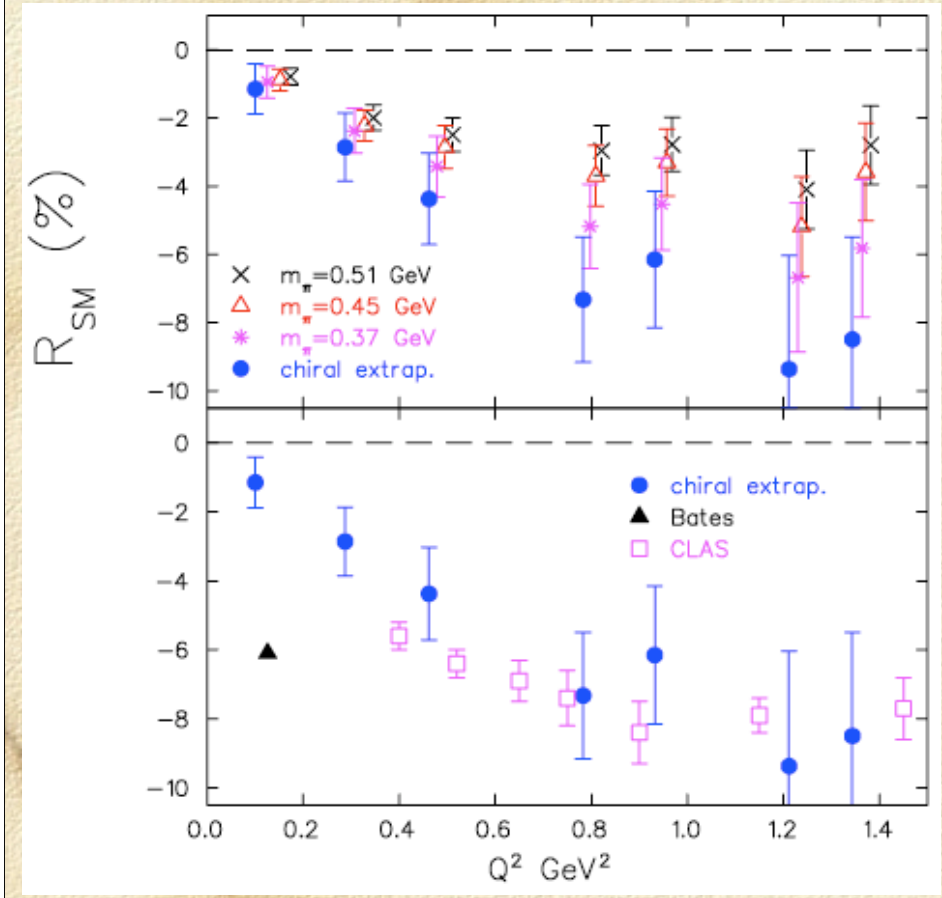


- Infer deformation experimentally from N→Δ transition form factor
  - Dominant transition M1
  - C2 and E2 would vanish if nucleon and Δ spherical

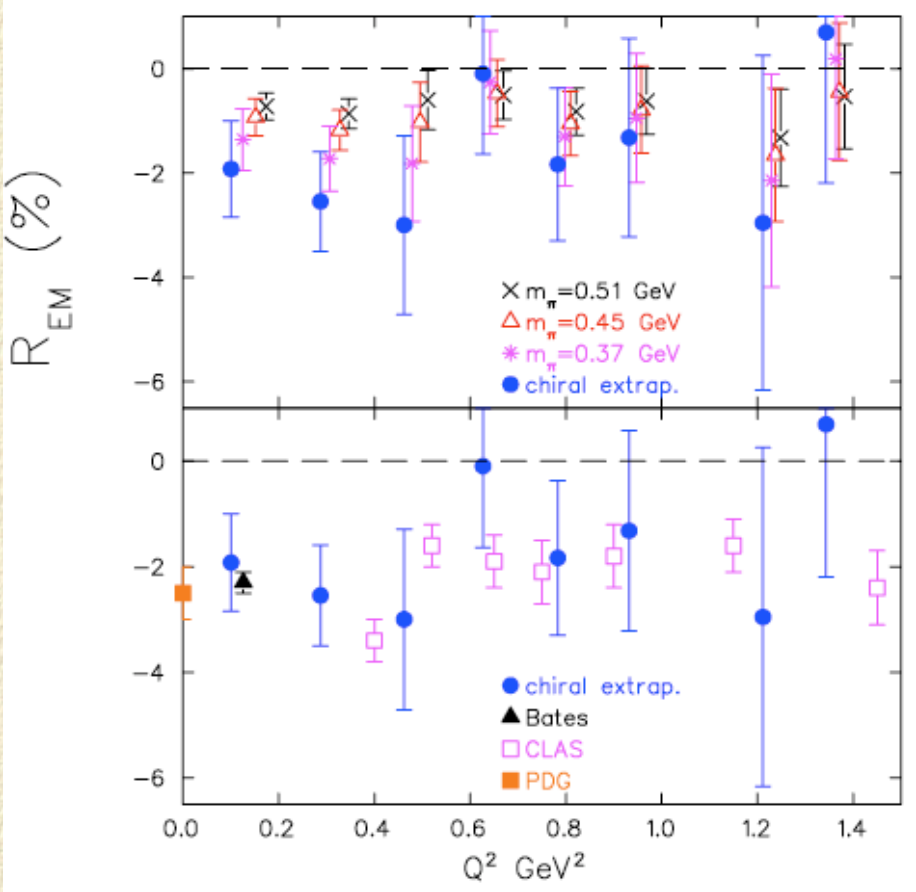
M1 form factor



Electric and Coulomb transitions



C2/M1



E2/M1

MILC Configuration Landscape

$a = 0.13$ fm, $N_T = 32$

$\frac{m_l}{m_s}$	N_L	m_π (MeV)	configs avail	hrs/CG	F	N	π	Δ	NP	DD	Total hrs
1	20	775	407	.092	0	0	0	0	0	0	0
0.8	20	699	344	.106	0	0	0	0	0	0	0
0.6	20	605	559	.130	0	0	0	0	5	0	130
0.4	20	498	485	.179	0	0	0	3	0	0	161
0.2	20	359	642	.325	4	2	0	0	5	100	7314
0.2	28	359	275	.892	0	4	0	3	0	0	1338
0.14	20	300	449	.450	2	6	1	3	5	0	1666
0.1	24	254	397	1.07	2	8	1	3	0	0	3198
Total hrs					1300	2317	455	2329	906	6501	13809

$a = 0.09$ fm, $N_T = 48$

$\frac{m_l}{m_s}$	N_L	m_π (MeV)	configs avail	hrs/CG	F	N	π	Δ	NP	DD	Total hrs
1	28	775	500	.258	1	4	1	0	0	0	310
0.4	28	498	514	.565	1	4	1	3	0	0	1186
0.2	28	359	512	1.08	1	4	1	3	0	0	2258
0.1	40	254	500	6.11	1	4	1	0	0	0	7332
Total hrs					2402	4805		2402	1476		11085

$a = 0.06$ fm, $N_T = 72$

$\frac{m_l}{m_s}$	N_L	m_π (MeV)	configs avail	hrs/CG	F	N	π	Δ	NP	DD	Total hrs
0.4	48	498	500	4.27	1	4	0	0	0	0	3840
0.2	48	359	600	8.12	0	0	0	0	0	0	0
Total hrs					1280	2560				1	3840

Current QCDOC Production

- 28^3 x96 MILC lattices at a=0.09 fm
- Formidable Challenges - stress many QCDOC features
  - 28^3 x96 lattice
    - 14 motherboards, 2 x 7 x 7 x 24 local vol.
    - Bagel assembly code currently 18% of peak
  - Heavy IO
  - Matrix elements require global sums
- Outstanding cooperative effort to meet challenges
  - Dru Renner, Robert Edwards
  - Bob Mawhinney, Chulwoo Jung, ...
- Learning to exploit QCDOC on difficult applications

Summary

- Entering era of quantitative solution in chiral regime
  - Quark distributions: $g_A, \Sigma, \langle x \rangle$
  - Form factors
  - Transverse structure
  - Origin of nucleon spin
  - Baryon shapes
- Gaining insight into how QCD works
  - Overlap with trial functions
  - Dependence on m_q, N_c, N_f
  - Exotic states, diquark correlations

Challenges in the SciDAC/QCDOC era

- Hybrid QCD $a = 0.125 \text{ fm} : m_\pi = 300, 250 \text{ MeV}$
  $a = 0.09, 0.06 \text{ fm}$
- Extrapolation based on hybrid chiral perturbation theory
- Nonperturbative renormalization
- Disconnected diagrams
- Gluon observables
- Exotic Baryons and Mesons
- Full QCD with chiral fermions