25 years

QCD thermodynamics on the lattice

– from the early days to QCDOC –

It's a long time and a long way to go
What was once mine isn't mine anymore

Todd Rundgren

Confinement and deconfinement

confinement

- stick together, find a comfortable separation
- controlled by confinement potential

$$V(r) = -\frac{4}{3}\frac{\alpha(r)}{r} + \sigma r$$

deconfinement

- free floating in the croud
- average distance always smaller than r_{af}:

$$r_{af} = \sqrt{\frac{4}{3}\frac{\alpha(r)}{\sigma}} \simeq 0.25\,\mathrm{fm}$$

Confinement and deconfinement

confinement

- stick together, find a comfortable separation
- controlled by confinement potential

$$V(r) = -\frac{4}{3}\frac{\alpha(r)}{r} + \sigma r$$

$$\alpha(r) \equiv \frac{g^2(r)}{4\pi} \sim \frac{1}{\ln(1/r\Lambda)}$$

deconfinement

- free floating in the croud
- average distance always smaller than r_{af}:

$$r_{af} = \sqrt{\frac{4}{3}\frac{\alpha(r)}{\sigma}} \simeq 0.25\,\mathrm{fm}$$

A Short History of the Quark Gluon Plasma

- Finite size of hadrons must lead to critical density
  $\Rightarrow$ end of hadron physics I.Ya. Pomeranchuk (1951)
- Copious resonance production leads to an exponential mass spectrum
  $\Rightarrow$ limiting temperature R. Hagedorn (1965)
- QCD is asymptotically free $\Rightarrow$ thermodynamics at high temperature should approach ideal gas $\Rightarrow$ phase transition N. Cabibbo and G. Parisi (1975)
- perturbation theory at high temperatures suffers from infrared problems
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serious problem for QCD thermodynamics

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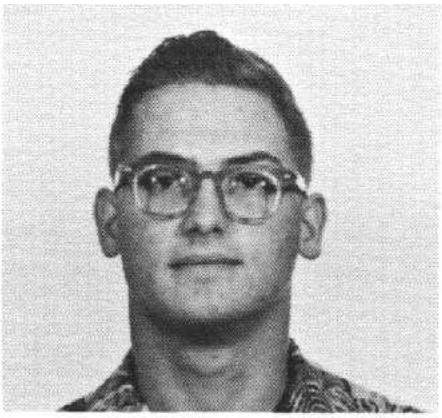
rescue(r) came in the same year



non-perturbative numerical calculations based on

the lattice formulation of QCD

M. Creutz, Phys. Rev. D21 (1980) 2308

The 1980 quantum jump

M. Creutz, Phys. Rev. D21 (1980) 2308

lattice size: 10^4

number of iterations $\mathcal{O}(30)$!!

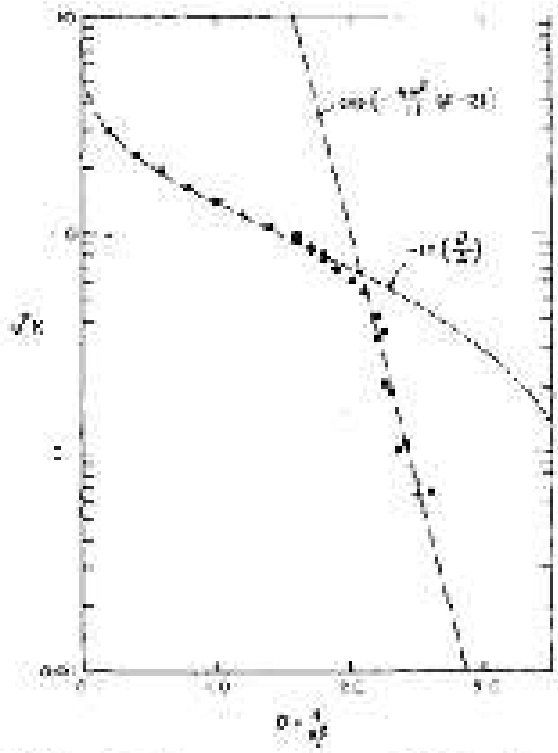


FIG. 6. The cutoff squared times the string tension as a function of β. The solid lines are the strong- and weak-coupling limits.

behavior of Eq. (3.22) occurs rather sharply over a range of about 10% in β about $\beta = 2$. This appearance of the confinement mechanism occurs at

$$\frac{c_0^2}{4\pi} = 0.16 . \qquad (5.1)$$

The rapid evolution out of the perturbative regime may be responsible for the remarkable phenomenological successes of the bag model.[21] High-temperature-series results,[12] as well as semi-classical treatments,[9] have also suggested an abrupt onset of confinement.

Our analysis allows a determination of the renormalization scale of the coupling in terms of the string tension. Using the observed asymptotic normalization

$$\sigma^2 R \underset{\beta \to \infty}{\sim} \exp\left(-\frac{6\pi^2}{11}(\beta - 2)\right) , \qquad (5.2)$$

we can solve for c_0^2 to give

$$\frac{c_0^2}{4\pi} \simeq \frac{3\pi}{11 \ln(1/\sigma A)} , \qquad (5.3)$$

where the renormalization scale is

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..equilibration is essentially complete after 20 iterations. !!!

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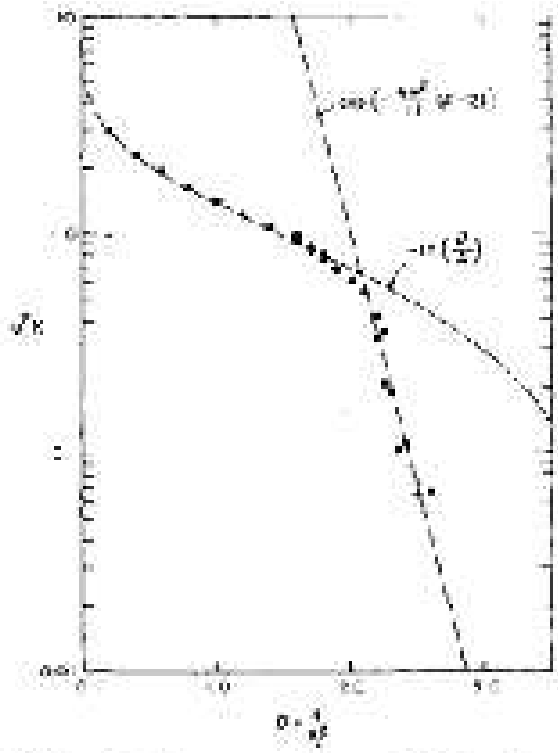

FIG. 6. The [illegible] squared times the string tension as a function of β. The solid lines are the strong- and weak-coupling limits.

behavior of Eq. (3.22) occurs rather sharply over a range of about $10^{\frac{1}{2}}$ in β about $\beta = 2$. This appearance of the confinement mechanism occurs at

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  B. Svetitsky and L. McLerran (1981);

  J. Kuti, J. Polonyi and K. Szlachanyi (1981);

  first presented at the conference on

  Statistical Mechanics of Quarks and Hadrons

  Bielefeld, August 1980

Deconfinement in an SU(2) gauge theory

J. Kuti, J. Polonyi and K. Szlachanyi, 1980/81

The calculation of this correction based on the fact that g_{eff} can be readed from the D_{oo} finite temperature gluon propagator in Coulomb gauge:

$$\frac{g^2_{eff}(a,T)}{\vec{p}^2}\Big|_{\vec{p}^2=\frac{1}{a^2}} = g_s^2\, D_{oo}\,(p^o=0,\ |\vec{p}| = \frac{1}{a},\ T,\ s) \quad , \tag{13}$$

where s is a substraction point. Using the Schwinger-Dyson equation one can sum up to the 1-loop bubble diagrams. The result is[10]:

$$g^2(a,T) = \frac{g^2(a_2,\ T_2)}{1 + g^2(a_1,T_1)\ [\ \frac{11N}{24\pi}\ \ln\frac{a_1}{a_2} + \frac{N}{18}(T_1^2\ a_1^2 - T_2^2\ a_2^2)\ ]} \quad , \tag{14}$$

restricted ourselves to the leading terms in $T \cdot a$.

Our results are summarized in Fig. 7. The calculated points of the critical temperature follow the renormalization group relation

$$T_c \cdot a = \text{const.}\ (g^2)^{-\frac{51}{121}}\ e^{-\frac{12\pi^2}{11g^2}} \quad , \tag{15}$$

for $4/g^2 > 2$ (solid line). The finite temperature correction to g is really negligable for $4/g^2 > 2$ (dashed line). Using the same as at zero temperature we get $T_c = (0.35 \pm 0.05) \cdot 10^{-} \cdot \sigma^{1/2}$. The errors come partly from the inaccuracy of ρ' in [2] and from the errors of the determination of $T_c \cdot a$ values which are shown as horizontal error bars in Fig. 7. Taking $\sigma = 0.2(\text{GeV})^2$ one arrives to the $T_c = 160 \pm 30$ MeV result.

The relation between T_c and σ agrees with one of the Hamiltonian method where we have used the same ρ' value. This is an indication of the equality of the Λ parameter defined in the two methods.

REFERENCES

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9) K.G. Wilson, J. Kogut, Phys. Rep. 12, 75 (1974). M. Creutz, Phys. Rev. D15, 1128 (1977).

10) The details of our calculation will be published elsewhere.

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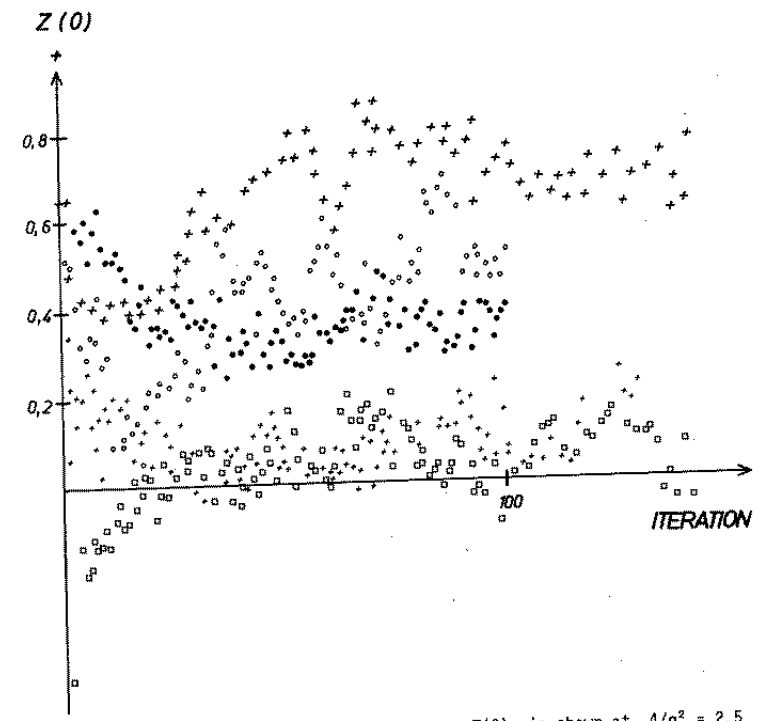


Fig. 1 : The evolution of the order parameter $Z(0)$ is shown at $4/g^2 = 2.5$ for various values of N_T as the function of the number of iterations. The lattice is $8^3 \cdot N_T$, values of N_T are the following: +: 4 , o: 5, •: 6, +: 7, □: 8. The relative fluctuations increase at $N_T \approx 7\text{-}8$. In these cases one can see flip-flop phenomenon between a finite value 0.13 and the noise 0.01 indicating the evolution of free or confinement domains.

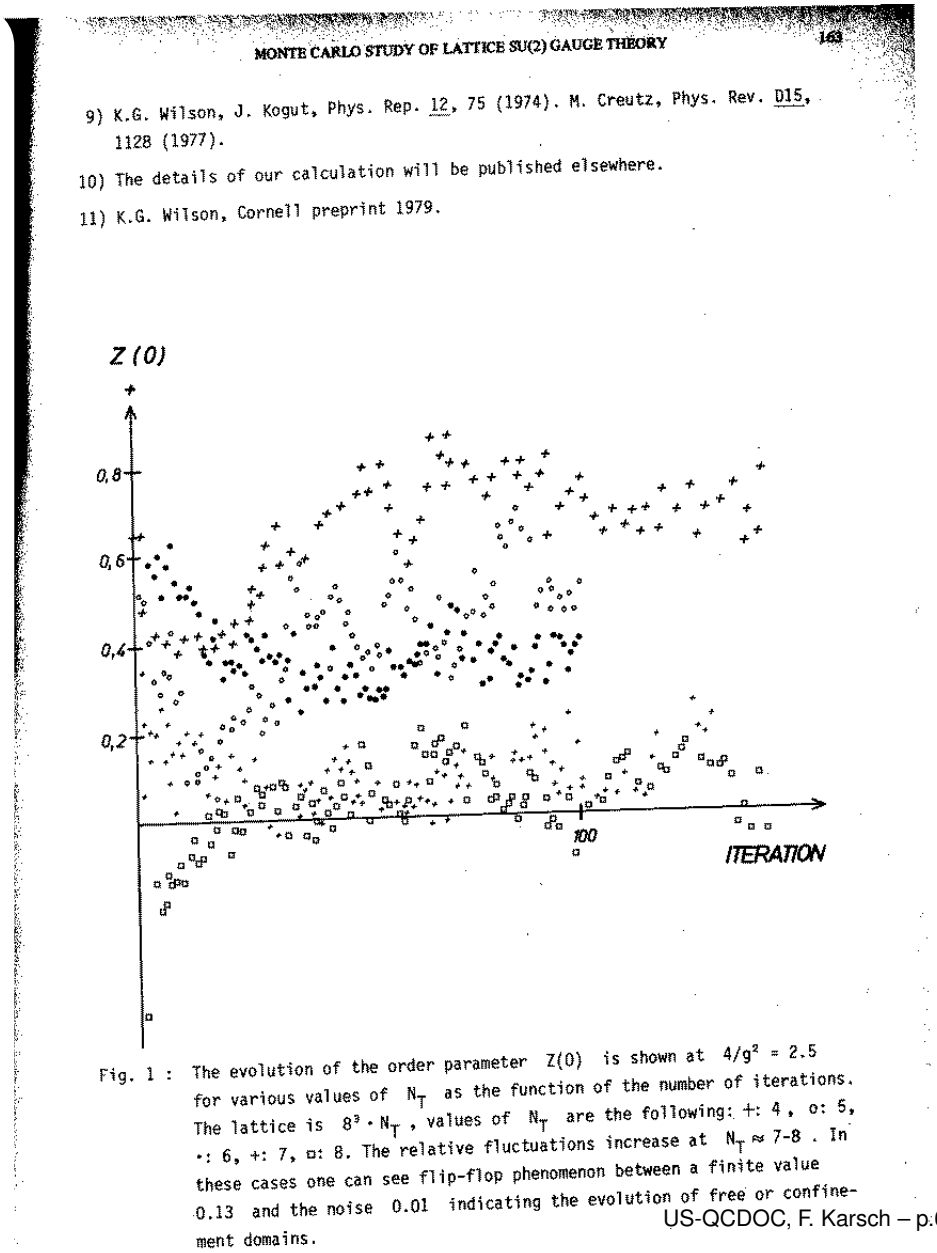
Deconfinement in an SU(2) gauge theory

J. Kuti, J. Polonyi and K. Szlachanyi, 1980/81

lattice size: $8^3 \times N_\tau$

$$N_\tau = 4 - 10$$

number of iterations $\mathcal{O}(100)$



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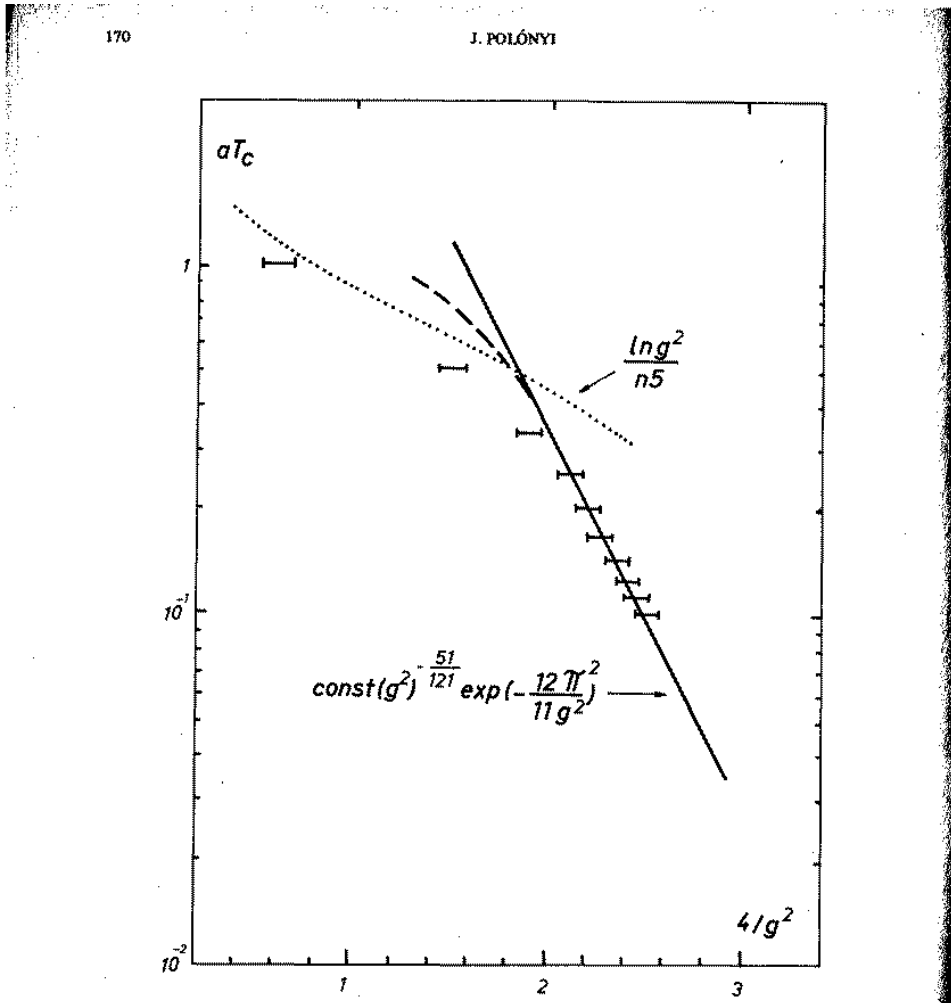

Fig. 8 : Critical temperature measured at various g^2 . Above $4/g^2 = 2$ our results follow the renormalization group relation (solid line). The dashed line includes the finite temperature correction. The dotted line is a strong coupling estimate of $a \cdot T_c$ [10].

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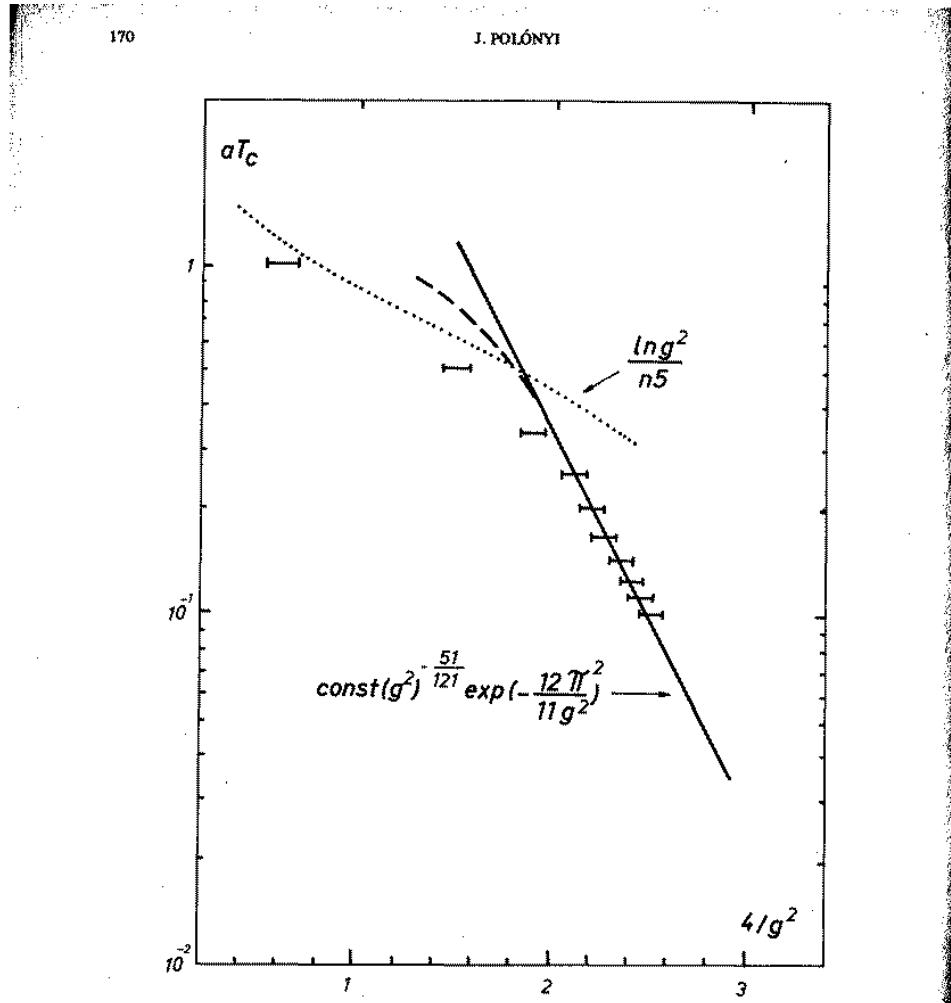
find asymptotic scaling !!

$$T_c = (0.35 \pm 0.05)\sqrt{\sigma}$$

$$\Downarrow$$

$$T_c = (160 \pm 30) \text{ MeV}$$



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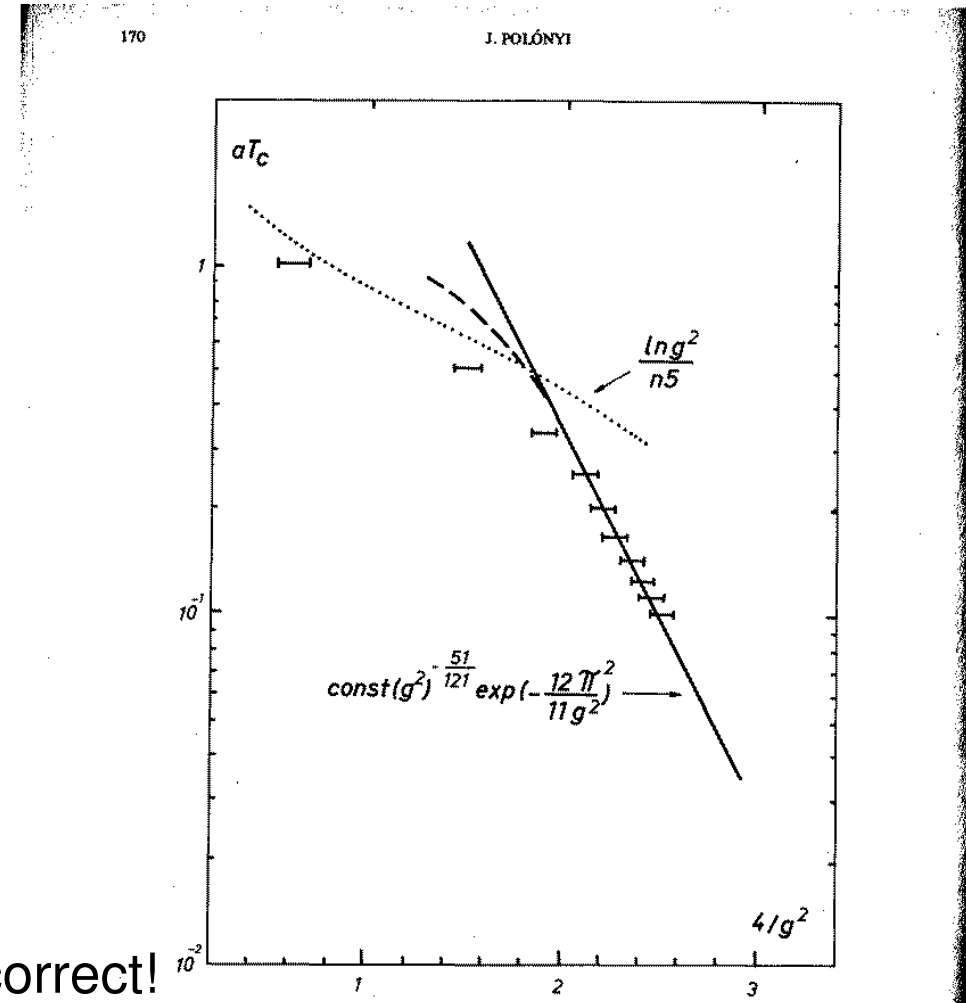
Hagedorn temperature !! $\Rightarrow$ must be correct!



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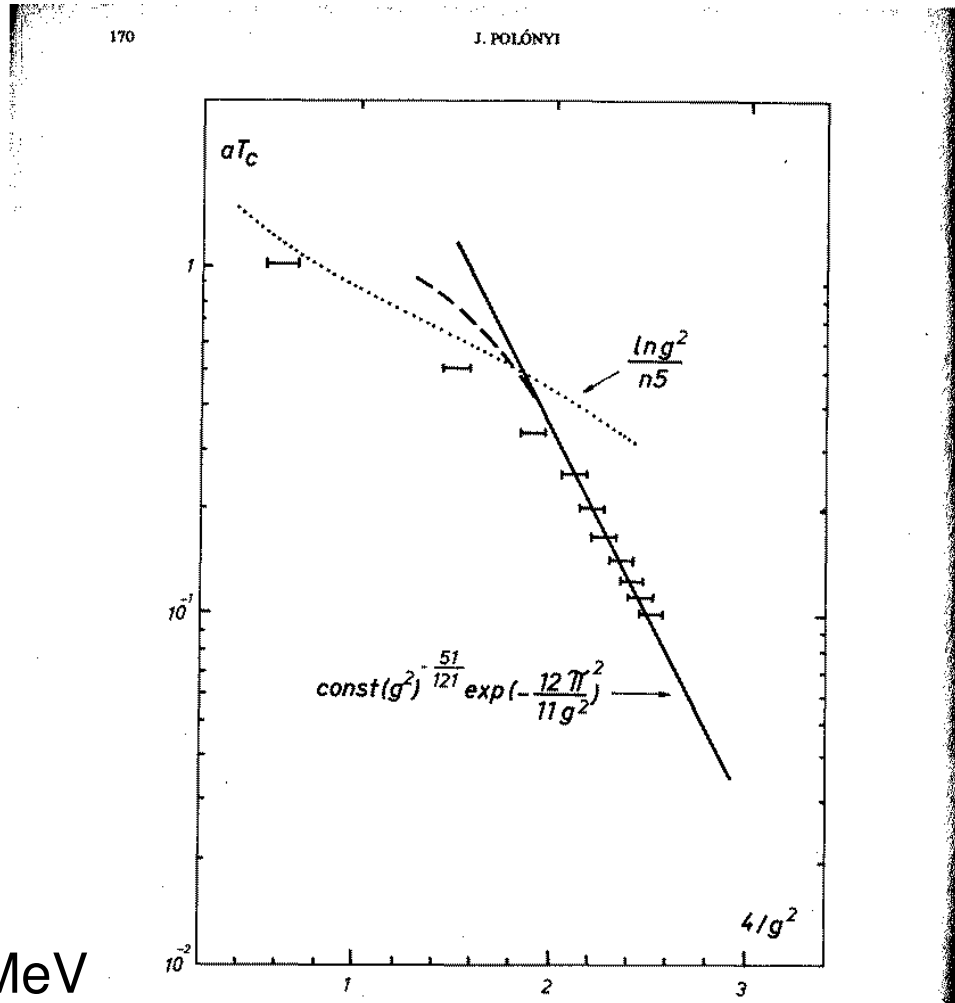
today: $T_c[SU(2)] = 0.69\sqrt{\sigma} \simeq 290$ MeV



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Deconfinement in an SU(2) gauge theory

L. McLerran and B. Svetitsky, 1980/81

lattice size: $7^3 \times 3$

order parameter for deconfinement

critical exponents: $|L| \sim |T - T_c|^\gamma$

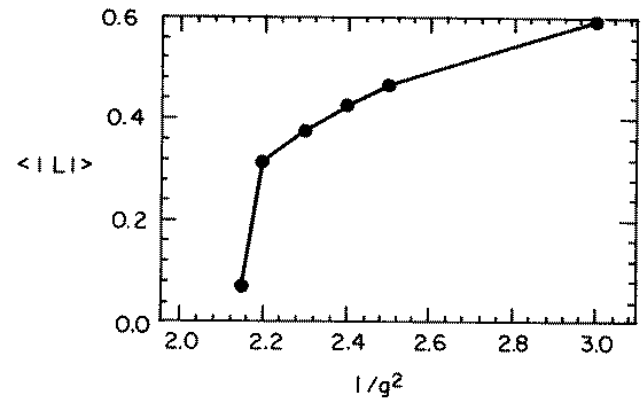

Figure 2
The order parameter <L> as a function of $1/g_0^2$ for SU(2) in a lattice where $N_t = 3$, $N_s = 7$.

where $\gamma = .21 \pm .01$ for $N_t = 3$, $N_s = 7$.[9]

The energy of separation of a two quark system is

$$e^{-\beta V(\vec{r})} = \langle L(\vec{r}) L(\vec{o}) \rangle \quad . \tag{21}$$

For SU(2), the quarks are in a real representation, and there is no difference between this system and a quark antiquark system. For SU(N), the energy of a quark-antiquark system is

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where

$$L^\dagger \equiv \frac{1}{N} \mathrm{tr} \left(P\, e^{i \int_0^\beta dt\, \tau \cdot A^0(\vec{r},t)} \right)^\dagger \quad . \tag{23}$$

Using the generalizations of Eqs. (21)-(23) for static multiquark configurations, it is possible to construct an expression for the energy of a multi-quark state. The Z_N transformation of Eq. (17) may then be applied to conclude that the only states which may have finite energy are those with the number of quarks minus the number of antiquarks equal to an integer multiple of N. These states are states possessing singlet configurations of quarks. In the confined phase where the Z_N symmetry is manifest, only singlet configurations of quarks may have finite energy.

Another interesting fact is that near the phase transition the measured value of <L> may remain constant for a very large number of iterations and then change

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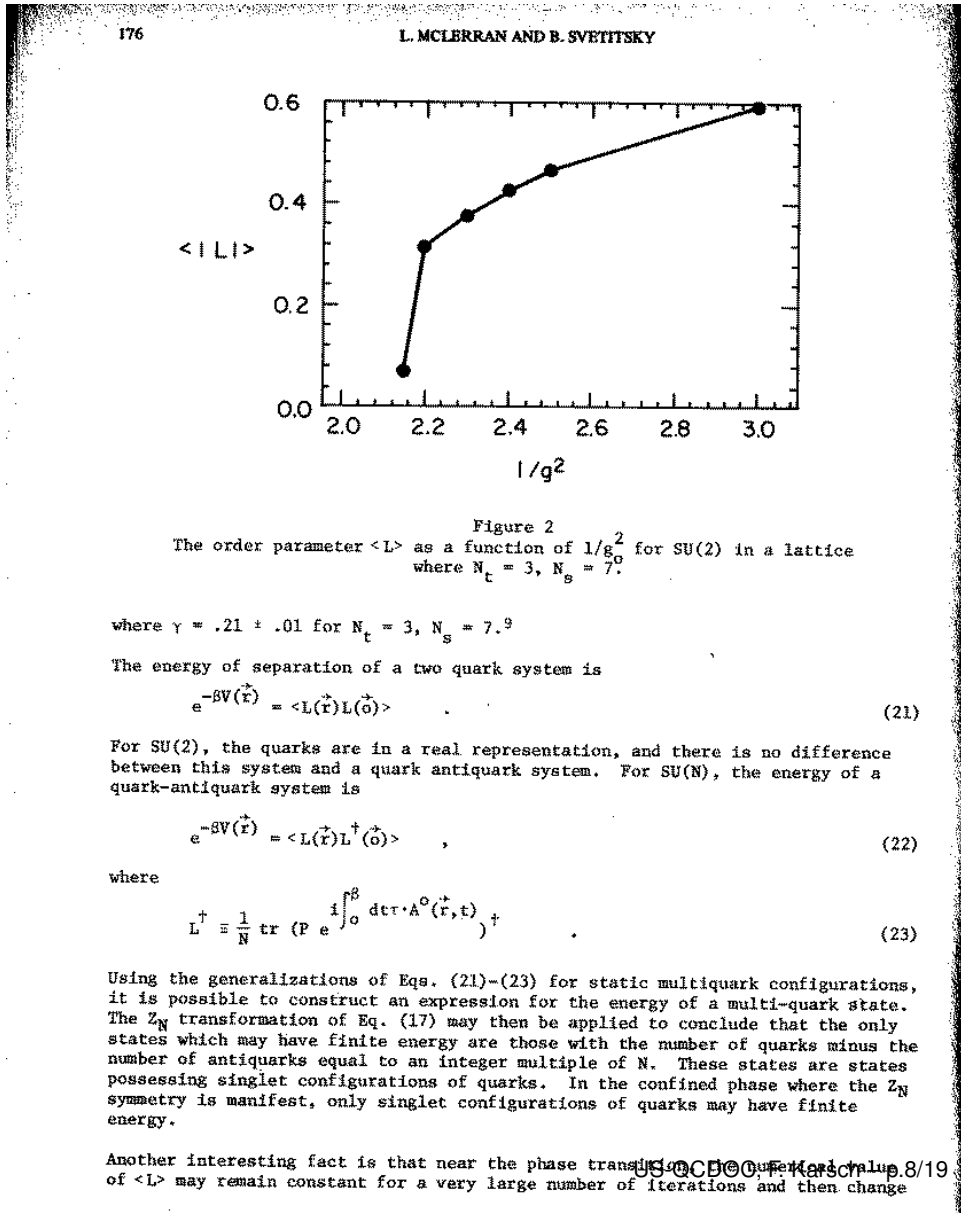
$T_c \simeq 200$ MeV



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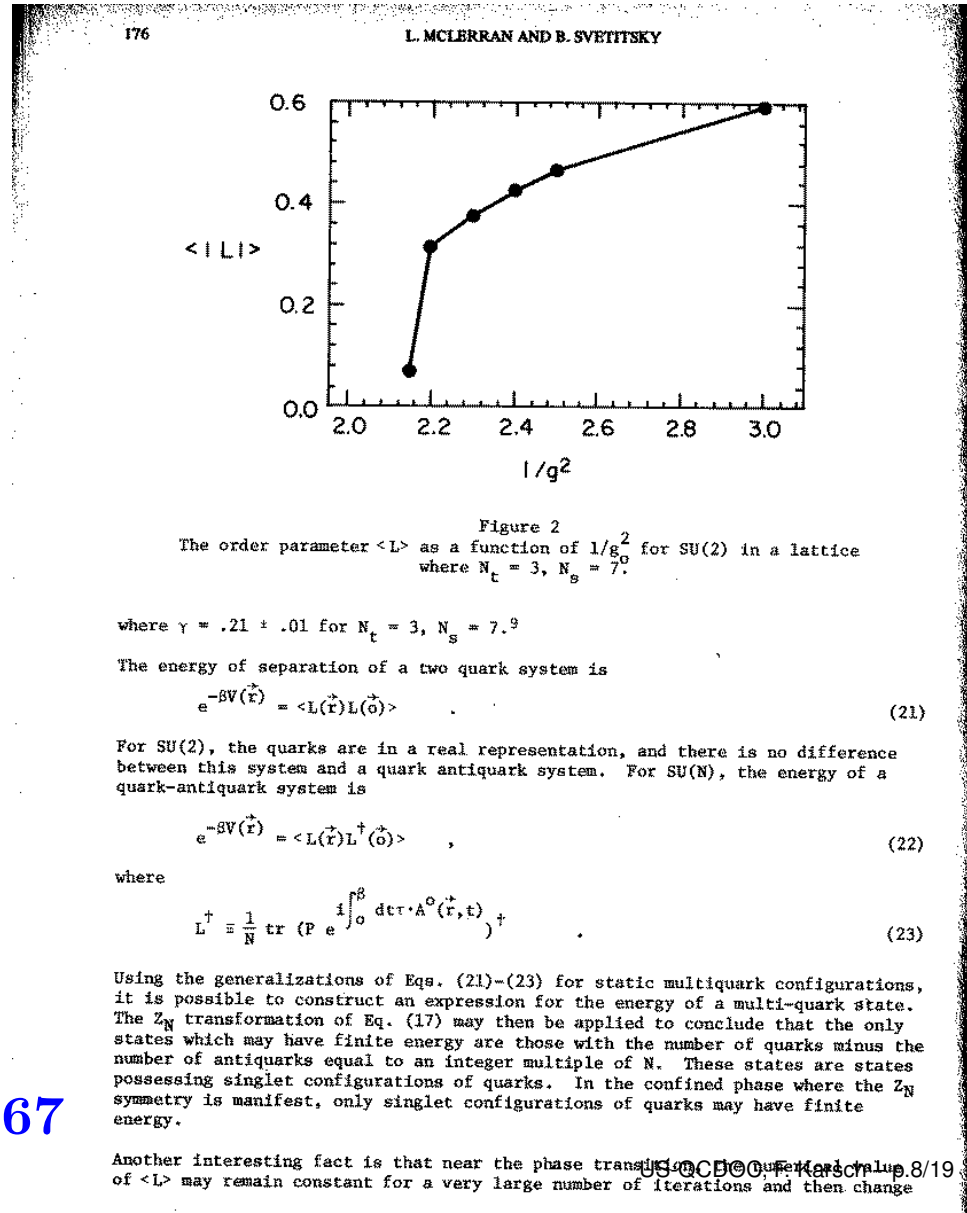
today:

learned about universality classes

L.G. Yaffe, B. Svetitsky, 1982

R. Pisarski, F. Wilczek, 1984

Ising universality class: $\gamma \equiv \beta = 0.3267$



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  N.H. Christ et al. (1991)

QCD Thermodynamics with light quarks

lattice size: $8^3 \times 4$

equation of state, 4 flavor QCD

first order phase transition

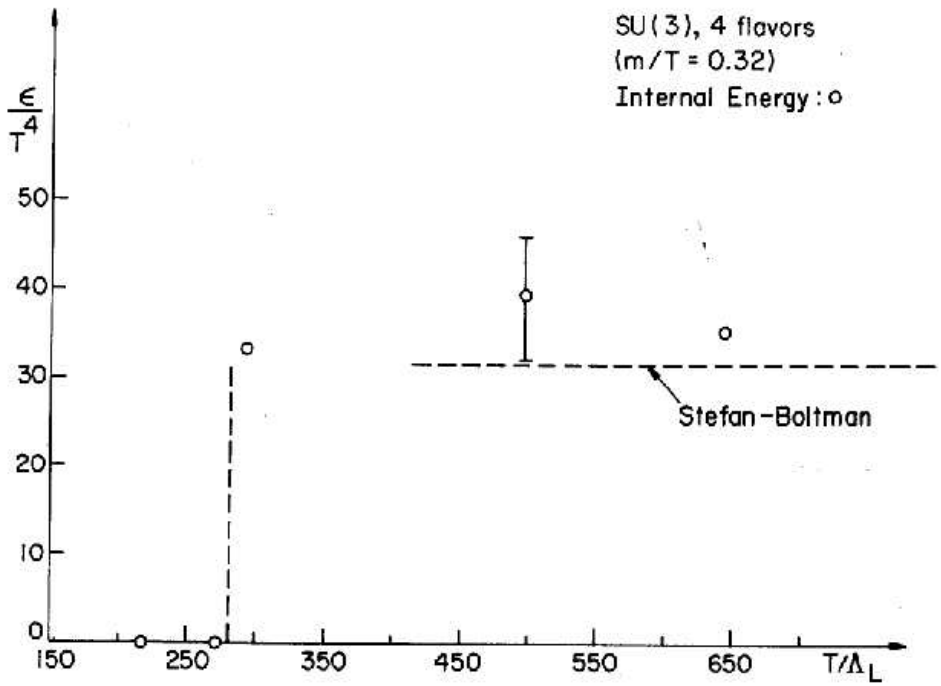

Fig. 1

J.B. Kogut et. al., 1984

lattice size: $16^3 \times 4$

QCD phase diagram

flavour dependence

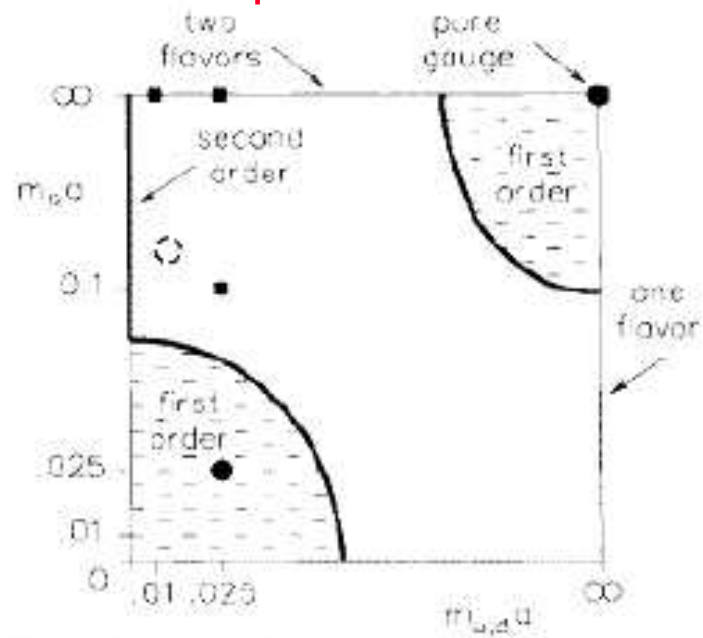

FIG. 1. Presence and absence of the finite-temperature QCD phase transition as a function of $m_{u,d}a$ and $m_s a$. Mass values for which the transition is and is not seen on a $16^3 \times 4$ lattice are denoted respectively by solid circles and squares. The physical point, indicated roughly by the dashed circle, lies in the region of no transition.

N.H. Christ et. al., 1991

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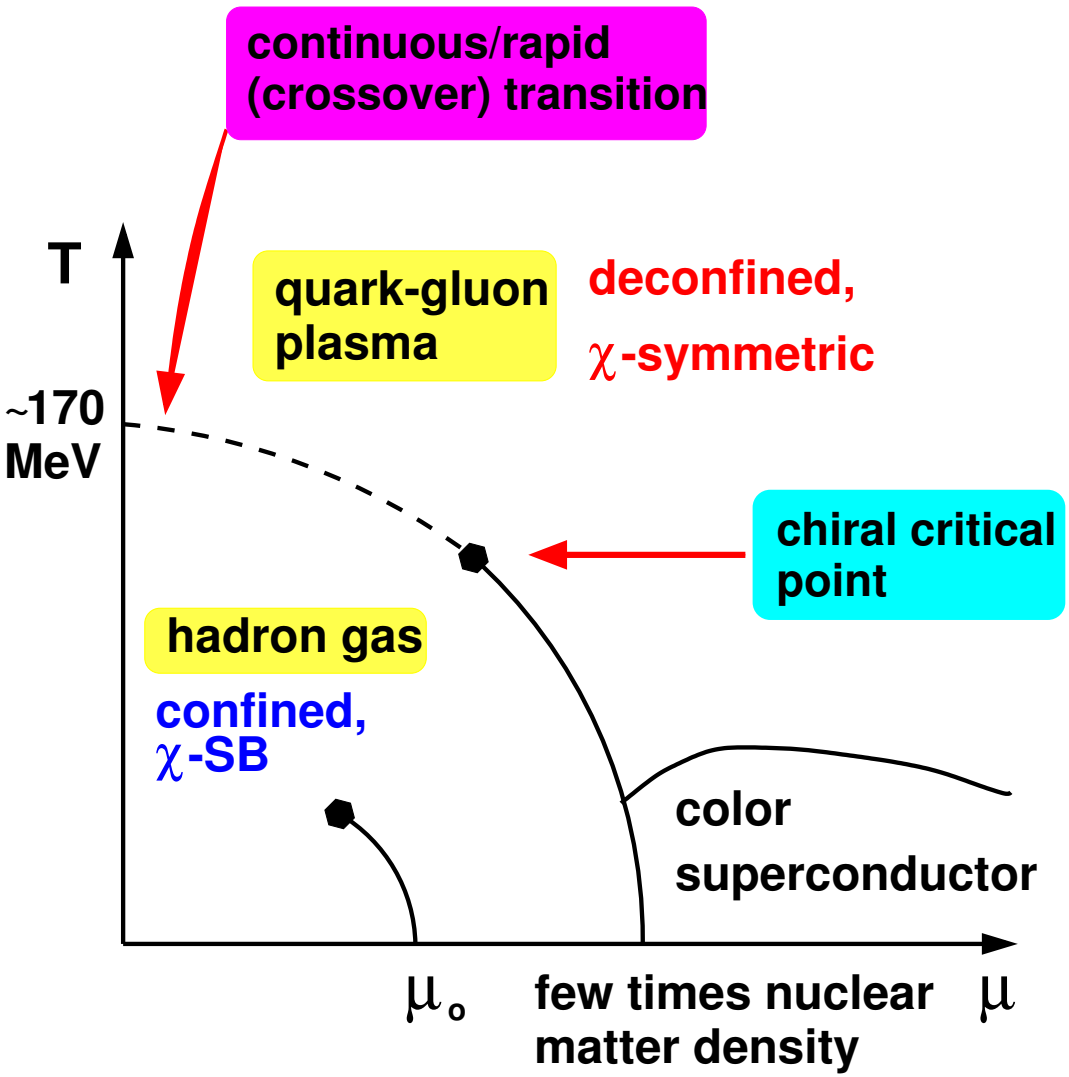
  N.H. Christ et al. (1991)
- ..and grows exponential at non-zero baryon number density
  first evidence for existence of a (chiral) critical point at non-zero baryon number density from lattice calculations Z. Fodor and S. Katz (2002)

Critical behavior in hot and dense matter: QCD phase diagram



continuous transition for small chemical potential and small quark masses at

$$T_c \simeq 170\,MeV$$
$$\epsilon_c \simeq 0.7\,GeV/fm^3$$

2nd order phase transition; Ising universality class

$T_c(\mu)$ under investigation

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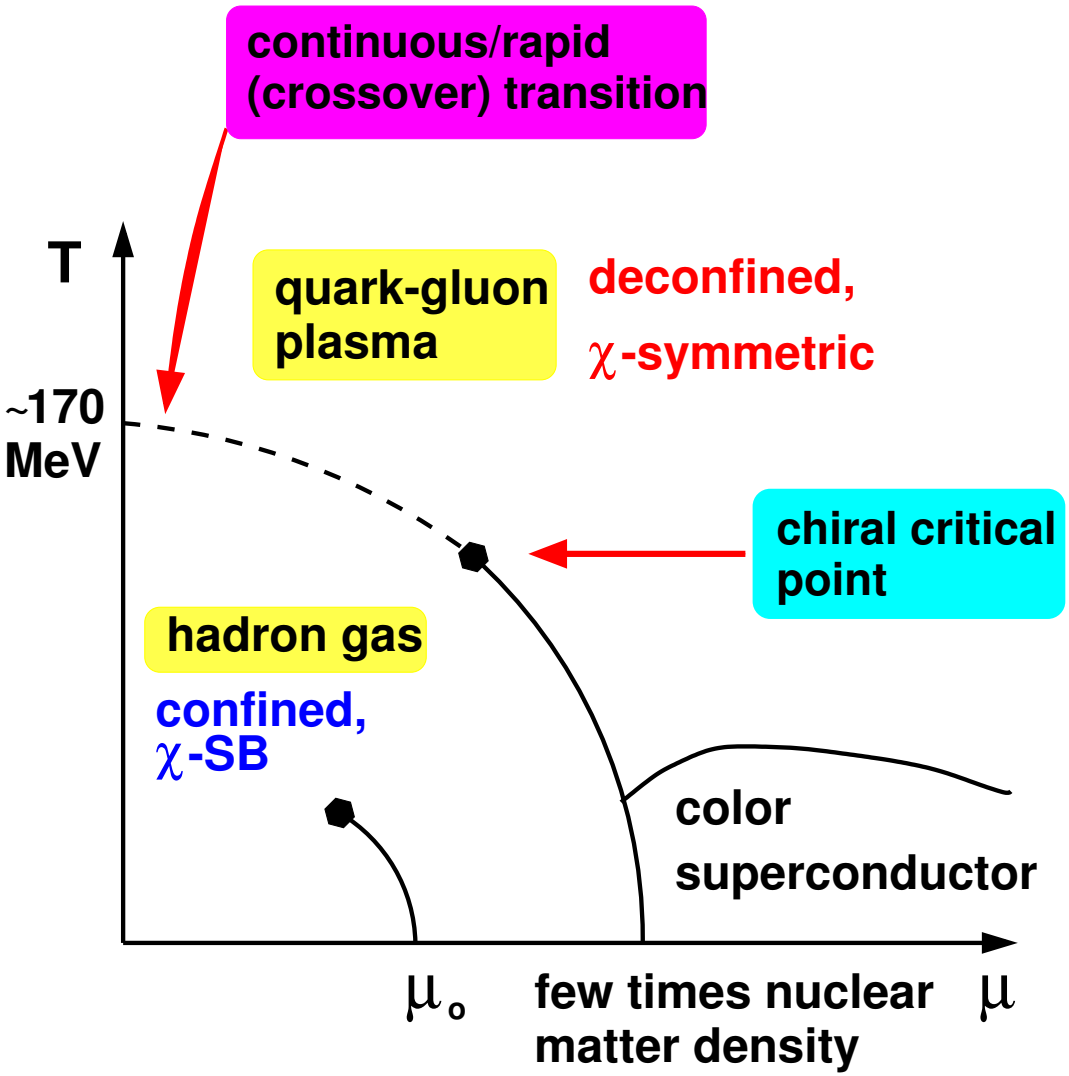

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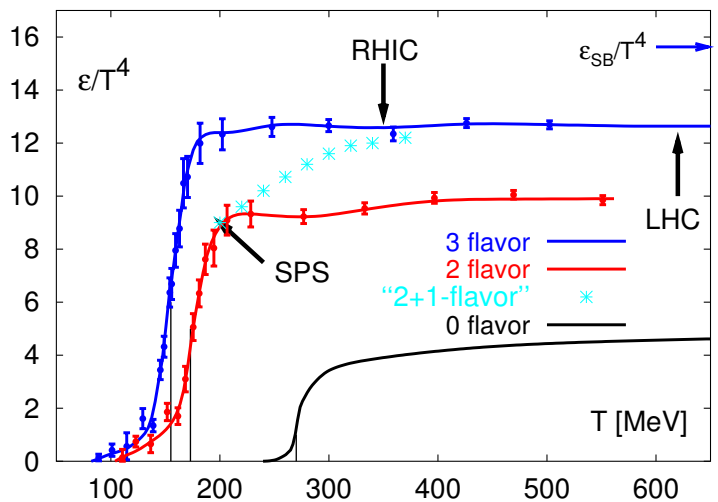
$T_c(\mu)$ under investigation

recent doubts on order of transition
A. Di Giacomo et al., hep-lat/0503030

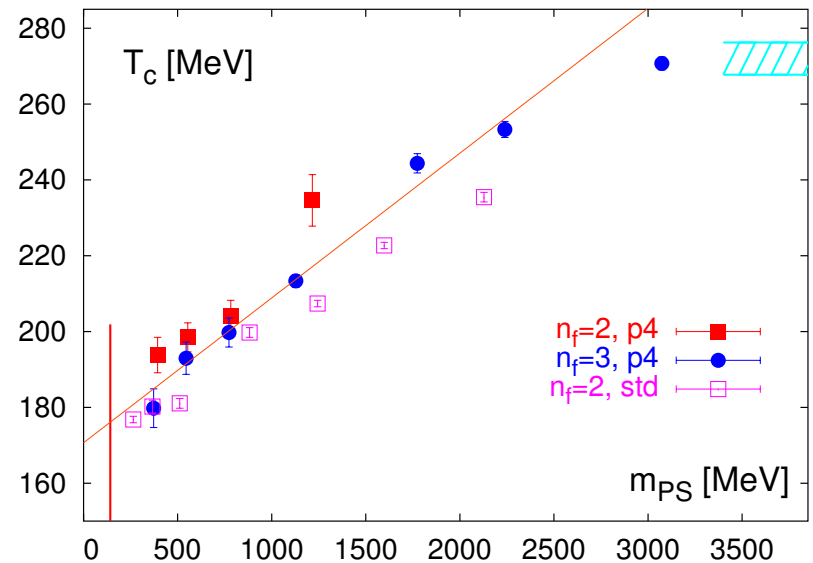
location of CCP uncertain;
volume dependence (Fodor/Katz)

improving accuracy on $T_c, \epsilon_c, ..$ needed to make contact to HIC phenomenology

$\mu = 0$: Equation of State and T_c



QCD EoS



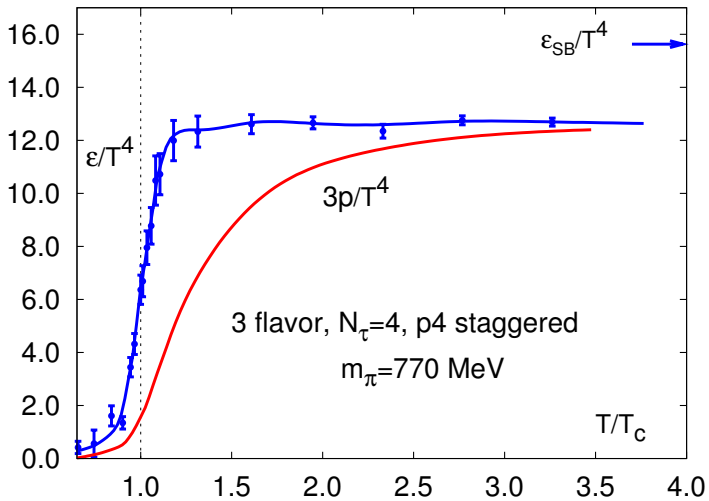
transition temperature

- ϵ/T^4 for $m_\pi \simeq$ 770 MeV; ($m_\pi/m_\rho \simeq 0.7$, $TV^{1/3} = 4$) $\epsilon_c/T_c^4 = 6 \pm 2$ $\Rightarrow$
- $T_c = (173 \pm 8 \pm sys)$ MeV (T_c for $m_\pi \gtrsim$ 300 MeV) $\epsilon_c = (0.3 - 1.3)$GeV/fm^3

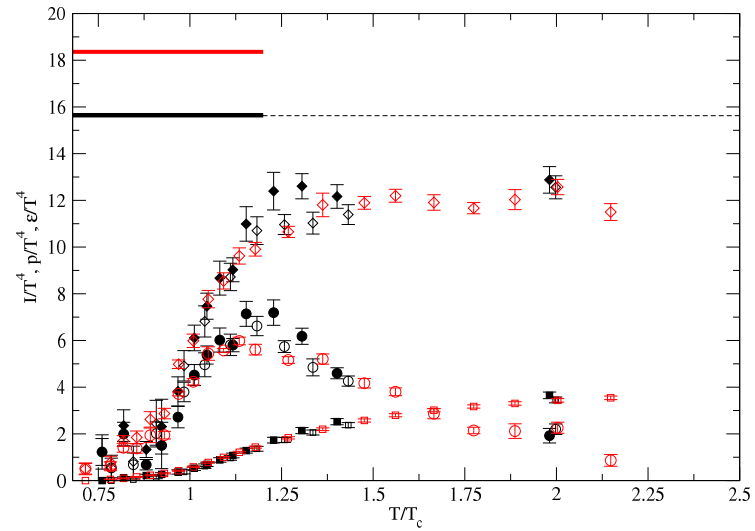
- improved staggered fermions but still on rather coarse lattices: $N_\tau = 4$, i.e. $a^{-1} \simeq 0.8$ GeV

FK, E. Laermann, A. Peikert, Nucl. Phys. B605 (2001) 579

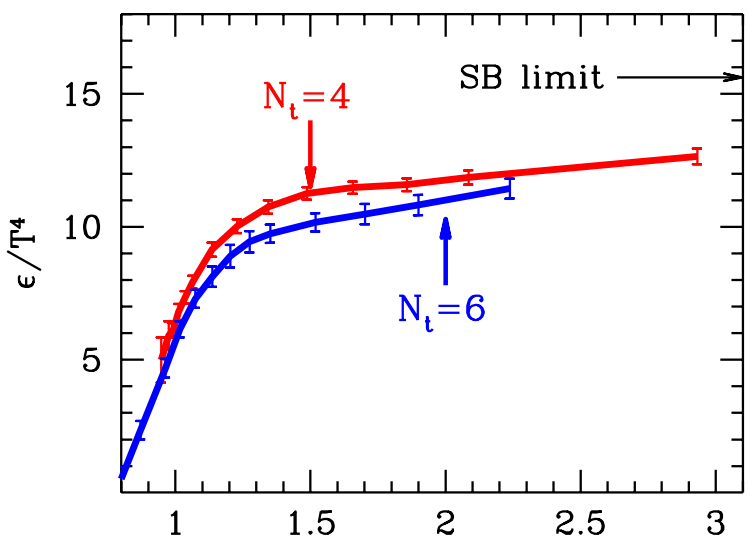
recent results on QCD EoS



old Bielefeld result, 2001
improved staggered (p4), $N_\tau = 4$
3-flavor, $m_\pi \simeq$ 770 MeV



MILC-collaboration, hep-lat/0509053
$\mathcal{O}(a^2)$ improved staggered, $N_\tau = 4,\ 6$
(2+1)-flavor, $m_\pi \gtrsim 250$ MeV



$\epsilon_c/T_c^4 \simeq 6$ insensitive to m_π and a^{-1}
HOWEVER: thermodynamic limit??
$TV^{1/3} \simeq 2$

Y. Aoki et al., hep-lat/0510084
standard staggered, $N_\tau = 4,\ 6$
(2+1)-flavor, $m_\pi \longrightarrow 140$ MeV (extrap.)
ϵ/T^4 rescaled with $(\epsilon_{SB}/T^4)(N_\tau)$

Thermodynamics on QCDOC

US QCDOC (DOE funded)

RBRC QCDOC

RIKEN-BNL computing environment:
2x10 TFlops QCDOC

$\sim$ 5 TFlops devoted to QCD Thermodynamics

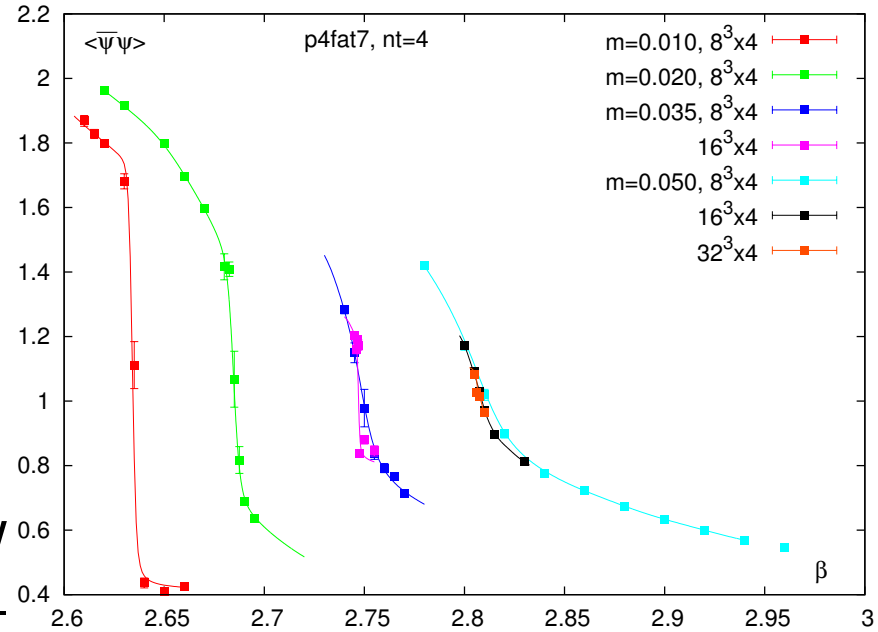
$\Rightarrow$ towards a calculation of QCD thermodynamics with a realistic quark mass spectrum

T_c and EoS in (2+1)-flavor QCD

goal: (2+1)-flavor QCD with a physical strange quark mass and "almost" physical light quarks ($m_\pi \simeq 200$ MeV) in the thermodynamic limit $VT^3 \simeq 4$

1st step: 3-flavor QCD, calculation of critical couplings for several quark masses and lattice sizes ($N_\tau = 4,\ 6,\ N_\sigma = 8,\ 16,\ 32$)

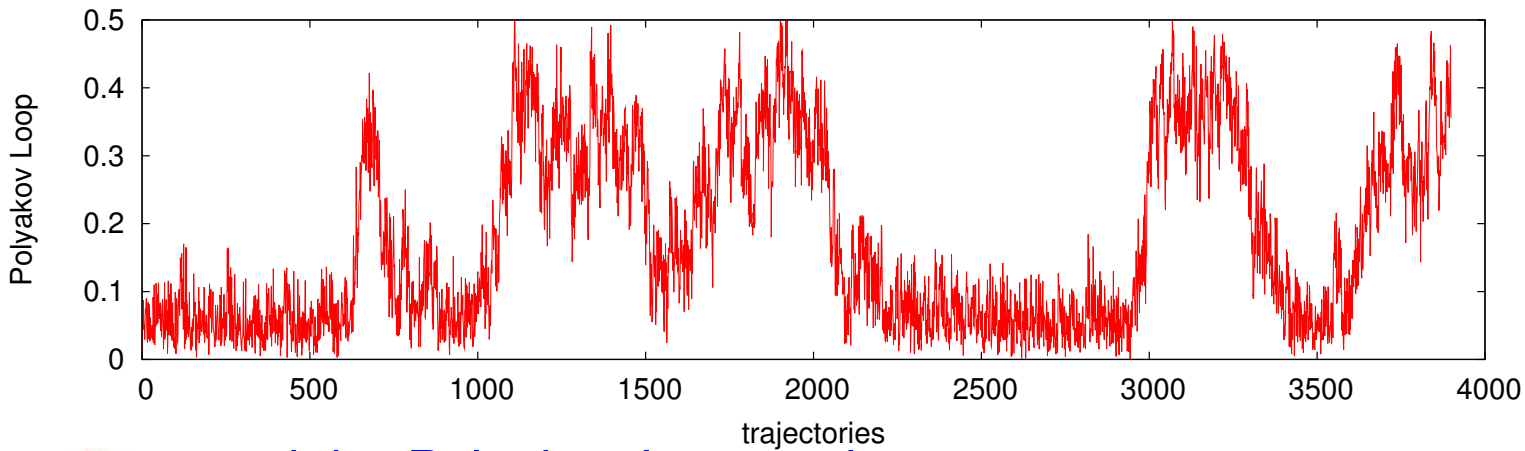
- chiral condensate
- reached small quark masses $m_{PS}/m_V < 0.4$
- achieved high statistics up to 7000 traj. per β-value
- small β separations allow Ferrenberg-Swendsen analysis also for $N_\tau = 6$



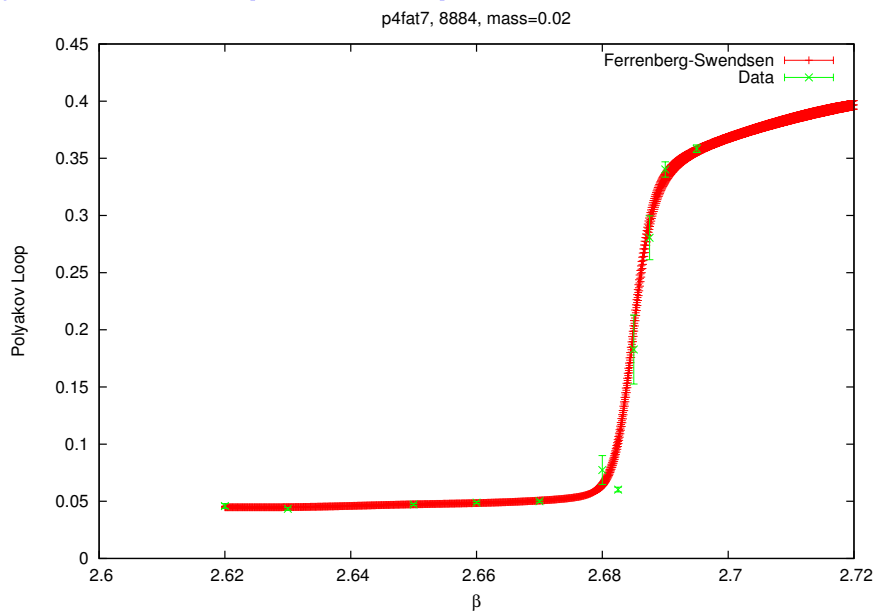
3-flavor QCD: transition becomes 1st order for small quark masses

Time histories and Polyakov loops

- still looking at time histories...



- ...and the Polyakov Loop order parameter:



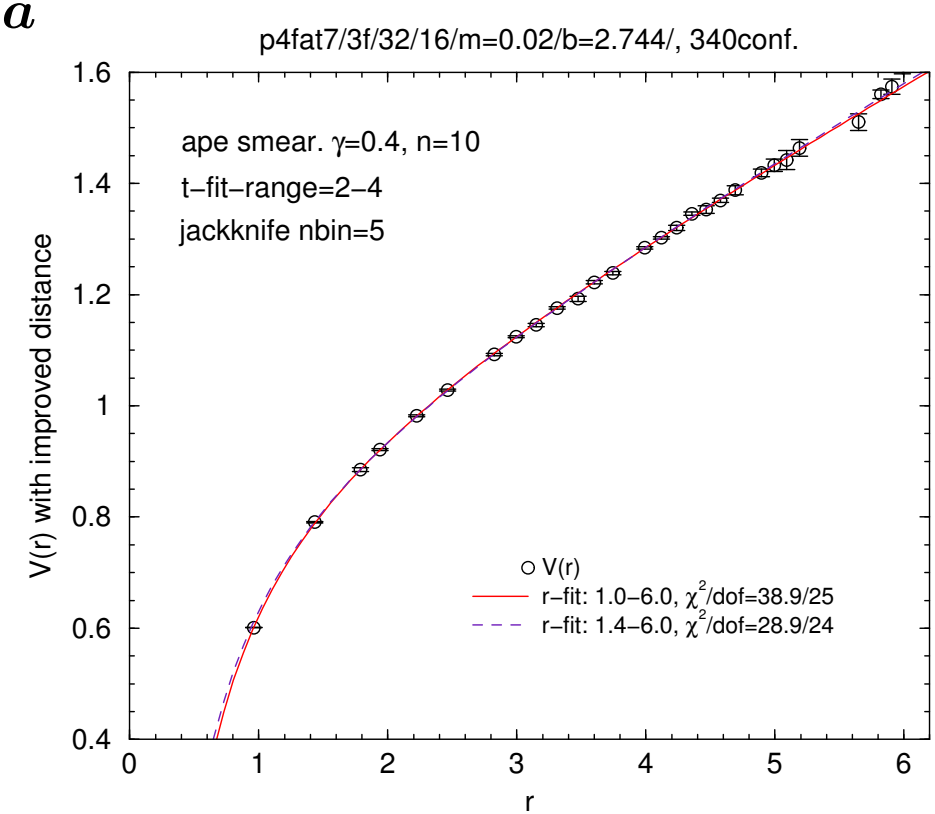
Determination of T_c (3-flavor QCD)

2nd step: determine the scale at β_c, e.g. from heavy quark potential

$$1/T_c = N_\tau a(\beta_c(N_\tau)) \;\Rightarrow\; T_c/\sqrt{\sigma} = 1/(N_\tau\sqrt{\sigma})$$

or use "r_0": $\quad T_c r_0 = N_\tau r_0/a$

- scale determined on $16^3 \times 32$ lattices
- will check consistency of different actions
- aim at consistent picture from different scales



T=0: heavy quark potential

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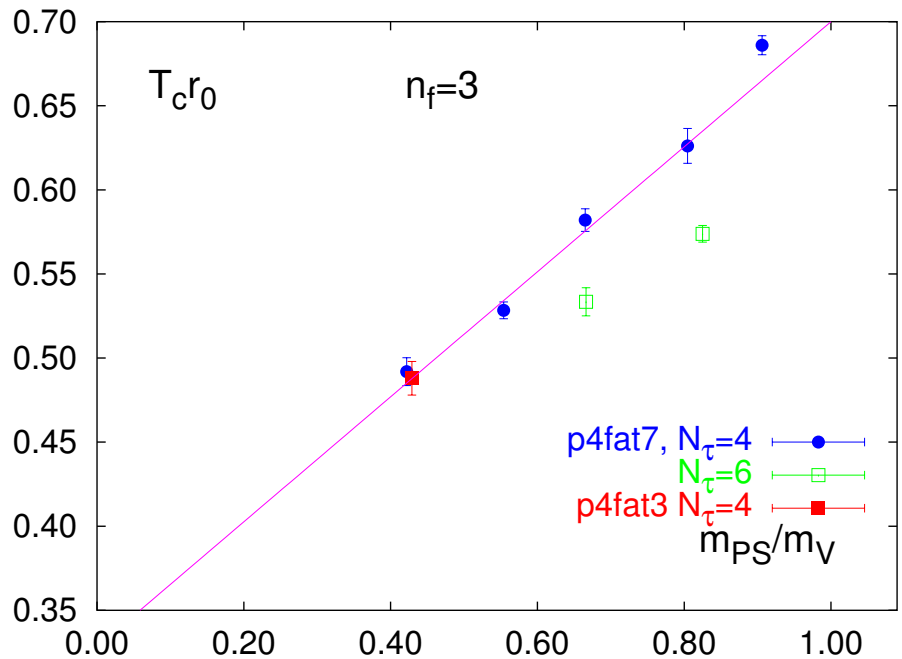
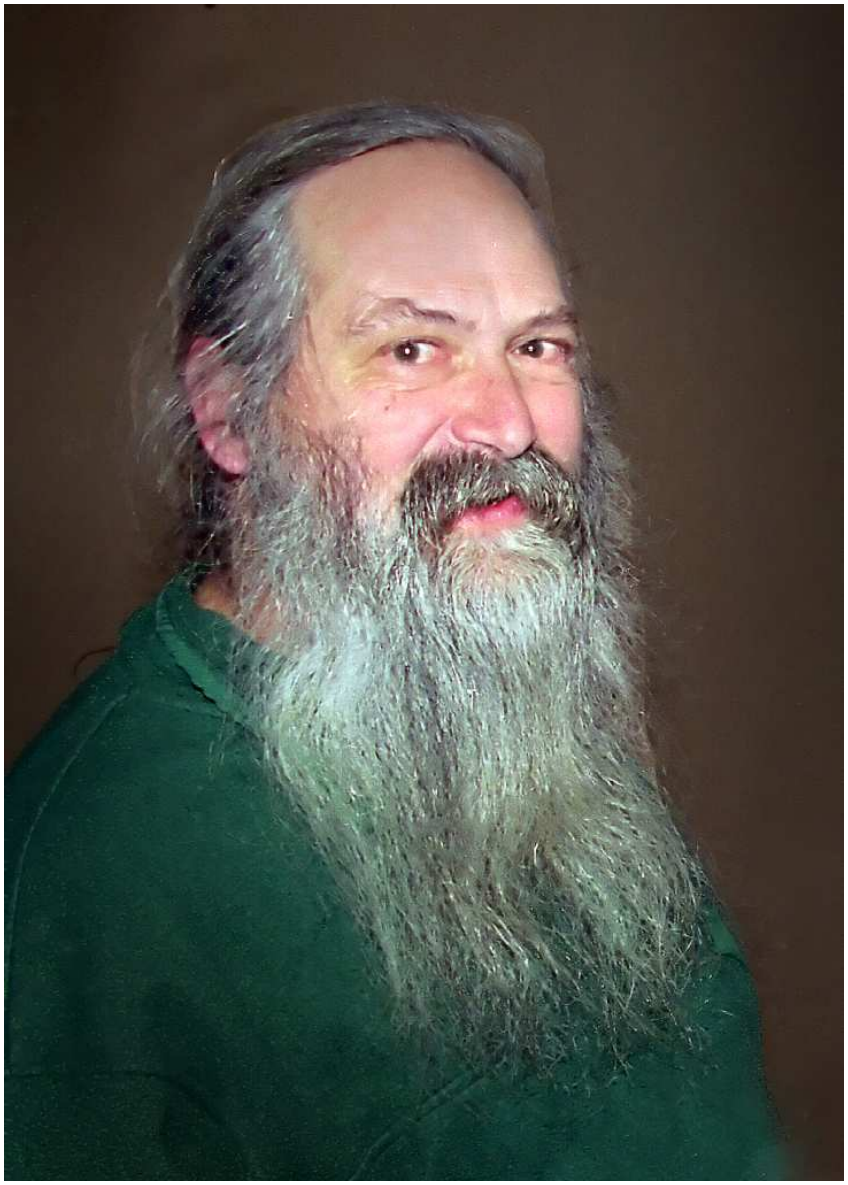


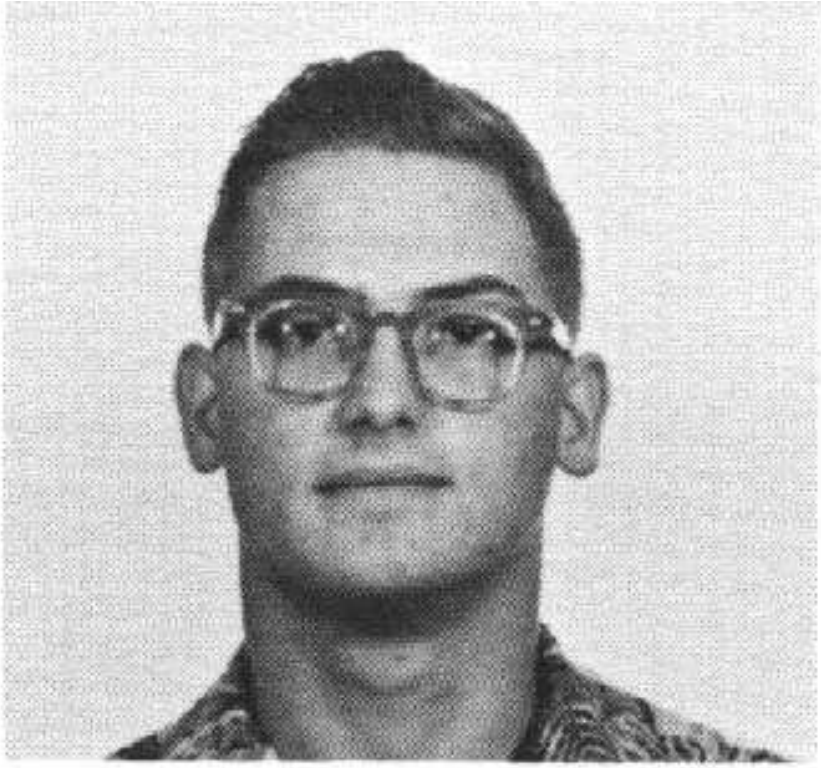
critical temperature from r_0
chiral extrapolation (11/29/05):
$T_c = (140 \pm 10)$ MeV

25 years of lattice calculations (at BNL)

...would not have happend without Mike's outstanding contributions...

25 years of lattice calculations (at BNL)





Michael J. Creutz
San Dieguito Un. H. S
Encinitas, Calif. C

25 years of lattice calculations (at BNL)

...would not have happend without Mike's outstanding contributions...

...and would not be this healthy in the US without the joint american-japanese adventure that made the QCDOC possible

25 years of lattice calculations (at BNL)

an american-japanese present for Mike

made possible by Atsushi Nakamura

25 years of lattice calculations (at BNL)

an american-japanese present for Mike

The SHARP-Soroban ($\sim$ 1985)

The mechanic world meets the electronic world:

An electronic pocket calculator and a soroban are both fitted in one frame. The soroban is made of plastic, it has 13 rods with the system (1+4). This curious construction indicates the change from the use of mechanical calculation devices to electronic computers.