The pressure of deconfined QCD for all temperatures and quark chemical potentials

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Strong & Electroweak Matter, May 11, 2006

Pressure for all T/μ up to g^4

- Based on:
  A. Ipp, K. Kajantie, A. Rebhan, A. Vuorinen,
  hep-ph/0604060

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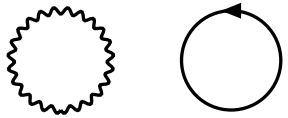
- 44 slides

The problem

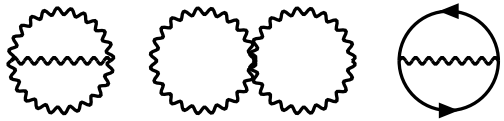
Thermodynamic potential:

$$\Omega = \frac{-1}{\beta V} \ln \int \mathcal{D}A \mathcal{D}\bar{\psi} \mathcal{D}\psi e^{-\mathcal{S}_{\mathrm{QCD}}}$$

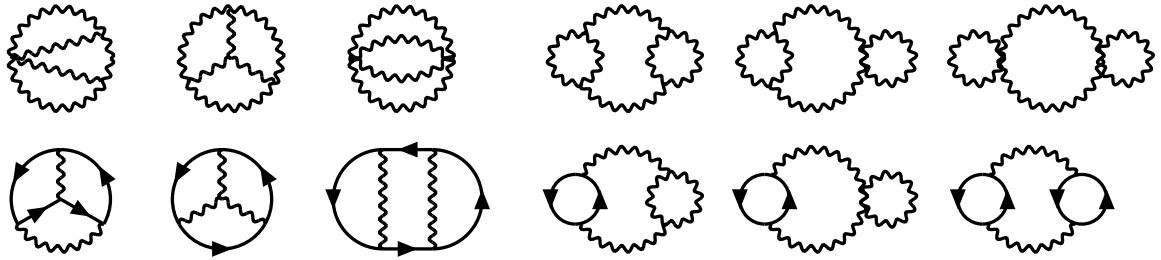
g^0



g^2



g^4



which of these diagrams cause trouble?

(+ ghost diagrams)

massless QCD Lagrangian

$$\mathcal{L}_{\mathrm{QCD}} = \tfrac{1}{4} F^a_{\mu\nu} F^{\mu\nu,a} + \bar{\psi} \not{D} \psi$$

$$\mathcal{S}_{\mathrm{QCD}} = \int_0^\beta d\tau \int d^3x \left(\mathcal{L} + \mu \bar{\psi} \gamma_0 \psi \right)$$

pressure

$$p_{\mathrm{QCD}}(T, \mu, g) = -\frac{\Omega}{V}$$

dimensional reduction:

$$p_{\mathrm{QCD}} = p_E + p_M + p_G$$

$$= .. + \# g^4 \mu^4 \ln \frac{T}{\bar{\mu}_{\overline{\mathrm{MS}}}}$$

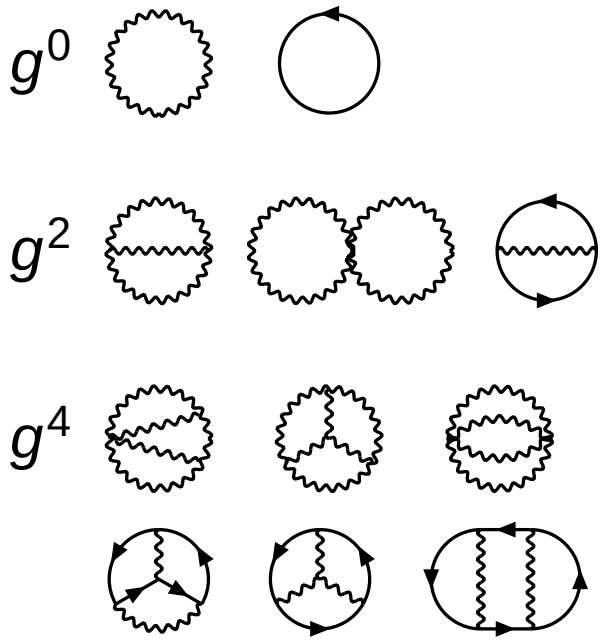
diverges at $T \to 0$

?

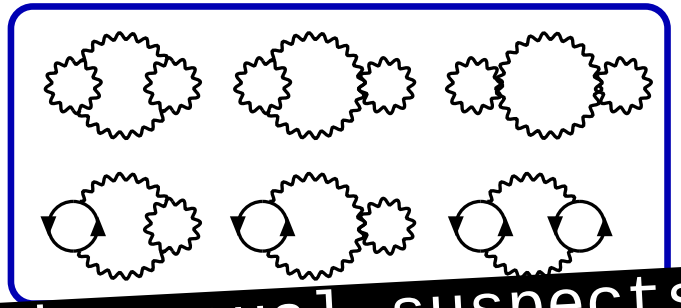
The problem

Thermodynamic potential:

$$\Omega = \frac{-1}{\beta V} \ln \int \mathcal{D}A \mathcal{D}\bar{\psi} \mathcal{D}\psi e^{-\mathcal{S}_{\mathrm{QCD}}}$$



2GR (two-gluon-reducible)



The usual suspects

which o

(+ ghost diagrams)

massless QCD Lagrangian

$$\mathcal{L}_{\mathrm{QCD}} = \frac{1}{4} F^a_{\mu\nu} F^{\mu\nu,a} + \bar{\psi} \not{D} \psi$$

$$\mathcal{S}_{\mathrm{QCD}} = \int_0^\beta d\tau \int d^3x \left(\mathcal{L} + \mu \bar{\psi} \gamma_0 \psi \right)$$

pressure

$$p_{\mathrm{QCD}}(T, \mu, g) = -\frac{\Omega}{V}$$

dimensional reduction:

$$p_{\mathrm{QCD}} = p_E + p_M + p_G$$

$$= .. + \# g^4 \mu^4 \ln \frac{T}{\bar{\mu}_{\overline{\mathrm{MS}}}}$$

diverges at $T \to 0$

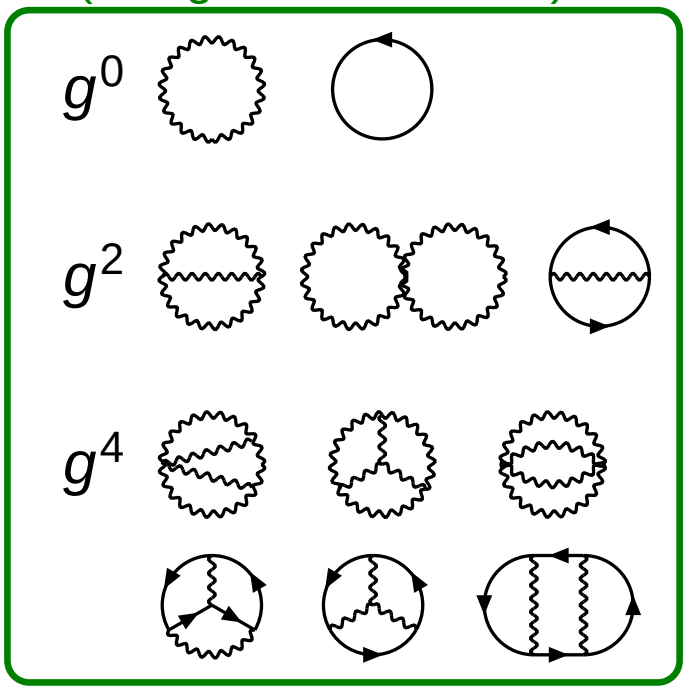
?

The idea: Resum 2GR separately

$$p_{\mathrm{QCD}} = p_{\mathrm{2GI}} + p_{\mathrm{2GR}}^{\mathrm{resummed}} \text{ up to } \mathcal{O}(g^4)$$

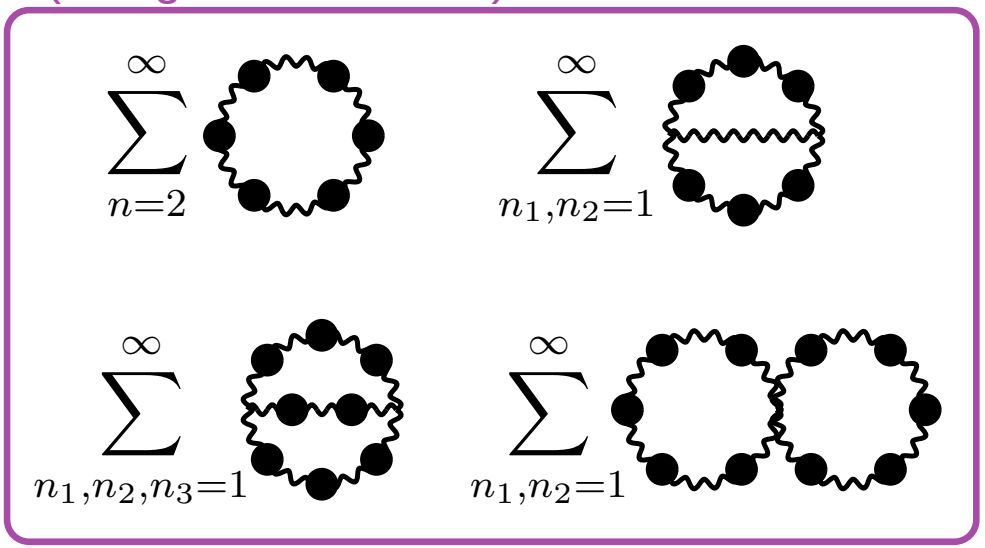
for all T and μ.

2GI
(two-gluon-irreducible)

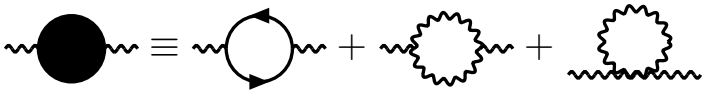


(+ ghost diagrams)

2GR resummed
(two-gluon-reducible)



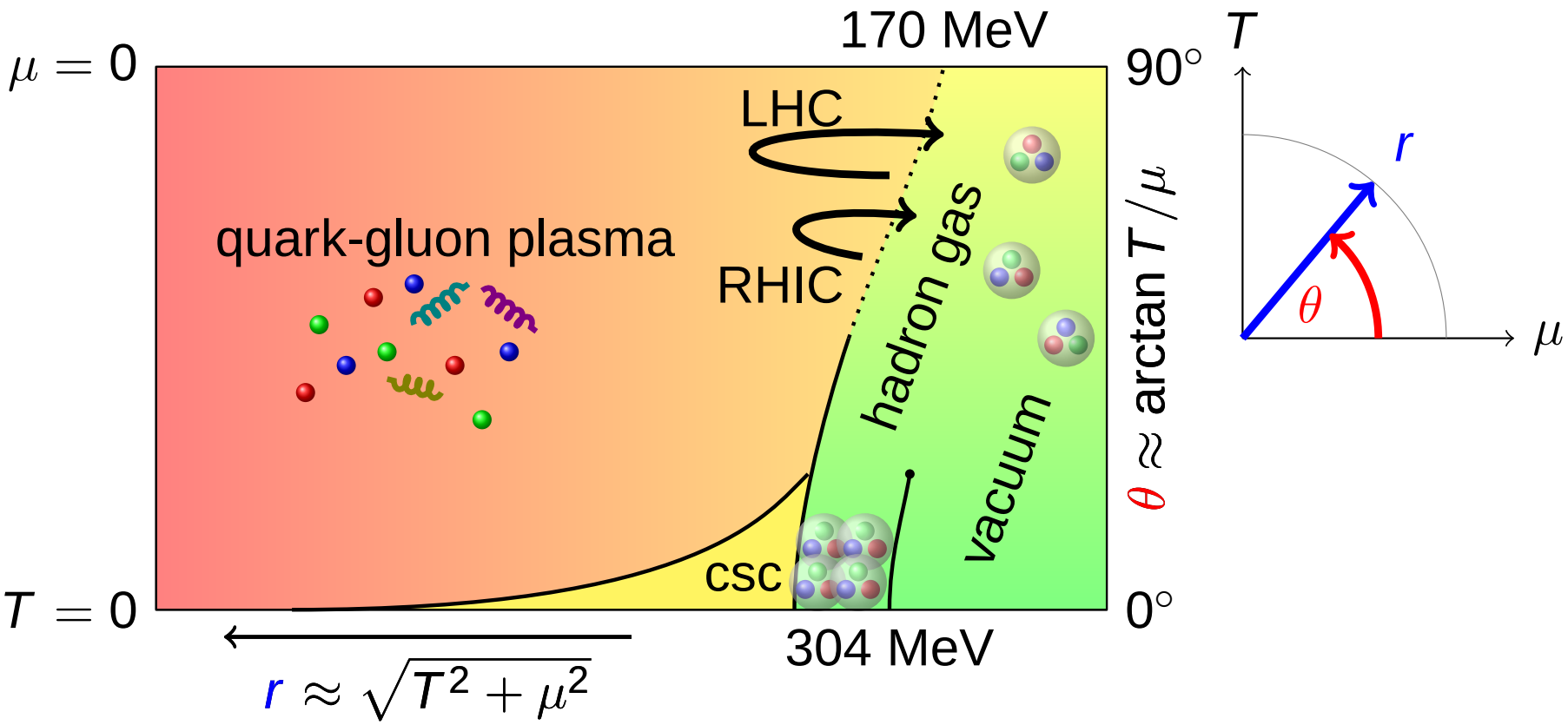
with gluon self-energy



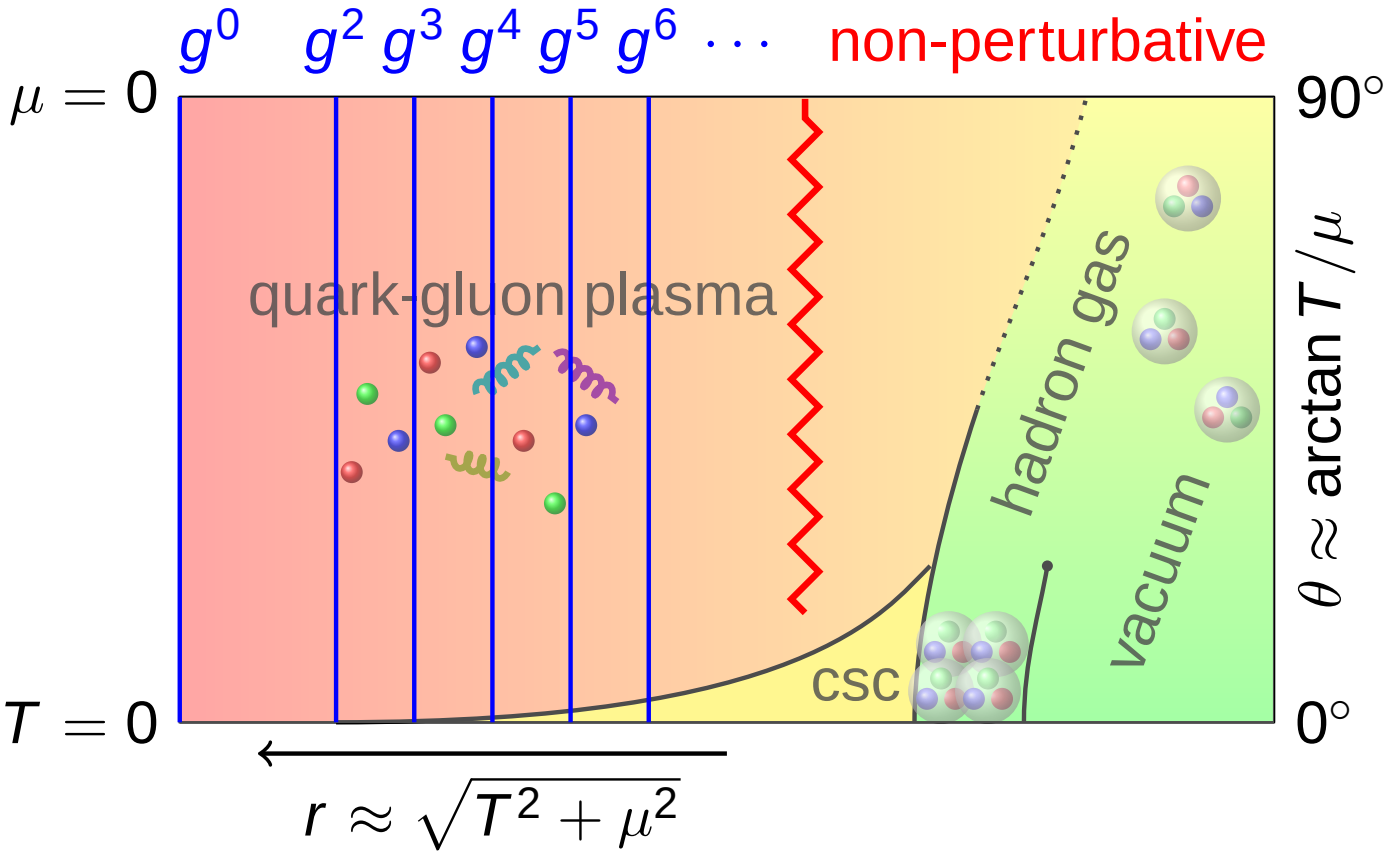
Outline

1. Introduction
   - Phase diagram
   - Previous approaches
2. New approach
   - Problem of dimensional reduction
   - Our solution
   - Technical details
3. Numerical results
   - Agreement with existing approaches
   - Beyond existing approaches
4. Summary

QCD phase diagram (in polar coordinates)



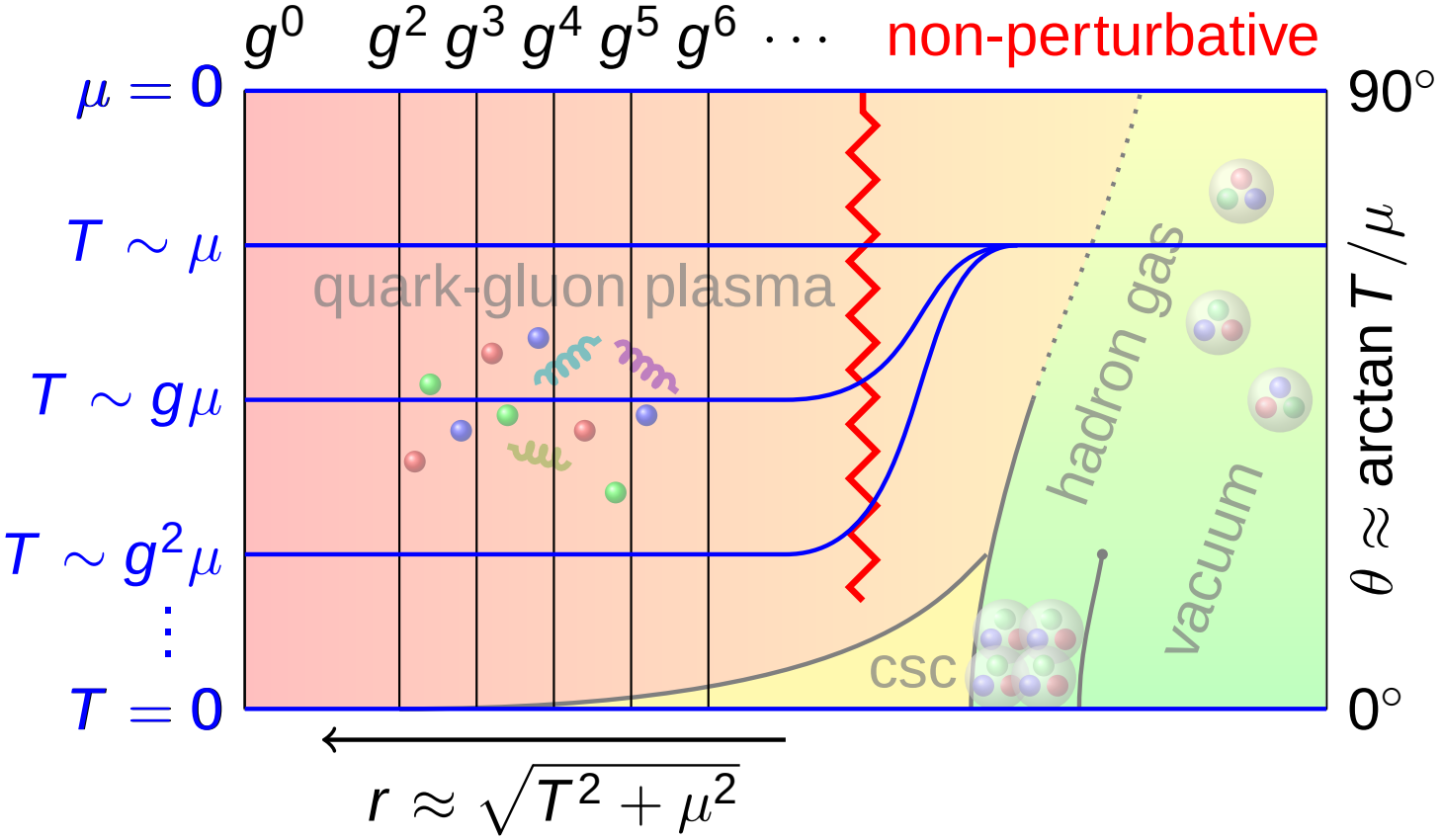
QCD phase diagram (in polar coordinates)



$$\beta(g) \equiv \mu \frac{dg}{d\mu} = -\frac{2g^3}{(4\pi)^2}\left(\frac{11}{3}N_c - \frac{2}{3}N_f\right) + \mathcal{O}(g^5)$$

Gross, Wilczek, Politzer (1973)

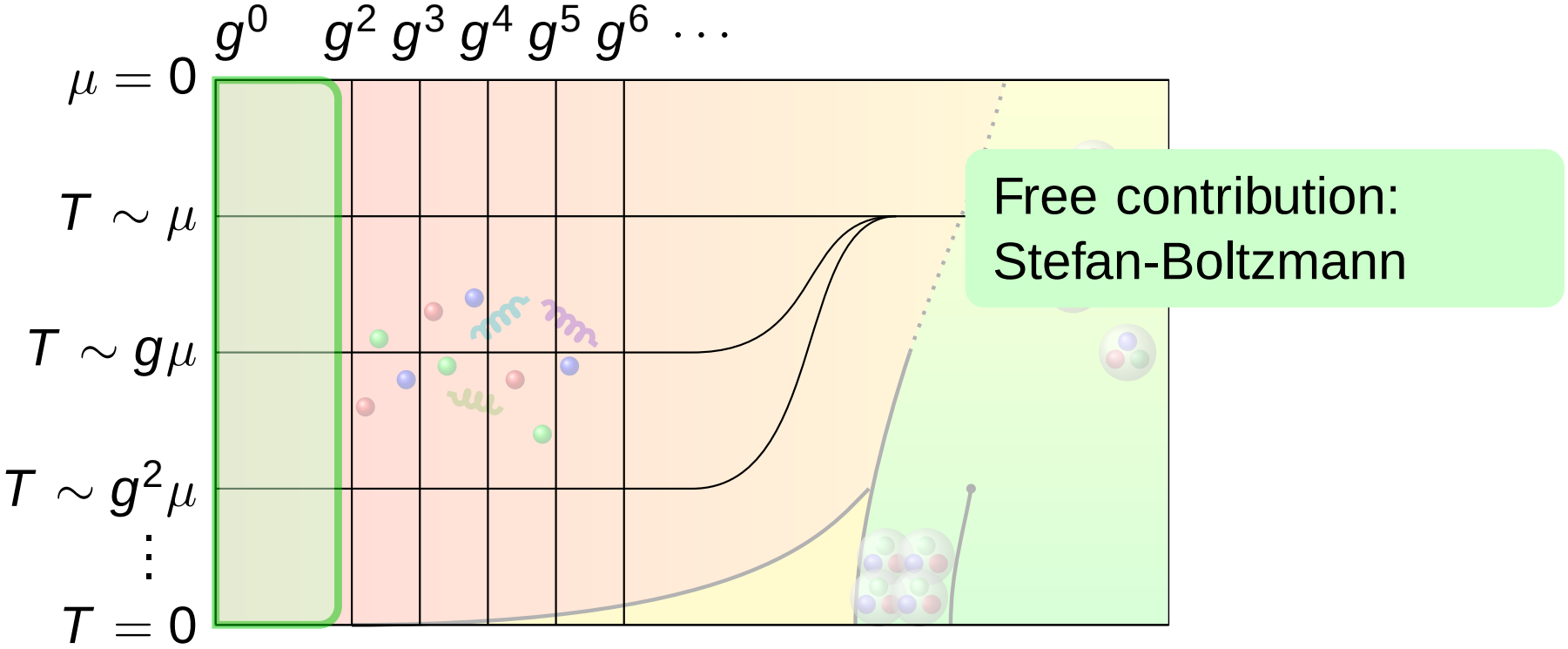
QCD phase diagram (in polar coordinates)



color superconducting gap $\phi \sim T_c \sim \mu g^{-5} \exp \frac{-3\pi^2}{g\sqrt{2}}$

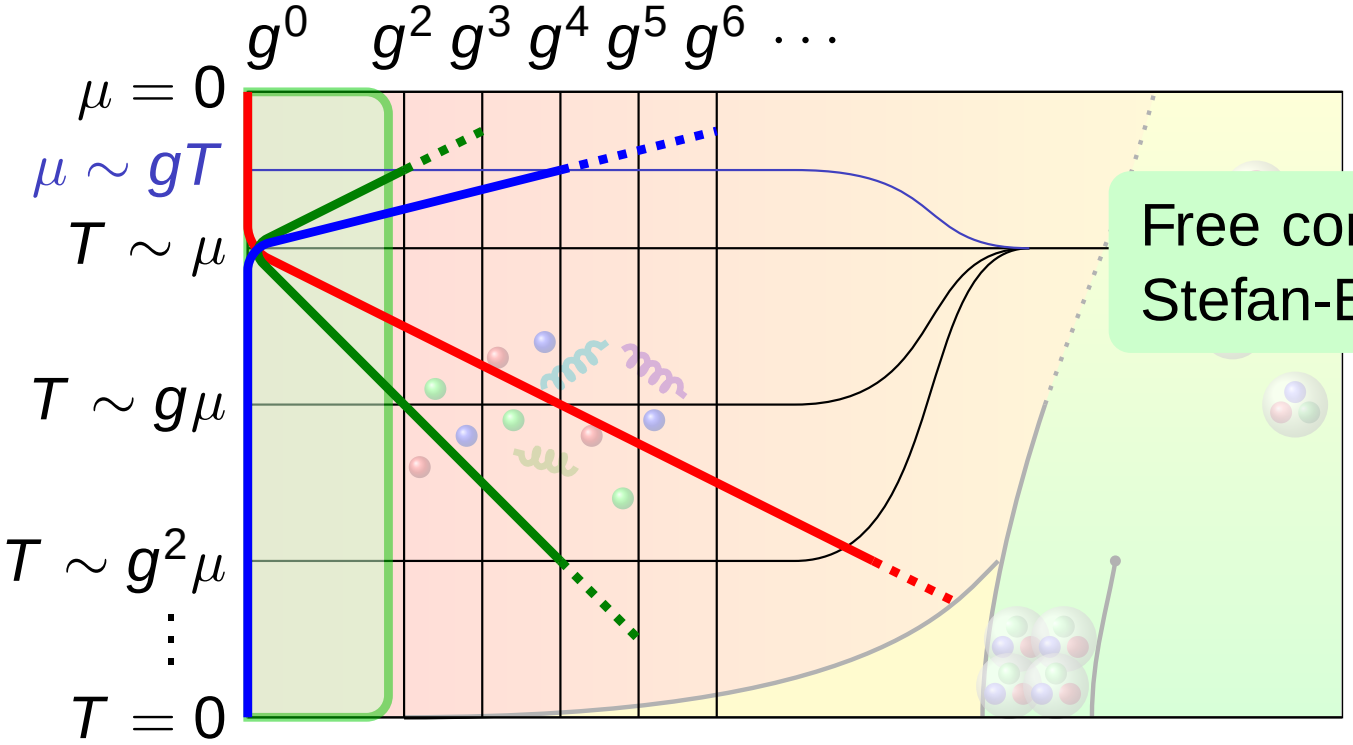
Son (1999)

Previous approaches for the pressure $p(T, \mu, g)$



$$p_{\mathrm{SB}}(T, \mu) = \left(\frac{\pi^2 N_g}{45} + \frac{7\pi^2 N_c N_f}{180}\right) T^4 + \frac{N_c N_f}{6} T^2 \mu^2 + \frac{N_c N_f}{12\pi^2} \mu^4 + \mathcal{O}(g^2)$$

Previous approaches for the pressure $p(T, \mu, g)$



Free contribution:
Stefan-Boltzmann

$$p_{\mathrm{SB}}(T, \mu) = \left(\frac{\pi^2 N_g}{45} + \frac{7\pi^2 N_c N_f}{180}\right) T^4 + \frac{N_c N_f}{6} T^2 \mu^2 + \frac{N_c N_f}{12\pi^2} \mu^4 + \mathcal{O}(g^2)$$

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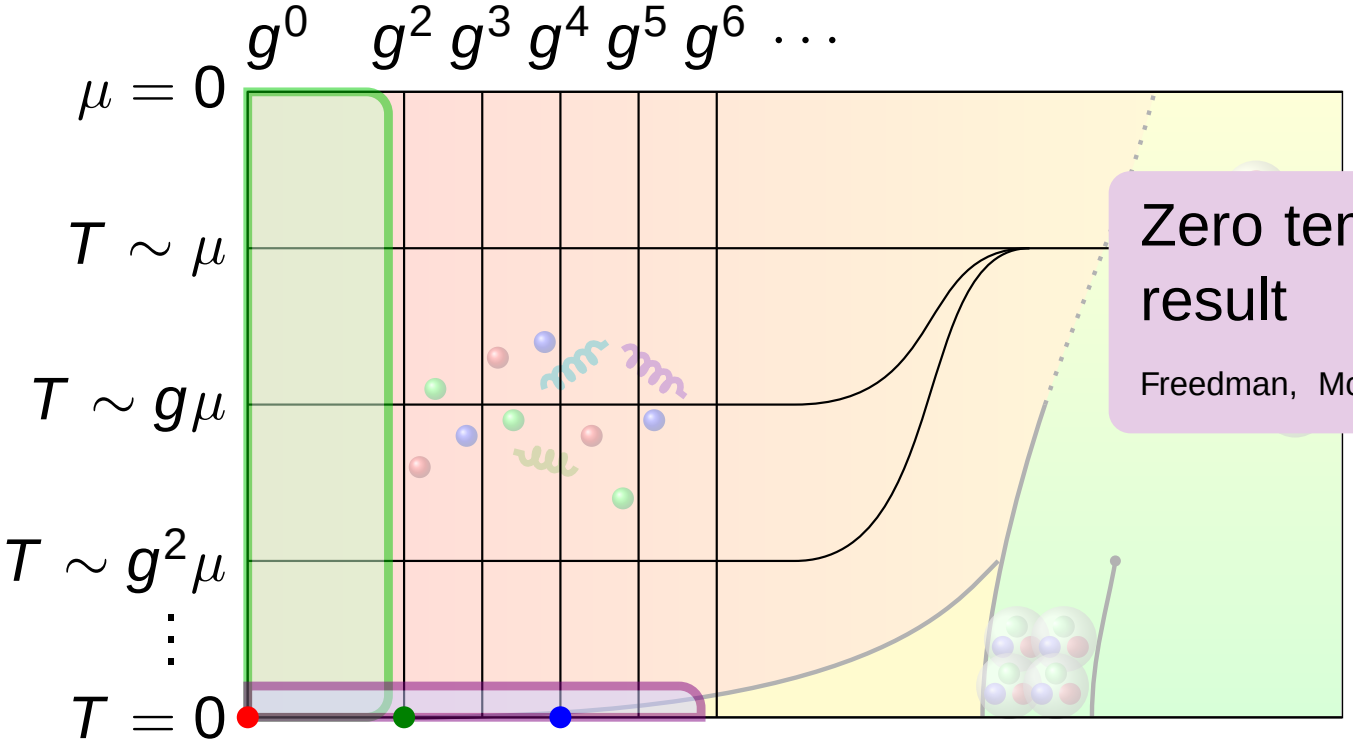
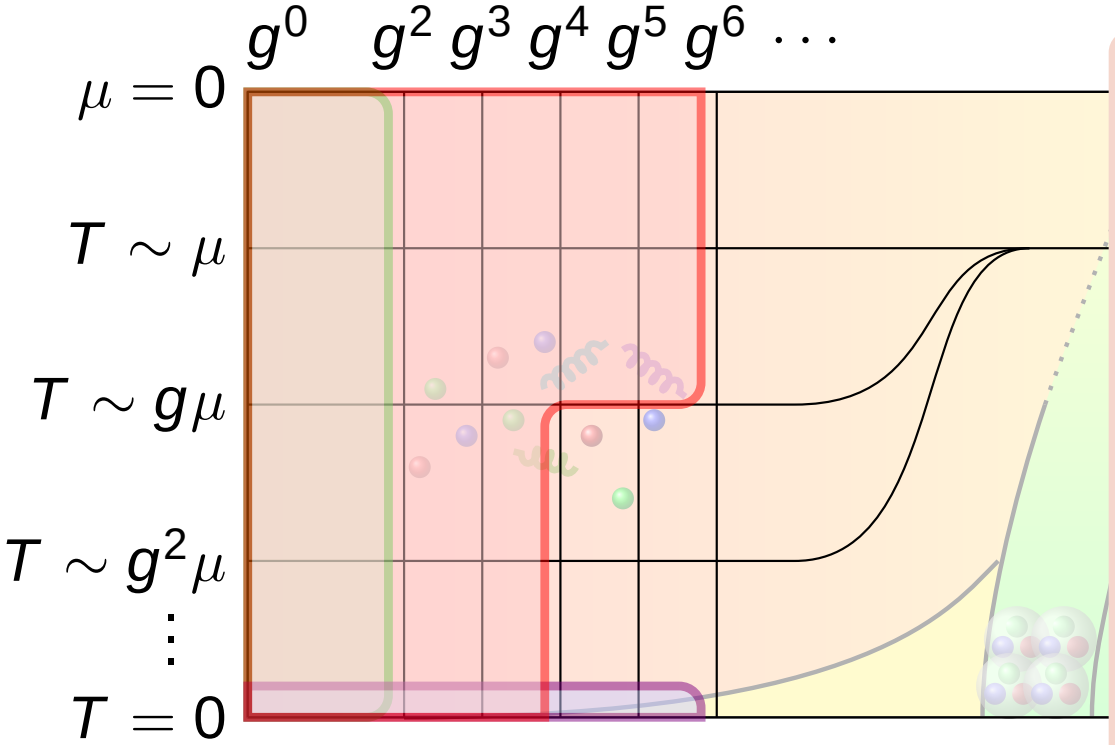


Zero temperature result

Freedman, McLerran (1977)

$$p = \frac{\mu^4}{2\pi^2}\left(1 - \frac{g^2}{2\pi^2} - \frac{g^4}{16\pi^4}\left(3.3385 + 4\ln g - \frac{29}{3}\ln\frac{\mu}{\bar{\mu}_{\overline{\mathrm{MS}}}}\right) + \mathcal{O}(g^6 \ln g)\right)$$

Previous approaches for the pressure $p(T, \mu, g)$



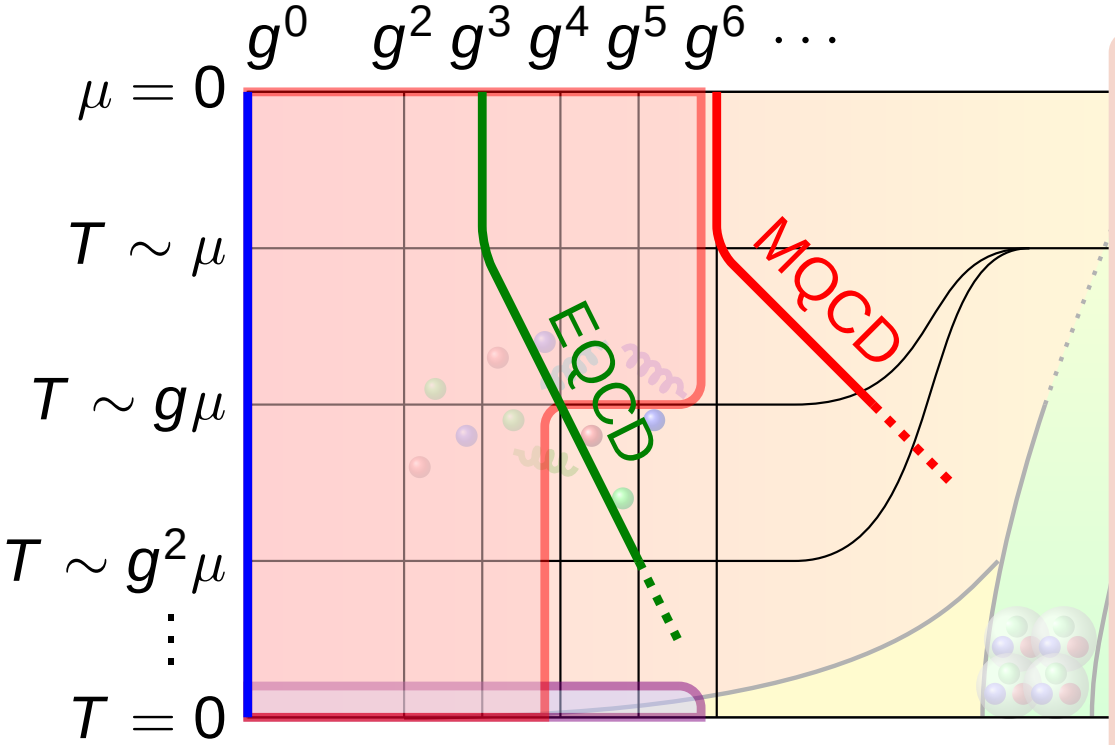
Perturbation theory

g^2: Shuryak; Chin (1978)
g^3: Kapusta (1979)
$g^4 \ln g$: Toimela (1983)
g^4: Arnold, Zhai (1994)
g^5: Zhai, Kastening (1995)

Dimensional reduction

g^5: Braaten, Nieto (1996)
$g^6 \ln g$: Kajantie, Laine
Rummukainen, Schröder (2002)
$\mu > 0$ for g^5, $g^6 \ln g$: Vuorinen (2003)
g^6: Di Renzo, Laine, Miccio,
Schröder, Torrero (2006)
Laine, Schröder, Vuorinen,... (200?)

Previous approaches for the pressure $p(T, \mu, g)$



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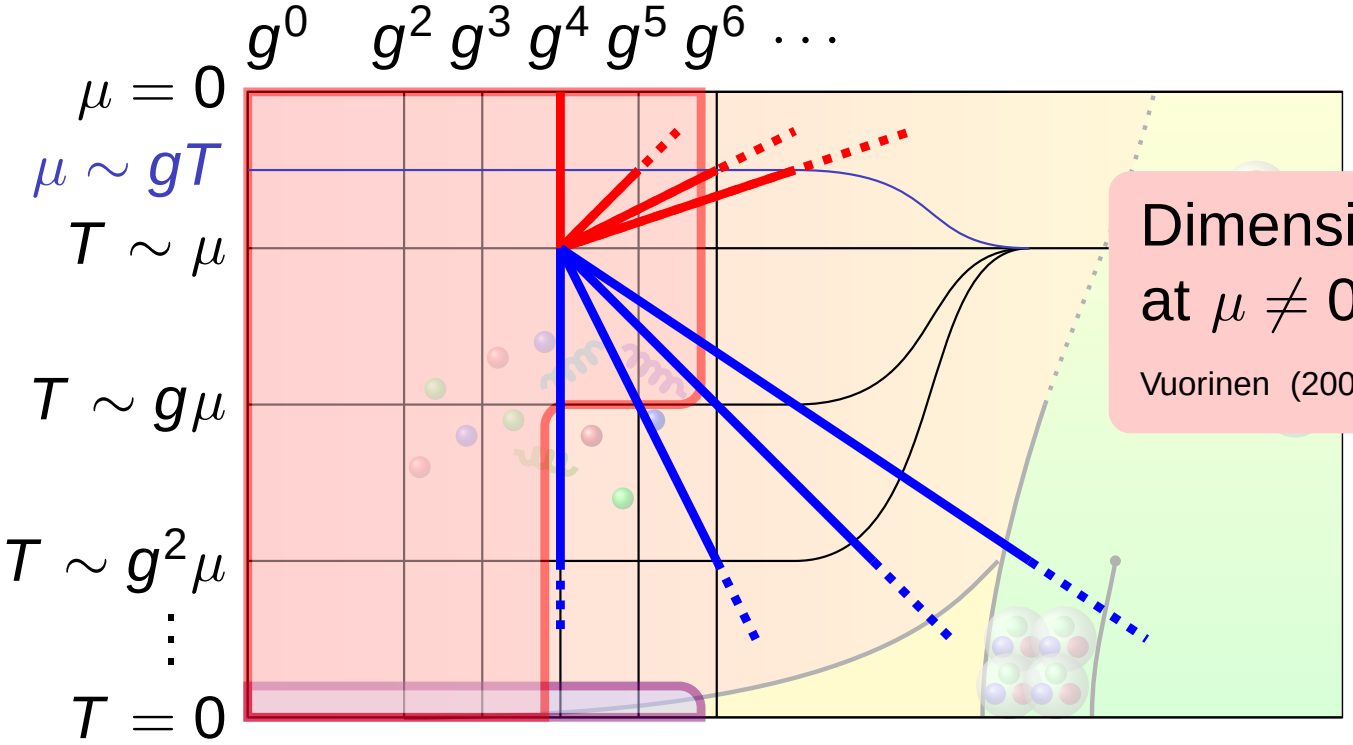
Separation of scales:

$$T \gg m_E \sim gT \gg m_{\mathrm{mag}} \sim g^2 T$$

$$p = p_E(T, g) + p_M(m_E^2, g_E, \lambda_E, \ldots) + p_G(g_M, ..)$$

Previous approaches for the pressure $p(T, \mu, g)$

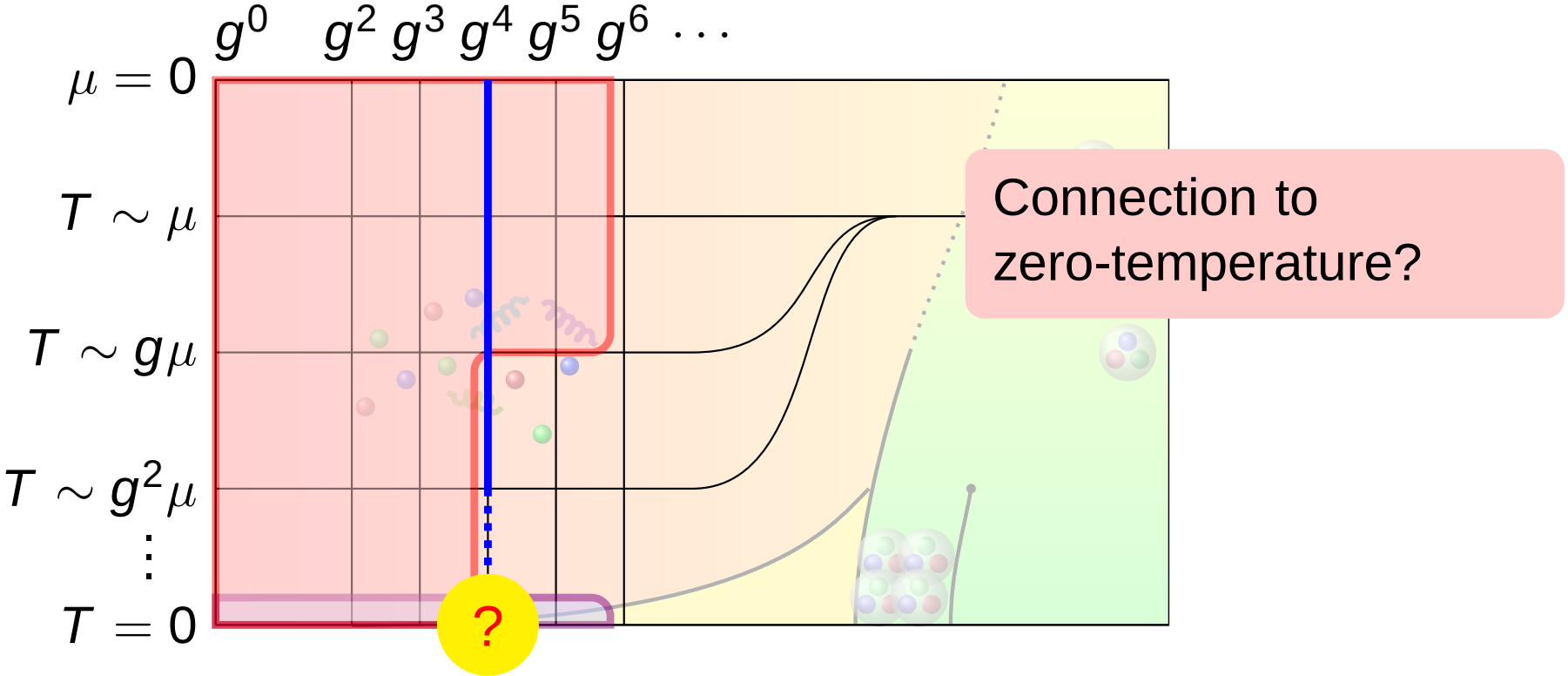


Dimensional reduction at $\mu \neq 0$

Vuorinen (2003)

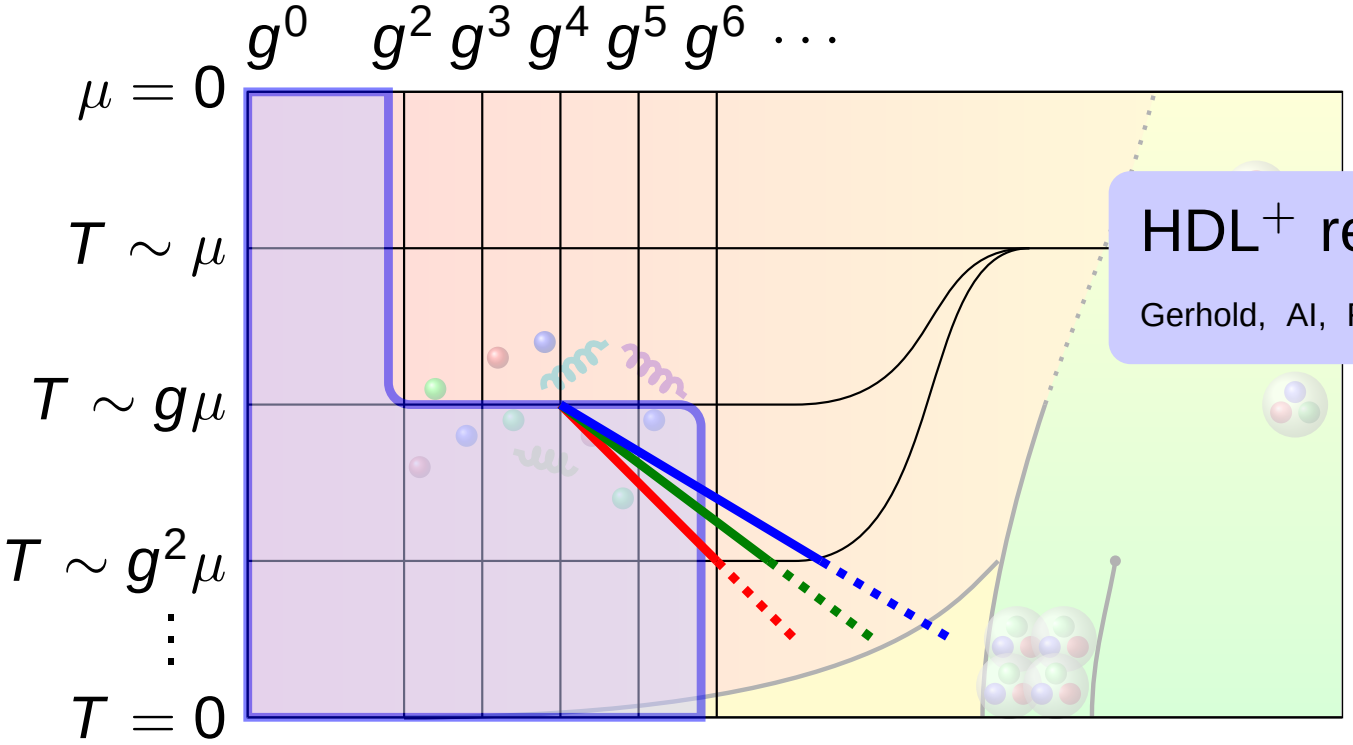
Order g^4 contains special functions $\aleph(..\mu/T)$ that can be expanded for small and large μ/T.

Previous approaches for the pressure $p(T, \mu, g)$



$$p_{\mathrm{DR}} = \dots + \#g^4\mu^4 \ln\frac{T}{\bar{\mu}_{\overline{\mathrm{MS}}}} + \dots$$

Previous approaches for the pressure $p(T, \mu, g)$



HDL$^+$ resummation

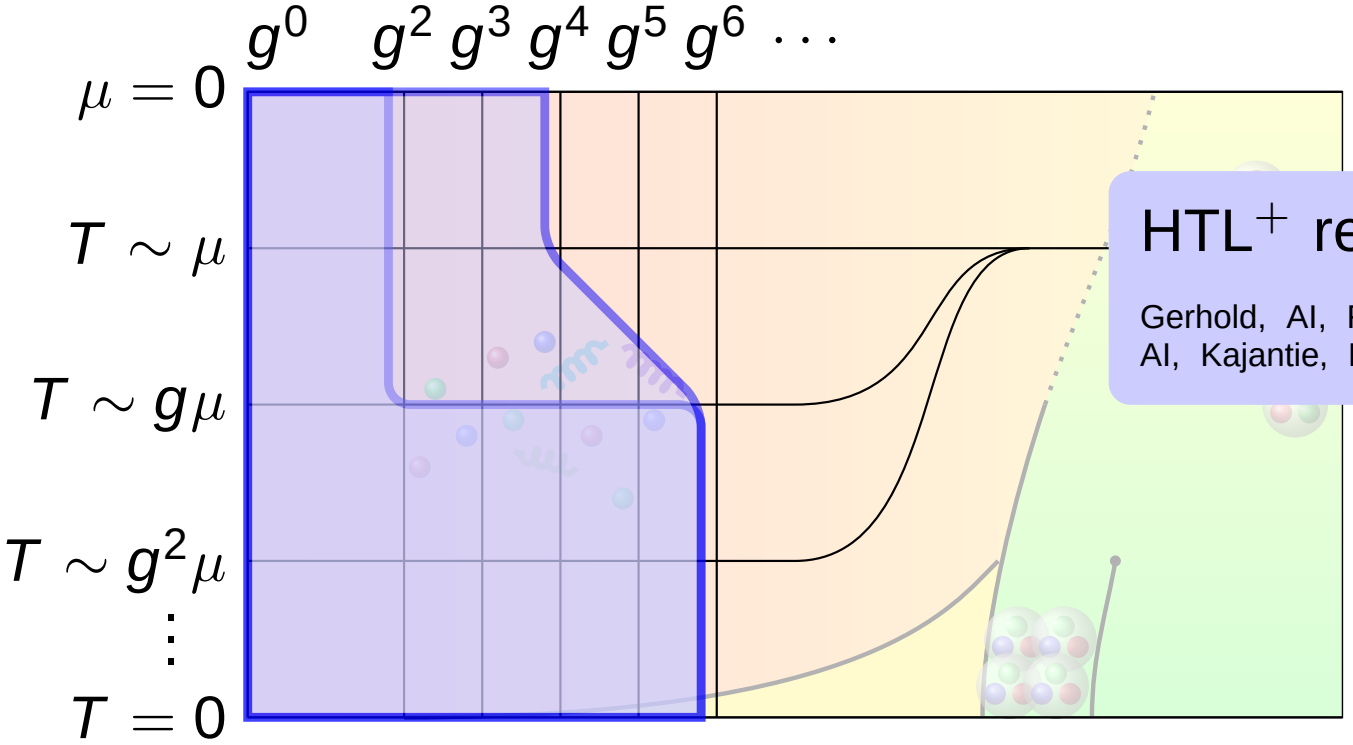
Gerhold, AI, Rebhan (2004)

$$\delta p \equiv p - p_{\mathrm{SB}} - (p - p_{\mathrm{SB}})|_{T=0}$$

$$\delta p = g^2 T^2 \mu^2 \left\{ \# \ln \frac{T}{g\mu} + \# + \# \left(\frac{T}{g\mu}\right)^{2/3} + \# \left(\frac{T}{g\mu}\right)^{4/3} + \dots \right\}$$

Non-Fermi-liquid behavior in entropy & specific heat

Previous approaches for the pressure $p(T, \mu, g)$

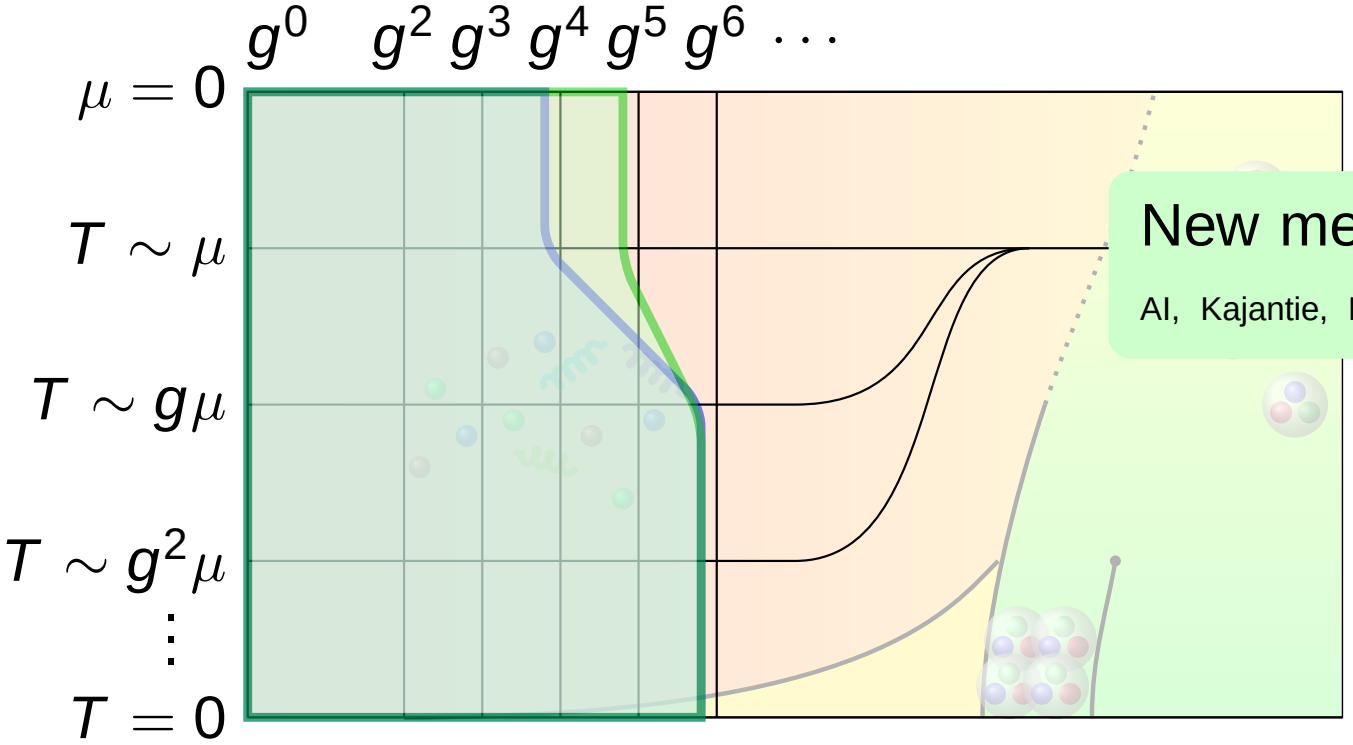


HTL$^+$ resummation

Gerhold, AI, Rebhan (2004)
AI, Kajantie, Rebhan, Vuorinen (2006)

$$\frac{1}{N_g}\delta p^{\mathrm{HTL}^+} = -\frac{g^2 T_F}{48\pi^2}\mu^2 T^2 + \frac{g^2(2C_A - T_F)}{288} T^4 - \frac{1}{2\pi^3}\int_0^\infty dq_0\, n_b(q_0)\int_0^\infty dq\, q^2$$
$$\times \left[2\,\mathrm{Im}\ln\left(\frac{q^2 - q_0^2 + \Pi_T^{\mathrm{HTL}}}{q^2 - q_0^2}\right) + \mathrm{Im}\ln\left(\frac{q^2 - q_0^2 + \Pi_L^{\mathrm{HTL}}}{q^2 - q_0^2}\right)\right]$$

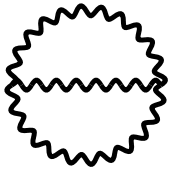
Previous approaches for the pressure $p(T, \mu, g)$



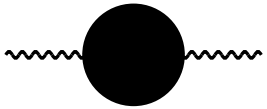
New method: p_{IV}

AI, Kajantie, Rebhan, Vuorinen (2006)

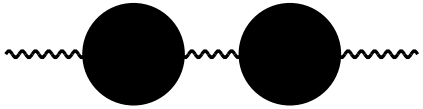
Problem of dimensional reduction



$$\frac{1}{K^2}$$

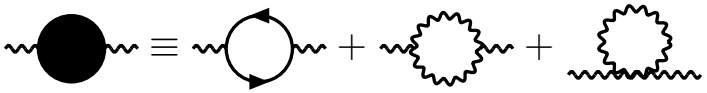


$$-\frac{1}{K^2}\Pi\frac{1}{K^2}$$



$$+\frac{1}{K^2}\Pi\frac{1}{K^2}\Pi\frac{1}{K^2}$$

HTL: $g^2\mu^2$ g^2T^2 g^2T^2



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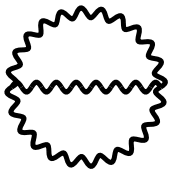
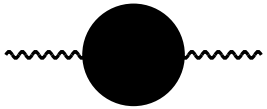


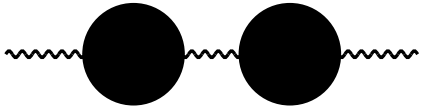
hard regime

$K \sim T$

$$\frac{1}{K^2} \sim \frac{1}{T^2}$$

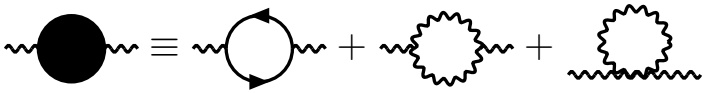


$$-\frac{1}{K^2}\Pi\frac{1}{K^2} \sim \frac{g^2}{T^2}$$

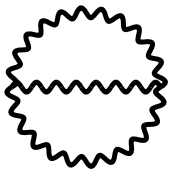


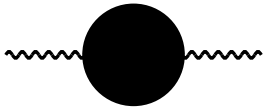
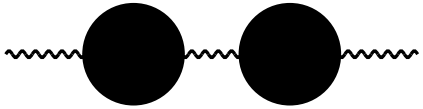
$$+\frac{1}{K^2}\Pi\frac{1}{K^2}\Pi\frac{1}{K^2} \sim \frac{g^4}{T^2}$$

HTL: $g^2\mu^2$ g^2T^2 g^2T^2



Problem of dimensional reduction



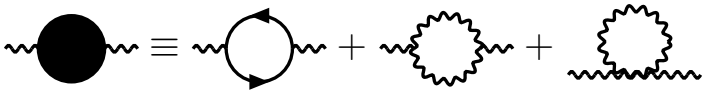
		hard regime $K \sim T$	soft regime $K \sim gT$
	$\frac{1}{K^2}$	$\sim \frac{1}{T^2}$	$\sim \frac{1}{g^2T^2}$
	$-\frac{1}{K^2}\Pi\frac{1}{K^2}$	$\sim \frac{g^2}{T^2}$	$\sim \frac{1}{g^2T^2}$
	$+\frac{1}{K^2}\Pi\frac{1}{K^2}\Pi\frac{1}{K^2}$	$\sim \frac{g^4}{T^2}$	$\sim \frac{1}{g^2T^2}$

$$- + \dots$$

$$= \frac{1}{K^2 + \Pi}$$

For soft $K \sim gT$ all these diagrams contribute at the same perturbative order and have to be resummed.

HTL: $g^2\mu^2$ g^2T^2 g^2T^2



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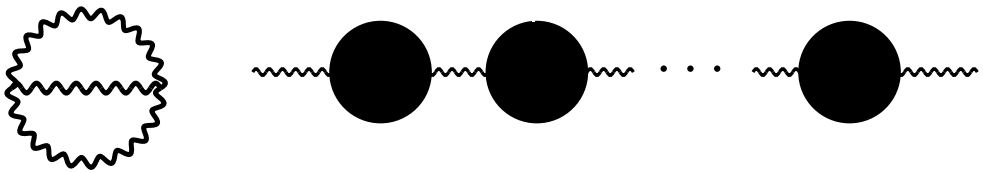
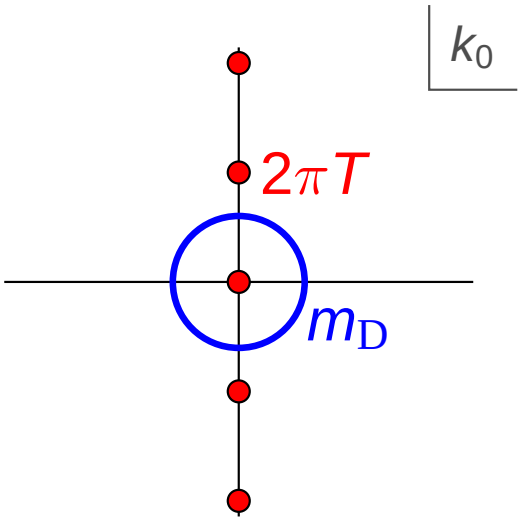


$$\rightarrow \frac{1}{-k_0^2 + \mathbf{k}^2 + \Pi}$$

Matsubara sum:
$ik_0 = 0, \pm 2\pi T, \pm 4\pi T, \ldots$

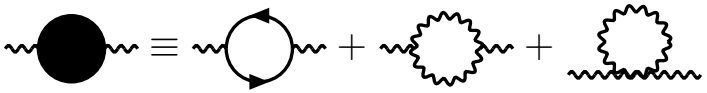
$2\pi T \gtrsim m_{\mathrm{D}}$



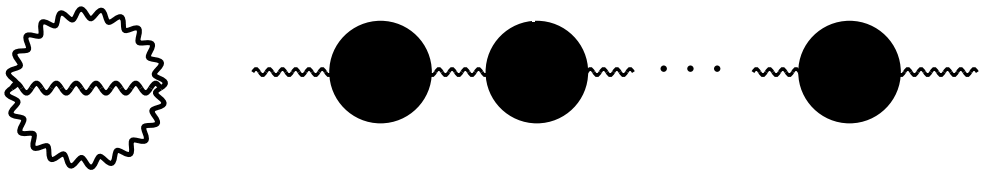
integrate out hard modes, effective 3-D theory for $k_0 = 0$

HTL: $g^2\mu^2$ g^2T^2 g^2T^2

$m_{\mathrm{D}}^2 =$



Problem of dimensional reduction



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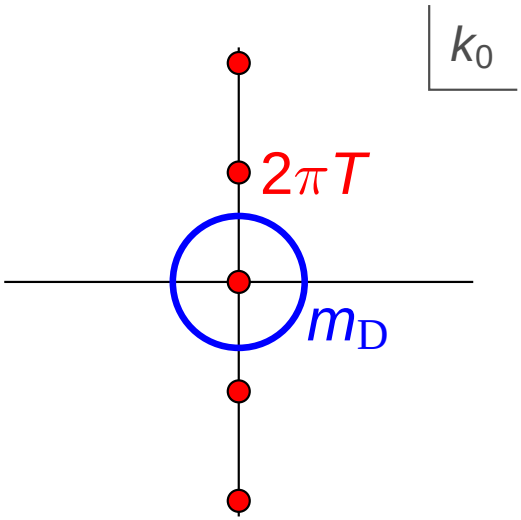
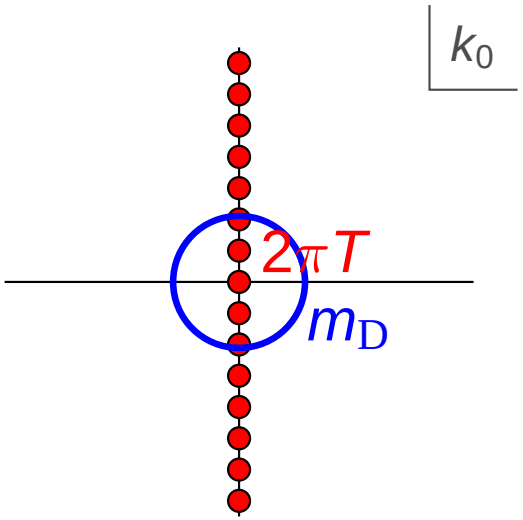
$2\pi T \gtrsim m_{\mathrm{D}}$



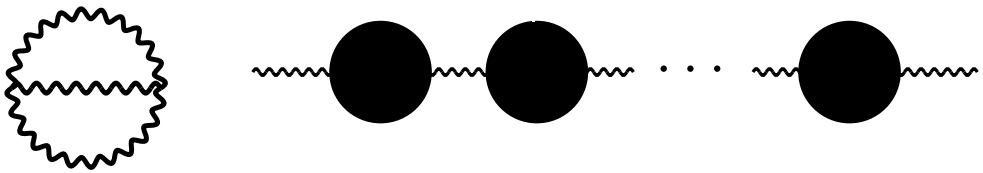
integrate out hard modes, effective 3-D theory for $k_0 = 0$

$2\pi T \lesssim m_{\mathrm{D}} \sim g\mu/\pi$



DR fails, have to use 4D integration $\sum_{\omega_n} \rightarrow \int dk_0 n(k_0)$

Problem of dimensional reduction



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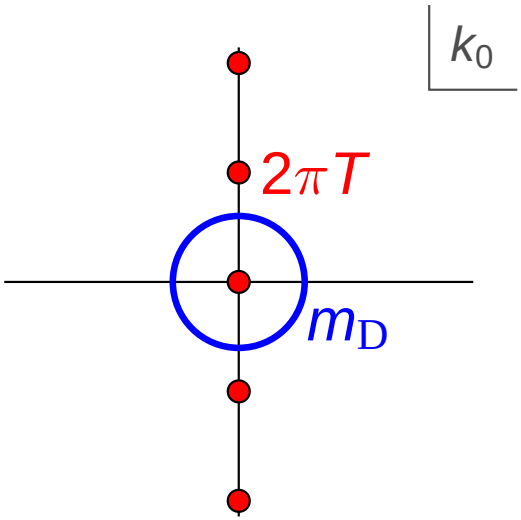

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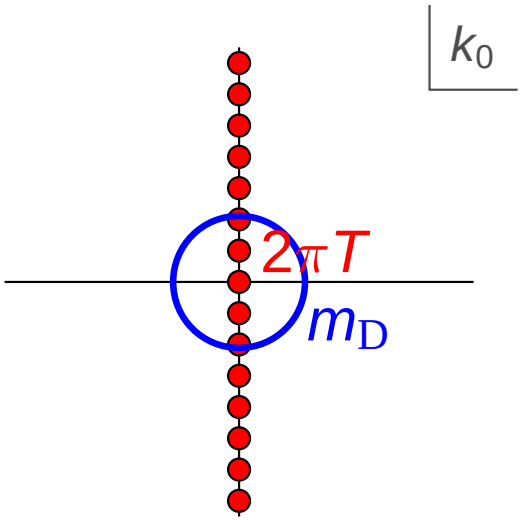

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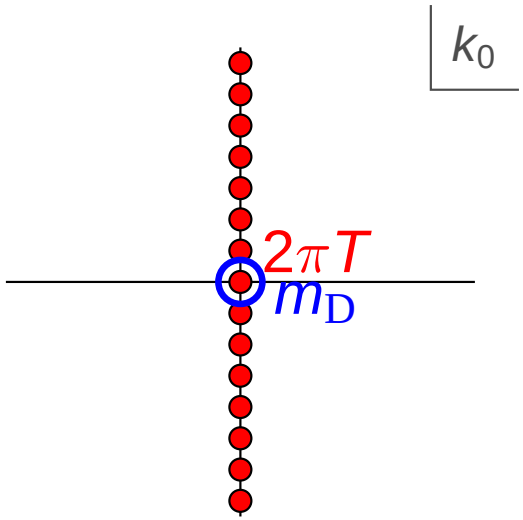
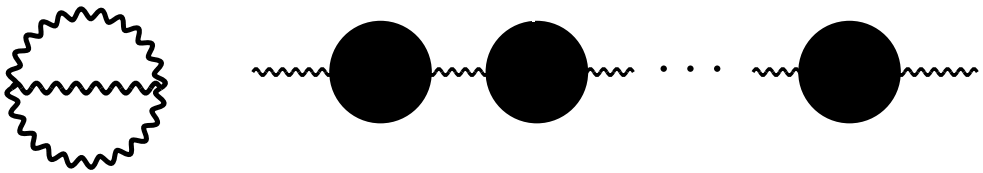


perturbatively can always choose smaller g

Problem of dimensional reduction



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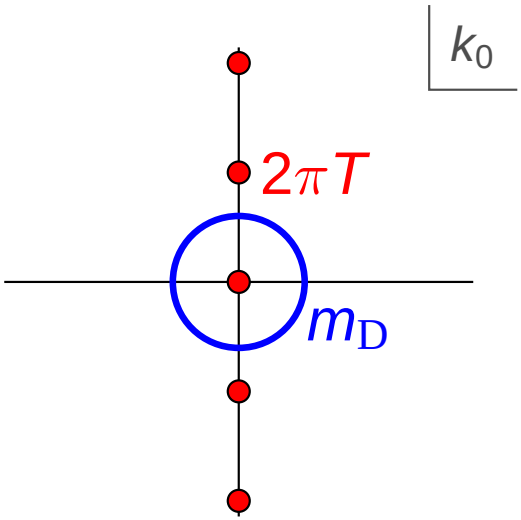

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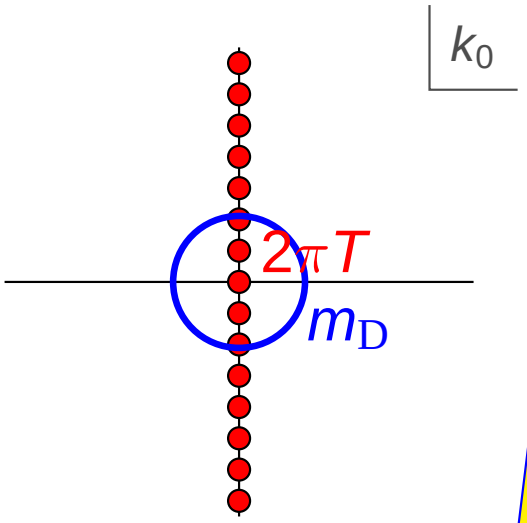

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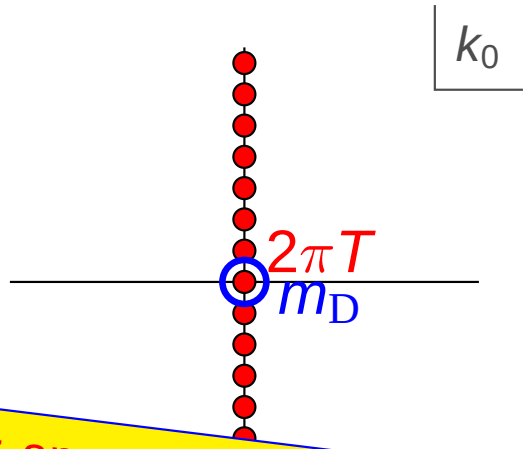
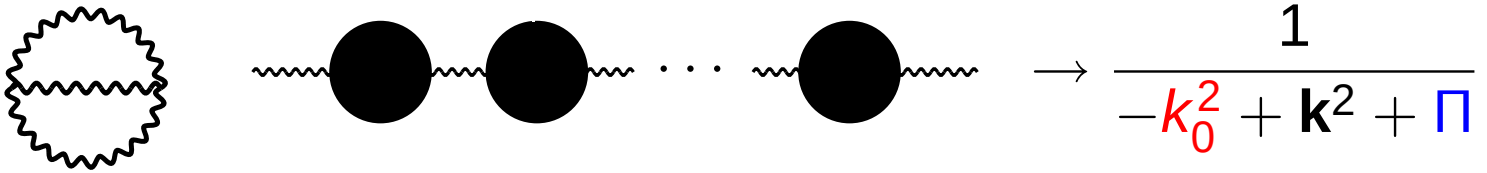


For any given T/μ dimensional reduction is valid, if g is chosen small enough.

smaller g

Problem of dimensional reduction



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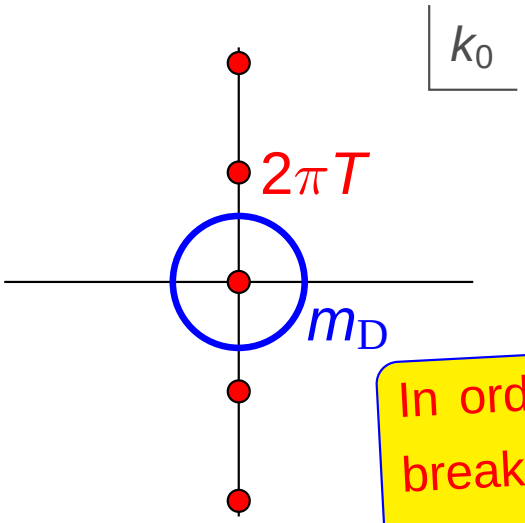

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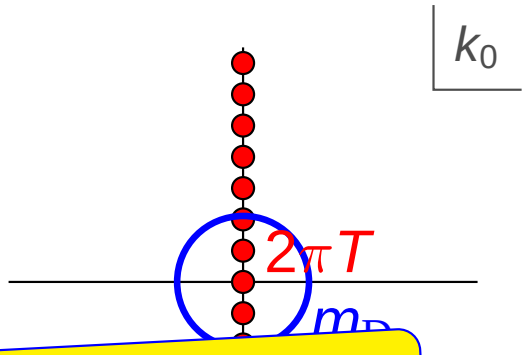
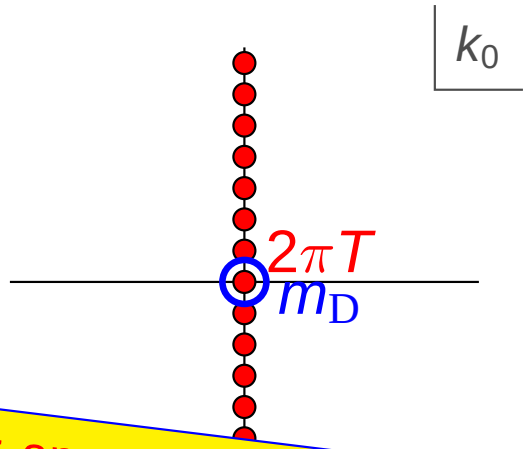


In order to observe the breakdown of DR, we have to study a parametrically small temperature region $T \sim g\mu$.

$2\pi T \gtrsim m_{\mathrm{D}} \sim g\mu/\pi$



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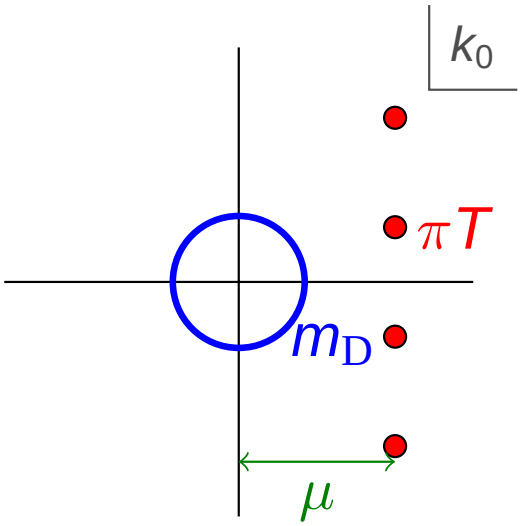
smaller g

Problem of dimensional reduction

for fermions:

$\pi T \gtrsim m_{\mathrm{D}}$



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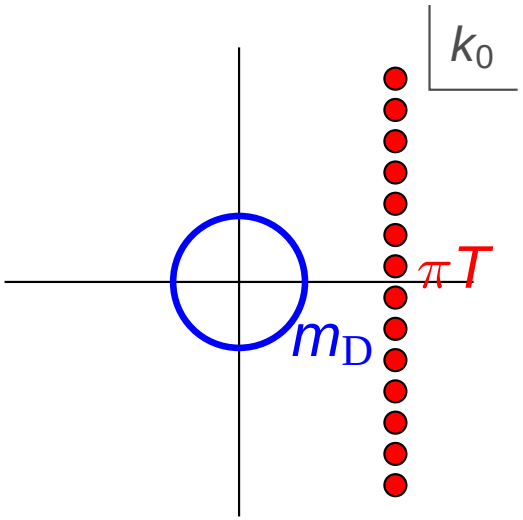


Matsubara sum for fermions:
$ik_0 = \pm\pi T + i\mu,$
$\pm 3\pi T + i\mu, \ldots$

only need to resum gluon lines:

$\Rightarrow$ 2GR

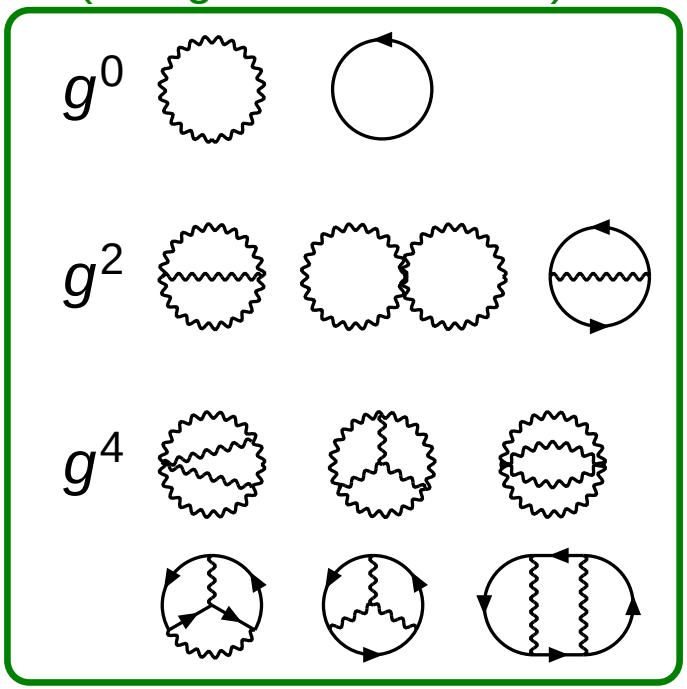
There is no problem with fermions, as μ regulates the IR region.

Our solution

$$p_{\mathrm{QCD}} = p_{\mathrm{2GI}} + p_{\mathrm{2GR}}^{\mathrm{resummed}} \text{ up to } \mathcal{O}(g^4)$$

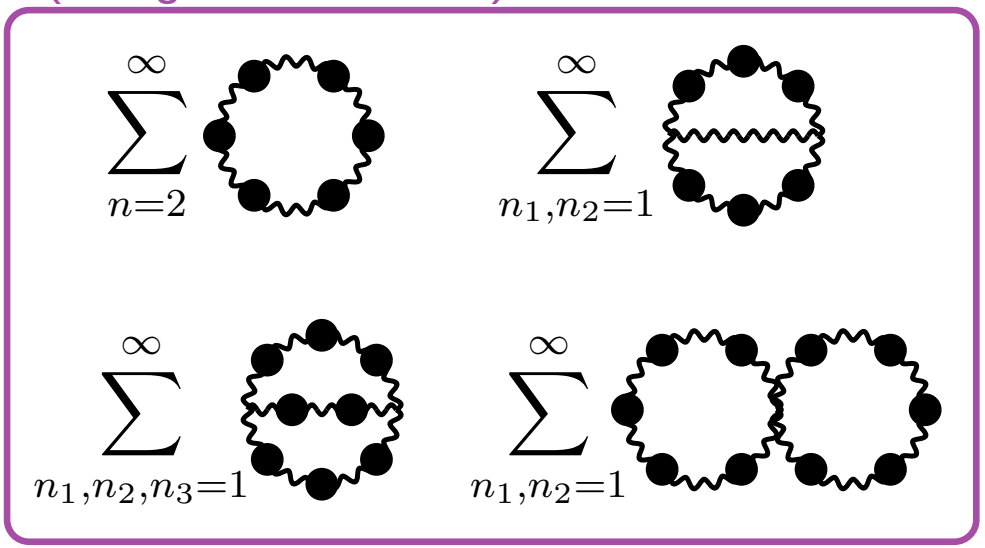
for all T and μ.

2GI
(two-gluon-irreducible)

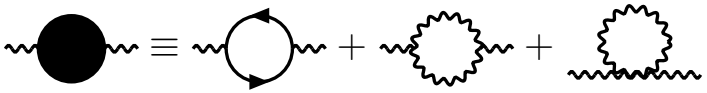


(+ ghost diagrams)

2GR resummed
(two-gluon-reducible)



with gluon self-energy



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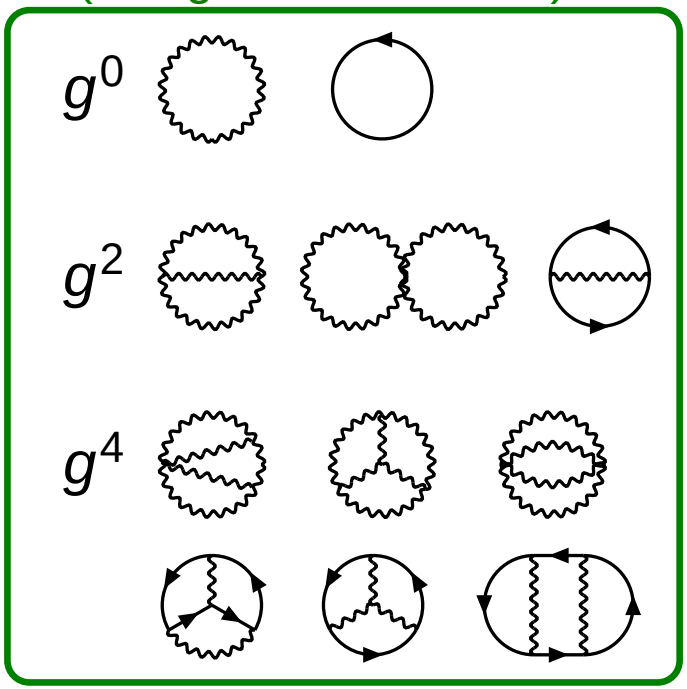

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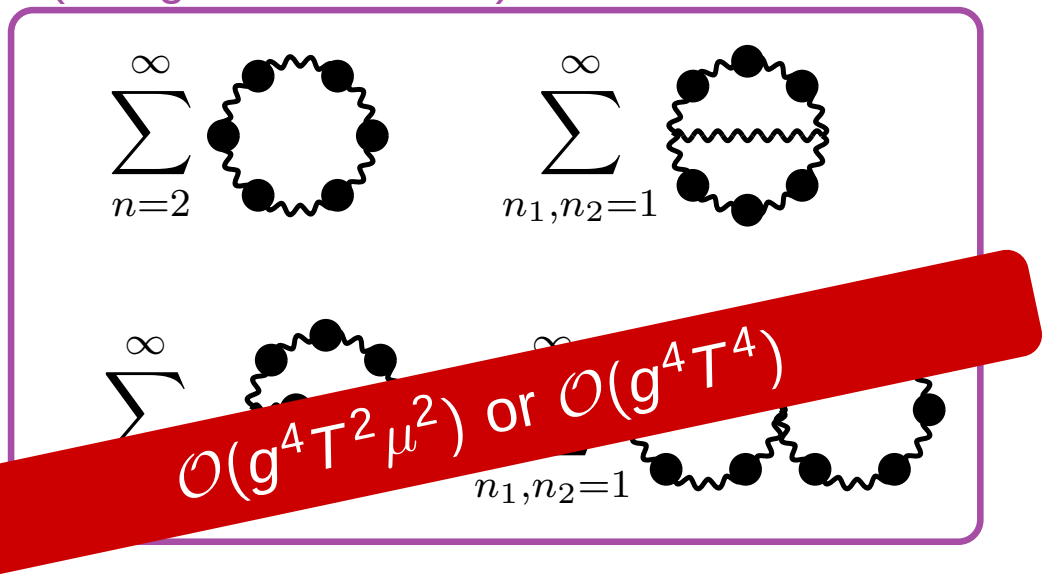

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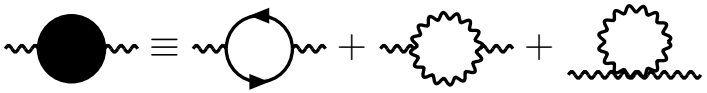

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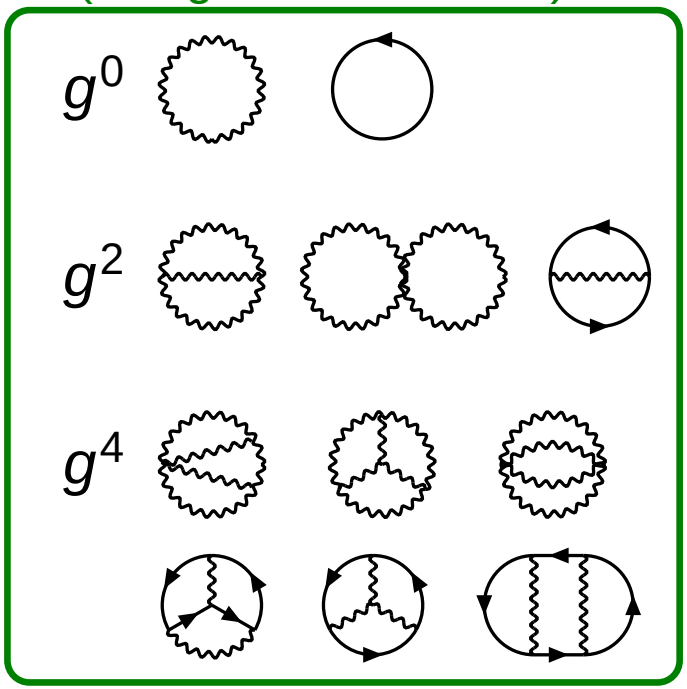

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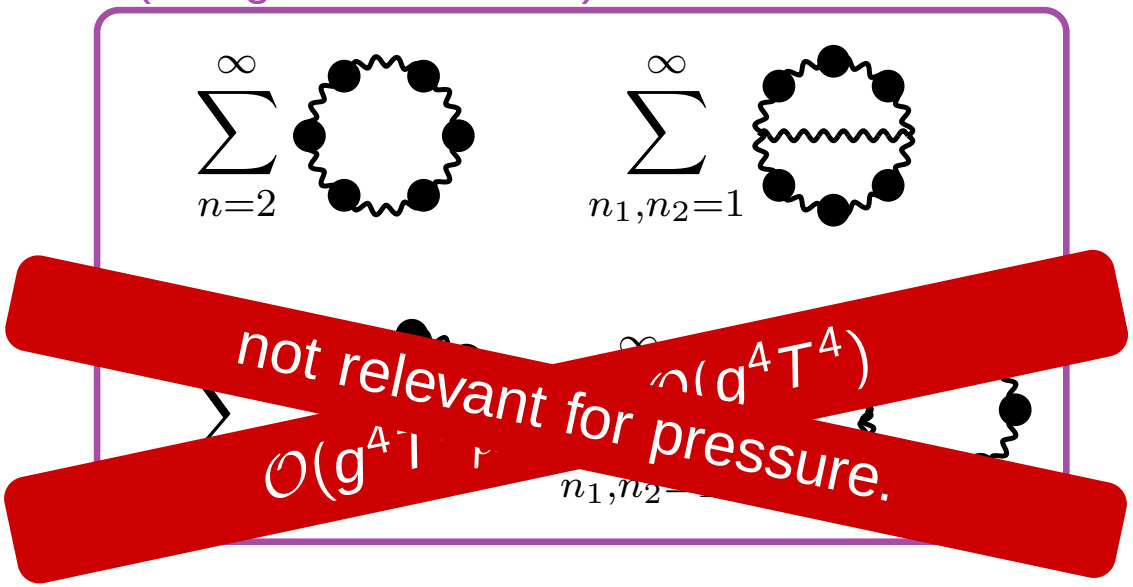

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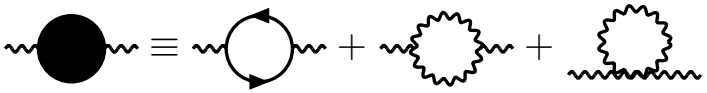

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only contributes in DR regime.

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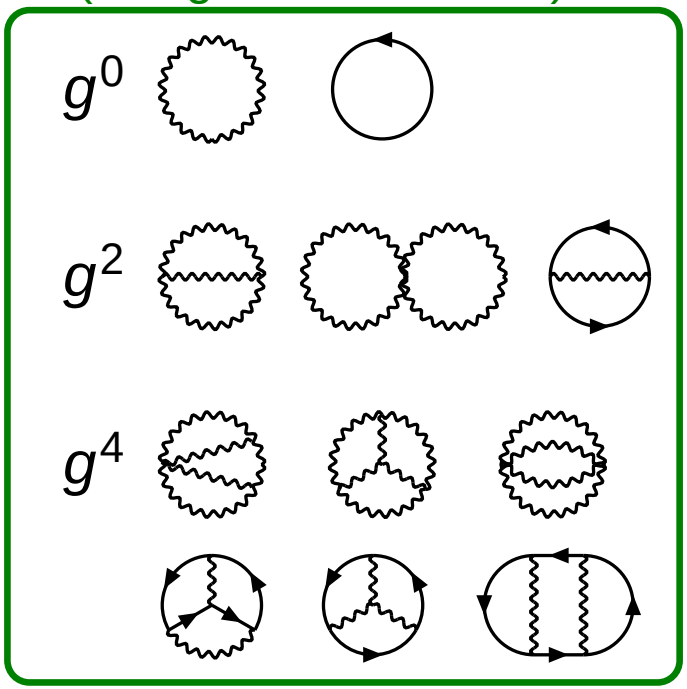

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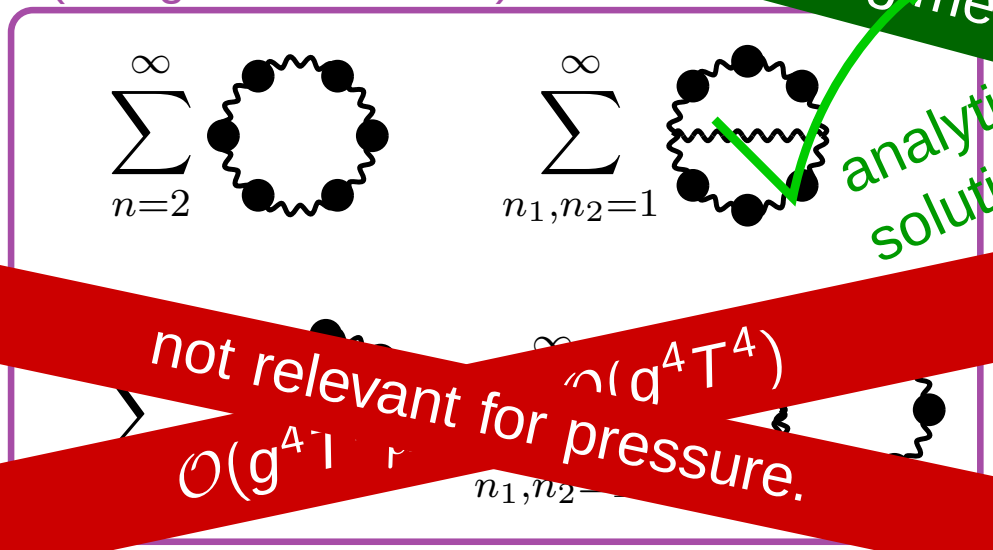
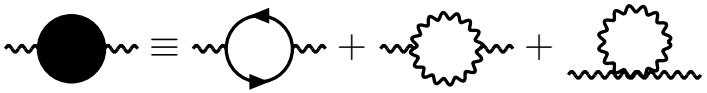


analytic solution

not relevant for pressure.

$\mathcal{O}(g^4T^4)$

with gluon self-energy



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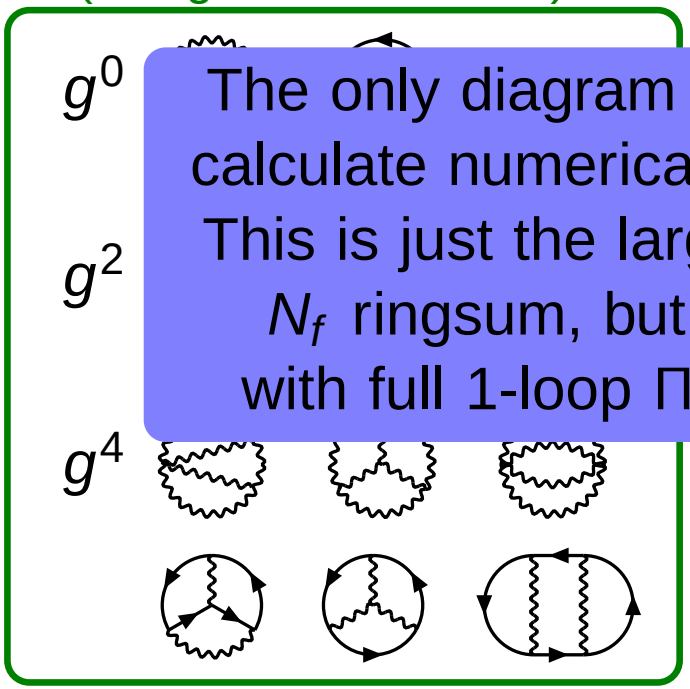

The only diagram to calculate numerically. This is just the large N_f ringsum, but with full 1-loop Π.

(+ ghost diagrams)

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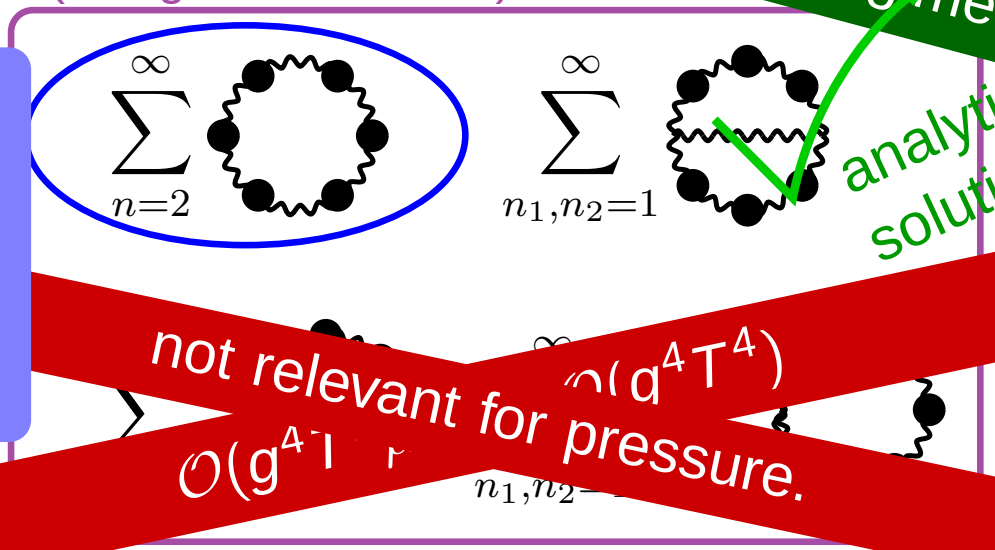

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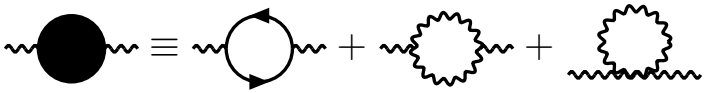
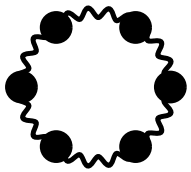


Technical details

$\sum_{n=2}^{\infty}$ 

$$p_{\text{ring}}^{\text{finite}} = -\frac{d_A}{2}\sum\!\!\!\!\!\!\int_P \Big\{ \ln\Big[1+\Pi_{\text{L}}(P)/P^2\Big] - \Pi_{\text{L}}(P)/P^2 + C_A^2 g^4 T^4/(72(P^2+m^2)^2)$$
$$+2\Big(\ln\Big[1+\Pi_{\text{T}}(P)/P^2\Big] - \Pi_{\text{T}}(P)/P^2 + C_A^2 g^4 T^4/(72(P^2+m^2)^2)\Big)\Big\}$$

Split off vacuum & regulate UV

$$p_{\text{ring}} = p_{\text{ring}}^{\text{finite}} + p_{\text{ring}}^{\text{IR}} + p_{\text{ring}}^{\text{UV}}$$

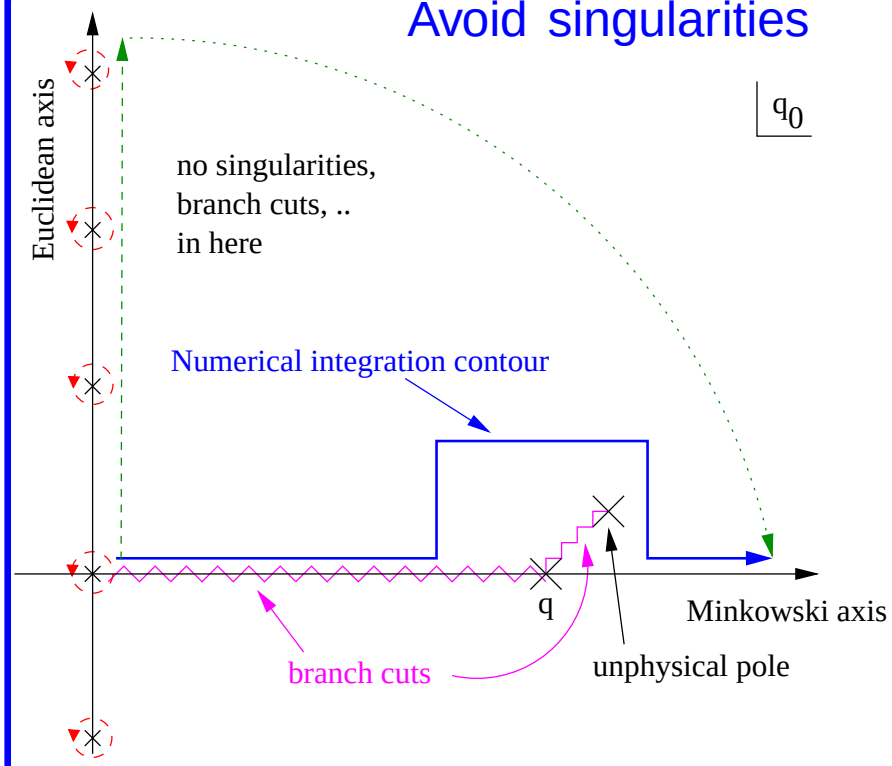
$$p_{\text{ring}}^{\text{IR}} \equiv \frac{d_A C_A^2 g^4 T^4}{48}\sum\!\!\!\!\!\!\int_P \left\{\frac{1}{(P^2+m^2)^2} - \frac{1}{P^4}\right\}$$

$$p_{\text{ring}}^{\text{UV}} = \frac{1}{4}(d-1)d_A(\Pi_{\text{UV}})^2\sum\!\!\!\!\!\!\int_P \frac{1}{P^4}$$

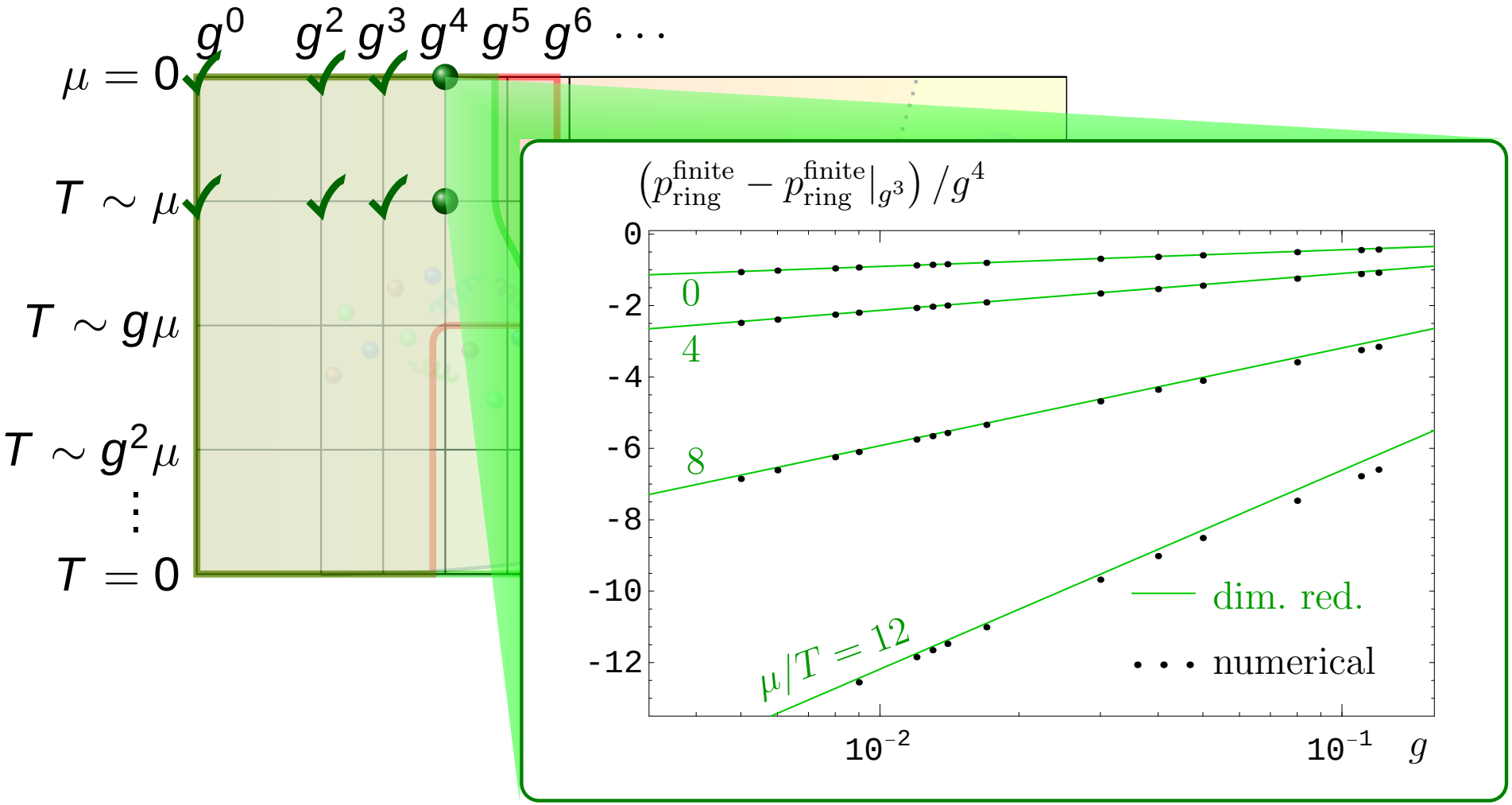
Magnetic mass: $m_{\text{mag}} = c_f \frac{3}{32} g^2 C_A T$

$\Pi_{\text{T}}(P) \to \Pi_{\text{T}}(P) + m_{\text{mag}}^2$

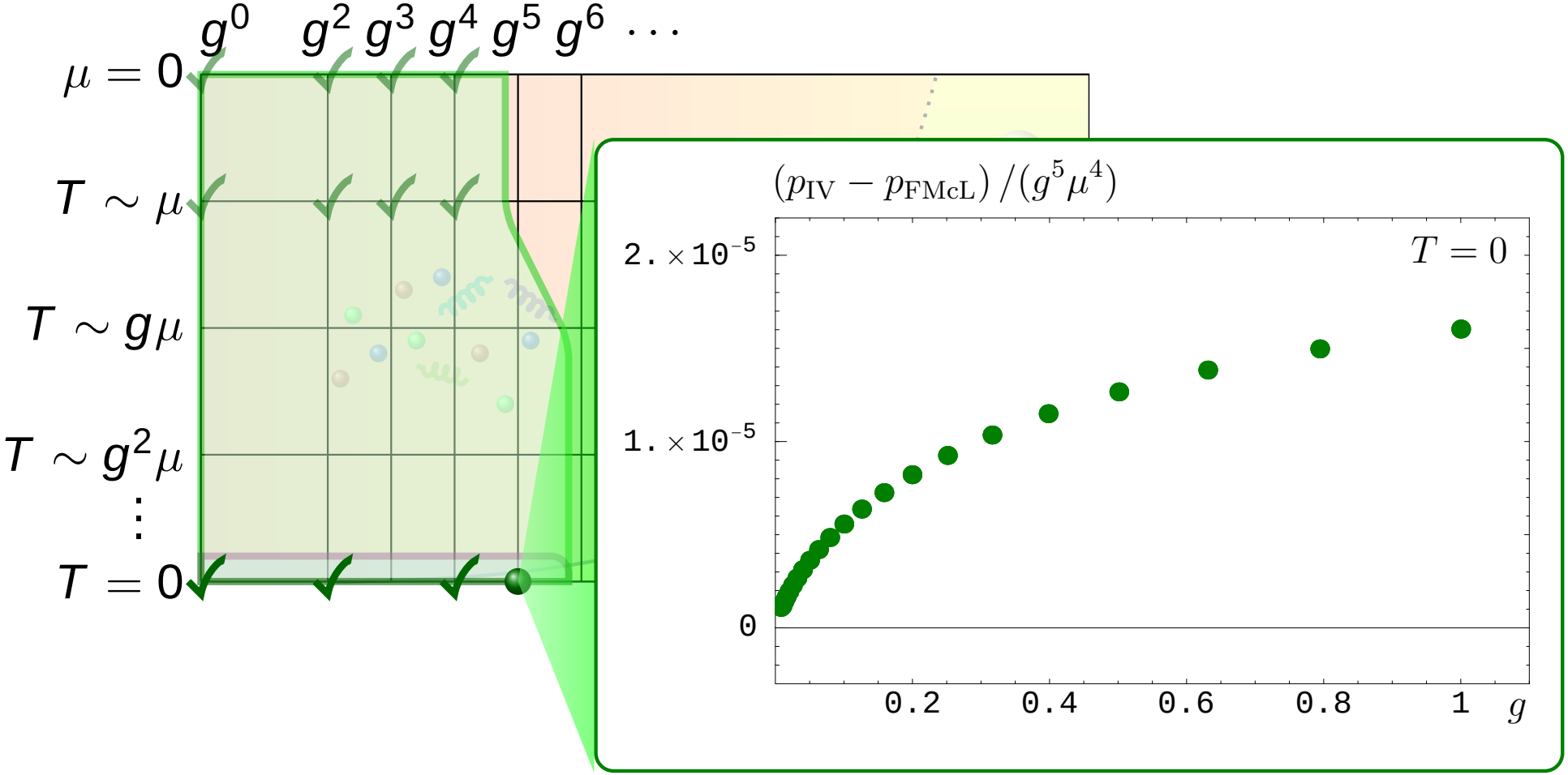
Avoid singularities



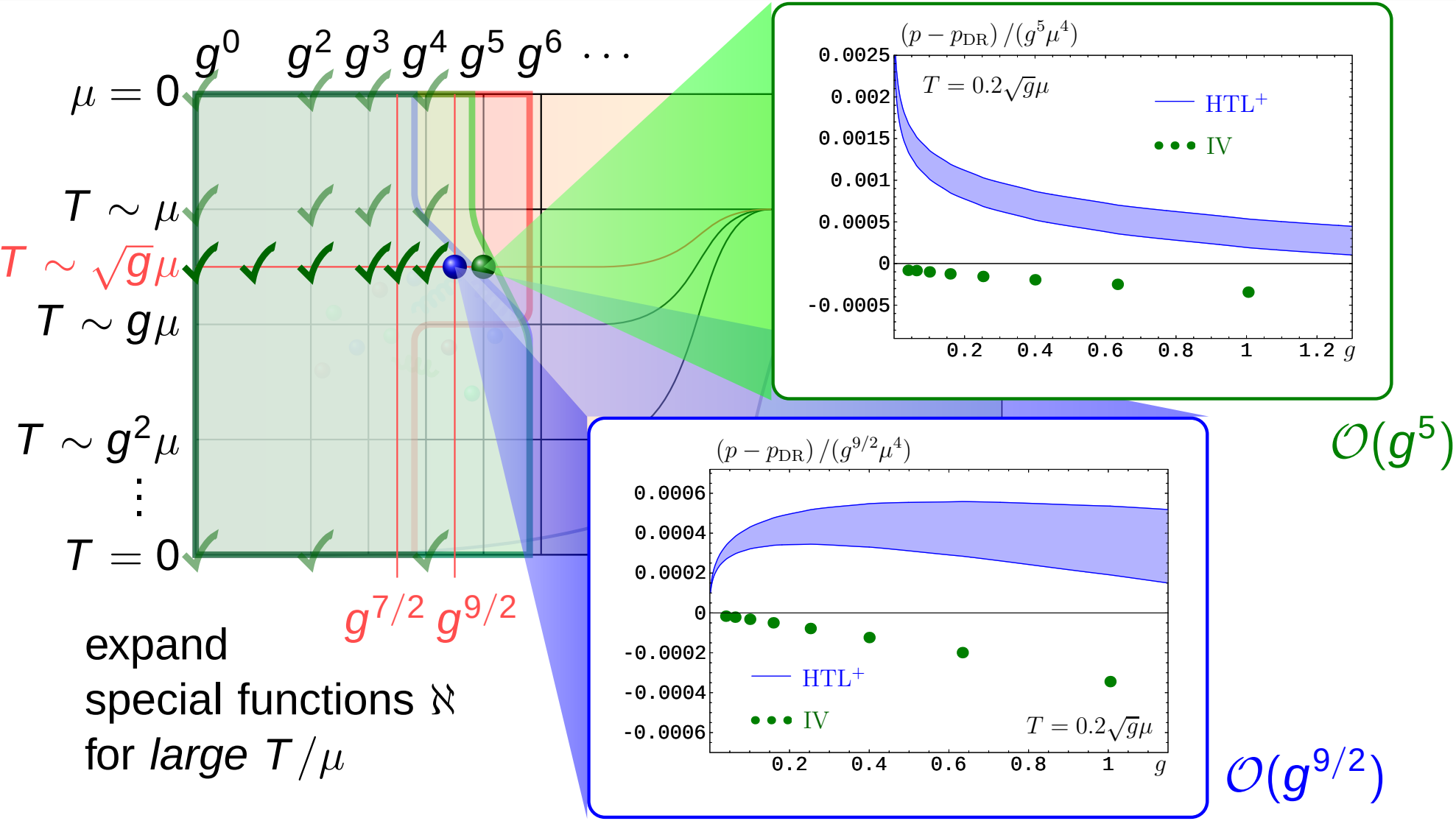
Dimensional reduction OK



Zero temperature OK

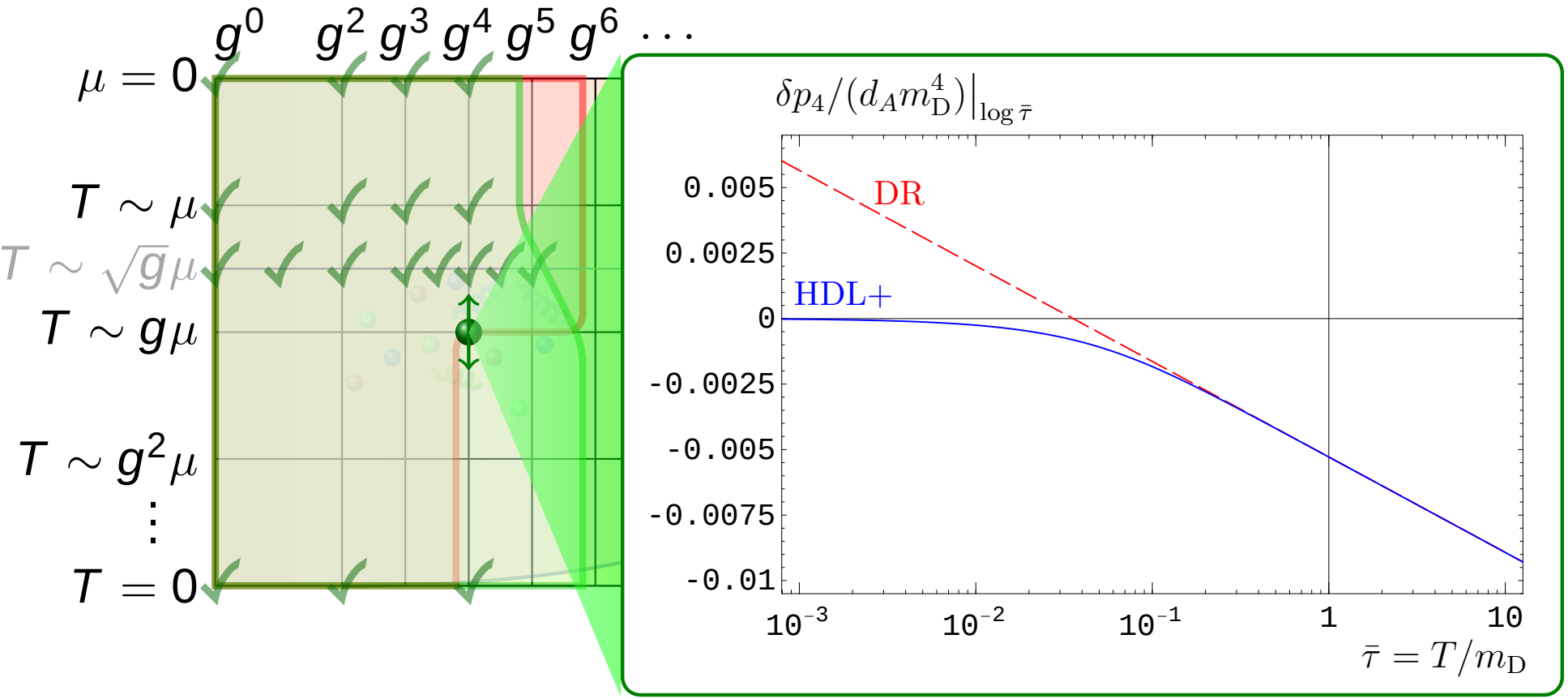


Dimensional reduction at $T \sim \sqrt{g}\mu$ OK

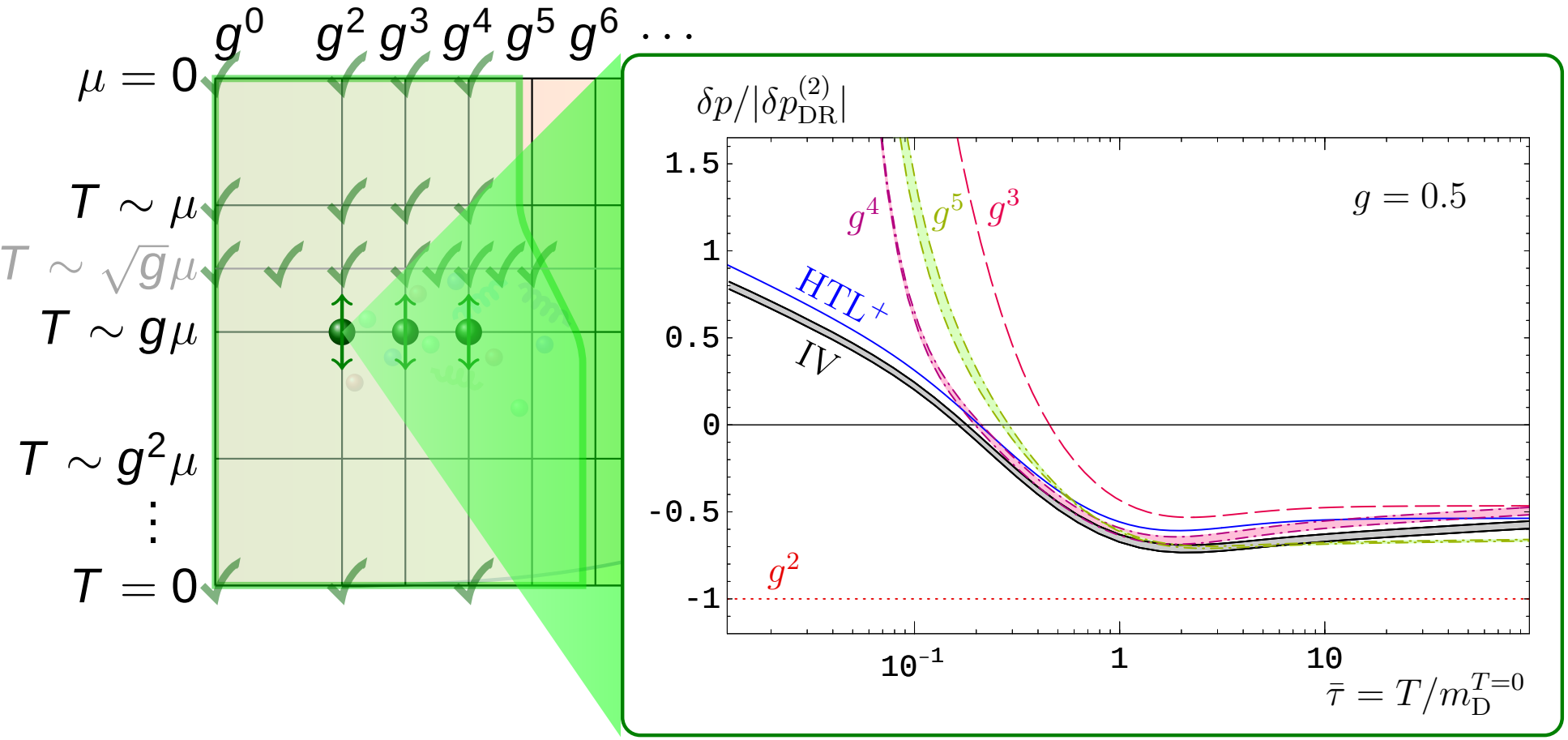


expand
special functions ℵ
for *large* T/μ

HDL$^+$ resummation OK



Interaction pressure δp



Summary

- Dimensional reduction works when $T \gtrsim 0.2 m_{\mathrm{D}}$.
- New approach: Treat 2GI analytically and 2GR numerically.
- Full perturbative understanding for all T and μ up to $O(g^4)$.
- Outlook: Minimal 2GI resummation motivates full 2PI resummation at larger g.

For Further Reading I

Please find all references in

A. Ipp, K. Kajantie, A. Rebhan, A. Vuorinen
The pressure of deconfined QCD for all temperatures and quark chemical potentials
hep-ph/0604060