Small Field Fluctuations

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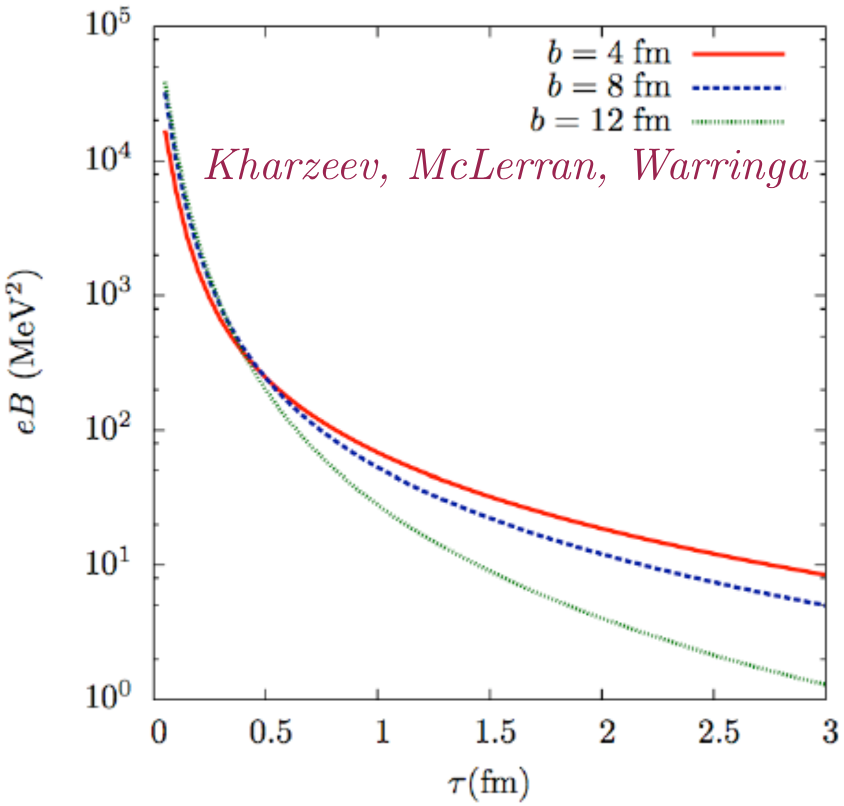
P- and CP-odd Effects in Hot and Dense Matter

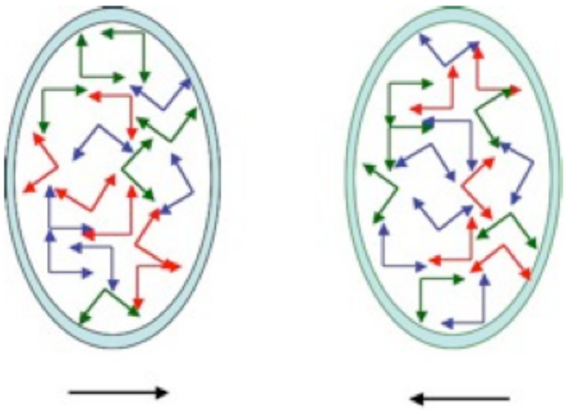
Chiral Magnetic Effect



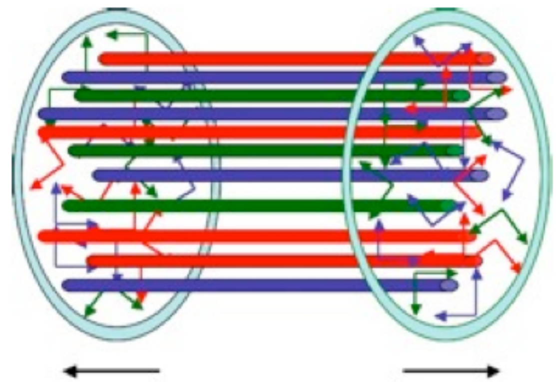
1. Expect the largest contribution to CME at early times (i.e. during the Glasma phase).

Glasma Flux Tube Picture

- Before



After



$$N_{\text{f.t.}} \sim \alpha_s \frac{dN}{dy}$$

$$\sim 300 \text{ in Au-Au}$$

This result has been verified through numerical simulations.

In the classical limit these solutions are boost invariant and therefore sphaleron transitions do not take place.

Kharzeev, Krasnitz, Venugopalan

The CGC Framework

- Physical observable given as average of source distribution

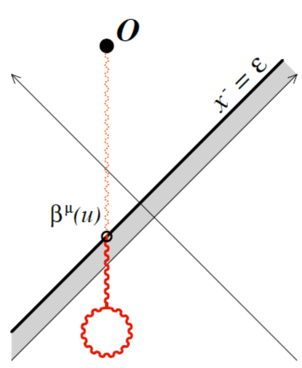
$$\langle \mathcal{O} \rangle_Y \equiv \int [D\rho]\; W_Y[\rho]\; \mathcal{O}[\rho]$$

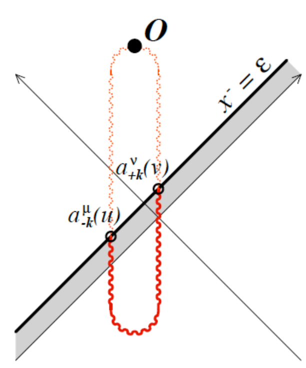
- and obeys JIMWLK RGE: $\dfrac{\partial \langle \mathcal{O} \rangle_Y}{\partial Y} = \langle \mathcal{H} \mathcal{O} \rangle_Y$

- Perturbative Expansion: $\mathcal{O} = \dfrac{1}{g^2}\left[c_0 + c_1 g^2 + c_2 g^4 + \cdots\right]$

$$\mathcal{O}_{\mathrm{LO}} = \frac{c_0}{g^2}$$

$$\mathcal{O}_{\mathrm{LLog}} = \frac{1}{g^2} \sum_n d_n g^{2n} \log^n \left(\frac{1}{x_{1,2}} \right)$$





Multi-particle production at leading log order

- These logarithms can be re-summed
- Inclusive observables can be expressed in factorized form

$$\left\langle\mathcal{O}\right\rangle_{\text{LLog}} = \int \left[D\Omega_1(\bar{y},\mathbf{x}_\perp) D\Omega_2(\bar{y},\mathbf{x}_\perp)\right] W\left[\Omega_1(\bar{y},\mathbf{x}_\perp)\right] W\left[\Omega_2(\bar{y},\mathbf{x}_\perp)\right] \mathcal{O}_{\text{LO}}$$

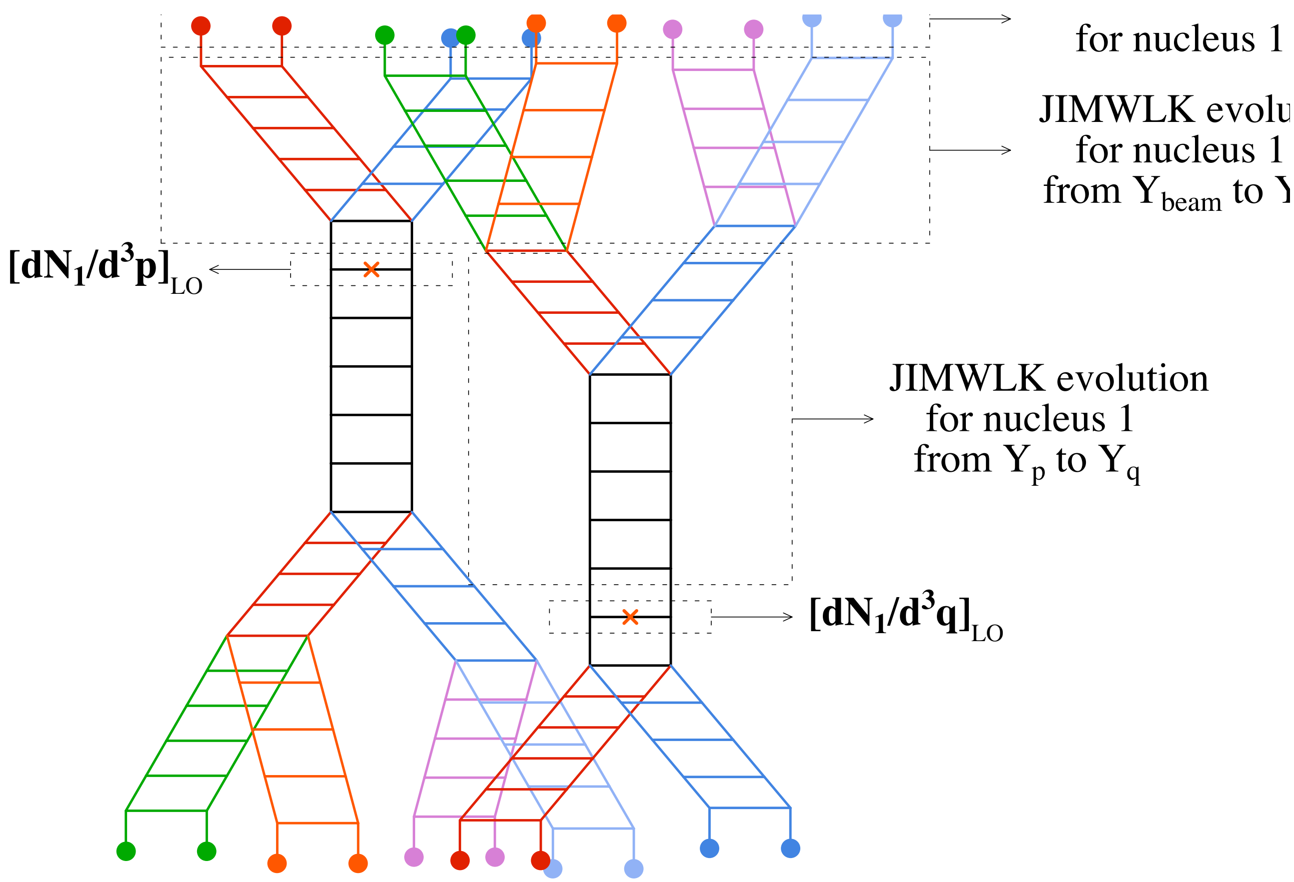
$$\Omega_{1,2}(\bar{y},\mathbf{x}_\perp) \equiv \mathrm{P}\,\exp ig \int_0^{x_y^\mp} \mathrm{d}z^\mp \,\frac{1}{\nabla_\perp^2}\rho_{1,2}(z^\mp,\mathbf{x}_\perp)$$

F. Gelis, T. Lappi and R. Venugopalan,
High energy factorization and long range rapidity correlations in the Glasma I, II, III
PRD, aX:0810.4829 [hep-ph]

Example: Double Inclusive Spectra

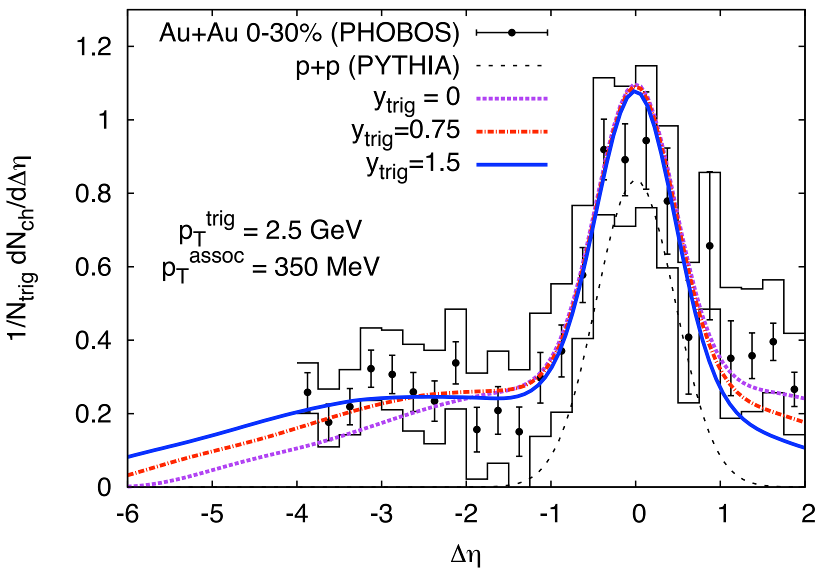
$$\left\langle \frac{dN_n}{d^2p_{\perp,1}dy_1 \cdots d^2p_{\perp,n}dy_n} \right\rangle = \int [d\rho_A d\rho_B] W_{y_{\text{beam}}-Y}[\rho_A] W_{y_{\text{beam}}+Y}[\rho_B]$$

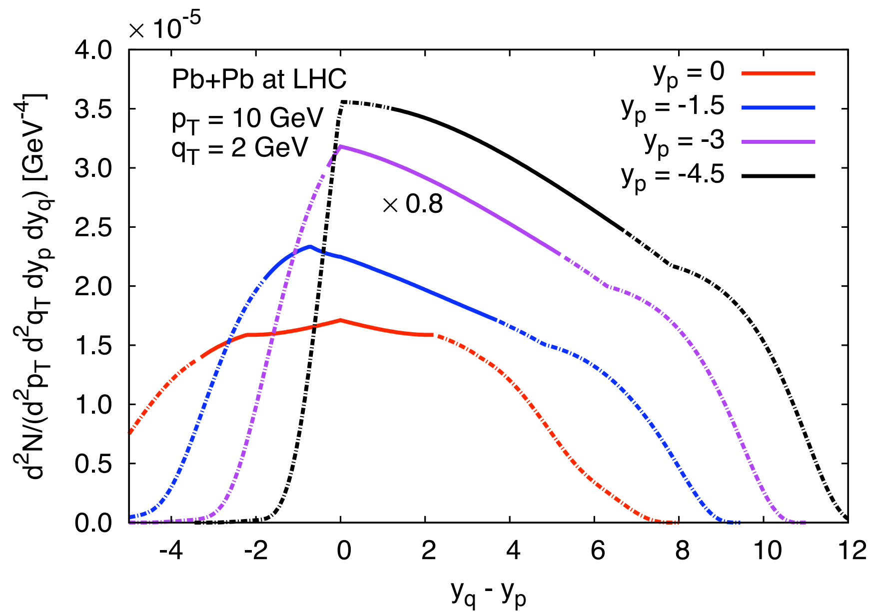
$$\times \quad \frac{dN_{\text{LO}}}{d^2p_{\perp,1}dy_1}(\rho_A,\rho_B) \cdots \frac{dN_{\text{LO}}}{d^2p_{\perp,n}dy_n}(\rho_A,\rho_B)$$



Double Inclusive Spectra

$$C(\mathbf{p},\mathbf{q}) = \frac{\alpha_s^2}{16\pi^{10}} \frac{N_c^2(N_c^2-1)S_\perp}{d_A^4\ \mathbf{p}_\perp^2 \mathbf{q}_\perp^2} \int \mathrm{d}^2\mathbf{k}_{1\perp} \times$$
$$\Big\{ \Phi_{A_1}^2(y_p,\mathbf{k}_{1\perp})\Phi_{A_2}(y_p,\mathbf{p}_\perp-\mathbf{k}_{1\perp})\left[\Phi_{A_2}(y_q,\mathbf{q}_\perp+\mathbf{k}_{1\perp})+\Phi_{A_2}(y_q,\mathbf{q}_\perp-\mathbf{k}_{1\perp})\right]$$
$$+\Phi_{A_2}^2(y_q,\mathbf{k}_{1\perp})\Phi_{A_1}(y_p,\mathbf{p}_\perp-\mathbf{k}_{1\perp})\left[\Phi_{A_1}(y_q,\mathbf{q}_\perp+\mathbf{k}_{1\perp})+\Phi_{A_1}(y_q,\mathbf{q}_\perp-\mathbf{k}_{1\perp})\right]\Big\}$$





K.D., Francois Gelis, Tuomas Lappi, Raju Venugopalan

arXiv:0911.2720, NPA

Two classes of fluctuations

- Zero Modes: $p_\eta = 0$
  - what we just looked at

- Non-Zero Modes: $p_\eta \neq 0$
  - lead to secular divergences

$$\mathcal{O}_{\text{LLog+LInst.}} = \frac{1}{g^2} \sum_n \sum_m d_{nm} g^{2n} e^{(m-n)\sqrt{\mu\tau}} \log^n \left(\frac{1}{x_{1,2}} \right)$$

- Can these be re-summed in a similar manner?

$$\begin{aligned} \left\langle \mathcal{O} \right\rangle_{\text{LLog+LInst.}} &= \int \left[D\Omega_1(\bar{y}, \mathbf{x}_\perp) D\Omega_2(\bar{y}, \mathbf{x}_\perp) \right] W\left[\Omega_1(\bar{y}, \mathbf{x}_\perp) \right] W\left[\Omega_2(\bar{y}, \mathbf{x}_\perp) \right] \\ &\times \int \left[D\omega(\bar{y}, \mathbf{x}_\perp) \right] Z\left[\omega(\bar{y}, \mathbf{x}_\perp) \right] \mathcal{O}_{\text{LO}} \left(\Omega_1 + \omega, \Omega_2 + \omega \right) \end{aligned}$$

- What is the spectrum of small fluctuations?

Classical Preheating and De-coherence
D. T. Son, hep-ph/9601377

Initial Singularity of the Little Bang
K. Fukushima, F. Gelis and L. McLerran
NPA, hep-ph/0610416

Conclusions

- Small field fluctuations control both
  - Rapidity dependence of Ridge
  - and generation of topological susceptibility

- Work in progress on a first principle calculation of topological susceptibility generation in heavy ion collisions

Leading Order form

- Leading order single inclusive spectra with fixed sources

$$\left.\frac{\mathrm{d}N_1\left[\rho_1,\rho_2\right]}{\mathrm{d}^2\mathbf{p}_\perp\mathrm{d}y_p}\right|_{\mathrm{LO}} = \frac{1}{16\pi^3}\lim_{x_0,y_0\to+\infty}\int \mathrm{d}^3\mathbf{x}\,\mathrm{d}^3\mathbf{y}\; e^{ip\cdot(x-y)}\;(\partial_x^0 - iE_p)(\partial_y^0 + iE_p)$$
$$\times\sum_{\lambda,a}\epsilon_\lambda^\mu(\mathbf{p})\epsilon_\lambda^\nu(\mathbf{p})\; A_\mu^a(x)[\rho_1,\rho_2]\; A_\nu^a(y)[\rho_1,\rho_2]\;.$$

- at large $\mathrm{k_T}$ use perturbative solution to YM eqn.

$$p^2 A_a^\mu(\mathbf{p}) = -if_{abc}\,g^3\int\frac{\mathrm{d}^2\mathbf{k}_\perp}{(2\pi)^2}\,L^\mu(\mathbf{p},\mathbf{k}_\perp)\frac{\tilde{\rho}_1^b(\mathbf{k}_\perp)\tilde{\rho}_2^c(\mathbf{p}_\perp-\mathbf{k}_\perp)}{\mathbf{k}_\perp^2\,(\mathbf{p}_\perp-\mathbf{k}_\perp)^2}$$

- with non-local source correlations

$$\left\langle\widetilde{\rho}^a(\mathbf{k}_\perp)\widetilde{\rho}^b(\mathbf{k}'_\perp)\right\rangle = (2\pi)^2\mu_A^2(y)\,\delta^{ab}\delta(\mathbf{k}_\perp-\mathbf{k}'_\perp)$$

Color Glass Condensate

- Parton of size $\dfrac{1}{k_\perp^2}$ has cross section $\sigma \sim \alpha_s \dfrac{1}{k_\perp^2}$
- For a hadron of size $S_\perp$ geometric overlap occurs when $\dfrac{dN}{dy}\sigma \sim S_\perp$
- Saturation momentum: $k^2_{\perp,\mathrm{max}} \equiv Q_S^2 \sim \dfrac{\alpha_s}{S_\perp}\dfrac{dN}{dy}$