Parity breaking in dense nucler matter

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Spontaneous parity breaking in dense nuclear matter

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Phys.Lett.B678:416, 2009.

We conjecture it to occur at low temperature but large baryon number density due to condensation of parity-odd mesons (pions, kaons,… and their radial excitations)

How large?

Beyond the range of validity of pion-nucleon effective Lagrangian but not large enough for quark percolation, *i.e.* in the hadronic phase with heavy meson excitations playing an essential role

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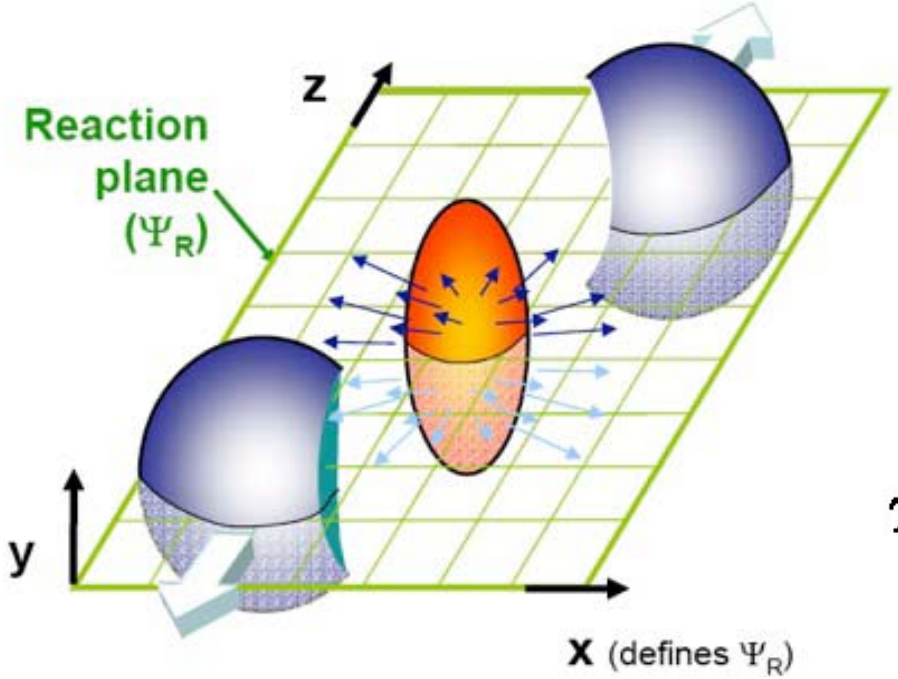
We conjecture it to occur at low temperature but large baryon number density due to condensation of parity-odd mesons (pions, kaons,… and their radial excitations)

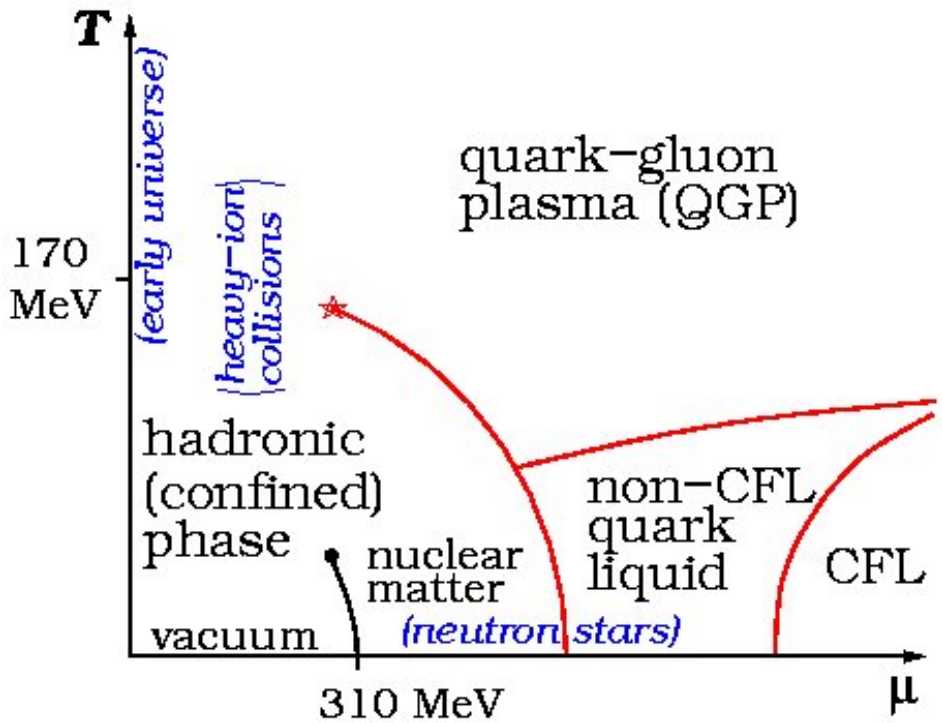
$$\rho_B \sim (3 - 10)\, \rho_N \qquad \rho_N \sim 0.17\ \mathrm{fm}^{-3} = (0.18\ \mathrm{fm})^{-3}$$

<u>Beyond the range of validity</u> of pion-nucleon effective Lagrangian but <u>not large enough</u> for quark percolation, *i.e.* in the hadronic phase with heavy meson excitations playing an essential role



CME is (obviously) a non-equilibrium phenomenon

Inhomogeneities are essential



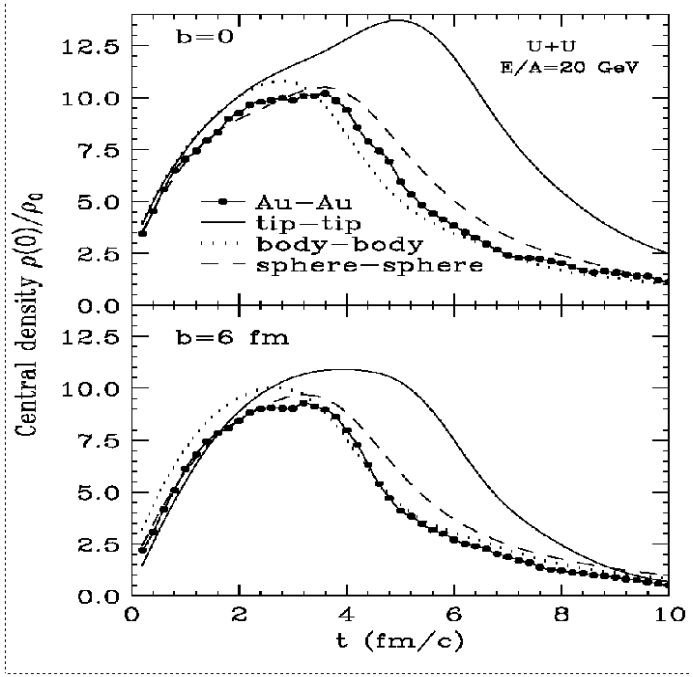
The possible mechanism of spontaneous symmetry breaking of parity that will be discussed here is an equilibrium (or quasi-equilibrium) phenomenon

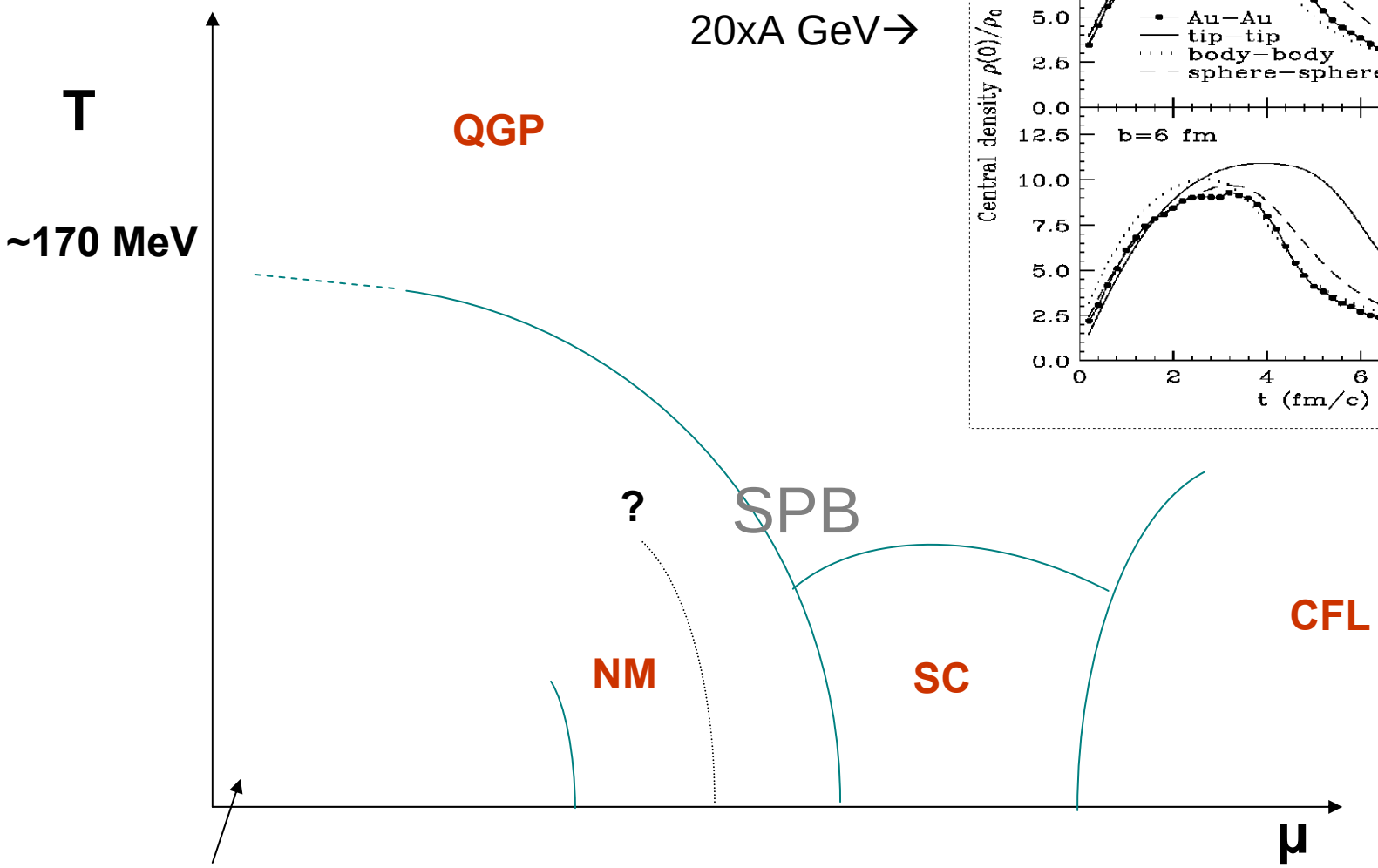
Phenomenon is translationally invariant.

Best reproduced by central collisions

Possibility of parity breaking in QCD ?

Vafa-Witten theorem is not valid at μ > 0





Usual QCD

Converter of collider energies per N into phase diagram

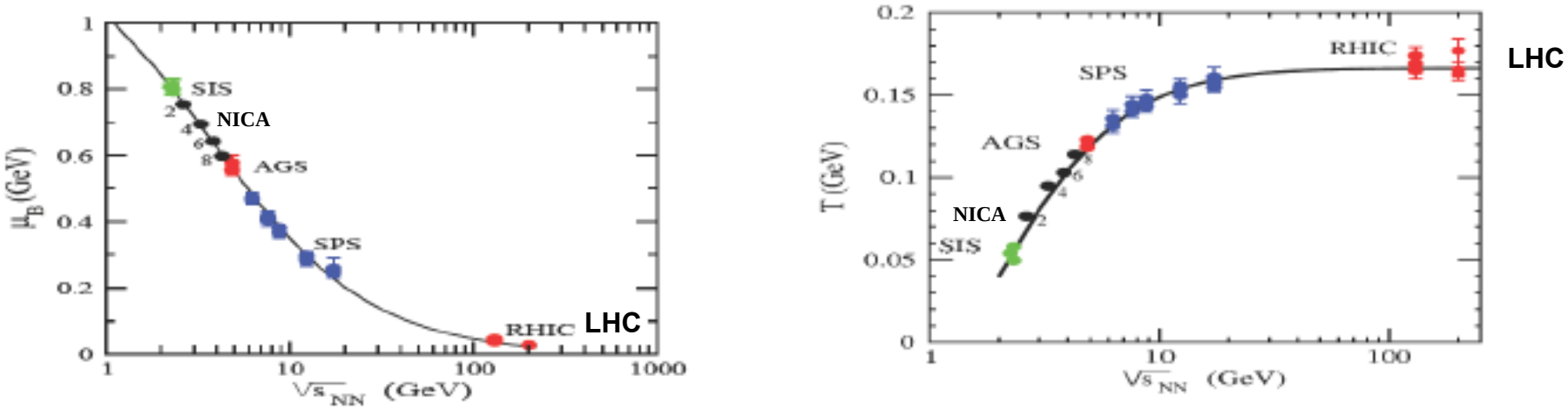
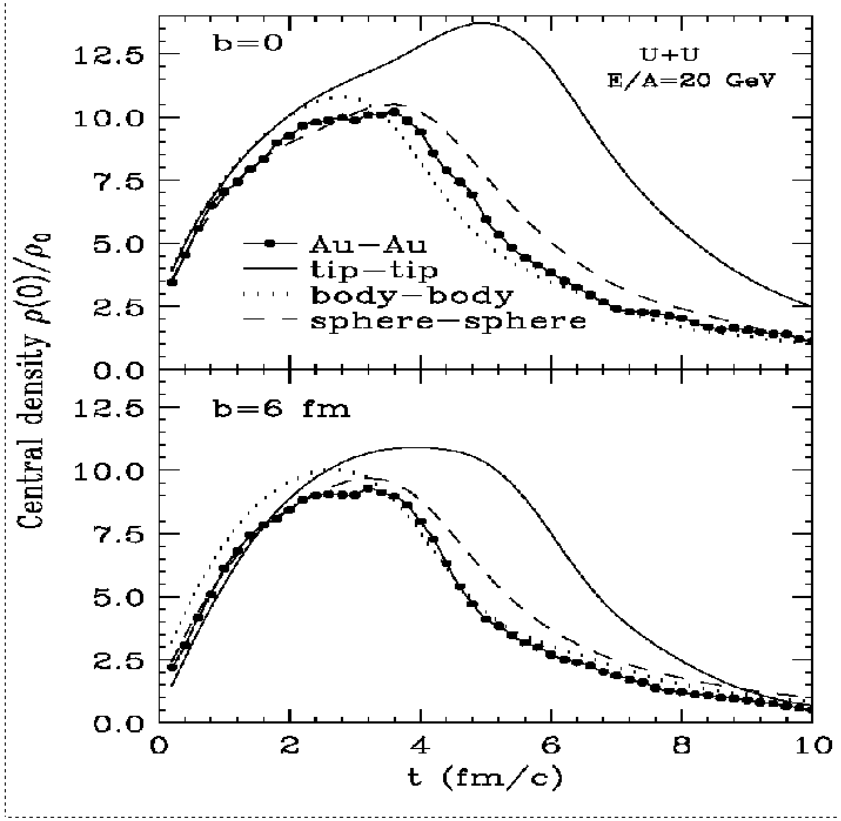


FIG. 1: (Color online) $\sqrt{s_{NN}}$ dependence of μ_B (top panel) and T obtained from chemical fits [20] to particle yield ratios obtained at different accelerator facilities as indicated. The solid lines are fits to these data.

Both phenomena lead to a similar effective description of the electromagnetic phenomena present in HI collisions in terms of Chern-Simons electrodynamics



$$\mathcal{L}_{\rm MCS} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - A_\mu J^\mu + \frac{c}{4}\, P_\mu J^\mu_{CS}$$

$$J^\mu_{CS} = \epsilon^{\mu\nu\rho\sigma} A_\nu F_{\rho\sigma} \qquad P_\mu = \partial_\mu \theta$$

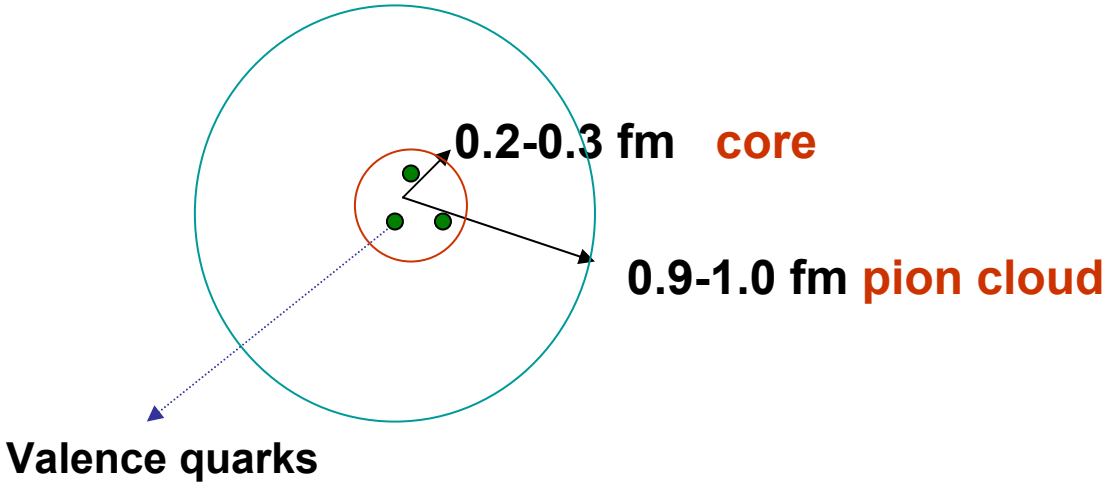
CME: $\theta = \theta(x,t)$

SPB: $\theta = \theta(x,t)$,

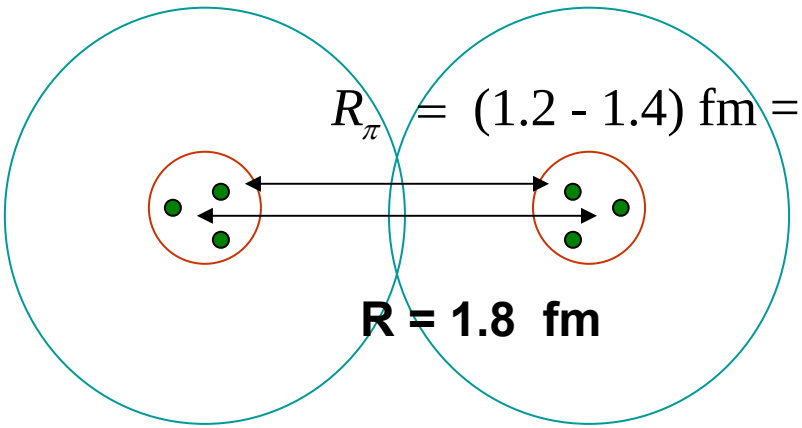
adiabaticity in t needs to be assumed = approximate local thermal equilibrium

See talk by A. Andrianov on possible experimental consequences

Nucleons



Normal nuclear matter



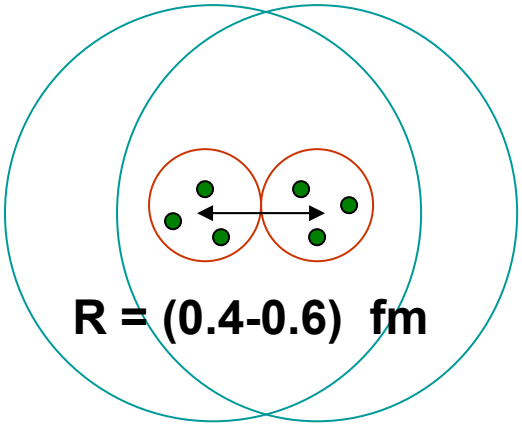
$$R_\pi = (1.2 - 1.4)\ \text{fm} = (140\text{-}160\ \text{MeV})^{-1} \approx (m_\pi)^{-1}$$

Pion-nucleon realm

Superdense nuclear matter

$$\varrho_B = (25 \div 100)\varrho_N$$
$$\simeq (0.4 \div 0.6 fm)^{-3}$$

Quark stars?

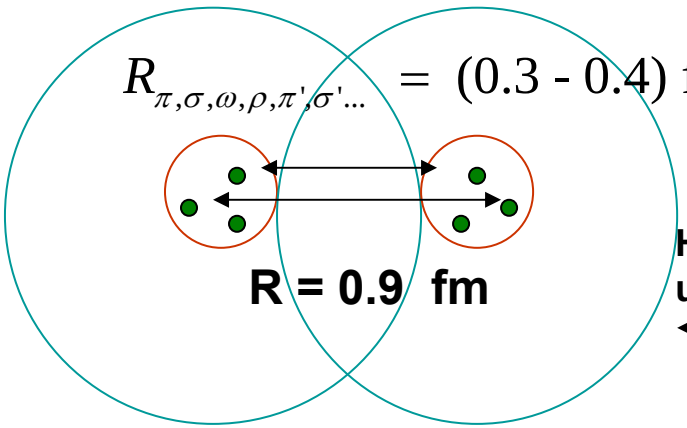


Quark percolation
2SC, CFL …

Dense nuclear matter

$$\varrho_B = 8\varrho_N \simeq (0.9 fm)^{-3}$$

Neutron stars



Higher-mass mesons +
undressed nucleons
⟷ dressed quarks

A bit of history:
pion condensation in symmetric nuclear matter $\rho_p = \rho_n$

A. Migdal (1971)

$$\omega^2 = 1 + k^2 - 4\pi n F(k)$$

$\hbar = c = m_\pi = 1$

where n is the nucleon density and $F(k)$ is the forward pion-nucleon scattering amplitudewhich for both π^+ and π^- mesons, has the sign corresponding to attraction ($F > 0$), and therefore at sufficient-density the frequency can vanish, meaning instability of the pion field. However, $F(k)$ is small at small k and instability sets in at $k = k_0$, which corresponds to the minimal value of $k^2 - 4\pi n F(k)$. The instability condition is $\omega^2 = 0$ or

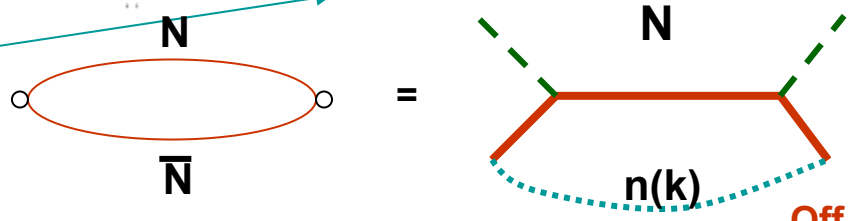
$$1 + k_0^2 = 4\pi n F(k_0)$$

In this approach a pion condensate is spatially inhomogeneous!

A more precise calculation includes the particle-hole excitations of the nuclear medium

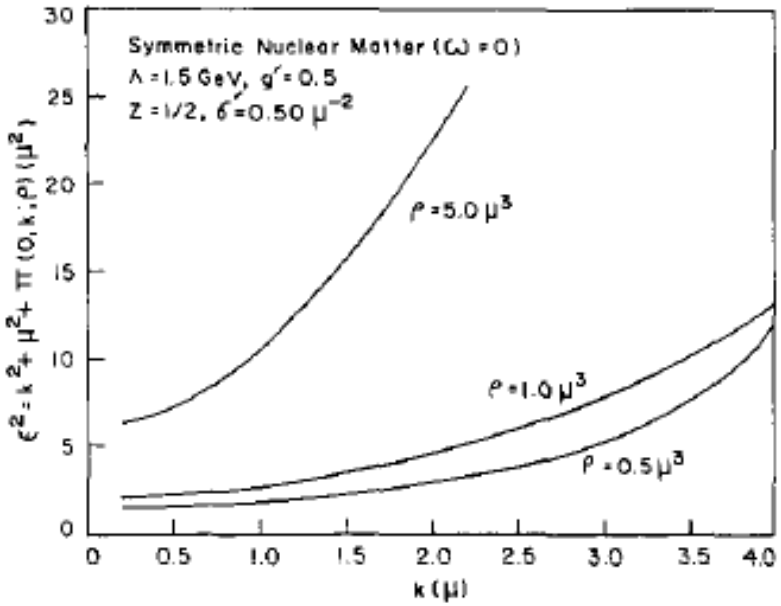
$$\omega^2 = 1 + k^2 + \Pi(k, \omega)$$

Polarization operator =



Off-shell amplitudes from an effective lagrangian

T Shamsunnahar et al.
J. Phys. G. 17 (1991) 887



No pion condensate

Simplest sigma model is not rich enough to provide SPB

Extended linear sigma model

with two multiplets of scalar and pseudoscalar mesons (as close to QCD as possible)

$$H_j = \sigma_j \mathbf{I} + i\hat{\pi}_j, \quad j = 1,2; \quad H_j H_j^\dagger = (\sigma_j^2 + (\pi_j^a)^2)\mathbf{I}, \qquad \hat{\pi}_j \equiv \pi_j^a \tau^a$$

Chiral limit $\longrightarrow$ $SU_L(2) \times SU_R(2)$ **symmetry**

$$\begin{aligned} V_{\rm eff} \;=\; & \frac{1}{2}{\rm tr}\Big\{ -\sum_{j,k=1}^{2} H_j^\dagger \Delta_{jk} H_k \\ & +\lambda_1 (H_1^\dagger H_1)^2 + \lambda_2 (H_2^\dagger H_2)^2 + \lambda_3 H_1^\dagger H_1 H_2^\dagger H_2 \\ & +\frac{1}{2}\lambda_4 (H_1^\dagger H_2 H_1^\dagger H_2 + H_2^\dagger H_1 H_2^\dagger H_1) + \frac{1}{2}\lambda_5 (H_1^\dagger H_2 + H_2^\dagger H_1) H_1^\dagger H_1 \\ & +\frac{1}{2}\lambda_6 (H_1^\dagger H_2 + H_2^\dagger H_1) H_2^\dagger H_2 \Big\} + \mathcal{O}(\frac{|H|^6}{\Lambda^2}), \end{aligned}$$

9 real constants

$\Delta_{jk} \sim \lambda_A \sim N_c$

Chiral expansion in $1/\Lambda$
in hadron phase of QCD

$$\Lambda \simeq 4\pi F_\pi \sim 4 M_{dyn}$$

Coupling to matter

$$\mathcal{L}_{int} = -(\bar{q}_R H_1 q_L + \bar{q}_L H_1^\dagger q_R)$$

Chirally symmetric parametrization

$$H_1(x) = \sigma_1(x) U(x) = \sigma_1(x)\xi^2(x); \qquad \langle H_1 \rangle = \langle \sigma_1 \rangle > 0$$

$$H_2(x) = \xi(x)\big(\sigma_2(x) + i\hat{\pi}_2(x)\big)\xi(x) = \sigma_2(x)U(x) + i\xi(x)\hat{\pi}_2(x)\xi(x)$$

Effective potential

$$\begin{aligned} V_{\text{eff}} \;=\; & -\sum_{j,k=1}^{2} \sigma_j \Delta_{jk} \sigma_k - \Delta_{22}(\pi_2^a)^2 \\ & +\lambda_1\sigma_1^4 + \lambda_2\sigma_2^4 + (\lambda_3+\lambda_4)\sigma_1^2\sigma_2^2 + \lambda_5\sigma_1^3\sigma_2 + \lambda_6\sigma_1\sigma_2^3 \\ & +(\pi_2^a)^2\Big((\lambda_3-\lambda_4)\sigma_1^2 + \lambda_6\sigma_1\sigma_2 + 2\lambda_2\sigma_2^2\Big) + \lambda_2\Big((\pi_2^a)^2\Big)^2 \end{aligned}$$

Vacuum state for $\rho = 0$

Neutral pseudoscalar condensate breaking P?

No in QCD at zero quark density

Gap equations:

$$
\begin{aligned}
2(\Delta_{11}\sigma_1 + \Delta_{12}\sigma_2) &= 4\lambda_1\sigma_1^3 + 3\lambda_5\sigma_1^2\sigma_2 + 2(\lambda_3+\lambda_4)\sigma_1\sigma_2^2 + \lambda_6\sigma_2^3 \\
&\quad +\rho^2\Big(2(\lambda_3-\lambda_4)\sigma_1 + \lambda_6\sigma_2\Big), \qquad\qquad \pi_2^a = \delta^{a0}\rho \\
2(\Delta_{12}\sigma_1 + \Delta_{22}\sigma_2) &= \lambda_5\sigma_1^3 + 2(\lambda_3+\lambda_4)\sigma_1^2\sigma_2 + 3\lambda_6\sigma_1\sigma_2^2 + 4\lambda_2\sigma_2^3 \\
&\quad +\rho^2\Big(\lambda_6\sigma_1 + 4\lambda_2\sigma_2\Big), \\
0 &= 2\pi_2^a\Big(-\Delta_{22} + (\lambda_3-\lambda_4)\sigma_1^2 + \lambda_6\sigma_1\sigma_2 + 2\lambda_2\sigma_2^2 + 2\lambda_2\rho^2\Big)
\end{aligned}
$$

Sufficient condition to avoid P-parity breaking in normal QCD vacuum (μ = 0)

$$
\boxed{(\lambda_3-\lambda_4)\sigma_1^2 + \lambda_6\sigma_1\sigma_2 + 2\lambda_2\sigma_2^2 > \Delta_{22}}
$$

Second variation for $\rho = 0$

$$\frac{1}{2}V_{11}^{(2)} = -\Delta_{11} + 6\lambda_1\sigma_1^2 + 3\lambda_5\sigma_1\sigma_2 + (\lambda_3 + \lambda_4)\sigma_2^2,$$

$$V_{12}^{(2)} = -2\Delta_{12} + 3\lambda_5\sigma_1^2 + 4(\lambda_3 + \lambda_4)\sigma_1\sigma_2 + 3\lambda_6\sigma_2^2,$$

$$\frac{1}{2}V_{22}^{(2)} = -\Delta_{22} + (\lambda_3 + \lambda_4)\sigma_1^2 + 3\lambda_6\sigma_1\sigma_2 + 6\lambda_2\sigma_2^2$$

$$\frac{1}{2}V_{ab}^{(2)\pi} = \delta_{ab}\Big(-\Delta_{22} + (\lambda_3 - \lambda_4)\sigma_1^2 + \lambda_6\sigma_1\sigma_2 + 2\lambda_2\sigma_2^2\Big) > 0$$

Mass of π_2 **if** $\rho = 0 \rightarrow$
necessary condition

$$\boxed{(\lambda_3 - \lambda_4)\sigma_1^2 + \lambda_6\sigma_1\sigma_2 + 2\lambda_2\sigma_2^2 > \Delta_{22} > 0}$$

Condition is necessary and sufficient

In addition $\sigma_2 = 0$, $\sigma_1 = 0$ **should be a maximum (or at least a saddle point) for chiral symmetry breaking to occur**

Landscape of solutions

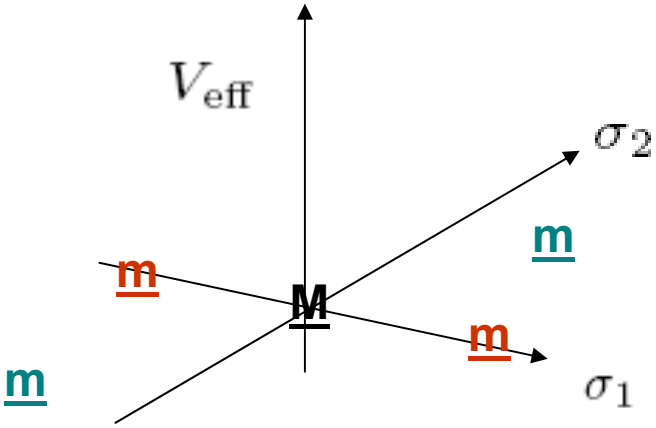
The effective potential is invariant under $\sigma_i \rightarrow -\sigma_i$

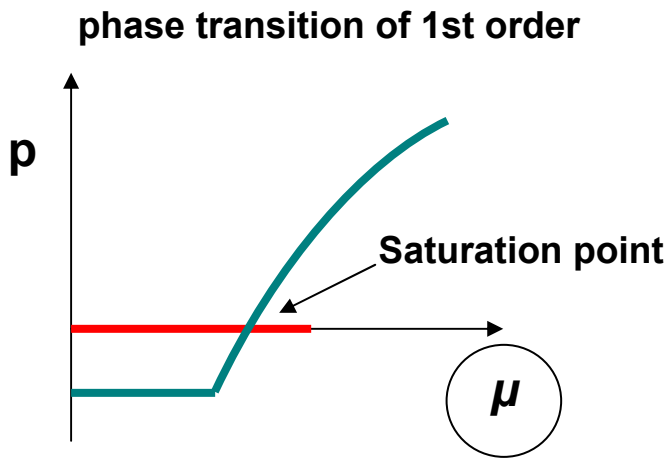
$\sigma_2 = 0, \sigma_1 \neq 0$ **is a solution (a minimum) if** $\lambda_3 \pm \lambda_4 > 2\lambda_1 > 0.$

If $\lambda_6 < 0, \quad (\lambda_3 + \lambda_4)\Delta_{22} > 2\lambda_2\Delta_{11}, \quad \lambda_3^2 - \lambda_4^2 > 4\lambda_1\lambda_2.$

another set of solutions appears…

9 extrema = 4 minima + 4 saddle points + 1 maximum at the origin





Repulsive vector meson interaction needed for satisfactory description

Embedding a chemical potential

$$\int dx\, \mu \left(\bar{q}\gamma_0 q(x) - \rho_B \right)$$

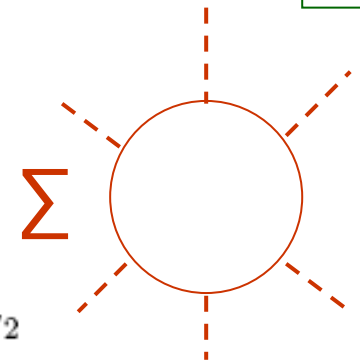
Local coupling to quarks

$$\mathcal{L}_{int} = -(\bar{q}_R H_1 q_L + \bar{q}_L H_1^\dagger q_R)$$

Superposition of physical meson states
(quark matter=nuclear matter)

$$\Delta V_{\rm eff}(\mu) = \frac{\mathcal{N}}{2}\Theta(\mu - |H_1|)\left[\mu|H_1|^2\sqrt{\mu^2 - |H_1|^2} - \frac{2\mu}{3}(\mu^2 - |H_1|^2)^{3/2} - |H_1|^4 \ln\frac{\mu + \sqrt{\mu^2 - |H_1|^2}}{|H_1|}\right]\left(1 + O\left(\frac{\mu^2}{\Lambda^2}\right)\right)$$

$$\mathcal{N} \equiv \frac{N_c N_f}{4\pi^2}$$



Density and Fermi momentum

$$\varrho_B = -\frac{1}{3}\partial_\mu \Delta V_{\rm eff}(\mu) = \frac{N_c N_f}{9\pi^2} p_F^3 = \frac{N_c N_f}{9\pi^2}(\mu^2 - |\langle H_1\rangle|^2)^{3/2}$$

The calculation can be easily extended to finite T
no thermal loops for mesons
(OK in large N-limit)

$$\Delta V_{\rm eff}(\mu) \longrightarrow \Delta V_{\rm eff}(T, \mu)$$

Finite density modifies the minima of the effective potential

Only the first equation for stationary points is modified

$$2(\Delta_{11}\sigma_1 + \Delta_{12}\sigma_2) = 4\lambda_1\sigma_1^3 + 3\lambda_5\sigma_1^2\sigma_2 + 2(\lambda_3+\lambda_4)\sigma_1\sigma_2^2 + \lambda_6\sigma_2^3$$
$$+\rho^2\Big(2(\lambda_3-\lambda_4)\sigma_1 + \lambda_6\sigma_2\Big) + 2\mathcal{N}\Theta(\mu-\sigma_1)\left[\mu\sigma_1\sqrt{\mu^2-\sigma_1^2} - \sigma_1^3\ln\frac{\mu+\sqrt{\mu^2-\sigma_1^2}}{\sigma_1}\right]$$

The matrix of second variations depends on μ only through its first element

$$\frac{1}{2}V_{11}^{(2)\sigma} = -\Delta_{11} + 6\lambda_1\sigma_1^2 + 3\lambda_5\sigma_1\sigma_2 + (\lambda_3+\lambda_4)\sigma_2^2 + (\lambda_3-\lambda_4)\rho^2$$
$$+\mathcal{N}\Theta(\mu-\sigma_1)\left[\mu\sqrt{\mu^2-\sigma_1^2} - 3\sigma_1^2\ln\frac{\mu+\sqrt{\mu^2-\sigma_1^2}}{\sigma_1}\right]$$

Conditions to approach a P-violating phase

$$(\lambda_3 - \lambda_4)\sigma_1^2 + \lambda_6\sigma_1\sigma_2 + 2\lambda_2\sigma_2^2 > \Delta_{22} \longrightarrow \partial_\mu\Big[(\lambda_3 - \lambda_4)\sigma_1^2 + \lambda_6\sigma_1\sigma_2 + 2\lambda_2\sigma_2^2\Big] < 0$$

or

$$\Big(\lambda_6\sigma_1 + 4\lambda_2\sigma_2\Big)V^{(2)}_{\sigma_1\sigma_2} < \Big(2(\lambda_3 - \lambda_4)\sigma_1 + \lambda_6\sigma_2\Big)V^{(2)}_{\sigma_2\sigma_2}$$

One condition for 9 parameters:
P-violation is not exceptional but rather typical!

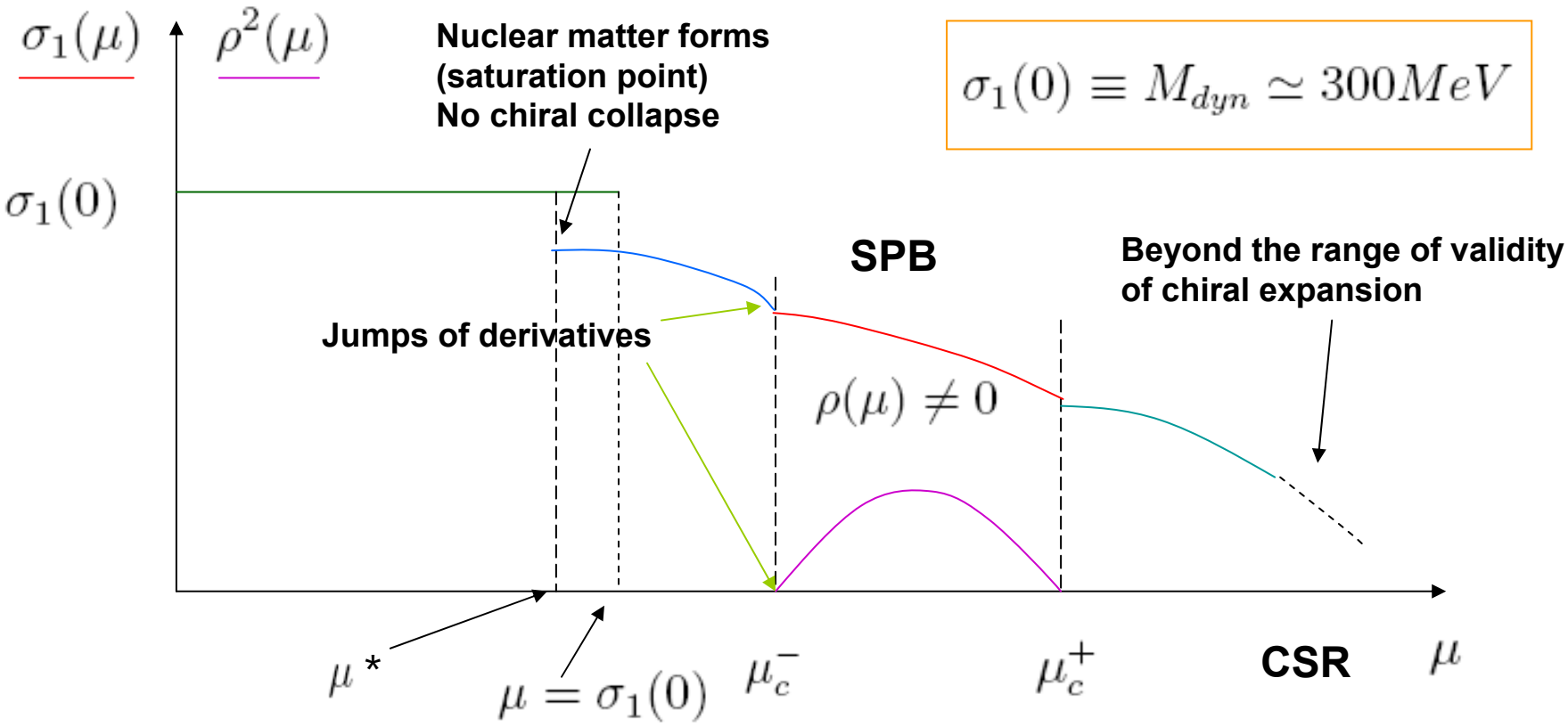
Critical points μ_c **where** $\rho(\mu_c) = 0$

$$(4\lambda_2\Delta_{12} - \lambda_6\Delta_{22})r^2 + (2\lambda_6\Delta_{12} - 4\lambda_4\Delta_{22})r + 2(\lambda_3 - \lambda_4)\Delta_{12} - \lambda_5\Delta_{22} = 0$$

There are in general two solutions

$$r \equiv \frac{\sigma_2}{\sigma_1}$$

Spontaneous P-parity breaking (2nd order phase transition)



$$\sigma_1(0) \equiv M_{dyn} \simeq 300 MeV$$

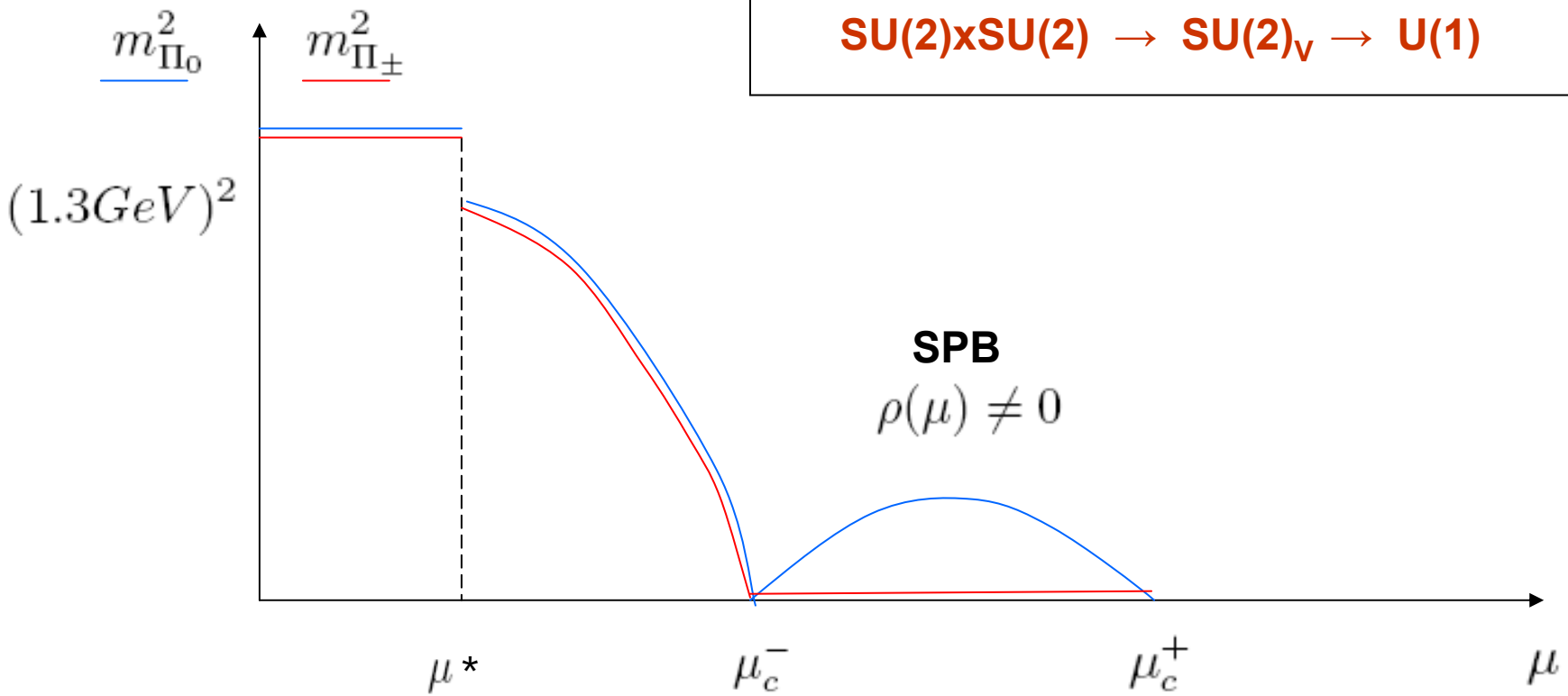
With increasing μ one enters SPB phase and leaves it before (?) encountering any new phase (CSR, CFL …)

Order of the phase transition?

Mass spectrum of “pseudoscalar” states

(parity no longer a good quantum number in strong interactions !)

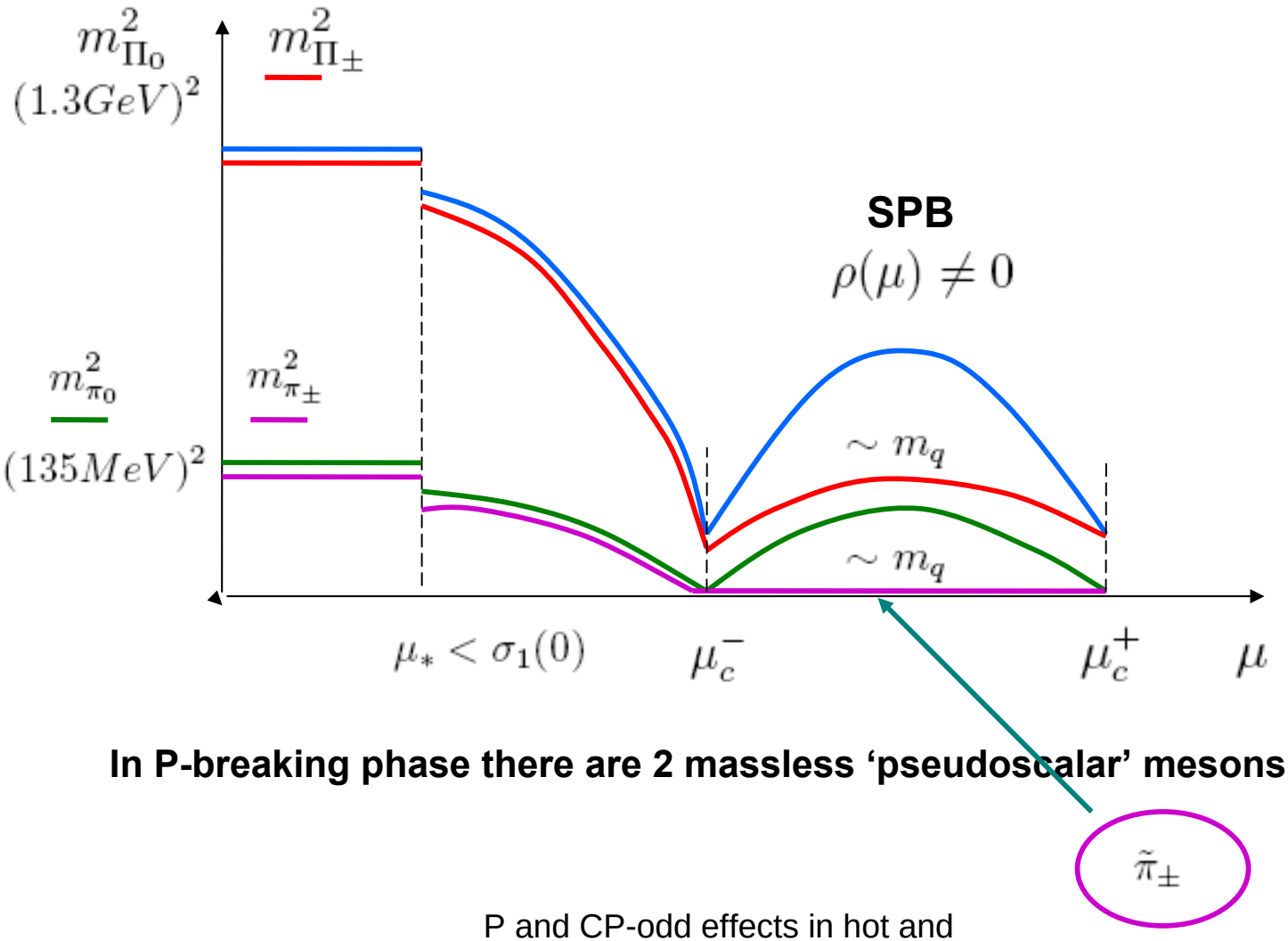
SU(2)xSU(2) → SU(2)$_V$ → U(1)



pseudoscalar mesons

$\pi_0 \quad \pi_\pm \quad \Pi_\pm$

Mass spectrum of "pseudoscalar" states (beyond chiral limit)



In P-breaking phase there are 2 massless 'pseudoscalar' mesons

Kinetic terms

Symmetric under $SU_L(2) \times SU_R(2)$

$$\mathcal{L}_{kin} = \frac{1}{4} \sum_{j,k=1}^{2} A_{jk} \mathrm{tr} \left\{ \partial_\mu H_j^\dagger \partial^\mu H_k \right\}$$

Chirally symmetric parametrization

$$H_1(x) = \sigma_1(x) U(x) = \sigma_1(x) \xi^2(x)$$

$$H_2(x) = \xi(x)(\sigma_2(x) + i\hat{\pi}_2)(x)\xi(x) = \sigma_2(x) U(x) + i\xi(x)\hat{\pi}_2(x)\xi(x)$$

Expanding around a vacuum configuration…

$$U = 1 + i\hat{\pi}/F_0 + \cdots \qquad \xi = 1 + i\hat{\pi}/2F_0 + \cdots$$

$$\sigma_j \equiv \bar{\sigma}_j + \Sigma_j \qquad \hat{\pi} = \tau_3 \rho + \hat{\Pi}$$

P-breaking phase

Mixing with massless pions is different for neutral and charged ones because vector isospin symmetry is broken

$$\tilde{\pi}^{\pm} = \pi^{\pm} + \zeta\Pi^{\pm}, \qquad \tilde{\pi}^{0} = \pi^{0} + \frac{F_0^2}{F_0^2 + A_{22}\rho^2}\Big(\zeta\Pi^0 - \frac{\rho}{F_0}\sum_{j=1}^{2} A_{j2}\partial_\mu\Sigma_j\Big).$$

Partially diagonalized kinetic term

$$\begin{aligned}\mathcal{L}^{(2)}_{kin} \;=\; & \partial_\mu\tilde{\pi}^{\pm}\partial^\mu\tilde{\pi}^{\mp} + (A_{22}-\zeta^2)\partial_\mu\Pi^{\pm}\partial^\mu\Pi^{\mp} + \frac{1}{2}\Big(1+\frac{A_{22}\rho^2}{F_0^2}\Big)\partial_\mu\tilde{\pi}^0\partial^\mu\tilde{\pi}^0 \\ & + \frac{1}{2}\Big(A_{22} - \frac{F_0^2}{F_0^2+A_{22}\rho^2}\zeta^2\Big)\partial_\mu\Pi^0\partial^\mu\Pi^0 + \frac{1}{2}\sum_{j,k=1}^{2}\frac{A_{jk}F_0^2 + \rho^2\det A\,\delta_{1j}\delta_{1k}}{F_0^2+A_{22}\rho^2}\partial_\mu\Sigma_j\partial^\mu\Sigma_k \\ & - \frac{F_0\rho}{F_0^2+A_{22}\rho^2}\zeta\partial_\mu\Pi^0\sum_{j=1}^{2}A_{j2}\partial^\mu\Sigma_j\end{aligned}$$

Isospin breaking $\quad SU_V(2) \to U(1)$

Further diagonalization $\Pi^0 \quad \Sigma_1 \quad \Sigma_2 \quad \Longrightarrow \quad \tilde{\Pi}^0 \quad \tilde{\Sigma}_1 \quad \tilde{\Sigma}_2$

mixes neutral pseudoscalar and scalar states

Therefore genuine mass states don't possess a definite parity in decays

$$\tilde{\Pi}^0 \quad \tilde{\Sigma}_1 \quad \tilde{\Sigma}_2 \;\longrightarrow\; \pi_0\ \pi_0\ \pi_0 \qquad \pi_0\ \pi_+\ \pi_- \qquad \pi_0\ \Pi_+\ \Pi_- \ldots$$

$$\tilde{\Pi}^0 \quad \tilde{\Sigma}_1 \quad \tilde{\Sigma}_2 \;\longrightarrow\; \pi_0\ \pi_0 \qquad \pi_+\ \pi_- \qquad \Pi_+\ \Pi_- \ldots$$

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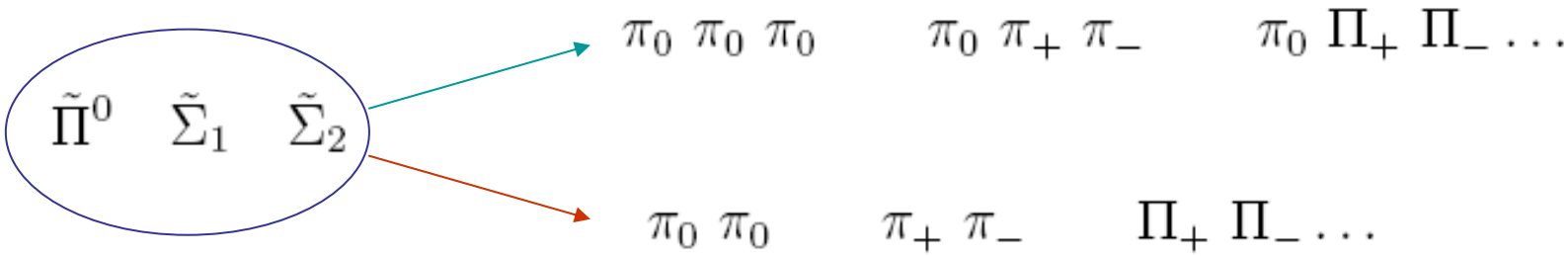
IS THIS A REAL POSSIBILITY ?

Further diagonalization $\Pi^0 \quad \Sigma_1 \quad \Sigma_2 \quad \Longrightarrow \quad \tilde{\Pi}^0 \quad \tilde{\Sigma}_1 \quad \tilde{\Sigma}_2$

mixes neutral pseudoscalar and scalar states

Therefore genuine mass states don't possess a definite parity in decays



IS THIS A REAL POSSIBILITY ?

- **Lattice simulations**
- **Experimental discovery !**

Possible experimental/observational signatures of P-parity breaking

Resonances do not have a definite parity and the same heavy resonance can decay both in two and three pions . It will look like doubling of states.

At the very point of the phase transition leading to parity breaking one has six massless pion-like states. Throughout the P-breaking phase charged massless pions exist and their production is enhanced.

Substantial modifications in the equation of state (neutron stars)

PCAC relations for massless charged pions are modified giving enhancement of electroweak decays (more leptons)

Additional strong isospin breaking: $f_{\pi_0} \neq f_{\pi_\pm}$

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However some of these signals may be quite subtle to interpret or to have a good theoretical control over them.

Ideally we would like a signal that involves `easy' particles such as photons, electrons or muons.

See A. Andrianov's talk…..

Back-up slides

Search for stable nuclear matter

$$\tilde{H}_j = \sum_{k=1,2} L_{jk} H_k.$$

$$\Delta_{jk} = \Delta\delta_{jk} \text{ and } \lambda_5 = 0$$

One of the solution of the simplified equations is $\sigma_2 = 0$ and $\sigma_1^2 = \Delta/2\lambda_1$ provided that $\lambda_3 \pm \lambda_4 > 2\lambda_1 > 0$. **It is a minimum**

If $\sigma_2 \neq 0$, another set of solutions for σ_i given in term of the ratio $x \equiv \sigma_2/\sigma_1$

$$2\lambda_1 - (\lambda_3 + \lambda_4) - \frac{3}{2}\lambda_6 x + (\lambda_3 + \lambda_4 - 2\lambda_2)x^2 + \frac{1}{2}\lambda_6 x^3 = 0.$$

This equation may have one or three real roots.

We need three roots!

Repulsive forces are needed for stabilization

Traditionally chemical potential triggers condensation of omega mesons

$$\Delta\mathcal{L}_\omega = -\frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - g_{\omega\bar{q}q}\bar{q}\gamma_\mu\omega^\mu q,$$

Constant v.e.v. $g_{\omega\bar{q}q}\langle\omega_0\rangle \equiv \bar{\omega}.$ **Shift** $\mu \to \mu + \bar{\omega} \equiv \bar{\mu}.$

Modification of effective potential

$$\Delta V_\omega = -\frac{1}{2}m_\omega^2\langle\omega_0^2\rangle = -\frac{1}{2}\frac{(\bar{\mu}-\mu)^2}{G_\omega}, \quad G_\omega \equiv \frac{g_{\omega\bar{q}q}^2}{m_\omega^2} \simeq \mathcal{O}(\frac{1}{N_c}).$$

Stationary solution

$$\frac{\bar{\mu}-\mu}{G_\omega} = -N_c\varrho_B(\mu) = -\frac{N_cN_f}{3\pi^2}(\bar{\mu}^2 - \sigma_1(\bar{\mu})^2)^{3/2}.$$

Saturation point **(phase transition to p = 0)**

$$\sum_{j,k=1}^{2}\left(\sigma_j^0\Delta_{jk}\sigma_k^0 - \sigma_j^*\Delta_{jk}\sigma_k^*\right) = \frac{N_cN_f}{6\pi^2}\bar{\mu}^*p_F^3(\bar{\mu}^*) + G_\omega\frac{N_c^2N_f^2}{9\pi^4}p_F^6(\bar{\mu}^*)$$

$$= \frac{N_c}{2}\bar{\mu}^*\varrho_B(\mu^*) + G_\omega N_c^2\varrho_B^2(\mu^*),$$

$$\mu_* \simeq 303MeV$$

$$G_\omega \sim (10 \div 15)GeV^{-2}$$

Absence of chiral collapse

In many models at the saturation point chiral symmetry is restored and all hadron states in normal nuclear matter are massless in the chiral limit.(collapse!)

But the second variation

$$\frac{1}{2}V^{(2)}_{\sigma_1\sigma_1} = -\Delta_{11} + \mathcal{N}\mu^2,\ V^{(2)}_{\sigma_1\sigma_2} = -2\Delta_{12},\ \frac{1}{2}V^{(2)}_{\sigma_2\sigma_2} = -\Delta_{22}.$$

$$\Delta_{22} > 0$$

Thus there is no any value of chemical potential which would make this matrix positively defined!

No chiral collapse in two-multiplet models with CSB!

Remarkably the condition $\Delta_{22} > 0$ **is also necessary for spontaneous P-breaking!**

Minimal model admitting SPB

$$\lambda_5 = \lambda_6 = 0 \Rightarrow \Delta_{12} = 0$$ **from consistency**

Such an effective Lagrangian is symmetric under $Z_2 \times Z_2$

$$H_1 \to -H_1 \quad \text{or} \quad H_2 \to -H_2$$

Fit on hadron phenomenology

$$A_{11} = \frac{1}{9} = A_{22}, \quad F_0 = 100MeV, \quad \sigma_1 = 300MeV = 3F_0,$$

$$m_{s,1} = 0.7GeV = 7F_0, \quad m_p = 1.3GeV = 13F_0, \quad m_{s,2} = 1.5GeV = 15F_0,$$

$$\Delta_{11} \simeq 2.7F_0^2, \quad \lambda_1 \simeq 0.15, \quad \lambda_4 \simeq 0.35.$$

For $\varrho_{B,crit} \simeq 3\varrho_{B,nuclear}$ → $\mu_c \simeq 4.3F_0$ ↔ $\sigma_{1,crit} \simeq 1.8F_0$

$\lambda_3 \simeq 3.6 \qquad \Delta_{22} \simeq 11F_0^2$

$m_{s,1} \simeq 1.7F_0, \; m_{s,2} \simeq 4.5F_0$