Magnetic catalysis in a holographic approach.

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Application of the AdS-CFT to condensed matter theory

1. AdS/CFT correspondense

$$< e^{\int_{\partial M} \Phi_0 \hat{O}} >= e^{-S_{cl}[\Phi]} \quad (1)$$

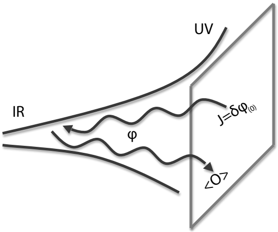

Figure: S. Hartnol, 2009

Application of the AdS-CFT to condensed matter theory

2. Physics in the bulk, e.g. SC instability of the black holes

$$q^2 g_F^2 \geq 2\Delta(\Delta - 3) + 3 \tag{2}$$

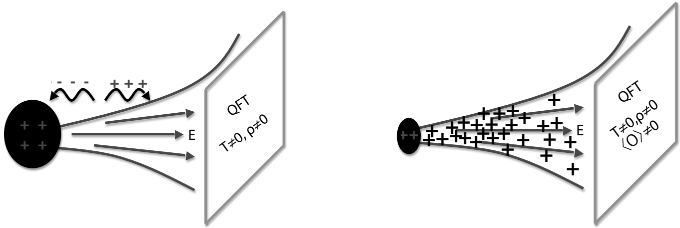

Figure: S. Hartnoll, 2009

Magnetic catalysis in the field theory

$B \neq 0 \;\rightarrow\; \bar{\psi}\Delta\gamma^3\gamma^5\psi$: in $(2+1)$-d P, T odd m; in $(3+1)$-d $k^3 \rightarrow k^3 \pm \Delta$, $< j_5^3(u) >= -tr[\gamma^3\gamma^5 G(u,u)]$

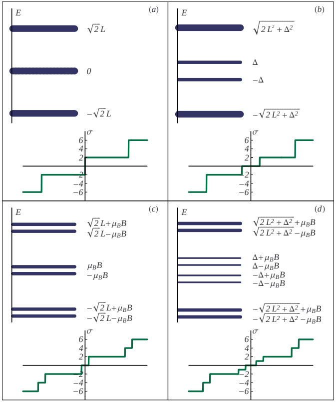

Figure: I. A. Shovkovy, 2006

Calculation set up. Geometry

$$ds^2 = g_{MN}dx^M dx^N = \frac{r^2}{R^2}(-f dt^2 + d\vec{x}^2) + \frac{R^2}{r^2}\frac{dr^2}{f}$$

$$f(r_h) = 0$$

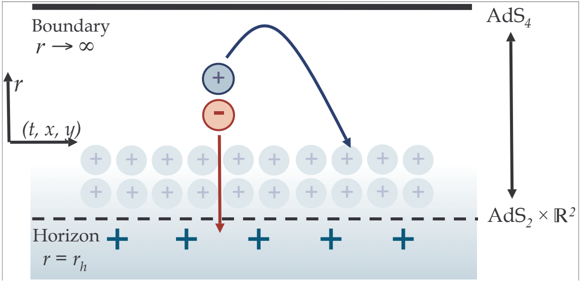

Figure: T. Faulkner et al, 2010

Calculation set up. Black hole

$$
\begin{aligned}
f &= 1 + \frac{Q^2 + H^2}{r^4} - \frac{M}{r^3} \\
A_t &= \mu \left(1 - \frac{r_h}{r}\right), \quad A_x = -\mathcal{H} y \\
\mu &= \frac{g_F Q}{R^2 r_h}, \quad \mathcal{H} = \frac{g_F H}{R^4}, \quad T \sim |f'(r_h)|/4\pi
\end{aligned}
$$

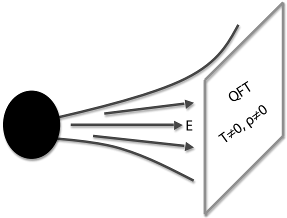

Figure: T. Faulkner et al, 2010

Quasinormal modes of black holes. Fermionic response

$$\begin{aligned} G_R(\omega,k) &= \frac{h_1}{k-k_F-\frac{\omega}{v_F}-\Sigma(\omega,k_F)} \\ \Sigma(\omega,k_F) &= hG^{IR}(\omega,k_F) = hc(k_F)\omega^{2\nu_{k_F}} \\ \omega_c(k) &= \omega_*(k) - i\Gamma(k) \end{aligned}$$

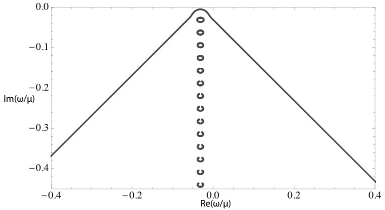

Figure: S. Hartnoll, 2009

Quasinormal modes of black holes. Fermionic response

$$\nu < \frac{1}{2}, \quad \omega_*(k) = (k - k_F)^{\frac{1}{2\nu_{k_F}}}, \quad \frac{\Gamma(k)}{\omega_*(k)} = const$$

$$\nu = \frac{1}{2}, \quad \Sigma(\omega) \approx \tilde{c}_1 \omega \log \omega + i d_1 \omega, \quad \frac{d_1}{\tilde{c}_1} = -\frac{\pi}{1 + \mathrm{e}^{-\frac{2\pi q}{\sqrt{12}}}}$$

$$\nu > \frac{1}{2}, \quad \omega_*(k) = v_F(k - k_F) + \ldots, \quad \frac{\Gamma(k)}{\omega_*(k)} \sim (k - k_F)^{2\nu_{k_F} - 1} \to 0$$

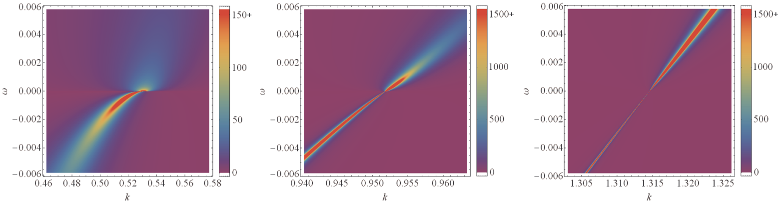

Figure: T. Faulkner et al, 2010

Estimate of magnetic catalysis. Variational calculations

$$S = i\int d^4x\sqrt{-g}\left(i\bar{\psi}\Gamma^M\mathcal{D}_M\psi - m\bar{\psi}\psi - G_{int}(\bar{\psi}i\Gamma^{\hat{2}}\Gamma^{\hat{5}}\psi\right)$$

$$\Delta = 2G_{int} < \bar{\psi}i\Gamma^{\hat{2}}\Gamma^{\hat{5}}\psi >$$

$$\begin{aligned} S_{eff} &= \frac{V_2}{T}\int dr\sqrt{-g}\left(\frac{|\Delta(r)|^2}{4G_{int}}\right. \\ &- \left.\frac{T}{2}\sum_n\int\frac{d^2k}{(2\pi)^2}dr'\operatorname{tr}\ln G^{-1}(i\omega_n,k,r,r')\right) \end{aligned}$$

$$\begin{aligned} D(z,l)\psi &= \lambda(z,l),\quad \lambda(z_*(l),l) = 0 \to z_*(l) \\ S_{eff} &= \frac{V_2}{T}\left(\frac{|\Delta|^2}{4G_{int}} + \frac{T|q\mathcal{H}|}{2\pi R}\sum_{z_*(l)}\ln\left(\frac{1}{2\pi}|\Gamma(\frac{iz_*}{2\pi T}+\frac{1}{2})|^2\right)\right) \end{aligned}$$

Variational calculations of Δ

$$\Delta = \frac{G_{int}|q\mathcal{H}|}{\pi R}\frac{1}{\pi}\left(\frac{\partial\omega_*}{\partial\Delta}\mathrm{Im}\Psi(\frac{iz_*}{2\pi T}+\frac{1}{2})-\frac{\partial\Gamma}{\partial\Delta}\mathrm{Re}\Psi(\frac{iz_*}{2\pi T}+\frac{1}{2})\right)$$

$$\nu > \frac{1}{2}: \quad \frac{\partial\omega_*}{\partial\Delta} = 1, \quad \frac{\partial\Gamma}{\partial\Delta} \sim (\frac{\Delta}{v_F} - k_F)^{2\nu_{k_F}-1} \to 0,$$

$$\frac{\Gamma}{\omega_*} \sim (\Delta - v_F k_F)^{2\nu_{k_F}-1} \to 0$$

$$\Delta = \frac{1}{2\pi}\frac{G_{int}|q\mathcal{H}|}{R}$$

$$\nu < \frac{1}{2}: \quad \frac{\partial\omega_*}{\partial\Delta} \sim \frac{\partial\Gamma}{\partial\Delta} \sim (\frac{\Delta}{v_F} - k_F)^{\frac{1}{2\nu_{k_F}}-1} \to 0$$

$$\Delta = 0$$

Variational calculations of T_c

$$\nu_{k_F} > \tfrac{1}{2}$$

$$\begin{aligned}
n &= \frac{|q\mathcal{H}|}{2\pi R}\frac{1}{\pi}\sum_{z_*}\Bigg(\frac{\partial\omega_*}{\partial(v_F k_F)}\mathrm{Im}\Psi(\frac{iz_*}{2\pi T}+\frac{1}{2}) \\
&- \frac{\partial\Gamma}{\partial(v_F k_F)}\mathrm{Re}\Psi(\frac{iz_*}{2\pi T}+\frac{1}{2})\Bigg) \\
\Delta &= \frac{G_{int}|q\mathcal{H}|}{\pi R}\frac{1}{\pi}\sum_{z_*}\Bigg(\frac{\partial\omega_*}{\partial\Delta}\mathrm{Im}\Psi(\frac{iz_*}{2\pi T}+\frac{1}{2}) - \frac{\partial\Gamma}{\partial\Delta}\mathrm{Re}\Psi(\frac{iz_*}{2\pi T}+\frac{1}{2})\Bigg)
\end{aligned}$$

$$T_c = \frac{G_{int}|q\mathcal{H}|}{2\pi R}(1-\eta_{\mathcal{H}}^2), \quad \eta_{\mathcal{H}} = \frac{2\pi R n}{|q\mathcal{H}|} \equiv \frac{\mathcal{H}_c}{\mathcal{H}}$$

Microscopic calculation of magnetic catalysis

$$D(\Omega,k)\mathcal{G}^R(r,r',\Omega,k)=\frac{1}{\sqrt{-g}}i\delta(r,r'),\quad D(\Omega,k)\psi_{radial}(r)=0$$

$$\mathcal{G}^R_\alpha(r,r')=\frac{G_\alpha(\Omega,k)}{R^3}\times\begin{cases}-\psi^{bdy}_\alpha(r)\tilde{\psi}^{in}_\alpha(r') & r>r'\\ -\psi^{in}_\alpha(r)\tilde{\psi}^{bdy}_\alpha(r') & r<r'\end{cases},\quad \tilde{\psi}_\alpha=i\psi^T_\alpha\sigma^1$$

$$\psi^{bdy}_\alpha = r^{\frac{3}{2}-mR}\begin{pmatrix}1\\0\end{pmatrix},\quad \psi^{in}_\alpha=\frac{1}{G_\alpha}r^{\frac{3}{2}+mR}\begin{pmatrix}0\\1\end{pmatrix}+r^{\frac{3}{2}-mR}\begin{pmatrix}1\\0\end{pmatrix}$$

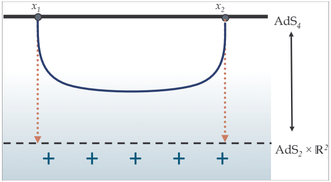

Figure: T. Faulkner et al, 2010

Ginsburg-Landau in the AdS_4 bulk

$$S^{(2)} = \frac{V_2}{T}\int dr\sqrt{g(r)}\left(\frac{|\Delta|^2}{4G_{int}} + \int dr'\sqrt{g(r')}\Delta(r)\Delta(r')^* F(r,r')\right)$$

$$F(r,r') = -2T\sum_n \int \frac{d^2k}{(2\pi)^2}\mathrm{tr}\mathcal{G}(r',r,i\omega_n,\vec{k})\Gamma\mathcal{G}(r,r',-i\omega_n,-\vec{k})\bar{\Gamma}$$

$$F(r,r') = \frac{i}{R^6}\int\frac{d^2k}{(2\pi)^2}\int_{-\infty}^{\infty}\frac{d\Omega}{\pi}\tanh\frac{\Omega}{2T}G_1(\Omega,\vec{k})^*G_1(-\Omega,-\vec{k})\times$$

$$\begin{cases} \psi_2^{in}(r',\Omega)^\dagger\sigma^1\psi_2^{in}(r',-\Omega)\psi_1^{bdy}(r,\Omega)^\dagger\sigma^1\psi_1^{bdy}(r,-\Omega) & r>r' \\ \psi_2^{bdy}(r',\Omega)^\dagger\sigma^1\psi_2^{bdy}(r',-\Omega)\psi_1^{in}(r,\Omega)^\dagger\sigma^1\psi_1^{in}(r,-\Omega) & r<r' \end{cases}$$

$$+(1\leftrightarrow 2)$$

$$T \ll \Omega \ll \mu$$

Critical temperatute T_c

$$\nu > \frac{1}{2}: \quad F(r,r') = -\frac{|q\mathcal{H}|h_1^2 v_F^2}{2\pi^2 R^6}\frac{1}{T}\,\psi^0(r)^\dagger\sigma^1\psi^0(r)\psi^0(r')^\dagger\sigma^1\psi^0(r')$$

$$\nu > \frac{1}{2}: \quad F(r,r') = -\frac{|q\mathcal{H}|h_1^2}{2\pi^2 h_2^2 R^6}\cos(2\pi\nu)\frac{\mu^{1-4\nu} - T^{1-4\nu}}{1-4\nu}$$

$$\psi^0(r)^\dagger\sigma^1\psi^0(r)\psi^0(r')^\dagger\sigma^1\psi^0(r')$$

$$\nu > \tfrac{1}{2}$$

$$\frac{\Delta(r)}{2G_{int}} + \int dr'\sqrt{-g(r')}\Delta(r')F(r',r) = 0, \quad \Delta^0(r) \sim \psi^0(r)^\dagger\sigma^1\psi^0(r)$$

$$T_c = \frac{G_{int}|q\mathcal{H}|h_1^2 v_F^2}{\pi^2 R^6}\int dr\sqrt{-g(r)}(\psi^0(r)^\dagger\sigma^1\psi^0(r))^2$$

Solving for zero modes

$$\psi^0 = (\psi_1^0, \psi_2^0)^T,\ \psi_1^0 = \tfrac{1}{2}(\tilde{y}_1^0, \tilde{y}_2^0)^T$$

$\omega = 0$, $m = 0$, $r = r_0 \frac{1}{1-z}$, $z = 0$ is the horizon, $z = 1$ is the boundary

$$\begin{aligned} \tilde{y}^0_{1;2} &= N_{1;2}(z-1)^{\frac{3}{2}} z^{-\frac{1}{2}+\nu_\lambda}(z-\bar{z}_0)^{-\frac{1}{2}-\nu_\lambda}\left(\frac{z-z_0}{z-\bar{z}_0}\right)^{\frac{1}{4}(-1\mp\sqrt{2}q\mu/z_0)} \\ &\times\ {}_2F_1\left(\frac{1}{2}+\nu_\lambda \mp \frac{\sqrt{2}}{3}q\mu, \nu_\lambda \pm i\frac{q\mu}{6}, 1+2\nu_\lambda, \frac{2i\sqrt{2}z}{3z_0(z-\bar{z}_0)}\right) \end{aligned}$$

$$\tilde{\eta}^0_{1;2} = \tilde{N}_{1;2}\left(\frac{\tilde{y}^0_{1;2}}{N_{1;2}}\ \text{with}\ \ \nu_\lambda \to -\nu_\lambda\right)$$

Matching the solutions

"Near" region, $z \ll 1$

$$\begin{aligned} \tilde{y}_{1;2}^{near} &= C_{1;2} z^{-\frac{1}{2}-\nu_\lambda} \mathrm{e}^{-\frac{i\omega}{6z}} {}_1F_1\left(\frac{1}{2} \mp \frac{1}{2} + \nu_\lambda + \frac{iq\mu}{6}, 1 + 2\nu_\lambda, \frac{i\omega}{3z}\right) \\ &+ D_{1;2}(\nu_\lambda \to -\nu_\lambda) \end{aligned}$$

"Far" region, $z \gg \omega$

$$\begin{aligned} \tilde{y}_{1;2}^{far} &= \tilde{N}_{1,2} S_{1;2}(\nu) z^{-\frac{1}{2}-\nu} + N_{1;2} S_{1;2}(-\nu) z^{-\frac{1}{2}+\nu} + O(\omega) \\ S_{1;2}(\nu) &= (-1)^{3/2} (-\bar{z}_0)^{-\frac{1}{2}+\nu_\lambda} \left(\frac{z_0}{\bar{z}_0}\right)^{-\frac{1}{4} \mp \frac{\sqrt{2} q\mu}{4 z_0}} \end{aligned}$$

$$G = \lim_{z \to 1} \frac{\tilde{y}_1^0 + i\tilde{y}_2^0 + \tilde{\eta}_1^0 + i\tilde{\eta}_2^0}{\tilde{y}_2^0 + i\tilde{y}_1^0 + \tilde{\eta}_2^0 + i\tilde{\eta}_1^0 + \omega(\tilde{y}_2^{(1)} + i\tilde{y}_1^{(1)}) + O(\omega^2)}$$

Fermi momentum k_F

$$\frac{{}_2F_1(1+\nu_{k_F}+\frac{iq\mu}{6}), \frac{1}{2}+\nu_{k_F}-\frac{\sqrt{2}q\mu}{3}, 1+2\nu_{k_F}, \frac{2}{3}(1-i\sqrt{2})}{{}_2F_1(\nu_{k_F}+\frac{iq\mu}{6}, \frac{1}{2}+\nu_{k_F}-\frac{\sqrt{2}q\mu}{3}, 1+2\nu_{k_F}, \frac{2}{3}(1-i\sqrt{2}))} = \frac{6\nu_{k_F}-iq\mu}{k_F(-2i+\sqrt{2})}$$

$$\nu_{k_F} = \frac{1}{6}\sqrt{6k_F^2-(q\mu)^2}$$

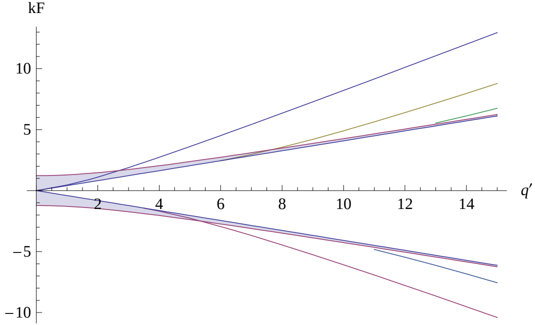

Figure: Fermi momentum k_F vs. charge of the fermion field $q' = \sqrt{3}q$. We set $r_* = 1$, $g_F = 1$, $\mu = \sqrt{3}$.

Wavefunction of a pairing mode Δ^0

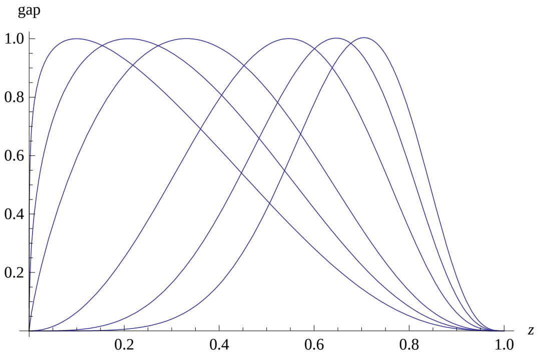

Figure: Wavefunction of a pairing mode $\Delta^0 = \psi^{0\dagger}\sigma^1\psi^0$ as a function of the radial coordinate z, with the horizon at $z = 0$ and the boundary at $z = 1$, for different values of the charge $q' = \sqrt{3}q$ for the first Fermi surface. We set $r_* = 1$, $g_F = 1$. From left to right the values of the charge are $q' = \{3, 3.4, 4, 6, 8, 10\}$.

Constant h_1 in the boundary Green function G

$$h_1 = \lim_{z \to 1} \frac{\tilde{y}_1^0 + i\tilde{y}_2^0}{\partial_k(\tilde{y_2^0} + i\tilde{y}_1^0)}$$

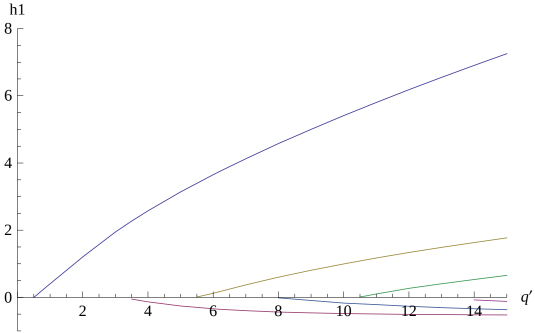

Figure: Constant h_1, reflecting the UV physics of the AdS_4 bulk, vs. charge $q' = \sqrt{3}q$. It vanishes at $\nu_{k_F} = 0$

Fermi velocity

$$v_F = \frac{1}{h_1}\left(\int_0^1 dz\sqrt{g/g_{tt}}\psi^{0\dagger}\psi^0\right)^{-1}\lim_{z\to 1}\frac{|\tilde{y}_1^0 + i\tilde{y}_2^0|^2}{(1-z)^3}$$

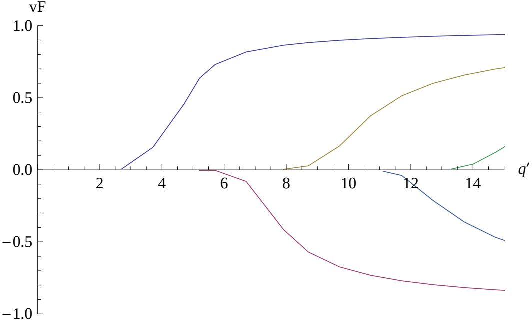

Figure: Constant v_F, reflecting the UV physics of the AdS_4 bulk, vs. charge $q' = \sqrt{3}q$. It vanishes at $\nu_{k_F} = \frac{1}{2}$. For the first Fermi surface it happens ($\nu_{k_F} = \frac{1}{2}$) at $q' = 2.71$

Critical temperature T_c

$$T_c = \frac{G_{int}|q\mathcal{H}|h_1^2 v_F^2}{\pi^2} \int dz \sqrt{-g}(\psi^0(z)^\dagger \sigma^1 \psi^0(z))^2$$

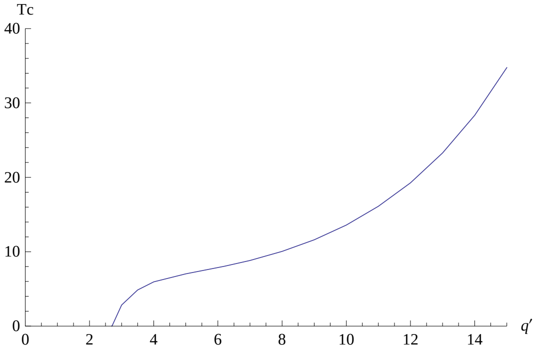

Figure: Critical temperature T_c vs. the charge $q' = \sqrt{3}q$ for the first Fermi surface.

Conformal dimension ν_{k_F}

$$\nu_{k_F} = \sqrt{\frac{k_F^2}{\sqrt{12}Q\sqrt{1+(H/Q)^2}} + \frac{\Delta^2}{6} - \frac{q^2 g_F^2}{12(1+(H/Q)^2)}} \qquad (3)$$

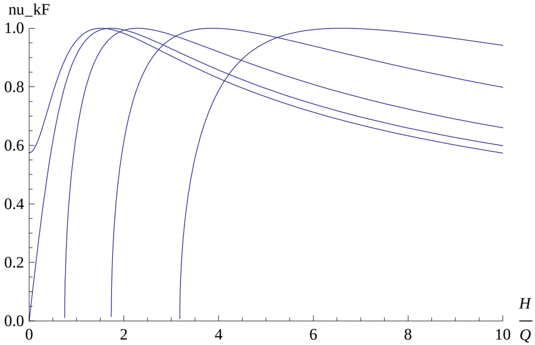

Figure: Conformal dimension ν_{k_F} as a function of the ratio $\frac{H}{Q}$ for different values for q'.

Breitenlochner-Freedman bound

$$\nu = \sqrt{\frac{1}{6}\left(\frac{2|q\mathcal{H}|l}{r_{**}^2} + m^2\right) - \frac{q^2 g_F^2}{12}\frac{r_*^4}{r_{**}^4} + \frac{1}{4}}$$

$$\Delta_\phi = \frac{3}{2} + \sqrt{m^2 + \left(\frac{3}{2}\right)^2}$$

$$q^2 g_F^2 \frac{r_*^4}{r_{**}^4} \geq 2\Delta_\phi(\Delta_\phi - 3) + 3 + \frac{4R^4|q\mathcal{H}|l}{r_{**}^2}$$

Conclusion

1. Modeling magnetic catalysis in the AdS_4 : pairing $\bar{\psi} i \Gamma^2 \Gamma^5 \psi$ is realized for the Fermi liquids, no paring for non-Fermi liquids – agrees with SC; for $\nu_{k_F} > \frac{1}{2}$, threshold $H > H_c$ (or $n < n_c$) to have $\Delta \neq 0$ and then $T_c \sim H$ – agrees with FT; for $\nu > \frac{1}{2}$ $\psi^{0\dagger} \sigma \psi^0$ is supported away from the horizon, for $\nu < \frac{1}{2}$ - near the horizon with $v_F \ll c$; BF bound signals $\bar{\psi} \Gamma \psi$ instability, easiest to achieve for LLL, important for σ – agrees with FT.
2. It may be possible to pair non-Fermi liquids. Solve Einstein-Maxwell-Scalar sector (with "free" scalars) to find a solution with nonzero $< \psi \Gamma \psi >$, the "hairy" black hole.
3. Chern-Simon $< J^{mu} = \frac{m}{2|m|} \frac{1}{4\pi} \varepsilon^{\mu\nu\rho} F_{\nu\rho} >$, the mass gap order in H may induce the charge in $(2+1)$-d and the electric current in the direction parallel to H in $(3+1)$-d – chiral magnetic effect.
4. Unified description of the Fermi and non-Fermi liquids. When the description using collective modes and order parameters break, the holographic approach may work.