P-odd Correlations in Quark Fragmentation

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P- and CP-odd Effects in Hot and Dense Matter
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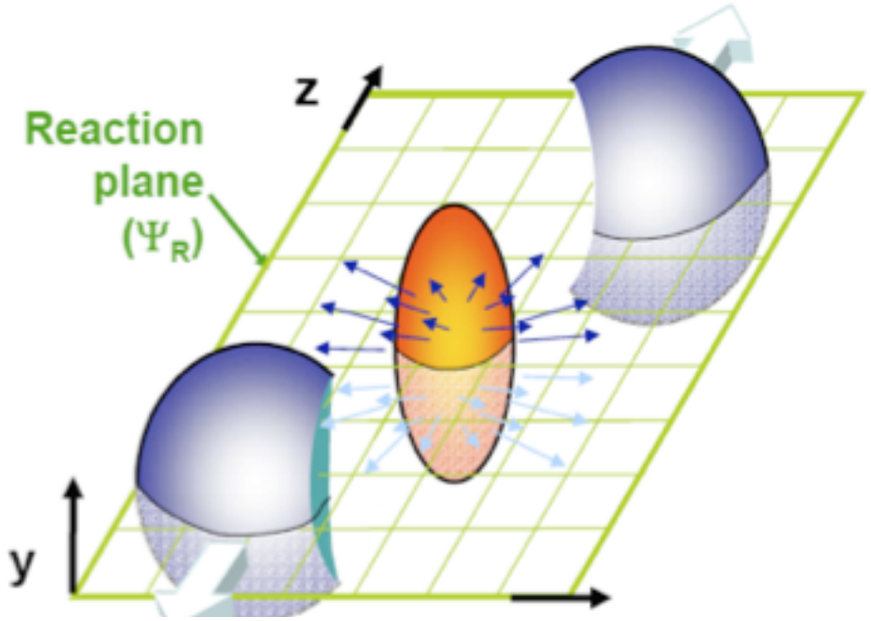
based on work with D. Kharzeev

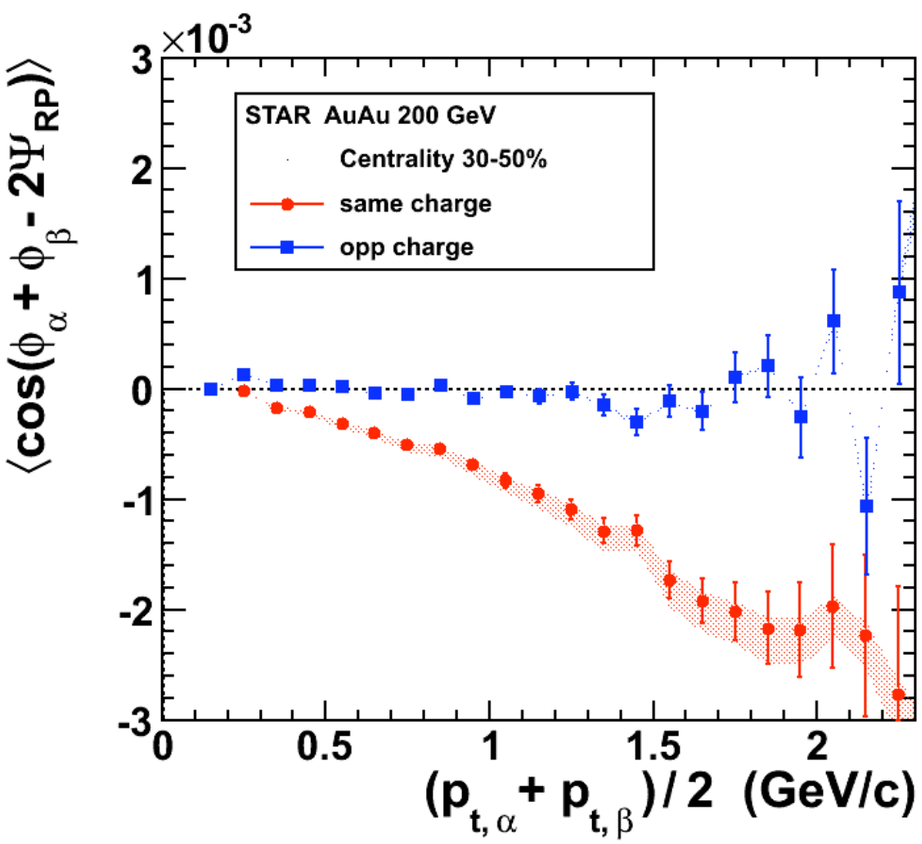
Outline

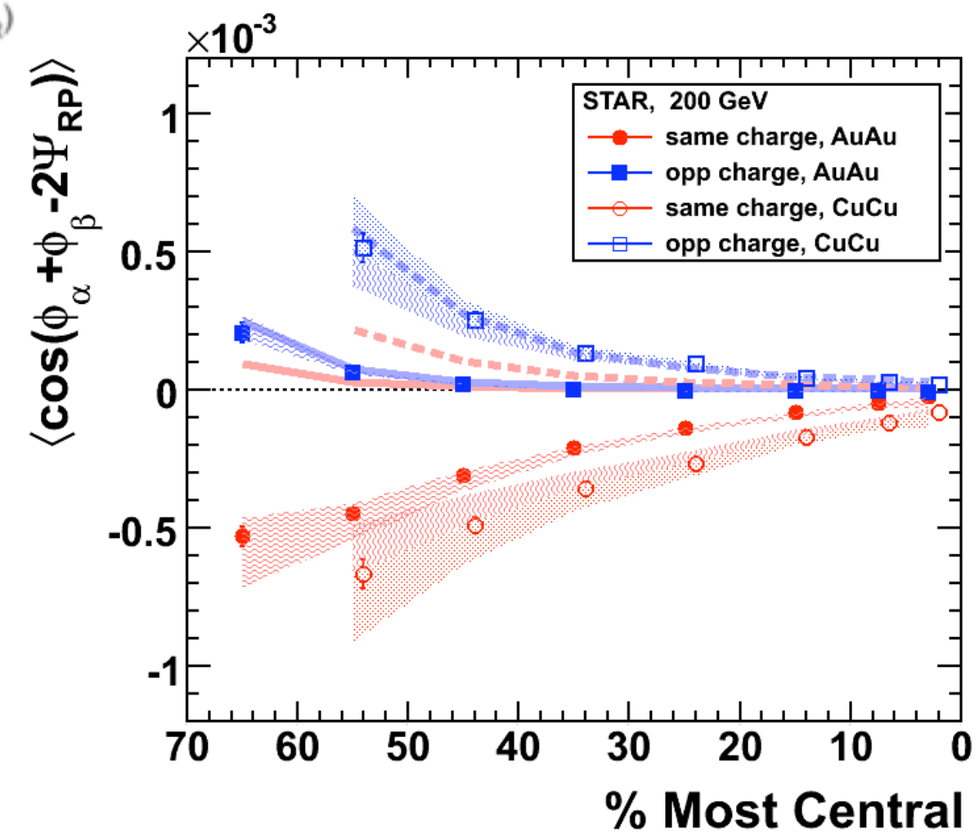
- Introduction
- Conventional picture: QCD factorization theorem
- How to incorporate P-odd effects in conventional picture
  - Operator definition for fragmentation functions
  - Generalize to P-odd fragmentation functions
- Relative size of P-odd fragmentation functions
- Observational effects
- Outlook

Charge separation observed in experiments

- Charge separation has been observed by both STAR and PHENIX





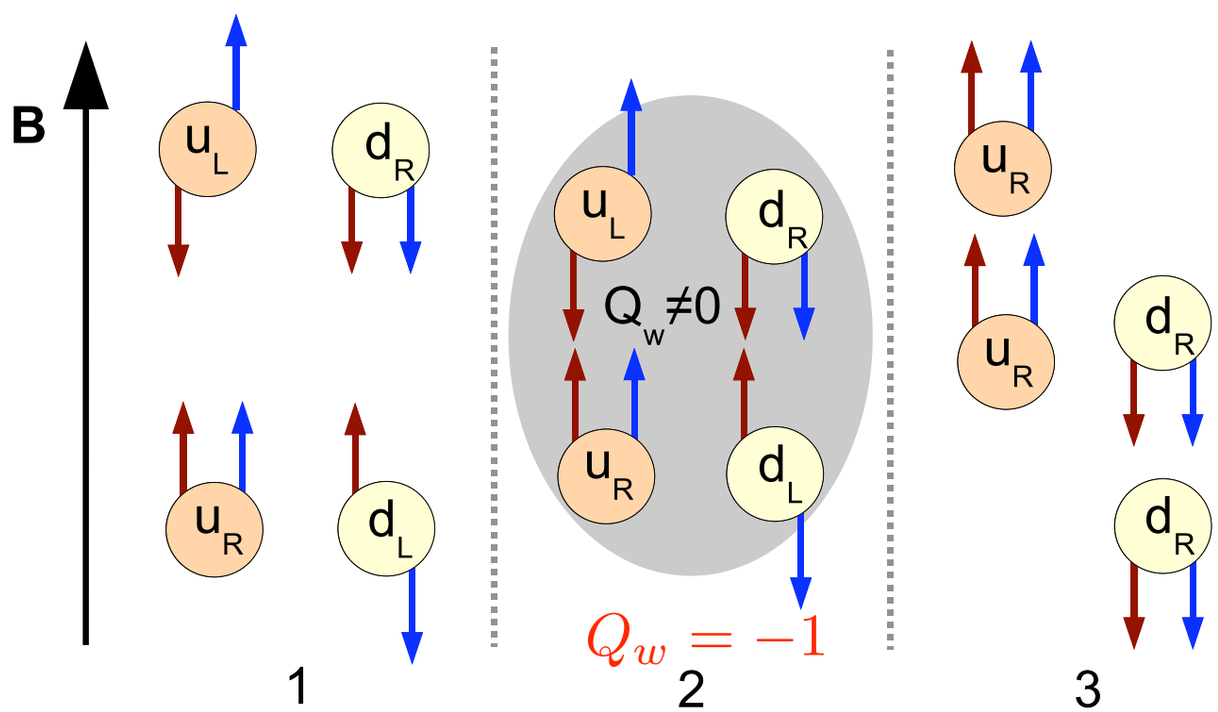


Chiral Magnetic Effect

- The fermions interact with a gauge field configuration with negative winding number Q_w (P-odd bubble or domain) will convert left-handed fermions to right-handed ones

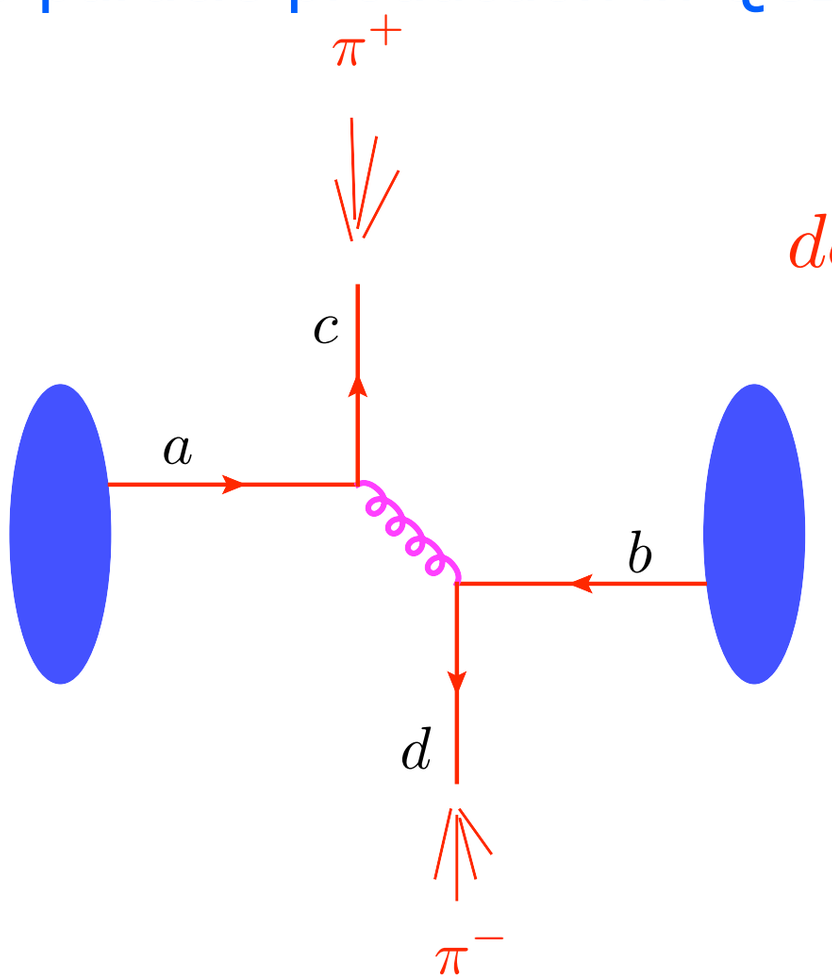
$$N_L^f - N_R^f = 2Q_w$$

  - More positively charged particles move parallel to magnetic field, negatively charged anti-parallel
  - P-odd effect



Particle production in QCD factorization picture

- Two particle production in QCD factorization picture

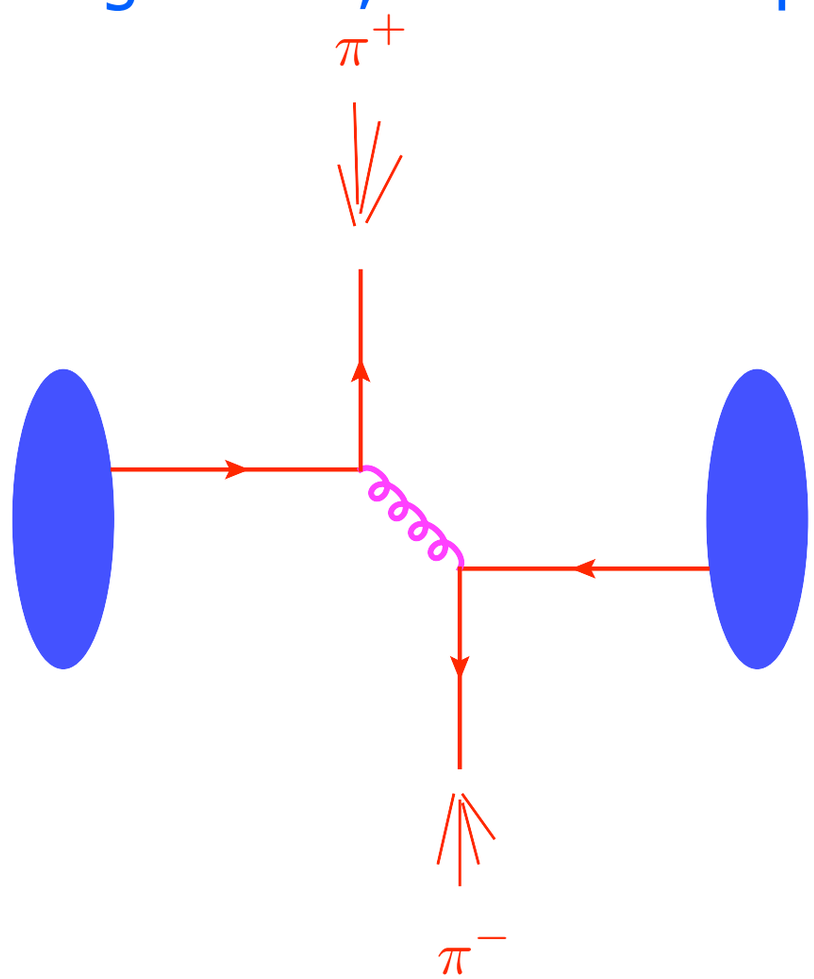


$$d\sigma_{\pi^+\pi^-} \quad \propto \quad \phi_{a/A} \otimes \phi_{b/B} \otimes H_{ab\to cd} \otimes D_{c\to\pi^+} \otimes D_{d\to\pi^-}$$

- QCD conserves C, P, T globally, QCD cross section is P-even
  - Partonic scattering conserves parity
  - All parton distribution functions, and fragmentation function are parity-even

Fragmentation inside P-odd bubble

- During the process while the outgoing parton penetrates non-trivial gauge field configuration, what will happen?



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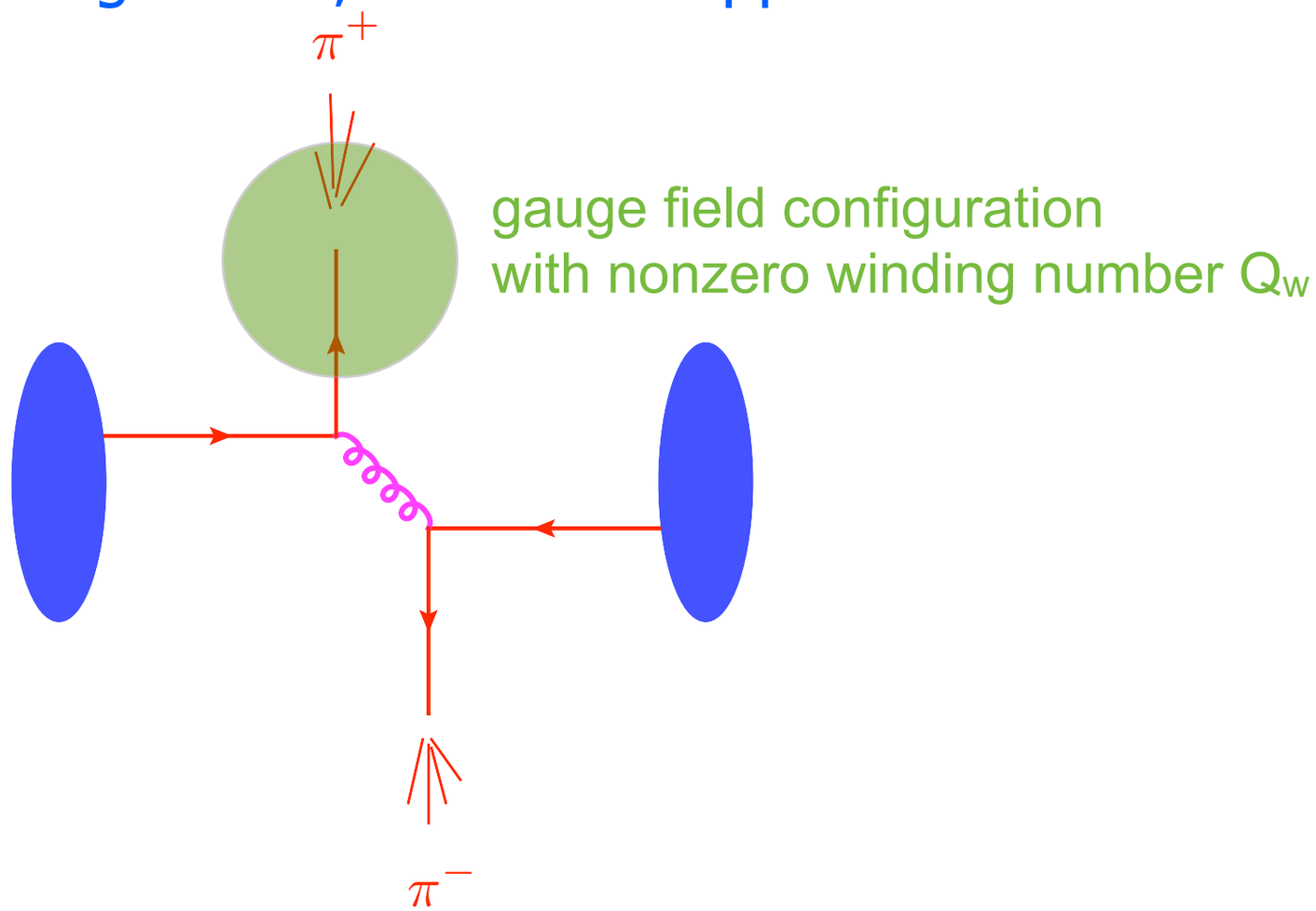

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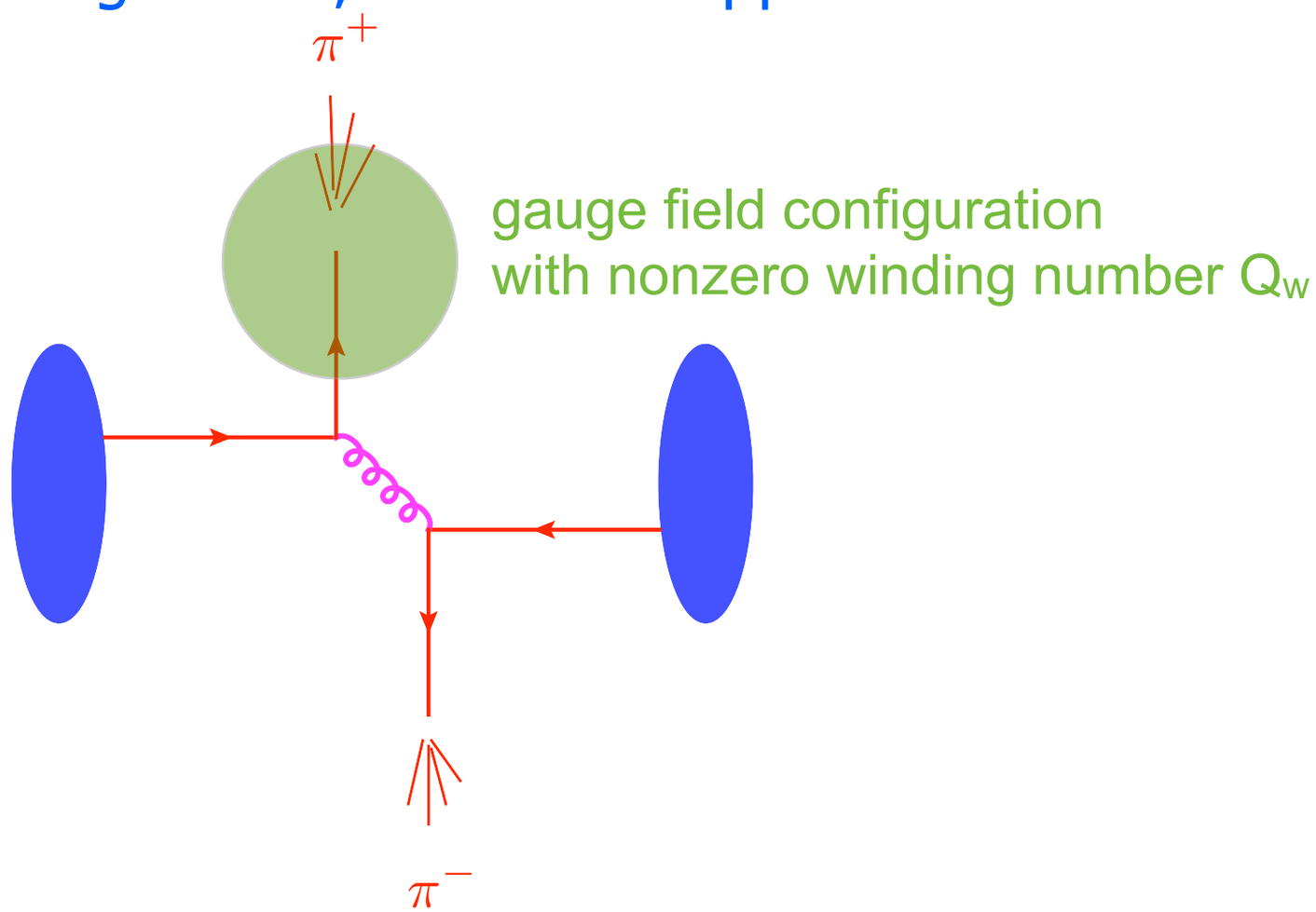
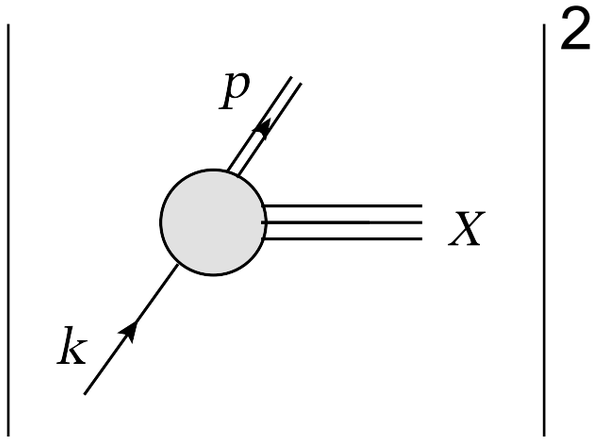


- P-odd modes in quark fragmentation could be populated

How is fragmentation function defined?

- Fragmentation function usually has a probability interpretation

$\Delta(p,k) =$



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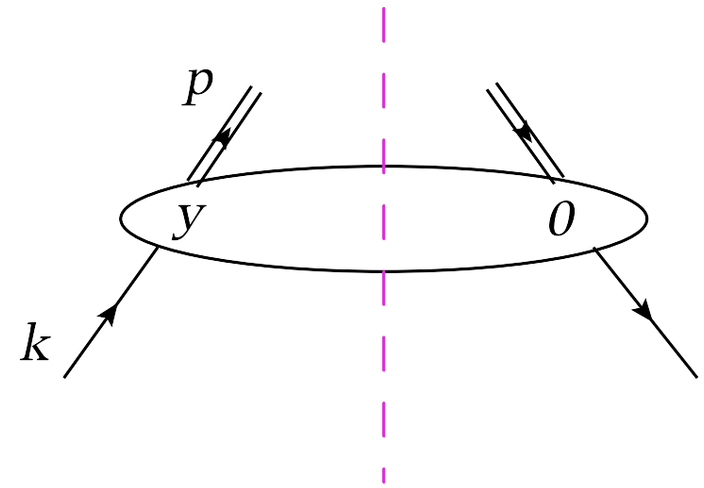


$$\Delta(p,k) \sim \int \frac{d^4y}{(2\pi)^4} e^{ik\cdot y} \langle 0|\psi(y)|PX\rangle\langle PX|\bar{\psi}(0)|0\rangle$$

- What are the usual parametrizations for fragmentation function?
  - Expand Δ(p, k) in spinor space
  - Impose certain constraints
  - Pick the leading projection or contribution

Fragmentation functions: parametrization (I)

- Δ(p, k) expansion in spinor space by using the Dirac basis matrices

$$\Gamma = \{1, \gamma^\mu, \gamma^\mu\gamma^5, \sigma^{\mu\nu}, i\gamma^5\}$$

  - Notice also Δ(p, k) is a Lorentz scalar, built up from two momenta p and k

$$\Delta(p,k) = \left[MA_1 1 + A_2 \not{p} + A_3 \not{k} + A_4 \sigma^{\mu\nu}\frac{k_\mu p_\nu}{M}\right] + \left[A_5 \not{p}\gamma^5 + A_6 \not{k}\gamma^5 + MA_7 i\gamma^5 + A_8 \sigma^{\mu\nu} i\gamma^5 \frac{k_\mu p_\nu}{M}\right]$$

  - M=hadron mass, used to make all A_i same dimension

- Constraints: Ralston & Soper, 79, Boer, Mulders & Pijlman, 03, ...
  - Hermiticity: $\Delta^\dagger(p,k) = \gamma^0 \Delta(p,k)\gamma^0$
  - Parity: $\Delta(p,k) = \gamma^0 \Delta(\bar{p},\bar{k})\gamma^0$ $\qquad \bar{p}^\mu = p_\mu = (p^0, -\vec{p})$
  - Time-reversal: $\Delta^*(p,k) = V_T \Delta(\bar{p},\bar{k}) V_T^{-1}$ $\qquad$ with $V_T = i\gamma^1\gamma^3$

Fragmentation functions: parametrization (II)

- How constraints affect the parametrization

$$\Delta(p,k) = \left[M A_1 1 + A_2 \not{p} + A_3 \not{k} + A_4 \sigma^{\mu\nu} \frac{k_\mu p_\nu}{M} \right] + \left[A_5 \not{p} \gamma^5 + A_6 \not{k} \gamma^5 + M A_7 i \gamma^5 + A_8 \sigma^{\mu\nu} i \gamma^5 \frac{k_\mu p_\nu}{M} \right]$$

  - Hermiticity: all A_i are real

$$A_i^* = A_i$$

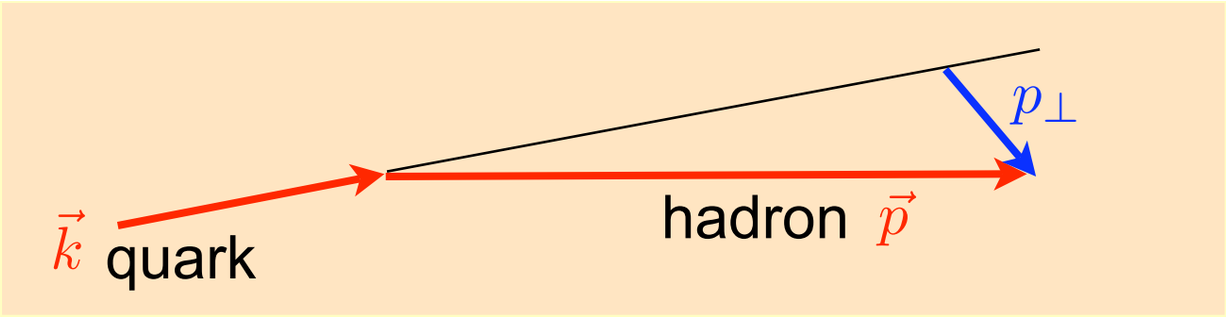
  - Parity: $A_5 = A_6 = A_7 = A_8 = 0$

  - Time-reversal doesn't give any further constraint since it changes out-state to in-state

Fragmentation functions: parametrization (III)

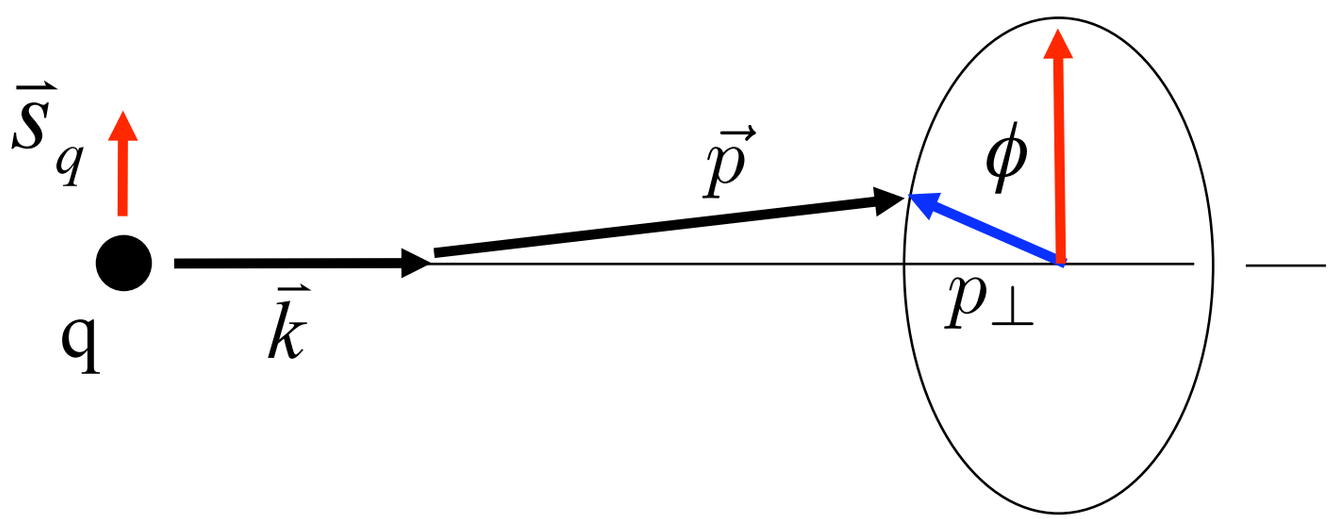
- Twist-expansion:

$$p^\mu \approx p^+ n^\mu \qquad k^\mu \approx (p^+ n^\mu - p_\perp^\mu)/z$$



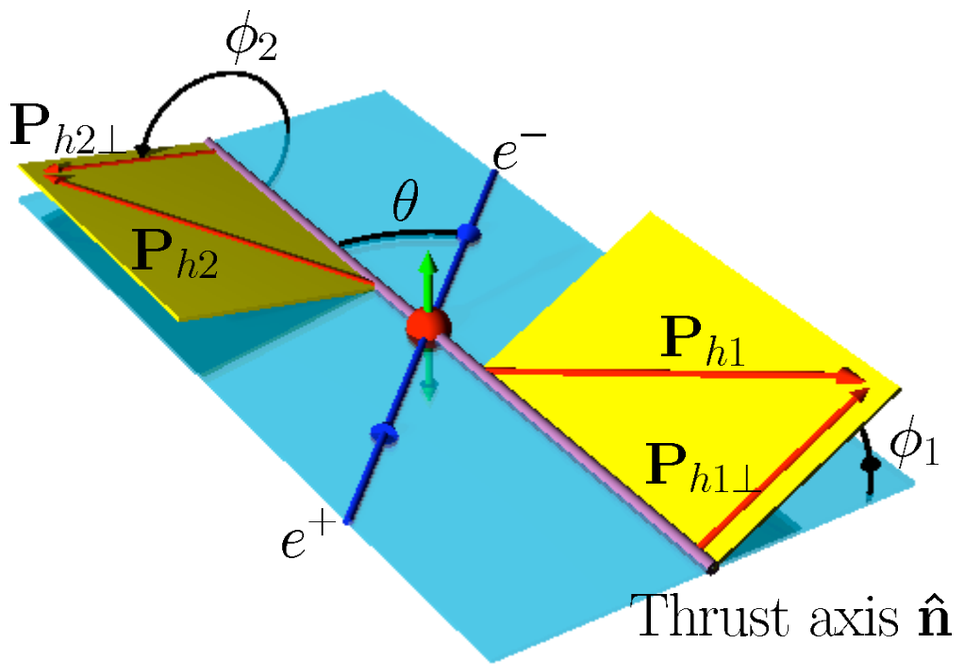
$$\Delta(z, p_\perp) \;=\; \frac{1}{2}\left[D(z, p_\perp^2)\not{n} + H_1^\perp(z, p_\perp^2)\sigma^{\mu\nu}\frac{p_{\perp\mu} n_\nu}{M}\right] + \mathcal{O}(M/p^+, p_\perp/p^+)$$

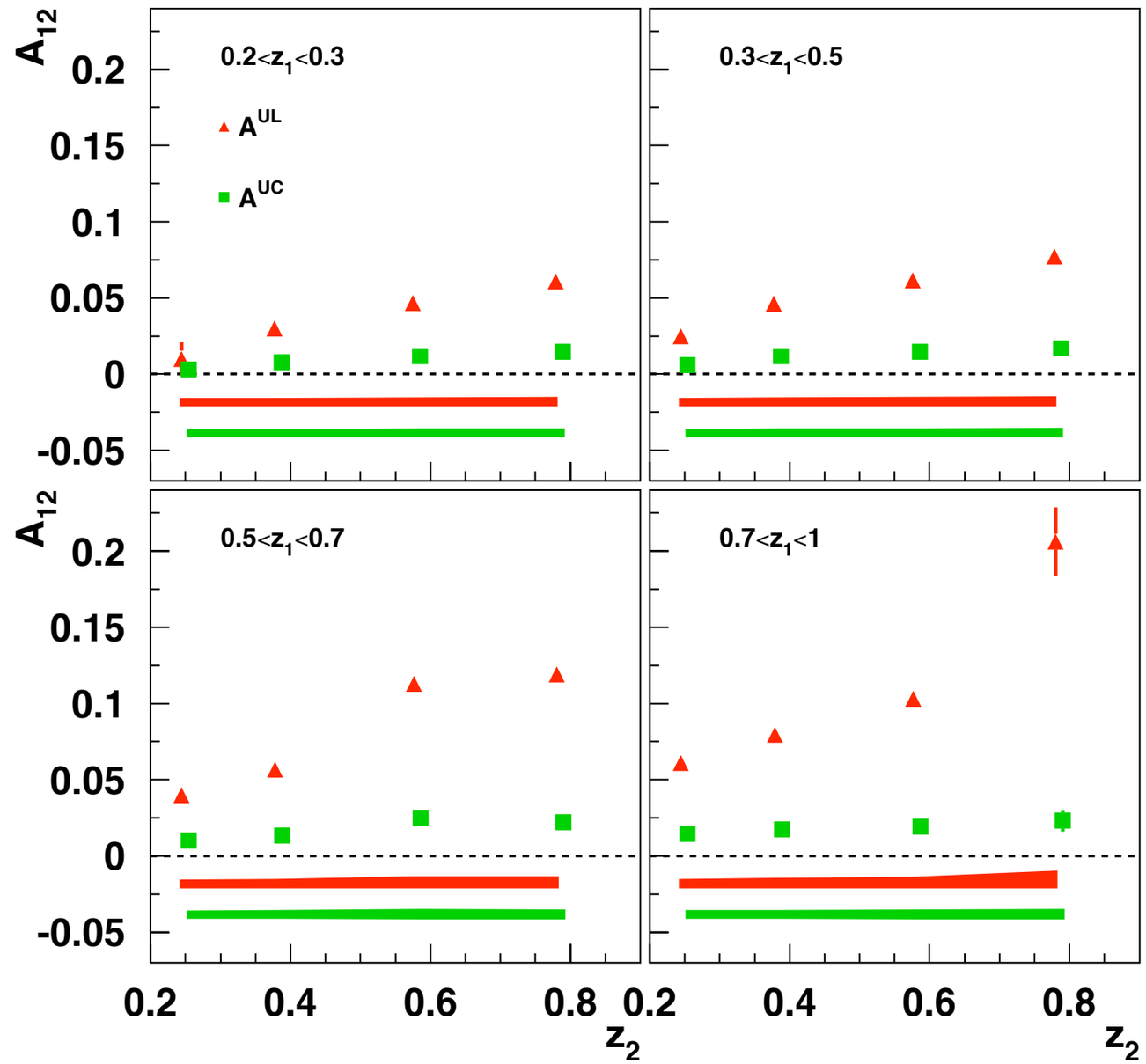
- $D(z, p_\perp^2)$ is unpolarized fragmentation function
- $H_1^\perp(z, p_\perp^2)$ is so-called Collins fragmentation function



Collins effect observed in HERMES, COMPASS, BELLE

- Collins effect observed by BELLE Collaboration: $\cos(\Phi_1+\Phi_2)$





PRL 96, 232002 (2006)
PRD 78, 032011 (2008)

Quark fragmentation in P-odd bubble

- If the fragmentation happens or is initiated in the P-odd bubble, then P-odd modes (or P-odd fragmentation functions) could be populated
  - Without parity constraint, A_5, A_6, A_7, and A_8 do not vanish any more

$$\Delta(p,k) = \left[M A_1 1 + A_2 \not{p} + A_3 \not{k} + A_4 \sigma^{\mu\nu} \frac{k_\mu p_\nu}{M}\right] + \left[A_5 \not{p}\gamma^5 + A_6 \not{k}\gamma^5 + M A_7 i\gamma^5 + A_8 \sigma^{\mu\nu} i\gamma^5 \frac{k_\mu p_\nu}{M}\right]$$

  - Twist-expansion again:

$$\Delta(z,p_\perp) = \frac{1}{2}\left[D(z,p_\perp^2)\not{n} + H_1^\perp(z,p_\perp^2)\sigma^{\mu\nu}\frac{p_{\perp\mu} n_\nu}{M}\right] + \frac{1}{2}\left[\widetilde{D}(z,p_\perp^2)\not{n}\gamma^5 + \widetilde{H}_1^\perp(z,p_\perp^2)\sigma^{\mu\nu} i\gamma^5 \frac{p_{\perp\mu} n_\nu}{M}\right]$$

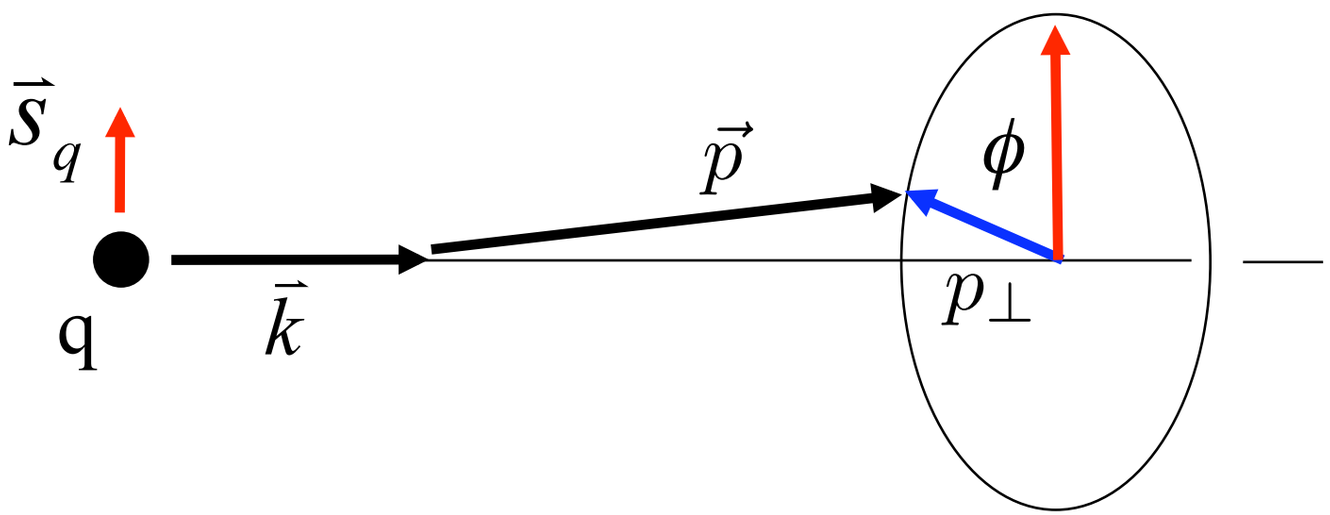
    - $D(z,p_\perp^2)$ and $H_1^\perp(z,p_\perp^2)$ are parity-even
    - $\widetilde{D}(z,p_\perp^2)$ and $\widetilde{H}_1^\perp(z,p_\perp^2)$ are parity-odd

Understanding the new fragmentation function

- A transversely polarized quark fragments into an unpolarized meson

$$D_{\pi/q^\uparrow}(z,p_\perp) = D(z,p_\perp^2) + H_1^\perp(z,p_\perp^2)\frac{(\hat{k}\times p_\perp)\cdot s_q}{M} + \widetilde{H}_1^\perp(z,p_\perp^2)\frac{p_\perp\cdot s_q}{M}$$

  - Illustration of Collins function: asymmetric azimuthal distribution w.r.t the momentum of the fragmenting quark

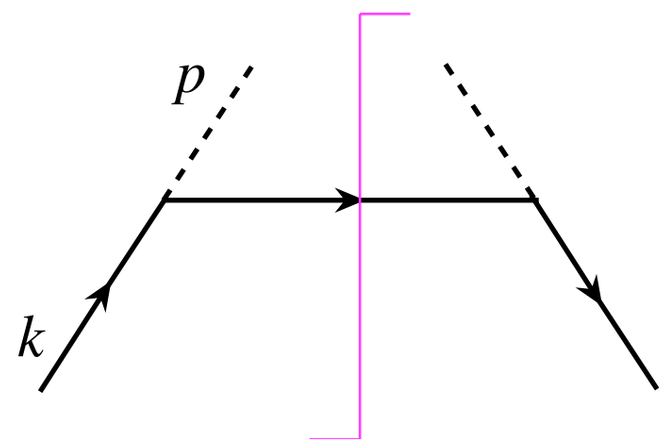


Estimate the size of the P-even fragmentation function

- Chiral quark model to estimate P-even fragmentation function
  - Point-like interaction between quark and pion meson: Manohar & Georgi, 82

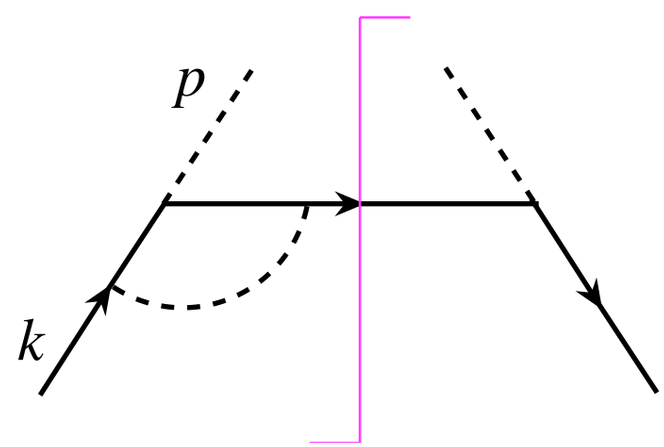
$$L_{qq\Pi} = -\frac{g_A}{2f_\pi}\bar{\psi}_q\gamma^\mu\gamma^5\vec{\tau}\cdot\partial_\mu\vec{\pi}\psi_q$$

  - Unpolarized FF can be calculated from the simple diagram



$$D(z,p_\perp^2) = \frac{g_A^2}{4f_\pi^2}\frac{1}{16\pi^3}\frac{1}{z}\left[1-\frac{4(1-z)z^2m_q^2m_\pi^2}{\left(p_\perp^2+z^2m_q^2+(1-z)m_\pi^2\right)^2}\right]$$

  - Similar for Collins FF (from pion loop diagrams):



$$H_1^\perp(z,p_\perp^2)$$

Estimate of the P-odd fragmentation functions - (I)

- $\widetilde{D}(z,p_\perp^2)$ and $\widetilde{H}_1^\perp(z,p_\perp^2)$ are P-odd, certainly can not exist in the normal Feynman diagram calculation
- They have to be generated by non-trivial gauge field configurations
  - The existence of gauge field configuration with nonzero winding number corresponds to add the following term in the Lagrangian

$$\frac{g^2}{32\pi^2}\theta(x,t)F_a^{\mu\nu}\tilde{F}_{\mu\nu}^a$$

  - By performing an axial U(1) rotation, it can be transformed into the fermionic contribution

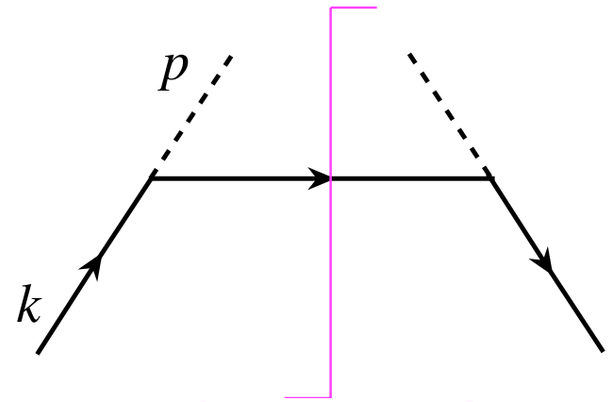
$$\frac{1}{2N_f}\partial_\mu\theta\bar{\psi}_q\gamma^\mu\gamma^5\psi_q \equiv \bar{\theta}_\mu\bar{\psi}_q\gamma^\mu\gamma^5\psi_q$$

  - Thus leads to a modification of the quark propagator Brown & Lee 78, Shuryak 82, ...

$$\begin{aligned} i\widetilde{S}(p,\bar{\theta}) &= \frac{i}{\not{p}-m+\not{\bar{\theta}}\gamma^5} \\ &= i\left[\mathcal{P}_\mathcal{R}S(p+\bar{\theta})+\mathcal{P}_\mathcal{L}S(p-\bar{\theta})\right]\left[1+m\gamma^5\left(S(p+\bar{\theta})-S(p-\bar{\theta})\right)\right] \\ &\quad\times\left[1+\frac{4m^2\bar{\theta}^2}{\left((p+\bar{\theta})^2-m^2\right)\left((p-\bar{\theta})^2-m^2\right)}\right]^{-1} \end{aligned}$$

Estimate of the P-odd fragmentation functions - (II)

- Quark propagation is changed when gauge field configurations exist
  - The P-odd fragmentation function can now be generated from the following simple diagram



  - When $\bar{\theta}^\mu$ along 0 or z direction, the size of the P-odd fragmentation function is suppressed by $\bar{\theta}/p^+$
  - Consider when $\bar{\theta}^\mu = \bar{\theta}_\perp \hat{p}_\perp^\mu$, to the leading order of $\bar{\theta}$ (neglect $\mathcal{O}\left(\bar{\theta}^2\right)$):

$$\widetilde{D}(z,p_\perp^2) = \frac{g_A^2}{4f_\pi^2}\frac{1}{16\pi^3}\frac{1}{z}\frac{4\bar{\theta}_\perp p_\perp}{p_\perp^2+z^2m_q^2+(1-z)m_\pi^2}\left[1-\frac{4(1-z)^2z^2m_q^2m_\pi^2}{\left(p_\perp^2+z^2m_q^2+(1-z)m_\pi^2\right)^2}-\frac{z}{2}\right]$$

$$\widetilde{H}_1^\perp(z,p_\perp^2) = \frac{g_A^2}{4f_\pi^2}\frac{m_q m_\pi}{8\pi^3}\frac{\bar{\theta}_\perp}{p_\perp}\frac{1}{\left(p_\perp^2+z^2m_q^2+(1-z)m_\pi^2\right)^3}$$
$$\times\left[(1-z)^2m_\pi^2\left[(3z-2)m_\pi^2-4(p_\perp^2-z^2m_q^2)\right]+\left(p_\perp^2+z^2m_q^2\right)^2(z-2)\right]$$

Relative size of the fragmentation functions

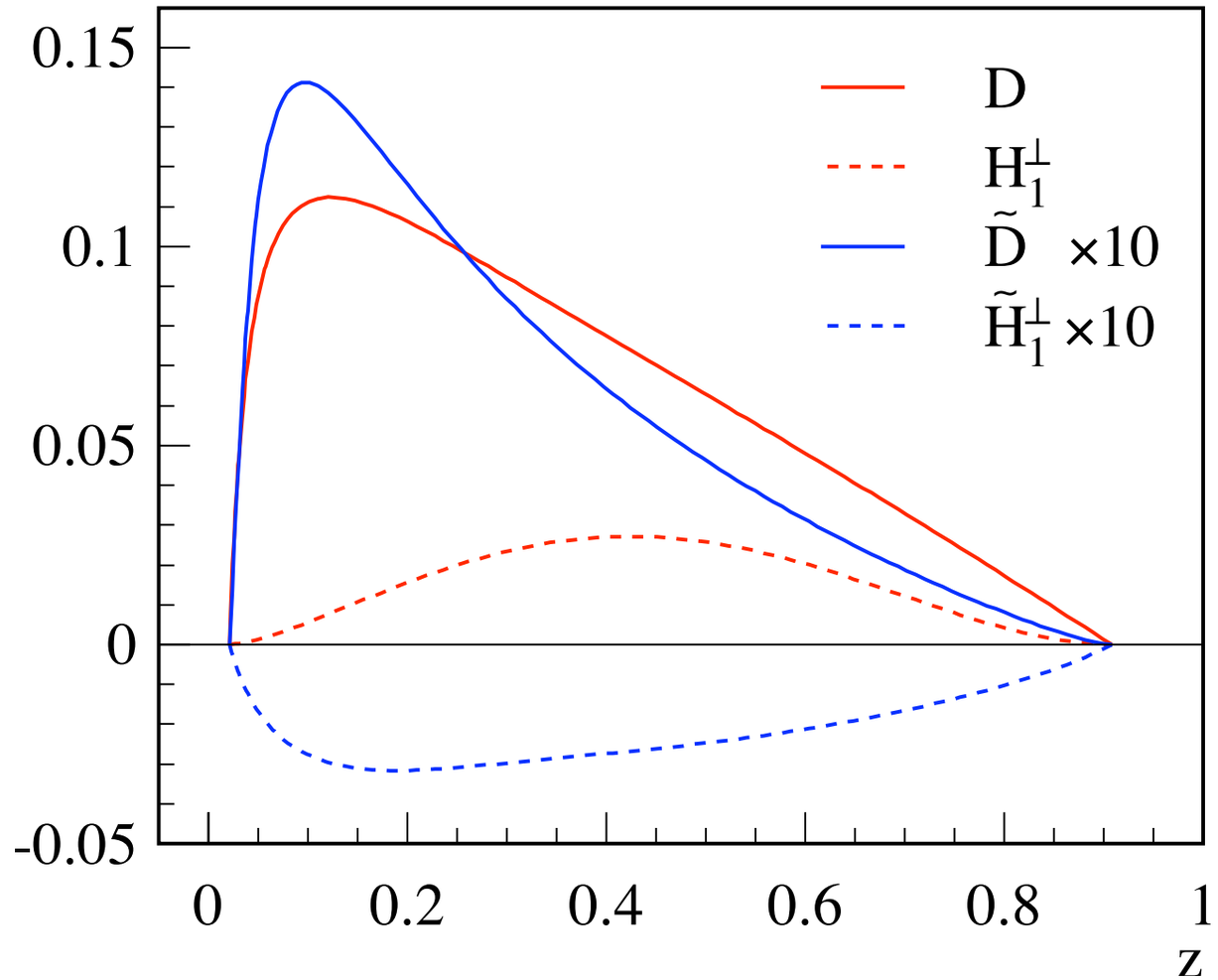
- The relative size of the P-odd and P-even fragmentation functions in chiral quark model

$$D(z) = \int^{\mu^2} d^2p_\perp D(z, p_\perp^2)$$

$$H_1^\perp(z) = \int^{\mu^2} d^2p_\perp \frac{|\vec{p}_\perp|}{M} H_1^\perp(z, p_\perp^2)$$

$$\widetilde{D}(z) = \int^{\mu^2} d^2p_\perp \widetilde{D}(z, p_\perp^2)$$

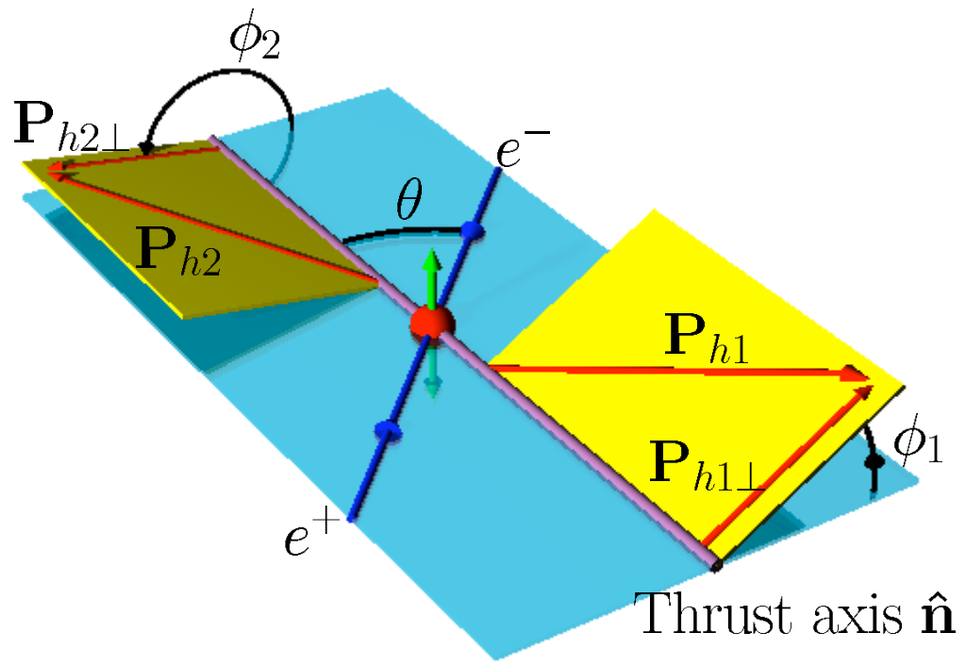
$$\widetilde{H}_1^\perp(z) = \int^{\mu^2} d^2p_\perp \frac{|\vec{p}_\perp|}{M} \widetilde{H}_1^\perp(z, p_\perp^2)$$



- $\bar{\theta}^\mu = \bar{\theta}_\perp \hat{p}_\perp^\mu$ with $\bar{\theta}_\perp = 10$ MeV
- scale for the fragmentation functions μ is chosen such that the virtuality of the fragmenting quark is equal to 1 GeV

Azimuthal correlation in $e^+e^- \to h_1 h_2 + X$ process

- The frame for di-hadron production in e^+e^- annihilation



- P-even case: Collins effect

$$d\sigma / dz_1 dz_2 d^2p_{1\perp} d^2p_{2\perp} d\cos\theta \sim \sigma_0 \sum_q e_q^2 \left\{ (1+\cos^2\theta) \left[D_q(z_1, p_{1\perp}^2) D_{\bar{q}}(z_2, p_{2\perp}^2) \right] \right.$$
$$\left. + \sin^2\theta \cos(\phi_1 + \phi_2) \frac{p_{1\perp} p_{2\perp}}{M_1 M_2} \left[H_1^{\perp q}(z_1, p_{1\perp}^2) H_1^{\perp \bar{q}}(z_2, p_{2\perp}^2) \right] \right\}$$

New azimuthal correlations in P-odd case

- The di-hadron cross section with both P-even and P-odd fragmentation functions
  - P-odd times P-odd gives P-even, thus the conventional P-even terms could receive contributions from the product of two P-odd fragmentation functions
  - P-odd alone generate new azimuthal correlations, which can only happen at the event-by-event

$$d\sigma/\,dz_1 dz_2 d^2p_{1\perp} d^2p_{2\perp} d\cos\theta \sim$$

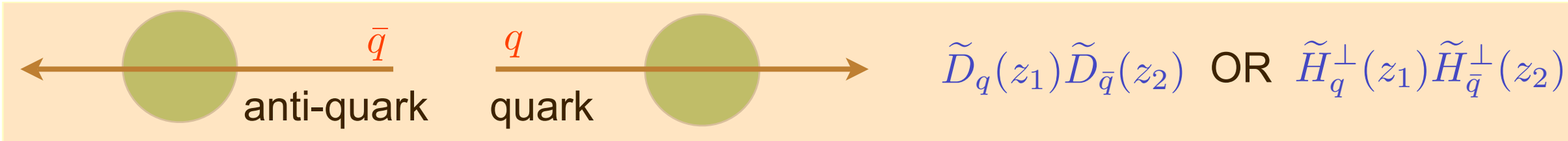
$$\sigma_0 \sum_q e_q^2 \Big\{ (1+\cos^2\theta)\left[D_q(z_1,p_{1\perp}^2)D_{\bar q}(z_2,p_{2\perp}^2) - \widetilde{D}_q(z_1,p_{1\perp}^2)\widetilde{D}_{\bar q}(z_2,p_{2\perp}^2)\right]$$
$$+\sin^2\theta\cos(\phi_1+\phi_2)\frac{p_{1\perp}p_{2\perp}}{M_1M_2}\left[H_1^{\perp q}(z_1,p_{1\perp}^2)H_1^{\perp\bar q}(z_2,p_{2\perp}^2) + \widetilde{H}_1^{\perp q}(z_1,p_{1\perp}^2)\widetilde{H}_1^{\perp\bar q}(z_2,p_{2\perp}^2)\right]$$
$$+\sin^2\theta\sin(\phi_1+\phi_2)\frac{p_{1\perp}p_{2\perp}}{M_1M_2}\left[H_1^{\perp q}(z_1,p_{1\perp}^2)\widetilde{H}_1^{\perp\bar q}(z_2,p_{2\perp}^2) - \widetilde{H}_1^{\perp q}(z_1,p_{1\perp}^2)H_1^{\perp\bar q}(z_2,p_{2\perp}^2)\right]\Big\}$$

Integrate over the moduli of transverse momenta

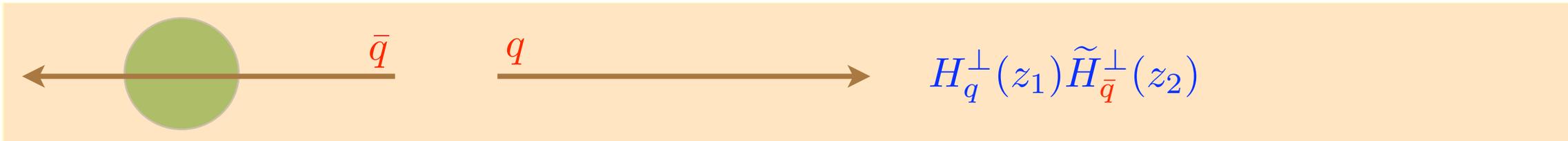
- The final azimuthal correlations

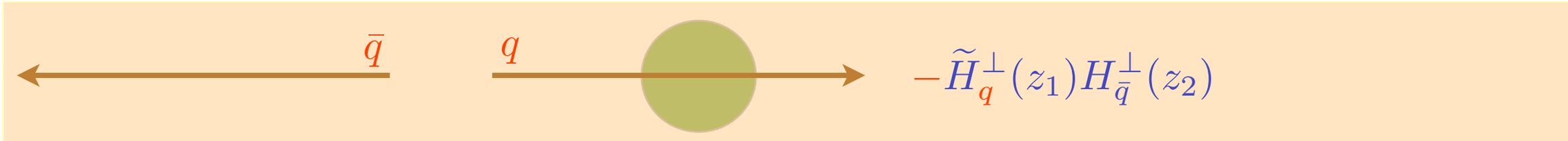
$$\frac{d\sigma}{dz_1 dz_2 d\cos\theta d(\phi_1+\phi_2)} = \sigma_0 \sum_q e_q^2 \Big\{ (1+\cos^2\theta)\left[D_q(z_1)D_{\bar q}(z_2) - \widetilde{D}_q(z_1)\widetilde{D}_{\bar q}(z_2)\right]$$
$$+\sin^2\theta\cos(\phi_1+\phi_2)\left[H_q^\perp(z_1)H_{\bar q}^\perp(z_2) + \widetilde{H}_q^\perp(z_1)\widetilde{H}_{\bar q}^\perp(z_2)\right]$$
$$+\sin^2\theta\sin(\phi_1+\phi_2)\left[H_q^\perp(z_1)\widetilde{H}_{\bar q}^\perp(z_2) - \widetilde{H}_q^\perp(z_1)H_{\bar q}^\perp(z_2)\right]\Big\}$$

- Physical pictures:
  - P-odd times P-odd terms:



  - P-odd term alone:





Size of event-by-event correlation

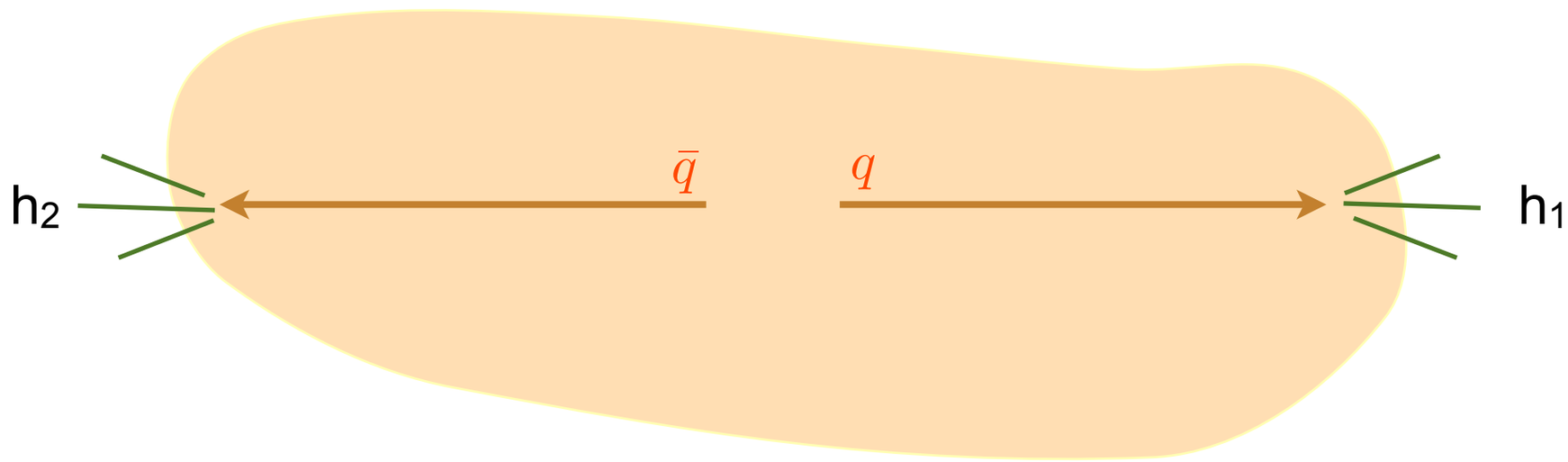
- Besides the kinematic factor $\sin^2\theta/(1+\cos^2\theta)$, the P-odd sin(Φ_1+Φ_2) correlation is controlled by the following factor

$$\frac{H_q^{\perp}(z_1)\widetilde{H}_{\bar{q}}^{\perp}(z_2)}{D_q(z_1)D_{\bar{q}}(z_2)-\widetilde{D}_q(z_1)\widetilde{D}_{\bar{q}}(z_2)}$$

  - It certainly depends the size of $\bar{\theta}^\mu = \partial^\mu\theta(x,t)$, which we have set along $\hat{p}_\perp^\mu$ such that $\bar{\theta}^\mu = \bar{\theta}_\perp \hat{p}_\perp^\mu$
  - Choose $\bar{\theta}_\perp \sim 10MeV$, we find the above factor gives around ~ 1-2% for $\langle z_1 \rangle \sim \langle z_2 \rangle \sim 0.5$ (typical for BELLE experiment)

Two hadrons fragment inside the same bubble

- So far we still consider two hadrons fragment independently, thus QCD factorization still holds
- If instead two hadrons fragment in the same bubble



- No longer independent fragmentation, final two hadrons know the whole history of the dynamics, the QCD factorization picture breaks down
- Further investigation needed and ongoing

Summary and outlook

- We generalize the current quark fragmentation functions to include P-odd modes
- The newly derived P-odd fragmentation functions lead to new azimuthal correlations
  - P-odd times P-odd contributions survive when averaging over different events
  - P-odd correlation alone leads to observable which exists only on event-by-event basis
- Independent fragmentation (even with P-odd FF) still follow the conventional wisdom of QCD factorization
  - There could be situation where QCD factorization breaks down
- How this formalism could be used to describe the charge separation is under the study

Summary and outlook

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- The newly derived P-odd fragmentation functions lead to new azimuthal correlations
  - P-odd times P-odd contributions survive when averaging over different events
  - P-odd correlation alone leads to observable which exists only on event-by-event basis
- Independent fragmentation (even with P-odd FF) still follow the conventional wisdom of QCD factorization
  - There could be situation where QCD factorization breaks down
- How this formalism could be used to describe the charge separation is under the study

Thank You!