Chiral and deconfinement transition in finite temperature QCD

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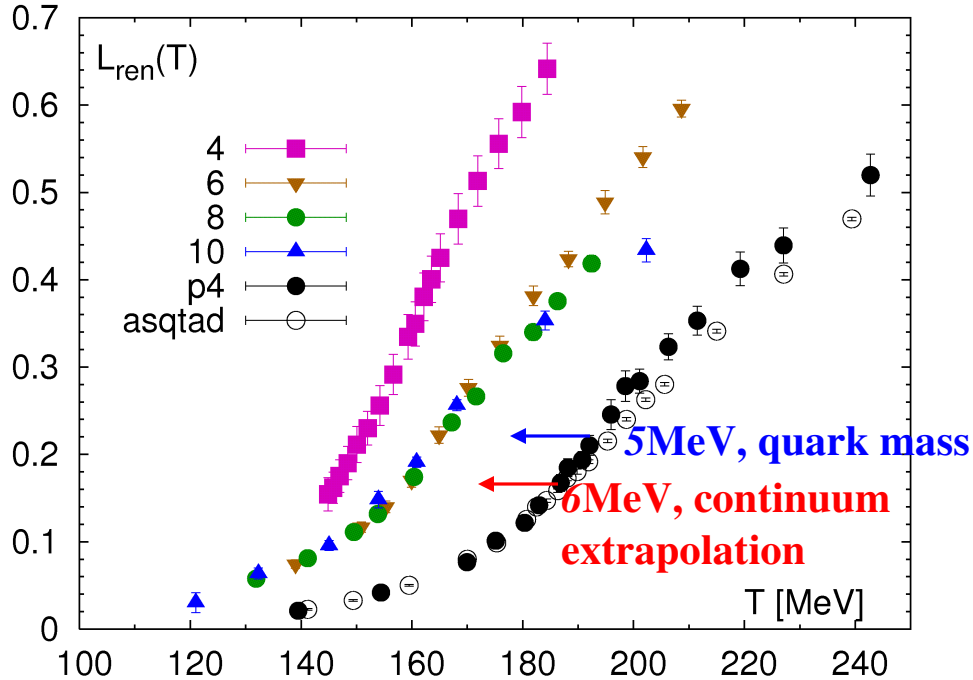
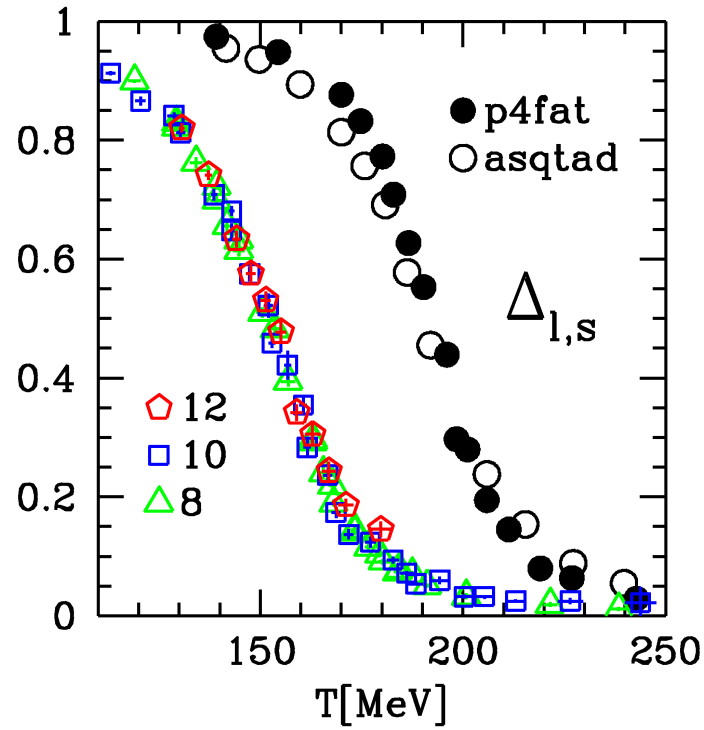
- Introduction :
  - ✓ Deconfiniment in high T QCD
  - ✓ A solved problem : SU(3) gauge theory
  - ✓ Chiral and deconfinement transition at T>0: lattice results and controversy
- Lattice calculations with HIGHLY IMPROVED STAGGERED QUARK (HISQ)
- Chiral aspect of the QCD transition and universal scaling
- Deconfinement transition in $T>0$ QCD, fluctuations of conserved charges and HRG
- Summary

DM 2010, Stellenbosch Institute for Advenced Studies, April 7, 2010

Deconfinement and chiral transition

stout : Budapest-Wuppertal Group, Aoki et al., PLB 643 (06) 46; arXiv:0903.4155





Renormalized Polyakov loop

$$L_{ren} = \exp(-F_Q(T)/T)$$

Renormalized chiral condensate

$$\Delta_{s,l}(T) = \frac{\langle\bar{q}q\rangle_T - \frac{m_q}{m_s}\langle\bar{s}s\rangle_T}{\langle\bar{q}q\rangle_{T=0} - \frac{m_q}{m_s}\langle\bar{s}s\rangle_{T=0}}$$

no qualitative change, but significant shift of the transition region toward smaller T

stout action is optimized to reduce the effect of flavor symmetry breaking, but not the quark the quark dispersion relation

Deconfinement and chiral transition

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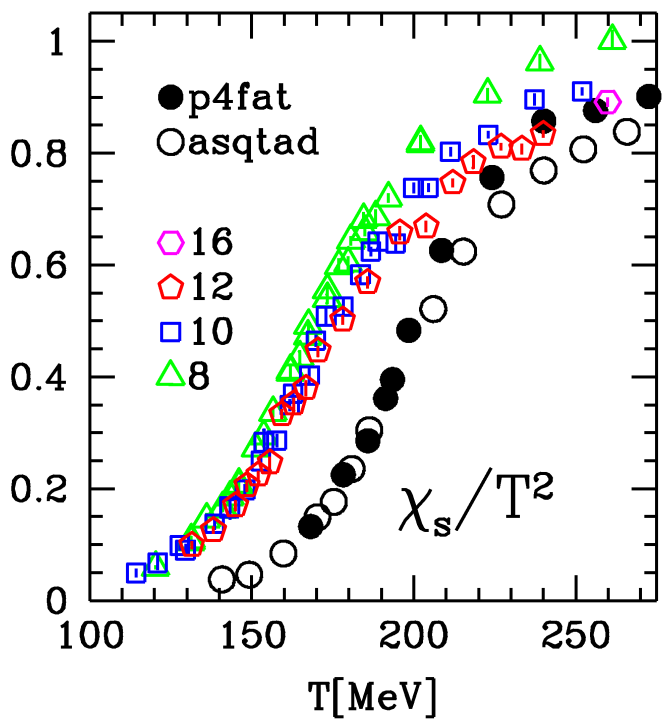
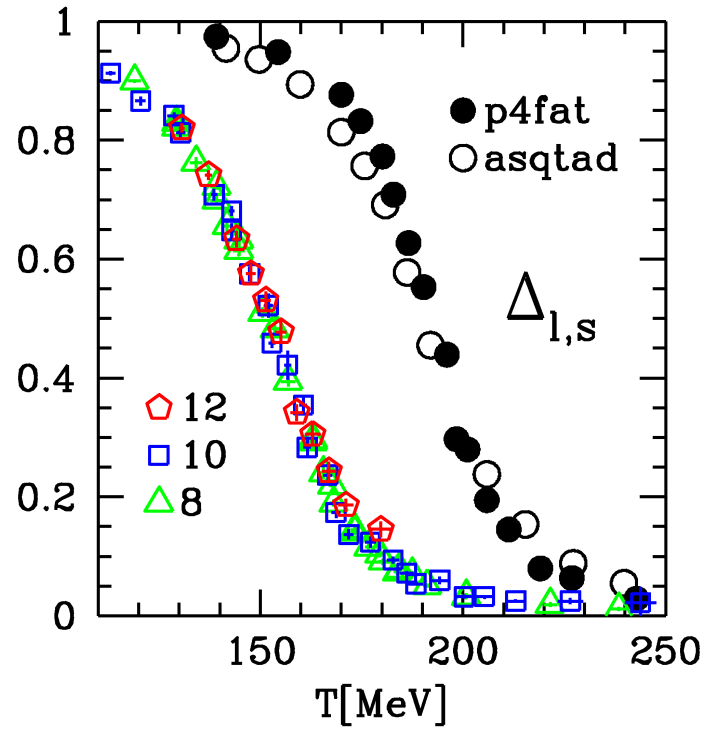


Strangeness fluctuations:

$$\chi_s/T^2 = < S^2 > - < S >^2$$



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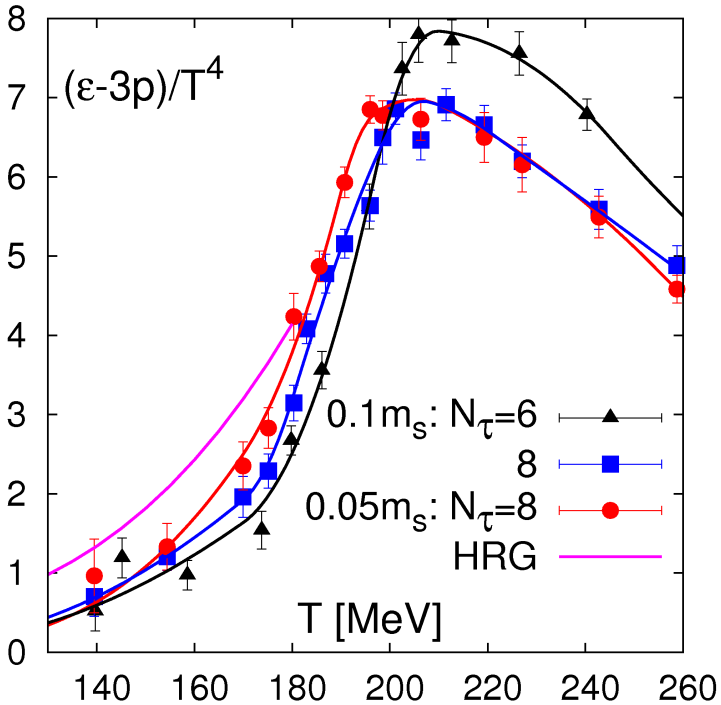
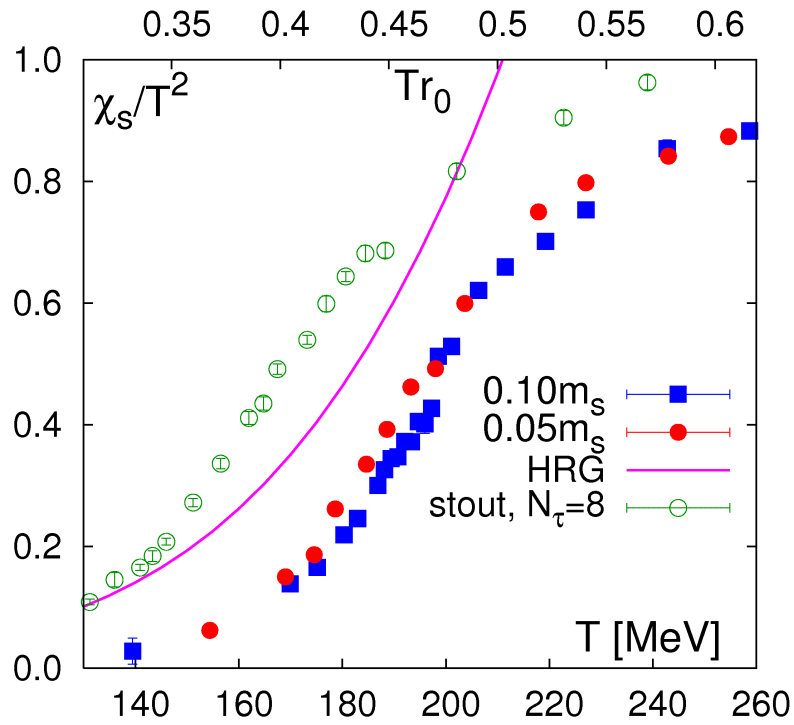
agreement between HotQCD results and Budapest-Wuppertal results at high T

Lattice results for "physical" quark masses

Lattice calculations at the physical quark mass and $N_\tau=8$, Cheng et al, arXiv:0911.2215





- Thermodynamics quantities are quark mass independent for $T>200$MeV
- The quark mass effect is also small at low temperature and is similar to cutoff effects
- Lattice results are significantly below the Hadron Resonance Gas

Improved staggered calculations at finite temperature

low T region
T<200 MeV

cutoff effects are different in :

high-T region
*T>200*MeV

$\mathcal{O}(\alpha_s^n(a\Lambda_{QCD})^2)$ errors

$a = 1/(TN_\tau)$

$\mathcal{O}((aT)^2)$ errors

a>0.125fm

$N_\tau = 8$

a<0.125fm

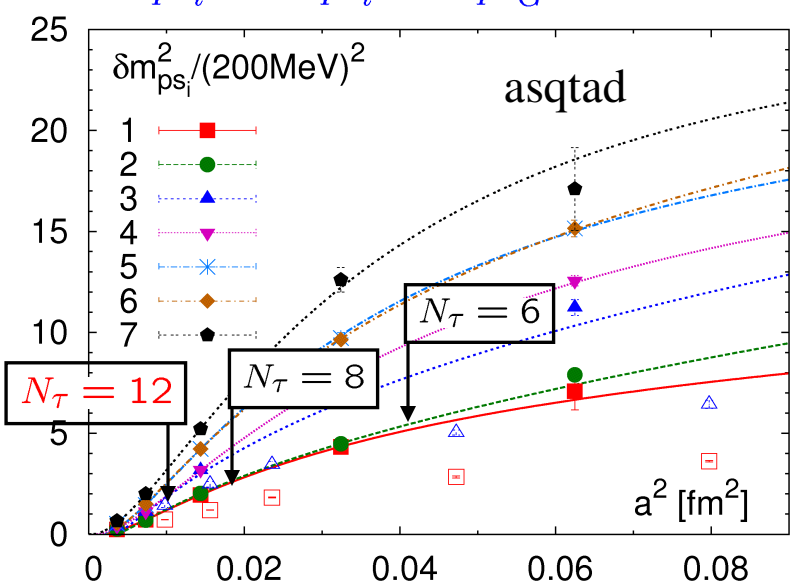
hadronic degrees of freedom

improvement of the flavor symmetry is important → fat links

quark degrees of freedom

quark dispersion relation

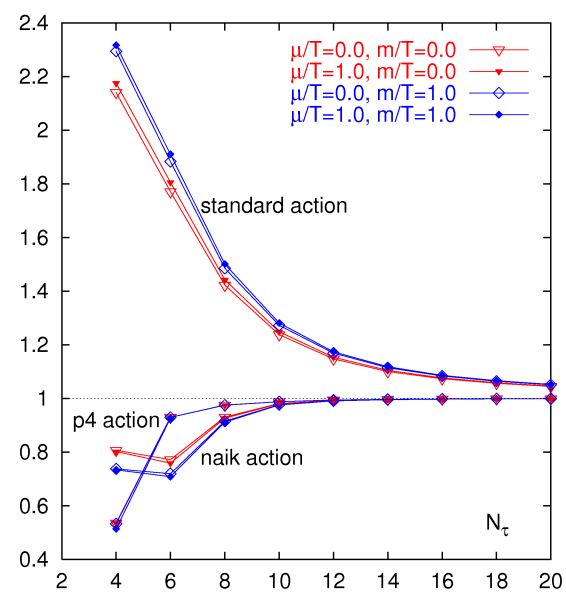
$$\delta m_{ps_i}^2 = m_{ps_i}^2 - m_{ps_G}^2 \sim \alpha_s a^2$$



stout

for #flavors < 4
rooting trick

$$detD \rightarrow (detD)^{\frac{n_f}{4}}$$



p4*, *asqtad*, *HISQ

The Highly Improved Staggered Quark (HISQ) Action

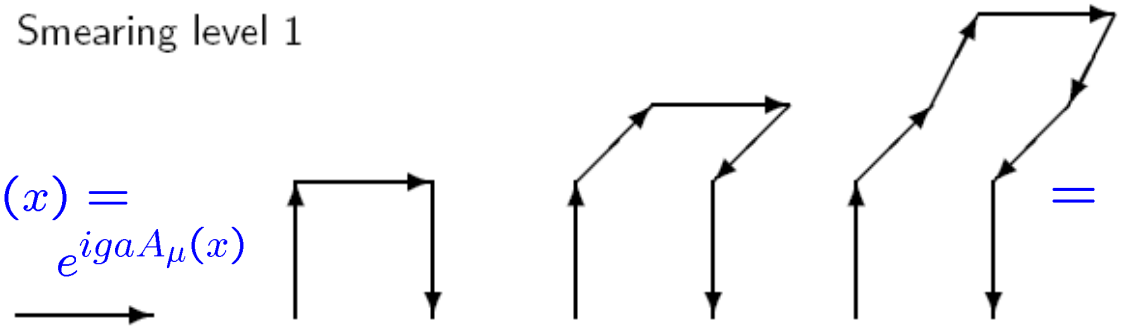
HISQ action

two levels of gauge field smearing with re-unitarization

Follana et al, PRD75 (07) 054502

Smearing level 1

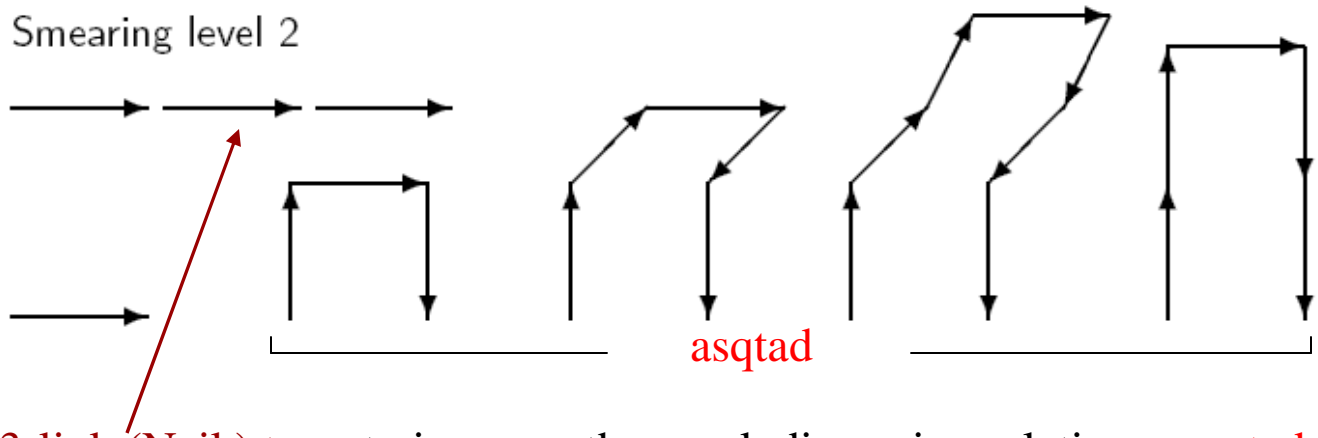
$U_\mu(x) = e^{igaA_\mu(x)}$



$= U_\mu^{fat7} \rightarrow \tilde{U}_\mu$

$= \frac{U_\mu^{fat7}}{\sqrt{U_\mu^{fat7} U_\mu^{fat7\dagger}}}$

Smearing level 2



projection onto U(3) improves flavor symmetry

Hasefratz, arXiv:hep-lat/0211007

3-link (Naik) term to improve the quark dispersion relation + asqtad smearing

T=0 calculations with HISQ action

$0.2m_s$, N_τ=6

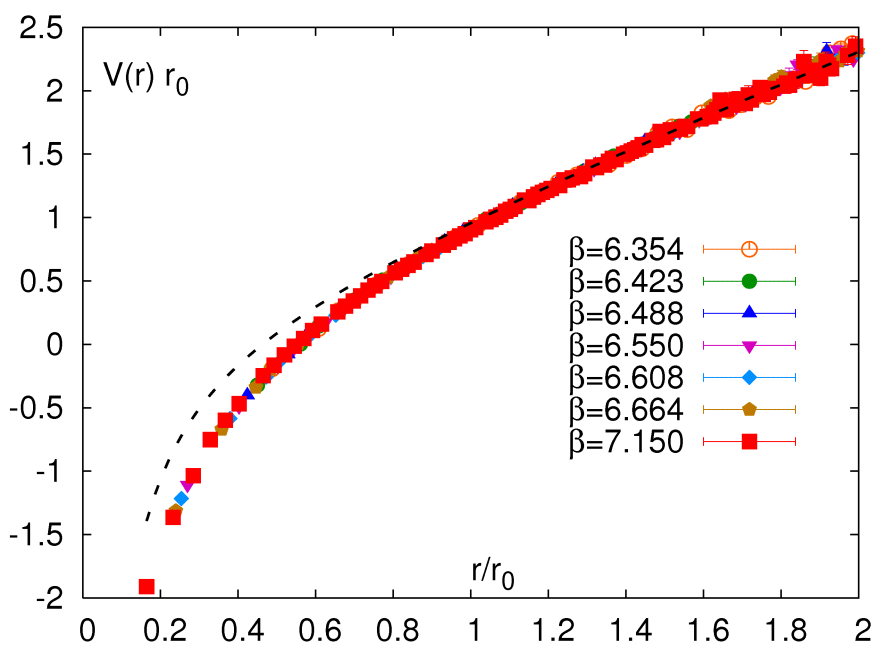
$m_\pi = 306 - 312$ MeV
$m_K = 522 - 532$ MeV
$m_\rho = 850 - 883$ MeV
$m_N = 1130 - 1183$ MeV

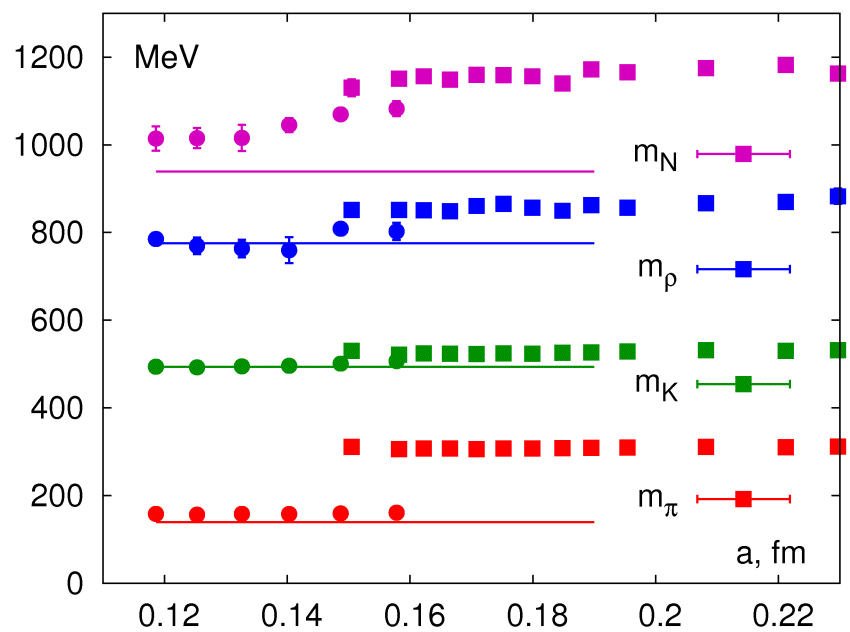
$0.05m_s$, N_τ=8

$m_\pi = 156 - 161$ MeV
$m_K = 493 - 501$ MeV
$m_\rho = 760 - 808$ MeV
$m_N = 1014 - 1083$ MeV

The lattice spacing is set through the calculation of static potential

$$\left(\frac{dV_{q\bar{q}}(r)}{dr}\right)_{r=r_0} = 1.65, \;\; r_0 = 0.469(7) fm$$

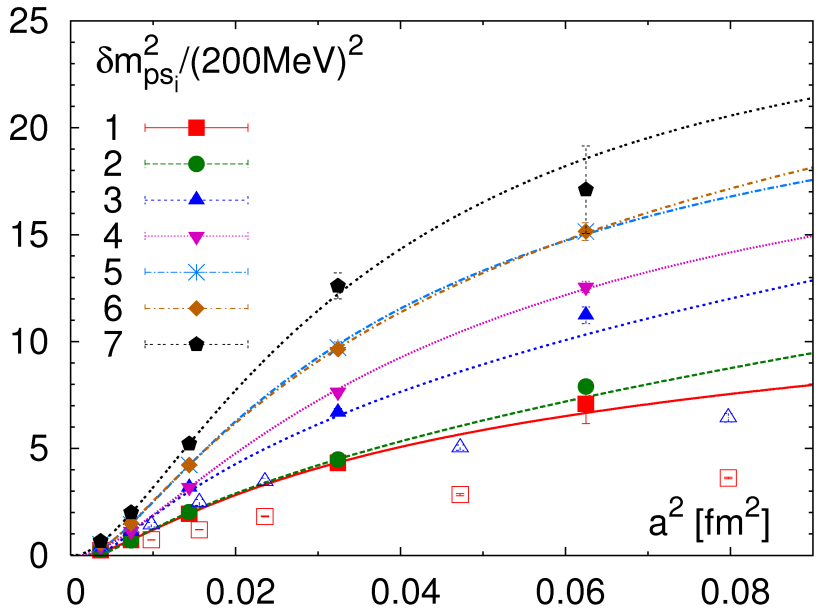




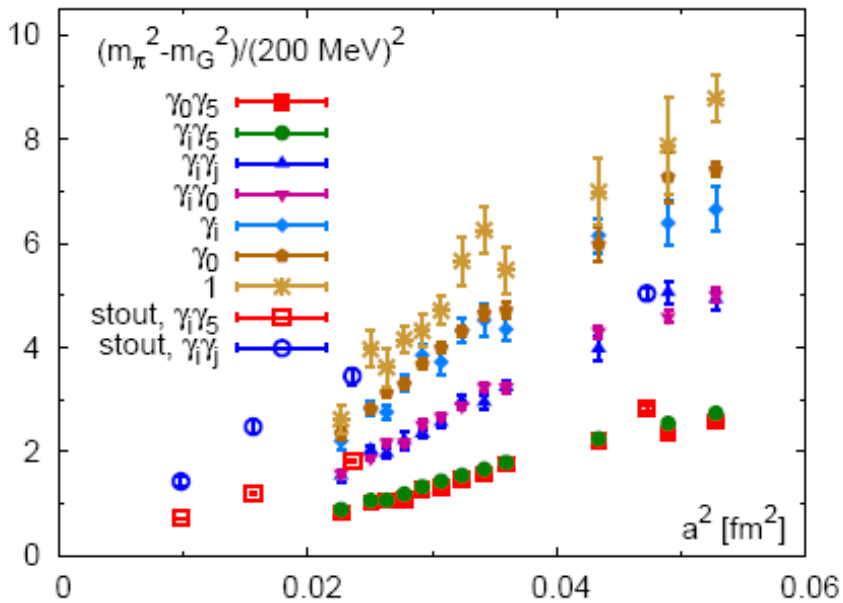
Mass splitting of pseudo-scalar mesons

Only one out of 16 PS mesons has zero mass in the chiral limit, the quadratic mass splitting is the measure of flavor symmetry breaking

asqtad



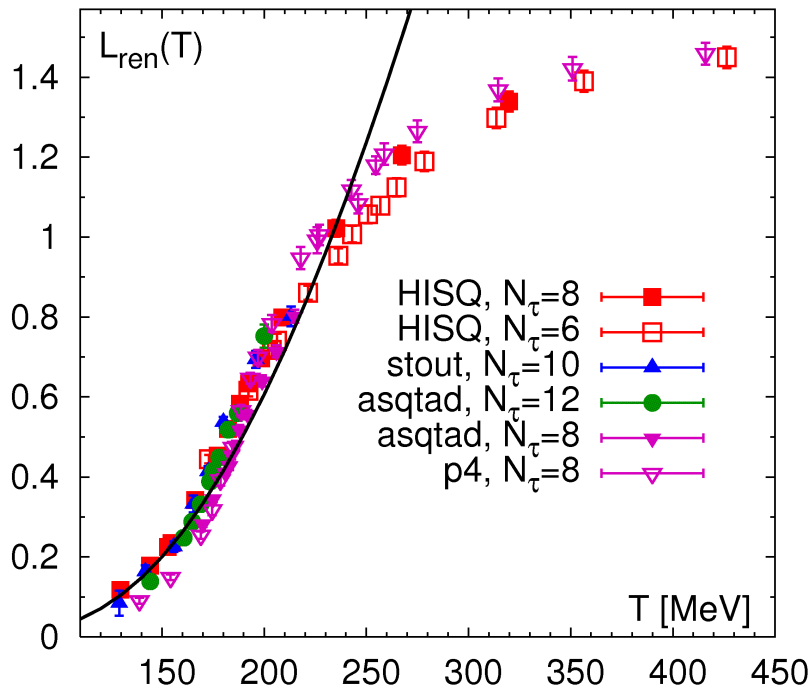
HISQ

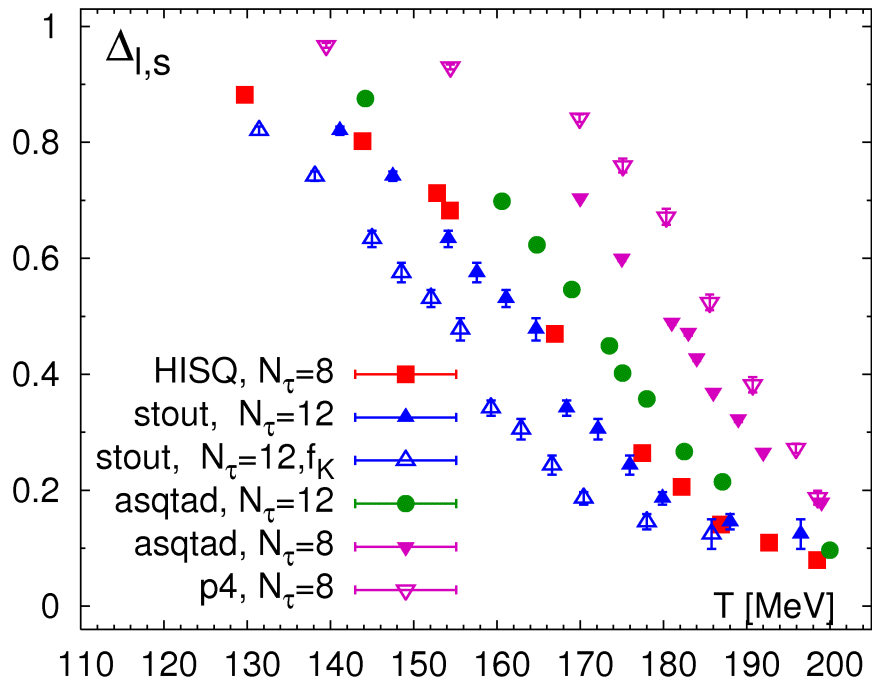


PS meson splittings in HISQ calculations are reduced by factor ~ 2.5 compared to asqtad at the same lattice spacing and are even smaller than for stout action => discretizations effects

for $N_\tau=8$ HISQ calculations are similar to those in $N_\tau=12$ asqtad calculations

Deconfinement and chiral transition





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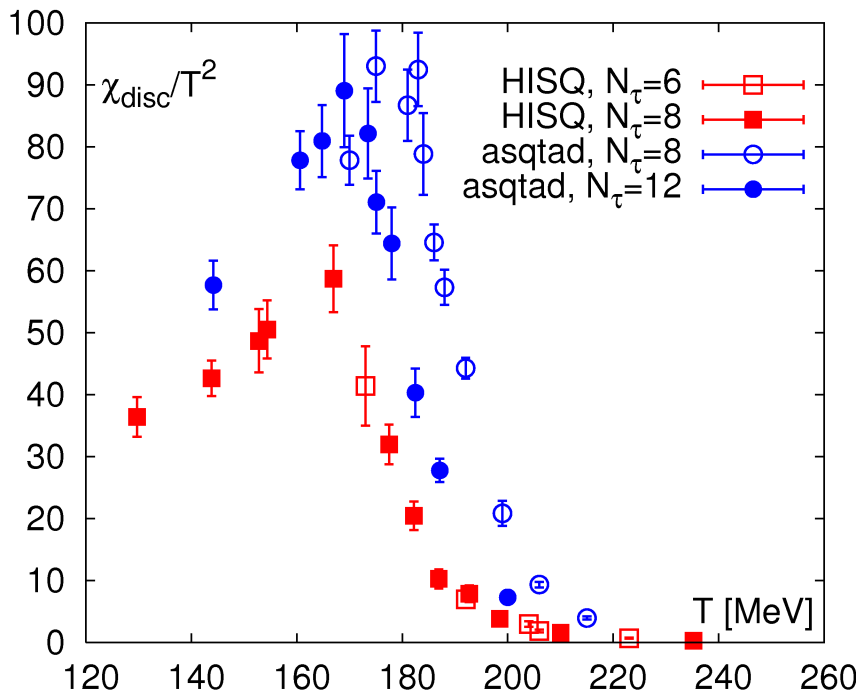
- Good agreement between stout and HISQ calculation at low T
- At high T HISQ results agree well with p4 results

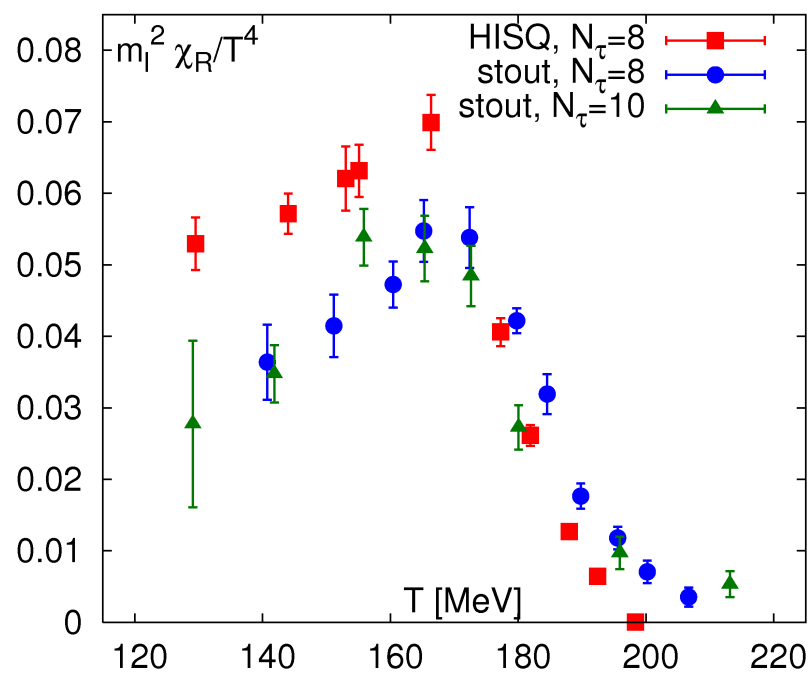
Chiral condensate susceptibility

$$\chi_l(T) = \frac{\partial \langle \psi\psi \rangle_l}{\partial m_l} = \frac{\partial^2 \ln Z(T)}{\partial m_l^2}$$
$$= \langle (\bar{\psi}\psi)_l^2 \rangle - \langle \bar{\psi}\psi \rangle_l^2 + 2\langle \mathrm{Tr} D_l^{-2} \rangle$$
$$= \chi_{l,disc} + 2\chi_{conn}$$

$$\chi_r(T) = \frac{m_l^2}{T^4}(\chi_l(T) - \chi_l(T=0)) = \frac{m_l^2}{T^4}\chi_R$$

RG invariant, but vanishes in the chiral limit





significant shift of the peak compared to N_τ=8 calculations

$T_c \approx 170 MeV$ both for stout and HISQ if r_0 is used to set the scale

Chiral condensate susceptibility

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$$\chi_r(T) = \frac{m_l^2}{T^4}\left(\chi_l(T) - \chi_l(T=0)\right) = \frac{m_l^2}{T^4}\chi_R$$

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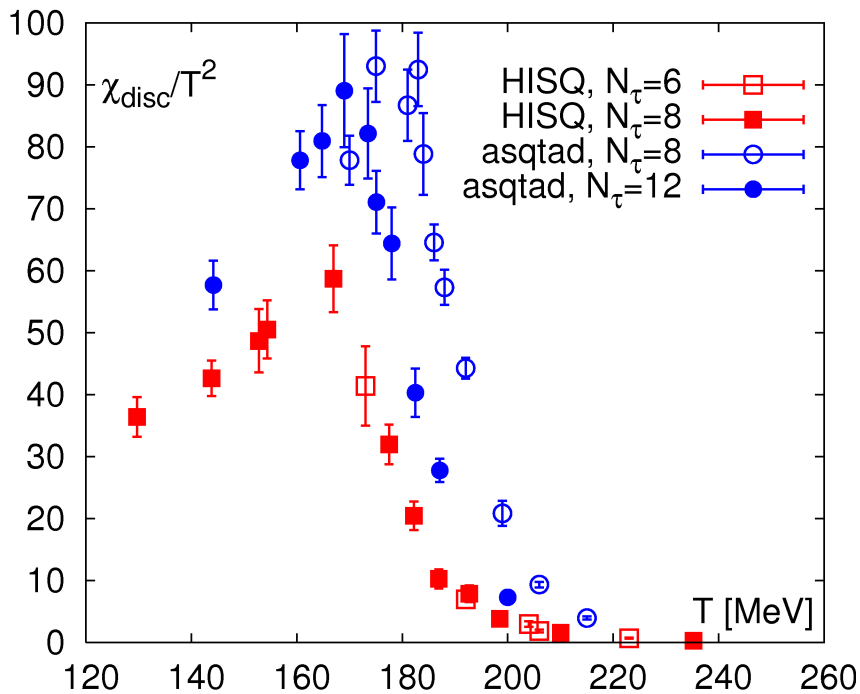
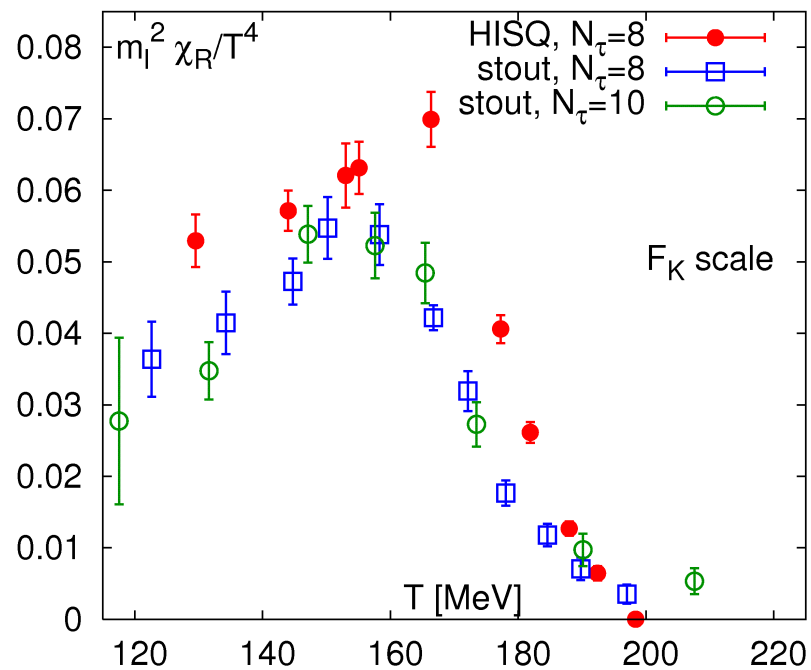




significant shift of the peak compared to N_τ=8 calculations

the peak position shifts to smaller T if f_K is used to set the scale

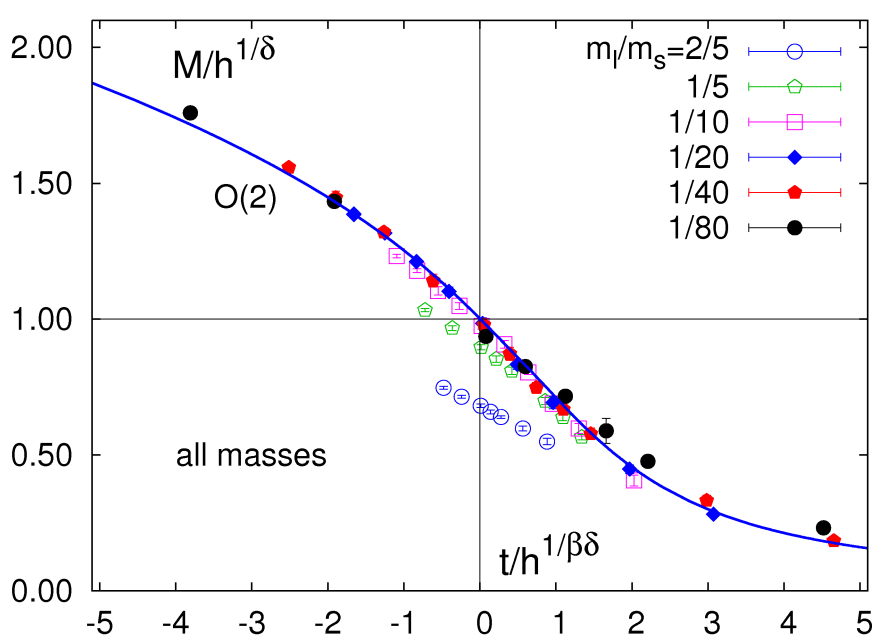
Chiral transition and universal scaling

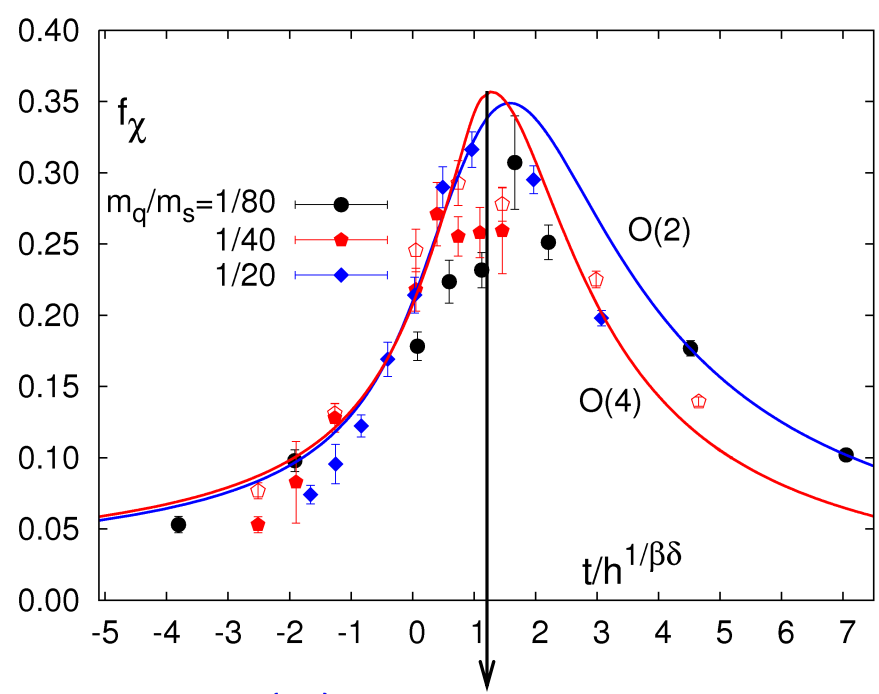
For sufficiently heavy strange quark and small light quark masses the chiral transition is governed by universal O(4) (O(2))scaling :

$$f(T, m_l, m_s) = f_{reg}(T, m_l, m_s) + f_s(t, h), \; t = \frac{1}{t_0}\frac{T - T_c}{T_c}, \; H = m_l/m_s, h = H/h_0$$

$$M = -\frac{\partial f_s(t,h)}{\partial H} = h^{1/\delta} f_G(z), \; z = t/h^{1/\beta\delta} \qquad f_\chi = m_s^2 \frac{\chi_l}{T^4} h_0^{1+1/\delta} (m_l/m_s)^{1-1/\delta}$$

N_τ=4





$$\frac{T_p(H) - T_c}{T_c} = \frac{z_p}{z_0} H^{1/\beta\delta}$$

Ejiri et al, PRD 80 (09) 094505

QCD thermodynamics at non-zero chemical potential

Taylor expansion :

$$\frac{p(T,\mu_B,\mu_Q,\mu_S,\mu_c)}{T^4} = \sum_{i,j,k} \frac{1}{i!j!k!l!} \chi_{ijk}^{BQS} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k \left(\frac{\mu_c}{T}\right)^l$$

hadronic

$$\frac{p(T,\mu_u,\mu_d,\mu_s,\mu_c)}{T^4} = \sum_{i,j,k} \frac{1}{i!j!k!l!} \chi_{ijkl}^{uds} \left(\frac{\mu_u}{T}\right)^i \left(\frac{\mu_d}{T}\right)^j \left(\frac{\mu_s}{T}\right)^k \left(\frac{\mu_c}{T}\right)^l$$

quark

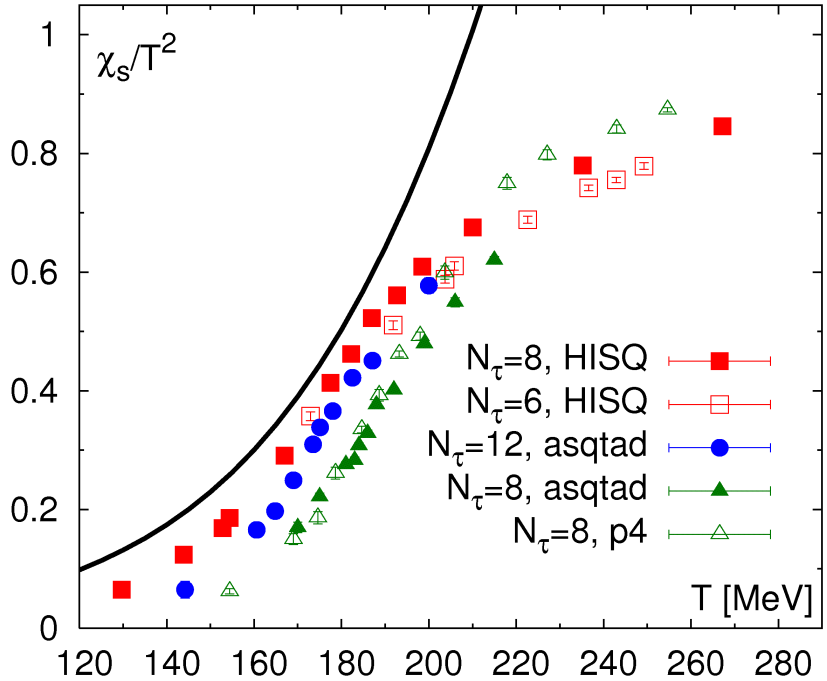
$$\chi_{ijk}^{abc} = \frac{\partial^i}{\partial\hat{\mu}_a^i}\frac{\partial^j}{\partial\hat{\mu}_b^j}\frac{\partial^k}{\partial\hat{\mu}_c^k}\frac{\partial^l}{\partial\hat{\mu}_d^l}\frac{1}{VT^3} \ln Z(T,V,\mu_a,\mu_b,\mu_c,\mu_d)|_{\mu_a=\mu_b=\mu_c=\mu_d=0},$$

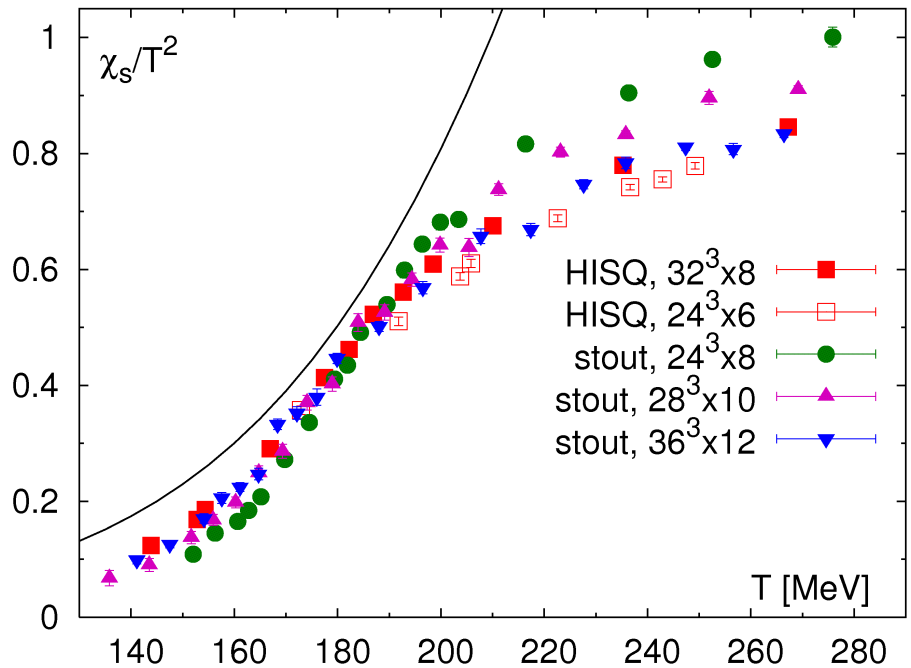
$$\hat{\mu}_X = \mu_X/T$$

Taylor expansion coefficients give the fluctuations and correlations of conserved charges, e.g.

$$\chi_2^X = \frac{1}{VT^3}(\langle X^2\rangle - \langle X\rangle^2) \qquad \chi_{11}^{XY} = \frac{1}{VT^3}(\langle XY\rangle - \langle X\rangle\langle Y\rangle)$$

Strangeness fluctuations and HRG

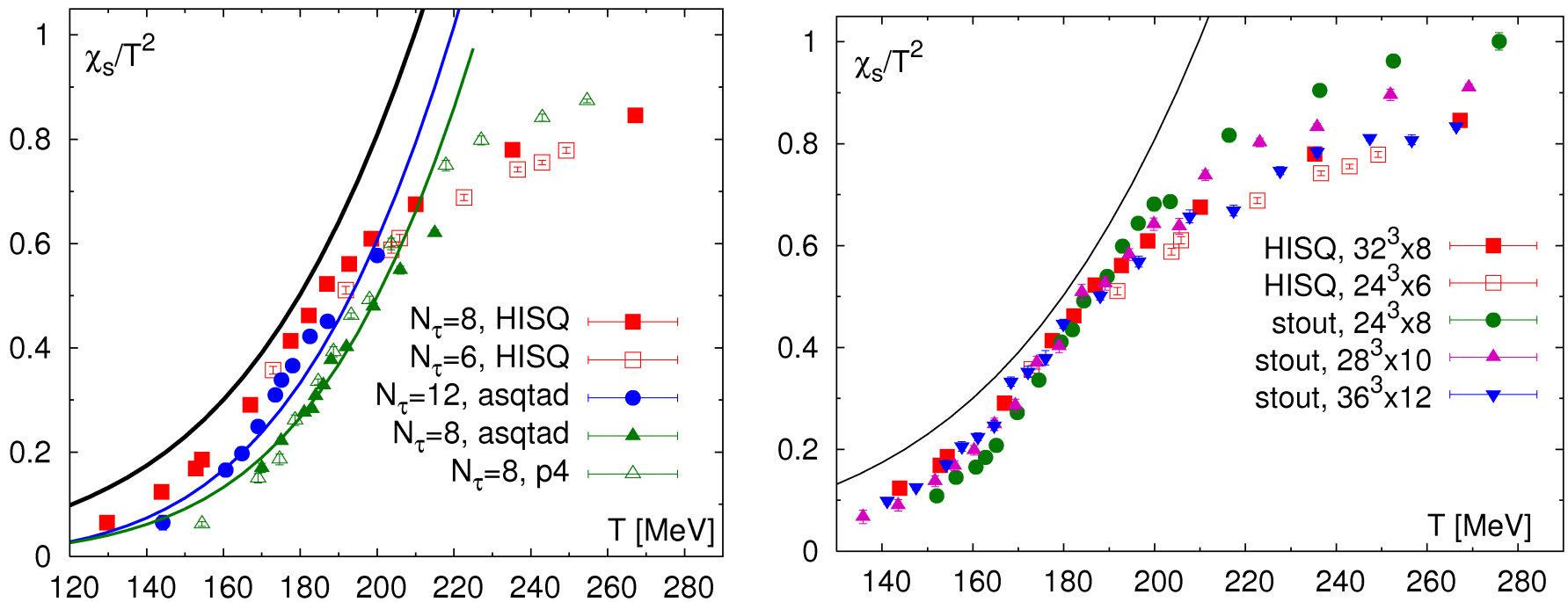




- enhancement of strangeness fluctuations compared to the calculations performed on $N_\tau=8$
- good agreement between HISQ and stout calculations

All the lattice results are below the HRG
Hadron masses on the lattice are large than physical due to a2 discretization effects this needs to be taken into account, Huovinen, P.P, arXiv:0912.2541

Strangeness fluctuations and HRG



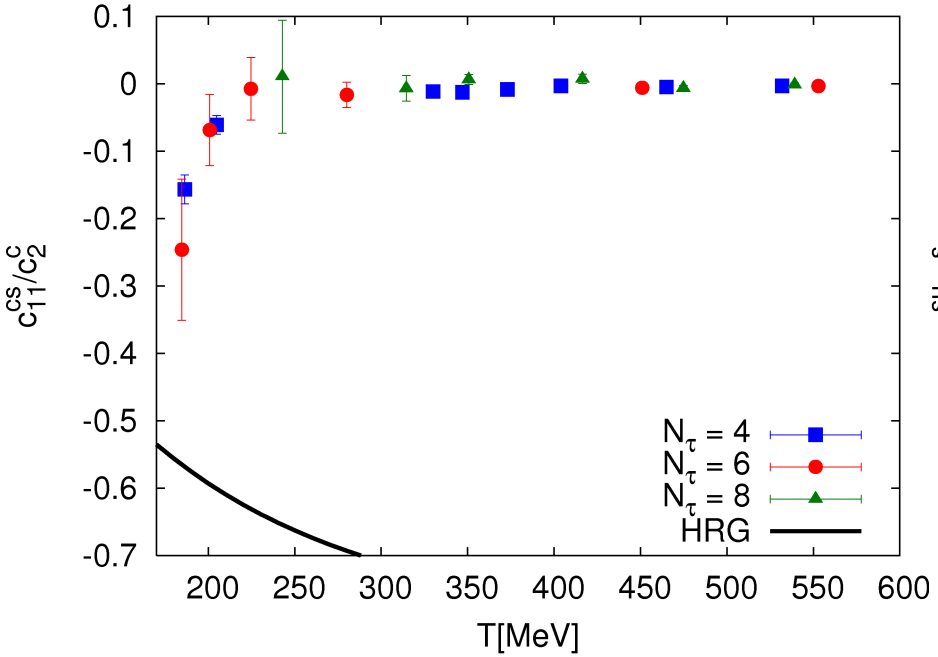
- enhancement of strangeness fluctuations compared to the calculations performed on $N_\tau=8$

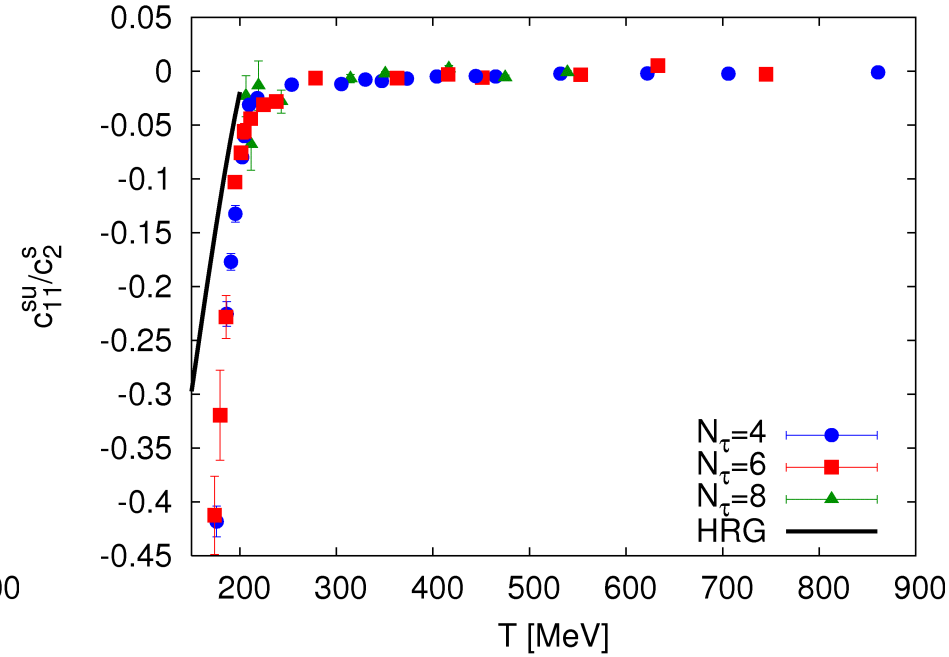
- good agreement between HISQ and stout calculations

HRG model with hadron spectrum modified by discretization effects describes the lattice results quite well
Huovinen, P.P, arXiv:0912.2541

Deconfinement : correlations of conserved charges

Cheng et al., arXiv:0811.1006, P.P, Hegde, Velytsky, PoS LAT2009 159



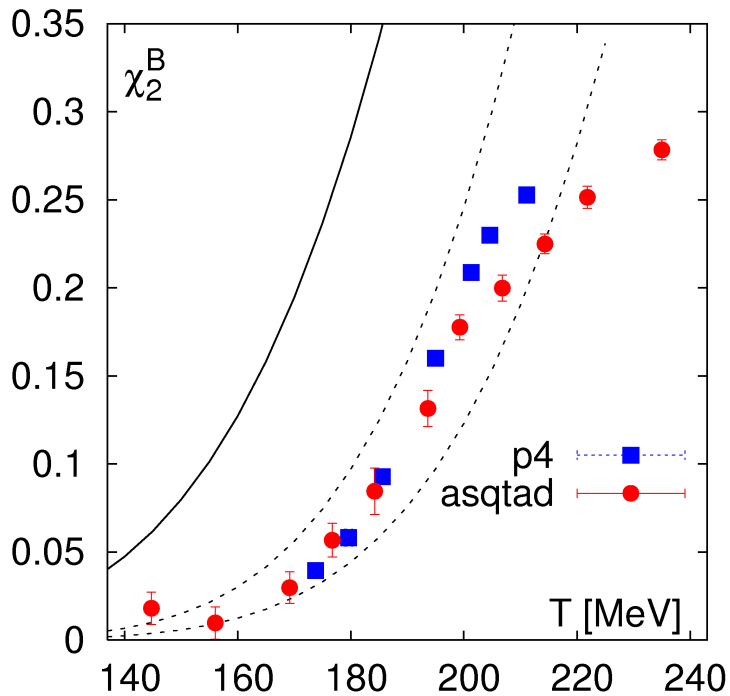


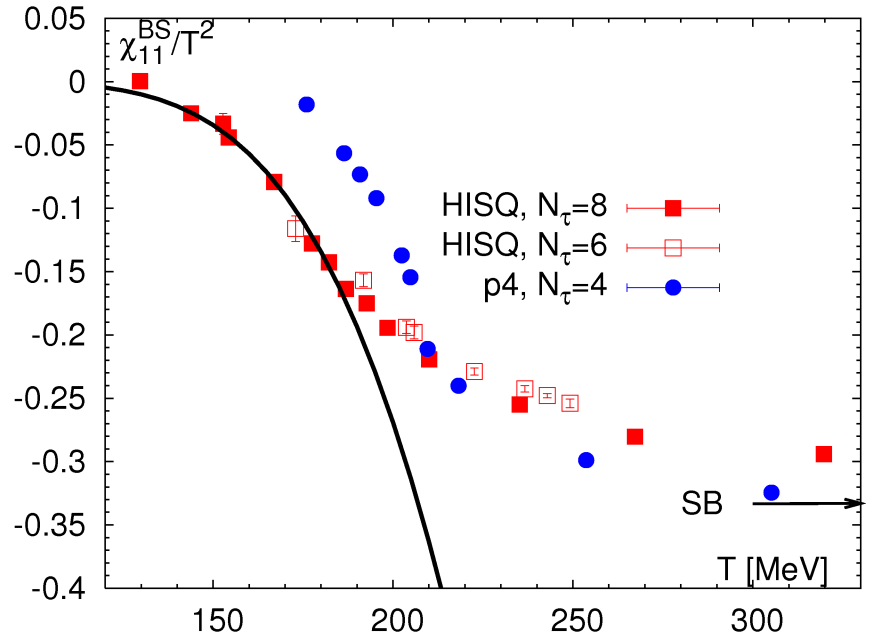
correlations of conserved charges at low temperatures can be qualitatively understood in terms of hadron resonance gas model

hadrons can carry different quark flavors => different quark flavors are correlated in the hadronic phase

quarks are un-bound at high temperatures => no correlations between different quark flavors

Baryon fluctuations and correlations

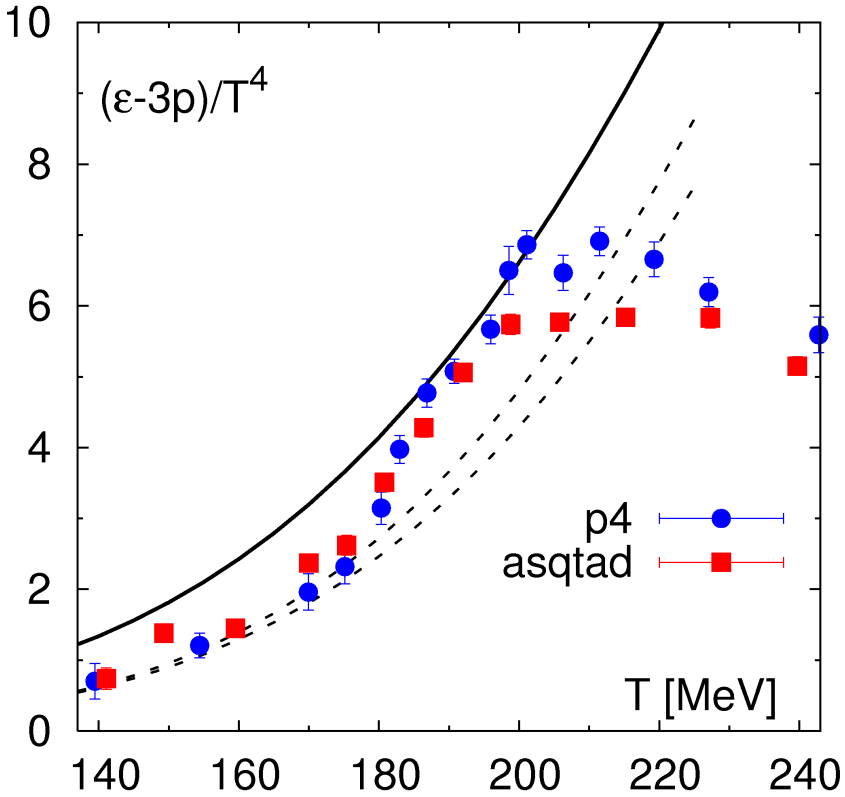


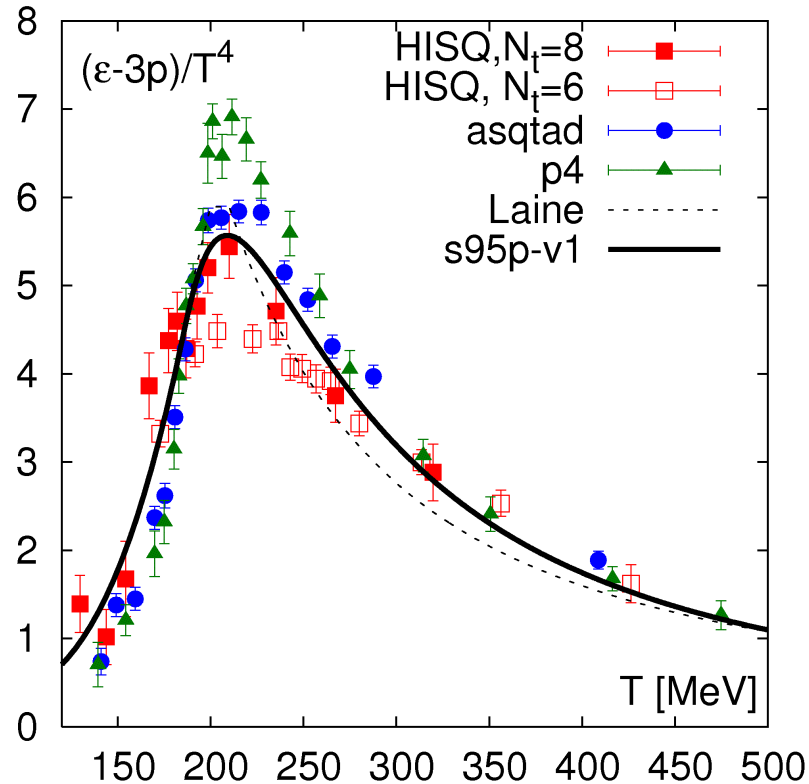


Agreement between HRG with modified spectrum and lattice obtained with p4 and asqtad on N_τ=8, Huovinen, P.P, arXiv:0912.2541

HISQ results are close to the physical HRG result

Trace anomaly in HRG and on the lattice





Agreement between HRG with modified spectrum and lattice obtained with p4 and asqtad on N_τ=8, Huovinen, P.P, arXiv:0912.2541

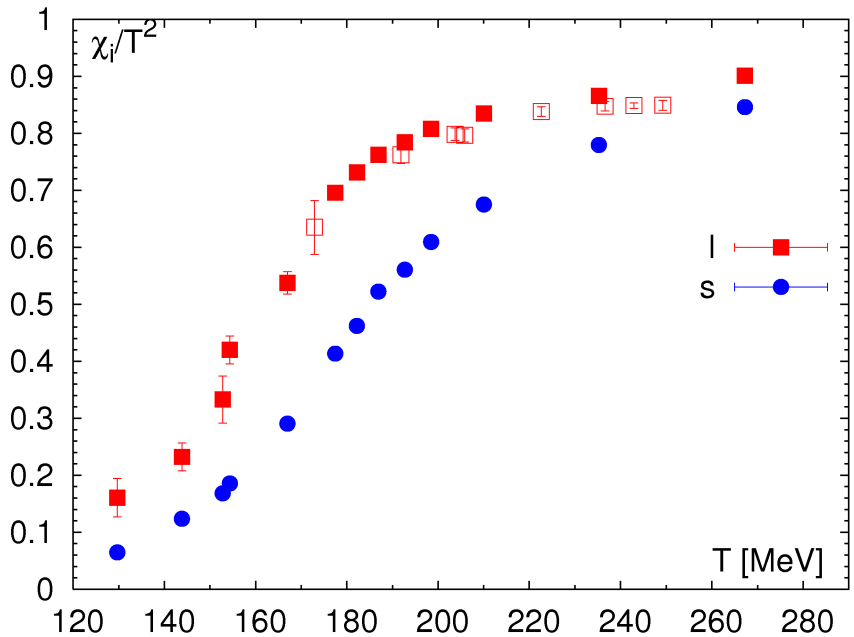
HISQ results are close to the s95p-v1 parameterization used in hydro models

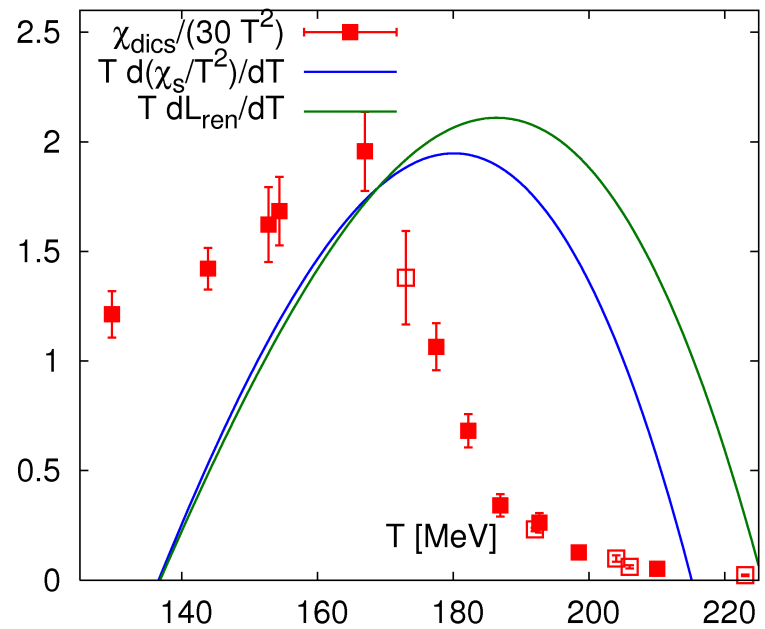
Chiral and deconfinement transition again

Light quark number fluctuations are sensitive to singular part of the free energy

$$f(T,\mu_l) = \frac{T}{V}\ln Z(T,\mu_l,V) = f_{sing}(t) + f_{reg}(T,\mu_l)$$

$$t = \frac{T-T_c}{T_c} + \kappa\left(\frac{\mu_l}{T}\right)^2, f_{sing}(t) \sim |t|^{2-\alpha}, \frac{\chi_l}{T^2} \sim \frac{\epsilon}{T^4} \sim A_\pm t^{1-\alpha} + regular$$





inflection points of Polyakov loop and strangeness susceptibility maybe higher than chiral transition temperature because they are dominated by the regular part

Conclusions

- Lattice discretization effects are large in the staggered fermion formulation at low T
these discretization effects can be largely reduced using HISQ action
(will be used by HotQCD collaboration in the future)

- Chriral transition temperature $T_c < 170MeV$

- At low T thermodynamics can be understood in terms of HRG
The deconfinement transition can understood as transition from HRG to quark gluon gas
it is gradual and analagous to ionized gas – plasma transition

- To study the interplay between chiral and deconfinement aspect of the transition
and precisely determine the transition temperature one has to consider
thermodynamic quantities that are sensitive to the singular part of the free energy density

- Many thanks to my collaborators in RBC-BI and HotQCD collaborations, especially to
A. Bazavov, C. DeTar, M. Cheng, P. Hegde, U. Heller,F. Karsch, E. Laermann, C. Miao,
S. Mukherjee, C. Schmidt, W. Soeldner, D. Toussaint