$Q\bar{Q}$ potential in an anisotropic QGP

arXiv:0711.4722

Adrian Dumitru
J.W. Goethe Univ., Frankfurt

Collaborators: Yun Guo, Mike Strickland

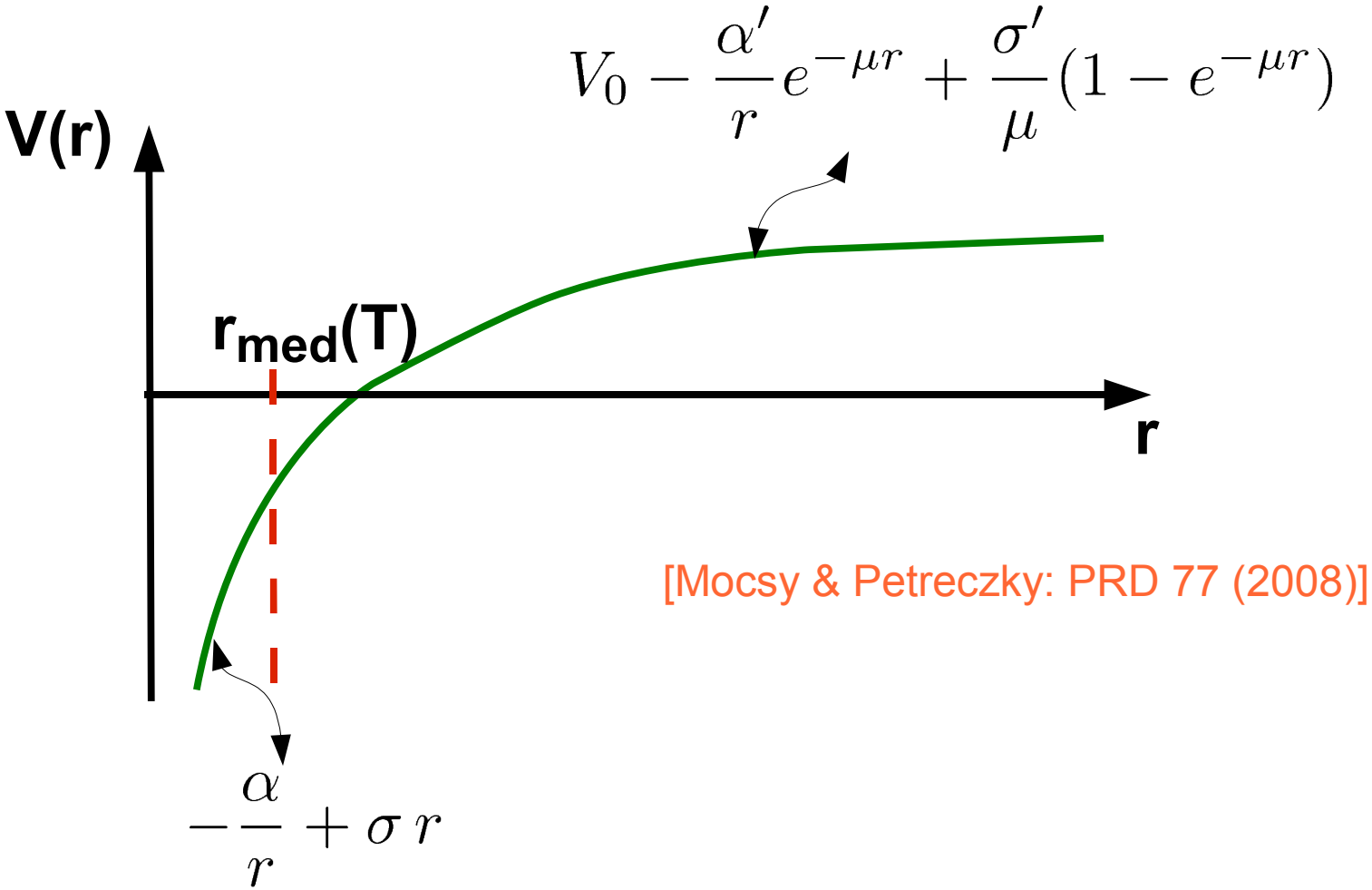
- Motivation
- Covariant-gauge gluon propagator in aniso. QGP
- Static limit: heavy-quark potential
- Results and discussion

- Motivation I:

  - $Q\bar{Q}$ bound states may survive above T_C
    (Jakovac et al, PRD 07; Aarts et al, PRD 07)
  - (p)QCD calculations possible
  - test potential models at T>0
    (Karsch, Mehr, Satz, ZPC 88;
    Petreczky and Mocsy, 2005-2008)
  - coalescence models for hadronization @ RHIC?

$$r_{\rm med}(T) \approx 0.5 \text{ fm } (T_C/T)$$

[Kaczmarek et al: PRD 70 (2004)]

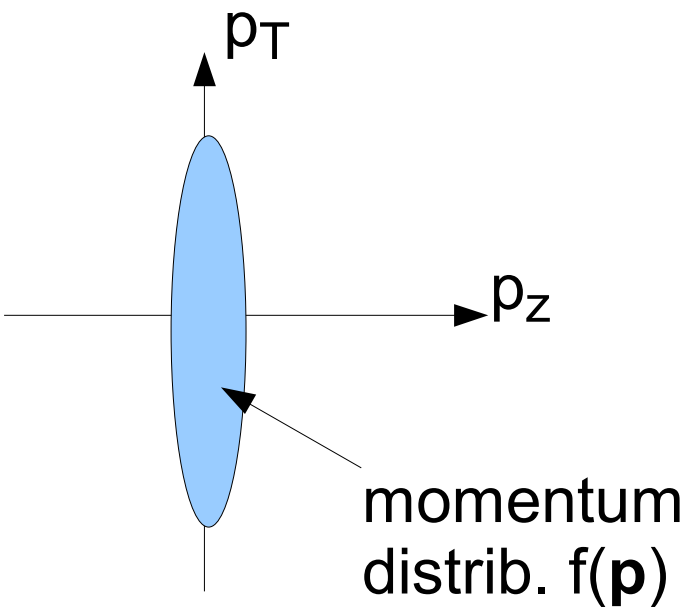


[Mocsy & Petreczky: PRD 77 (2008)]

- Motivation II: expanding plasma always (slightly) out of equilibrium

$$f(\mathrm{p}) = f_{iso}(\sqrt{\mathrm{p}^2 + \xi p_z^2})$$

anisotropy parameter



- Relation to viscosity: $\xi = \frac{10}{T\tau}\frac{\eta}{s}$

[e.g. Asakawa, Bass, Müller: Prog. Theor. Phys. 116 (2007) p. 725]

Covariant-gauge gluon propagator in anisotr. QGP

1) HTL (retarded) self-energy [Romatschke, Strickland: PRD 68 (2003)]

$$\Pi^{\mu\nu} = g^2 \int \frac{d^3k}{(2\pi)^3} v^\mu \frac{\partial f(\mathsf{k})}{\partial k^\beta} \left(g^{\nu\beta} - \frac{v^\nu p^\beta}{p \cdot v + i\epsilon} \right)$$

$$\left(\Delta^{-1}\right)^{\mu\nu}(p,\xi) = -p^2 g^{\mu\nu} + p^\mu p^\nu - \Pi^{\mu\nu}(p,\xi) - \frac{1}{\lambda} p^\mu p^\nu$$

invert: $(\Delta^{-1})^{\mu\sigma} \Delta_\sigma{}^\nu = g^{\mu\nu}$

static limit:

$$\Delta^{00}(\omega = 0, \mathsf{p}) = \frac{\mathsf{p}^2 + m_\alpha^2 + m_\gamma^2}{(\mathsf{p}^2 + m_\alpha^2 + m_\gamma^2)(\mathsf{p}^2 + m_\beta^2) - m_\delta^4}$$

"Mass" scales:

$$
\begin{aligned}
m_\alpha^2 &= -\frac{m_D^2}{2p_\perp^2\sqrt{\xi}}\left(p_z^2\arctan\sqrt{\xi} - \frac{p_z\mathsf{p}^2}{\sqrt{\mathsf{p}^2+\xi p_\perp^2}}\arctan\frac{\sqrt{\xi}p_z}{\sqrt{\mathsf{p}^2+\xi p_\perp^2}}\right)\\
m_\beta^2 &= m_D^2\frac{(\sqrt{\xi}+(1+\xi)\arctan\sqrt{\xi})(\mathsf{p}^2+\xi p_\perp^2)+\xi p_z\left(p_z\sqrt{\xi}+\frac{\mathsf{p}^2(1+\xi)}{\sqrt{\mathsf{p}^2+\xi p_\perp^2}}\arctan\frac{\sqrt{\xi}p_z}{\sqrt{\mathsf{p}^2+\xi p_\perp^2}}\right)}{2\sqrt{\xi}(1+\xi)(\mathsf{p}^2+\xi p_\perp^2)}\\
m_\gamma^2 &= -\frac{m_D^2}{2}\left(\frac{\mathsf{p}^2}{\xi p_\perp^2+\mathsf{p}^2} - \frac{1+\frac{2p_z^2}{p_\perp^2}}{\sqrt{\xi}}\arctan\sqrt{\xi} + \frac{p_z\mathsf{p}^2(2\mathsf{p}^2+3\xi p_\perp^2)}{\sqrt{\xi}(\xi p_\perp^2+\mathsf{p}^2)^{\frac{3}{2}}p_\perp^2}\arctan\frac{\sqrt{\xi}p_z}{\sqrt{\mathsf{p}^2+\xi p_\perp^2}}\right)\\
m_\delta^2 &= -\frac{\pi m_D^2\xi p_z p_\perp|\mathsf{p}|}{4(\xi p_\perp^2+\mathsf{p}^2)^{\frac{3}{2}}}\,.
\end{aligned}
$$

Note:

$$\text{for } \xi = 0: \quad m_\beta = m_D \quad , \quad m_{\alpha,\gamma,\delta} = 0$$

Potential: F.T. of static propagator

$$V(\mathsf{r},\xi) = -g^2 C_F \int \frac{d^3\mathsf{p}}{(2\pi)^3}\, e^{i\mathsf{p}\cdot\mathsf{r}} \Delta^{00}(\omega = 0, \mathsf{p}, \xi)$$

Limiting cases:

- $\xi \to 0$: Debye-screened potential

$$V(\mathsf{r},\xi = 0) \;=\; V_{\rm iso}(r) \;=\; -\frac{\alpha_s C_F}{r}\, e^{-r m_D}$$

- $r \to 0$: Coulomb potential

$$V(\mathsf{r} \to 0, \xi) \;=\; V_{\rm vac}(r) \;=\; -\frac{\alpha_s C_F}{r}$$

(no string ~σr from perturbation theory)

- weak anisotropy ξ«1: angular dependence of screening

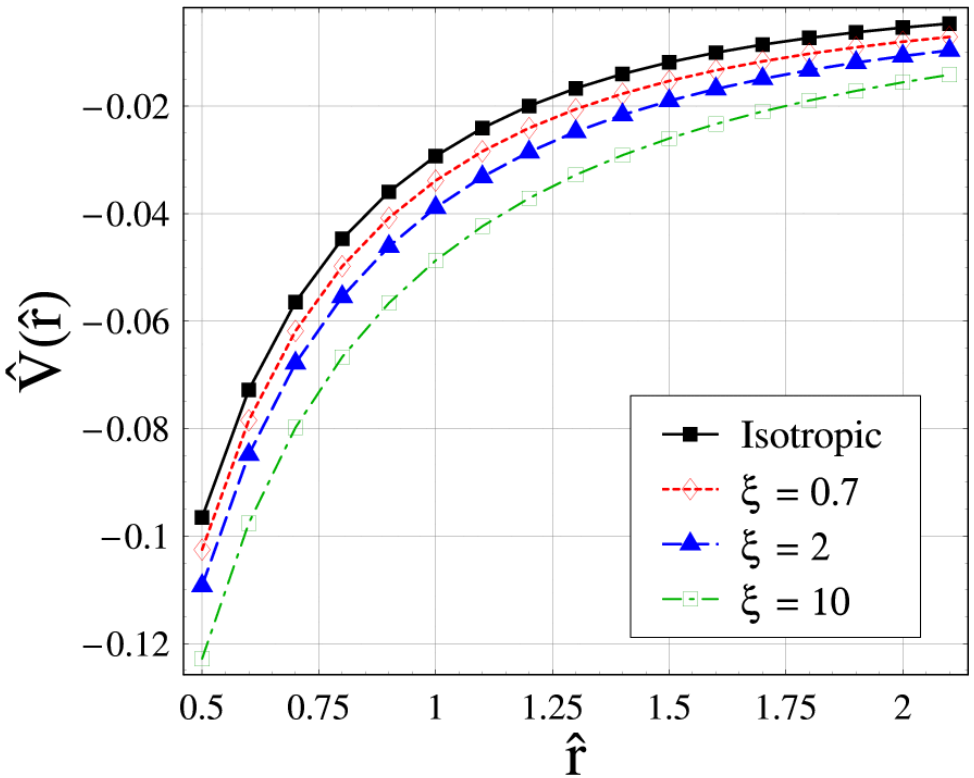
$$V(\mathbf{r} \parallel \hat{z}, \xi \ll 1) = V_{iso}(r)\left[1 + \xi \underbrace{\left(2\frac{e^{\hat{r}} - 1}{\hat{r}^2} - \frac{2}{\hat{r}} - 1 - \frac{\hat{r}}{6}\right)}_{= +0.27 \text{ for } \hat{r}=1}\right]$$

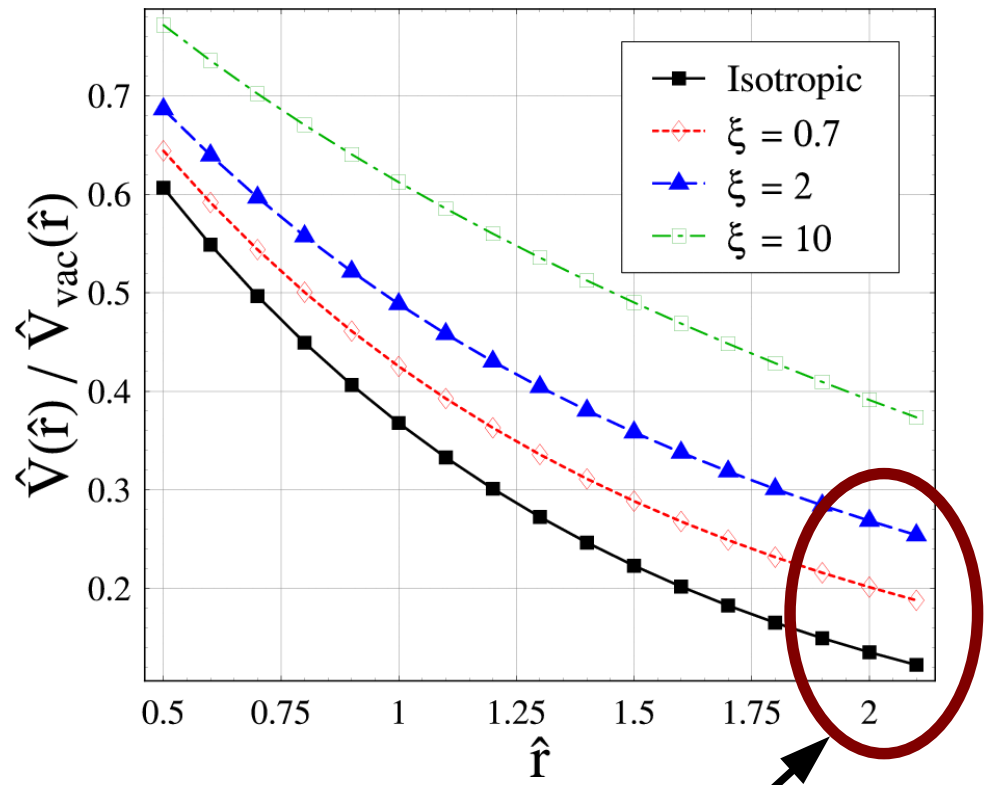
$$V(\mathbf{r} \perp \hat{z}, \xi \ll 1) = V_{iso}(r)\left[1 + \xi \underbrace{\left(\frac{1 - e^{\hat{r}}}{\hat{r}^2} + \frac{1}{\hat{r}} + \frac{1}{2} + \frac{\hat{r}}{3}\right)}_{= +0.12 \text{ for } \hat{r}=1}\right]$$

valid for $\hat{r} \equiv r\, m_D \lesssim 1$

- screening weaker for r ∥ z

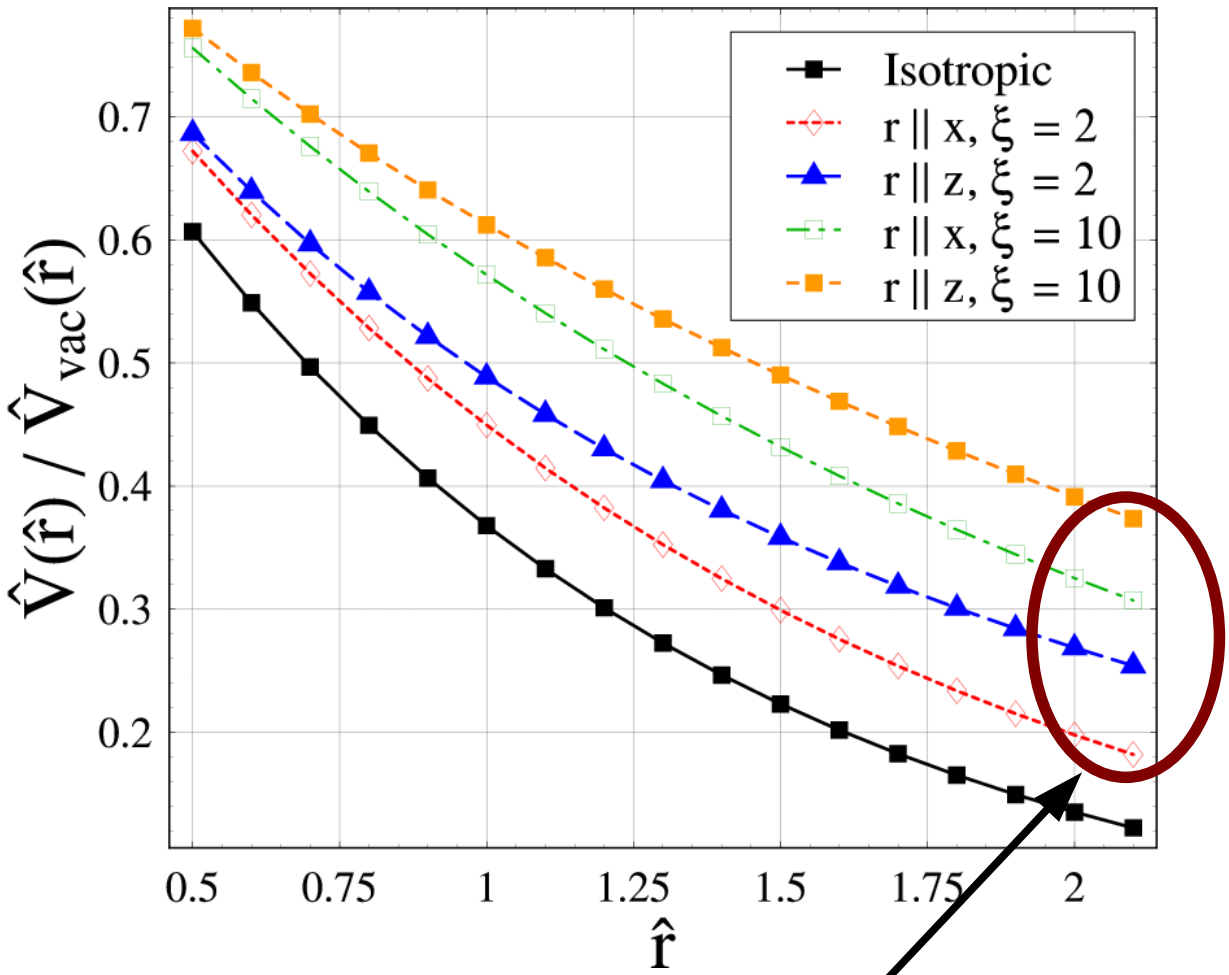
• numerical results for arbitrary anisotropy ξ and r ∥ z:





weakening of screening more pronounced at larger distance

• r ∥ z versus r ⊥ z:

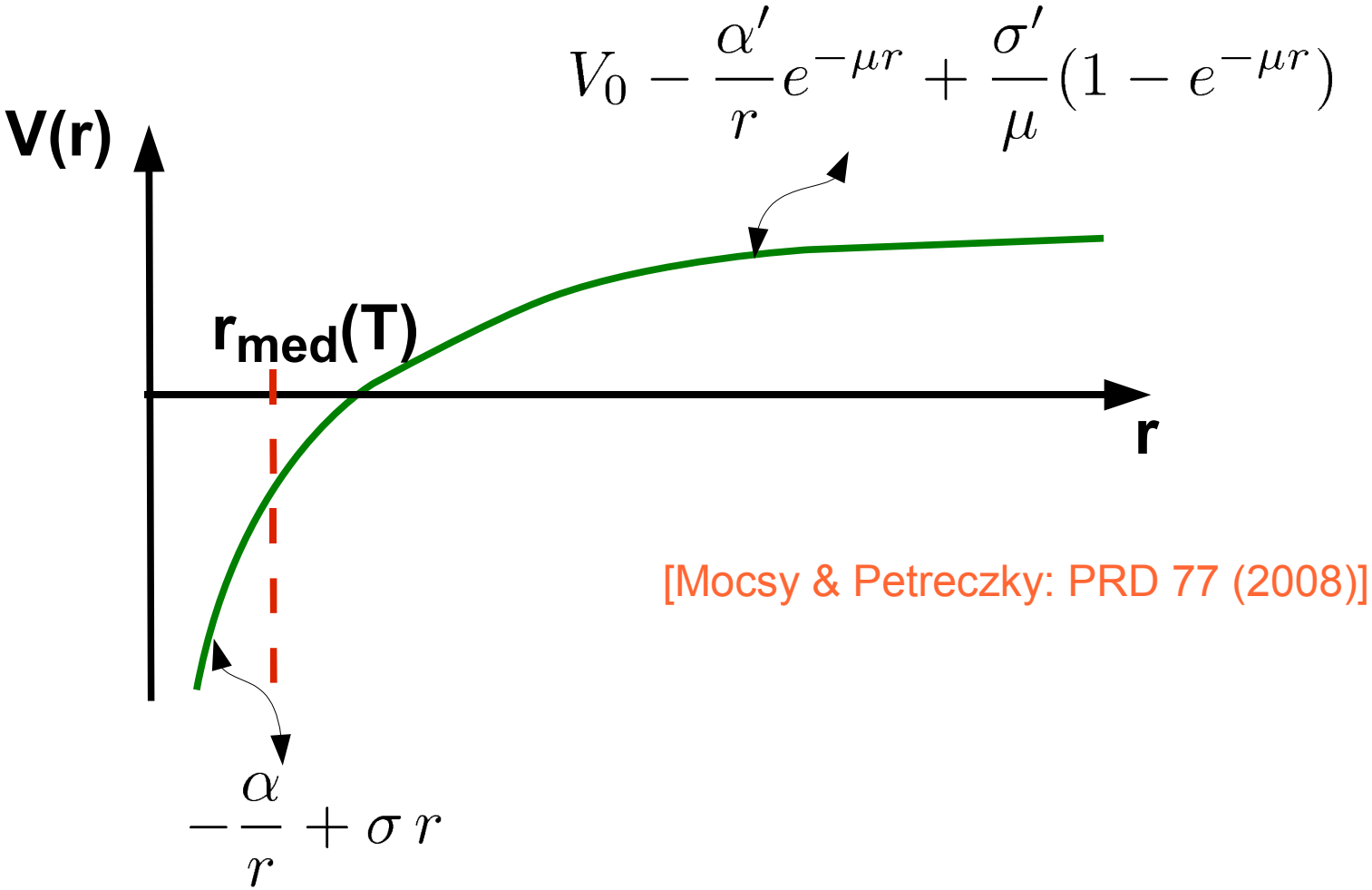


- angular dependence stronger at larger distance
- as r→0, angular dependence disappears

Discussion

$$r_{\rm med}(T) \approx 0.5 \text{ fm } (T_C/T)$$

[Kaczmarek et al: PRD 70 (2004)]



[Mocsy & Petreczky: PRD 77 (2008)]

- We determined only the one-gluon exchange contribution
- string can be neglected for $r \sim r_{med}$ if $r_{med} < \sqrt{(\alpha/\sigma)}$, $\leftrightarrow$ $T > 2T_C$

• consider $Q\overline{Q}$ state of size $\sqrt{\langle r^2 \rangle} \sim r_{med}$ and T sufficiently high so that $r_{med} < \sqrt{(\alpha/\sigma)}$:

screened Coulomb dominates, potential and its angular dependence can be determined from pQCD

• anisotropy (ξ>0) affects binding energy

ground state, $1 \gg \xi \gg m_D / (\alpha_s C_F m_Q)$:

$$\frac{\Delta E}{E_C} \approx \frac{4}{\alpha_s C_F} \frac{m_D}{m_Q} \left(-1 + \frac{\xi}{6} + \cdots \right)$$

(universal, all states)

• for larger states, shift of binding energy depends on angular momentum ℓ ! (not the same for η_b, Y, ...)

Outlook:

- determine wave functions, binding energies etc in anisotropic plasma from potential model

- imaginary part of potential (→ width of quarkonium states) in an anisotropic plasma ?

[Laine et al: JHEP (2007);
Mocsy, Petreczky: PRL 99 (2007)]