Matrix model for the “sQGP”

1. AA collisions at RHIC and the sQGP (strong Quark-Gluon Plasma)

Unambiguous signal(s) of new physics, from *high* momenta

AA collisions ≠ A (pp collisions): AA coll.’s *strongly* affect even *heavy* quarks.

2. “Strong” = strong coupling, right? Not necessarily.

Lattice simulations + thermal field theory => *moderate* coupling down to T_c

3. Matrix model for the sQGP

sQGP = deconfined QCD, at temperatures T from T_c to ~ 3 T_c .

Effective theory at large A_0 = (not so) random matrix model.

Confinement from *dynamical* generation of eigenvalue repulsion

AA collisions at RHIC and the sQGP

(Sometimes mythical creatures, like unicorns and the Quark-Gluon Plasma, do exist...)

QCD: "transition" to QGP at $T_c \sim 200$ MeV.

So: natural to look at soft momenta as signal, (transverse) momenta $\sim T_c$

RHIC experiments find, instead, that the cleanest signals of new physics are at *hard* momenta, ~ 2-20 GeV.

R_{AA}: *robust* signal of new physics

R_{AA}= for a given p_t, # particles central AA/($A^{4/3}$ # particles pp)

For π^0's, p_t : 2 -> 20 GeV, $R_{AA} \sim 0.2$. As if jets emitted *only* from surface!

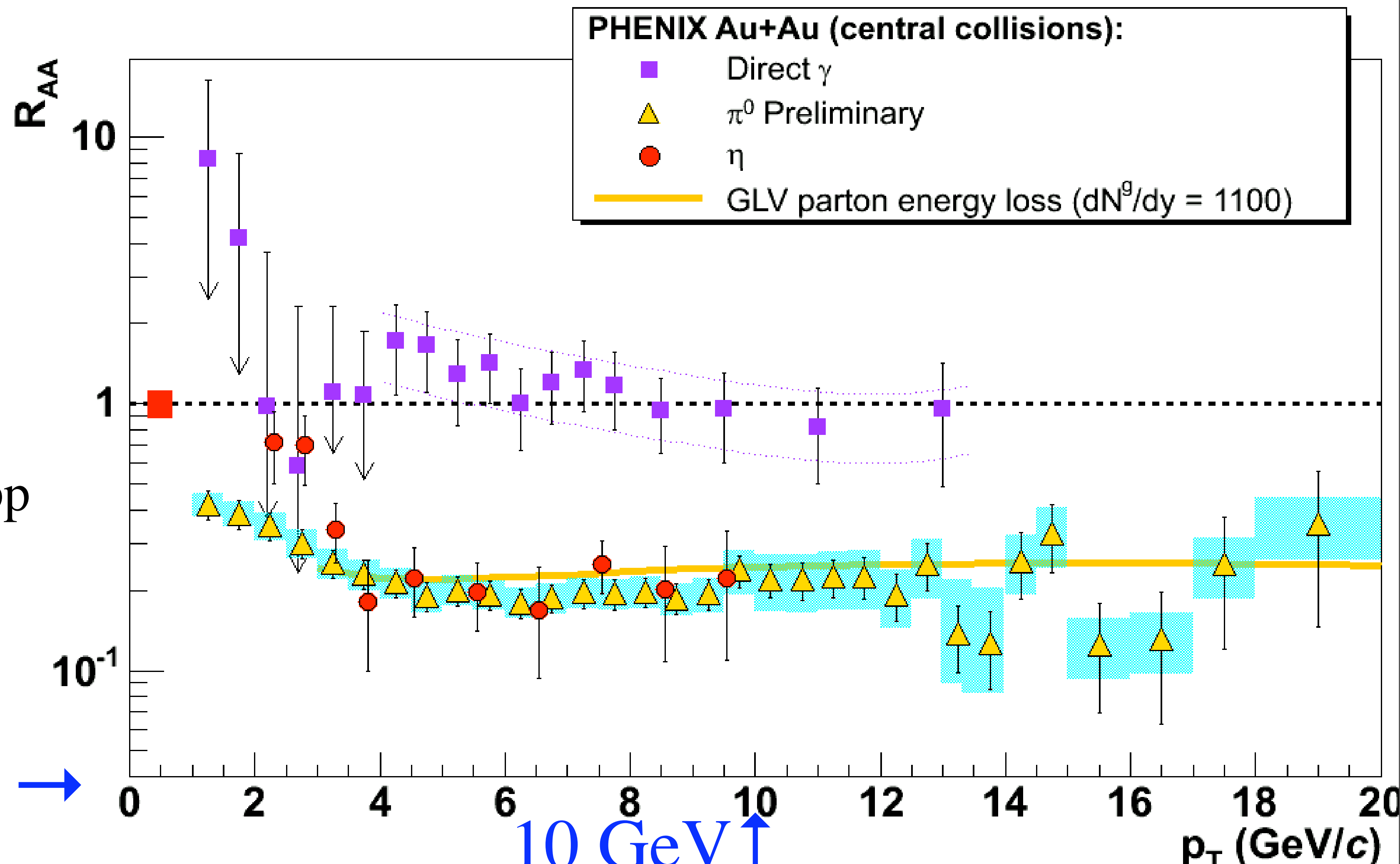
Due to "energy loss" in thermal medium?

$A^{4/3}$: *experimentally*: for γ's, $R_{AA} \sim 1.0$ π^0's "eaten", γ's not

R_{AA}: ↑

particles
central AA/
particles pp

A=200 =>



p_t →

10 GeV↑

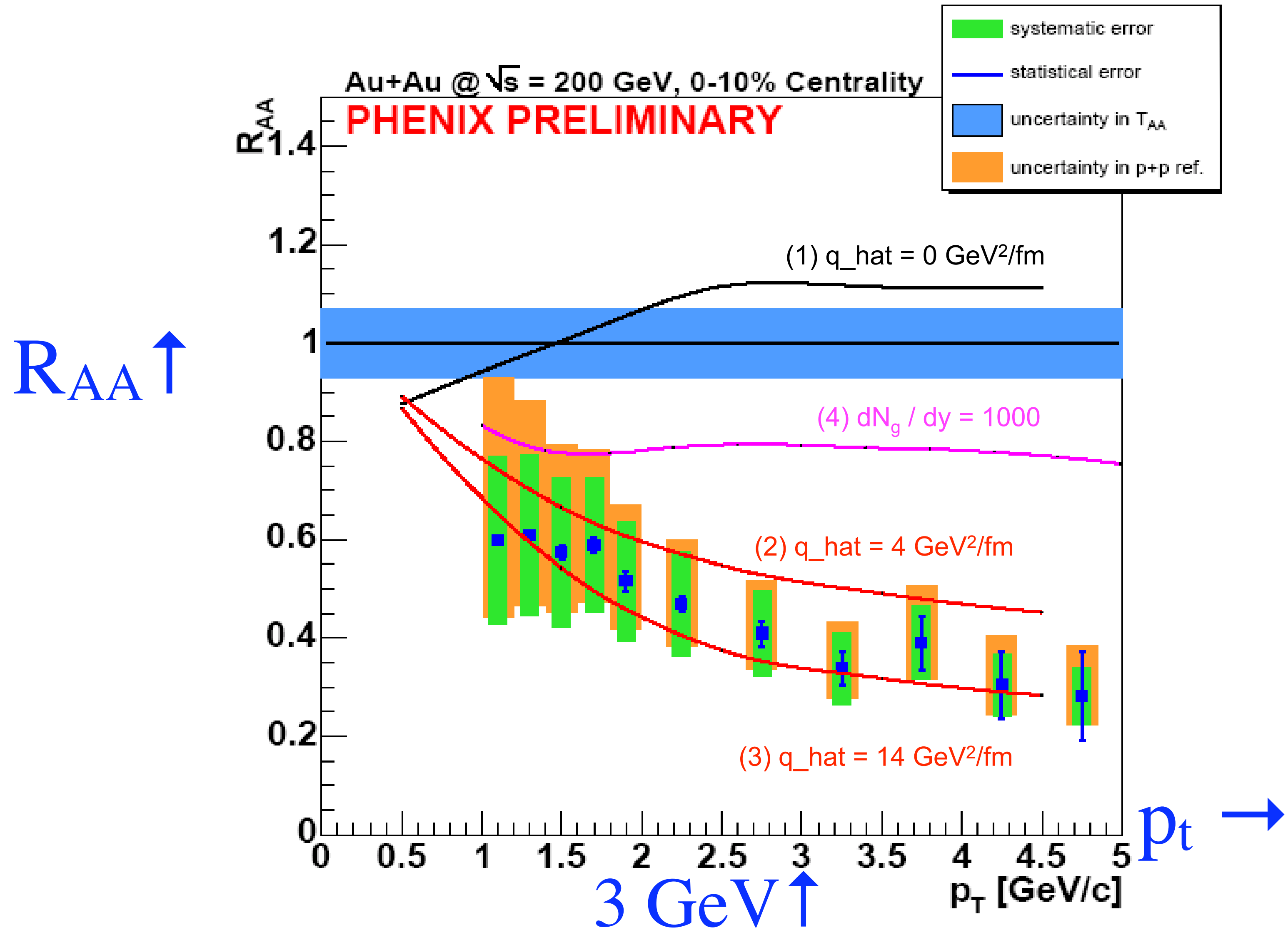
R_{AA} for heavy quarks: also suppressed!

PHENIX: R_{AA} for charm quarks ~ light quarks!

Mass of charm quark m_{charm} ~ 1.5 GeV; T ~ 200 MeV.

Heavy quark less sensitive to medium by T/m_{charm} ~ 1/8. *No* sign of that!

Experimental evidence for "sQGP": heavy quarks ~ same as light!



“Most Perfect Fluid on Earth”

Large # particles: try *ideal* hydrodynamics, with *zero* viscosity in QGP

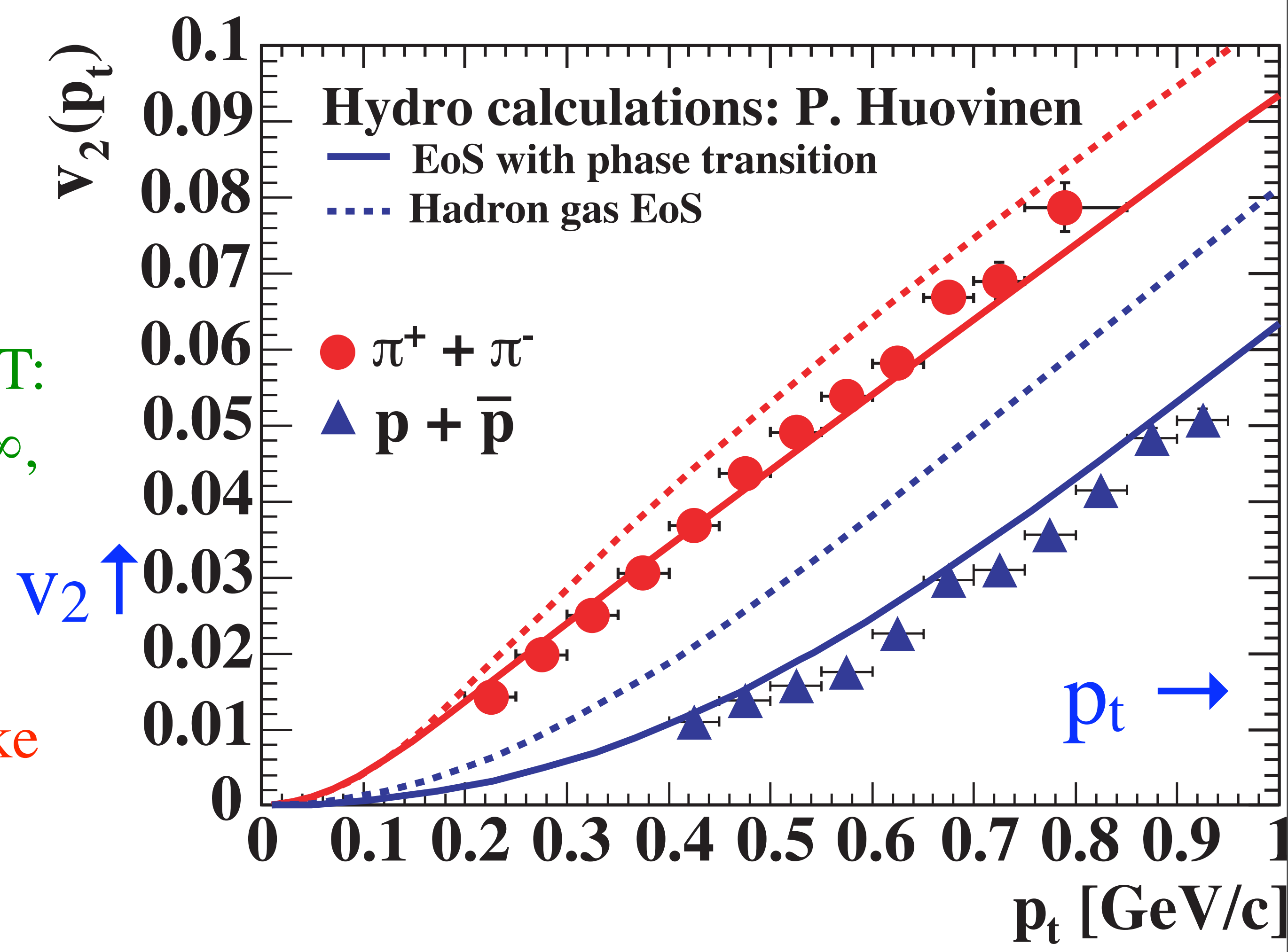
Need: initial time ~1 fm/c, hadronic “afterburner” (~ large hadronic viscosity)

Good fit to π’s, K’s, p’s.... for both single particle spectra and “elliptic flow” v_2

Viscosity ~ 1/cross section:
small viscosity =>
strong coupling?

$\mathcal{N}$ = 4 SUSY QCD + AdS/CFT:
At infinite coupling, & $N_c = \infty$,
viscosity/entropy = 1/(4 π).
Universal lower bound?

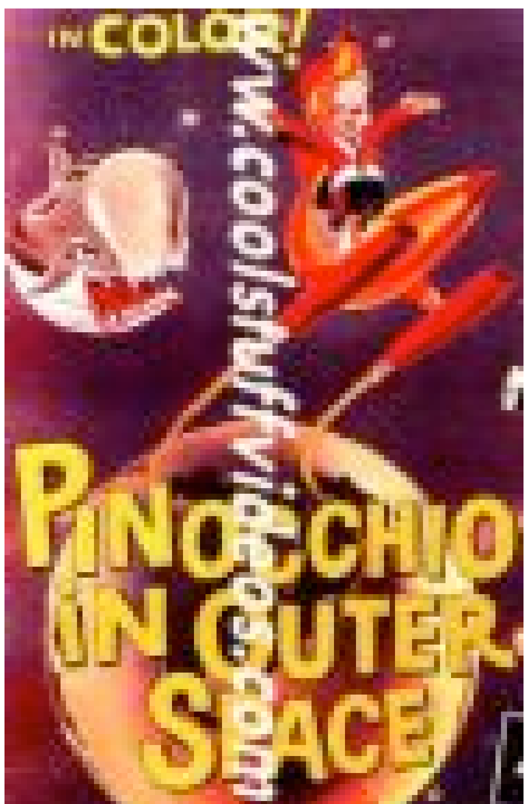
Exp.’y: charm quarks flow like
light quarks => sQGP



New York Times, October 20, 2006:

The Universe on a String: by Brian Greene

....And in a recent, particularly intriguing development, data now emerging from the Relativistic Heavy Ion Collider at the Brookhaven National Laboratory appear to be *more* accurately described using string theory methods than with more traditional approaches....



L. McLerran, Theory Summary at Quark Matter 2006, Shanghai:

"Pinocchio" award to Brian Greene...

For me: $\mathcal{N} = 4$ SUSY QCD + AdS/CFT = thermal field theory of *astounding* beauty

vs Google: "most reactionary physicists"

"Strong" = strong coupling, right?

In QCD, for momenta below 1 GeV, the coupling α_s is big,
assuredly one is in a non-perturbative regime.

Transition to QGP at $T_c \sim 200$ MeV. So strong coupling until $\sim 5\, T_c$?

Hunting for the "unicorn" in heavy ion collisions:
Unicorn = QGP. Hunters = experimentalists. So: "all theorists are dogs..."

“Helsinki” Program

Match original theory in 4D, to effective theory in 3D, for r > 1/T

$$\mathcal{L}^{eff} = \frac{1}{2}\,\mathrm{tr}\,G_{ij}^2 \;+\; \mathrm{tr}\,|D_i A_0|^2 + \; m_D^2\,\mathrm{tr}\,A_0^2 \;+\; \kappa\,\mathrm{tr}\,A_0^4$$

$m_{\mathrm{Debye}}{}^2 \sim g^2\, T^2$, $\kappa \sim g^4$, series in g^2 .

(First step in three: then resum m_{Debye} , m_{magnetic})

One resummation of perturbation theory, amongst others. Valid for *small* A_0

How does α_s^{eff} run? Braaten & Nieto ’96: $\alpha_s^{\mathrm{eff}}(2\,\pi\,T)$?

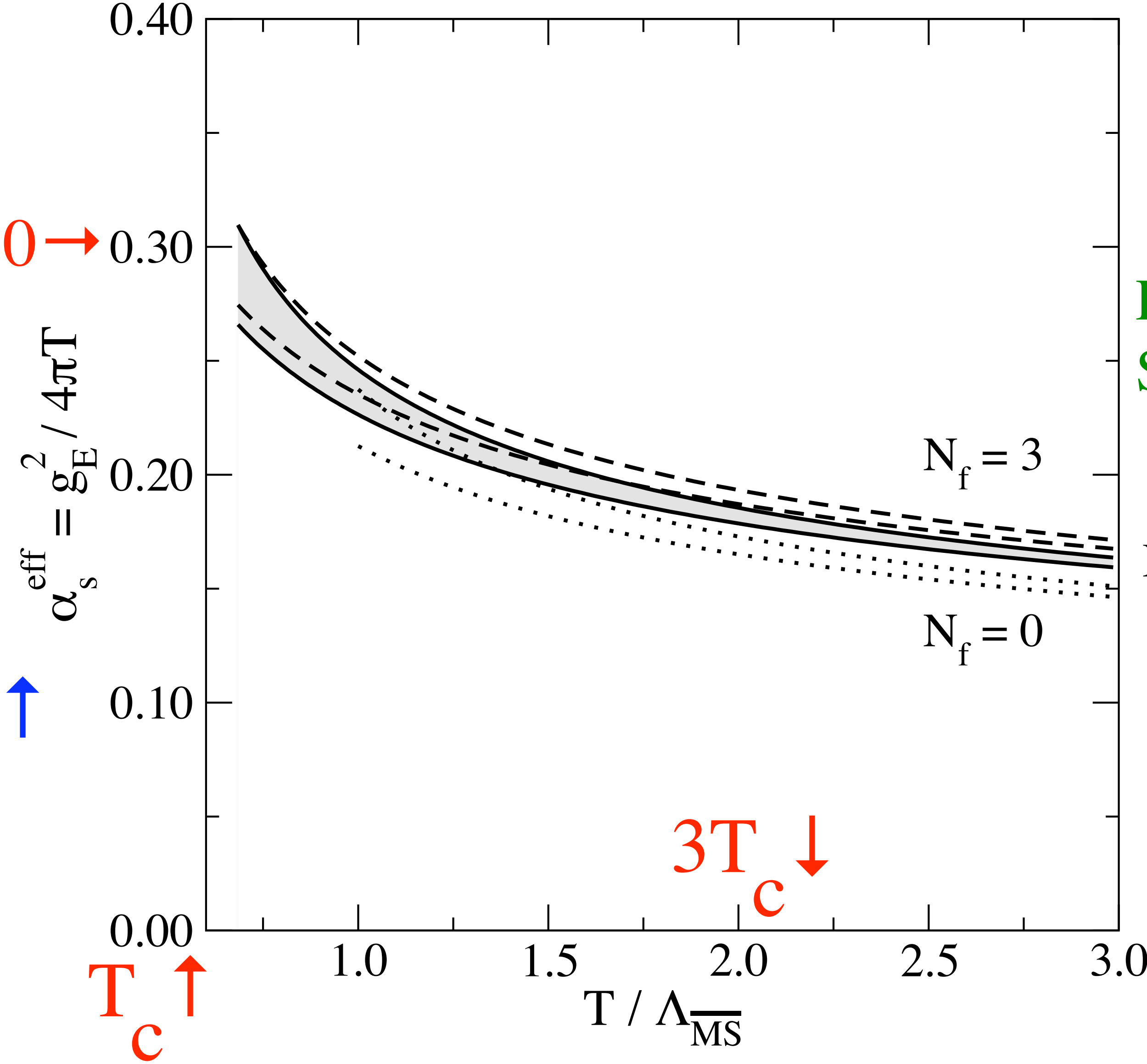
Even better! Laine & Schröder ’05: 2-loop calc. $\Rightarrow$ $\alpha_s^{\mathrm{eff}}(9\,T)$!

$T_c \sim 175$ MeV: $9\,T_c \sim 1.6$ GeV, $\alpha_s^{\mathrm{eff}}(9\,T_c) \sim 0.28$

$9\,(3\,T_c) \sim 4.8$ GeV : T_c to $\sim 3\,T_c$ *not* (so) strong coupling

α_s^{eff} is *not* so big, even at T_c

$0.30 \rightarrow$



Laine & Schröder '05

$\alpha_s^{eff}(T) \uparrow$

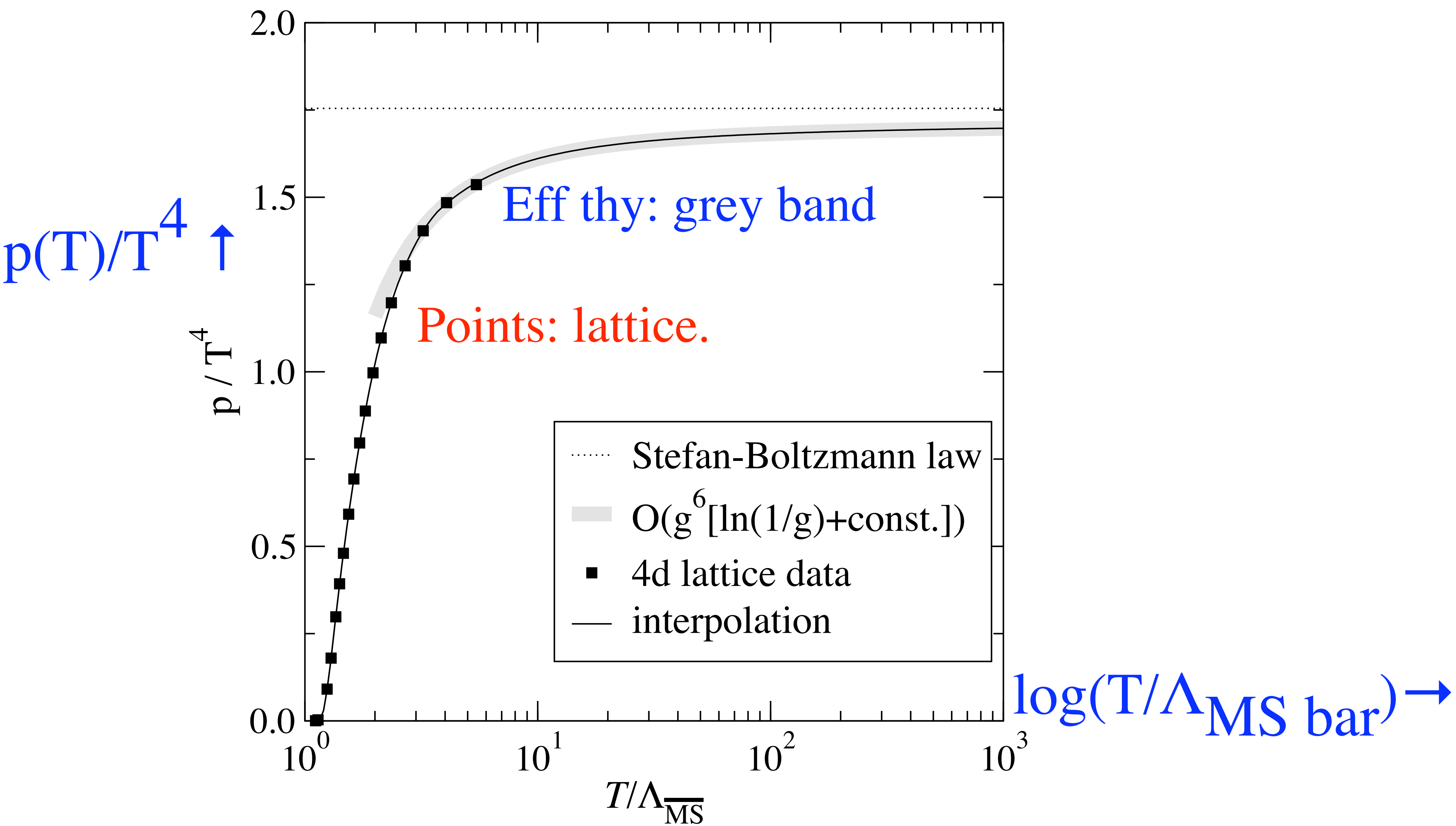
$T_c \uparrow$

$T/T_c \rightarrow$

$\alpha_s^{eff}(c\,T)$: $c \sim 2\,\pi \rightarrow 9$. Might have been $2\,\pi \rightarrow 2$.

If so, then strong coupling below $3\,T_c$. *Not* what happens.

Pressure: effective theory *fails* below ~ 3 T_c



If α_s^{eff} is not so big, why *doesn't* effective thy work for the pressure?

"Fuzzy" bag picture

With*out* dynamical quarks, can compute close to continuum limit for SU(3).

Looking at Bielefeld, lat/9602007, with new eyes: from T: 1.1 T_c to 4.0 T_c
(f_{pert} = constant)

$$p_{\mathrm{glue}}(T) \approx f_{pert}(T^4 - T_c^2 T^2) + \ldots$$

Suggests: *with* dynamical quarks: for T > 1.1 T_c , pressure a series in $1/T^2$:

$$p(T) \approx f_{pert}\, T^4 - B_{fuzzy}\, T^2 - B_{MIT} + \ldots$$

B_{fuzzy} "fuzzy" bag constant: dominates MIT bag constant, B_{MIT}, away from T_c

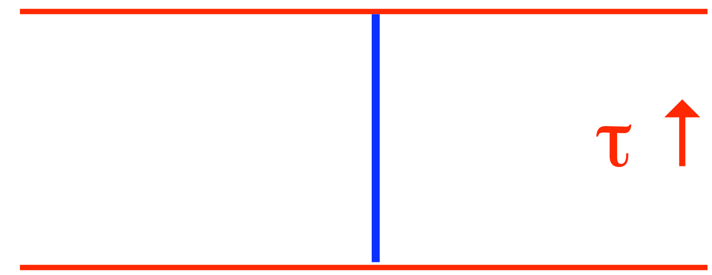
Only perturbative terms contribute to $f_{pert}(g^2)$: works down to T_c (Rebhan)

Perturbation theory fails because of *non*-perturbative terms, powers in $1/T^2$

Effective theory near T_c

Local quasiparticles? Here: use *non*local variable: straight, thermal Wilson line:

$$\mathbf{L}(x) = P\, e^{ig \int_0^{1/T} A_0(x,\tau)\, d\tau}$$



Under gauge transformations,

$$\mathbf{L}(x) \to \Omega(x, 1/T)^\dagger\, \mathbf{L}(x)\, \Omega(x, 0)$$

Trace gauge invariant.

= Polyakov loop, measures *fraction* of deconfinement.

$$\ell(x) = \mathrm{tr}\ \mathbf{L}/3$$

Can extract renormalized Polyakov loop from lattice.

Need to extract lattice "mass" renormalization from bare loop.

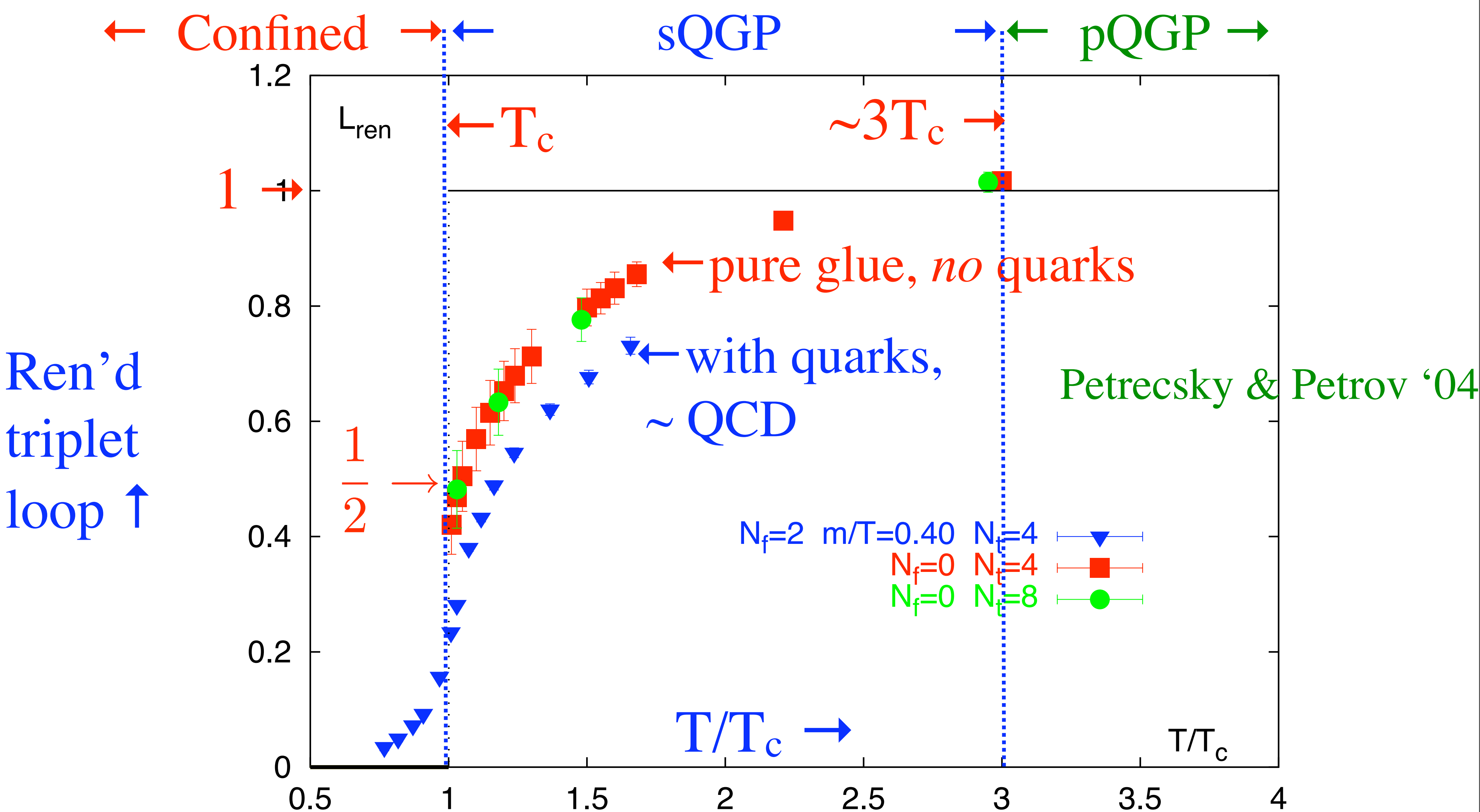
Perturbative regime: loop near one, ~ complete deconfinement. g A_0/T small.

Non-perturbative regime: loop < 1, *partial* deconfinement. g A_0/T large.

sQGP: *partially* deconfined

$T > 3\,T_c$: loop ~ 1, ~ perturbative QGP, “pQGP”. Eff. thy.: small A_0

$T_c \rightarrow 3\,T_c$: loop < 1, *partial* deconfinement, “sQGP” Eff. thy.: large A_0



Effective theory for large A_0

Symmetries? Certainly, invariance under static gauge transf.'s.
Plus: "large" gauge transformations - spatially constant, time *de*pendent. For SU(N):

$$U_c(\tau) = e^{2\pi i\, \tau T\, t_N/N} \quad , \quad t_N = \begin{pmatrix} \mathbf{1}_{N-1} & 0 \\ 0 & -(N-1) \end{pmatrix}$$

This $U_c(\tau)$ is *only* valid c/o quarks: $U_c(1/T) = \exp(2\,\pi\, i/N)\, U_c(0)$
Shows center symmetry of pure SU(N) glue: a global Z(N) symmetry

With quarks? Consider $U_c(\tau)$ to N^{th} power! $U_c(1/T)^N = \exp(2\,\pi\, i) U_c(0)^N = \mathbf{1}$.

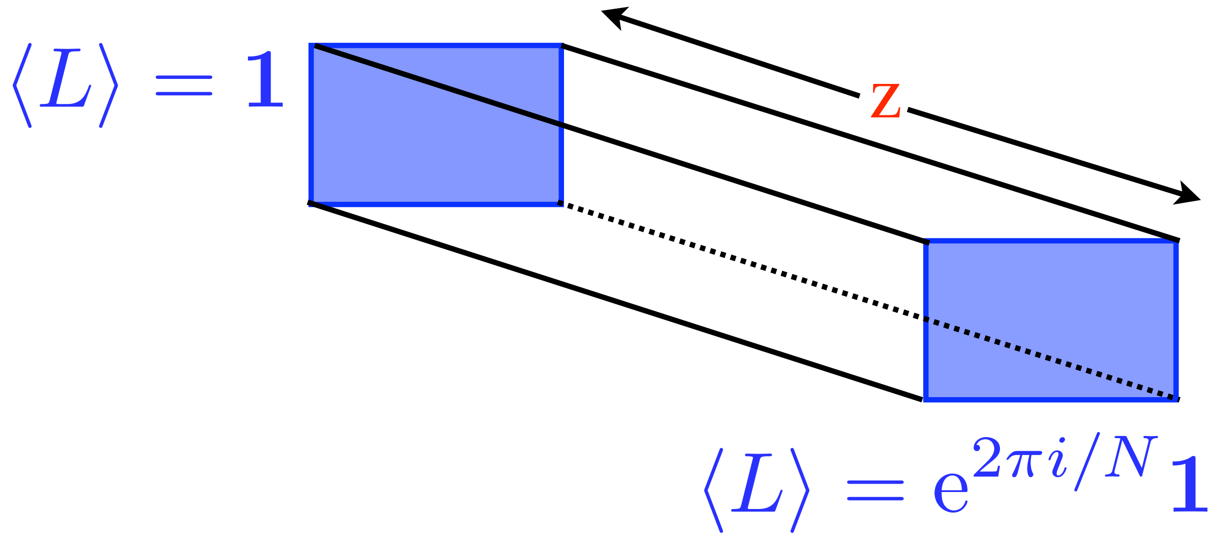
All theories must respect invariance under such *strictly* periodic gauge transf.'s.
For any gauge group, with any matter fields.
With center symmetry, or not. Even for QED.

Strictly periodic, but large gauge transf.'s place nontrivial constraints on a *non*abelian effective theory.

Z(N) interfaces

One way to probe large A_0: Z(N) interface related to gauge transformation, $U_c(\tau)$

Take a long box:



$$A_0 = \frac{2\pi T}{gN}\, q(z)\, t_N$$

Take $A_0 \sim t_N$, times "coordinate" q(z).

Even at large A_0, the (original) electric field is abelian: $E_i^{4D} \sim \partial_i A_0 \sim dq/dz$.

$\mathcal{L}_{\rm eff}$ = classical + 1 loop potential, for *constant* A_0

$$\mathcal{L}_{eff} = \mathrm{tr}\, E_i^2/2 + V_{1\,loop}(A_0) \sim \#(1/g^2(dq/dz)^2 + q^2(1-q)^2)$$

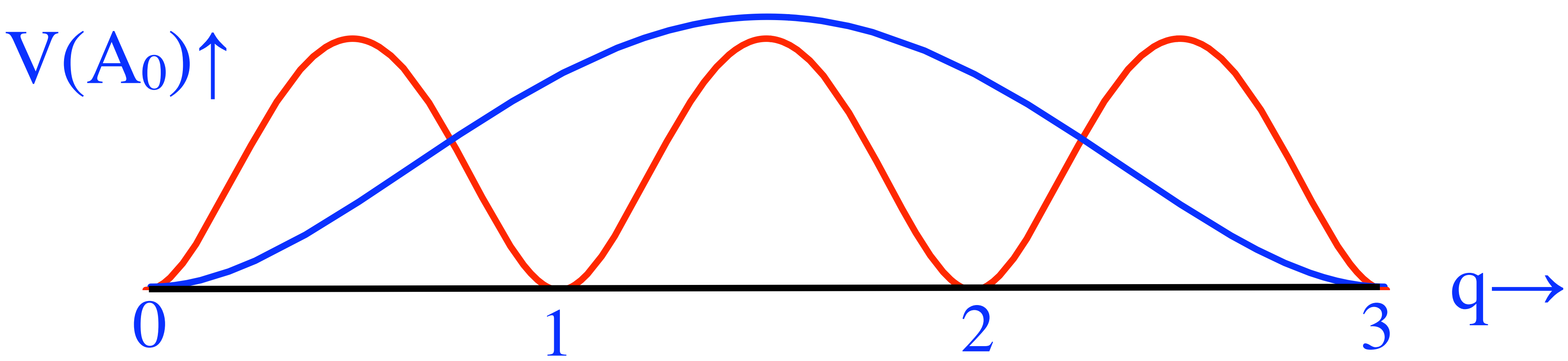
Usual tunneling problem: action ~ transverse area × # $T^2/(3\sqrt{g^2})$

Interface "fat": width ~ 1/(gT), so can use derivative expansion.

= $4\,\pi^2\,(N-1)T^2/\sqrt{(3N)}$. Compute semiclassically, now $(\sqrt{g^2})^3 \times \#$ Korthals Altes

U(1) interfaces

What if no center symmetry? QCD: SU(3) with dynamical quarks, G(2)...
Use "U(1)" interface for *strictly* periodic gauge transf. In QCD, $U_c(\tau)^3$



Red: potential for constant A_0 from SU(3) gluons
For integer q, $\langle L \rangle = \exp(2\pi i\, q/3)\, \mathbf{1}$. q = 0, 1, 2 are degenerate Z(3) vacua.

Blue: potential from quarks. Potential at q = 1, 2 ≠ q = 0 , 3: *no* Z(3) symmetry
Still have U(1) interface: $\langle L \rangle$: $\mathbf{1} \rightarrow \mathbf{1}$, but q(z): 0 → 3.

Use U(1) interfaces to probe large A_0 . Properties gauge invariant, physical.
Associated with U(1) topology in maximal torus.

Effective electric field?

Want 3D effective thy. for large $A_0 \sim T/g$.

Valid for $r > 1/T$, so A_0 varies slowly in space, momenta $p < T$.

Original electric field $E_i^{4D} = D_i A_0 - \partial_0 A_i$. So $E_i^{3D} = D_i A_0$?

For large gauge transf. $U_c(\tau)^N = \exp(2 \pi i T \tau t_N)$:

$$A_0^{diag} \rightarrow A_0^{diag} + \frac{2\pi T}{g} t_N \ , \ A_i \rightarrow \frac{1}{-ig} \Omega_c^\dagger(\tau) A_i \Omega(\tau)$$

Constant shift in A_0 , time *de*pendent rotation of A_i .

$D_i A_0 = (\partial_i - i g [A_i ,) A_0$ *not* invariant if $[A_i , t_N] \neq 0$.

Of course, E_i^{4D} invariant under $U_c(\tau)$.

$E_i^{3D} = D_i A_0$ at small A_0, but *not* at large A_0! Diakonov & Oswald '03, '04

Form E_i^{3D} from Wilson lines?

Electric field of Wilson lines

Wilson line SU(N) matrix, so diagonalize: $\mathbf{L}(x) = \Omega(x)^\dagger e^{\,i\lambda(x)}\,\Omega(x)$

Static gauge transf.'s: diagonal matrix λ invariant, Ω changes.

Strictly periodic $U_c(\tau)^N$: $\lambda_a \rightarrow \lambda_a + 2\,\pi \times$ integer: λ_a periodic. Of course!

Use just eigenvalues, $E_i^{3D} \sim \partial_i \lambda$? No, $E_i^{3D} \neq D_i A_0$ at small A_0

E_i^{3D} hermitean, so: $E_i^{3D}(x) = \dfrac{T}{ig}\mathbf{L}^\dagger(x) D_i \mathbf{L}(x)(1 + c_1 |\mathrm{tr}\mathbf{L}|^2 + \ldots)$

Small A_0 OK, but does *not* fix c_1, c_2...

Large but *abelian* A_0, $A_i = 0$: if $E_i^{3D} = \partial_i A_0$, *must* have $c_1=c_2=\ldots=0$.

Necessary for interfaces to match at *leading* order. Beyond: c_1, c_2 ... $\sim g^2$.

In general, *infinite* number of terms enter.

Calculable perturbatively, match through interfaces, Z(N) or U(1).

$L_{\rm eff}$ of Wilson lines at $0^{\rm th}$ order

To leading order, $$E_i^{3D} = \frac{T}{ig}\,\mathbf{L}^\dagger\, D_i\, \mathbf{L}$$

Gauge covariant "average" in time: $\mathbf{L}(\tau) = e^{ig\int_0^\tau A_o(\tau')d\tau'}$; $\mathbf{L} = \mathbf{L}(1/T)$

$$E_i^{3D}/T = \int_0^{1/T} d\tau\; \mathbf{L}(\tau)^\dagger\, \partial_i A_0(\tau)\, \mathbf{L}(\tau) - \mathbf{L}^\dagger[A_i, \mathbf{L}]$$

Math.'y: left invariant one form (Nair).

Lagrangian continuum form of
Banks and Ukawa '83, on lattice: $$\mathcal{L}^{eff}_{cl} = \frac{1}{2}\,\mathrm{tr}\, G_{ij}^2 + \frac{T^2}{g^2}\,\mathrm{tr}\,|\mathbf{L}^\dagger D_i \mathbf{L}|^2$$

To $0^{\rm th}$ order, Lagrangian for SU(N) principal chiral field.
Non-renormalizable in 3D, but only effective theory for r > 1/T.

Instanton number in 4D = winding number of **L** in 3D
Linear model: Vuorinen & Yaffe '06 (Match by imposing extra symmetry)

Confinement & adjoint Higgs phase?

Diagonalize $L = \Omega^\dagger \, e^{i\lambda} \, \Omega$

Static gauge transf.'s U: $e^{i\lambda}$ invariant, Ω not: $\Omega \to \Omega\,\mathcal{U} \; , \; D_i \to \mathcal{U}^\dagger \, D_i \, \mathcal{U}$

Electric field term:

$$\mathrm{tr}\,|\mathbf{L}^\dagger D_i \mathbf{L}|^2 \;=\; \mathrm{tr}\,(\partial_i \lambda)^2 \;+\; \mathrm{tr}\,|[\Omega\, D_i\, \Omega^\dagger, \mathrm{e}^{i\lambda}]|^2$$

1st term same as abelian

2nd term gauge *in*variant coupling of electric & magnetic sectors

$\langle e^{i\lambda} \rangle = 1$: no Higgs phase. True in perturbation theory, order by order in g^2

If $\langle e^{i\lambda} \rangle \neq 1$, Higgs phase,

In weak coupling, diagonal gluons massless,
off diagonal massive (a,b = 1...N)

$$m^2_{ab} = g^2 |\mathrm{e}^{i\lambda_a} - \mathrm{e}^{i\lambda_b}|^2$$

But for 3D theory, gluons couple *strongly*. Effects of Higgs phase?

N.B.: above 't Hooft's abelian projection for Wilson line.

Loop potential, perturbative & not.

U(N): constant **L**, 1 loop order:

$$\mathcal{L}^{eff}_{1\ loop} = -\,\frac{2\,T^4}{\pi^2}\,\sum_{m=1}^{\infty}\,\frac{1}{m^4}\,\left|\mathrm{tr}\,\mathbf{L}^m\right|^2\ .$$

Perturbative vacuum $<e^{i\lambda}> = 1$,
stable to leading order, to *any* finite order in g^2 .

Can compute corrections to effective Lagrangian at next to leading order, NLO.
At NNLO, $\sim g^3$, need to resum m_{Debye} . Eventually, $m_{magnetic}$

SU(3) lattice: near T_c , pressure(T) $\sim T^4$ *and* $\sim T^2$.

To represent: add, *by hand:*

$$\mathcal{L}^{eff}_{non-pert.}(\mathbf{L}) = +\,B_f\,T^2\ \left|\mathrm{tr}\,\mathbf{L}\right|^2$$

$B_f \sim \#\,T_c^2$ “fuzzy” bag const. Non-pert., infinity of possible terms.

$B_f \neq 0 \Rightarrow <e^{i\lambda}> \neq 1 \Rightarrow$ Higgs phase near T_c

Confinement in L_{eff}

SU(N), no quarks: in confined state, all Z(N) charged loops vanish:

$$\langle \mathrm{tr}\, \mathbf{L}^{j}_{\mathrm{conf}} \rangle = 0 \ , \ j = 1 \ldots (N-1)$$

Satisfied by "center symmetric" vacuum:

$$\mathbf{L}_{\mathrm{conf}} = \mathrm{diag}(1, z, z^2 \ldots z^{N-1}) \ , \ z = \mathrm{e}^{2\pi i/N} .$$

At finite N, perturbative pressure($\mathbf{L}_{\mathrm{conf}}$) *negative*. Not so good.

Large N: pressure($\mathbf{L}_{\mathrm{conf}}$) ~ 1, vs. ~ N^2 in deconfined phase.

At N=∞, center sym. state *can* represent confined vacuum.

$\mathbf{L}_{\mathrm{conf}}$ familiar from random matrix models:
completely *flat* eigenvalue distribution, from eigenvalue repulsion.

Where does eigenvalue repulsion arise *dynamically*?

Dynamical eigenvalue repulsion

Small volume: on *very* small sphere (R = radius, $g^2(R) << 1$ - Aharony et al.)

$L_{\rm eff}$ = random matrix model for constant mode. Measure gives eig. repulsion:

$$\mathcal{L}^{\rm eff}_{\rm Vandermonde} \sim - \sum_{a,b=1}^{N} \log(|e^{i\lambda_a} - e^{i\lambda_b}|^2)$$

Large volume: *no* sign of eigenvalue repulsion from perturbative loop potential.

From measure? But regularization dependent!

Eig. repulsion arises, *naturally*, from adjoint Higgs phase: $m^2_{ab} \sim |\mathrm{e}^{i\lambda_a} - \mathrm{e}^{i\lambda_b}|^2$

One loop order in 3D:

$$\mathcal{L}^{\rm eff}_{\rm 1\,loop} \sim -(m^2)^{3/2} \sim - \sum_{a,b=1}^{N} (g^2|e^{i\lambda_a} - e^{i\lambda_b}|^2)^{3/2}$$

Two loop: $L_{\rm Vandermonde}{}^{\rm eff}$?

But: 3D theory strongly coupled: magnetic glueballs *heavy*, not light.

In $L_{\rm eff}$, confinement arises *uniquely* from (dynamical) eigenvalue repulsion.

Could study numerically. Field theory of "not so" random matrices.

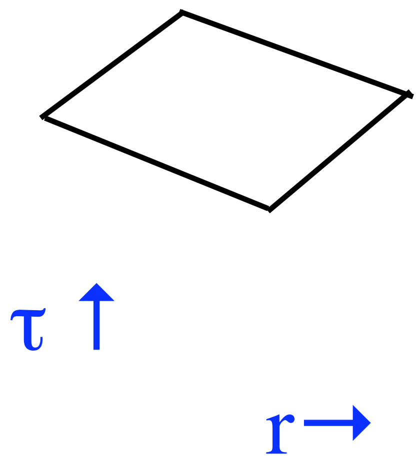
How to tell if adjoint Higgs phase?

No absolute, gauge invariant measure. Only differences qualitative.

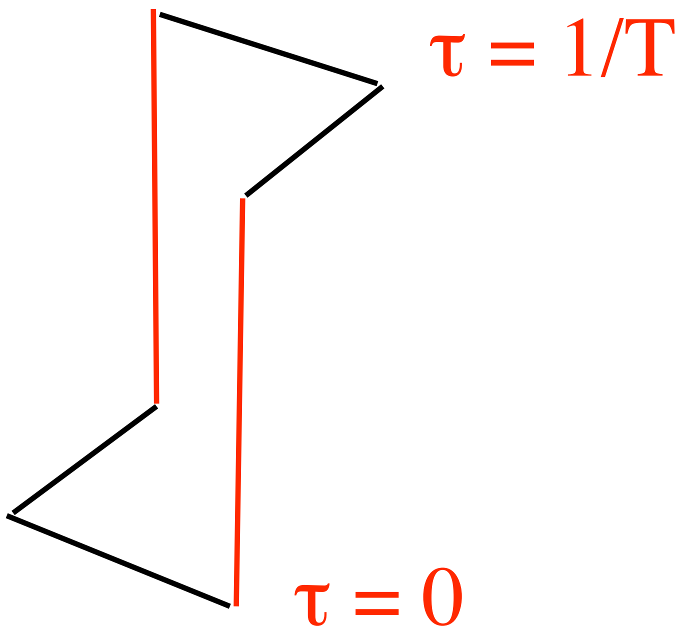
But: usually magnetic glueballs and Wilson line mix *very* little.
Higgs phase should *strongly* mix glueballs and Wilson line.

Maybe: measure magnetic glueballs from plaquettes "split" in time:

Usual spatial plaquette



"Split" spatial plaquette



Fuzzy bags and Wilson lines: credits

1. Helsinki program & renormalized loops

Resummation: Braaten & Nieto '96. Kraemmer & Rebhan '03. Andersen & Strickland '04.
Kajantie, Laine, Rummukainen, & Schröder '00, '02, & '03.
Giovannangeli '05. Laine & Schröder '05 & '06. Di Renzo, Laine +... '06

Renormalized loops: Kaczmarek, Karsch, Petreczky, & Zantow '02 Dumitru, Hatta... below.
Petreczky & Petrov '04. Gupta, Hubner, & Kaczmarek '06

2. (Some) large gauge transformations & interfaces

Large gauge transf.'s: Diakonov & Oswald '03 & '04. Megias, Ruiz Arriola, & Salcedo '03.
Center symmetry, G(2): Holland, Minkowski, Pepe, & Wiese '03. Pepe & Wiese '06.
Z(N) interfaces: Korthals-Altes et al '93, '99, '01, '02, '04

3. The electric field in terms of Wilson lines

Before: RDP '00. Dumitru & RDP '00-'02. Dumitru, Hatta, Lenaghan, Orginos & RDP '03
Dumitru, Lenaghan, & RDP '04. Oswald & RDP '05.
Linear model: Vuorinen & Yaffe '06. Here, non-linear model: RDP '06.
Lattice action: Banks & Ukawa '83. Bialas, Morel, & Petersson '04.

4. Confinement as an (adjoint) Higgs effect

Center symmetric vacuum: Weiss '82. Karsch & Wyld '86. Polchinski '91. Schaden '04.
Small sphere: Aharony, Marsano, Minwalla, Papadodimas, & Van Raamsdonk '03 & '05