Punchline:

Effective theory for deconfinement, *near* T_c.

There's *always* an effective theory.

Solidly based upon results from the lattice

Coming soon:

Real competition for AdS/CFT

http://www.anti-powerpoint-party.com

An effective (matrix) model for deconfinement

Lattice: SU(N) gauge theories, with*out* quarks.

Simulations show: $N = 3$ *close* to $N = \infty$. *Not* just the pressure.

Simple matrix model, valid in large N expansion (*no* small masses).

Fit to pressure for *all* N with 2 parameters

Good agreement with interface tensions

*Dis*agrees with the (renormalized) Polyakov loop - ?

New: adjoint Higgs phase, with *split* masses, for $T < 1.2\, T_c$.

Unexpected: transition region *very narrow,* $< 1.2\, T_c$

Dumitru, Guo, Hidaka, Korthals-Altes, & RDP, arXiv:1011.3820 + ...
Generalization of Meisinger, Miller, Ogilvie ph/0108009

Y. Hidaka & RDP, 0803.0453, 0906.1751, 0907.4609, 0912.0940.

What the lattice tells us

Weak dependence on # colors

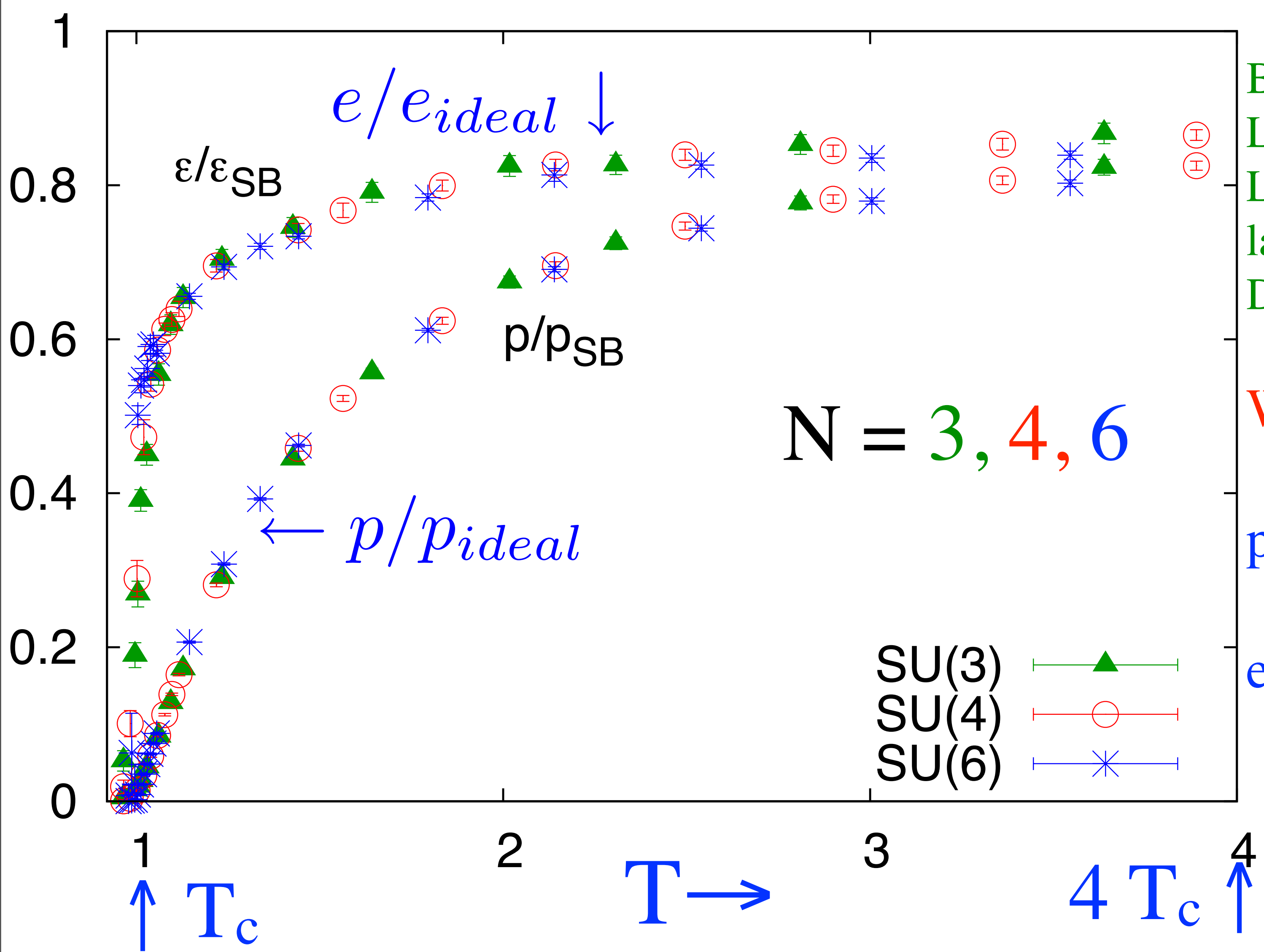
Not just the pressure...

Lattice: SU(N) thermodynamics ≈ *in*dependent of N

SU(N) gauge theories with*out* quarks, temperature T ≠ 0

Scaled by ideal gas, energy “e” and pressure “p” *approximately* independent of N.

e and p ≈ 0 below T_c: ~ N^2 - 1 gluons above T_c, vs ~ 1 hadrons below.



Boyd, Engels, Karsch, Laermann, Legeland, Luetgemeier, Petersson, lat/9602007

Datta & Gupta, 1006.0938

Width of “semi”-QGP?

pressure: *wide* ~ 4 T_c

energy: *narrow*, ~ 1.5 T_c

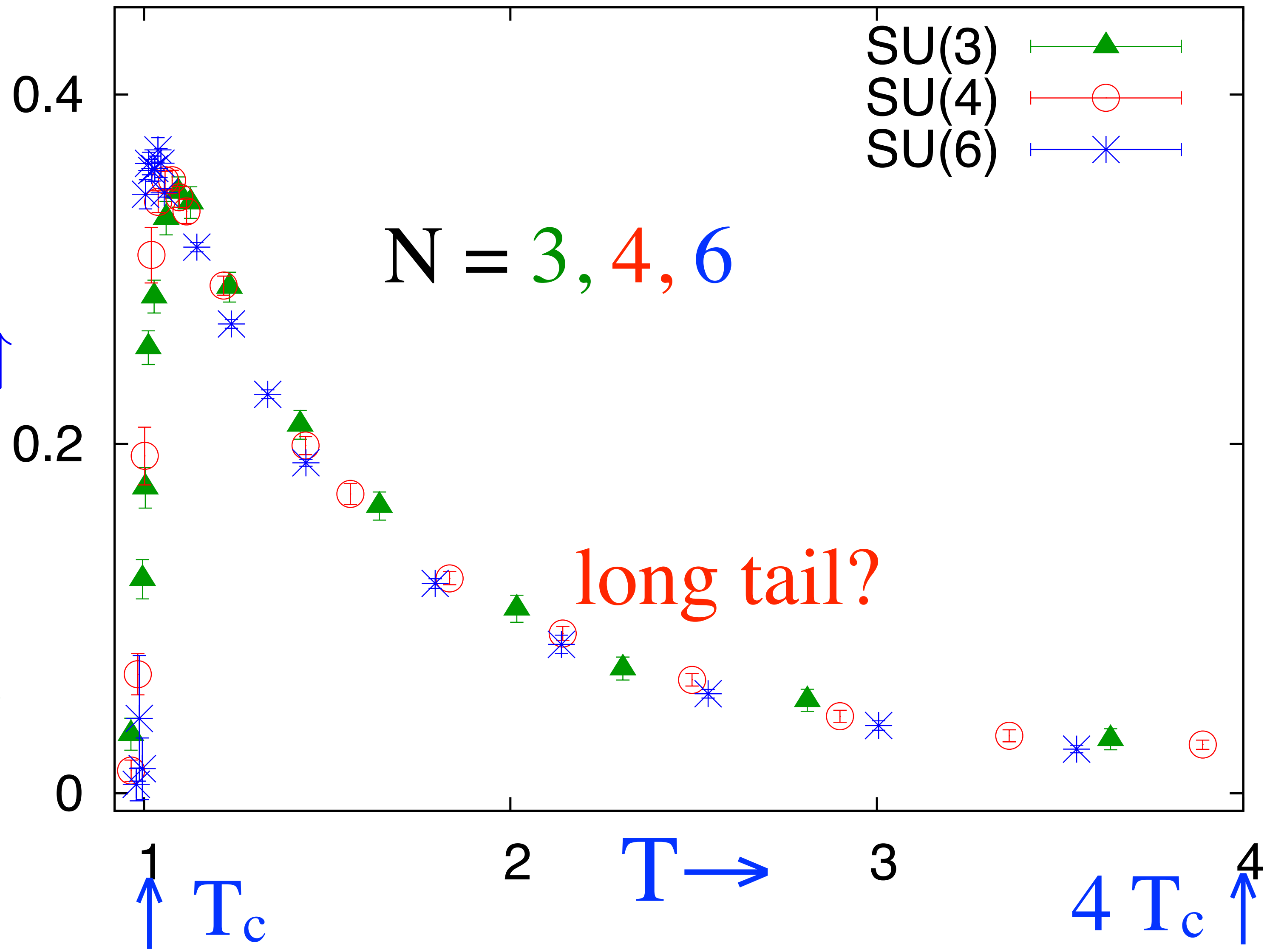
Lattice: peak in conformal anomaly

For SU(N), “peak” in $e\text{-}3p/T^4$ just above T_c. *Approximately* uniform in N.

Not near T_c: transition *2nd* order for N = 2, *1st* order for *all* N ≥ 3

N=3: *weakly* 1st order. N = ∞: *strongly* 1st order (latent heat ~ N^2)

$$\frac{1}{N^2-1}\,\frac{e-3p}{T^4}\uparrow$$



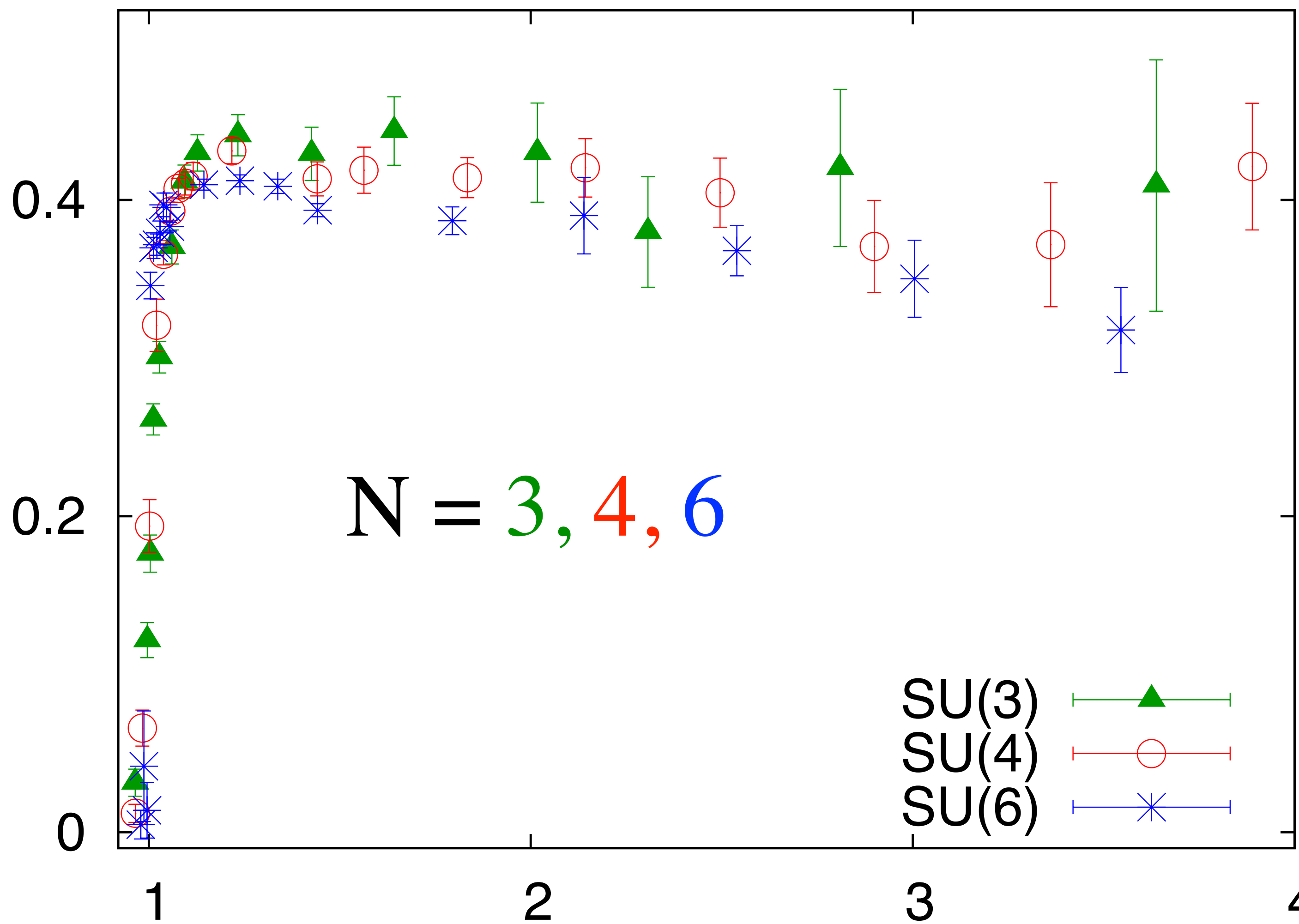
Datta & Gupta, 1006.0938

Lattice: tail in the conformal anomaly

Scaling: (e-3p)/T^2 *approximately* constant above 1.2 T_c: MMO '01; RDP, ph/0608242

Only true near T_c ; eventually, (e-3p)/T^4 ~ g^4(T)

$$\frac{1}{N^2-1}\ \frac{e-3p}{T^2\,T_c^2}\ \uparrow$$



↑ T_c T→ 4 T_c ↑

Datta & Gupta, 1006.0938

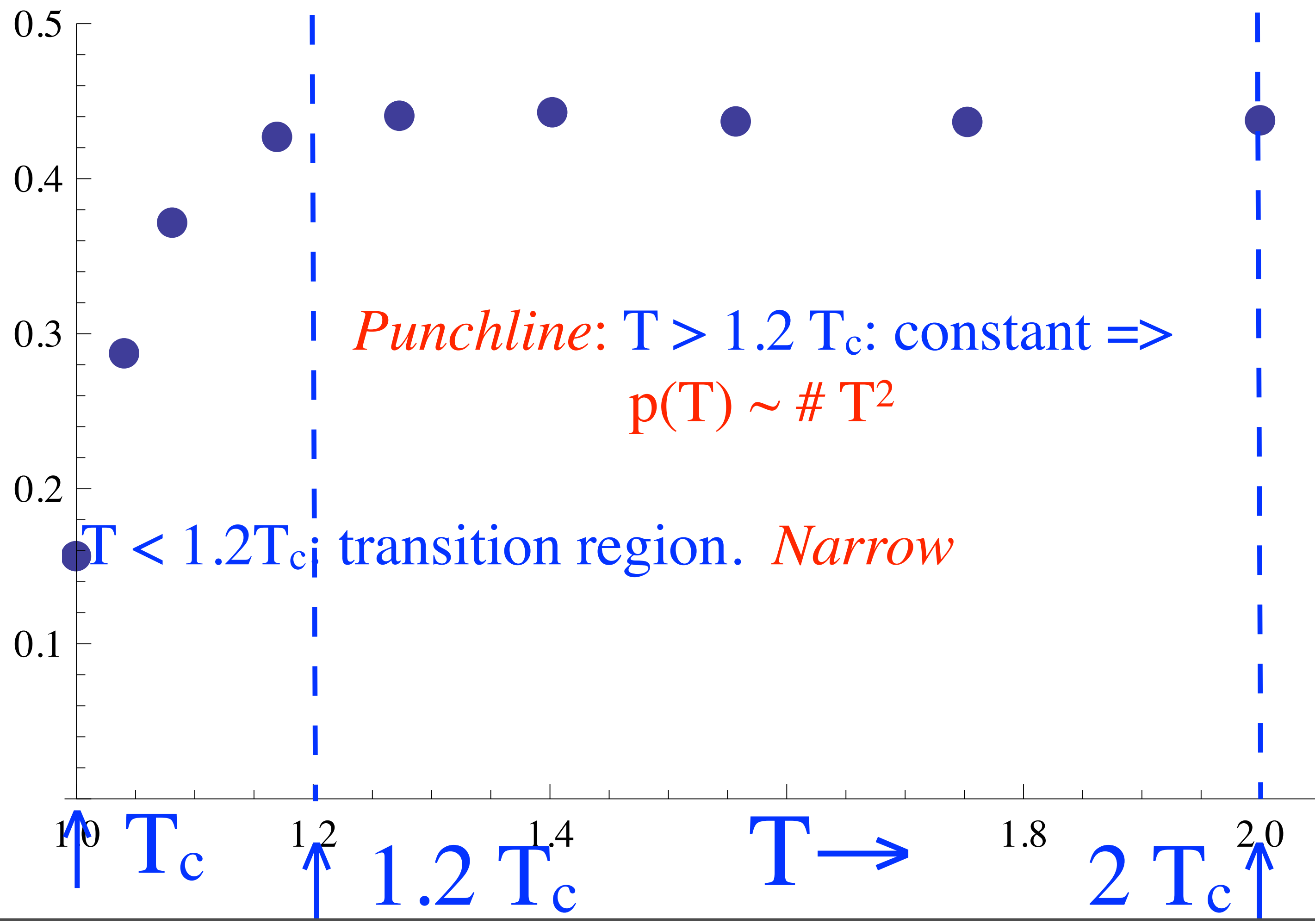
Lattice: *precise* results for three colors

Lattice: WHOT. Change # time steps at fixed lattice scale. Higher precision, ± 1%

$$T : 1.2 \rightarrow 2\,T_c : \frac{e-3p}{T^2} \approx (543\,\mathrm{MeV})^2 \pm 1\%$$

$$p(T) \approx \#\, T^2 (T^2 - c\,T_c^2)\,,\; c = 1.00 \pm .01$$

$$\frac{1}{8}\,\frac{e-3p}{T^2\,T_c^2} \uparrow$$



Umeda, Ejiri, Aoki, Hatsuda, Kanaya, Maezawa, Ohno, 0809.2842

At XQCD11:

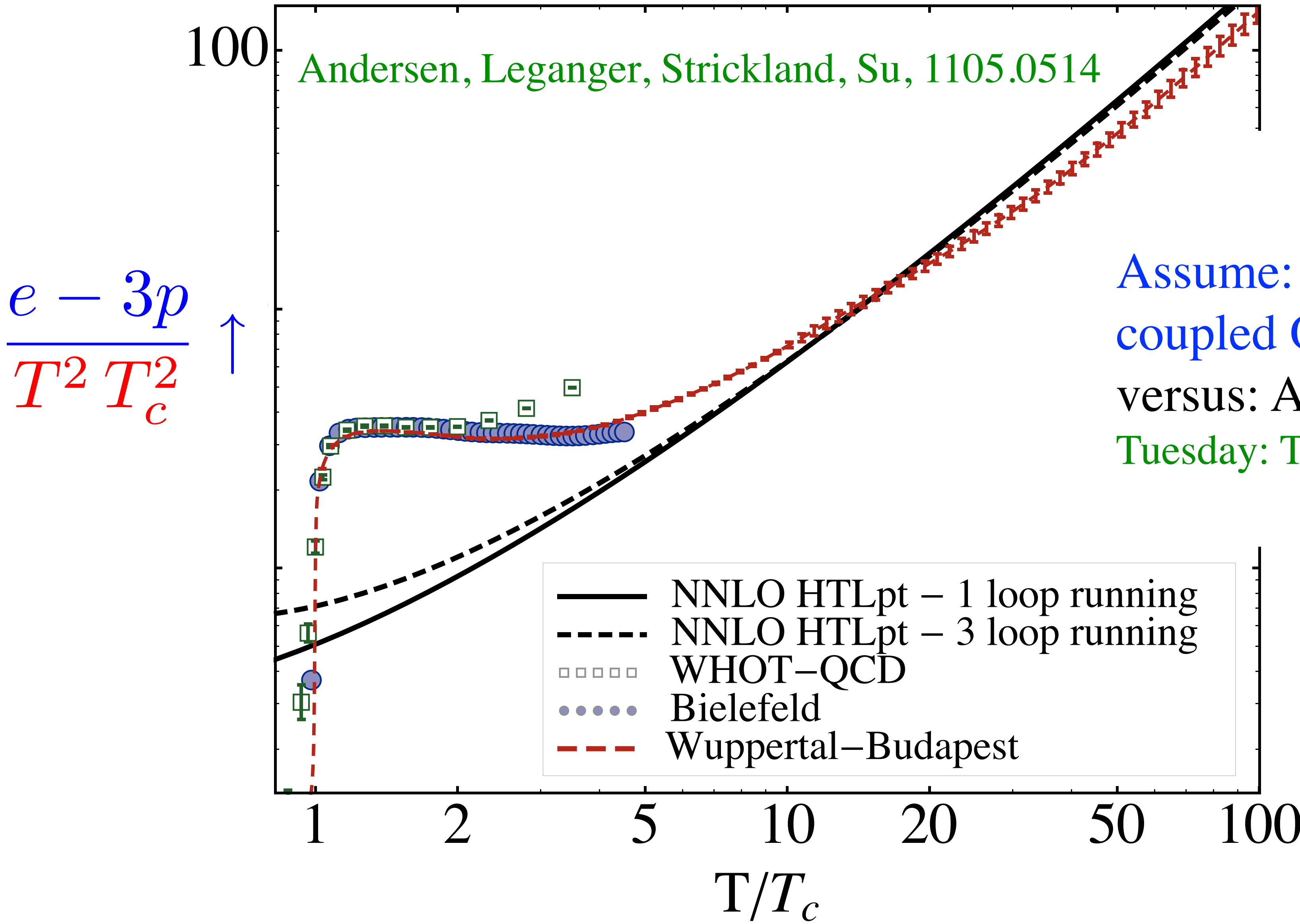
Shinji Ejiri, Kazayuki Kanaya

Not strong coupling, even at T_c

QCD coupling runs like $\alpha(2\pi T)$, *intermediate* at T_c, $\alpha(2\pi T_c) \sim 0.3$

Braaten & Nieto, hep-ph/9501375, Laine & Schröder, hep-ph/0503061 & 0603048

HTL resummed perturbation theory, NNLO, good to ~ 8 T_c :



Assume: *not* a strongly coupled QGP near T_c!

versus: AdS/CFT....

Tuesday: Takuya Saito

“Hidden” Z(2) spins in SU(2)

Consider *constant* gauge transformation:

$$U_c = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = -\mathbf{1}$$

As $U_c \sim \mathbf{1}$, locally gluons *invariant*:

$$A_\mu \to U_c^\dagger \, A_\mu \, U_c = +\, A_\mu$$

Nonlocally, Wilson *line* changes:

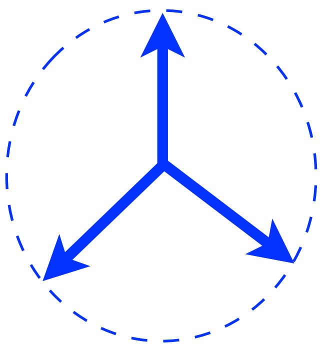
$$\mathbf{L} = \mathcal{P}\, \mathrm{e}^{ig \int_0^{1/T} A_0 \, d\tau} \to -\, \mathbf{L}$$

L ~ propagator for “test” quark.

SU(3): det $U_c = 1 \Rightarrow$

j = 0, 1, 2

$$U_c = \mathrm{e}^{2\pi i j/3}\, \mathbf{1}$$



N = 3

SU(N): $U_c = \mathrm{e}^{2\pi i j/N}\, \mathbf{1}$: Z(N) symmetry.

Z(N) spins of ‘t Hooft, with*out* quarks

Quarks ~ background Z(N) field, *break* Z(N) sym.

$$\psi \to U_c \psi = -\, \psi$$

Usual spins vs Polyakov Loop

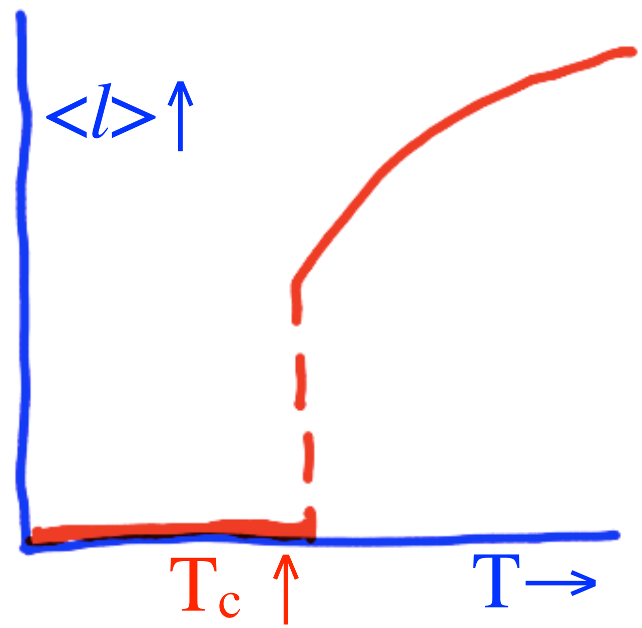
L = SU(N) *matrix*, Polyakov loop l ~ trace:

$$\ell = \frac{1}{N}\,\mathrm{tr}\,\mathbf{L}$$

Confinement: $F_{\text{test qk}} = \infty \Rightarrow \langle\, l \,\rangle = 0$

$$< \ell > \sim \mathrm{e}^{-F_{\text{test qk}}/T}$$

Above T_c, $F_{\text{test qk}} < \infty \Rightarrow \langle\, l \,\rangle \neq 0$



$\langle\, l \,\rangle$ measures ionization of color:

partial ionization when $0 < \langle\, l \,\rangle < 1$: “semi”-QGP

Svetitsky and Yaffe ’80:

SU(3) 1st order because Z(3) allows *cubic* terms:

$$\mathcal{L}_{\text{eff}} \sim \ell^3 + (\ell^*)^3$$

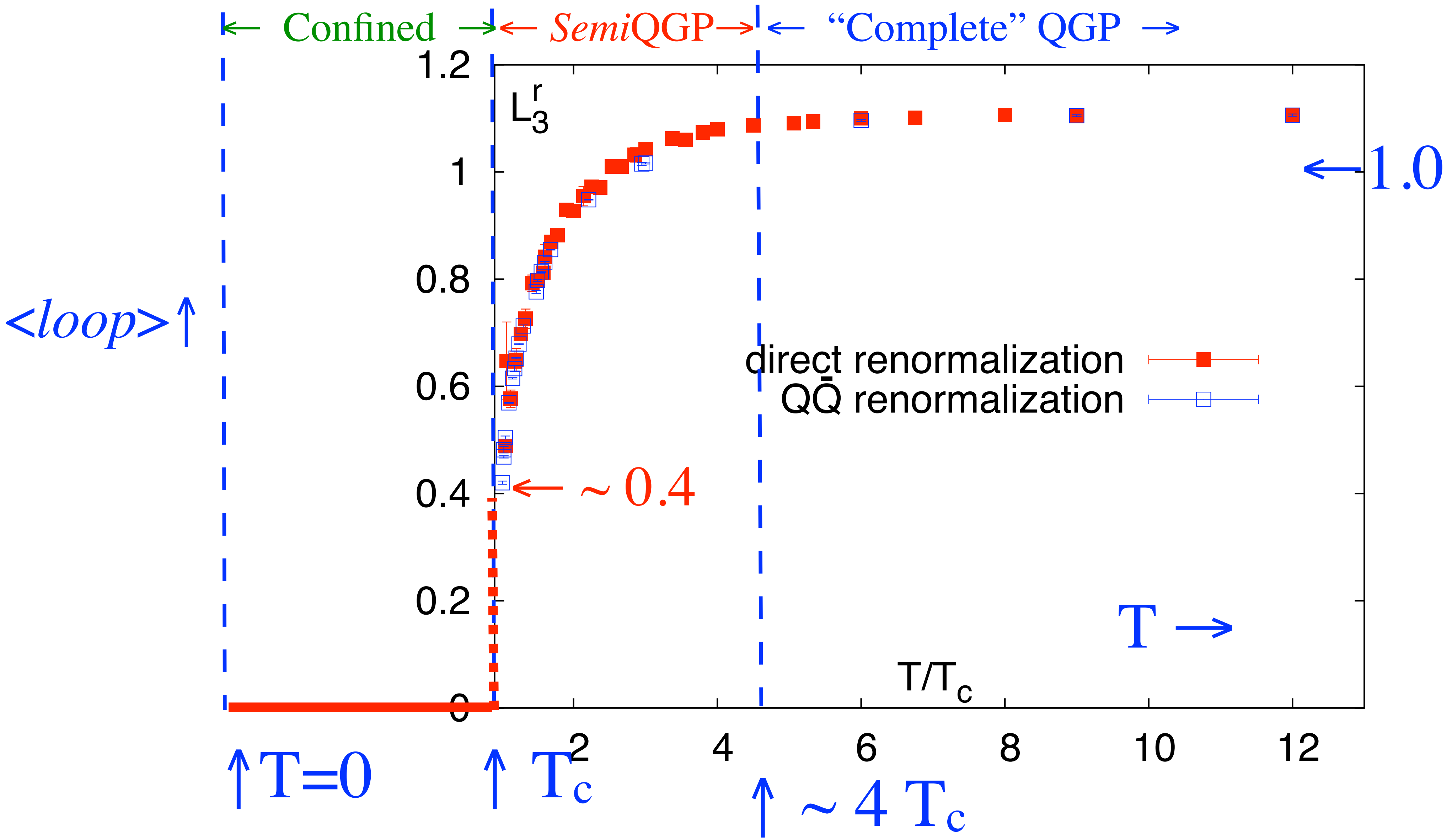
Does *not* apply for N > 3. *So why is deconfinement 1st order for all N ≥ 3?*

Polyakov Loop from Lattice: pure Glue, no Quarks

Lattice: (*renormalized*) Polyakov loop. Strict order parameter

Three colors: Gupta, Hubner, Kaczmarek, 0711.2251.

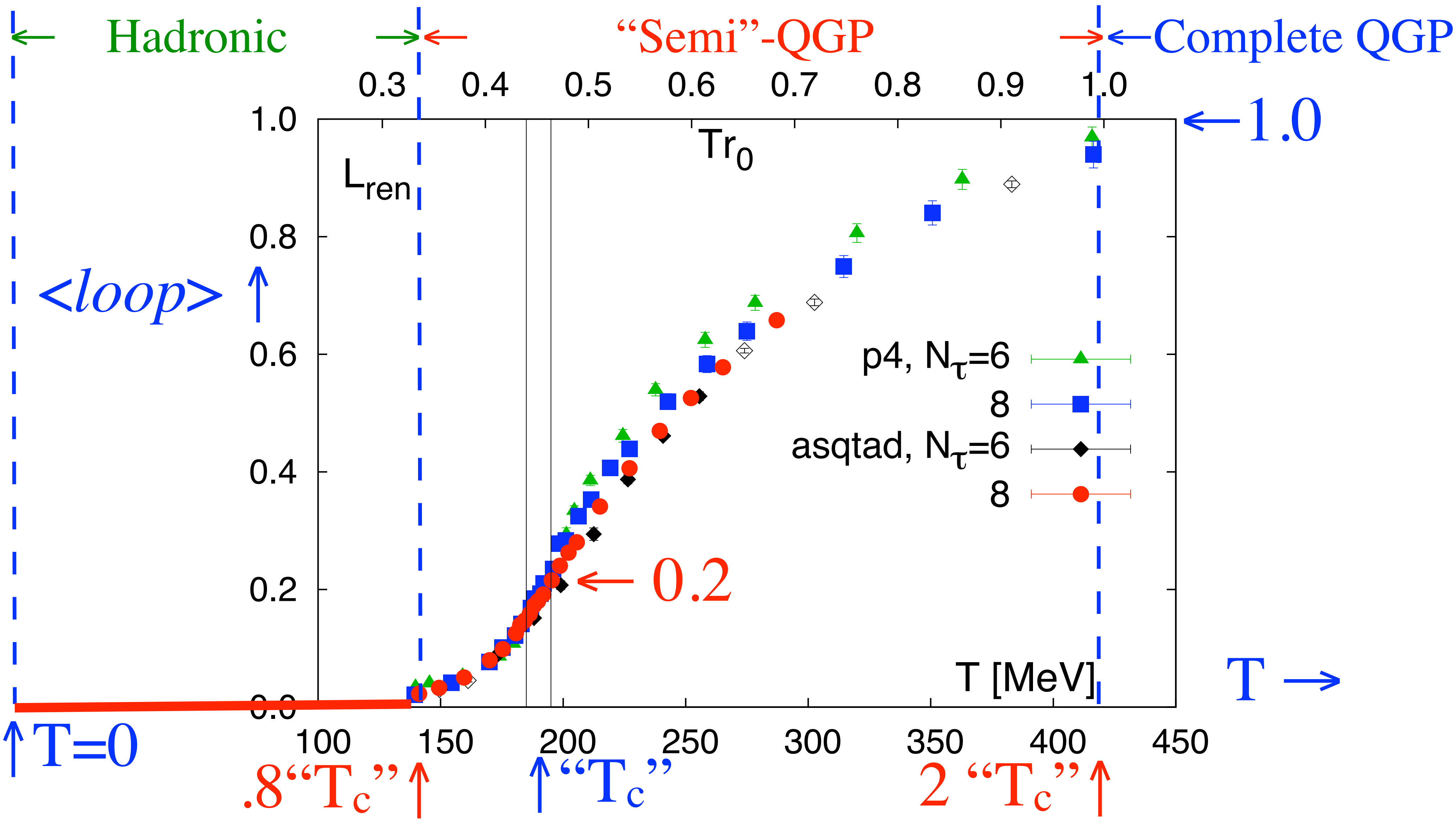
Suggests *wide* transition region, like pressure, to ~ 4 T_c.



Polyakov Loop from Lattice: Glue plus Quarks, "T_c"

Quarks ~ background Z(3) field. Lattice: Bazavov et al, 0903.4379.

3 quark flavors: *weak* Z(3) field, does *not* wash out approximate Z(3) symmetry.

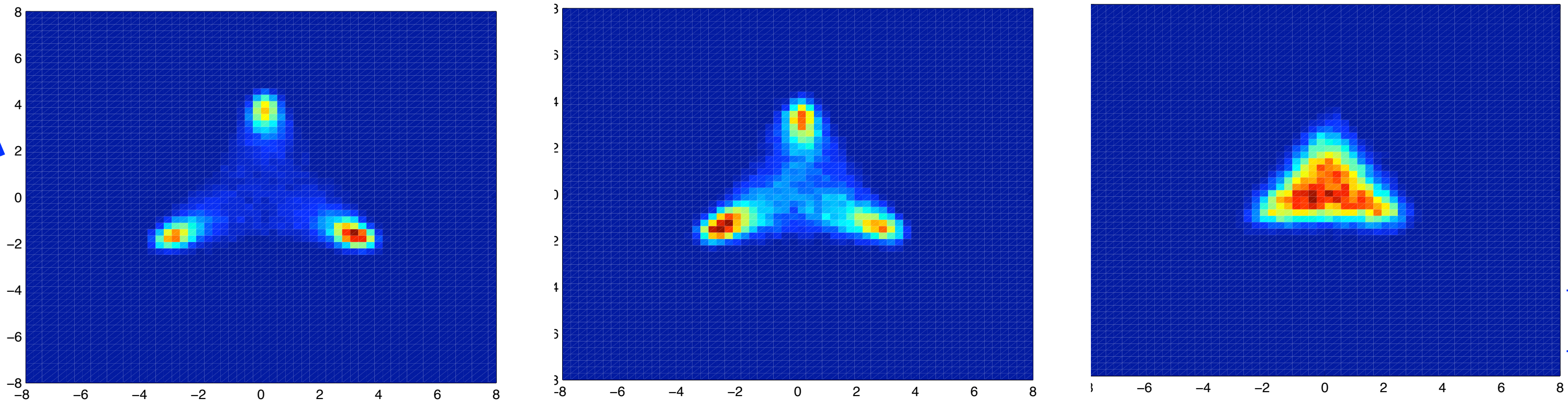


Interface tensions: order-order & order-disorder

Lattice, A. Kurkela, unpub.'d: 3 colors, loop l complex.

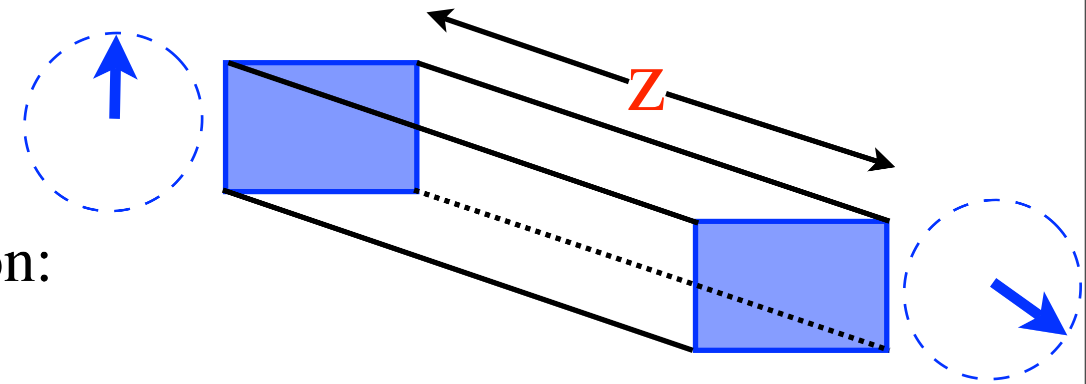
Distribution of loop shows Z(3) symmetry:

Im $l \uparrow$



Re $l \rightarrow$

$T >> T_c$ $T \sim T_c$ $T < T_c$



Interface tension: box long in z.

Each end: distinct but *degenerate* vacua.

Interface forms, action ~ interface tension:

$T > T_c$: order-order interface = ‘*t Hooft loop*:

measures response to *magnetic* charge

Korthals-Altes, Kovner, & Stephanov, hep-ph/9909516

$$Z \sim e^{-\sigma_{int} V_{tr}}$$

Also: *if* trans. 1st order, order-*dis*order interface *at* T_c .

Lattice: order-order interface tensions σ

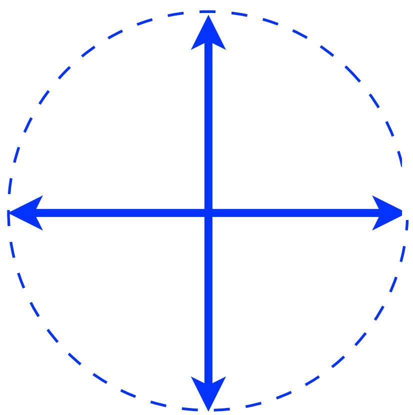
Lattice: de Forcrand & Noth, lat/0510081. σ ~ universal with N

Semi-classical σ : Giovanengelli & Korthals-Altes ph/0102022; /0212298; /0412322: *GKA '04*

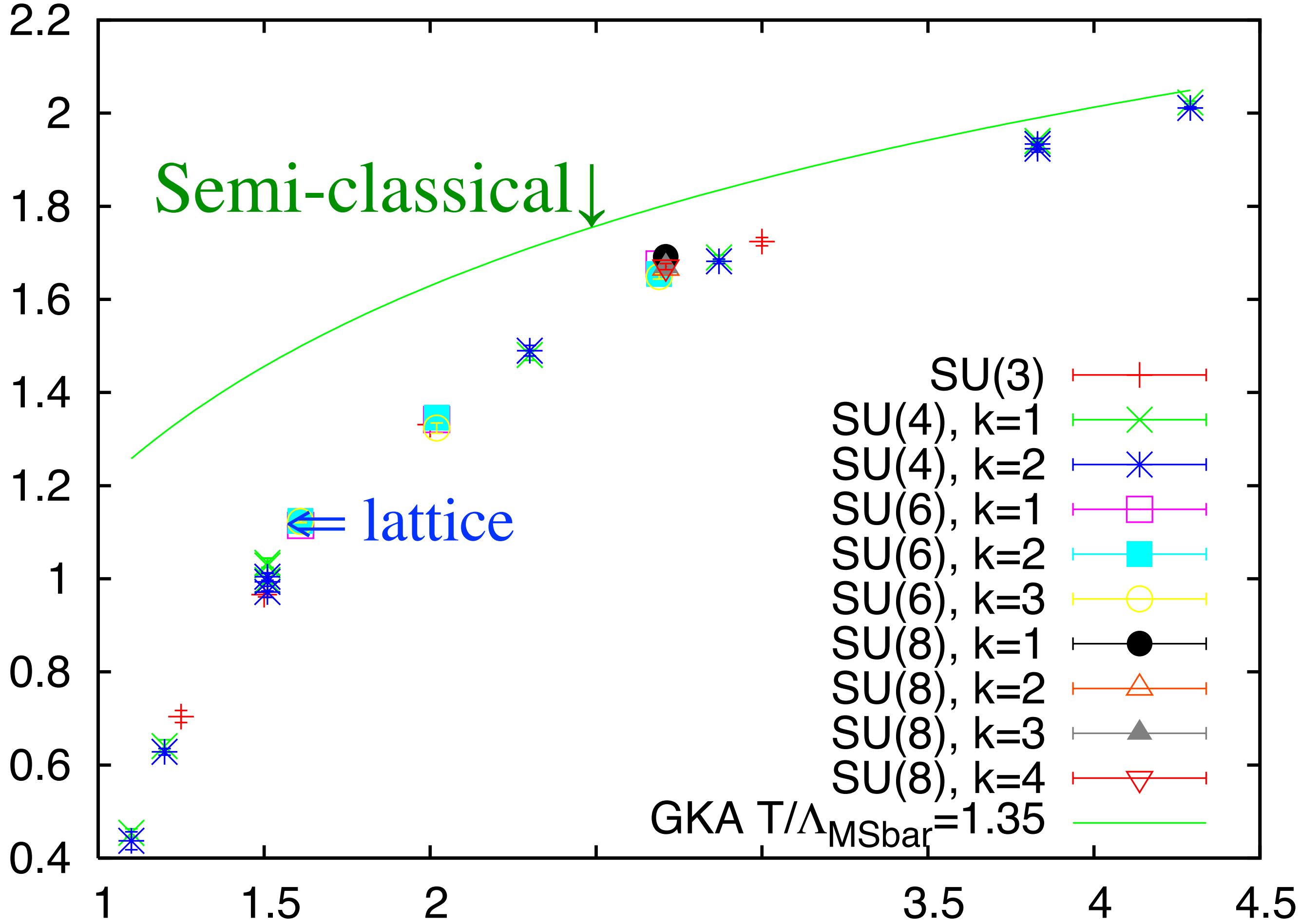
Above 4 T_c, semi-class σ ~ lattice. Below 4 T_c, lattice σ << semi-classical σ.

Even so, when N > 3, *all* tensions satisfy "Casimir scaling" at T > 1.2 T_c.

$$\frac{\sigma_k}{T^2\,k(N-k)} \uparrow$$



N = 4



↑ T_c T⟶ 4.5 T_c↑

Lattice: A_0 mass as $T \to T_c$ - *up or down*?

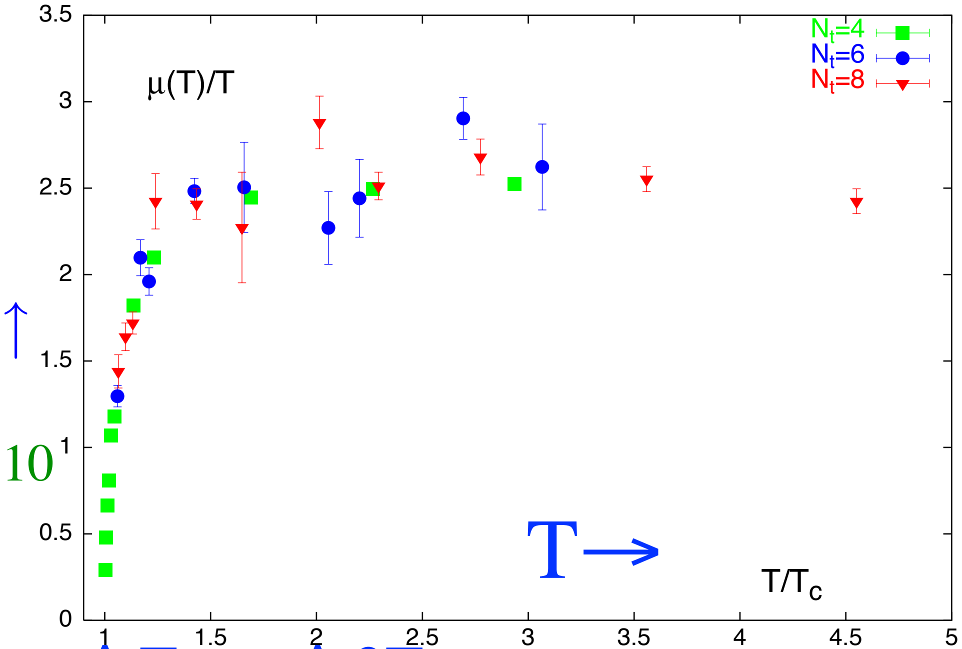
Gauge invariant: 2 pt function of loops:

$$\langle \mathrm{tr}\, \mathbf{L}^\dagger(x)\, \mathrm{tr}\, \mathbf{L}(0) \rangle \sim e^{-\mu x}/x^d$$

μ/T goes *down* as $T \to T_c$

Kaczmarek, Karsch, Laermann, Lutgemeier lat/9908010

$\frac{\mu}{T} \uparrow$



$\uparrow T_c \quad \uparrow 2T_c$

Gauge dependent: singlet potential

$$\langle \mathrm{tr}\, \left(\mathbf{L}^\dagger(x)\, \mathbf{L}(0) \right) \rangle \sim e^{-m_D x}/x$$

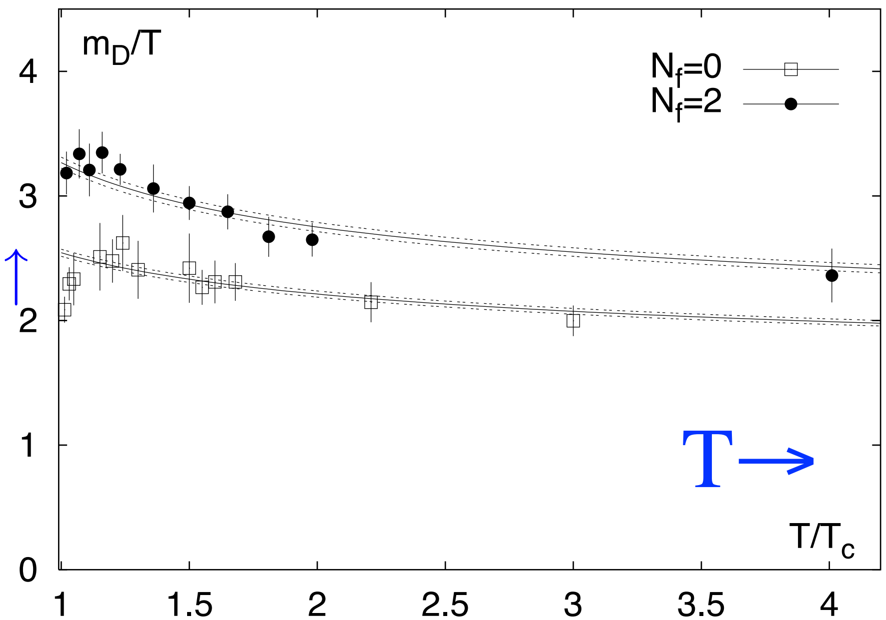
m_D/T goes *up* as $T \to T_c$

Cucchieri, Karsch, Petreczky lat/0103009,

Kaczmarek, Zantow lat/0503017

Tuesday: *Tereza Mendes*

$\frac{m_D}{T} \uparrow$



$\uparrow T_c \quad \uparrow 2T_c$

Which way do masses go as $T \to T_c$?

Both are constant above ~ 1.5 T_c.

Other models

Fit *only* the pressure, *not* interface tensions.

Masses as $T \rightarrow T_c^+$:

some go *up* (massive gluons)

some go *down* (Polyakov loops)

Models for the "s"QGP, T_c to 4 T_c

1. Massive gluons: Peshier, Kampfer, Pavlenko, Soff '96...Castorina, Miller, Satz 1101.1255
Castorina, Greco, Jaccarino, Zappala 1105.5902

Mass decreases pressure, so adjust m(T) to fit p(T). Simple model. Gluons *very* massive near T_c.

$$p(T) = \# T^4 - m^2 T^2 + \ldots$$

2. Polyakov loops: Fukushima ph/0310121...Hell, Kashiwa, Weise 1104.0572
Monday: *Masanobu Yahiro, Yuji Sakai*. Tuesday: *Takahiro Sasaki*

Effective potential of Polyakov loops.
Potential has 5 parameters...
With quarks, at $T \neq 0$, can go from $\mu = 0$ to $\mu \neq 0$

$$V_{eff}(T) \sim m^2 \ell^* \ell + T \log f(\ell^* \ell)$$

$$m^2 = T^4 \sum_{i=0}^{3} a_i (T_c/T)^i$$

3. AdS/CFT: Gubser, Nellore 0804.0434...Gursoy, Kiritsis, Mazzanti, Nitti, 0903.2859

Add potential for dilaton, φ, to fit pressure.
Only infinite N. Relatively simple potential,

$$V(\phi) \sim \cosh(\gamma\phi) + b\,\phi^2$$

4. Monopoles: Liao & Shuryak, 0804.0255. Vortices: Tuesday: *Takuya Saito*

Matrix model: two colors

Simple approximation

Two colors: transition 2nd order, vs 1st for $N \geq 3$

Using large N expansion at $N = 2$

Matrix model: SU(2)

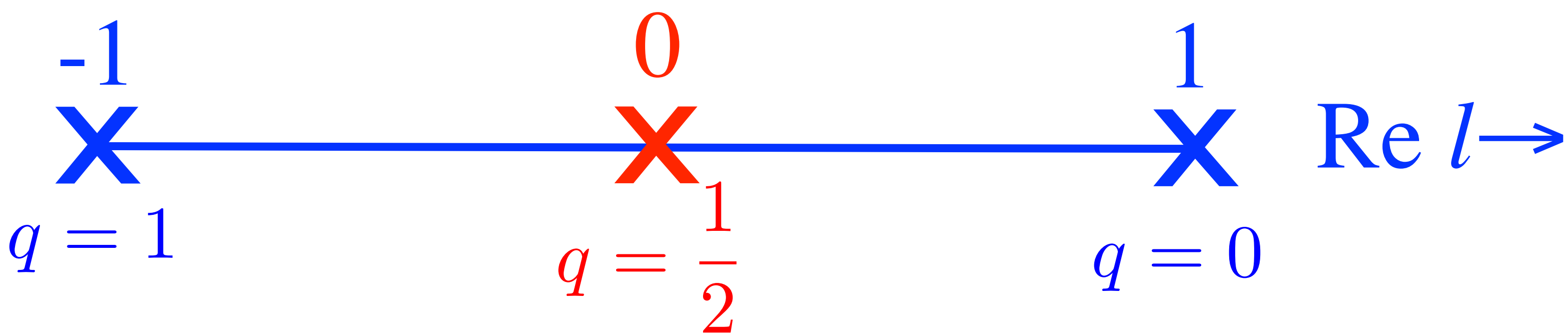
Simple approximation: *constant* $A_0 \sim \sigma_3$, *non*perturbative, ~ 1/g:

$$A_0^{cl} = \frac{\pi T}{g}\, q\, \sigma_3 \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad \mathbf{L}(q) = \begin{pmatrix} e^{i\pi q} & 0 \\ 0 & e^{-i\pi q} \end{pmatrix}$$

Single dynamical field, q

Loop *l* real. Z(2) degenerate vacua q = 0 and 1:

$$\ell = \cos(\pi q)$$



Point *half*way in between: q = ½ , *l* = 0 .

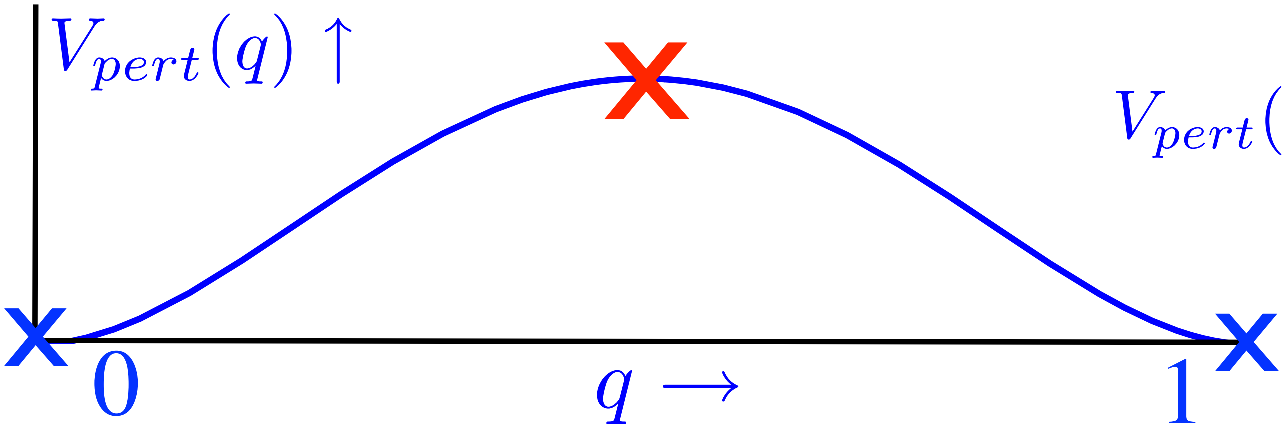
Confined vacuum, $\mathbf{L}_c$,

$$\mathbf{L}_c = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$$

Classically, A_0^{cl} has zero action: *no* potential for q.

Potential for q, interface tension

Computing to one loop order about A_0^{cl} gives a potential for q: Gross, RDP, Yaffe, '81



$$V_{pert}(q) = \frac{4\pi^2}{3} T^4 \, q^2 (1-q)^2$$

Use $V_{pert}(q)$ to compute σ: Bhattacharya, Gocksch, Korthals-Altes, RDP, ph/9205231.

$$V_{tot}(q) = \frac{2\pi^2 T^2}{g^2} \left(\frac{dq}{dz}\right)^2 + V_{pert}(q) \qquad \Rightarrow \sigma = \frac{4\pi^2}{3\sqrt{6}} \frac{T^2}{\sqrt{g^2}}$$

Balancing $S_{cl} \sim 1/g^2$ and $V_{pert} \sim 1$ gives $\sigma \sim 1/\sqrt{g^2}$ (*not* $1/g^2$).

Width interface ~ 1/g, justifies expansion about constant A_0^{cl}. GKA '04: $\sigma \sim ... + g^2$

Symmetries of the q's

Wilson line **L** *not* gauge invariant, $\mathbf{L} \rightarrow \Omega^\dagger \, \mathbf{L} \, \Omega$.
Its eigenvalues, $e^{\pm i \pi q}$, are.

$$\mathbf{L}(q) = \begin{pmatrix} e^{i\pi q} & 0 \\ 0 & e^{-i\pi q} \end{pmatrix}$$

O*rdering* of L's eigenvalues irrelevant.

Symmetries: $q \rightarrow q + 2$: q angular variable. Valid with quarks.

Pure glue: also, $q \rightarrow q + 1$, Z(2) transf., $\mathbf{L} \rightarrow -\mathbf{L}$

For pure glue, can restrict q: $0 \rightarrow 1$.

Then Z(2) transf. $q \rightarrow 1 - q$:
Z(2) transf., *plus* exchange of eigenvalues

$$\mathbf{L}(1-q) = -\begin{pmatrix} e^{-i\pi q} & 0 \\ 0 & e^{i\pi q} \end{pmatrix}$$

Any potential of q must be invariant under $q \rightarrow 1 - q$

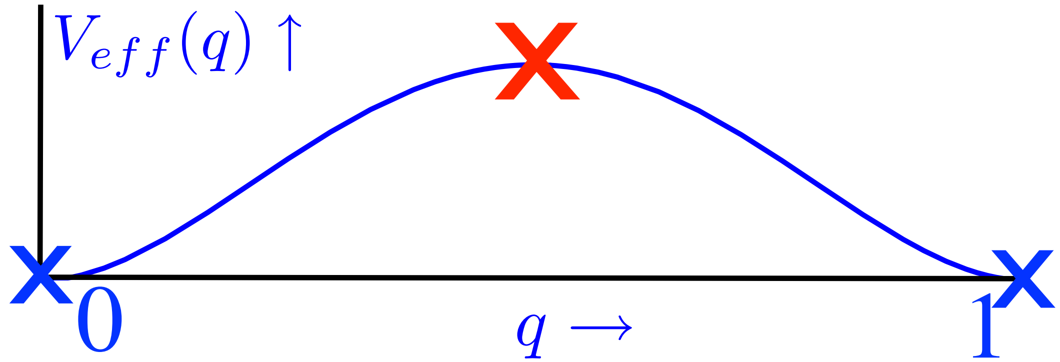
Potentials for the q's

Add *non*-perturbative terms, by *hand*, to generate <q> ≠ 0 :

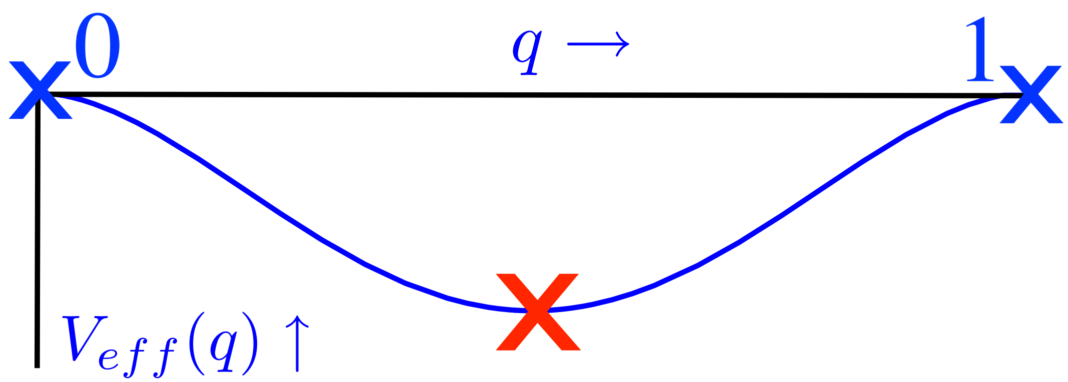
By hand? $V_{non}(q)$ from: monopoles, vortices...

$$V_{eff}(q) = V_{pert}(q) + V_{non}(q)$$

$T >> T_c$: $\langle q \rangle = 0,1 \rightarrow$



$T < T_c$: $\langle q \rangle = ½ \rightarrow$



Three possible "phases"

Two phases are familiar:

$\langle q \rangle = 0, 1$: $\langle l \rangle = \pm 1$: "Complete" QGP: usual perturbation theory. $T >> T_c$.

$\langle q \rangle = 1/2$: $\langle l \rangle = 0$: confined phase. $T < T_c$

There is also a *third* phase, "partially" deconfined

$0 < \langle q \rangle < 1/2$: $\langle l \rangle < 1$: "semi"-QGP. From some $x\, T_c > T > T_c$ *x?*

So *two* phase transitions are possible.

Lattice: *one* transition, to confined phase, at T_c. *No* other transition above T_c.
Still, there is an intermediate phase, the "semi"-QGP

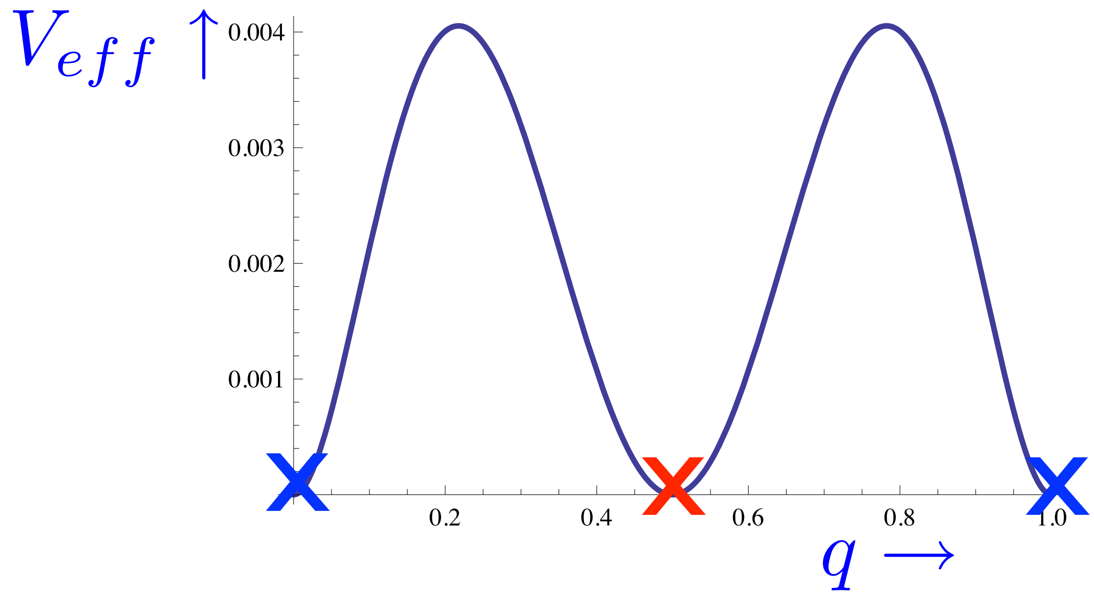
Strongly constrains possible non-perturbative terms, $V_{non}(q)$.

Getting three "phases", one transition

Simple guess: $V_{non} \sim loop^2$,

$$V_{eff} \sim \frac{a}{\pi^2}\left(\ell^2 - 1\right) + q^2(1-q)^2$$

$$\sim q^2(1-a) - 2q^3 + \ldots$$



1st order transition *directly* from complete QGP to confined phase, *not* 2nd

Generic if $V_{non}(q) \sim q^2$ at $q << 1$.

Easy to avoid, *if* $V_{non}(q) \sim q$ for small q. Then $\langle q \rangle \neq 0$ at all T.

Imposing the symmetry of $q \leftrightarrow 1 - q$, $V_{non}(q)$ *must* include

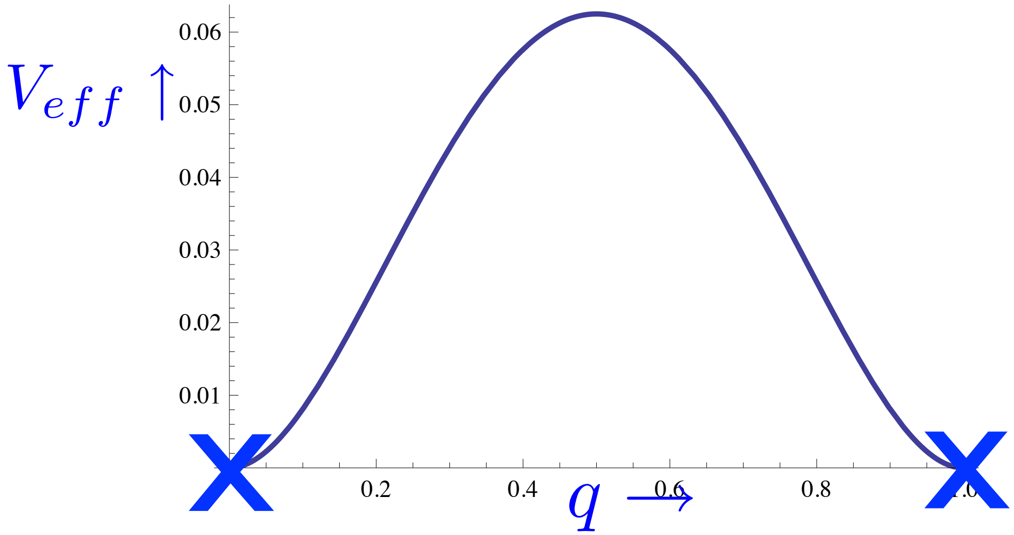
$$V_{non}(q) \sim q(1-q)$$

Cartoons of deconfinement

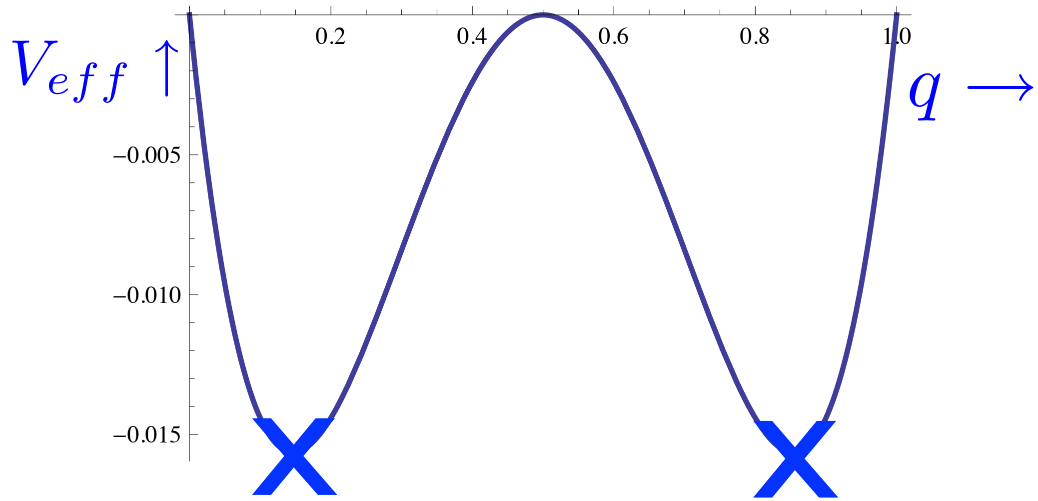
Consider:

$$V_{eff} = q^2(1-q)^2 - \; a\, q(1-q)\,,\; a \sim T_c^2/T^2$$

$\Downarrow$ a = 0: complete QGP



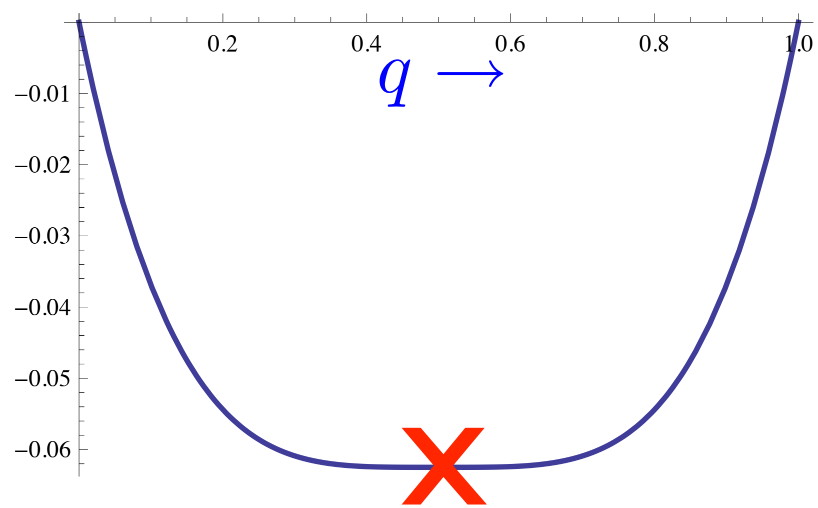
$\Downarrow$ a = ¼: semi QGP



a = ½: T_c=>

Stable vacuum at q = ½

Transition *second* order



0-parameter matrix model, N = 2

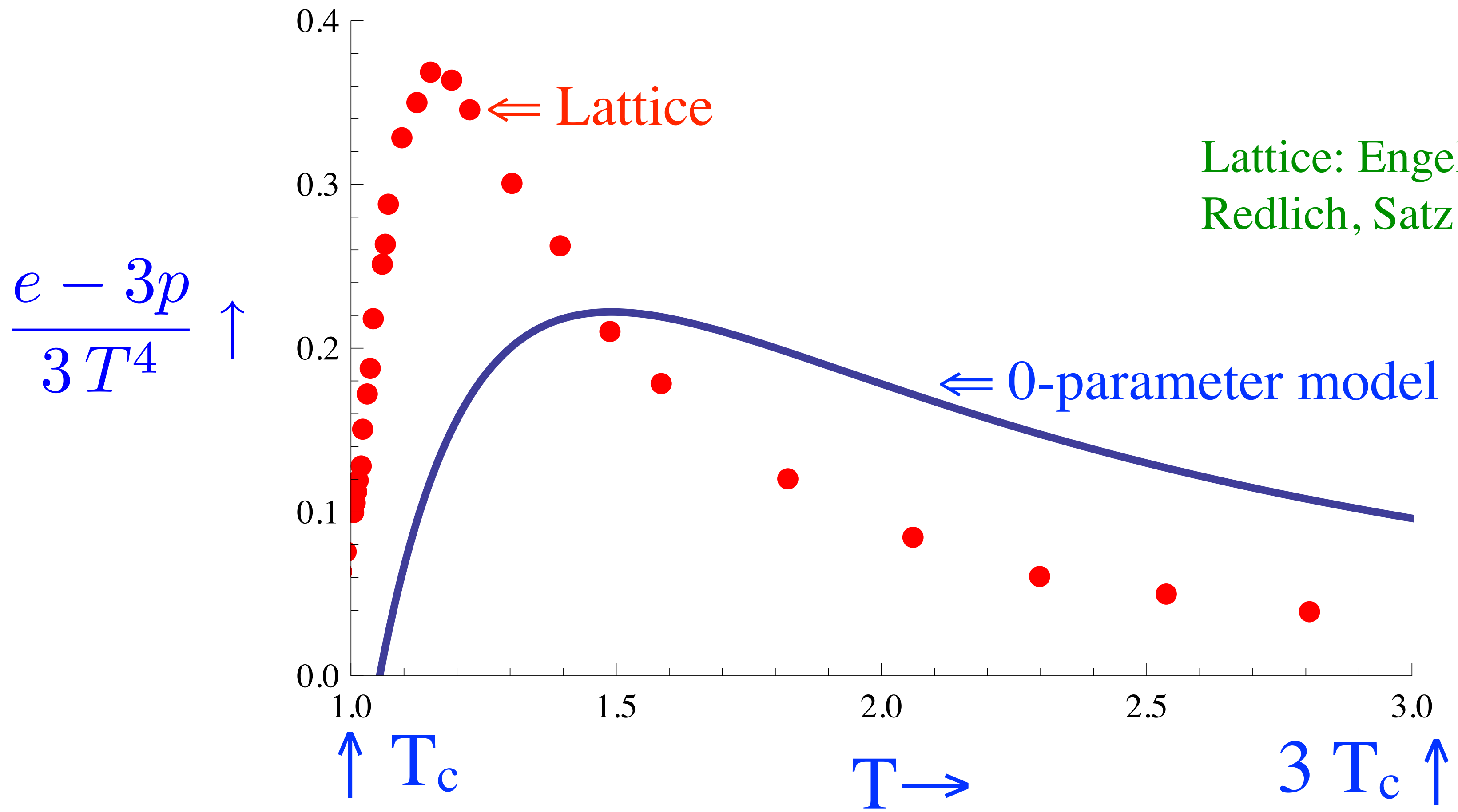
Meisinger, Miller, Ogilvie ph/0108009, MMO:

take $V_{non} \sim T^2$

$$V_{non}(q) = \frac{4\pi^2}{3} T^2 T_c^2 \left(-\frac{c_1}{5} q(1-q) + \frac{c_3}{15} \right)$$

Two conditions: transition occurs at T_c, pressure(T_c) = 0

Fixes c_1 and c_3, *no* free parameters. *Not* close to lattice data *(from '89!)*



1-parameter matrix model, N = 2

Dumitru, Guo, Hidaka, Korthals-Altes, RDP ‘10: to usual perturbative potential,

$$V_{pert}(q) = \frac{4\pi^2}{3} T^4 \left(-\frac{1}{20} + q^2(1-q)^2 \right)$$

Add - *by hand* - a non-pert. potential $V_{non} \sim T^2\, T_c^2$. *Also* add a term like V_{pert}:

$$V_{non}(q) = \frac{4\pi^2}{3} T^2\, T_c^2 \left(-\frac{c_1}{5}\, q(1-q) - c_2\, q^2(1-q)^2 + \frac{c_3}{15} \right)$$

Now just like any other mean field theory. $\langle q \rangle$ given by minimum of V_{eff}:

$$V_{eff}(q) = V_{pert}(q) + V_{non}(q) \qquad \left. \frac{d}{dq} V_{eff}(q) \right|_{q=\langle q \rangle} = 0$$

$\langle q \rangle$ depends nontrivially on temperature.

Pressure value of potential at minimum: $$p(T) = -V_{eff}(\langle q \rangle)$$

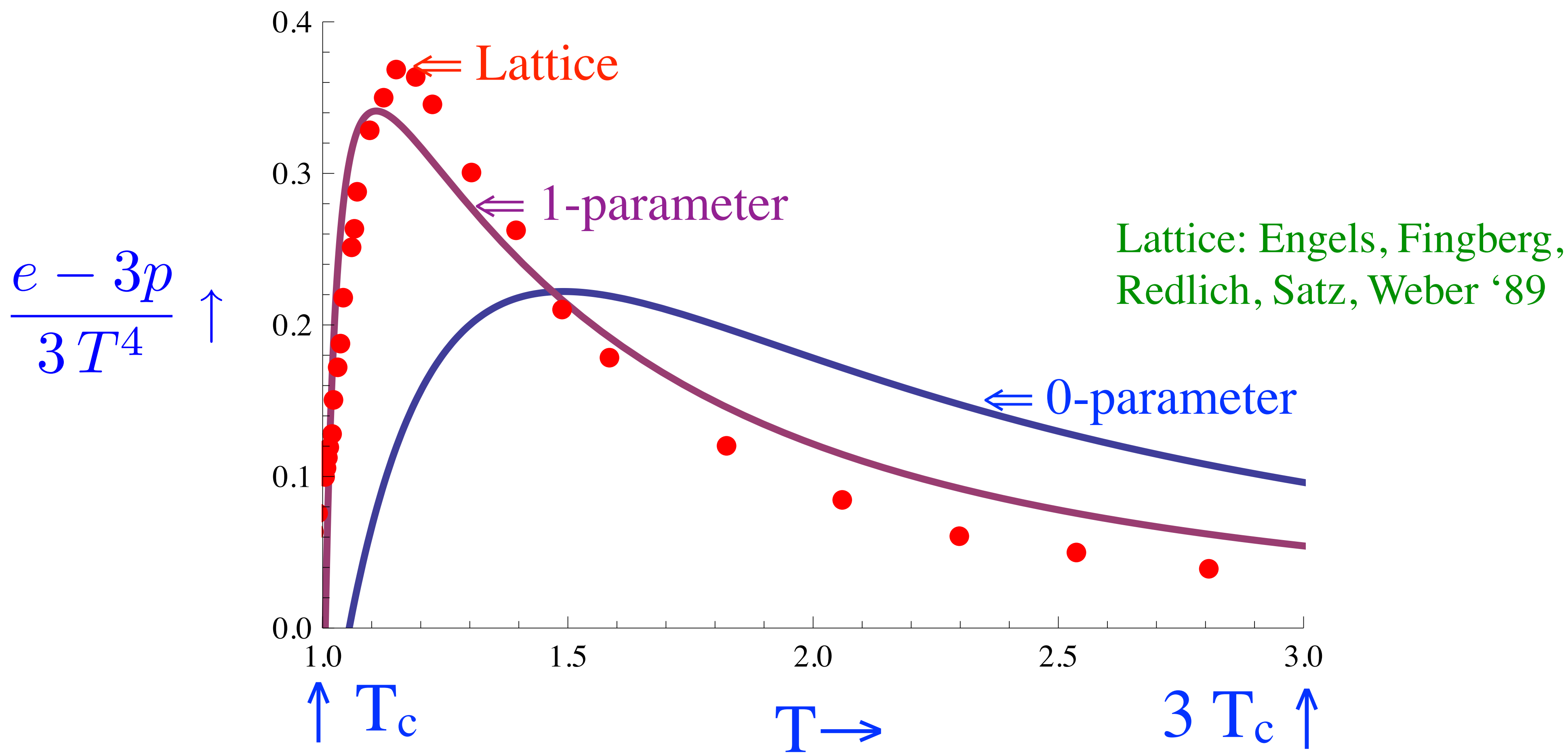
Lattice vs matrix models, N = 2

Choose c_2 to fit $e\text{-}3p/T^4$: optimal choice

$$c_1 = 0.23\,,\; c_2 = .91\,,\; c_3 = 1.11$$

Reasonable fit to $e\text{-}3p/T^4$; also to p/T^4, e/T^4.

N.B.: $c_2 \sim 1$. At T_c, terms $\sim q^2(1\text{-}q)^2$ *almost* cancel.



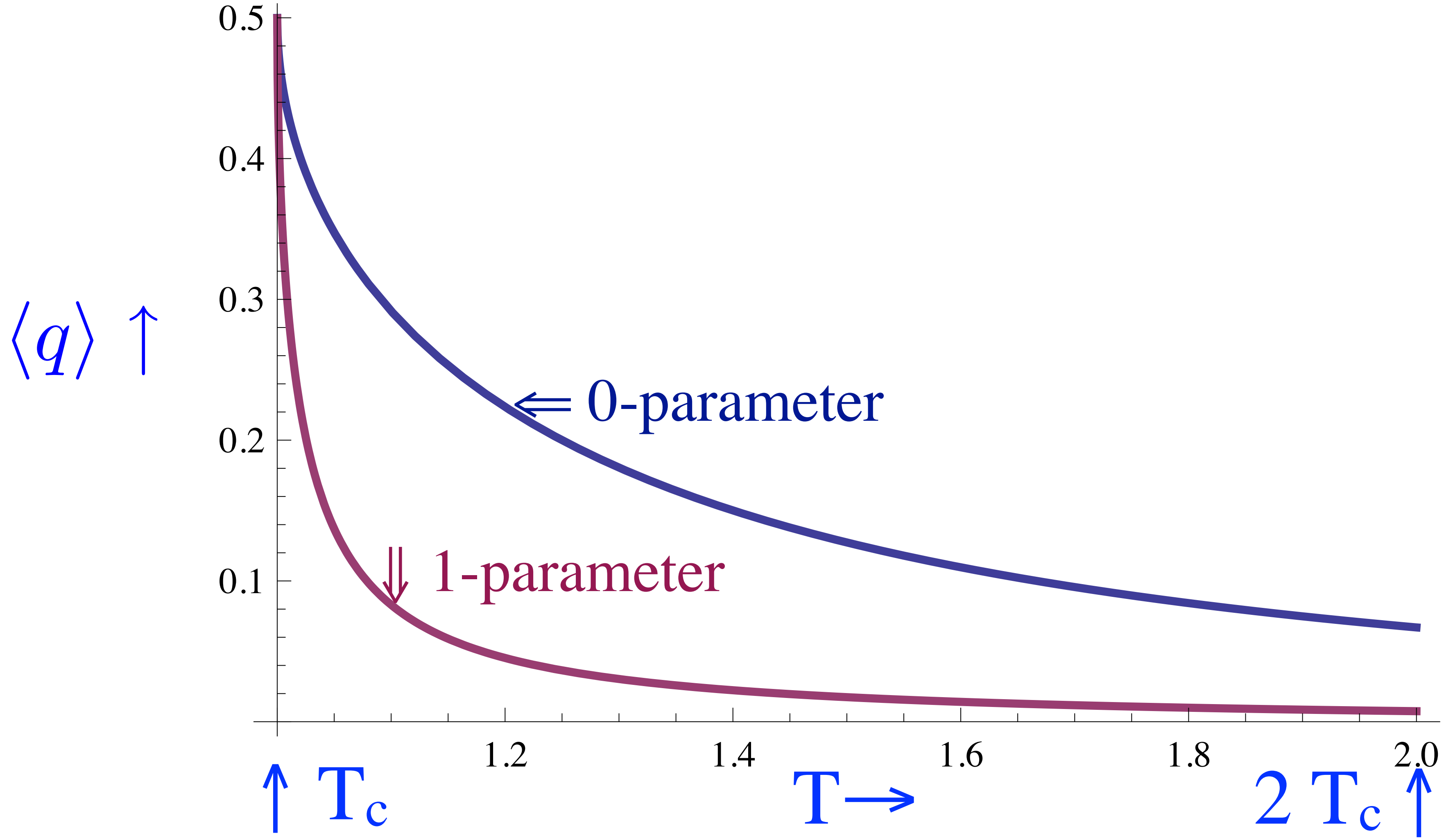
Width of transition region, 0- vs 1-parameter

1-parameter model: get sharper e-3p/T^4 because $\langle q \rangle$ -> 0 *much* quicker above T$_c$.

Physically: sharp e-3p/T^4 implies region where $\langle q \rangle$ is significant is *narrow*

N.B.: $\langle q \rangle \neq 0$ at all T, but numerically, *negligible* above ~ 1.2 T$_c$; p ~ $\langle q \rangle^2$.

Above ~1.2 T$_c$, the T^2 term in the pressure is due *entirely* to the *constant* term, c$_3$!



Polyakov loop: 1-parameter matrix model ≠ lattice

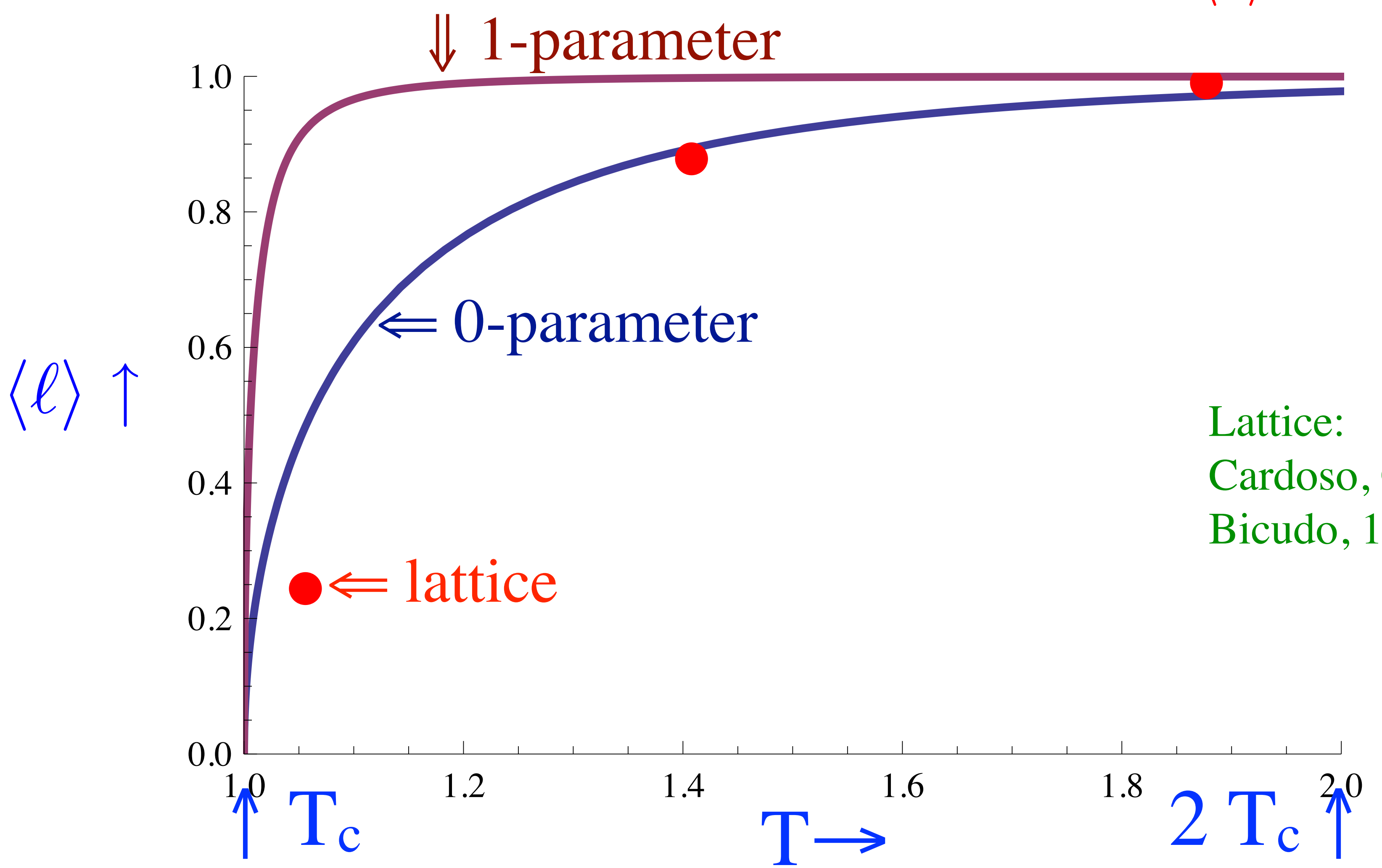
Lattice: *renormalized* Polyakov loop. Matrix model: $\langle l \rangle = \cos(\pi q/2)$

0-parameter model: close to lattice

1-parameter model: *sharp* disagreement. $\langle l \rangle$ rises to ~ 1 *much* faster - ?

Ambiguity of zero point energy?

$\langle \ell \rangle \to e^{-E_0/T} \langle \ell \rangle$?



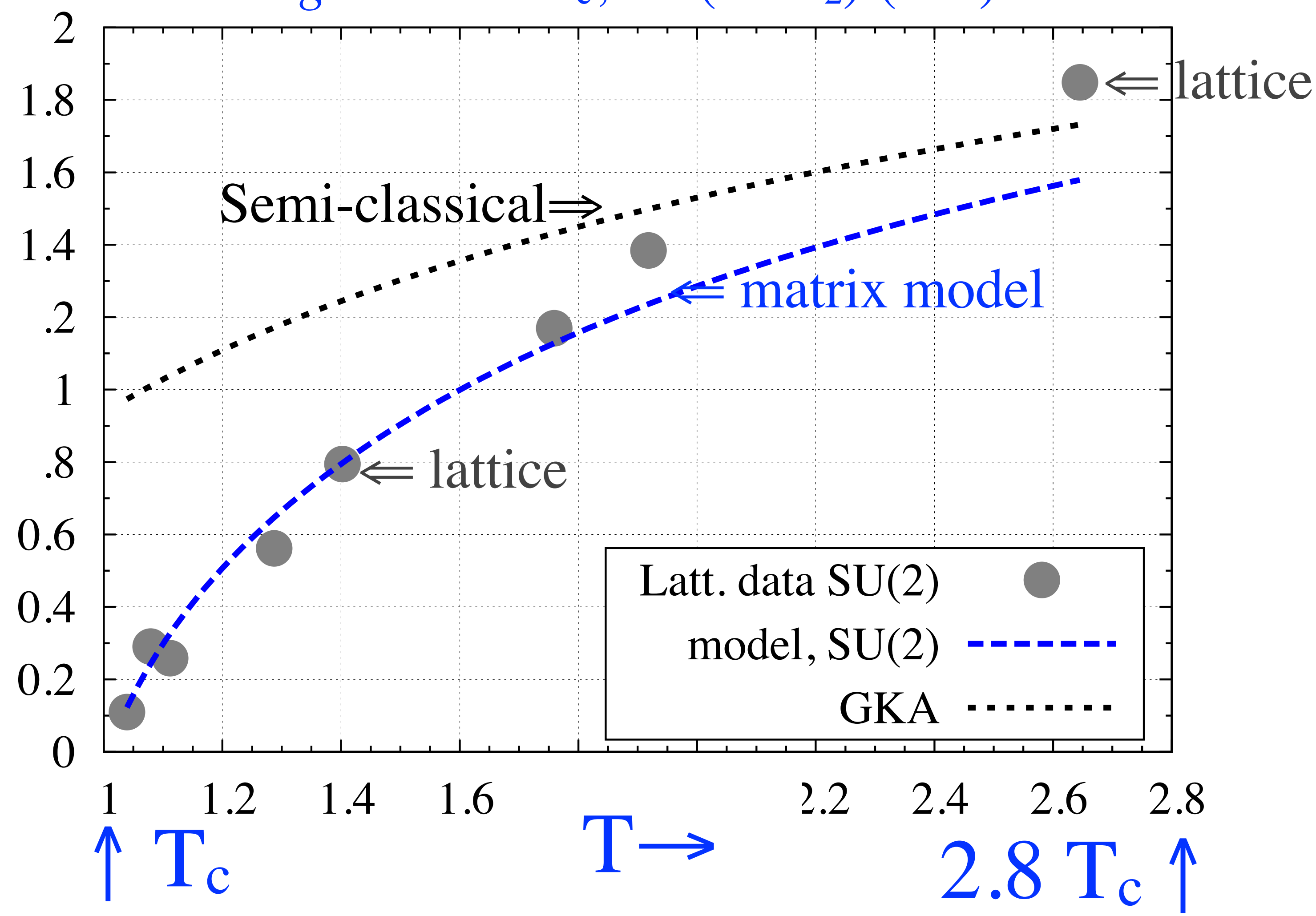
Interface tension, N = 2

σ vanishes as T→T_c , σ ~ $(t-1)^{2\nu}$.
Ising 2ν ~ 1.26; Lattice: ~ 1.32.
Matrix model: ~ 1.5: c_2 important.

$$\sigma(T) = \frac{4\pi^2 T^2}{3\sqrt{6g^2}} \frac{(t^2-1)^{3/2}}{t\,(t^2-c_2)} \;,\; t = \frac{T}{T_c}$$

Semi-class.: GKA '04. *Include* corr.'s ~ g^2 in matrix σ(T) (ok when T > 1.2 T_c)
N.B.: width of interface *diverges* as T→T_c, ~ $\sqrt{(t^2 - c_2)/(t^2-1)}$.

$\frac{\sigma}{T^2}$ ↑



Lattice:
de Forcrand,
D'Elia, Pepe,
lat/0007034

Adjoint Higgs phase, N = 2

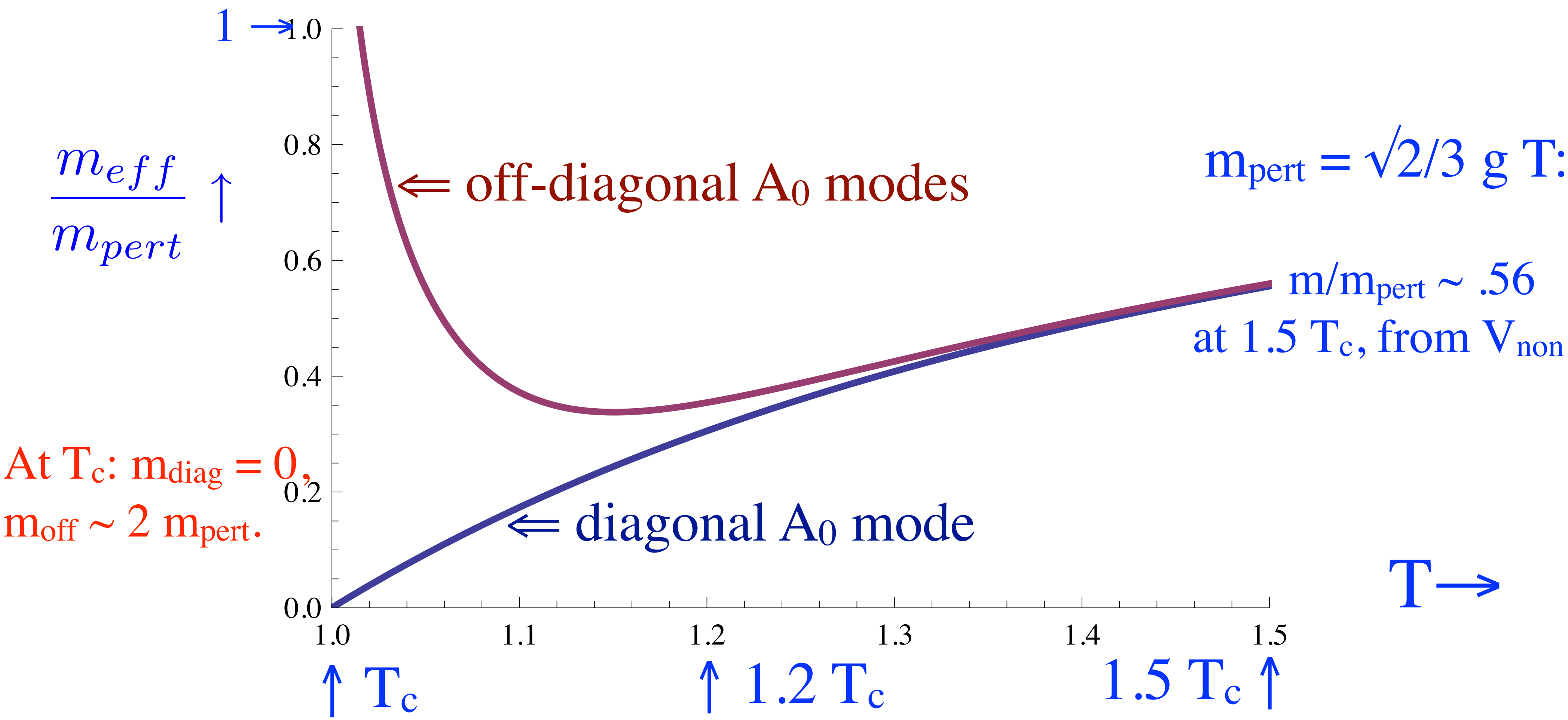
$A_0^{cl} \sim q\,\sigma_3$, so $\langle q \rangle \neq 0$ generates an (adjoint) Higgs phase:

RDP, ph/0608242; Unsal & Yaffe, 0803.0344, Simic & Unsal, 1010.5515

In background field, $A = A_0^{cl} + A^{qu}$: $D_0^{cl} A^{qu} = \partial_0 A^{qu} + i\, g\, [A_0^{cl}, A^{qu}]$

Fluctuations $\sim \sigma_3$ not Higgsed, $\sim \sigma_{1,2}$ Higgsed, get mass $\sim 2\pi T \langle q \rangle$

Hence when $\langle q \rangle \neq 0$, for $T < 1.2\, T_c$, *splitting* of masses:



Matrix model: N ≥ 3

Why the transition is *always* 1st order

One parameter model

Path to Z(3), three colors

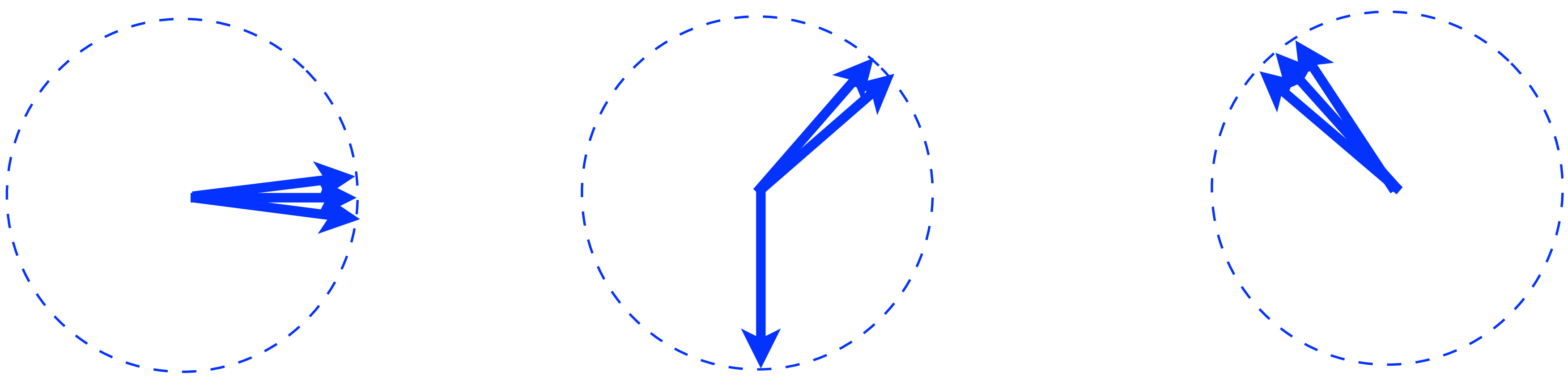
SU(3): *two* diagonal λ's, so *two* q's:

$$\lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \; ; \; \lambda_8 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

$$A_0 = \frac{2\pi T}{3\, g} \left(q_3\, \lambda_3 + q_8\, \lambda_8 \right)$$

Z(3) paths: move along λ_8, not λ_3: $q_8 \neq 0$, $q_3 = 0$.

$$\mathbf{L} = \mathrm{e}^{2\pi i q_8 \lambda_8 / 3}$$



$q_8 = 0$ $\quad$ $q_8 = 3/8$ $\quad$ $q_8 = 1$

$\mathbf{L} = \mathbf{1}$ $\quad$ $\mathbf{L} = \mathrm{e}^{2\pi i/3}\, \mathbf{1}$

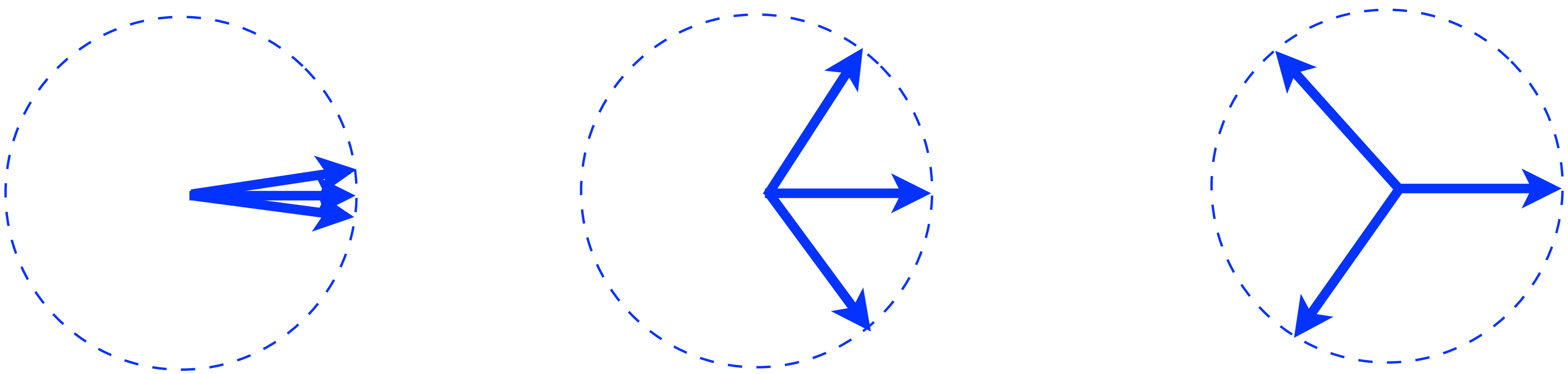
Path to confinement, three colors

Now move along λ_3: $\mathbf{L} = e^{2\pi i q_3 \lambda_3/3}$

In particular, consider $q_3 = 1$:
Elements of $e^{2\pi i/3}$ $\mathbf{L}_c$ *same* as those of $\mathbf{L}_c$.
Hence tr $\mathbf{L}_c$ = tr $\mathbf{L}_c^2 = 0$: $\mathbf{L}_c$ *confining* vacuum

$$\mathbf{L}_c = \begin{pmatrix} e^{2\pi i/3} & 0 & 0 \\ 0 & e^{-2\pi i/3} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Path to confinement: along λ_3, not λ_8, $q_3 \neq 0$, $q_8 = 0$.



$q_3 = 0$, $\ell = 1$ | $q_3 = 3/8$, $\ell \approx .8$ | $q_3 = 1$, $\ell = 0$

General potential for any SU(N)

Ansatz: constant, diagonal matrix
i, j = 1…N

$$A_0^{ij} = \frac{2\pi T}{g}\, q_i\, \delta^{ij} \qquad \mathbf{L}_{ij} = \mathrm{e}^{2\pi i\, q_j}\, \delta_{ij}$$

For SU(N), $\Sigma_{j=1\ldots N}\, q_j = 0$. Hence N-1 independent q_j's, = # diagonal generators.

At 1-loop order, the perturbative potential for the q_j's is

$$V_{pert}(q) = \frac{2\pi^2}{3} T^4 \left(-\frac{4}{15}(N^2-1) + \sum_{i,j} q_{ij}^2 (1-q_{ij})^2 \right) \;,\; q_{ij} = |q_i - q_j|$$

As before, *assume* a non-perturbative potential ~ $T^2\, T_c^2$:

$$V_{non}(q) = \frac{2\pi^2}{3} T^2 T_c^2 \left(-\frac{c_1}{5} \sum_{i,j} q_{ij}(1-q_{ij}) - c_2 \sum_{i,j} q_{ij}^2 (1-q_{ij})^2 + \frac{4}{15} c_3 \right)$$

Path to confinement, four colors

Move to the confining vacuum along *one* direction, q_j^c:

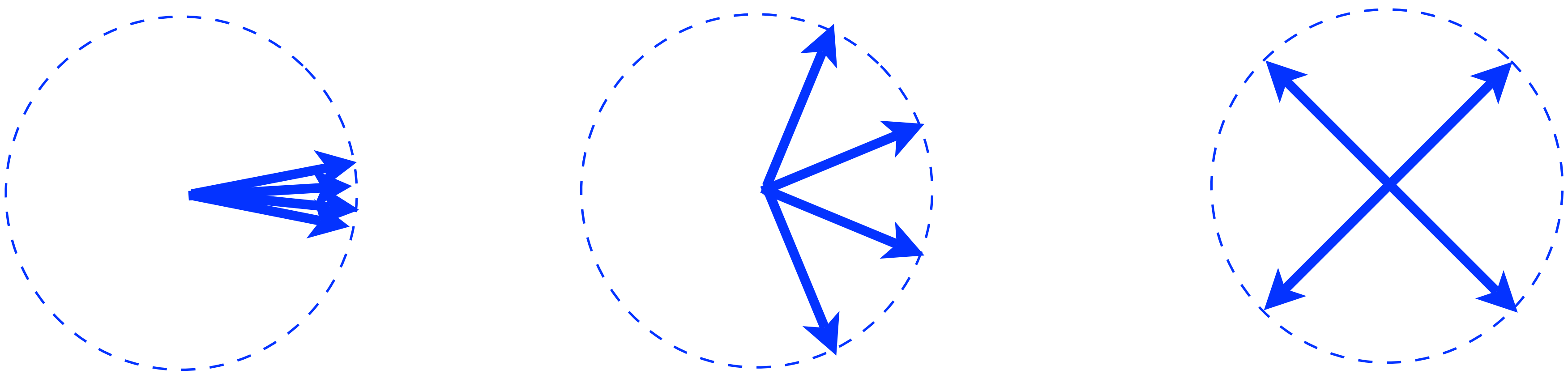
(For general interfaces, need *all* N-1 directions in q_j space)

Perturbative vacuum: q = 0.

Confining vacuum: q = 1.

Four colors:

$$q_j^c = \left(\frac{2j - N - 1}{2N} \right) q \, , \; j = 1 \ldots N$$



$q = 0$ $\quad$ $\ell = 1$

$q = 1/2$ $\quad$ $\ell \approx .65$

$q = 1$ $\quad$ $\ell = 0$

General N: confining vacuum = *uniform* distribution for eigenvalues of **L**

For infinite N, distribution flat.

Cubic term for *all* N ≥ 3, so transition first order

Define $\phi = 1 - q$,
Confining point $\phi = 0$

$$V_{tot} = \frac{\pi^2 (N^2 - 1)}{45} \, T_c^4 \, t^2 \, (t^2 - 1) \, \widetilde{V}(\phi, t) \, , \; t = \frac{T}{T_c}$$

$$\widetilde{V}(\phi, t) = - \, m_\phi^2 \, \phi^2 - 2 \left(\frac{N^2 - 4}{N^2} \right) \phi^3 + \left(2 - \frac{3}{N^2} \right) \phi^4$$

$$m_\phi^2 = 1 + \frac{6}{N^2} - \frac{c_1}{t^2 - c_2}$$

No term linear in ϕ. Cubic term in ϕ for *all* N ≥ 3; vanishes for N = 2.

Existence of cubic term generic.
Along q^c, about $\phi = 0$ there is *no* symmetry of $\phi \rightarrow - \phi$ for *any* N ≥ 3.
Hence terms ~ ϕ^3, and so a first order transition, are *ubiquitous*.

Special to matrix model, with the q_i's elements of Lie *algebra*.

Svetitsky and Yaffe '80: V_{eff}(loop) => 1st order *only* for N=3; loop element Lie *group*

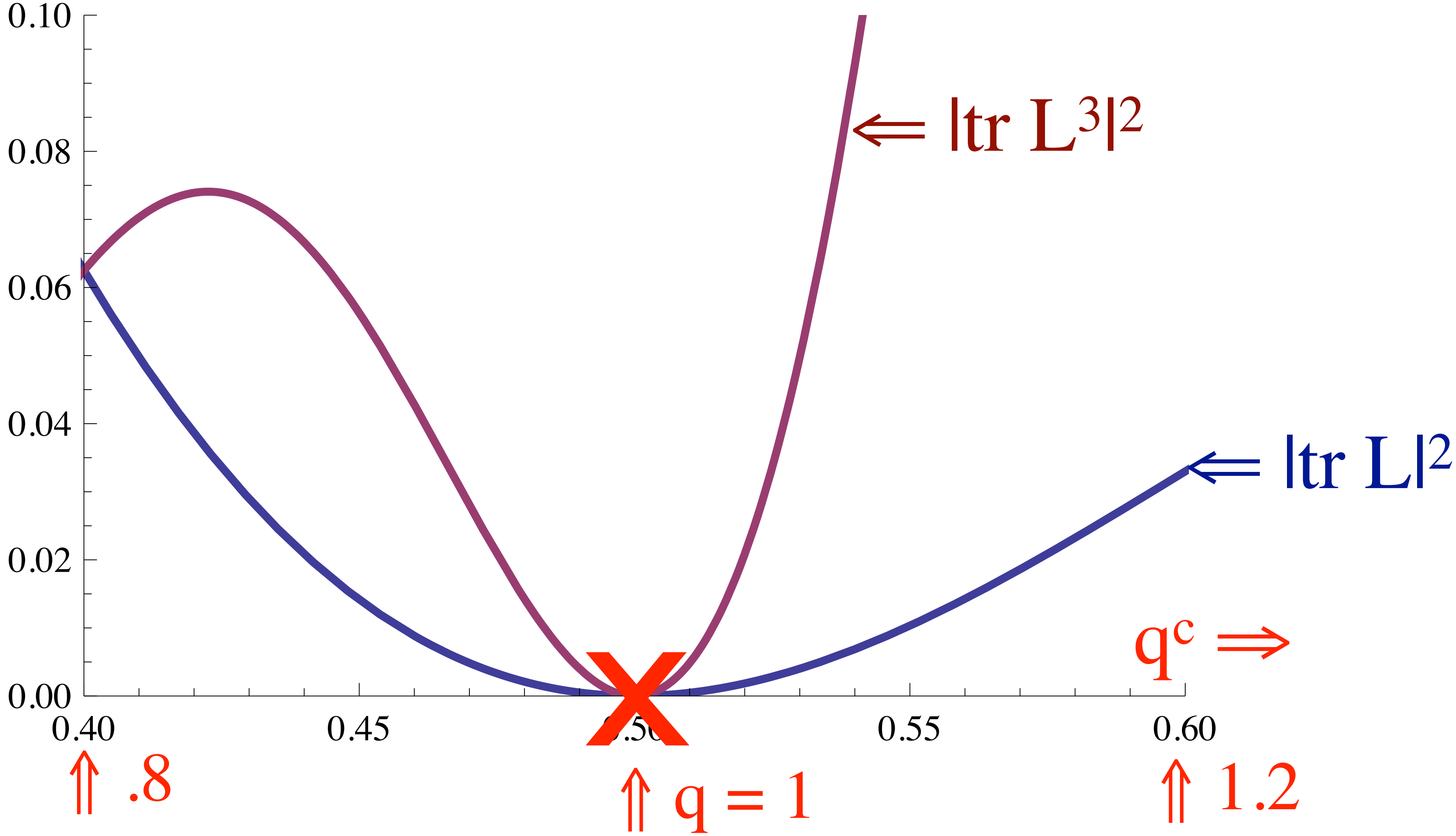
Cubic term for four colors

Construct V_{eff} either from q's, or equivalently, loops: tr **L**, tr $\mathbf{L}^2$, tr $\mathbf{L}^3$....

N = 4: $|\mathrm{tr}\,\mathbf{L}|^2$ and $|\mathrm{tr}\,\mathbf{L}^3|^2$ *not* symmetric about q = 1, so cubic terms, ~ $(q - 1)^3$.

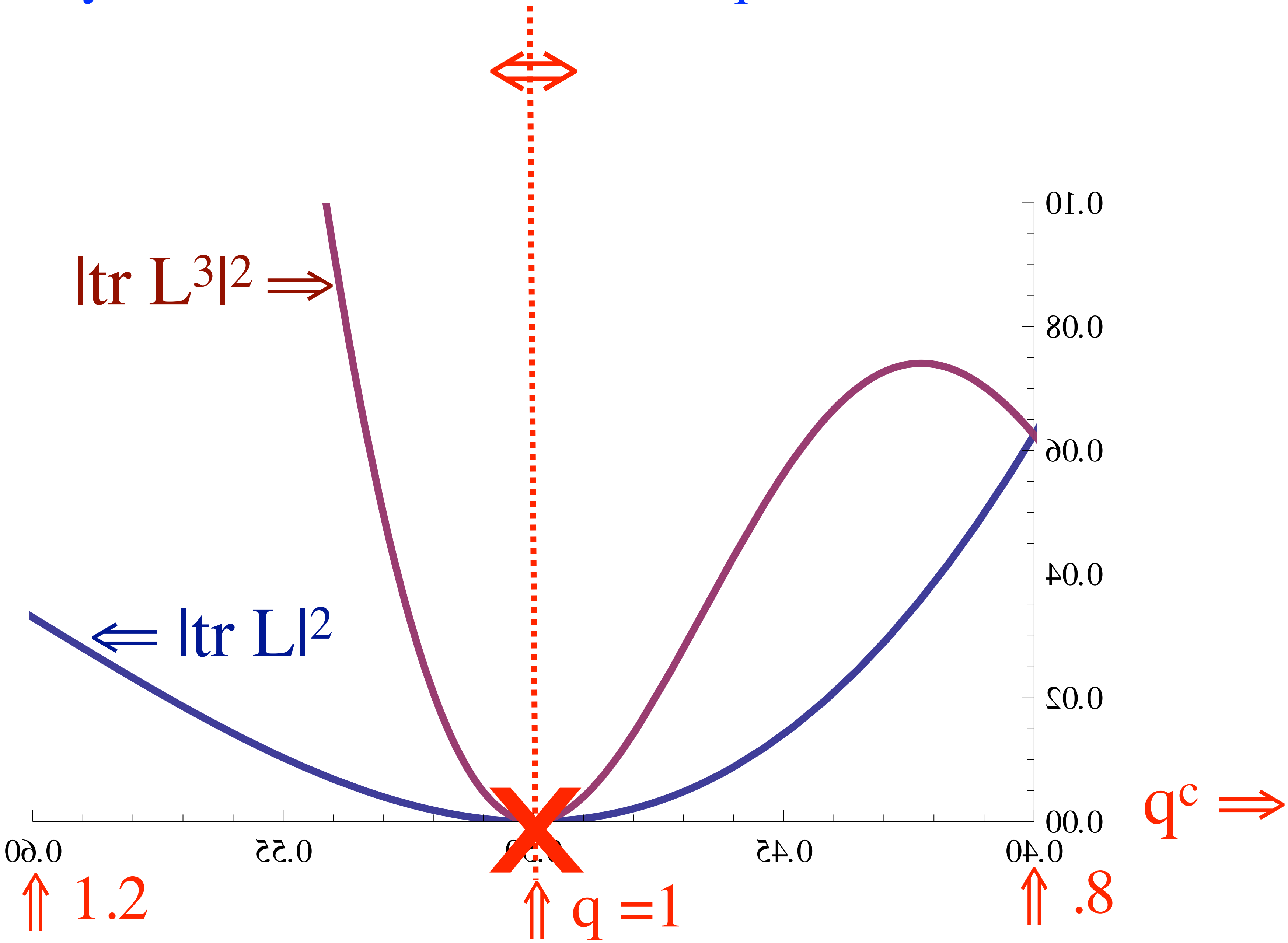
($|\mathrm{tr}\,\mathbf{L}^2|^2$ symmetric, residual Z(2) symmetry)

Cubic terms *special* to moving along q_c in a *matrix* model. *Not* true in loop model



Cubic term for four colors

Asymmetric in reflection about q = 1



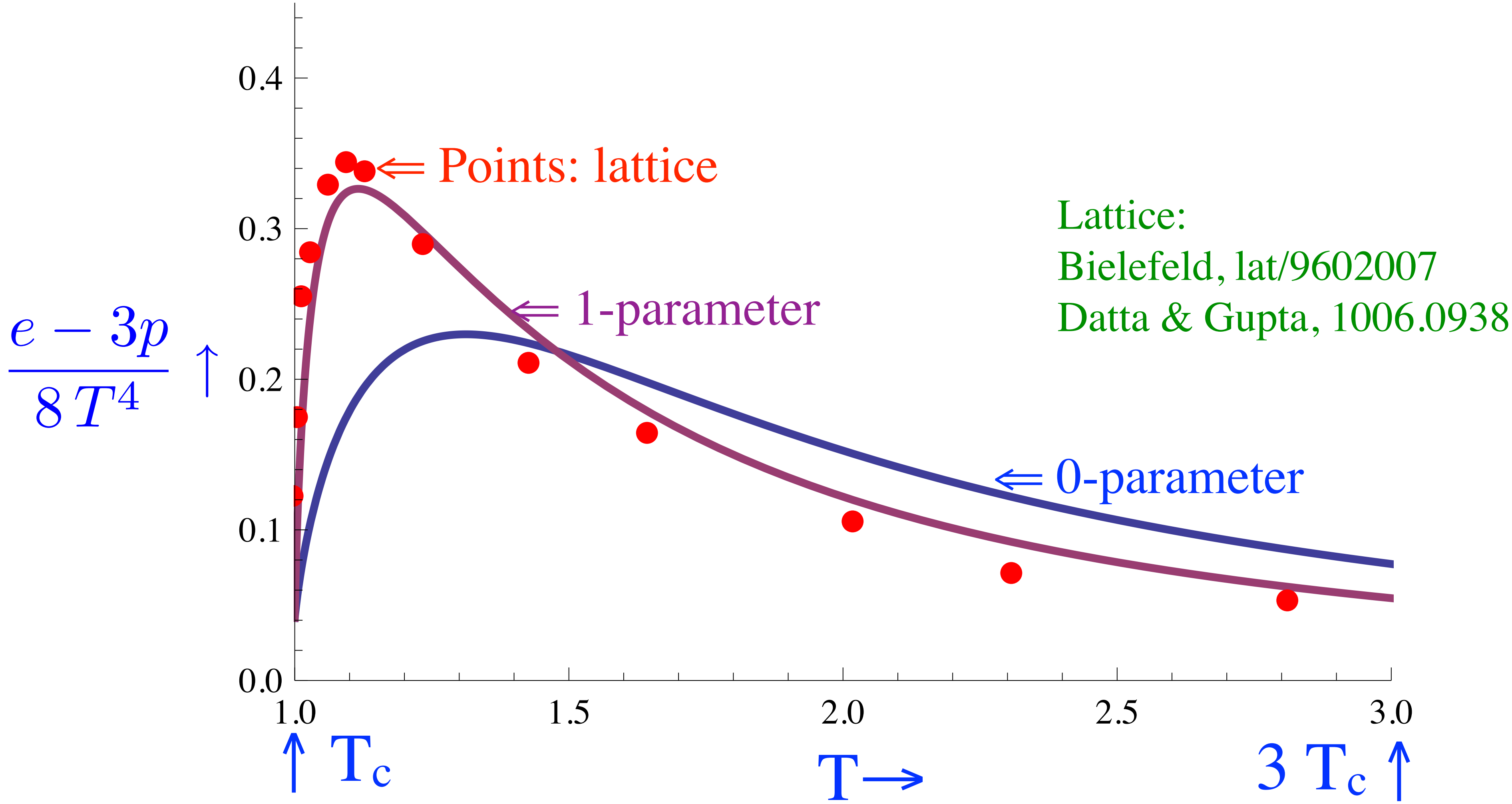
Lattice vs 0- and 1- parameter matrix models, N = 3

Results for N=3 similar to N=2.

0-parameter model way off.

Good fit $e-3p/T^4$ for 1-parameter model, $c_1 = 0.32\,,\ c_2 = 0.83\,,\ c_3 = 1.13$

Again, $c_2 \sim 1$, so at T_c, terms $\sim q^2(1-q)^2$ *almost* cancel.

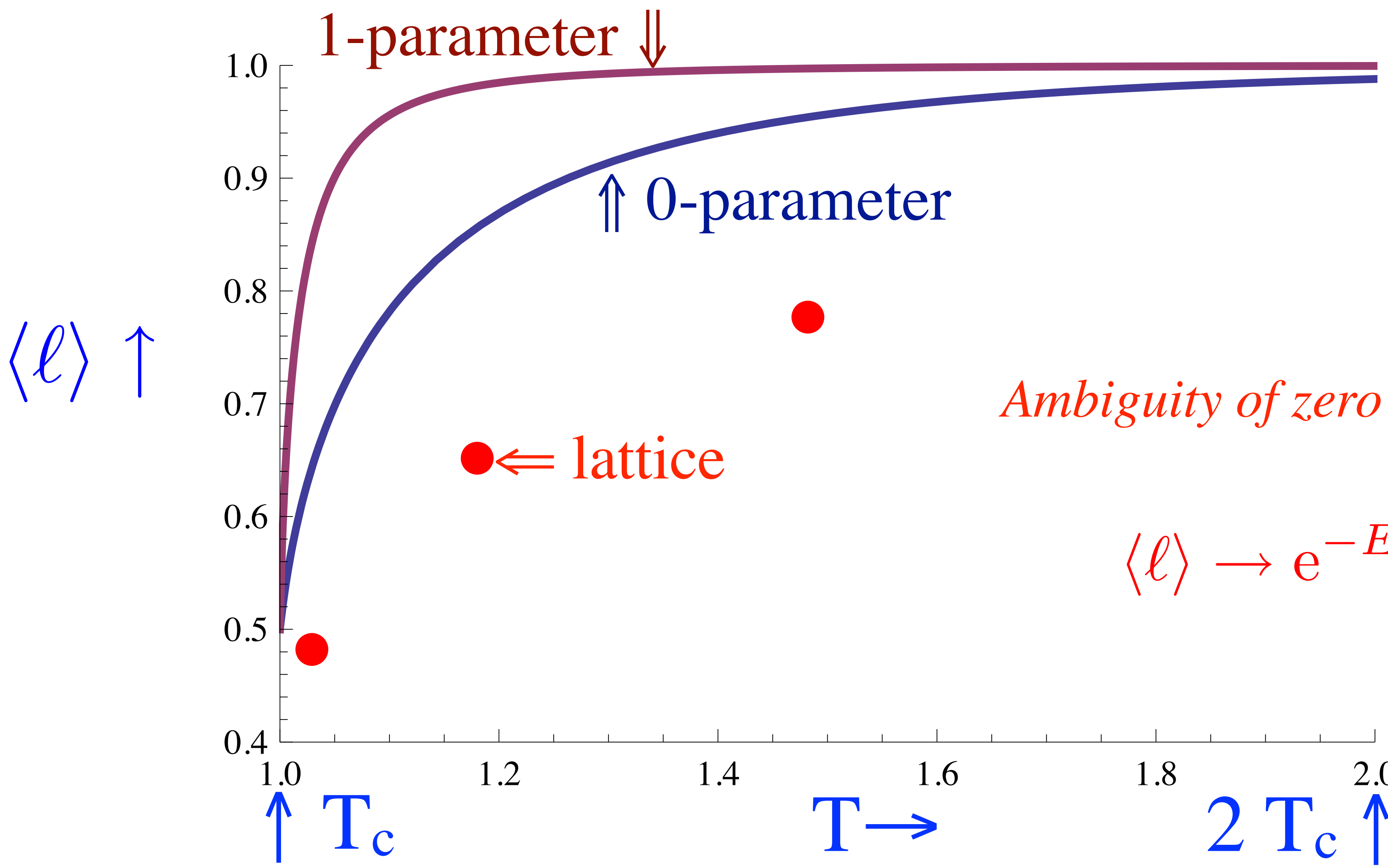


Polyakov loop: matrix models ≠ lattice

Renormalized Polyakov loop from lattice does *not* agree with *either* matrix model

$\langle l \rangle - 1 \sim \langle q \rangle^2$: By 1.2 T_c, $\langle q \rangle \sim .05$, negligible.

Again, for T > 1.2 T_c, the T^2 term in pressure due *entirely* to the *constant* term, c_3!



Ambiguity of zero point energy?

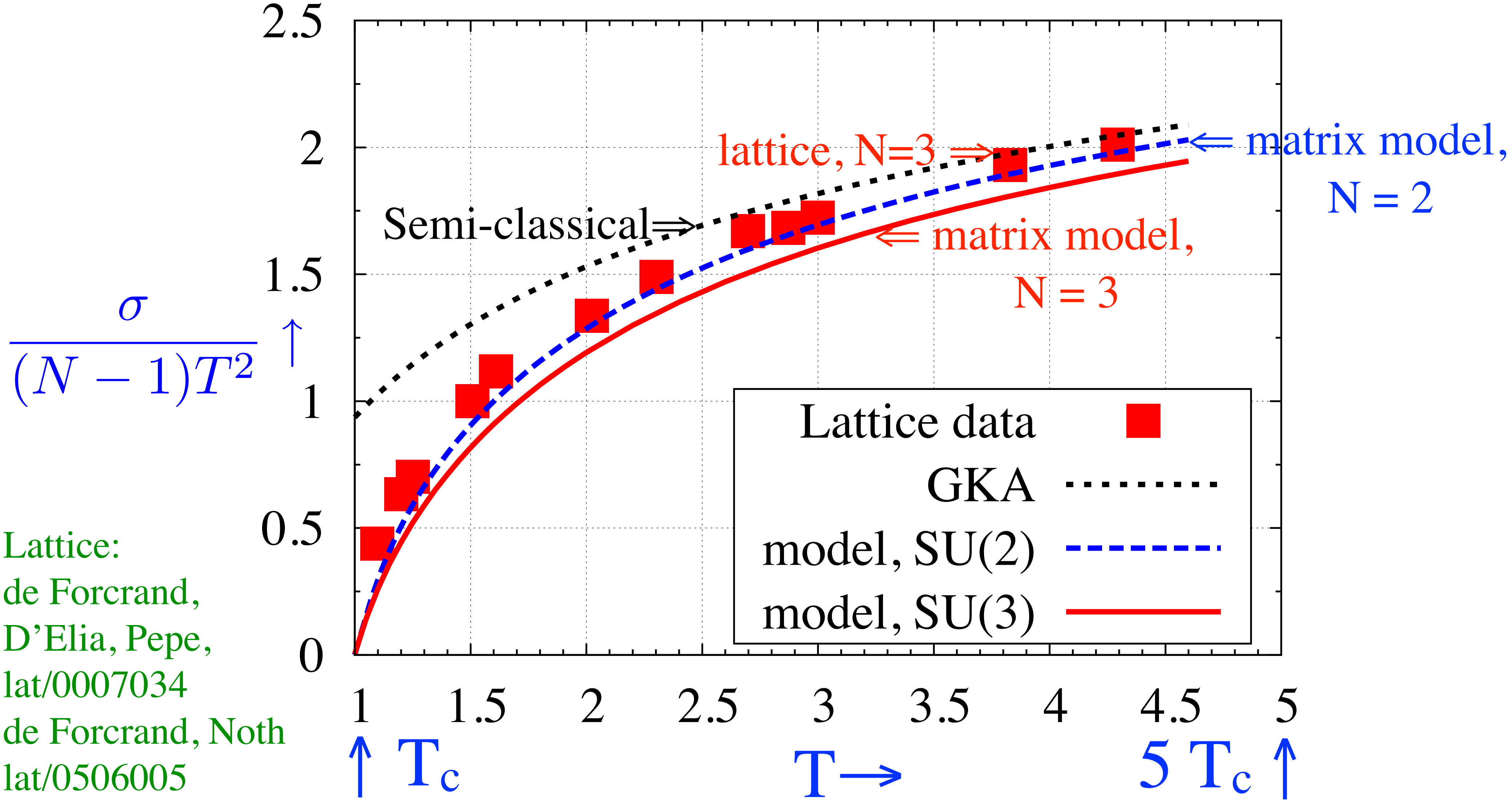
$$\langle \ell \rangle \rightarrow e^{-E_0/T} \langle \ell \rangle \, ?$$

Interface tension, N = 2 and 3

Order-order interface tension, σ, from matrix model close to lattice.

For $T > 1.2\, T_c$, path along λ_8; for $T < 1.2\, T_c$, along *both* λ_8 and λ_3.

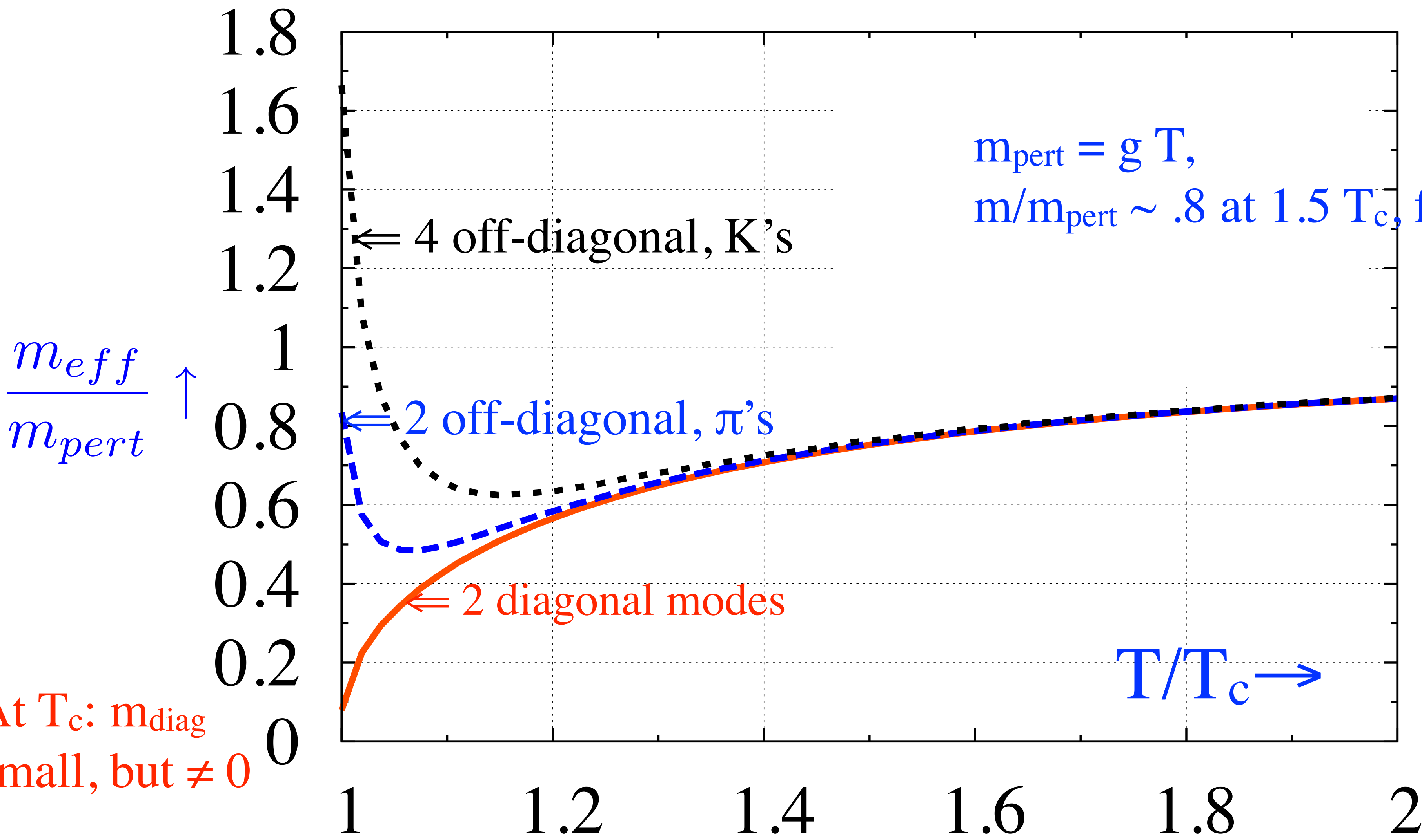
$\sigma(T_c)/T_c^2$ nonzero but *small*, ~ .02. Results for N =2 and N = 3 similar - ?



Lattice:
de Forcrand, D'Elia, Pepe, lat/0007034
de Forcrand, Noth lat/0506005

Adjoint Higgs phase, N = 3

For SU(3), deconfinement along $A_0^{cl} \sim q\,\lambda_3$. Masses $\sim [\lambda_3, \lambda_i]$: two off-diagonal.
Splitting of masses only for $T < 1.2\,T_c$:
Measureable from singlet potential, $\langle \mathrm{tr}\, L^\dagger(x)\, L(0)\rangle$, over *all* x.



At T_c: m_{diag} small, but $\neq 0$

Matrix model: $N \geq 3$

To get the latent heat right, two parameter model.

Thermodynamics, interface tensions improve

Latent heat, and a 2-parameter model

Latent heat, $e(T_c)/T_c^4$: 1-parameter model too small:

1-para.: 0.33. BPK: 1.40 ± .1; DG: 1.67 ± .1.

$$c_3(T) = c_3(\infty) + \frac{c_3(1) - c_3(\infty)}{t^2}\,,\; t = \frac{T}{T_c}$$

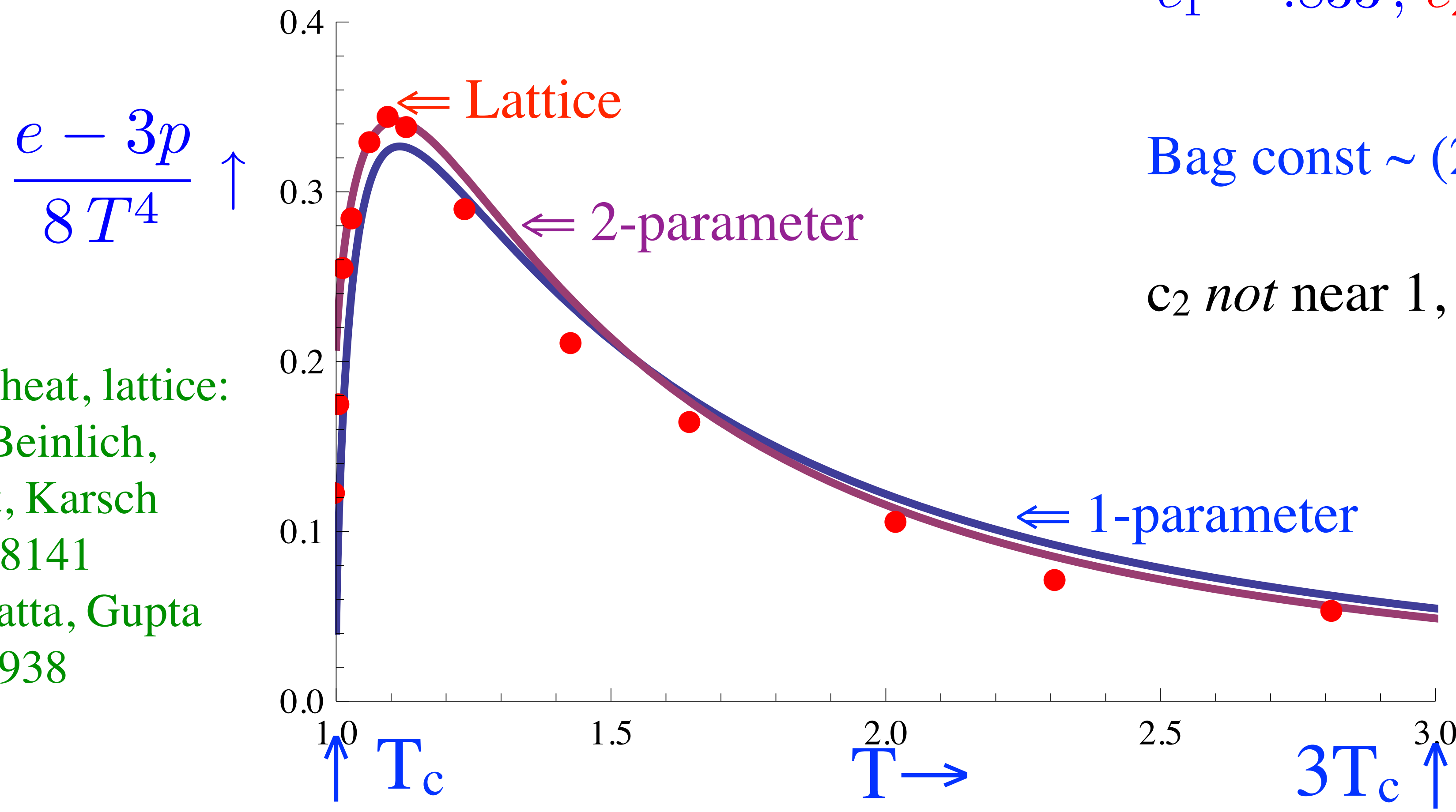
2-parameter model, $c_3(T)$. Like MIT bag constant

WHOT: $c_3(\infty) \sim 1$. *Fit* $c_3(1)$ to DG latent heat

Fits lattice for $T < 1.2\,T_c$, overshoots above.

$$c_3(1) = 1.33\,,\; c_3(\infty) = .95$$

$$c_1 = .833\,,\; c_2 = .552$$

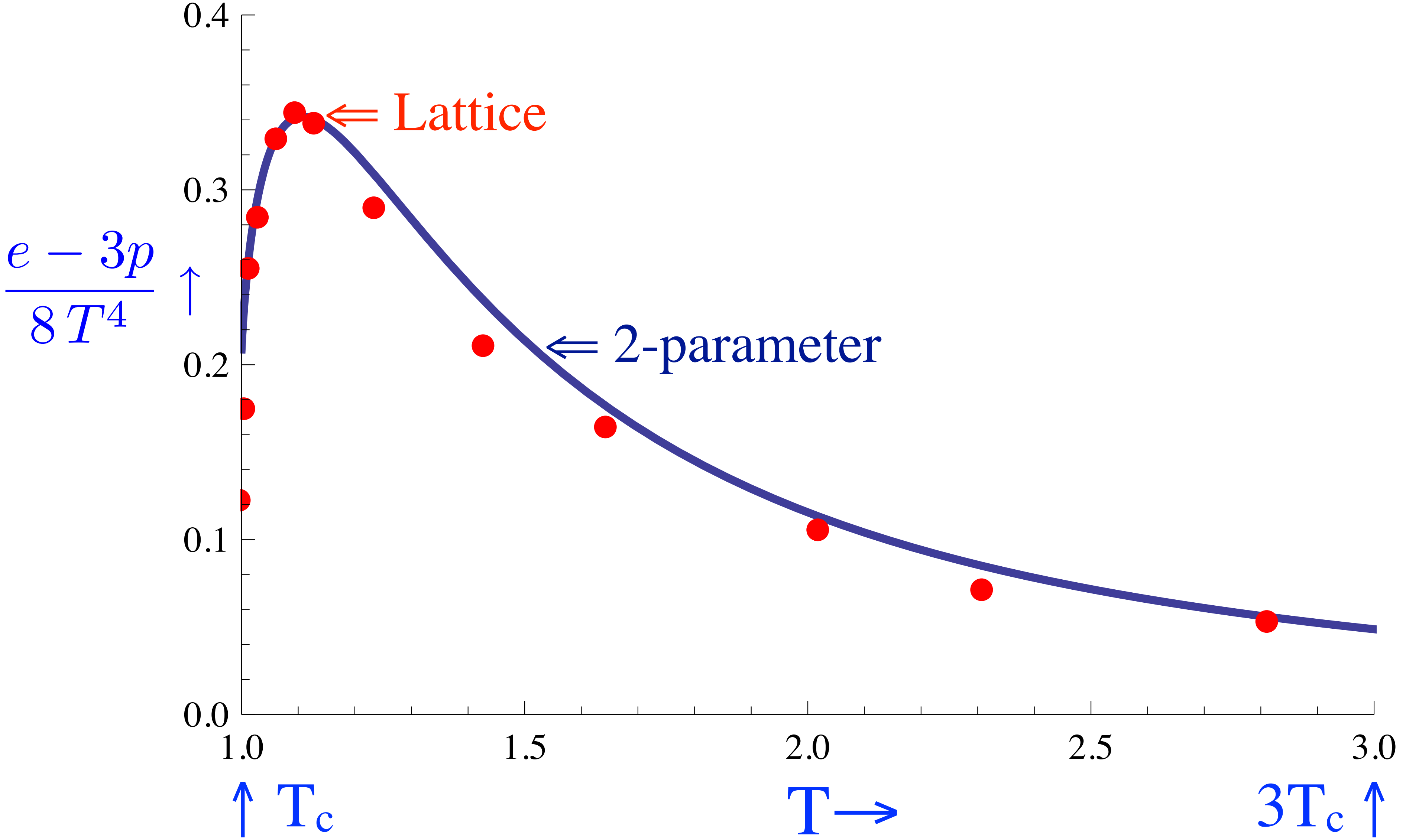


Bag const ~ (262 MeV)^4

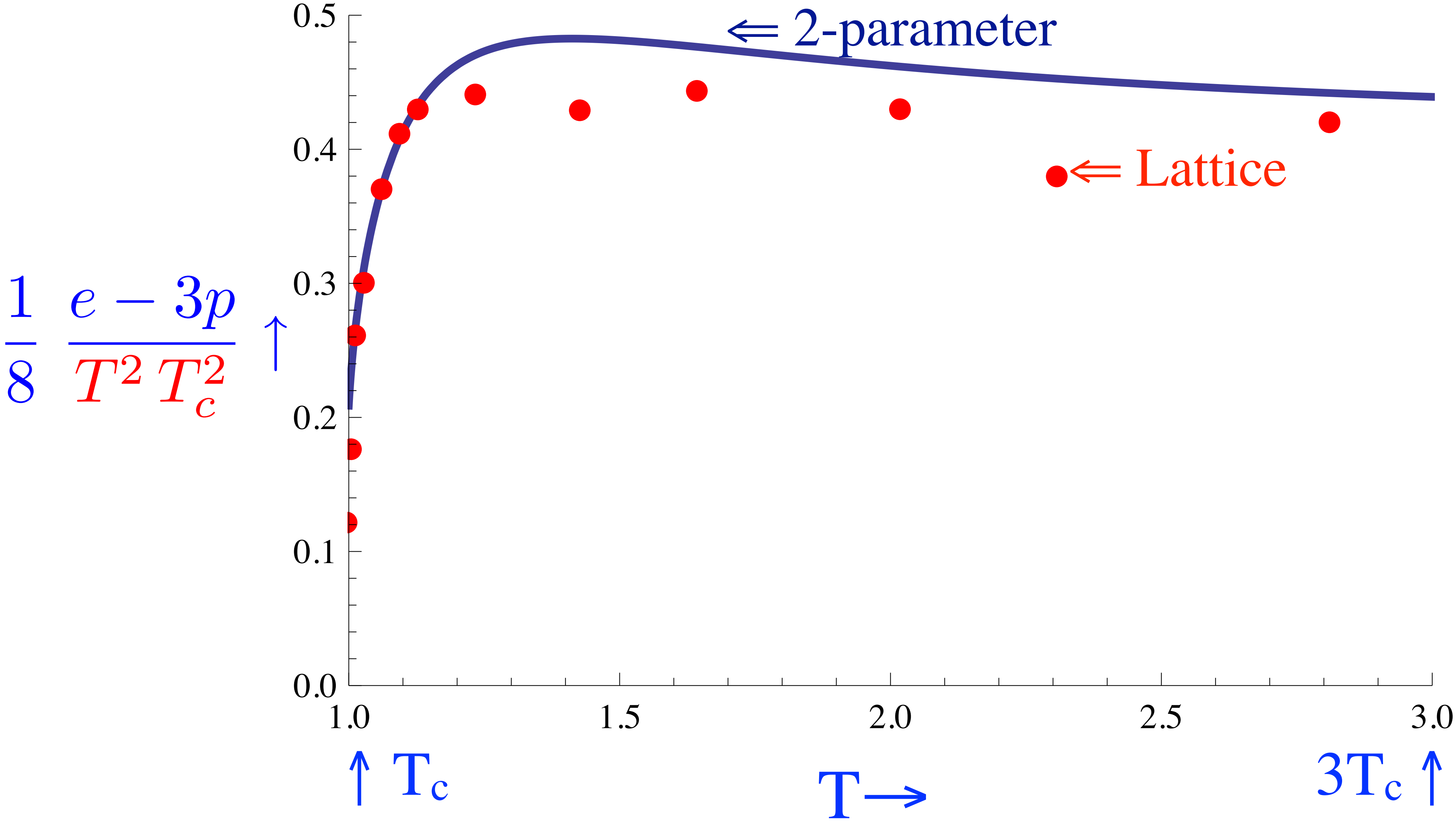
c_2 *not* near 1, vs 1-para.

Latent heat, lattice:
BPK: Beinlich, Peikert, Karsch
lat/9608141
DG: Datta, Gupta
1006.0938

Anomaly: 2-parameter model vs lattice

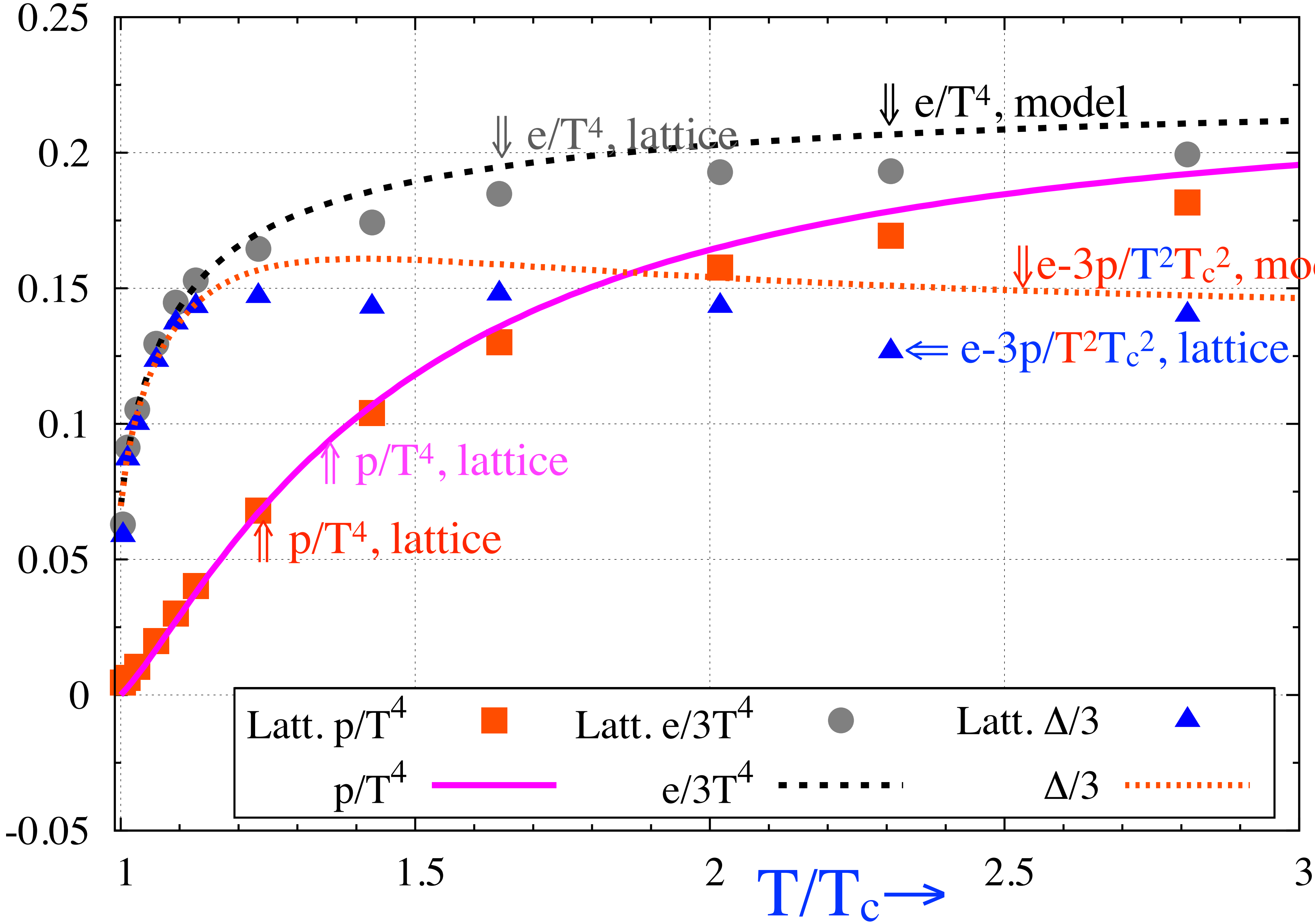


Anomaly times T²: 2-parameter model vs lattice



Thermodynamics of 2-parameter model, N = 3

$c_3(1) = 1.33\,,\ c_3(\infty) = .95\,,\ c_1 = .833\,,\ c_2 = .552$



Interface tensions, 2-parameter model, N = 3

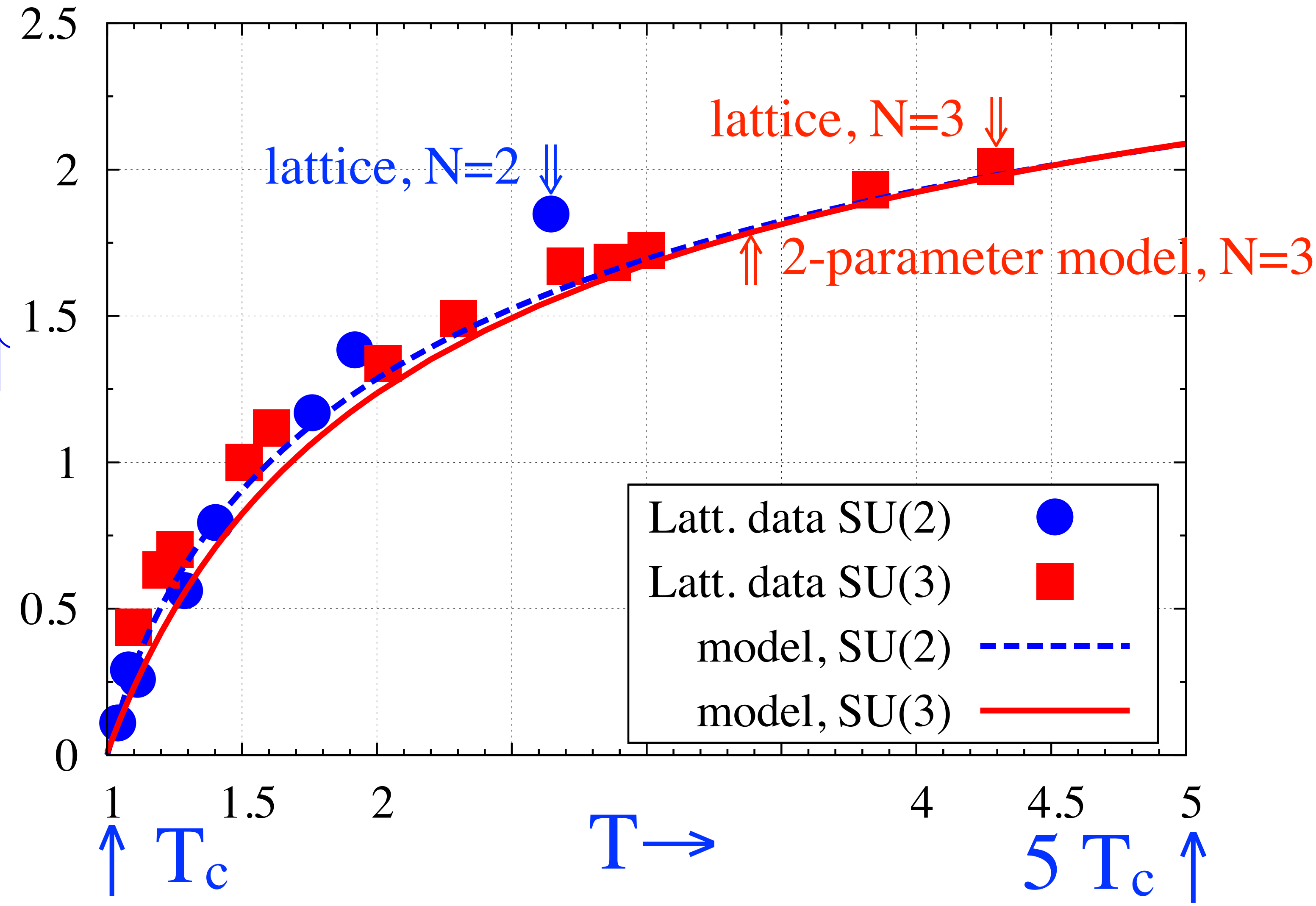
Order-order interface tension, σ, close to lattice. Order-order $\sigma(T_c)/T_c^2 \sim .043$.

1st order transition, so can compute order-disorder $\sigma(T_c)/T_c^2 \sim .022$, vs

Lattice: Lucini, Teper, Wegner, lat/0502003, .019 Beinlich, Peikert, Karsch lat/9608141 0.16

Order-order σ:

$$\frac{\sigma}{(N-1)T^2} \uparrow$$



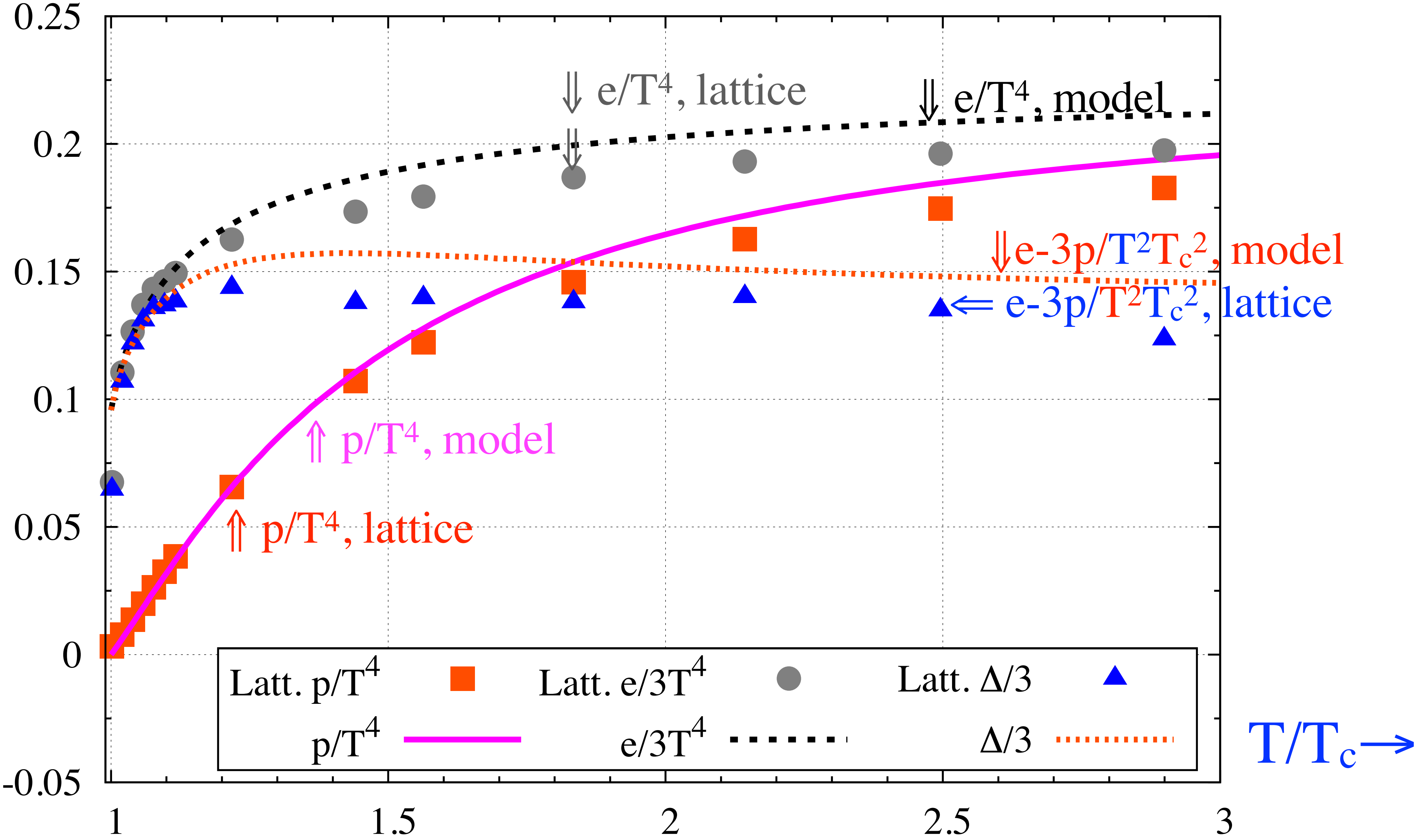
Lattice:
de Forcrand, D'Elia, Pepe, lat/0007034
de Forcrand, Noth lat/0506005

2-parameter model, N = 4

Assume $c_3(\infty) = 0.95$, like N=3. Fit $c_3(1)$ to latent heat, Datta & Gupta, 1006.0938

Order-disorder $\sigma(T_c)/T_c^2 \sim .08$, vs lattice, .12, Lucini, Teper, Wegner, lat/0502003

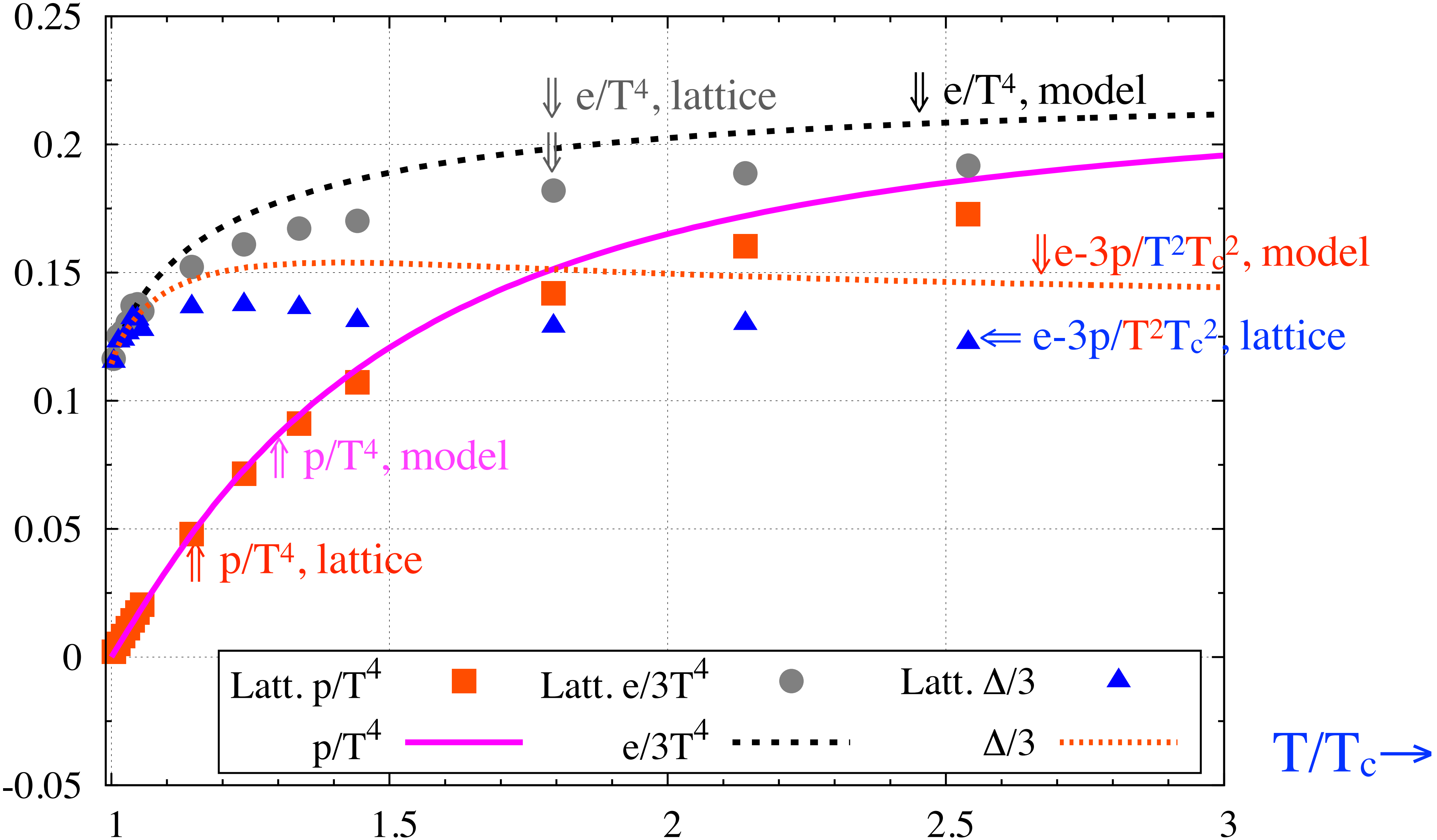
$$c_3(1) = 1.38\,,\; c_3(\infty) = .95\,,\; c_1 = 1.025\,,\; c_2 = 0.39$$



2-parameter model, N = 6

Order-disorder $\sigma(T_c)/T_c^2 \sim .25$, vs lattice, .39, Lucini, Teper, Wegner, lat/0502003

$c_3(1) = 1.42\,,\; c_3(\infty) = .95\,,\; c_1 = 1.21\,,\; c_2 = 0.23$



Conclusions

Transition region *narrow*: for pressure, $< 1.2\ T_c$!

For interface tensions, $< 4\ T_c$...

Above $1.2\ T_c$, pressure dominated by *constant* term $\sim T^2$.

What does this term come from? Gluon mass (for spatial gluons)?

In 2+1 dimensions, ideal T^3. Caselle + ...: *also* T^2 term in pressure.

But mass would be $m^2\ T$, not $m\ T^2$.

T^2 term like free energy of massless fields in 2 dimensions: string? Above T_c?

Need to include quarks!

Can then compute temperature dependence of:

shear viscosity, energy loss of light quarks, damping of quarkonia...

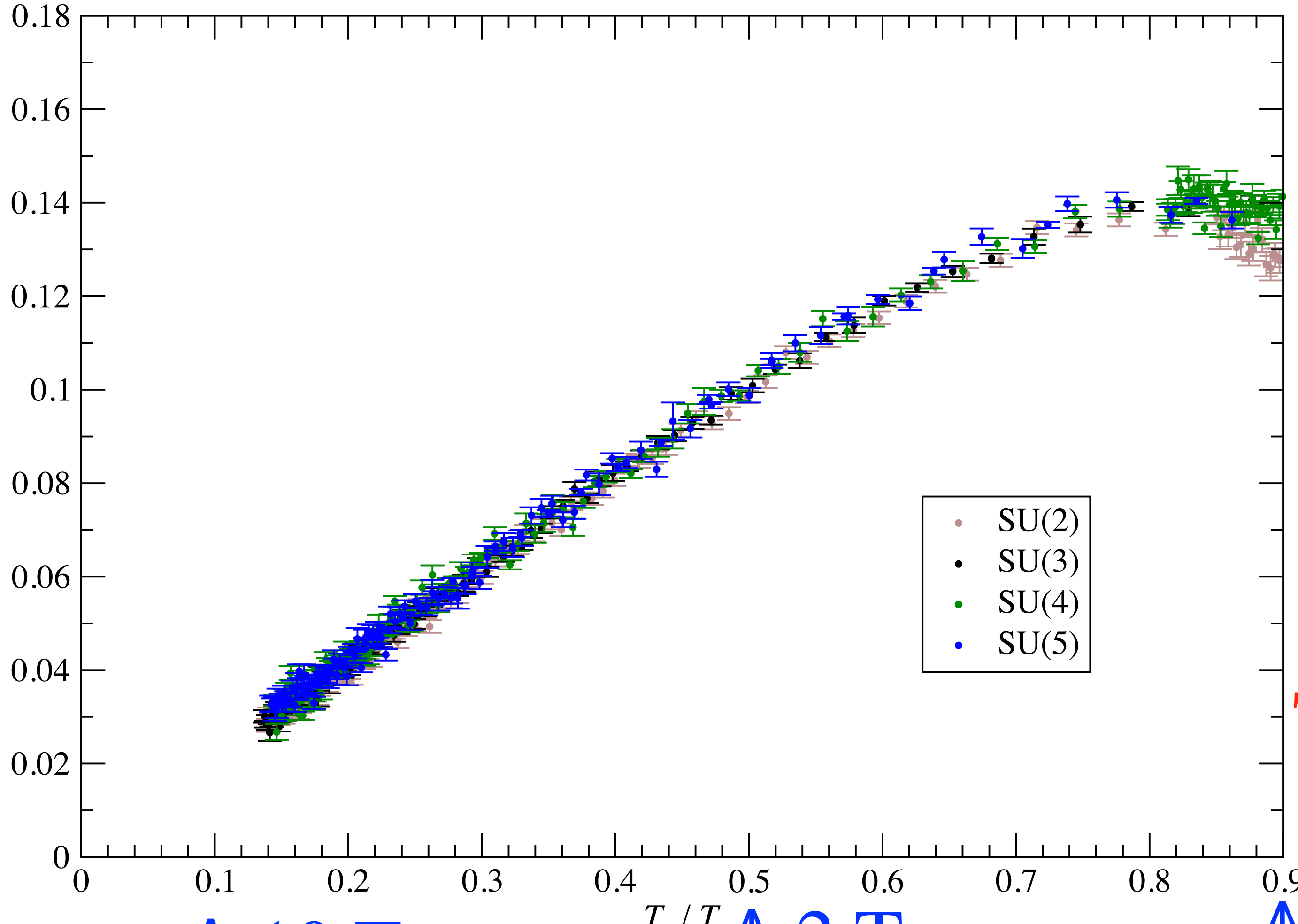
Lattice: SU(N) in 2+1 dimensions

Caselle, Castagnini, Feo, Gliozzi, Panero, $T < T_c$, 1105.0359, $T > T_c$, below, unpublished

SU(N) for N = 2, 3, 4, 5. # time steps = 6.

$$p(T) \approx \# \, T^2 (T - c\,T_c)\,, \; c \approx 1.$$

$$\frac{1}{N^2-1}\,\frac{e-2p}{T^3} \uparrow$$



$T_c/T \rightarrow$

↑ 10 T_c ↑ 2 T_c ↑ 1.1 T_c