Deconfinement: Renormalized Polyakov Loops to Matrix Models

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1. Review: Phase Transitions at $T \neq 0$, Lattice Results for N=3, nf = $0 \rightarrow 3$

   (N = # colors, nf = # flavors)

2. *Bare* Polyakov Loops, $\forall$ representations R: factorization for $N \rightarrow \infty$

3. *Renormalized* Polyakov Loops

4. Numerical results from the lattice:

   N=3, nf=0: R = 3, 6, 8 (10?)

5. Effective (matrix!) model for *renormalized* loops

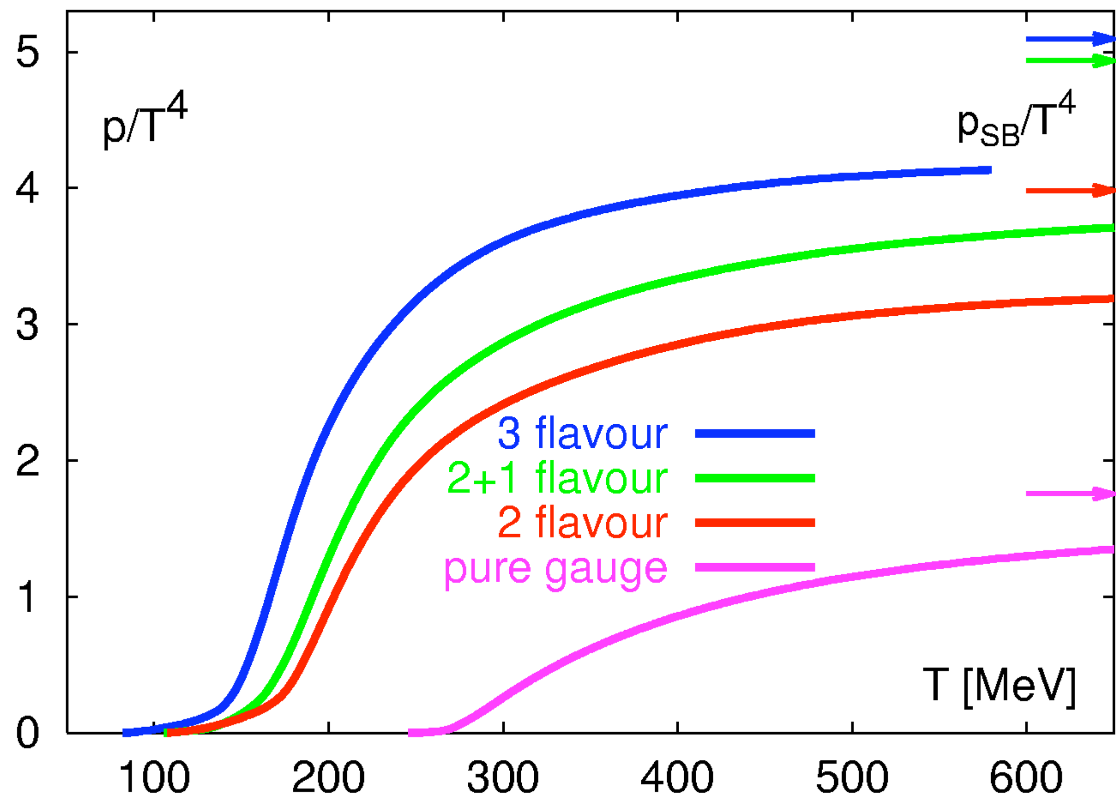
Review of Lattice Results: N=3, nf = 0, 2, 2+1, 3

nf = 0: T_deconf ≈ 270 MeV

pressure *small* for T < T_d

like N→∞: p ~ 1 for T<T_d, p ~ N^2 for T>T_d (Thorn, 81)

pressure/T^4 ↑



T=>

Bielefeld

nf ≠ 0: as nf ↑, p_ideal ↑, T_chiral ↓

BIG change: between nf = 0 and nf = 3,

p_ideal: 16 to 48.5 x ideal m=0 boson Tc: 270 to 175 MeV!

Even the order changes: first for nf=0 to "crossover" for nf = 2+1

Three colors, pure gauge: *weakly* first order

Latent heat: $\Delta\epsilon/\epsilon_{ideal}$ ~ 1/3 vs 4/3 in bag model. So?

Look at gauge-inv. correlation functions: (2-pt fnc. Polyakov loop)

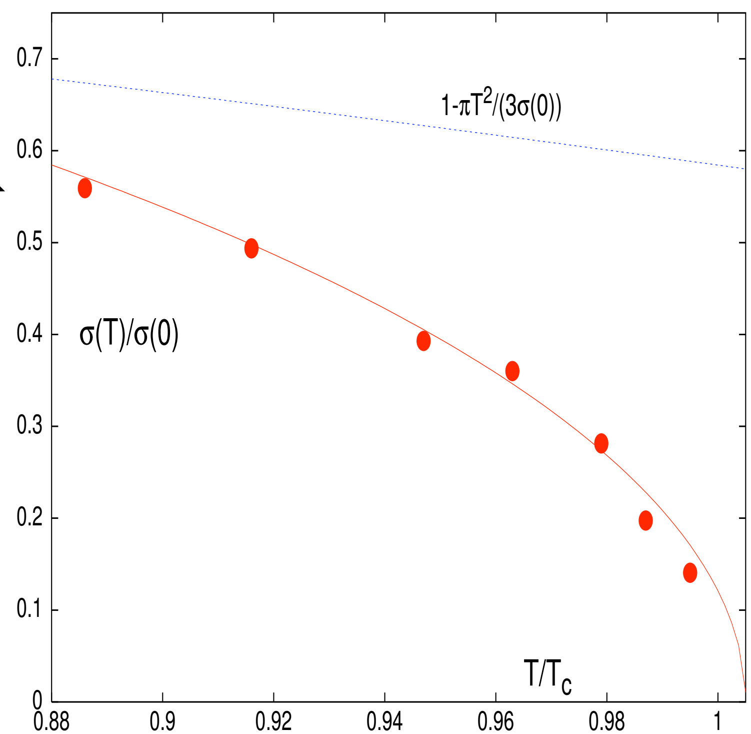
$$\langle \ell^*(x)\ell(0)\rangle - |\langle \ell \rangle|^2 \sim e^{-mx}/x \, , x \to \infty$$

T < T_d : m = σ/T

T > T_d: m = m_Debye

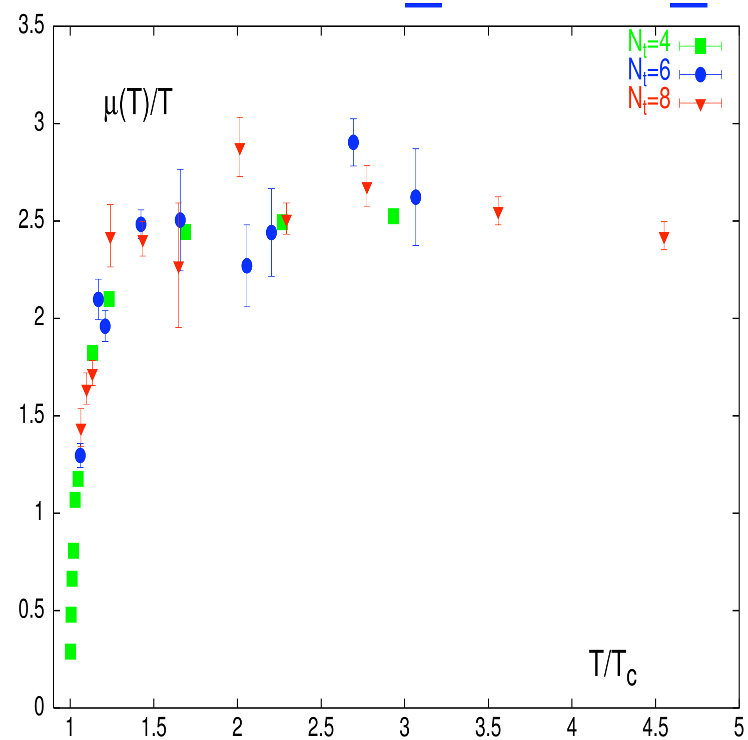
T-dep. string tension ↑

$$\frac{\sigma(T)}{\sigma(0)}$$



.9 T_d ↑ T_d ↑

FIG. 1:



↑ T_d ↑ 4 T_d

FIG. 1:

↑Debye mass

$$\frac{m_{Debye}(T)}{m_{Debye}(0)}$$

Bielefeld

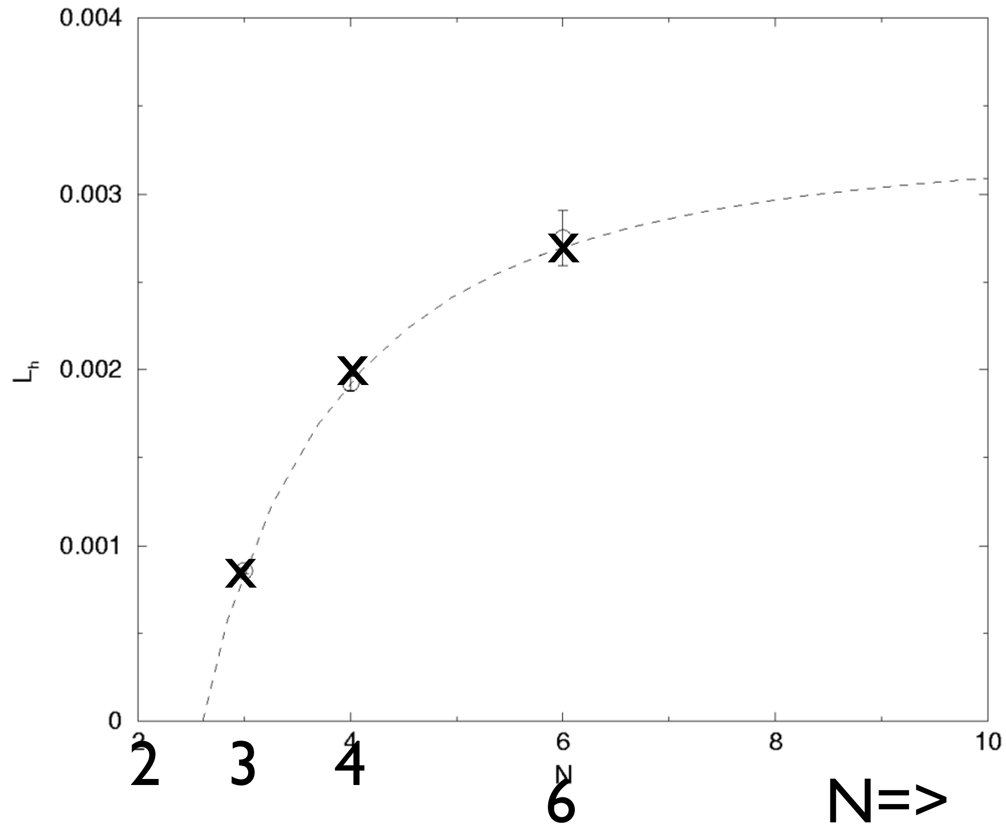
(Some) correlation lengths grow by ~ 10!

$$\frac{\sigma(T_d^-)}{\sigma(0)} \approx \frac{m_{Debye}(T_d^+)}{m_{Debye}(1.5T_d)} \approx \frac{1}{10}$$

Deconfining Transition vs N: First order ∀ N ≥ 4

Lucini, Teper, Wenger '03: latent heat ~ N^2 for N= 3, 4, 6, 8

latent heat/N^2



N=2: *second* order
=> no latent heat

N=3: "weakly" 1st

N ≥ 4: *strongly* first order.

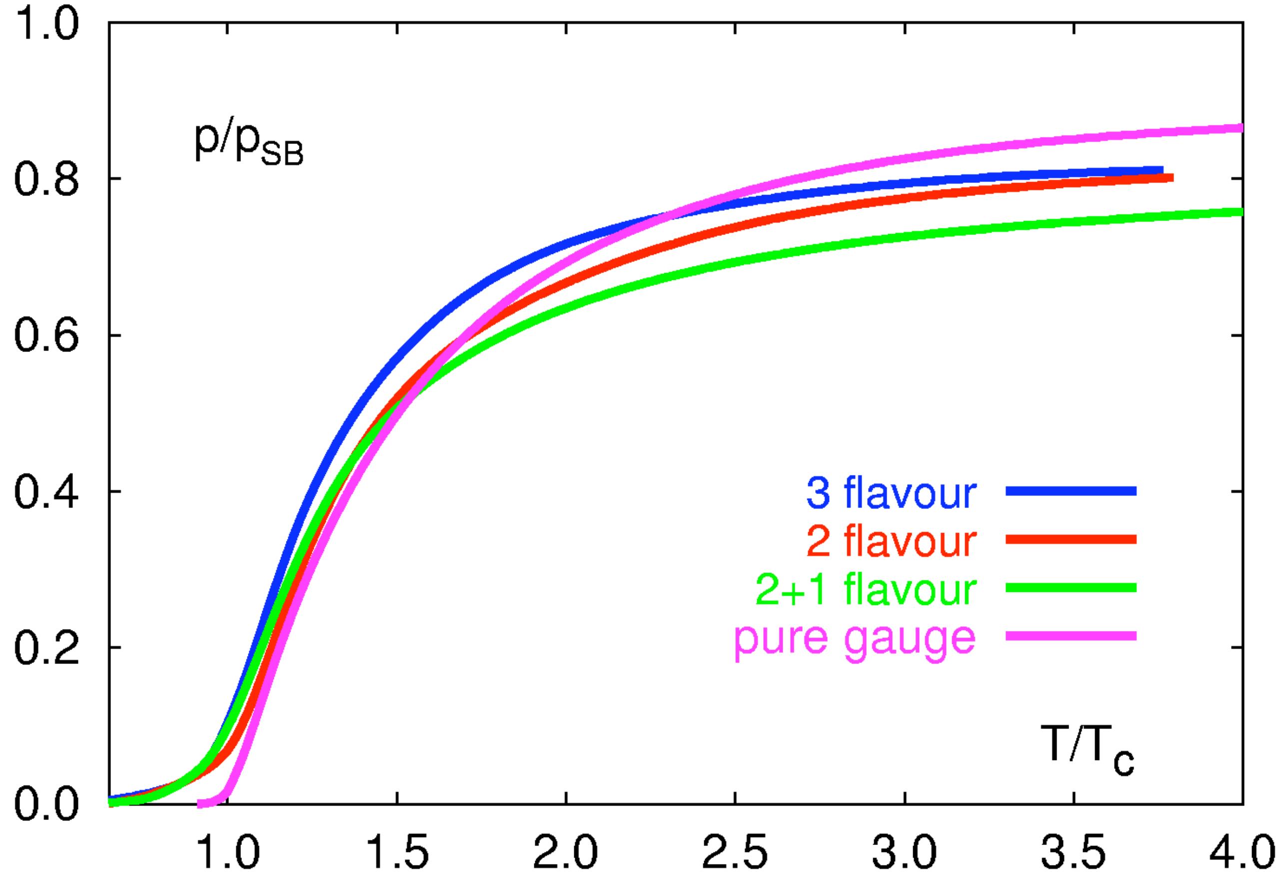
No data for $\sigma(T_d^-)$, $m_{Debye}(T_d^+)$

Is N → ∞ Gross-Witten?

Gross-Witten - First order *but:* $\sigma(T_d^-) = m_{Debye}(T_d^+) = 0$

Flavor Independence

$$\frac{p}{p_{ideal}}\left(\frac{T}{T_c}\right) \approx universal$$



Bielefeld

Perhaps: even for nf ≠ 0, "transition" *dominated* by gluons

At T ≠ 0: thermodynamics dominated by Polyakov loops

Wilson Lines at T ≠ 0

Always: "pure" SU(N) gauge, *no* dynamical quarks (nf = 0)

Imaginary time formalism: $\tau : 0 \rightarrow 1/T$

Wilson line in fundamental representation:

$$\mathbf{L}_N(\vec{x},\tau) = \mathcal{P}\exp\left(ig\int_0^\tau A_0^a(\vec{x},\tau')t_N^a d\tau'\right)$$

= *propagator* for "test quark"at x, moving up in (imaginary) time

$$D_0\mathbf{G}_N = \delta(\tau) => \mathbf{G}_N = \mathbf{L}_N\theta(\tau)$$

$\mathbf{L}_{\overline{N}} = \mathbf{L}_N^\dagger$ = propagator "test anti-quark" at x, moving back in time

$\mathbf{L}_N \,\epsilon\, SU(N):\ \mathbf{L}_N^\dagger\mathbf{L}_N = \mathbf{1}_N\ ,\ \det(\mathbf{L}_N) = 1$ (Mandelstam's constraint)

Polyakov Loops

Wrap *all* the way around in τ: $\mathbf{L}_N(\vec{x}, 1/T) \to \mathbf{L}_N$

Polyakov loop = *normalized* loop = gauge invariant

$$\ell_N = \frac{1}{N}\,\mathrm{tr}\,\mathbf{L}_N$$

Confinement: test quarks don't propagate

$$\langle \ell_N \rangle = 0 \quad , \quad T < T_{deconf}$$

Deconfinement: test quarks propagate

$$\langle \ell_N \rangle = e^{i\theta} |\langle \ell_N \rangle| \neq 0 \quad , \quad T > T_d$$

$e^{iN\theta} = 1 :$ Spontaneous breaking of global Z(N) = center SU(N)

't Hooft '79, Svetitsky and Yaffe, '82

Adjoint Representation

Adjoint rep. = "test meson"

$$\mathrm{tr}\,\mathbf{L}_{adj} = |\mathrm{tr}\,\mathbf{L}_N|^2 - 1$$

Note: both coefficients ~ 1

Check: $\mathbf{L}_N = \mathbf{1}_N \to \mathrm{tr}\,\mathbf{L}_{adj} = N^2 - 1$ = dimension of the rep.

Adjoint loop: divide by dim. of rep.

$$\ell_{adj} = \frac{1}{N^2 - 1}\,\mathrm{tr}\,\mathbf{L}_{adj}$$

At large N,

$$\ell_{adj} = |\ell_N|^2 + O\left(\frac{1}{N^2}\right)$$ = *factorization*

Two-index representations

2-index rep.'s = "di-test quarks" = symmetric or anti-sym.

$$\mathrm{tr}\mathbf{L}_{(N^2\pm N)/2} = \frac{1}{2}\left(\left(\mathrm{tr}\mathbf{L}_N\right)^2 \pm \mathrm{tr}\mathbf{L}_N^2\right)$$

Di-quarks: two qks wrap once in time, or one qk wraps twice

Again: both coeff's ~ 1. Subscript = dimension of rep.'s = (N^2 ± N)/2

For *arbitrary* rep. R, if d_R = dimension of R,

$$\ell_R \equiv \frac{1}{d_R}\,\mathrm{tr}\,\mathbf{L}_R$$

For 2-index rep.'s ±, as N → ∞,

$$\ell_\pm \sim \ell_N^2 + O\left(\frac{1}{N}\right)$$ corr.'s 1/N, *not* 1/N^2: $\sim \frac{1}{N^2}\mathrm{tr}\mathbf{L}_N^2$

Loops at Infinite N

"Conjugate" rep.'s of Gross & Taylor '93: $\mathbf{L}_N$ and $\mathbf{L}_{\overline{N}} = \mathbf{L}_N^\dagger$

If all test qks and test anti-qks wrap once and *only* once in time,

$$\mathrm{tr}\, \mathbf{L}_R = \#(\mathrm{tr}\, \mathbf{L}_N)^{p_+} (\mathrm{tr}\, \mathbf{L}_N^\dagger)^{p_-} + \ldots$$

Many other terms:

$$\#' \; \mathrm{tr}\, \mathbf{L}_N^2 \; (\mathrm{tr}\, \mathbf{L}_N)^{p_+ - 2} (\mathrm{tr}\, \mathbf{L}_N^\dagger)^{p_-} + \ldots$$

dimension R = $d_R \sim N^{p_+ + p_-}$

As N → ∞, *if* #, #'... are all of order 1, *first* term dominates, and:

$$\ell_R \sim (\ell_N)^{p_+} \; (\ell_{\overline{N}})^{p_-} + O\left(\frac{1}{N}\right)$$

Normalization: if $\quad \ell_N = \ell_{\overline{N}} = 1 \; , \; \ell_R = 1 \; \forall \; R$

Factorization at Infinite N

In the deconfined phase, the fundamental loop condenses:

$$\langle \ell_N \rangle = e^{i\theta} |\langle \ell_N \rangle| \neq 0 \quad , \quad T > T_d \quad , \quad e^{iN\theta} = 1$$

Makeenko & Migdal '80: at N=∞, *expectation values* factorize:

$$\langle \ell_R \rangle = \langle \ell_N \rangle^{p_+} \langle \ell_{\overline{N}} \rangle^{p_-} + O\left(N^{-1}\right)$$
$$= e^{i e_R \theta} |\langle \ell_N \rangle|^{p_+ + p_-} + O(N^{-1})$$

Phase trivial: $e_R = p_+ - p_-$ = Z(N) charge of R, defined modulo N

Magnitude *not* trivial: highest powers of $|\langle \ell_N \rangle|$ win.

At infinite N, *any* loop order parameter for deconfinement: *even* if e_R = 0! E.g.: adjoint loop (Damgaard, '87) vs adjoint screening.

N.B.: $\langle \ell_{test\ baryon} \rangle = \langle \ell_N \rangle^{N-1} \langle \mathrm{tr}\ \mathbf{L}_N^2 / N \rangle$

“Mass” renormalization for loops

Loop = propagator for infinitely massive test quark.
Still has mass renormalization, proportional to length of loop:

$$\langle \ell_R \rangle = \exp\left(-m_R/T\right) \quad , \quad m_R \equiv f_R^{div}/a$$

a = lattice spacing. m_R = 0 with dimensional regularization, but so what?

To 1-loop order in 3+1 dimensions:

$$\langle \ell_R \rangle - 1 \sim -\left(\frac{1}{T}\right) \mathcal{C}_R g^2 \int^{1/a} \frac{d^3k}{k^2} \sim -\frac{\mathcal{C}_R g^2}{aT}$$

Divergences order by order in g^2. *Only* power law divergence for straight loops in 3+1 dim.'s. ; corrections ~aT.

In 3+1 dim.'s, loops with cusps *do* have logarithmic divergence. (Dokshitzer: ~ *classical* bremstrahlung)

In 2+1 dim.'s, straight loops also have log. div.'s. (cusps do not)

Renormalization of Wilson Lines

Gervais and Neveu '80; Polyakov '80; Dotsenko & Vergeles '80 ....

For *irreducible* representations R, *renormalized* Wilson line:

$$\widetilde{\mathbf{L}}_R = \mathbf{L}_R / Z_R \quad , \quad Z_R \equiv \exp(-m_R/T)$$

Z_R = renormalization constant for Wilson line

As R's irreducible, different rep's don't mix under renormalization

For straight lines in 3+1 dimensions, only:

No anomalous dim. for Wilson line: *no* condition to fix Z_R at some scale

For *all* local, composite operators, Z's *independent* of T

Wilson line = *non*-local composite operator:

Z_R temperature dependent: from 1/T, and m_R (found numerically)

Not really different: a m_R = func. *renormalized* g^2, and so T-dependent.

Renormalization of Polyakov Loops

Renormalized loop: $\widetilde{\ell}_R = \ell_R / Z_R$

Constraint for bare loop: $|\ell_R| \leq 1$

For renormalized loop: $|\widetilde{\ell}_R| \leq Z_R^{-1}$

If $m_R > 0 \; \forall \, T, \; Z_R \to 0$ as a → 0, no constraint on ren.'d loop

E.g.: as T → ∞, ren'd loops approach 1 from *above*: (Gava & Jengo '81)

$$\langle \widetilde{\ell}_R \rangle - 1 \sim - \left(\frac{1}{T} \right) \mathcal{C}_R g^2 \int \frac{d^3 k}{k^2 + m^2_{Debye}} \sim (-) C_R g^2 (-) (m^2_{Debye})^{1/2}$$

$$\langle \widetilde{\ell}_R \rangle \approx \exp \left(+ \frac{C_R}{N} \frac{(g^2 N)^{3/2}}{8\pi\sqrt{3}} \right)$$

=> *negative* "free energy" for loop

Smooth large N limit: $C_R \approx \frac{N}{2} (p_+ + p_-) + O(1)$

Why all representations?

Previously: concentrated on loops in fund. and adj. rep.'s, esp. with cusps. At T ≠ 0, natural for loops, at a given point in space, to wrap around in τ many times. Most general gauge invariant term:

$$\mathcal{G} = (\mathrm{tr}\ \mathbf{L}_{R_1^+}^{q_1^+})^{n_1^+} (\mathrm{tr}\ \mathbf{L}_{R_2^+}^{q_2^+})^{n_2^+} \ldots (\mathrm{tr}\ (\mathbf{L}_{R_1^-}^{\dagger})^{q_1^-})^{n_1^-} \ldots$$

By group theory (the character expansion):

$$\mathcal{G} = \Sigma_R\, c_R\, \ell_R$$

Only this expression, which is linear in the *bare* loops, can be consistently ren.'d

Note: If all Z_R => 0 as a=> 0, $\mathcal{G} = c_1\ , a = 0.$

Irrelevant: physics is in the ren.'d, not the bare, loops. Discovered num.'y:

$$\langle |\ell_3|^2 \rangle = \frac{8}{9} Z_8 \langle \widetilde{\ell_8} \rangle + \frac{1}{9} \rightarrow \frac{1}{9} + \ldots\ a \rightarrow 0.$$

Lattice Regularization of Polyakov Loops

Basic idea: compare two lattices. *Same* temperature, *different* lattice spacing

If a << 1/T, ren'd quantities the same.

N_t = # time steps = 1/(aT) changes between the two lattices: get Z_R

N_s = # spatial steps; keep N_t/N_s fixed to minimize finite volume effects

$$\log\left(|\langle \ell_R \rangle|\right) = -f_R^{div} N_t + f_R^{cont} + f_R^{lat} \frac{1}{N_t} + \ldots$$

$f_R^{div} \to Z_R$ $\quad f_R^{cont} \to \langle \widetilde{\ell}_R \rangle$ $\quad$ Numerically, $f_R^{lat} \approx 0$

Each f_R is computed at fixed T. As such, there is *nothing* to adjust.

N.B.: also finite volume corrections from "zero" modes; *to be computed.*

Explicit exp. of divergences to ~g^4 at a≠0: Curci, Menotti, & Paffuti, '85

Representations, N=3

Label rep.'s by their dimension:

fundamental = 3

adjoint = 8

symmetric 2-index = 6

special to N=3: anti-symmetric 2-index = $\overline{3}$

"test baryon" = 10:

$$\ell_{10} = \frac{1}{10}\left(\mathrm{tr}\,\mathbf{L}_3\;\mathrm{tr}\,\mathbf{L}_3^2 + 1\right)$$

Measured 3, 6, 8, & 10 on lattice

Lattice Results

Standard Wilson action, three colors, quenched.

$$N_t = 4, 6, 8, \&10. \;\; N_s = 3N_t$$

Lattice coupling constant $\beta = 6/g^2$

β_d = coupling for deconfining transition: $= \beta_d(N_T)!$

Non-perturbative renormalization:

$$\log(T/T_d) = 1.7139(\beta - \beta_d) + \ldots$$

To get the same T/T_d @ different N_t, must compute at *different* β!

Calculate grid in β, interpolate to get the same T/T_d at different N_t

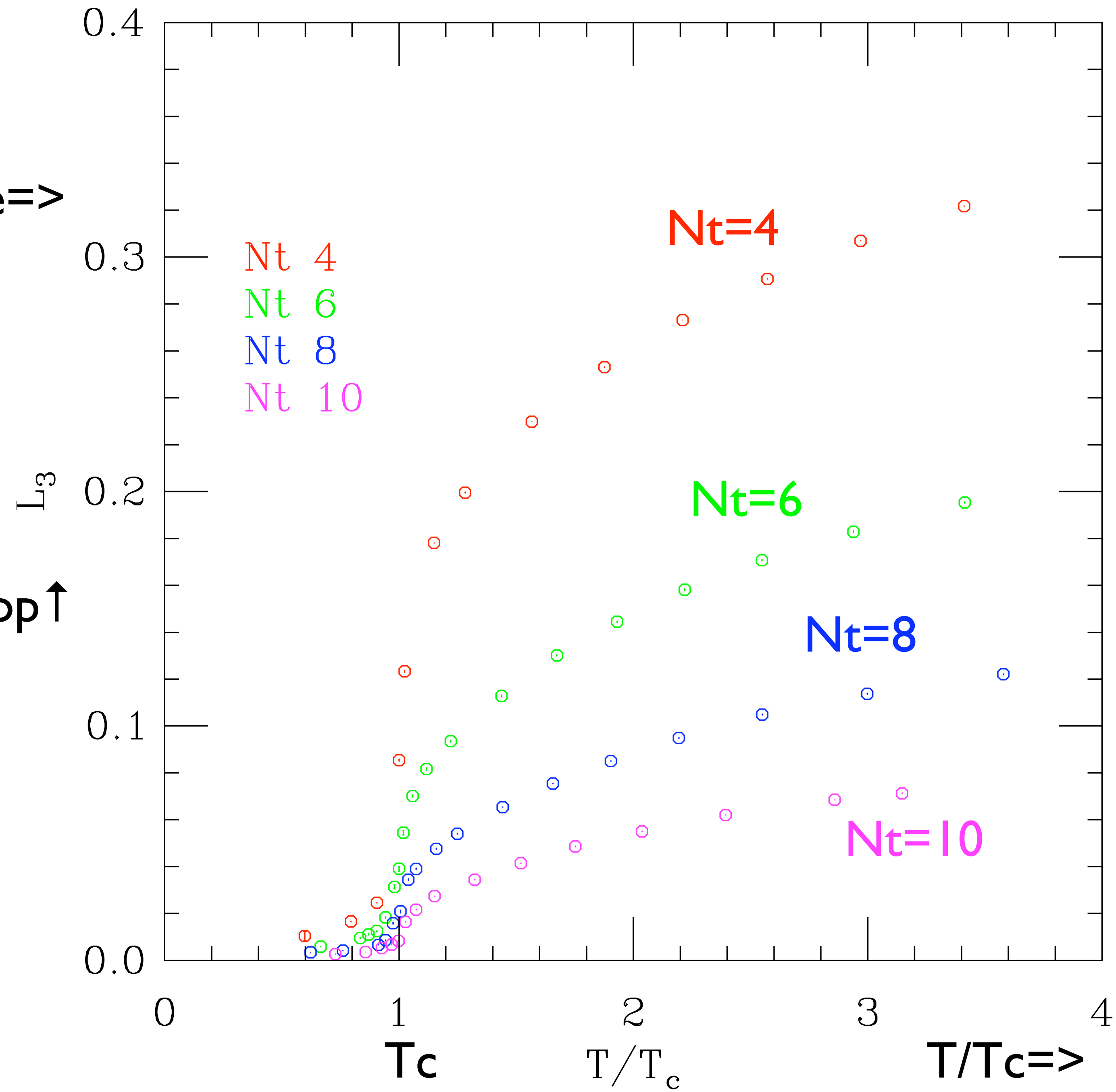
N.B. : Method same with dynamical quarks

Measured $\ell_3, \; \ell_6, \; \ell_8, \; \& \; \ell_{10}$ (No signal for 10 for N_t > 4)

Bare triplet loop vs T, at different Nt

Note scale=>
~ .3

Triplet loop↑



Tc

T/Tc=>

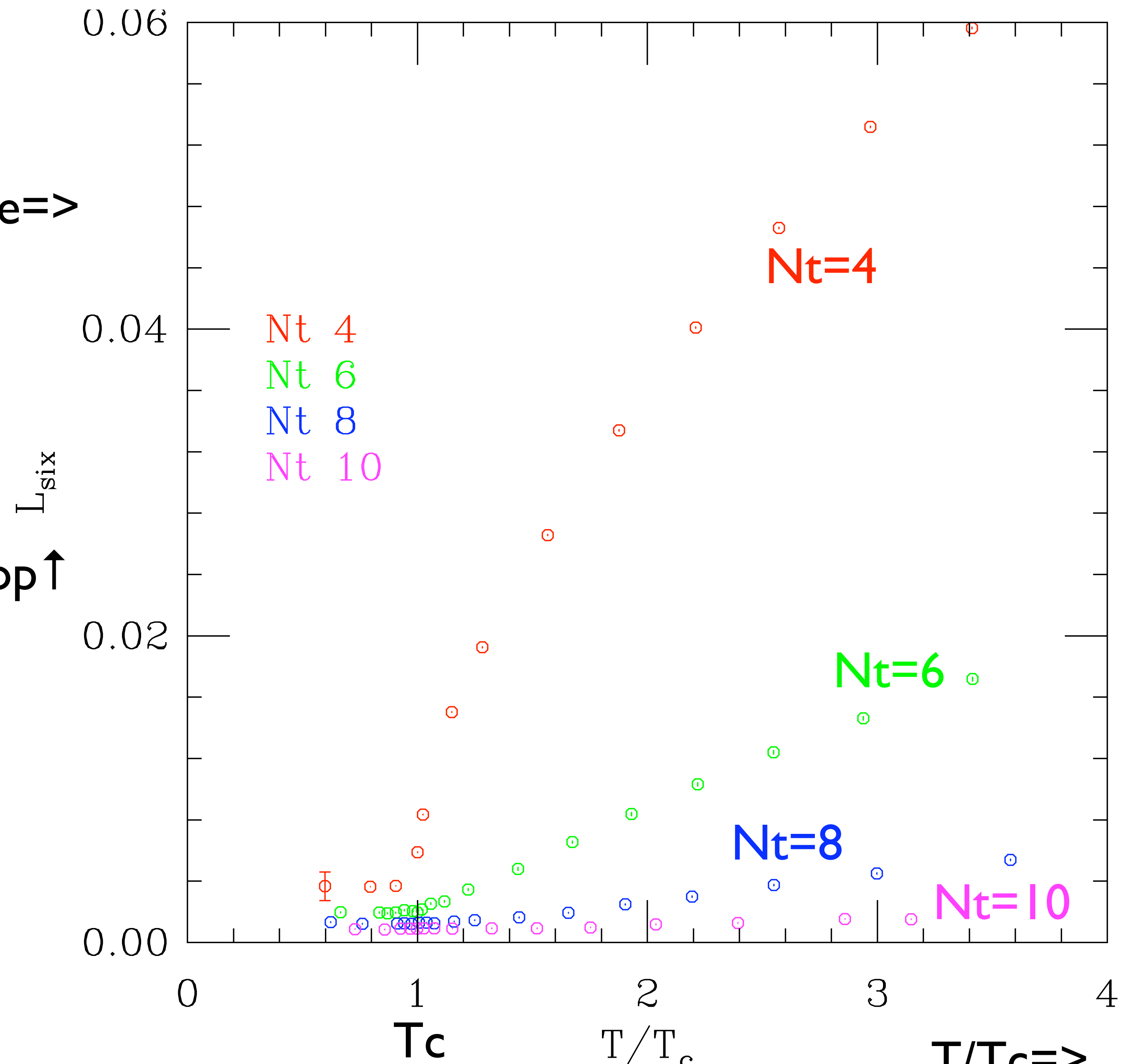
Nt = # time steps.

Bare loop vanishes as Nt → ∞

Bare sextet loop vs T, at different Nt

Note scale=>
~ .04

Sextet loop↑



Tc

T/Tc=>

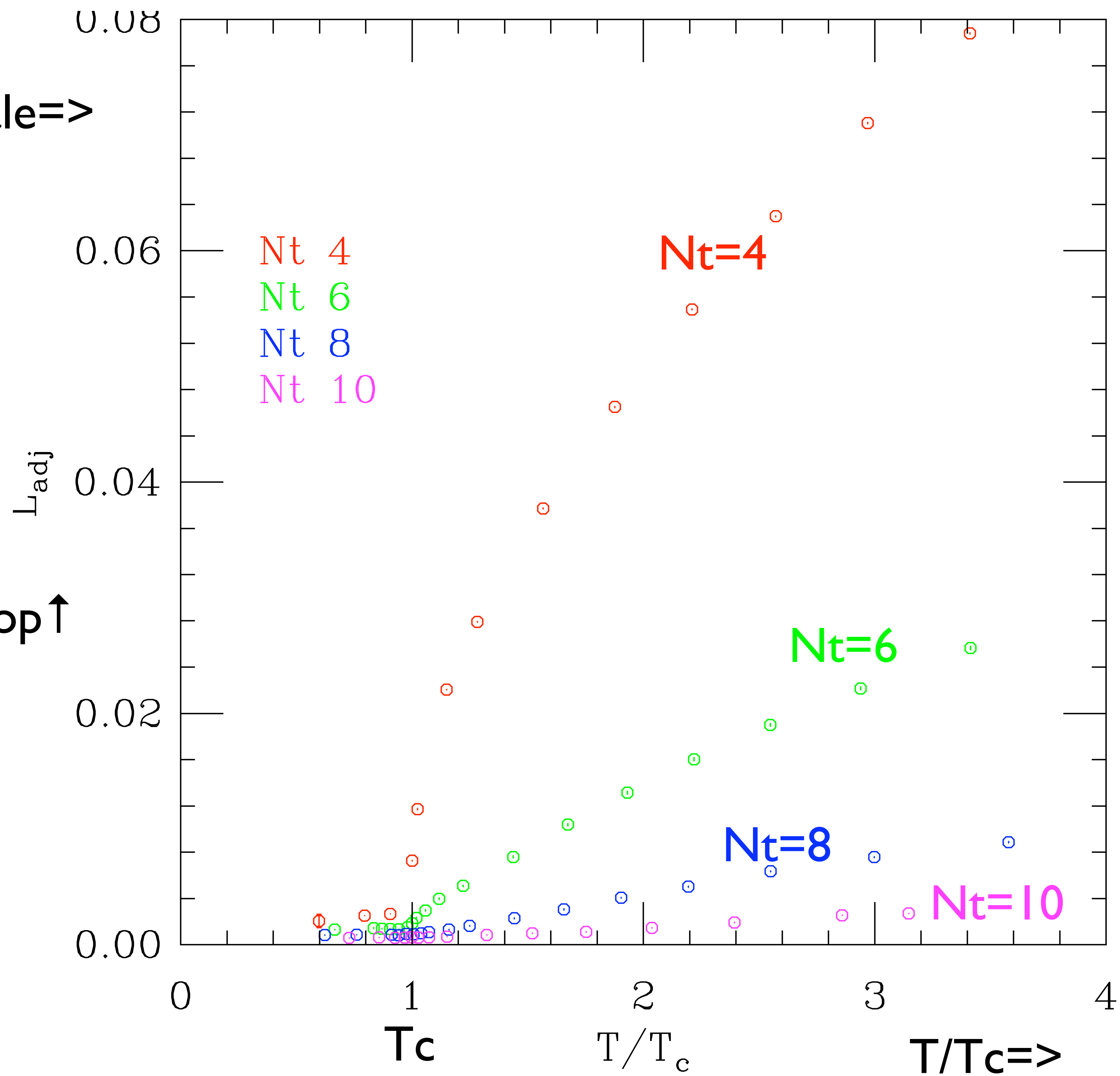
Nt = # time steps.

Bare loop vanishes more quickly as Nt → ∞

Bare octet loop vs T, at different Nt

Note scale=>
~ .06

Octet loop↑



Tc

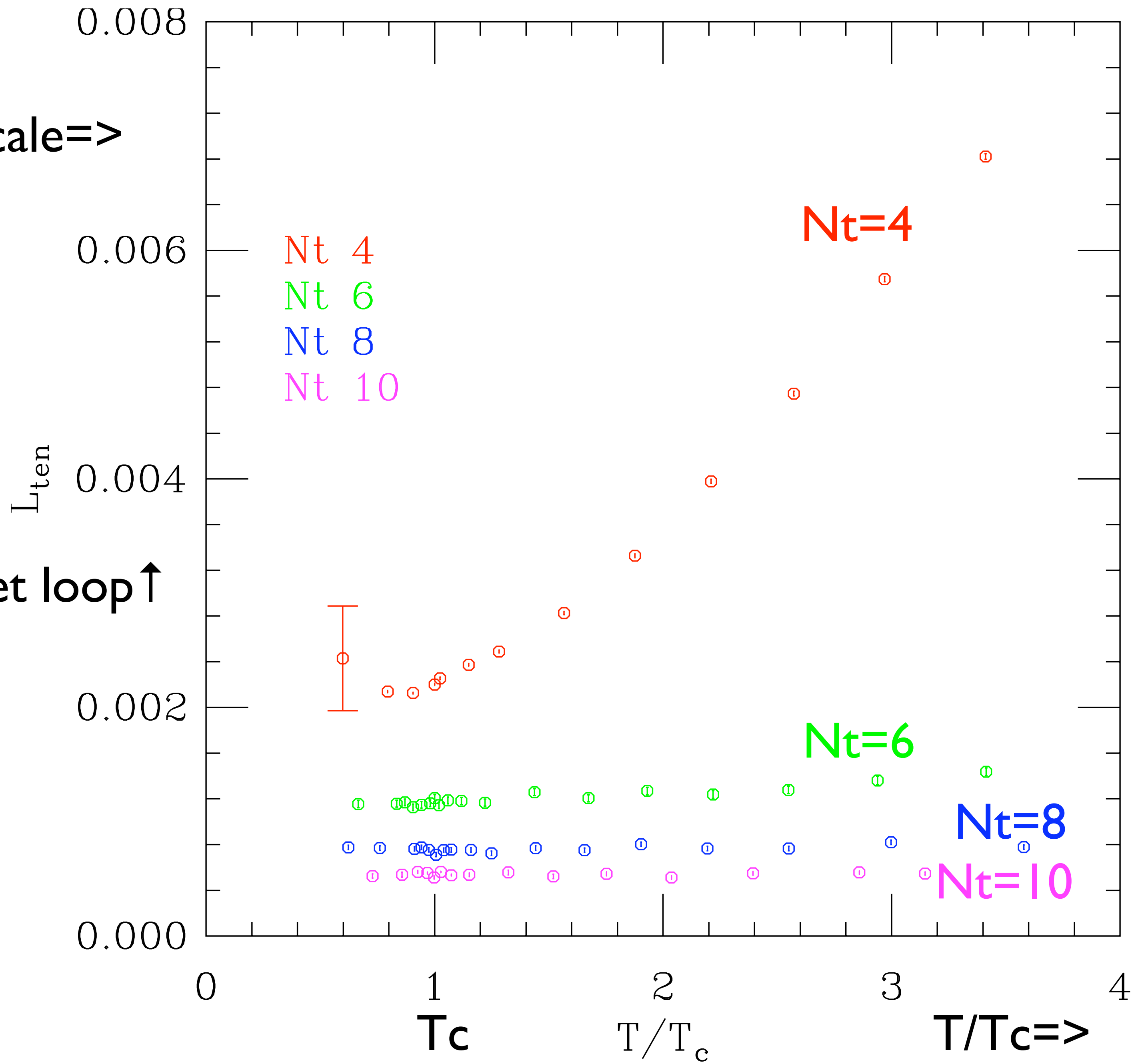
T/Tc=>

Very similar to sextet loop

Bare decuplet loop vs T, at different Nt

Note scale=>
~ .006

Decuplet loop↑

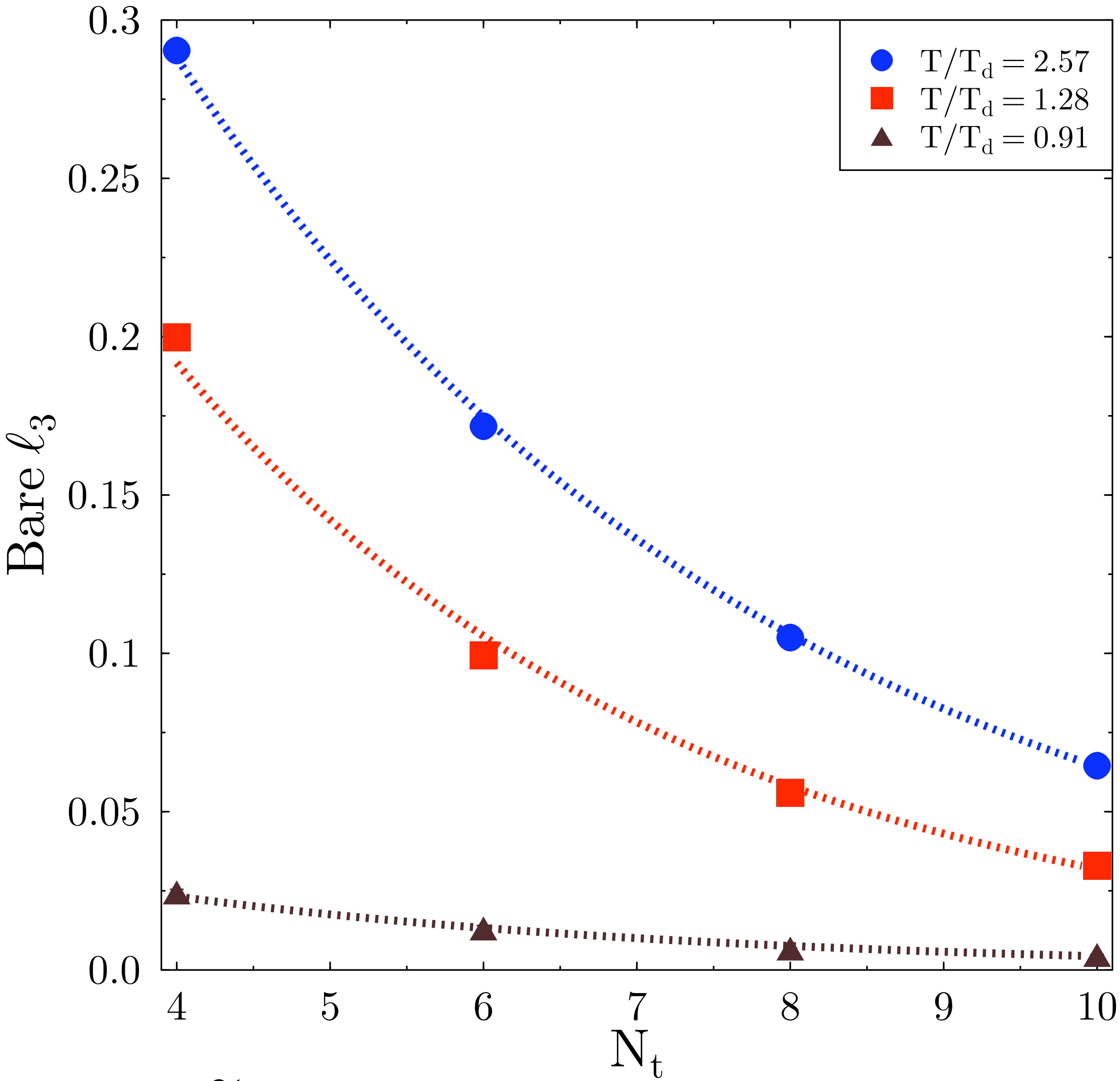


Tc

T/Tc=>

No stat.'y significant signal for decuplet loop above Nt=4.

Bare $|\ell_3|$ vs N_t

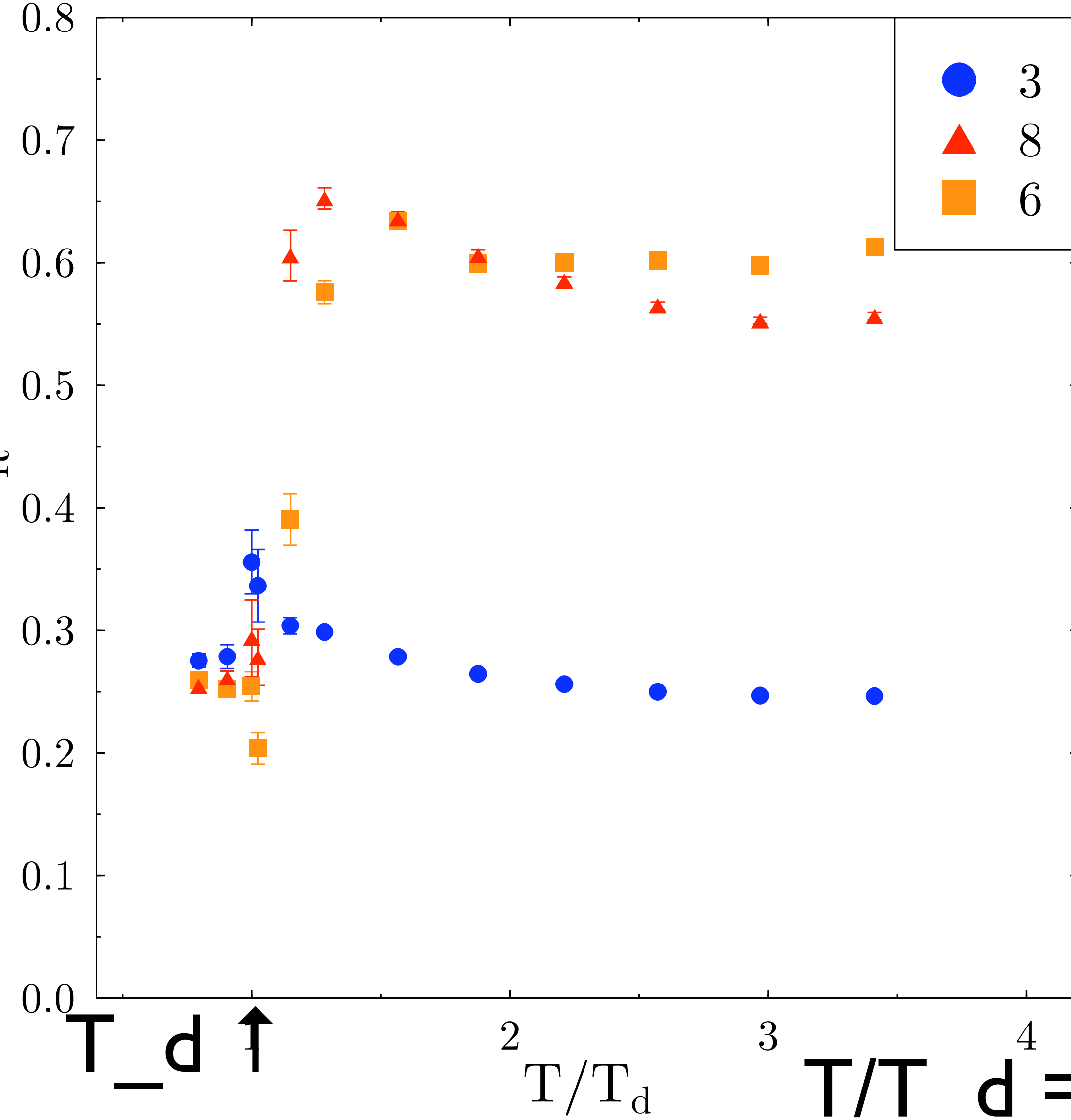


$$|\langle \ell_3 \rangle| \equiv \exp(-m_3/T)|\langle \widetilde{\ell_3} \rangle|$$

Divergent mass $m_R(T)$

a m_R looks like usual "mass": smooth function of ren.'d g^2 =>smooth func. of T: except near T_d! One loop: m_R ~ C_R; OK for T ~ 3 T_d. Fails for T < 1.5 T_d

$a\, m_R > 0 \;\forall\, T$



Casimir scaling for: a m_R ↓

$10/3 = C_6$

$3 = C_8$

$\frac{4}{3} = C_3$

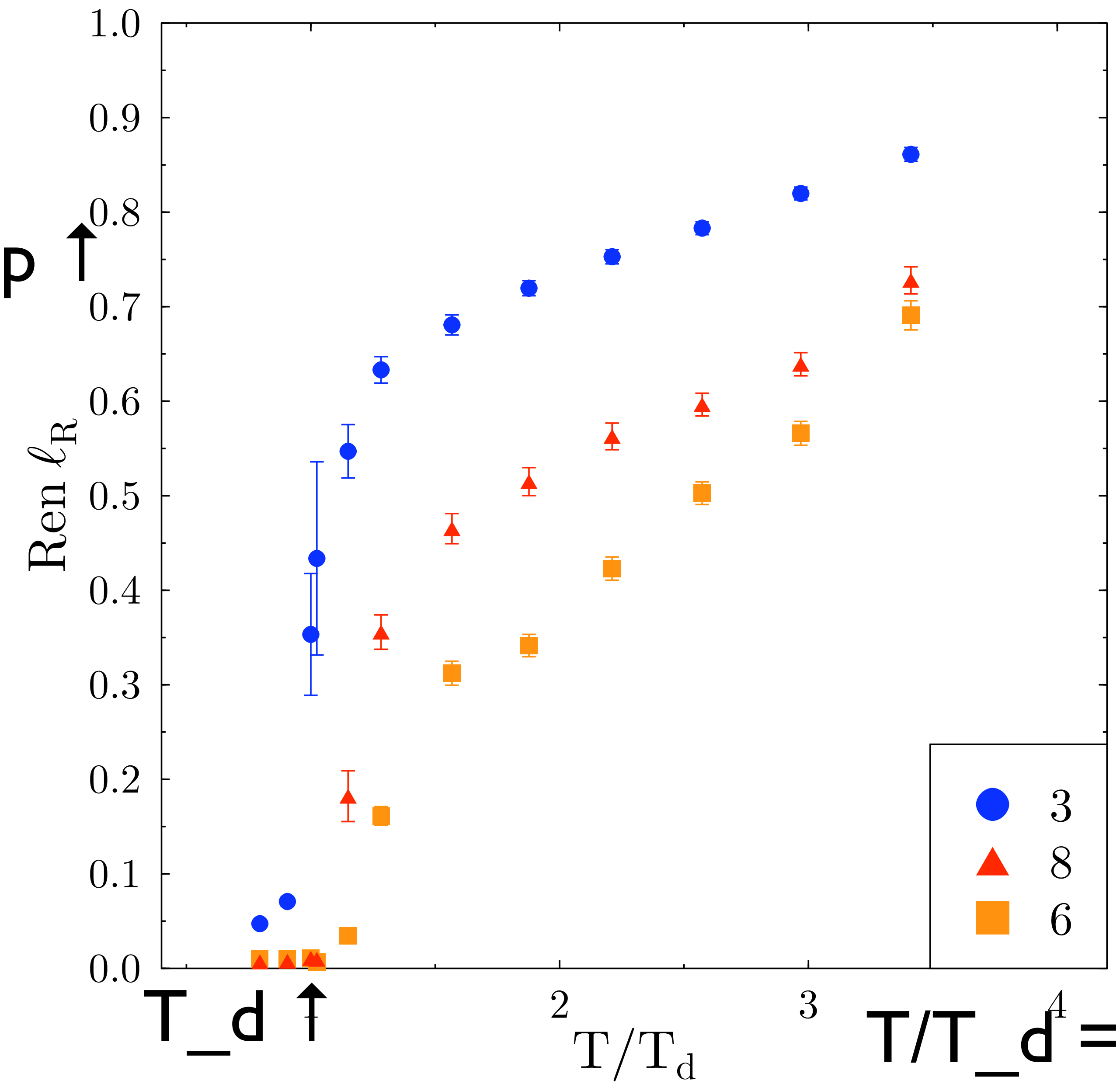
a m_R ↑

T_d ↑

T/T_d =>

Renormalized Polyakov Loops

Ren'd loop ↑



T_d ↑

T/T_d =>

Find: ren.'d triplet loop, but also significant octet and sextet loops, as well.

$$\widetilde{\ell}_3 > \widetilde{\ell}_8 > \widetilde{\ell}_6$$

No signal of decuplet loop at N_t > 4; C_10 big, so bare loop small

Results for Ren'd Polyakov Loops

$$T < T_d : Z(3) \text{ symmetry} \Rightarrow \langle \widetilde{\ell}_3 \rangle = \langle \widetilde{\ell}_6 \rangle = 0$$

$$\text{But } Z(3) \text{ charge } e_8 = 0 \Rightarrow \langle \widetilde{\ell}_8 \rangle \neq 0 \text{ for } T < T_d .$$

$$\text{Numerically} : \langle \widetilde{\ell}_8 \rangle = \; small \, \# \frac{1}{N^2} \approx 0 \; , \; T < T_d$$

Like large N: Greensite & Halpern '81, Damgaard '87...

(Similar to measuring adjoint string tension in confined phase)

Transition first order →ren.'d loops *jump* at T_d:

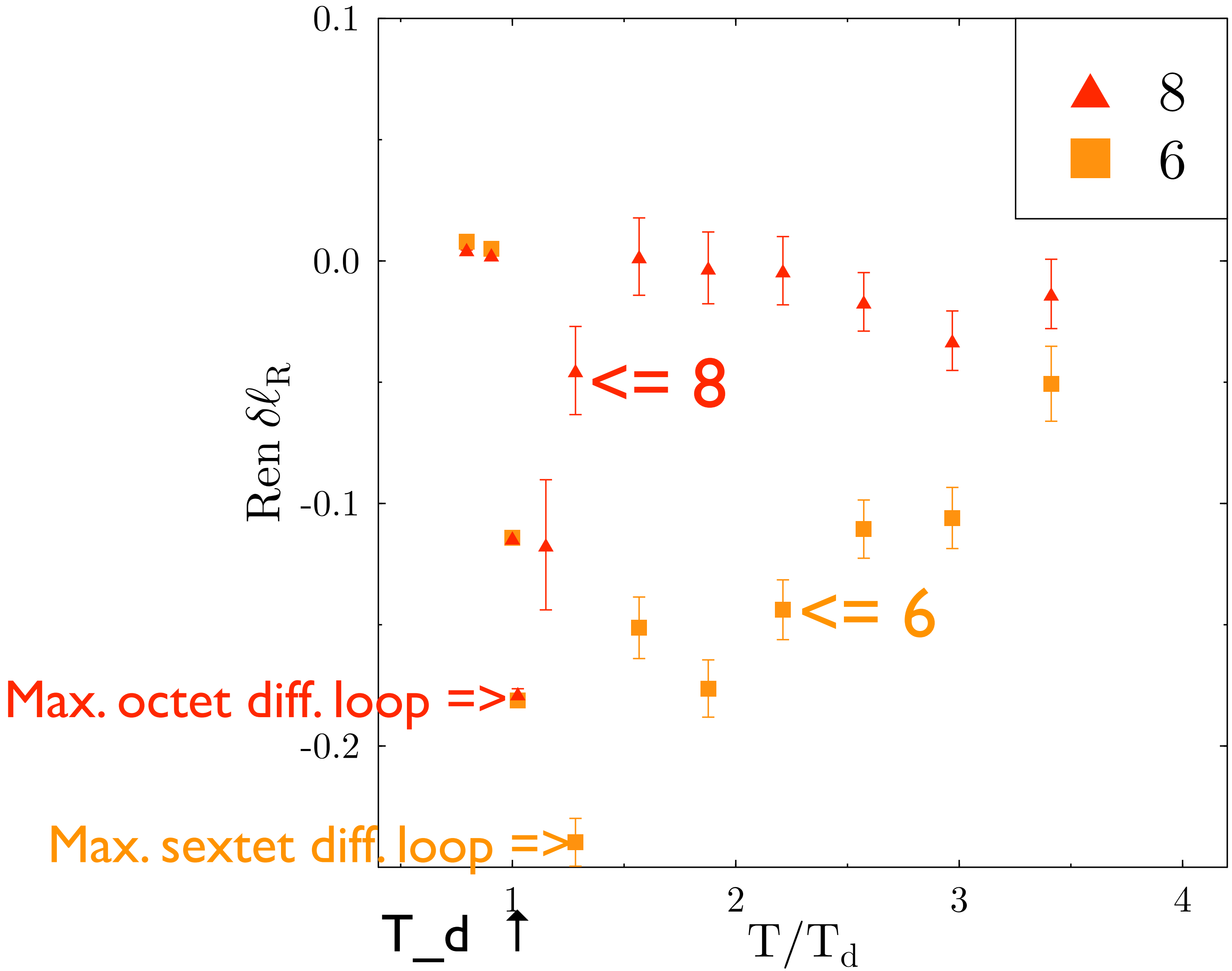
$$|\langle \widetilde{\ell}_3 \rangle| \approx .4 \pm .05 \; , \; T = T_d^+$$

T > T_d: Find ordering: 3 > 8 > 6. But compute difference loops:

$$\delta \widetilde{\ell}_6 \equiv \langle \widetilde{\ell}_6 \rangle - \langle \widetilde{\ell}_3 \rangle^2 \sim O(1/N)$$

$$\delta \widetilde{\ell}_8 \equiv \langle \widetilde{\ell}_8 \rangle - |\langle \widetilde{\ell}_3 \rangle|^2 \sim O(1/N^2)$$

Difference Loops: *Test* of Factorization at N=3



Details of spikes near T_d?

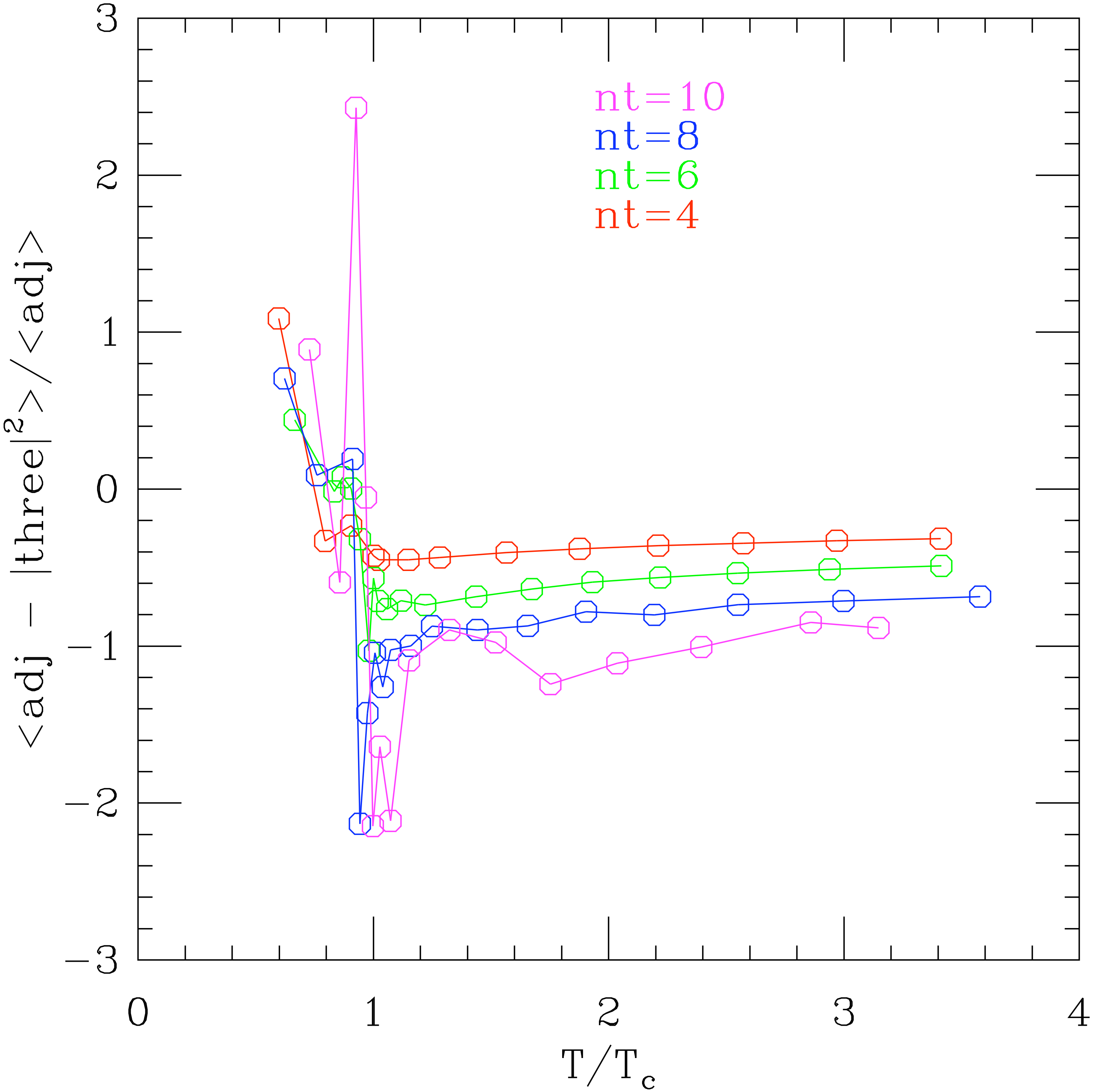
Sharp octet spike

Broad sextet spike

$$|\delta\widetilde{\ell_8}| \sim O(1/N^2) \leq .2 \; ; \; |\delta\widetilde{\ell_6}| \sim O(1/N) \leq .25$$

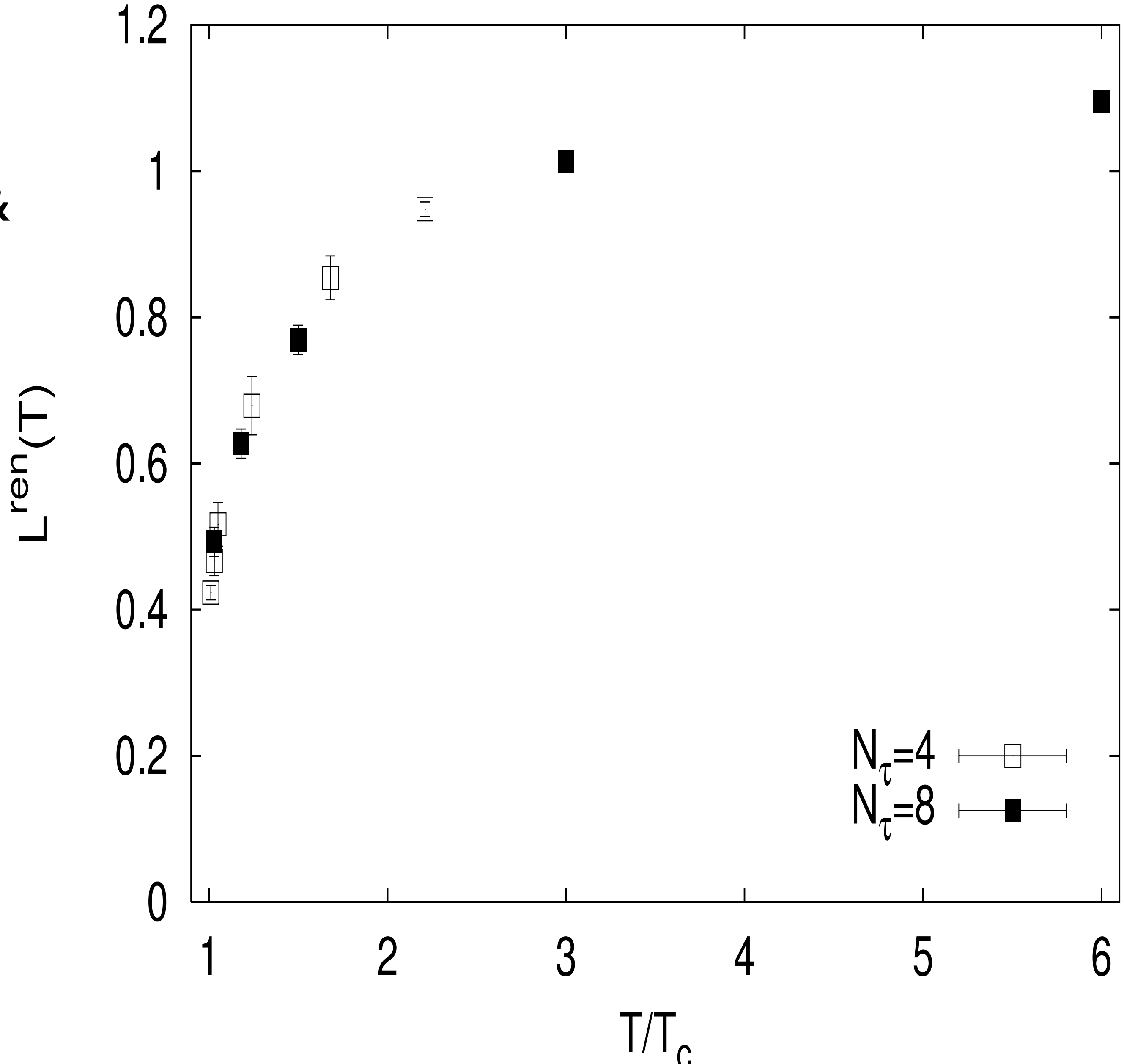
Bare Loops *don't* exhibit Factorization

Bare octet difference loop/bare octet loop: violations of factor. 50% @ Nt =4 200% @ Nt = 10.



Bielefeld's Renormalized Polyakov Loop

Kaczmarek, Karsch, Petreczky, & Zantow '02



$|\langle\widetilde{\ell_3}\rangle|$

Approx. agreement.

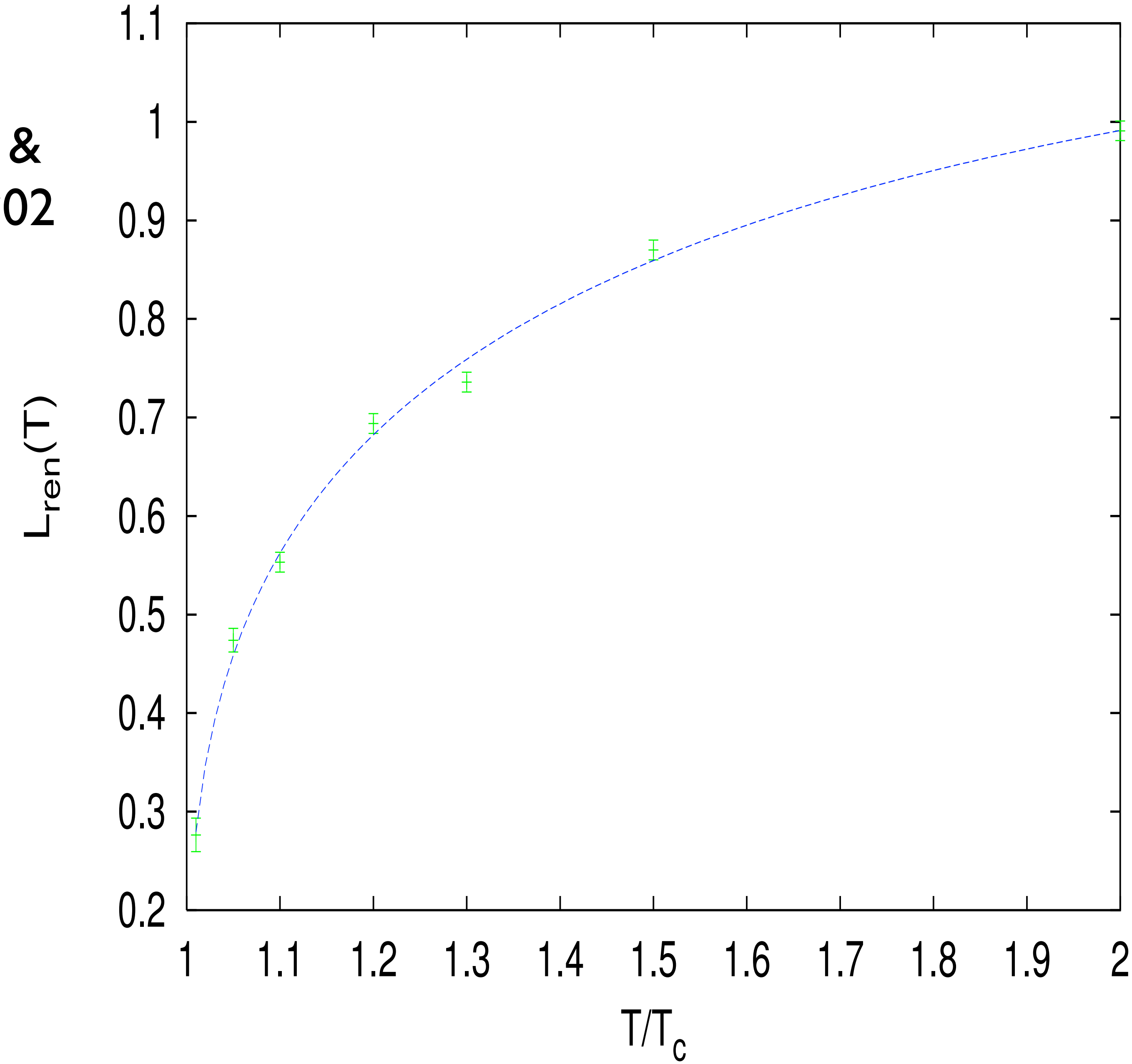
$$\langle \ell_3^*(x)\ell_3(0)\rangle - |\langle \ell_3\rangle|^2 \sim Z_3^2 \exp(-m_{Debye}|x|)$$

Need 2-pt fnc at one N_t vs 1-pt fnc at several N_t

Bielefeld's Ren'd Polyakov Loop, N=2

Digal, Fortunato, & Petreczky '02

$|\langle \widetilde{\ell}_2 \rangle|$



Transition second order: $\Rightarrow |\langle \widetilde{\ell}_2 \rangle| = 0 \ @ \ T = T_d^+$

Mean Field Theory for Fundamental Loop

At large N, if fundamental loop condenses, factorization ⇒ *all* other loops

This is a mean field type relation; implies mean field for $\langle \widetilde{\ell}_N \rangle$?

General effective lagrangian for *renormalized* loops:

Choose basic variables as Wilson lines, not Polykov loops: (i = lattice sites)

$$\mathcal{Z} = \int \Pi \, d\mathbf{L}_N(i) \; \exp\left(-\mathcal{S}(\ell_R(i))\right) \qquad\qquad \mathbf{L}_N(i) \, \epsilon \, SU(N)$$

Loops automatically have correct Z(N) charge, and satisfy factorization.

Effective action Z(N) symmetric. Potential terms (starts with adjoint loop):

$$\mathcal{W} = \Sigma_i \Sigma_R^{e_R=0} \; \gamma_R \ell_R(i)$$

and next to nearest neighbor couplings:

$$\mathcal{S}_R = -(N^2/3)\Sigma_{i,\hat{n}} \Sigma_{R,R'}^{e_R+e_{R'}=0} \; \beta_{R,R'} \; \mathrm{Re} \; \ell_R(i)\ell_{R'}(i+\hat{n}) \; .$$

In mean field approximation, that's it. (By using character exp.)

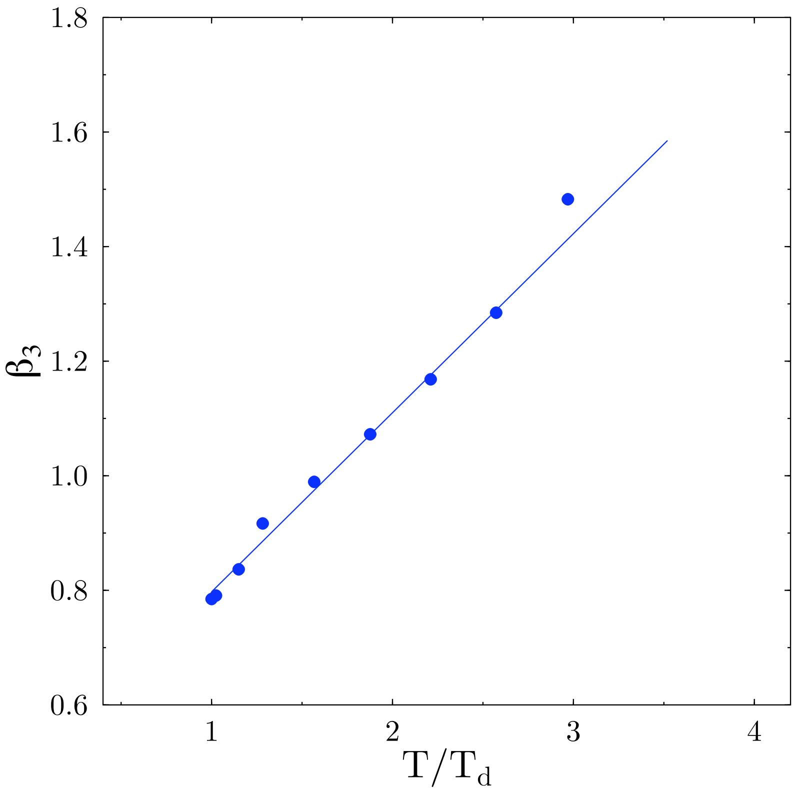
Matrix Model (=Mean Field) for N=3

Simplest possible model: only $\beta_{3,3^*} \equiv \beta_3 \neq 0$ (Damgaard, '87)

$$\langle \ell_3 \rangle = \int d\mathbf{L}\ \ell_3\ \exp\left(18\beta_3 \langle \ell_3 \rangle \mathrm{Re}\ell_3\right) \Big/ \int d\mathbf{L}\ \exp\left(18\beta_3 \langle \ell_3 \rangle \mathrm{Re}\ell_3\right)$$

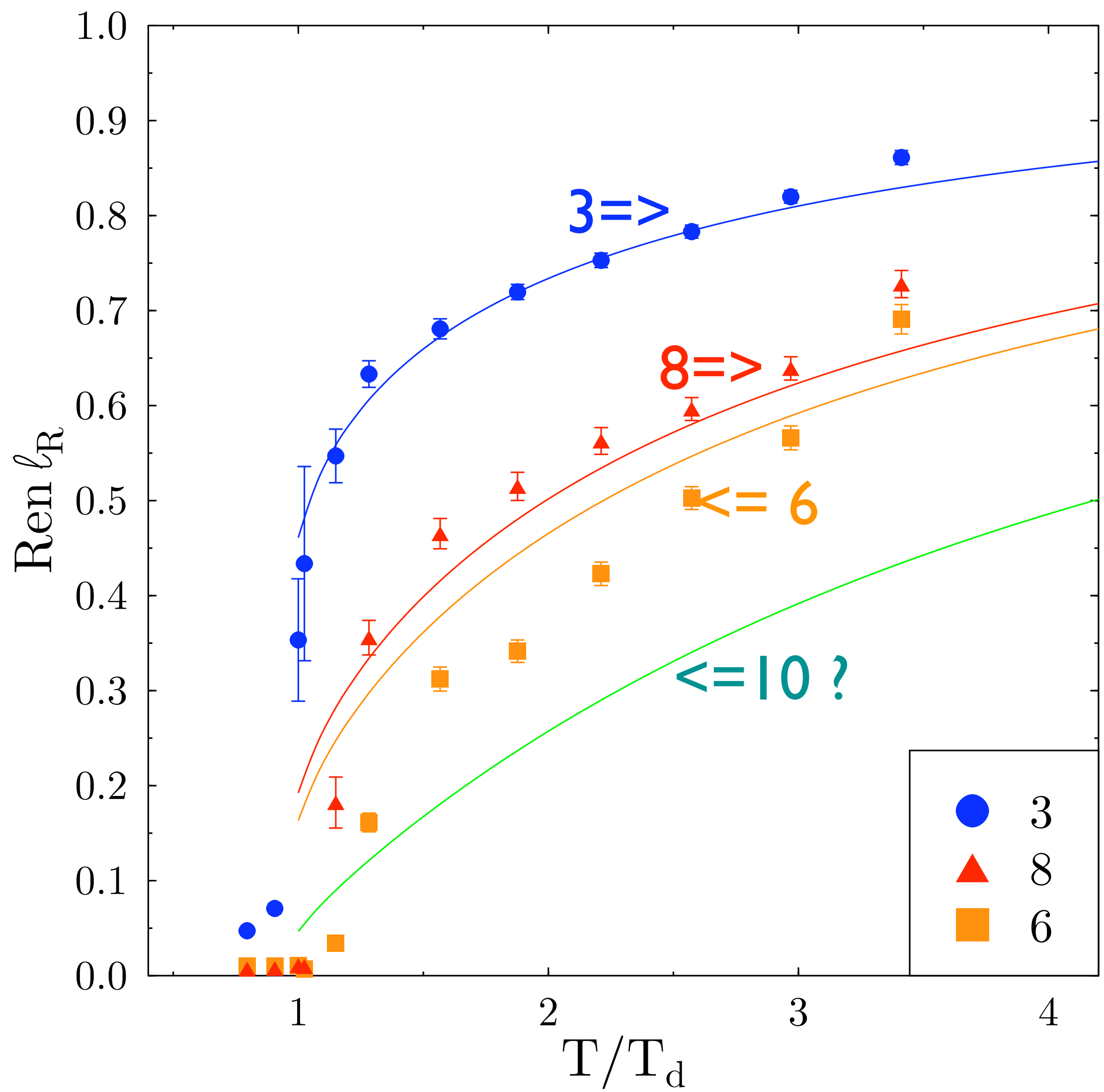
Fit $\beta_3(T)$ to get $\langle \widetilde{\ell_3} \rangle (T)$

Find $\beta_3(T)$ linear in T



Now compute loops in other representations using this $\beta_3(T)$

N=3: Lattice Results vs *Simple* Mean Field



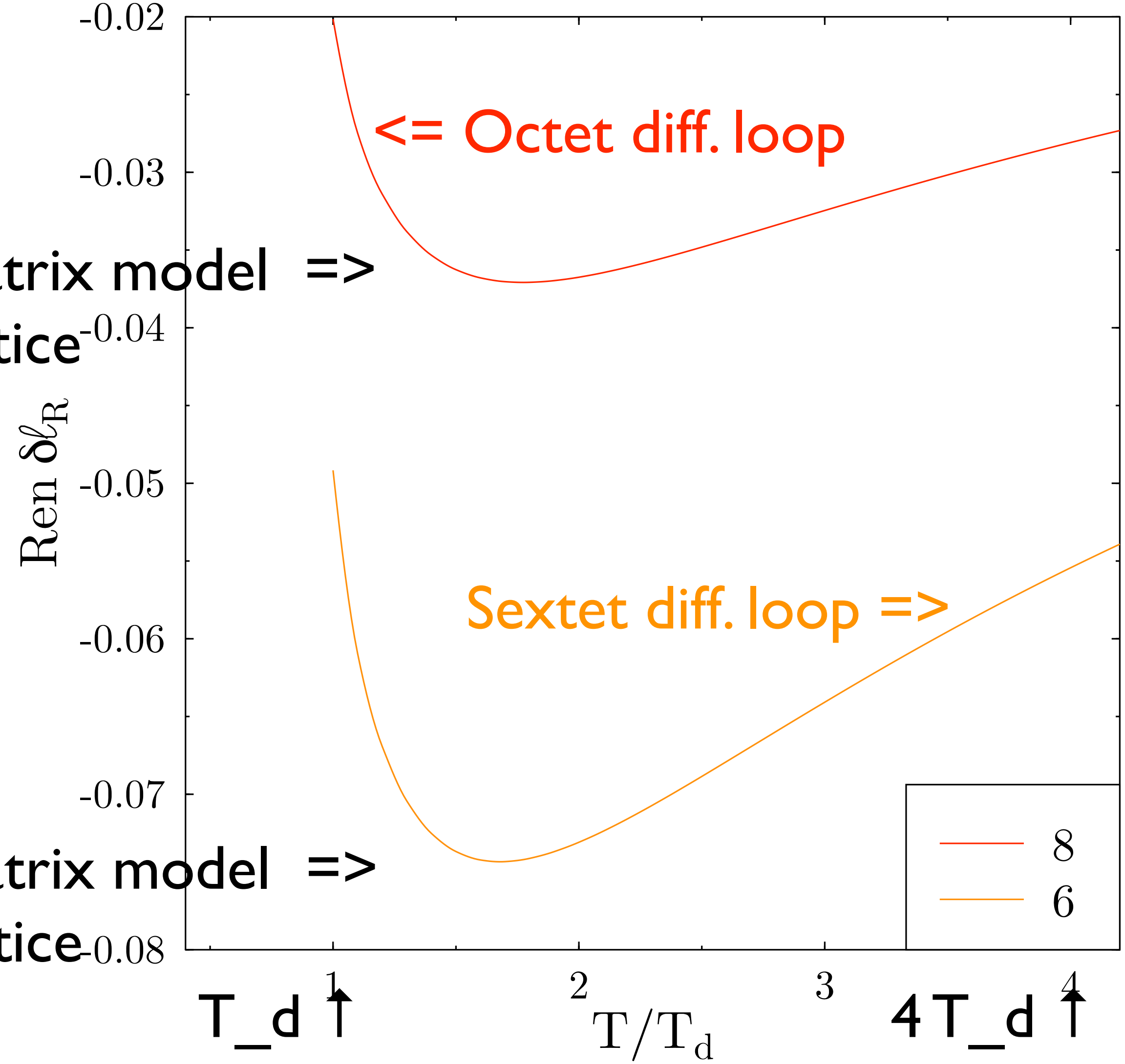
Solid lines = matrix model.

Points = lattice data for renormalized loops.

Approximate agreement for 6 & 8. Predicts signal for 10!

Difference Loops for Matrix Model, N=3

Max. ~ - .03 matrix model =>
vs -0.20 lattice



Max. ~ - .07 matrix model =>
vs -0.25 lattice

Curves = difference loops computed in matrix model

Large N => octet diff. loop < sextet diff. loop

Diff. loops: matrix model much broader & smaller than lattice data! => new physics in lattice.

Matrix Model, N=∞, and Gross-Witten

Consider mean field, where the *only* coupling is $\beta_{N,N^*} \equiv \beta$

Gross & Witten '80, Kogut, Snow, & Stone '82, Green & Karsch '84

At N=∞, mean field potential is non-analytic, given by two *different* potentials:

$$\mathcal{V}^{-}_{mf} = \beta(1-\beta)\ell^2 \quad , \quad \ell \leq 1/(2\beta)$$

$$\mathcal{V}^{+}_{mf} = -2\beta\ell + \beta\ell^2 + \frac{1}{2}\log(2\beta\ell) + \frac{3}{4} \quad , \quad \ell \geq 1/(2\beta)$$

For fixed β, the potential is everywhere continuous, but its *third* derivative is not, at the point $\ell = 1/(2\beta)$

$\beta \leq 1 : \langle \ell \rangle = 0$ = confined phase

$\beta \geq 1 : \langle \ell \rangle \neq 0$ = deconfined phase

$$\langle \ell \rangle = \frac{1}{2}\left(1 + \sqrt{\frac{\beta - 1}{\beta}}\right) :$$

$$\langle \ell \rangle = \frac{1}{2} \; , \; \beta = 1^+$$

$$\langle \ell \rangle \to 1 \; , \; \beta \to \infty$$

Gross-Witten Transition: "Critical" First Order

Transition first order. Order parameter jumps: 0 to 1/2. Also, latent heat ≠ 0:

$$\mathcal{V}_{mf}^{-} = 0 \; , \; \beta \leq 1 \quad , \quad \mathcal{V}_{mf}^{+} \approx -(\beta - 1)/4 \; , \; \beta \to 1^{+}$$

But masses vanish, asymmetrically, at the transition!

$$m_{-}^{2} \approx 2(1-\beta) \quad , \quad \beta \to 1^{-} \; . \qquad m_{+}^{2} \approx 4\sqrt{\beta - 1} \quad , \quad \beta \to 1^{+} \; .$$

If β~T, *and* the deconfining transition is Gross-Witten at N=∞, then the string tension and the Debye mass *vanish* at T_d as:

$$\sigma(T) \sim (T_d - T)^{1/2} \; , \; T \to T_d^{-}$$

$$m_{Debye}(T) \sim (T - T_d)^{1/4} \; , \; T \to T_d^{+}$$

But what about higher terms in the "potential"?

String related analysis @ large N

hep-th/0310285: Aharony, Marsano, Minwalla, Papadodimas, Van Raamsdonk
hep-th/0310286: Furuuchi, Schridber, & Semenoff

By integrating over vev, one can show model with mean field same as model with *just* adjoint loop in potential.

=> Most general potential @ large N:

$$\mathcal{W} = c_2|\ell|^2 + c_4(|\ell|^2)^2 + c_6(|\ell|^2)^3 + \ldots$$

Gross-Witten *simplest* model: c2 ≠ 0, c4=c6=...=0.

AMMPR: consider c4 ≠ 0. Work in small volume, => compute at small g^2.

AMMPR: *either:* 2nd order, or 1st order, $\langle \ell \rangle = \dfrac{1}{2}\,,\; T = T_d!$

Lattice for N=3: *close* to infinite N, with *small* c4, c6....?
=> close to Gross-Witten?

To do

Two colors: matching critical region near T_d to mean field region about T_d?

Higher rep.'s, factorization at N=2?

Three colors: better measurements, esp. near T_d: $\langle \widetilde{\ell}_3 \rangle (T_d^+) \ldots$

"Spikes" in sextet and octet loops? Fit to matrix model?

For decuplet loop, use "improved" Wilson line? $\int d\Omega_{\vec{n}} \sim$ HTL's

$$\mathbf{L}_{\rm imp} = \int d\Omega_{\vec{n}} \; \exp(\; ig \int (A_0 + \kappa a \vec{E} \cdot \vec{n}) d\tau)$$

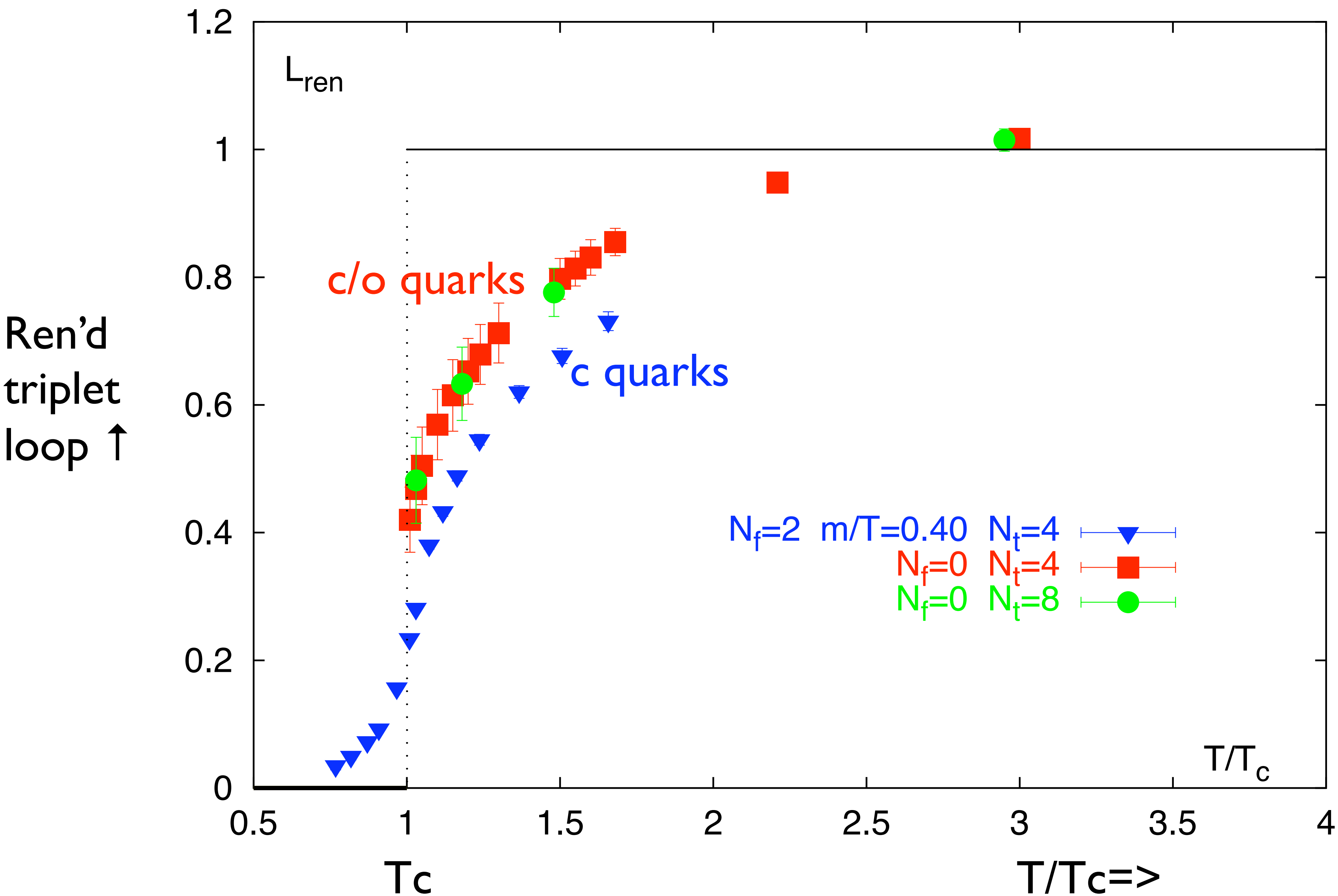
Four colors: is transition Gross-Witten? Or is N=3 an accident?

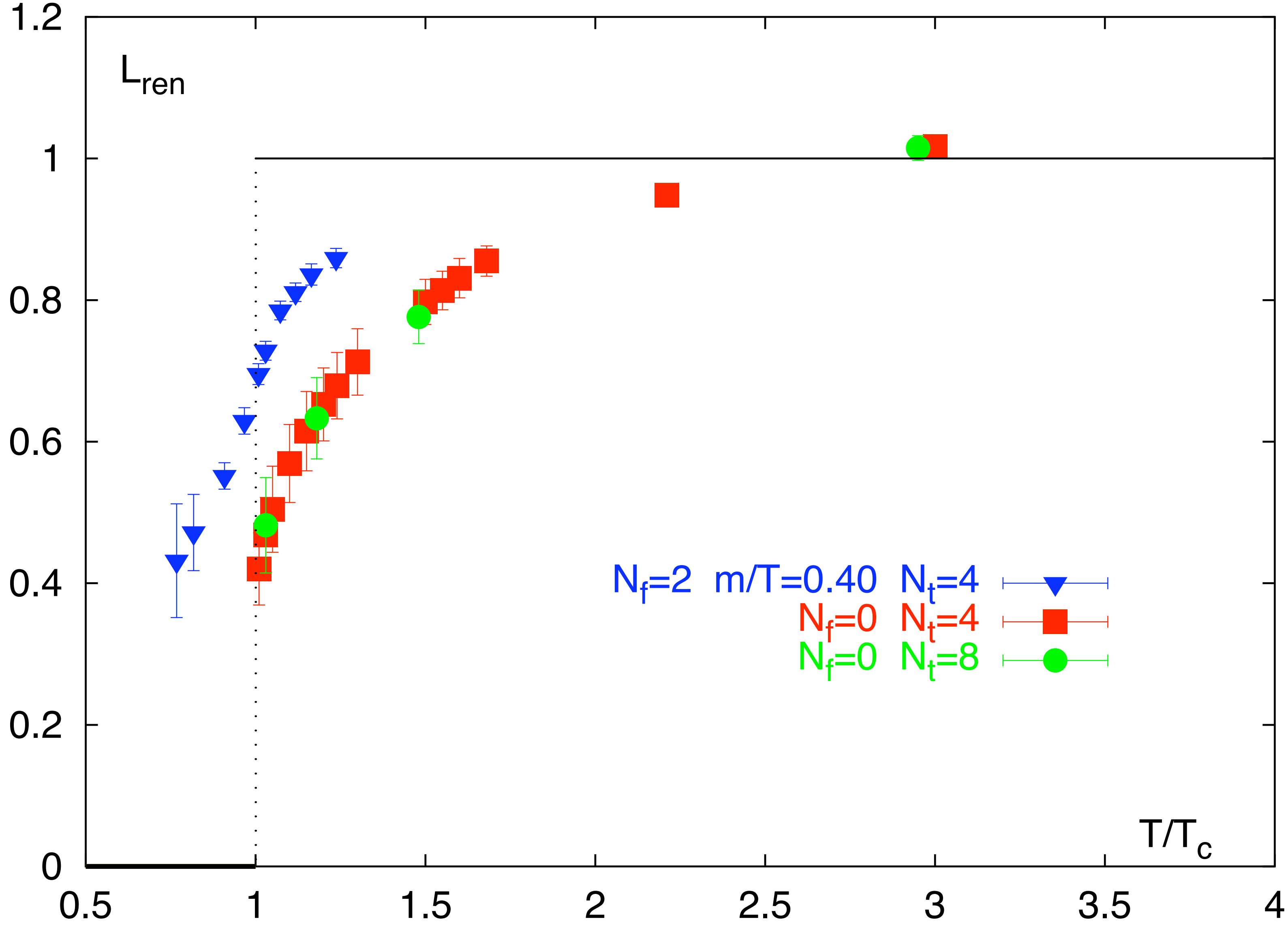
With dynamical quarks: method to determine ren.'d loop(s) *identical*

$$\text{Is } \langle \widetilde{\ell}_R \rangle \left(\frac{T}{T_c} \right)_{\textit{with} \text{ quarks}} \approx \langle \widetilde{\ell}_R \rangle \left(\frac{T}{T_d} \right)_{\text{pure gauge}} ?$$

Bielefeld: Ren'd loop *with* quarks. ≈ Same!

Kaczmarek et al: hep-lat/0312015





L_{ren}
T/T_c
$N_f=2$ $m/T=0.40$ $N_t=4$
$N_f=0$ $N_t=4$
$N_f=0$ $N_t=8$
0
0.2
0.4
0.6
0.8
1
1.2
0.5
1
1.5
2
2.5
3
3.5
4