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# Isovector Axial Charge of the Nucleon from Lattice QCD

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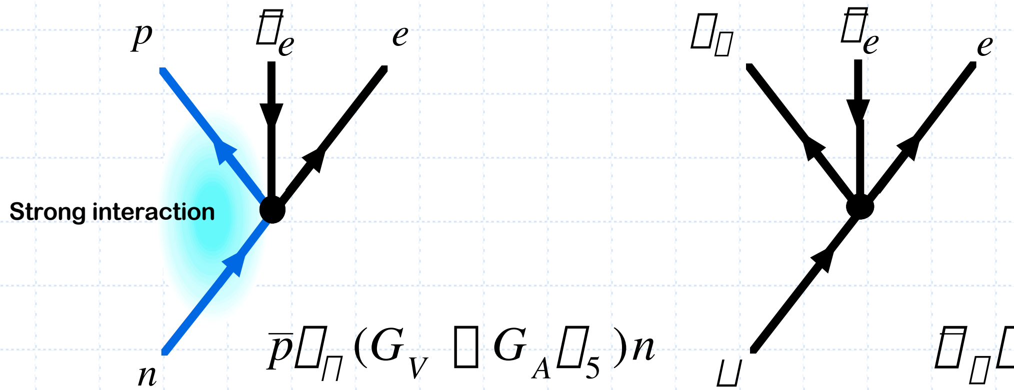
*As part of the RIKEN-BNL-Columbia-KEK QCD project*

# Nucleon axial charge: $g_A = \Delta u - \Delta d$

## Neutron beta decay




$$\bar p \gamma_\mu (G_V - G_A \gamma_5) n \qquad \bar\nu_\mu \gamma_\mu (1 - \gamma_5) \mu$$

- ✓ $G_V$ and $G_A$ are provided by the strong interaction
  - Experimental value: $G_A / G_V = 1.2670(35)$
- ✓ $G_V \approx 1$ (CVC) reflects the conservation of isospin
- ✓ Why $G_A > 1$ in contrast with $G_V \approx 1$?
  - ☞ the axial current is only partially conserved

☞ Isovector symmetry gives rise to $G_A / G_V = \Delta u - \Delta d$

$$\langle p | A_1^\mu + i A_2^\mu | n \rangle = \langle p | A_3^\mu | p \rangle - \langle n | A_3^\mu | n \rangle$$

# Why nucleon axial charge ?

- Well measured quantity in experiment.

  from neutron beta decay, $g_A$=1.2670(35)
- Strong relation to the axial vector symmetry
  - ✓ $g_A$ are supposed to respect the axial WT identity
- One of the simplest nucleon matrix elements.
  - ✓ lowest moment
  - ✓ zero momentum trasfer
  - ✓ no disconnected contribution

☞ A real good test of DWF towards studying the nucleon structure functions (□q, □q, etc)

# Lattice calculation of $g_A$

| type | group | fermion | lattice | □ | volume | configs | m□L | $g_A$ |
|---|---|---|---|---|---|---|---|---|
| quench | KEK [1] | Wilson | $16^3$x20 | 5.7 | (2.2 fm)$^3$ | 260 | ≥ 5.9 | 0.985(25) |
| | Kentuchy [2] | Wilson | $16^3$x24 | 6.0 | (1.5 fm)$^3$ | 24 | ≥ 5.8 | 1.20(10) |
| | DESY [3] | Wilson | $16^3$x32 | 6.0 | (1.5 fm)$^3$ | 1000 | ≥ 4.8 | 1.074(90) |
| | LHPC-SESAM [7] | Wilson | $16^3$x32 | 6.0 | (1.5 fm)$^3$ | 200 | ≥ 4.8 | 1.129(98) |
| | | | $16^3$x32 | 6.0 | (1.5 fm)$^3$ | O(500) | | |
| | QCDSF [4] | Wilson | $24^3$x48 | 6.2 | (1.6 fm)$^3$ | O(300) | | 1.14(3) |
| | | | $32^3$x48 | 6.4 | (1.6 fm)$^3$ | O(100) | | |
| | | | $16^3$x32 | 6.0 | (1.5 fm)$^3$ | O(500) | | |
| | QCDSF-UKQCD [5] | Clover | $24^3$x48 | 6.2 | (1.6 fm)$^3$ | O(300) | | 1.135(34) |
| | | | $32^3$x48 | 6.4 | (1.6 fm)$^3$ | O(100) | | |
| full | LHPC-SESAM [7] | Wilson | $16^3$x32 | 5.5 | (1.7 fm)$^3$ | 100 | ≥ 4.2 | 0.914(106) |
| | SESAM [6] | Wilson | $16^3$x32 | 5.6 | (1.5 fm)$^3$ | 200 | ≥ 4.5 | 0.907(20) |


1. M. Fukugita et al., Phys. Rev. Lett. 75 (1995) 2092.
2. K.F. Liu et al., Phys. Rev. D49 (1994) 4755.
3. M. Göckeler et al., Phys. Rev. D53 (1996) 2317.
4. S. Capitani et al., Nucl. Phys. B (Proc. Suppl.) 79 (1999) 548.
5. R. Horsley et al., Nucl. Phys. B (Proc. Suppl.) 94 (2001) 307.
6. S. Güsken et al., Phys. Rev. D59 (1999) 114502.
7. D. Dolgov et al., hep-lat/0201021.

# Low value of $g_A$ on lattice

- Possible systematic errors
  - ✓ Quenching

    $g_A^{Full} \lesssim g_A^{Quench}$ at a~0.1 fm ~ 5-10 % ↘
  - ✓ Finite lattice spacing

    $(g_A)_{at\ a \to 0} > (g_A)_{at\ a \sim 0.1\ fm}$ ~ 5 % ↗
  - ✓ Determination of $Z_A$

    $Z_A^{Non\text{-}pert} < Z_A^{Pert}$ (Clover) ~ 10 % ↘
  - ✓ Finite volume

    No estimation ?

# Determination of $Z_A$

- Renormalization of lattice operators is an essential ingredient to calculate the matrix element

$$O^{\,con}(\mu) = Z_O(a\mu)\, O^{\,latt}(a)$$

☞ Any matrix elements would suffer from the systematic error in determining the renormalization factor

- Need to know the nonperturbative renormalization factor in order to get the continuum value $g_A = Z_A g_A^{lattice}$

✓ DWF have a good chiral property

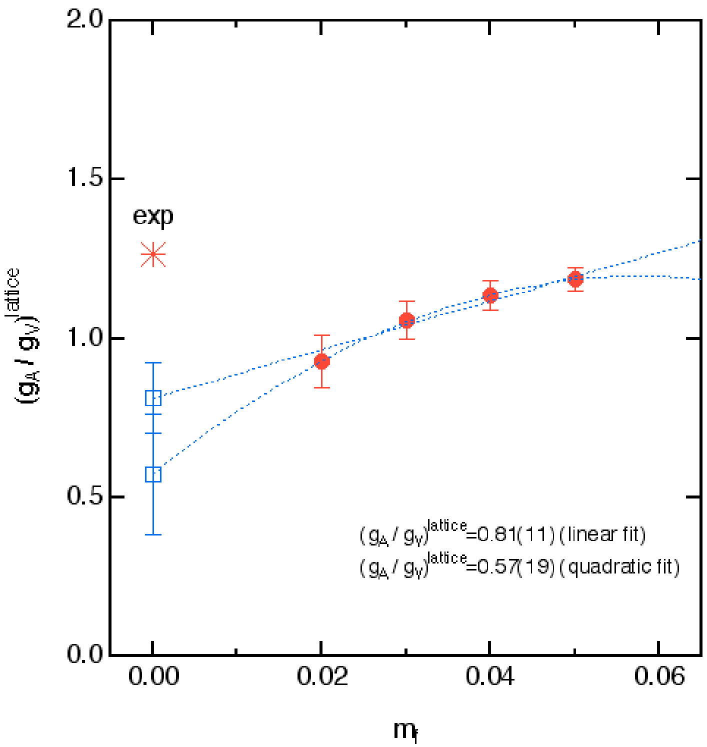
$Z_A=1$ for the conserved axial current

$Z_A=Z_V$ for the local current

☞ $g_A^{lattice} / g_V^{lattice}$ directly yields the continuum value $g_A$ with the fact that $g_V=1$ in the iso-symmetric calculation

# First DWF calculation of $g_A$

$\beta$=6.0, ($a^{-1} \approx$ 2 GeV), $16^3$ x 32 x16, $M_5$=1.8, 400 configs (hep-lat/0011011)




- Fairly strong dependence on $m_f$
  - ✓ There is some curvature.
- Chiral extrapolation yields the quite low value of $g_A$ at $m_f$=0
  - $g_A$=0.81(11) with linear fit
  - $g_A$=0.57(19) with quadratic fit
- Physical volume ~ (1.5 fm)$^3$
  - ✓ Corresponding to 2 * $r_A^{RMS}$

    $r_A^{RMS} = \langle r_A^2 \rangle^{1/2} \approx 0.7$ fm
  - ✓ Not large enough

☞ Need to check the finite volume effect

# RG improved gauge action

Need the large physical volume > $(2.0\ \mathrm{fm})^3$

☞ Simulation is employed on coarser lattice, but using RG improved gauge action

$$S_G \quad c_0 \sum_{\mathrm{plaq}} \mathrm{Tr}P(1\times1) + c_1 \sum_{\mathrm{rect}} \mathrm{Tr}P(1\times2)$$

DBW2 action: $c_1 = -1.4069$ with $c_0 + 8c_1 = 1$

QCD-TARO Collaboration, Nucl. Phys.B577 (2000) 263

1. Smooth gauge field
   - ☞ Keep good chiral properties of DWF

     K. Orginos (RBC Collaboration), hep-lat/0110074

2. Near renormalization trajectory
   - ☞ Provide good scaling behaviors with DWF

     Y. Aoki (RBC Collaboration) , hep-lat/0110143

# Details of the simulation

Gauge: DBW2 action ($c_1$= –1.4069)

□=0.87, $a^{-1} \approx 1.3$ GeV

two lattice sizes $16^3$x32, V≈(2.4 fm)$^3$, 100configs

$8^3$x24, V≈(1.2 fm)$^3$, 400configs

Fermion: Domain Wall Fermions (quench)

$L_s$=16, $M_5$=1.8

6 quark masses ($m_{\square}/m_{\square}$=0.62 - 0.89)

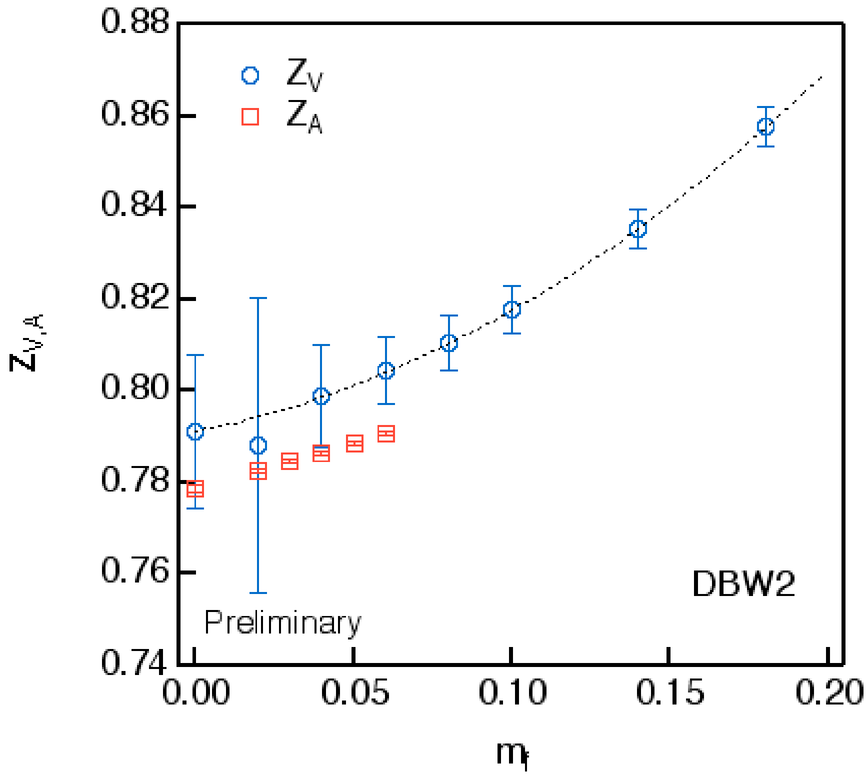
$m_{\square}L > 6$ (for the larger volume) and $m_{\square} > 500$MeV

Basic results: $m_{\square}a$=0.603(20), $m_N/m_{\square}$=1.30(6)

$m_{res}a=5.7(3)\times10^{-4}$

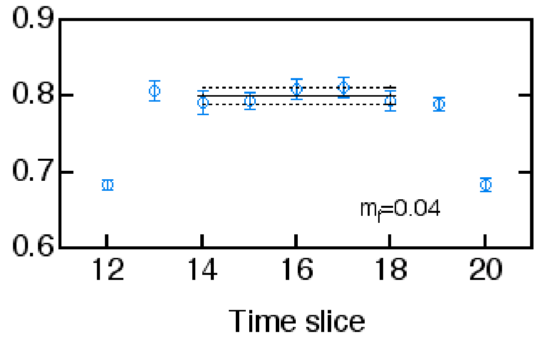
# Checking the relation $Z_V=Z_A$

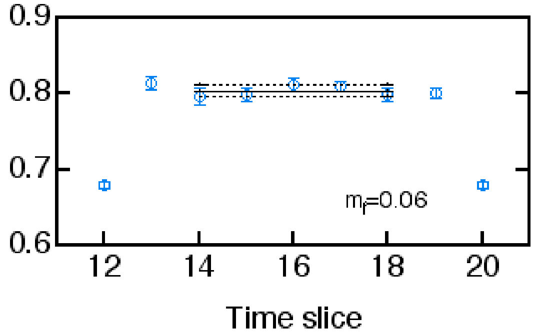
**$\beta$=0.87 (DBW2),$16^3$x32x16, $M_5$=1.8**




$$Z_V = 1/g_V^{\text{lattice}} \text{ since } g_V = 1$$

$$Z_A \text{ from } \langle A_\mu^{\text{conserved}}(t)P(0)\rangle = Z_A \langle A_\mu^{\text{local}}(t)P(0)\rangle$$



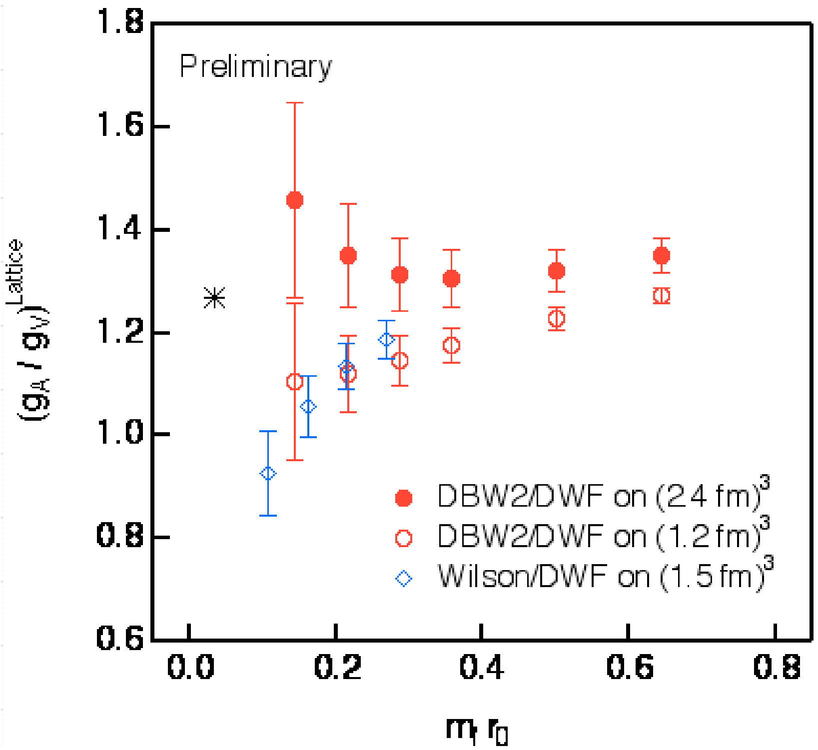



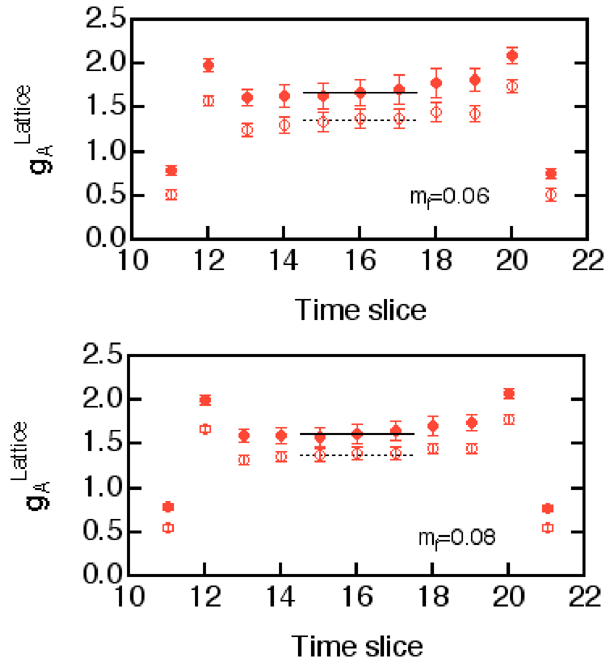

source at t=11, sink at 21, current insertions in between

✓ **Keep good chiral properties even at coarser lattice.**

# Large finite volume effect

$\beta$=0.87 (DBW2) ,$16^3$x32x16 (100 configs) and $8^3$x24x16 (400 configs)

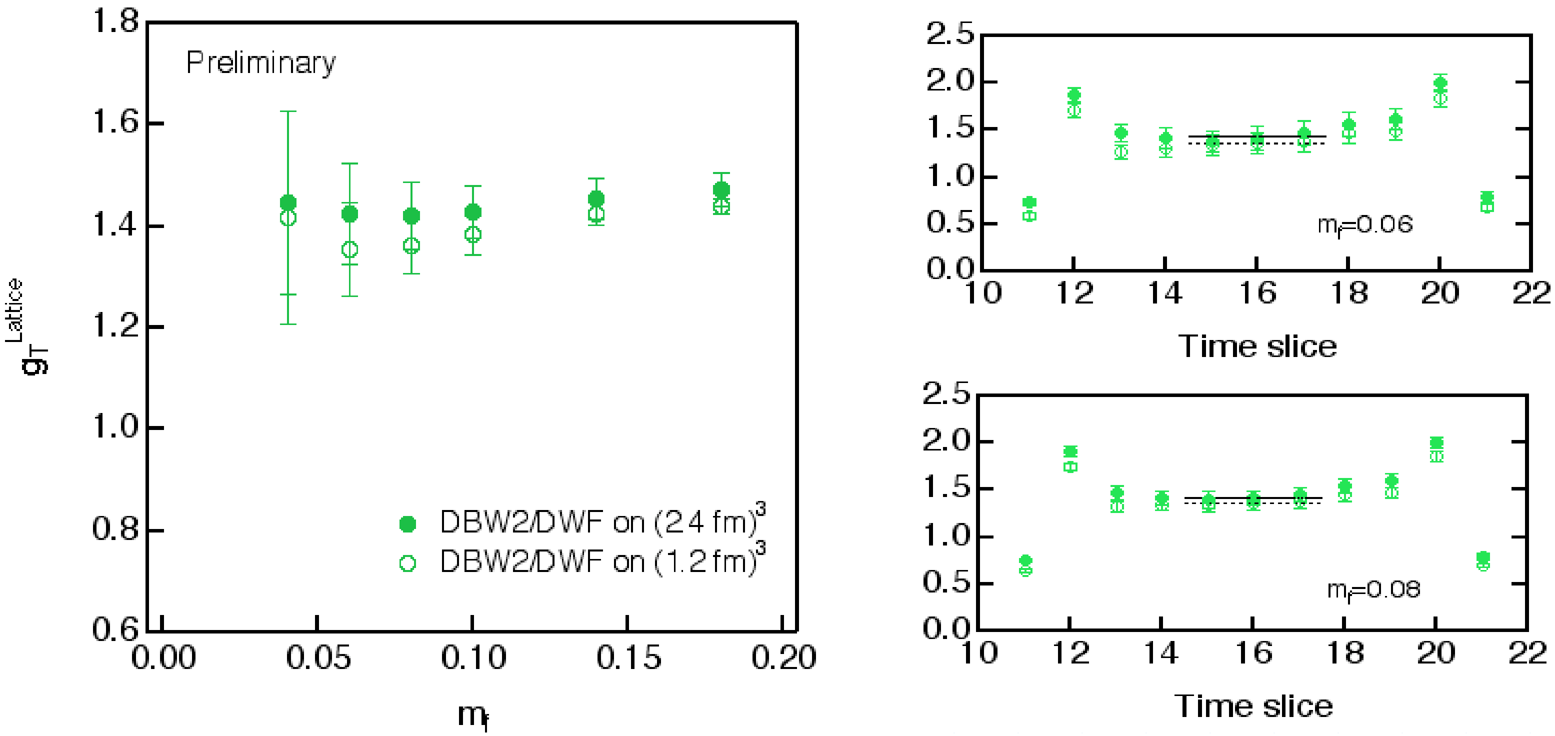



☞ **A clear volume dependence is observed**

# Tensor charge: $g_T = \delta u - \delta d$

$\beta = 0.87$ (DBW2) , $16^3 \times 32$ and $8^3 \times 24$, $L_s = 16$, $M_5 = 1.8$

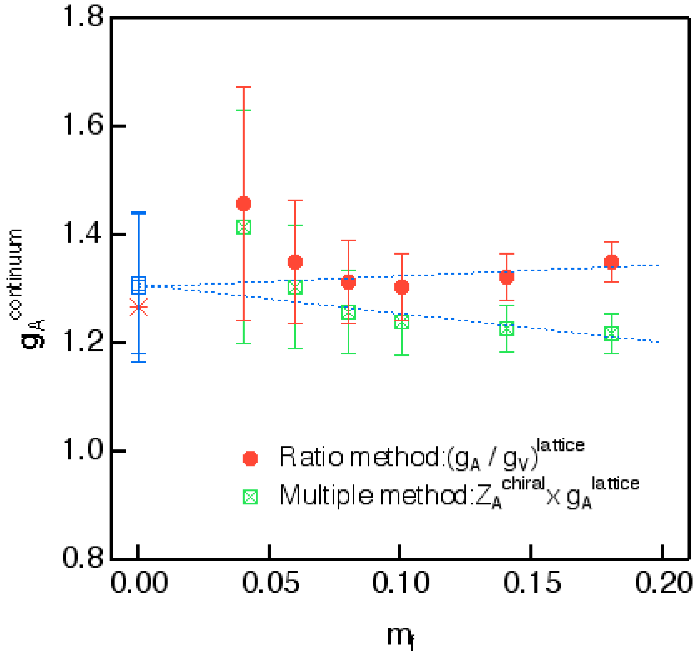



☞ **Need the renormalization factor $Z_T$**

# Values of $g_A$ from two methods

## Fully non-perturbative determination of $g_A$

DBW2+DWF, $16^3$x32x16, 100 configs




- Ratio method:

  $g_A^{cont} = (g_A / g_V)^{lattice}$

- Multiple method:

  $g_A^{cont} = Z_A^{chiral} \times g_A^{lattice}$

✓ Mild dependence on $m_f$

✓ Resultant values at $m_f$=0 are consistent; $g_A = 1.30(14)$

# Summary

Simulation is employed on coarse lattice ($a \sim 0.15$ fm),

large physical volume ~ $(2.4\ \text{fm})^3$

using DBW2 gauge action and Domain Wall Fermions

□=0.87, two lattice sizes $16^3$x32 and $8^3$x24, $L_s$=16, $M_5$=1.8

- ✓ Relevant three-point functions are well behaved.
- ✓ Confirm that $Z_A$=$Z_V$ is well satisfied even at coarse lattice.
- ✓ Determine $g_A$ in a fully non-perturbative way.
- ✓ Observe the significant finite volume effect on $g_A$.