NLO QED corrections to deep inelastic scattering

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DESY

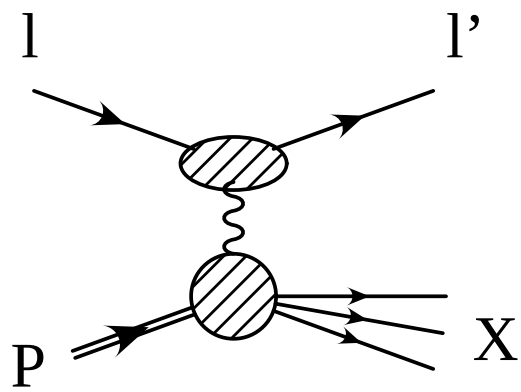
1. **Introduction**
2. **One loop result revisited**
3. **Non–leptonic Corrections**
4. **Two loop contributions**
5. **Numerical Results**
6. **Conclusion**

Work in common with Hiroyuki Kawamura

Phys. Lett. **B553** (2003) 242 and in preparation.

1. Introduction

Deep inelastic scattering :



QED corrections are large: particularly leptonic corrections

$$\frac{d^2\sigma}{dydQ^2} = \frac{d^2\sigma^{(0)}}{dydQ^2} + \sum_{k=1}^{\infty} \frac{d^2\sigma^{(k)}}{dydQ^2}$$

Born: $$\frac{d^2\sigma^{(0)}}{dydQ^2} = \frac{2\pi\alpha^2}{yQ^4}\left[2y^2 x F_1(x,Q^2) + 2(1-y)F_2(x,Q^2)\right]$$

corrections: $$\frac{d^2\sigma^{(k)}}{dydQ^2} = \sum_{l=0}^{k}\left(\frac{\alpha}{2\pi}\right)^k \log^{k-l}\left(\frac{Q^2}{m_e^2}\right) C^{(k,l)}(y,Q^2)$$

$l=0:$ $C^{(k,0)}(y,Q^2)$ leading logs. are analytically known to $(\alpha L)^5$ for Singlet, NS, for polarized and unpolarized nucleons.

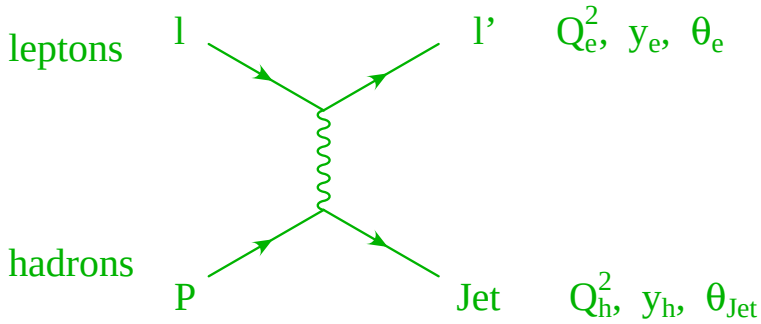
Blümlein and Kawamura, 2002

$k=1:$ all terms are known.

$k=2:$ only the l=0 terms have been known until recently.

Main issue : Calculation of $O\left(\alpha^2 \log\left(\frac{Q^2}{m_e^2}\right)\right)$

Radiative corrections to differential cross section strongly depend on the measurement of the kinematic variables, e.g. y, Q^2;

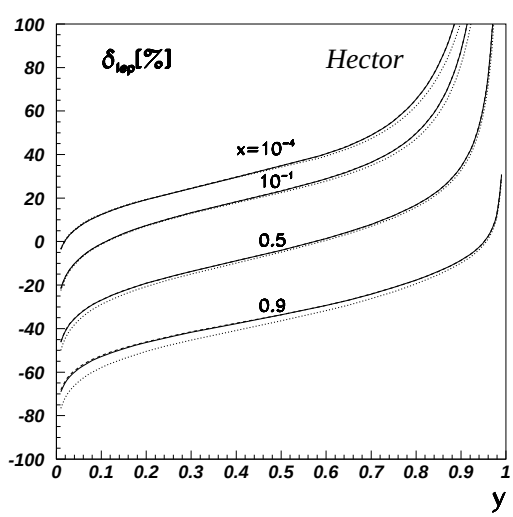


Exp. variables

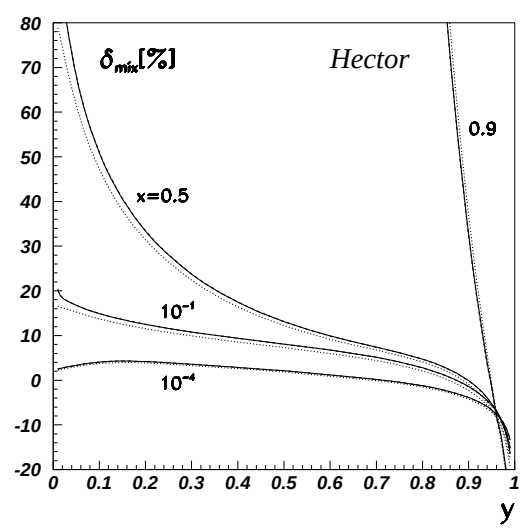
1. leptonic variables Q_e^2, y_e
2. mixed variables Q_e^2, y_h
3. hadronic variables Q_h^2, y_h
4. JB variables Q_{JB}^2, y_{JB}
5. double angle θ_e, θ_{Jet}
6. θ_e, y_{JB} θ_e, y_{JB}
7. Σ method Q_Σ, y_Σ
8. eΣ method $Q_l^2, y_{e\Sigma}$

J. Blümlein, 1994
HECTOR Collab.
Comp.Phys.Comm.94(1996)128.

leptonic variables



mixed variables



$O(\alpha)$ corrections are large $\longrightarrow$ $O(\alpha^2)$ needed.

2. $O(\alpha)$ corrections revisited

Example: Mixed variables

Kinematic separation in the case for $Q_h^2 \leq Q_l^2, \;\; Q_h^2 \geq Q_l^2$

$$\frac{d^2\sigma^{(1)}}{dydQ^2} = \frac{\alpha}{2\pi}\log\left(\frac{Q^2}{m_e^2}\right)\int_0^1 dz P_{ee}^0(z)\left[\theta(z-z_0^I)\frac{d^2\widetilde{\sigma}^{(0)}}{dydQ^2}J^I(z) - \frac{d^2\sigma^{(0)}}{dydQ^2}\right]$$
$$+\frac{\alpha}{2\pi}\log\left(\frac{Q^2}{m_e^2}\right)\int_0^1 dz P_{ee}^0(z)\left[\theta(z-z_0^F)\frac{d^2\widetilde{\sigma}^{(0)}}{dydQ^2}J^F(z) - \frac{d^2\sigma^{(0)}}{dydQ^2}\right]$$
$$+\frac{\alpha}{2\pi}C^{(1,1)}(y,Q^2)$$

$$\widetilde{F}(x,y,s) = F(\hat{x},\hat{y},\hat{s})$$

- Rescaling;

(i) $Q_h^2 < Q_l^2 \rightarrow$ ISR: $z = \frac{Q_h^2}{Q_l^2}, \;\; \hat{y} = y_h/z, \;\; \hat{Q}^2 = zQ_l^2, \;\; \hat{x} = zx_m$
$J^I(z) = 1$

(ii) $Q_h^2 > Q_l^2 \rightarrow$ FSR: $z = \frac{Q_l^2}{Q_h^2}, \;\; \hat{y} = y_h, \;\; \hat{Q}^2 = Q_l^2/z, \;\; \hat{x} = x_m/z$
$J^F(z) = 1/z,$

This choice of the rescaling preserves the structure :

$$P_{ee}^0(z) = \left(\frac{1+z^2}{1-z}\right)$$

- Integration boundary:

(i) ISR

Avoid the Compton peak $\Rightarrow Q_h^2 \geq Q_0^2 \geq M_P^2$

$$\Longrightarrow z \geq z_0^I = max\left\{\frac{Q_0^2}{Q_l^2}, y_h\right\}$$

· Only leptonic collinear singularities

· Avoid Non-DIS structures

(ii) FSR

Kinematic boundary: $z \geq x_m \equiv \dfrac{Q_l^2}{y_h S}$

$C^{(1,1)}(y_h, Q_l^2)$

$$c^{(1,1,i)}(y_h, Q_l^2, z) = \frac{1}{z^2}\left[\left\{P_{ee}^0(z)\left[\ln\left(\frac{1-y_h/z}{1-z}\right) - \frac{1}{z}\ln\left(\frac{1-y_h}{1-z}\right) - 1\right]\right.\right.$$
$$\left. +2\ln\left(\frac{z}{y_h}\right) + 1 - z\right\} y_h^2 2x_m F_1(x_h, Q_h^2)$$
$$+2\left\{(z-y_h)P_{ee}^0(z)\left[\ln\left(\frac{1-y_h/z}{1-z}\right) - \frac{1-y_h}{z^2(z-y_h)}\ln\left(\frac{1-y_h}{1-z}\right) - 1\right]\right.$$
$$+\left(\frac{1}{z} + \frac{3}{2} + 3z + 2z^2\right) - y_h\left(\frac{1}{z^2} + \frac{2}{z} + 4 + 3z\right)$$
$$\left.\left. + \frac{y_h^2}{2}\left(\frac{1}{z^2} + \frac{2}{z} + 2\right)\right\} F_2(x_h, Q_h^2)\right]$$

$$c^{(1,1,f)}(y_h, Q_l^2, z) = \left\{P_{ee}^0(z)\left[\ln\left(\frac{1-y_h}{1-z}\right) - \frac{1}{z}\ln\left(\frac{1-y_h z}{1-z}\right) - 1\right]\right.$$
$$\left. +2\ln\left(\frac{1}{y_h}\right) + 1 - z\right\} y_h^2 2x_m F_1(x_h, Q_h^2)$$

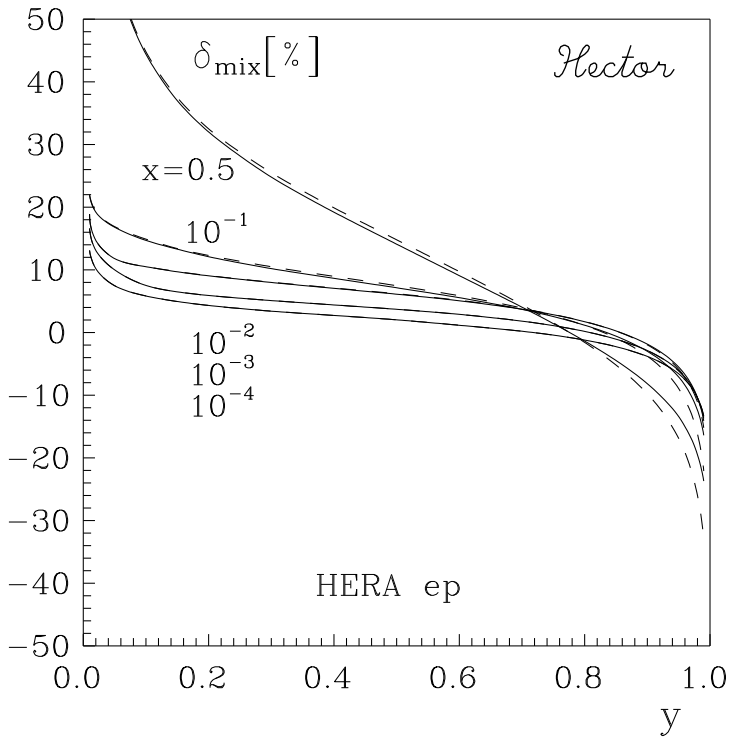
$$+2\left\{z(1-y_h)P^0_{ee}(z)\left[\ln\left(\frac{1-y_h}{1-z}\right)-\frac{1-y_hz}{z^3(1-y_h)}\ln\left(\frac{1-y_hz}{1-z}\right)-1\right]\right.$$
$$+\left(\frac{1}{z}+\frac{3}{2}+3z+2z^2\right)-y_h\left(\frac{1}{z}+2+4z+3z^2\right)$$
$$\left.+\frac{y_h^2}{2}\left(1+2z+2z^2\right)\right\}F_2(x_h,Q_h^2)$$

$$\begin{aligned}
C^{(1,1)}(y_h,Q_l^2) &= \frac{2\pi\alpha^2}{y_hQ_l^4}\int_{z_0^I}^1 dz\left\{c^{(1,1,i)}(y_h,Q_l^2,z)-c_{IR}^{(1,1,i)}(y_h,Q_l^2,z)\right\}\\
&+ \frac{2\pi\alpha^2}{y_hQ_l^4}\int_{z_0^F}^1 dz\left\{c^{(1,1,f)}(y_h,Q_l^2,z)-c_{IR}^{(1,1,f)}(y_h,Q_l^2,z)\right\}\\
&+ \delta_{VR}(y_h,Q_l^2)C^{(0,0)}(y_h,Q_l^2)
\end{aligned}$$

We confirm the results of Akhundov et. al. Fortschr. Phys. 44 (1996) 373.

3. Non–leptonic Corrections

l-q Interference term $O(\alpha)$



Comparison of the radiative corrections $\delta^{e+i}_{mix} = \delta_e + \delta_i$ (broken line) with the leptonic corrections δ_e (solid line) at HERA in mixed variables, Bardin et al., hep-ph/0504423.

Hadronic Corrections

These corrections have to be dealt with together with the QCD corrections to the parton densities. Solve: AP-equations at LO, add NLO perturbations and Wilson coefficients.

Refs.:

J. Kripfganz & H. Perlt, Z. Phys. **C41** (1988) 319

J. Blümlein, Z. Phys. **C47** (1990) 89

H. Spiesberger, Phys. Rev. **D52** (1995) 4936.

The relative $O(\alpha)$ correction is growing towards larger values of x and reaches there about -2% @ $Q^2 \sim 5000$ GeV^2. $O(\alpha^2)$ corrections are even smaller.

4. Two–loop contribution

Method: use the RG equations for mass factorization and charge renormalization Berends, Burgers, van Neerven, 1988; e^+e^- Annih. ISR

RGE's for OME's and Wilson Coefficients: ISR and FSR

$$\left[\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}+\gamma^i_{ee}\right]\Gamma^i_{ee}+\gamma^i_{e\gamma}\Gamma^i_{\gamma e} = 0$$
$$\left[\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}+\gamma^f_{ee}\right]\Gamma^f_{ee}+\gamma^f_{\gamma e}\Gamma^f_{e\gamma} = 0$$

$$\left[\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}-\gamma^i_{ee}-\gamma^f_{ee}\right]\widehat{\sigma}_{ee}-\gamma^i_{\gamma e}\widehat{\sigma}_{e\gamma}-\gamma^f_{e\gamma}\widehat{\sigma}_{\gamma e}=0$$

$$\frac{da}{d\ln\mu^2}=-\sum_{k=0}^{\infty}\beta_k a^{k+2};\quad a=\alpha/(4\pi)$$

represent as series :

$$\Gamma^k_{ab}=\delta_{ab}+\sum_{l=1}^{\infty}a^l\sum_{m=0}^{l}ln^m(\mu^2/m_e^2)\Gamma^{k,l}_{ab,m}$$
$$\widehat{\sigma}_{ab}=\delta_{ab}+\sum_{l=1}^{\infty}a^l\sum_{m=0}^{l}ln^m(Q^2/\mu^2)\widehat{\sigma}^l_{ab,m}$$

Insert into (to $O(a^2)$) :

$$\sigma_{ee}=\Gamma^i_{ee}\otimes\widehat{\sigma}_{ee}\otimes\Gamma^f_{ee}+\Gamma^i_{\gamma e}\otimes\widehat{\sigma}_{e\gamma}\otimes\Gamma^f_{ee}+\Gamma^i_{ee}\otimes\widehat{\sigma}_{\gamma e}\otimes\Gamma^f_{e\gamma}$$

The logarithms match in the expansion and the final expression is independent of μ.

The final expression expands in $\alpha/2\pi$ and $\ln(Q^2/m_e^2)$.

2nd order cross section

$$\frac{d^2\sigma^{0+\mathrm{RC}}}{dy_h dQ_l^2} = \frac{d^2\sigma^0}{dy_h dQ_l^2} + \left(\frac{\alpha}{2\pi}\right)\left[\log\left(\frac{Q_l^2}{m_e^2}\right) C^{(1,0)} + C^{(1,1)}\right] + \left(\frac{\alpha}{2\pi}\right)^2\left[\log^2\left(\frac{Q_l^2}{m_e^2}\right) C^{(2,0)} + \log\left(\frac{Q_l^2}{m_e^2}\right) C^{(2,1)} + C^{(2,2)}\right]$$

$C^{(2,0)}(y, Q^2)$: leading log term: 2nd order
$\longrightarrow$ ISR, FSR, interfarence.

Kripfganz, Möhring and Spiesberger, 1991
J. Blümlein, 1994

$C^{(2,1)}(y, Q^2)$: Several OME's and Wilson coefficients combine to cross sections

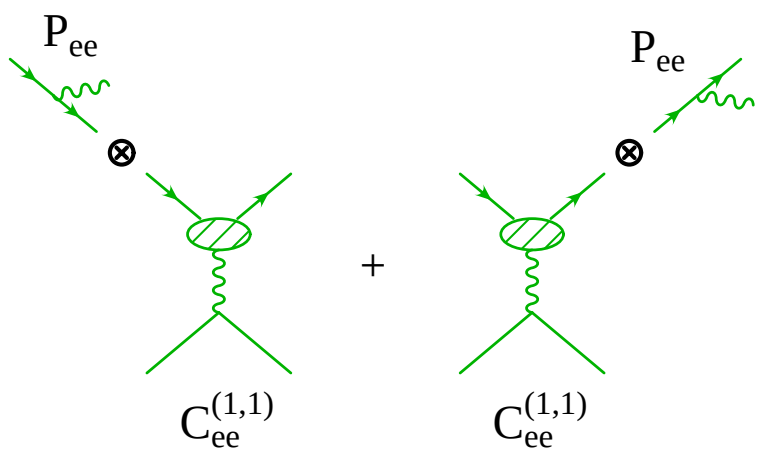
$$\left[\Gamma^{I,1}_{ee,0} + \Gamma^{F,1}_{ee,0} + \widehat{\sigma}^1_{ee,0}\right] = \sigma^1_{ee}$$

$$\left[\Gamma^{F,1}_{e\gamma,0} + \widehat{\sigma}^1_{e\gamma,0}\right] = \sigma^1_{e\gamma}$$

$$\left[\Gamma^{I,1}_{\gamma e,0} + \widehat{\sigma}^1_{\gamma e,0}\right] = \sigma^1_{\gamma e}$$

Individual Terms :

(A) $P^0_{ee} \otimes C^{(1,1)}_{ee}$, $C^{(1,1)}_{ee} \otimes P^0_{ee}$

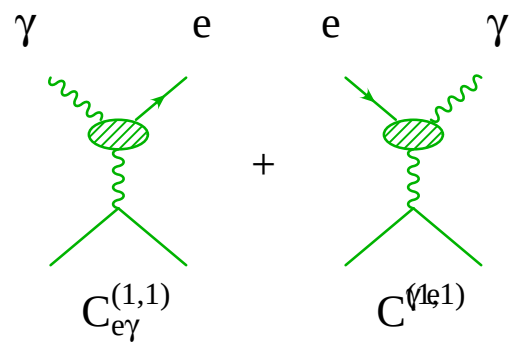


(A)

$$C_A^{(2,1)}(y_h, Q_l^2)$$
$$= \int_0^1 dz P^0_{ee}(z) \left[\theta(z - z_0^I)\widetilde{C}^{(1,1)}(y, Q^2) J^I(z) - C^{(1,1)}(y, Q^2)\right]$$
$$+ \int_0^1 dz P^0_{ee}(z) \left[\theta(z - z_0^F)\widetilde{C}^{(1,1)}(y, Q^2) J^F(z) - C^{(1,1)}(y, Q^2)\right]$$

(B). $C_{e\gamma}^{(1,0)} \otimes P_{\gamma e}^{0}$, (C). $C_{\gamma e}^{(1,0)} \otimes P_{e\gamma}^{0}$

New subprocesses:



$$\frac{d^2\sigma_{e\gamma}^{(1)}}{dydQ^2} = \frac{\alpha}{2\pi}\ln\left(\frac{Q^2}{m_e^2}\right) C_{e\gamma}^{(1,0)}(y,Q^2) + \frac{\alpha}{2\pi} C_{e\gamma}^{(1,1)}(y,Q^2)$$

$$\frac{d^2\sigma_{\gamma e}^{(1)}}{dydQ^2} = \frac{\alpha}{2\pi}\ln\left(\frac{Q^2}{m_e^2}\right) C_{\gamma e}^{(1,0)}(y,Q^2) + \frac{\alpha}{2\pi} C_{\gamma e}^{(1,1)}(y,Q^2)$$

$$C_{e\gamma}^{(1,0)}(y,Q^2) = \int_{z_0^I}^{1} dz P_{e\gamma}^{0}(z) J^I(z) \widetilde{C}^{(0,0)}(y,Q^2)$$

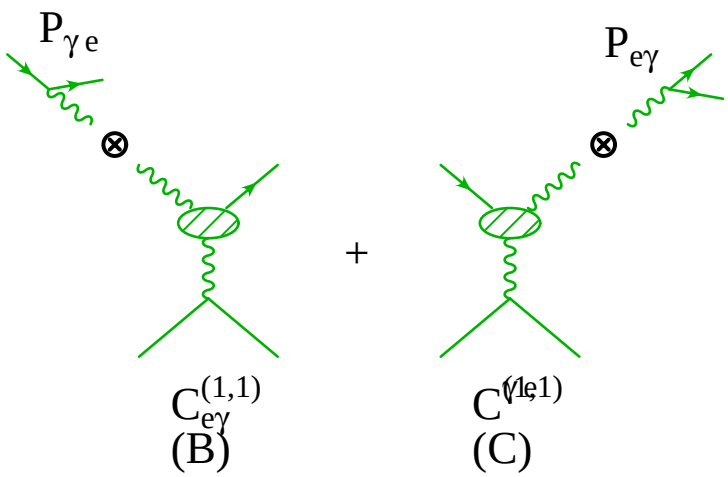
$$C_{\gamma e}^{(1,0)}(y,Q^2) = \int_{z_0^F}^{1} dz P_{\gamma e}^{0}(z) J^F(z) \widetilde{C}^{(0,0)}(y,Q^2)$$

$$\begin{aligned}
&C_{e\gamma}^{(1,1)}(y,Q^2) \\
&\quad = \frac{2\pi\alpha^2}{y_h Q_l^4}\int_{z_0^I}^{1}\frac{dz}{z^2}\left[\left\{P_{e\gamma}^{0}(z)\ln\left(\frac{(1-z)(z-y_h)}{z}\right)\right.\right. \\
&\quad -(1+2z)\ln\left(\frac{z}{y_h}\right) - 1 + \frac{z}{y_h} + 2z(1-z)\Big\} y_h^2 2x_m F_1(x_h,Q_h^2) \\
&\quad +2\left\{(z-y_h)P_{e\gamma}^{0}(z)\ln\left(\frac{(1-z)(z-y_h)}{z}\right) - y_h\ln\left(\frac{z}{y_h}\right)\right. \\
&\quad \left.\left.+(z-y_h)\left(\frac{y_h}{z} - 1 - 2y_h z + 8z + 2y_h z - 8z^2\right)\right\} F_2(x_h,Q_h^2)\right]
\end{aligned}$$

$$+\frac{2\pi\alpha^2}{y_h Q_l^4}\int_{z_0^F}^1 dz\left[\left\{\frac{1}{z}P^0_{\gamma e}(z)\ln\left(\frac{1-y_h z}{1-z}\right)-\left(1+\frac{2}{z}\right)\ln\left(\frac{1}{y_h}\right)\right.\right.$$
$$\left.-\frac{1}{z}+\frac{1}{y_h z}\right\}y_h^2 2x_m F_1(x_h,Q_h^2)$$
$$+2\left\{-y_h\ln\left(\frac{1}{y_h}\right)+\frac{1}{z}\left(\frac{1}{z}-y_h\right)P^0_{\gamma e}(z)\ln\left(\frac{1-y_h z}{1-z}\right)\right.$$
$$\left.\left.+\frac{1-y_h^2}{z}-2\frac{1-y_h}{z^2}\right\}F_2(x_h,Q_h^2)\right]$$

$$C^{(1,1)}_{\gamma e}(y,Q^2)$$
$$=\frac{2\pi\alpha^2}{y_h Q_l^4}\int_{z_0^I}^1\frac{dz}{z^2}\left[\left\{z\ln\left(\frac{z}{y_h}\right)+\frac{1}{z}P^0_{e\gamma}(z)\ln\left(\frac{1-y_h}{1-z}\right)\right.\right.$$
$$\left.-1+\frac{z}{y_h}\right\}y_h^2 2x_m F_1(x_h,Q_h^2)$$
$$+2\left\{-y_h\ln\left(\frac{z}{y_h}\right)+(1-y_h)\frac{1}{z^2}P^0_{e\gamma}(z)\ln\left(\frac{1-y_h}{1-z}\right)\right.$$
$$\left.\left.-\frac{z-y_h}{2z^2}(2-y_h-3z+2y_h z)\right\}F_2(x_h,Q_h^2)\right]$$
$$+\frac{2\pi\alpha^2}{y_h Q_l^4}\int_{z_0^F}^1 dz\left[\left\{P^0_{\gamma e}(z)\ln\left(\frac{(1-y_h)(1-z)}{z^2}\right)+\frac{1}{z}\ln\left(\frac{1}{y_h}\right)\right.\right.$$
$$\left.+2-\frac{3}{z}+\frac{1}{y_h z}\right\}y_h^2 2x_m F_1(x_h,Q_h^2)$$
$$+2\left\{z(1-y_h)P^0_{\gamma e}(z)\ln\left(\frac{(1-y_h)(1-z)}{z^2}\right)-y_h\ln\left(\frac{1}{y_h}\right)\right.$$
$$\left.\left.-\frac{1-y_h}{2}(7-y_h-14z+4y_h z+6z^2-2y_h z^2)\right\}F_2(x_h,Q_h^2)\right]$$

$O(\alpha^4)$ Results :



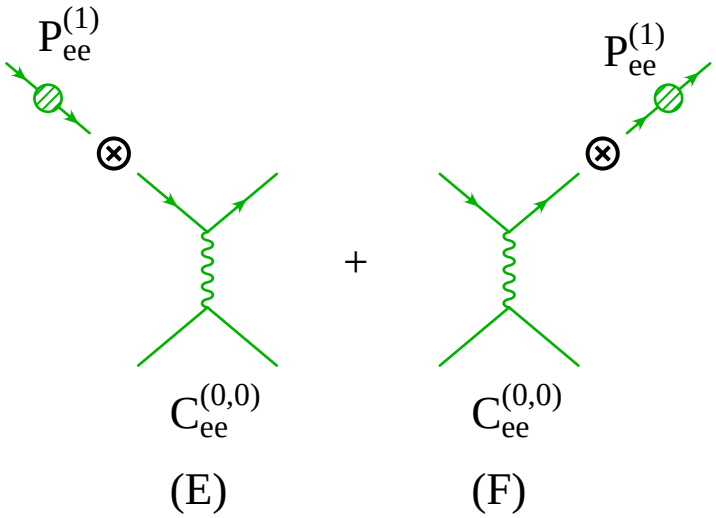
$$C_B^{(2,1)}(y,Q^2) = \int_{z_0^I}^1 dz P_{\gamma e}^0(z) J^I(z) \widetilde{C}_{e\gamma}^{(1,1)}(y,Q^2)$$

$$C_C^{(2,1)}(y,Q^2) = \int_{z_0^F}^1 dz P_{e\gamma}^0(z) J^F(z) \widetilde{C}_{\gamma e}^{(1,1)}(y,Q^2)$$

(D). Running Coupling Contribution

$$C_D^{(2,1)} = -\frac{\beta_0}{2} C^{(1,1)}(y,Q^2); \qquad \beta_0 = -\frac{4}{3}$$

(E) & (F). NLO splitting functions



$$C_E^{(2,1)}(y,Q^2) = \int_0^1 dz P_{ee,NS}^{(1,S)}(z) \left[\theta(z-z_0^I)\frac{d^2\widetilde{\sigma}^0}{dydQ^2}J^I(z) - \frac{d^2\sigma^0}{dydQ^2}\right] + \int_{z_0^I}^1 dz P_{ee,PS}^{(1,S)}(z)\frac{d^2\widetilde{\sigma}^0}{dydQ^2}J^I(z)$$

$$C_F^{(2,1)}(y,Q^2) = \int_0^1 dz P_{ee,NS}^{(1,T)}(z) \left[\theta(z-z_0^F)\frac{d^2\widetilde{\sigma}^0}{dydQ^2}J^F(z) - \frac{d^2\sigma^0}{dydQ^2}\right] + \int_{z_0^F}^1 dz P_{ee,PS}^{(1,T)}(z)\frac{d^2\widetilde{\sigma}^0}{dydQ^2}J^F(z)$$

$$P_{ee,\mathrm{NS}}^{(1,S)}\Big|_{\mathrm{OM}}(z) = P_{ee,\mathrm{NS}}^{(1,S)}\Big|_{\overline{\mathrm{MS}}}(z) + \frac{\beta_0}{2}\Gamma_{0,ee}^{(S)}(z)$$

$$P_{ee,\mathrm{NS}}^{(1,T)}\Big|_{\mathrm{OM}}(z) = P_{ee,\mathrm{NS}}^{(1,T)}\Big|_{\overline{\mathrm{MS}}}(z) + \frac{\beta_0}{2}\Gamma_{0,ee}^{(T)}(z)$$

$$P_{ee,\mathrm{NS}}^{(1,S)}\Big|_{\overline{\mathrm{MS}}}(z) = P_{ee,\mathrm{F}}^{1,S}(z) + P_{ee,N_f}^{1,S}(z)$$

$$P_{ee,NS}^{(1,T)}\Big|_{\overline{\mathrm{MS}}}(z) = P_{ee,\mathrm{F}}^{1,T}(z) + P_{ee,N_f}^{1,T}(z)$$

Operator Matrix Elements

$$\Gamma^{(S)}_{0,ee}(z) = \Gamma^{(T)}_{0,ee}(z) = -2\left[\frac{1+z^2}{1-z}\left(\ln(1-z)+\frac{1}{2}\right)\right]_+$$

S: Light Cone Expansion; T: Cut vertex method.

$\overline{\mathrm{MS}}$ Splitting Functions:

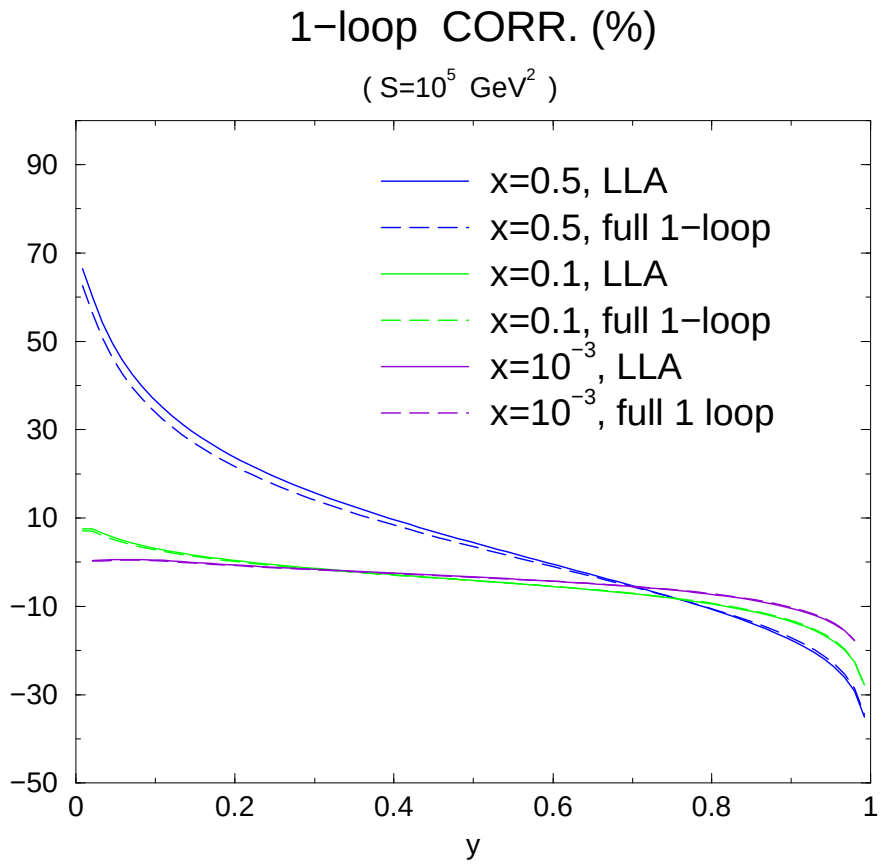
$$\begin{aligned}
P^{\mathrm{S}}_{ee,\mathrm{F}}(z) &= -2P^0_{ee}(z)\ln(z)\left[\ln(1-z)+\frac{3}{4}\right] \\
&\quad -\frac{1}{2}(1+z)\ln^2(z) - \frac{1}{2}(3+7z)\ln(z) - 5(1-z) \\
P^{\mathrm{S}}_{ee,\mathrm{PS}}(z) &= -(1+z)\ln^2(z) + \left(1+5z+\frac{8}{3}z^2\right)\ln(z) \\
&\quad +\frac{2}{9}\left(\frac{1-z}{z}\right)\left(10+z+28z^2\right) \\
P^{\mathrm{T}}_{ee,\mathrm{F}}(z) &= 2P^0_{ee}(z)\ln(z)\left[\ln(1-z)-\ln(z)+\frac{3}{4}\right] \\
&\quad +\frac{1}{2}(1+x)\ln^2(z) - \frac{1}{2}(7+3z)\ln(z) - 5(1-z) \\
P^{\mathrm{T}}_{ee,\mathrm{PS}}(z) &= (1+z)\ln^2(z) - \left(5+9z+\frac{8}{3}z^2\right)\ln(z) \\
&\quad -\frac{4}{9}\left(\frac{1-z}{z}\right)\left(5+23z+14z^2\right) \\
P^{\mathrm{S,T}}_{ee,\mathrm{N}_f}(z) &= -\frac{2}{3}\left\{P^0_{ee}(z)\left[\ln(z)+\frac{5}{3}\right]+2(1-z)\right\}
\end{aligned}$$

Curci, Furmanski, Petronzio, 1980;

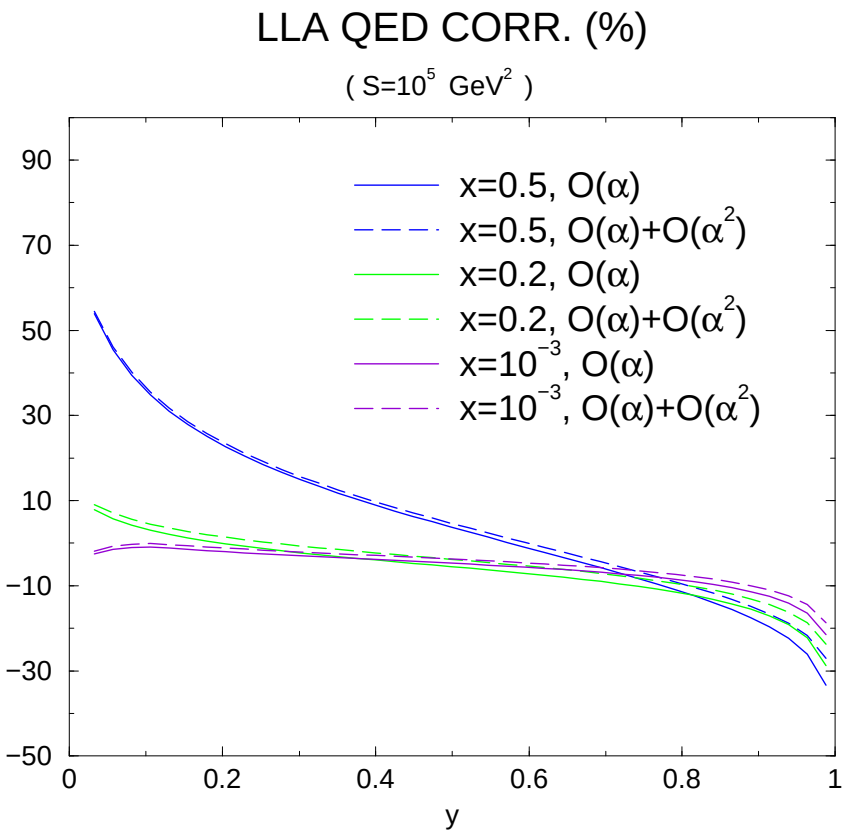
Furmanski and Petronzio, 1980;

Floratos, Kounnas, and Lacaze 1981

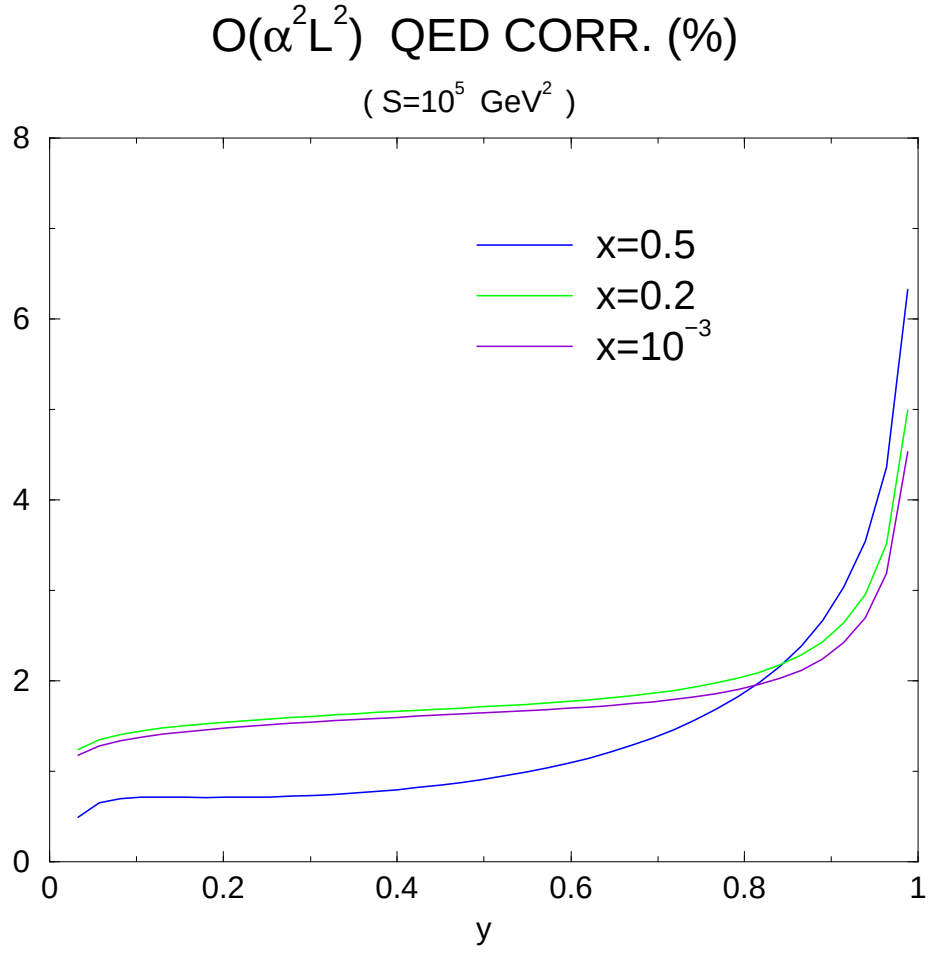
5. Numerical Results



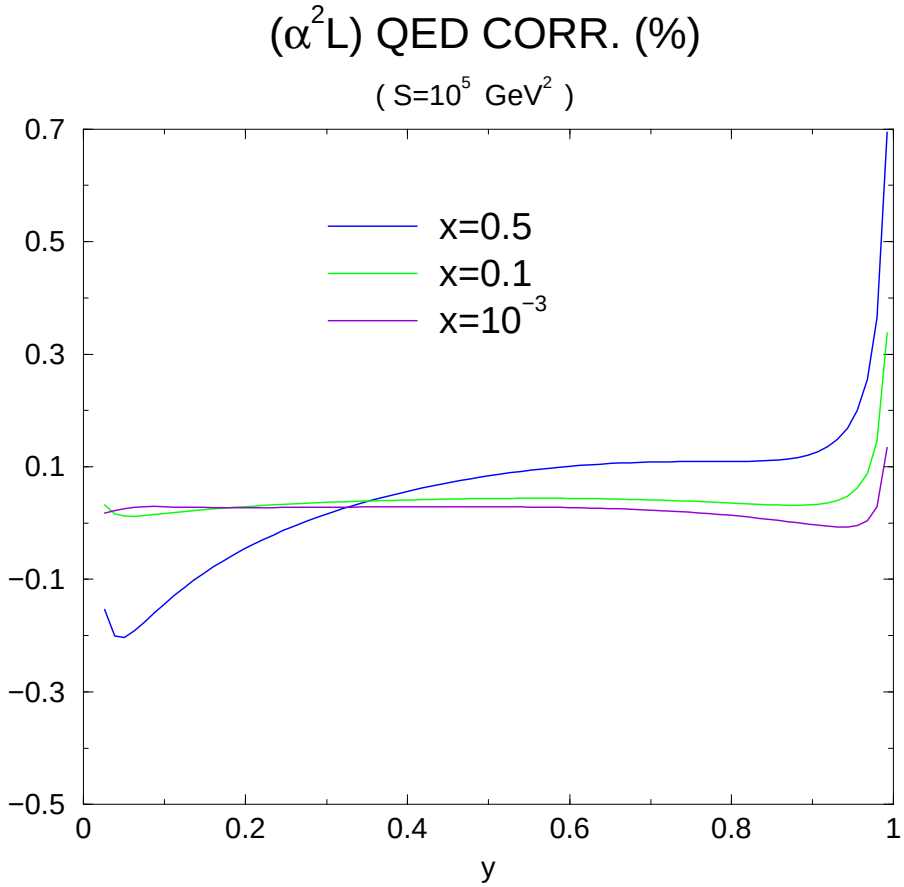
Comparison of the complete $O(\alpha)$ results with that of $O(\alpha L)$.



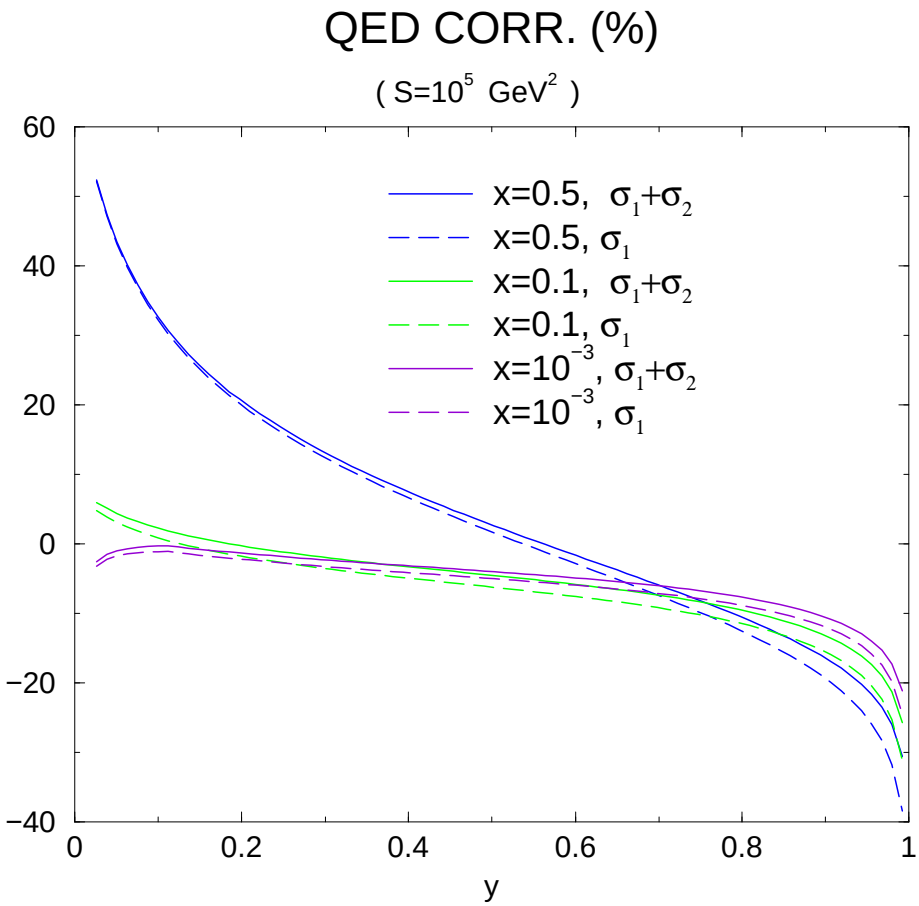
Comparison of the LLA results in $O(\alpha L)$ and $O(\alpha^2 L^2)$.



Contribution of the $O(\alpha^2 L^2)$ leptonic radiative corrections.



Contribution of $O(\alpha^2 L)$ to the leptonic radiative corrections.



Sum of the leptonic QED ccorrections to the Born cross section in $O(\alpha L + \alpha)$ (dashed lines) and $O(\alpha L + \alpha + \alpha^2 L^2 + \alpha^2 L)$ (full lines).

6. Conclusion

- QED corrections give large contributions to DIS cross sections, particularly from the leptonic side.

- With the help of the RGE–decomposition, the leptonic corrections of $O(\alpha^2 \mathrm{L})$ were calculated for the double differential cross section $d^2\sigma/dydQ^2$ of DIS in mixed variables.

- The calculation was carried out in the on-shell scheme for finite electron mass. This method generalizes earlier investigations for ISR in e^+e^- annihilation and includes both space–and timelike splitting functions and Wilson coefficients.

- The $O(\alpha^2 L)$ corrections are about a factor of 5 smaller than the $O(\alpha^2 L^2)$ corrections and are of the 1% level compared to the differential Born cross section in the whole kinematic domain.