Modification of particle distributions from covariant transport

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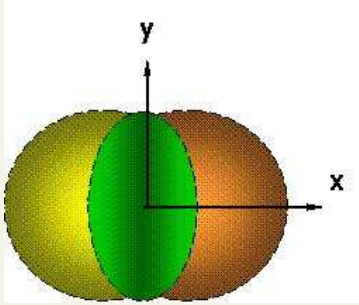
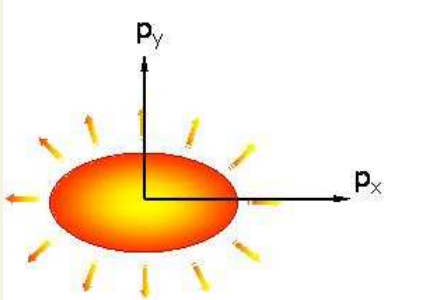
- **validity of Israel-Stewart hydro - Huovinen & DM, PRC79**
- **validity of Grad ansatz for δf - work in progress**

Thermalization at RHIC

efficient conversion of spatial eccentricity to **momentum anisotropy**

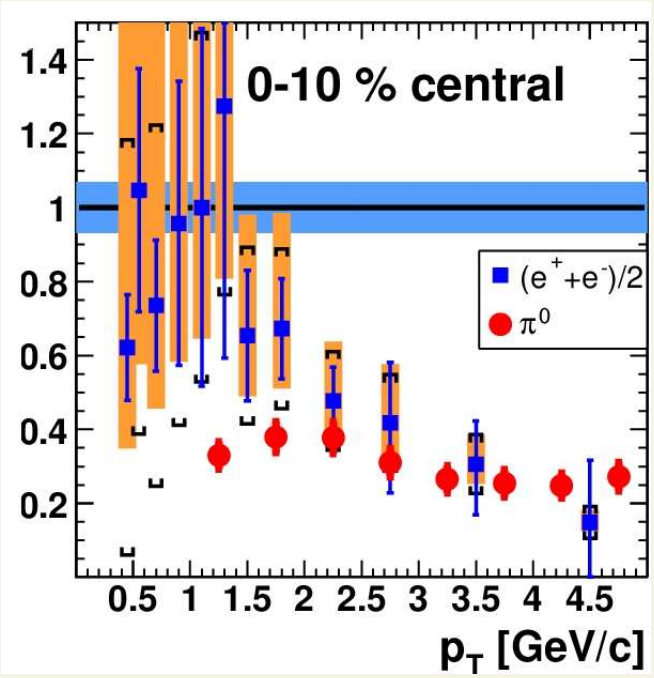


$\longrightarrow$



"elliptic flow"

$$\varepsilon \equiv \frac{\langle x^2 - y^2 \rangle}{\langle x^2 + y^2 \rangle} \qquad v_2 \equiv \frac{\langle p_x^2 - p_y^2 \rangle}{\langle p_x^2 + p_y^2 \rangle} \equiv \langle \cos 2\phi_p \rangle$$



very opaque - **large energy loss, even for heavy quarks**

$$R_{AA} = \frac{measured\ yield}{expected\ yield\ for\ dilute\ system}$$

Main dynamical frameworks

- **causal relativistic hydrodynamics** Israel, Stewart; ... Muronga, Rischke; Teaney et al; Romatschke et al; Heinz et al, DM & Huovinen

$$\partial_\mu T^{\mu\nu} = 0 \qquad\qquad (\mu_B \to 0)$$

$$T^{\mu\nu} = (e+p)u^\mu u^\nu - pg^{\mu\nu} + \pi^{\mu\nu} - \Pi\Delta^{\mu\nu}$$

$$\dot{\pi}^{\mu\nu} = F^{\mu\nu}(e,u,\pi,\Pi) \quad , \qquad \dot{\Pi} = G(e,u,\pi,\Pi)$$

e.g. Israel-Stewart theory

- **covariant transport** Israel, de Groot,... Zhang, Gyulassy, DM, Pratt, Xu, Greiner...

$$p^\mu \partial_\mu f = C_{2\to 2}[f] + C_{2\leftrightarrow 3}[f] + \cdots$$

fully causal and stable

near hydrodynamic limit, transport coefficients and relaxation times:

$$\eta \approx 1.2T/\sigma_{tr}, \qquad \tau_\pi \approx 1.2\lambda_{tr}$$

Applicability of IS hydro (in 0+1D)

Take longitudinal Bjorken expansion and ignore transverse expansion, $u^\mu(x) = (t,0,0,z)/\sqrt{t^2-z^2}$

$$T^{\mu\nu}_{LR} = \begin{pmatrix} e & & & \\ & p-\frac{\pi_L}{2} & & \\ & & p-\frac{\pi_L}{2} & \\ & & & p+\pi_L \end{pmatrix} = \begin{pmatrix} e & & & \\ & p_T & & \\ & & p_T & \\ & & & p_L \end{pmatrix}$$

importance of dissipation can be gauged via, e.g., the pressure anisotropy

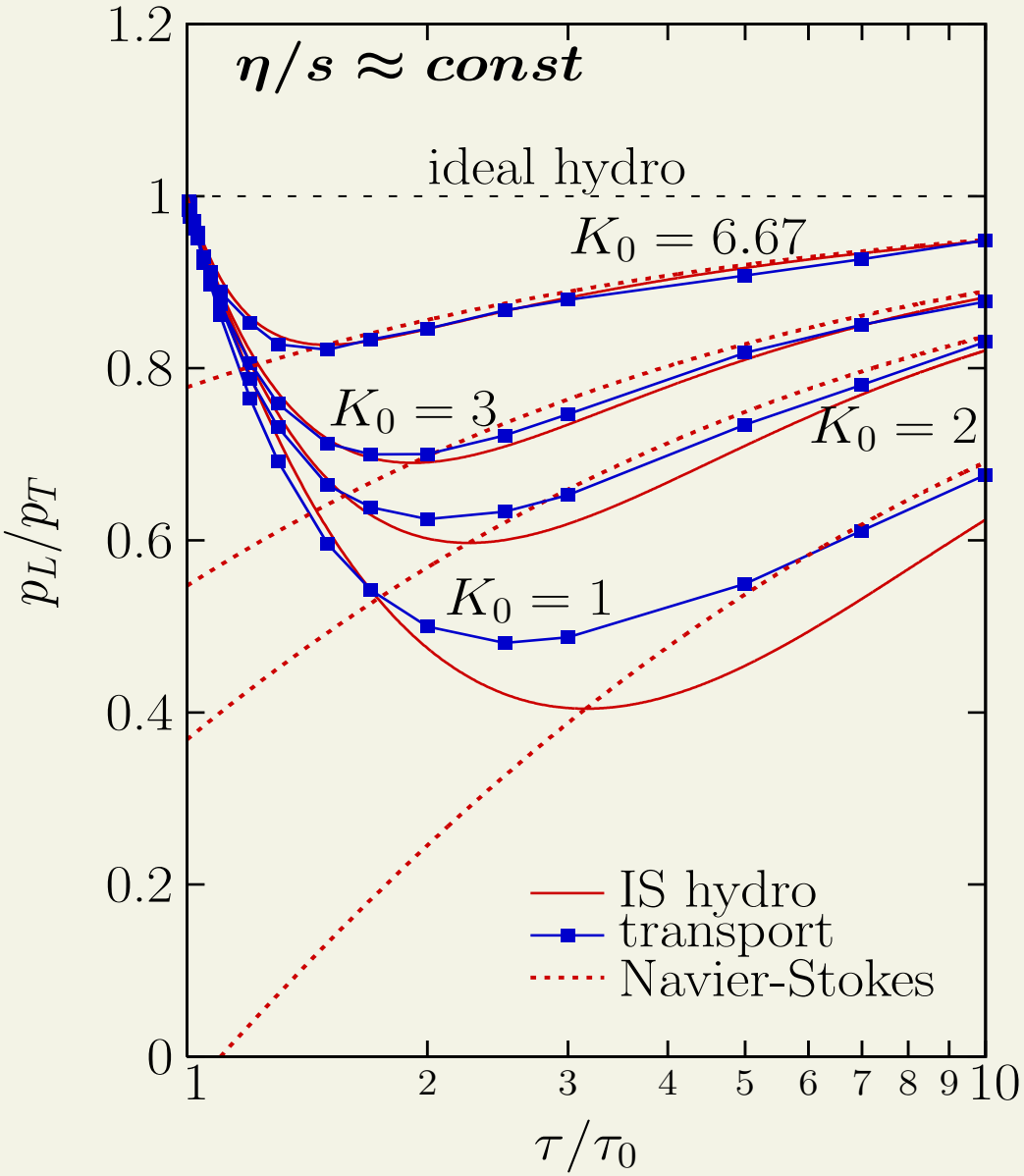
$$R \equiv \frac{p_L}{p_T} = \frac{p+\pi_L}{p-\pi_L/2} \qquad \text{(typically } \pi_L < 0 \Rightarrow R < 1\text{)}$$

study R from hydro AND transport as a function of the initial <u>inverse</u> mean free path $K_0 \equiv \tau_0/\lambda_{tr,0}$

IS should apply at large enough K_0

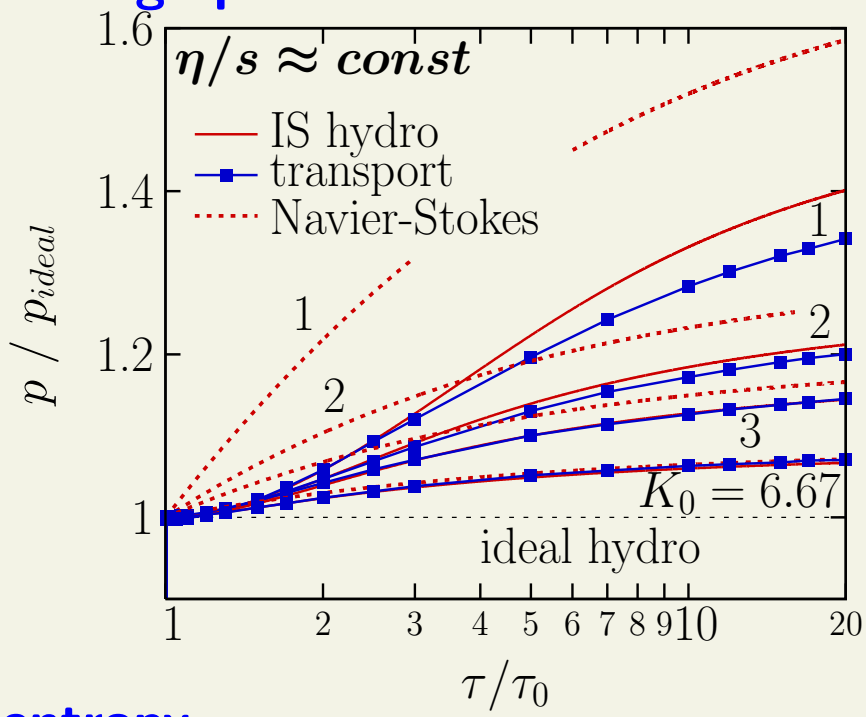
Validity of Israel-Stewart hydro (0+1D Bjorken)

pressure anisotropy T_{zz}/T_{xx}

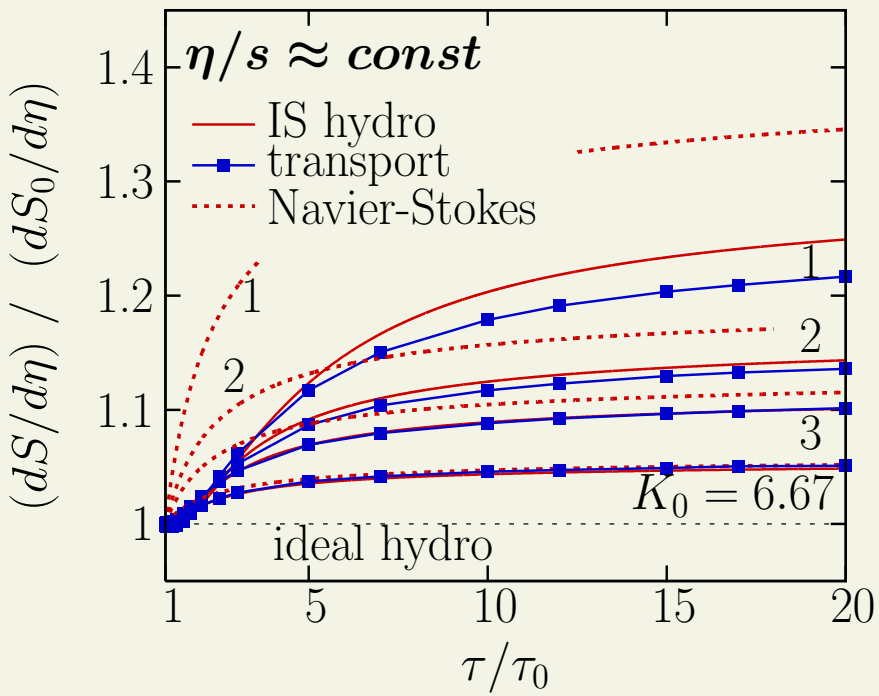


average pressure

Huovinen & DM, PRC79



entropy



IS is 10% accurate when $K_0 \equiv \tau_0/\lambda_{tr,0} \gtrsim 2$

In terms of viscosity

$$K_0 \approx \frac{T_0 \tau_0}{5} \frac{s_0}{\eta_{s,0}} \approx 12.8 \times \left(\frac{T_0}{1\ \mathrm{GeV}}\right) \left(\frac{\tau_0}{1\ \mathrm{fm}}\right) \left(\frac{1/(4\pi)}{\eta_0/s_0}\right) \tag{1}$$

for typical RHIC hydro initconds $T_0 \tau_0 \sim 1$, therefore

$$K_0 \gtrsim 2 - 3 \qquad \Rightarrow \qquad \frac{\eta}{s} \lesssim \frac{1-2}{4\pi} \tag{2}$$

I.e., shear viscosity cannot be many times more than the conjectured bound, for IS hydro to be applicable.

When IS hydro is accurate, dissipative corrections to pressure and entropy do not exceed 20% significantly (a necessary condition).

Freezeout and viscosity

heavy-ion applications in the end must match hydrodynamics to a particle description

- **in local equilibrium**

$$T^{\mu\nu}_{LR} = diag(e, p, p, p) \qquad \Leftrightarrow \qquad f_{eq} = \frac{g}{(2\pi)^3} e^{-p^\mu u_\mu / T}$$

- **near local equilibrium**

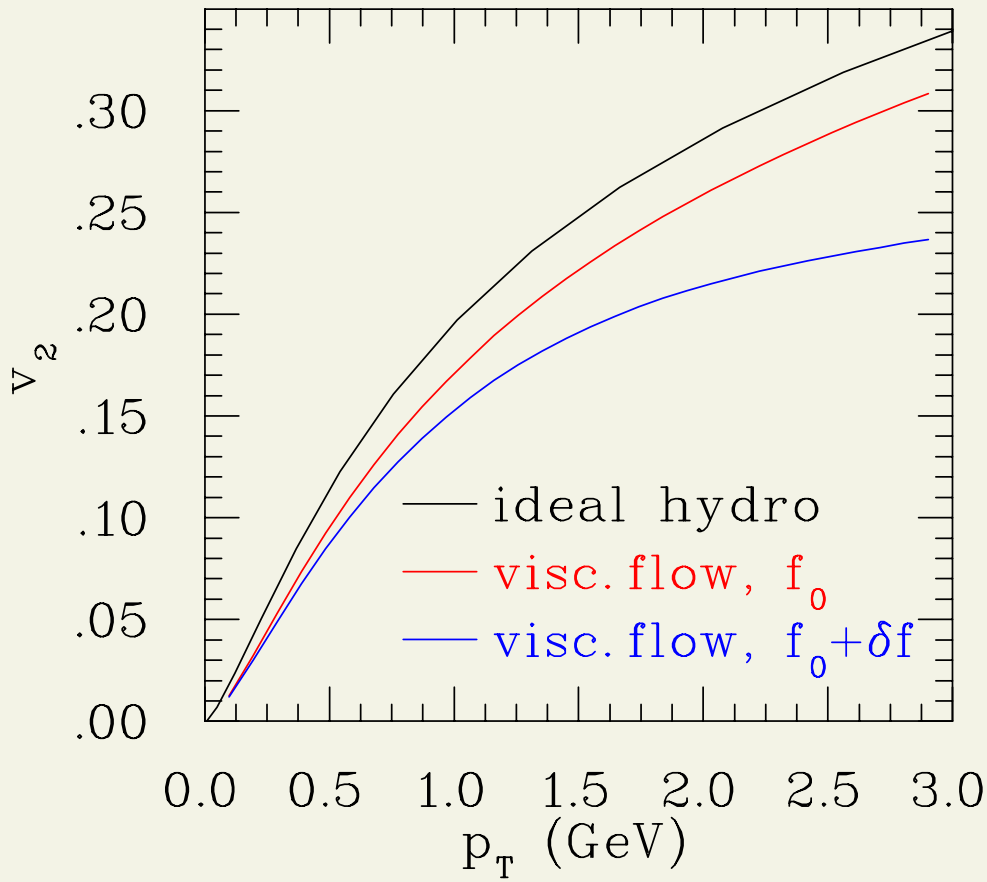
$$T^{\mu\nu} = T^{\mu\nu}_{ideal} + \delta T^{\mu\nu} \qquad \Leftrightarrow \qquad f = f_{eq} + \delta f$$

correction δf crucially affects basic observables - spectra, elliptic flow, ...

Viscous hydro elliptic flow

TWO effects: - **dissipative corrections to hydro fields** u^μ, T, n
- **dissipative corrections to thermal distributions** $f \to f_0 + \delta f$

$\boldsymbol{\eta/s \approx 1/(4\pi)}$ $\quad (\sigma \propto \tau^{2/3})$



Grad:
$\delta f = f_0 \left[1 + \frac{p^\mu p^\nu \pi_{\mu\nu}}{8nT^6}\right]$

quadratic (Grad) ansatz gives most significant correction

in general - for massless gas, with viscous shear only

$$\delta f \equiv f_{eq} \times C(\chi)\, \pi^{\mu\nu} \frac{p_\mu p_\nu}{T^2}\, \chi(\frac{p}{T})$$

from Grad's ansatz: $\chi \equiv 1$

this was a starting point in deriving IS hydro from kinetic theory

from linear response: $\chi(x) \sim x^\alpha$ **with** $-1 \lesssim \alpha \lesssim 0$ Dusling, Teaney, Moore, ('09)

δf **blows up at large momenta** $\Rightarrow$ **approximation breaks down**

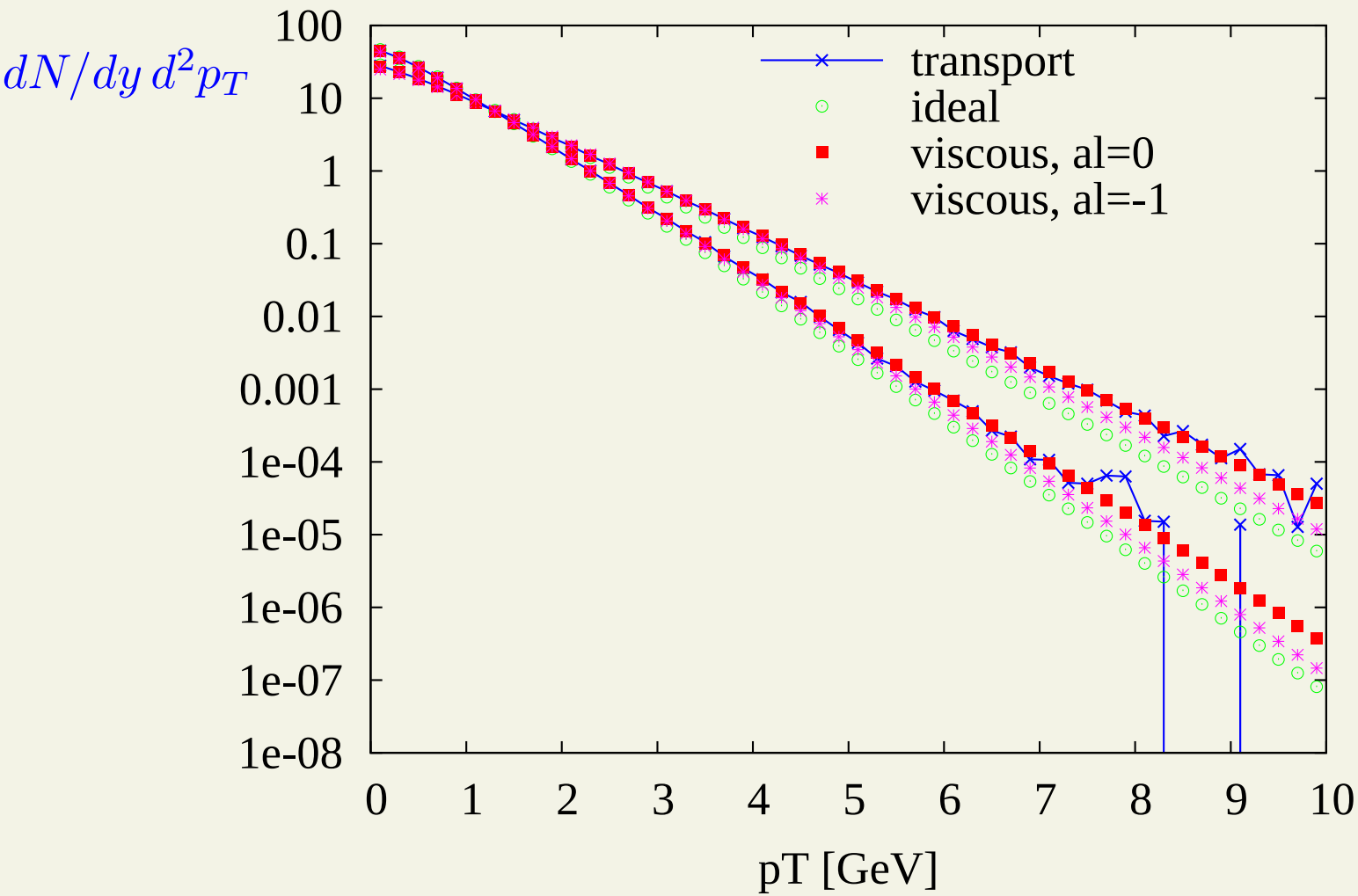
transport can tell how far in momenta we can trust these...

Again, do it in 0+1D - **in that case** $f = f(p_T, \xi, \tau)$**, where** $\xi \equiv \eta - y$

The goal is to compute f **from full nonequilibrium transport, which will also give us** $T^{\mu\nu}$**. Then we compare** f **with** $f_{eq} + \delta f$ **reconstructed from** $T^{\mu\nu}$ **alone using various ansatzes for** $\chi(p/T)$**.**

For simplicity, compare integrated quantities $dN(\tau)/dp_T^2 dy|_{y=0}$ **and** $dN(\tau)/d\xi$

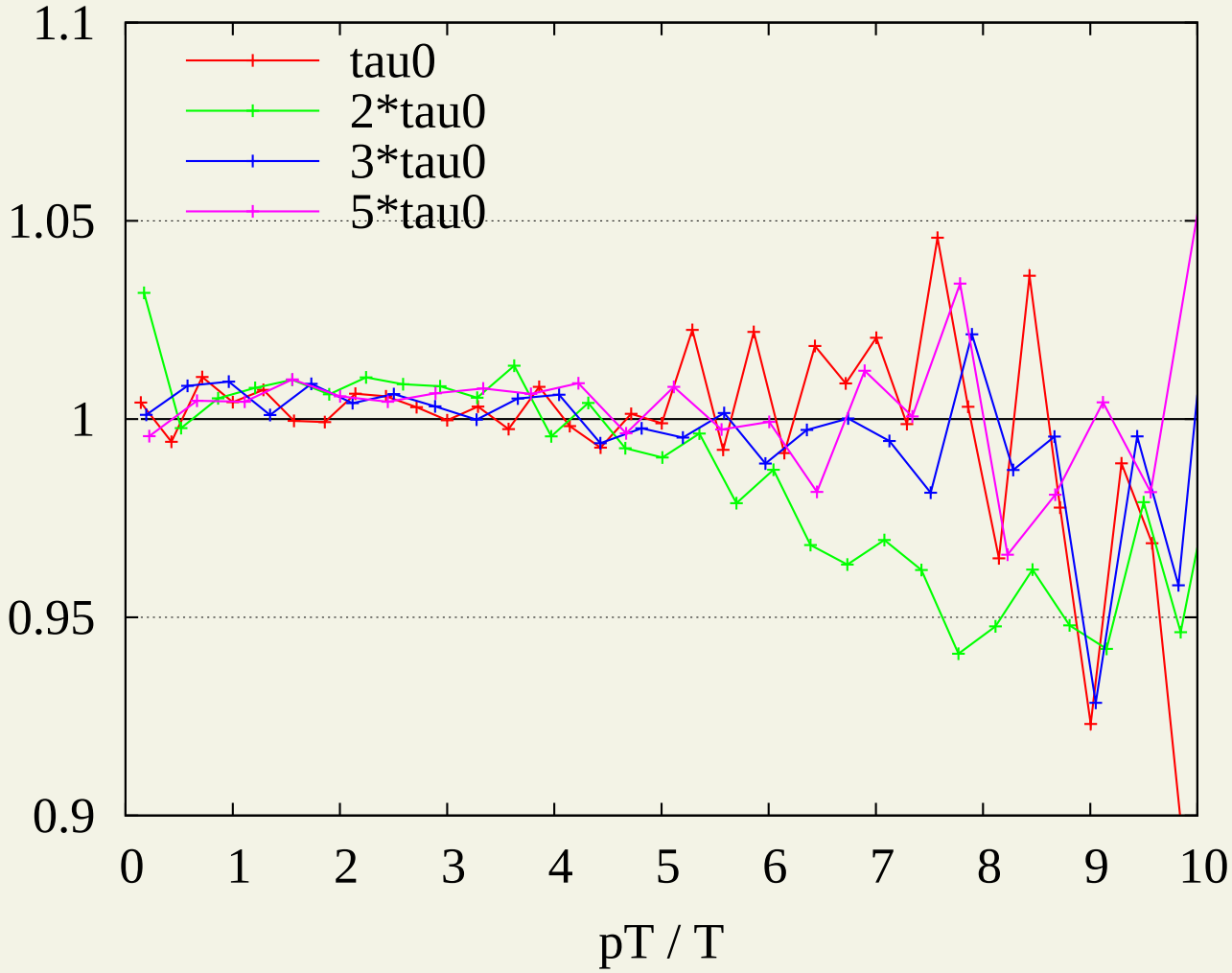
spectra in 0+1D Bjorken scenario at $\tau = 2\tau_0$ and $5\tau_0$
for $\eta/s \approx const$, $K_0 = 3$, local equilibrium initconds $\pi^{\mu\nu}(\tau_0) = 0$



Grad ansatz ($\alpha = 0$) works surprisingly well - on a log plot at least

ratio - transport spectra / Grad approximation

DM ('09):



spectra are few-percent accurate even at $p_T/T = 6$(!)

for $K_0 = 1$, i.e., three times higher η/s

ratio - transport spectra / Grad approximation

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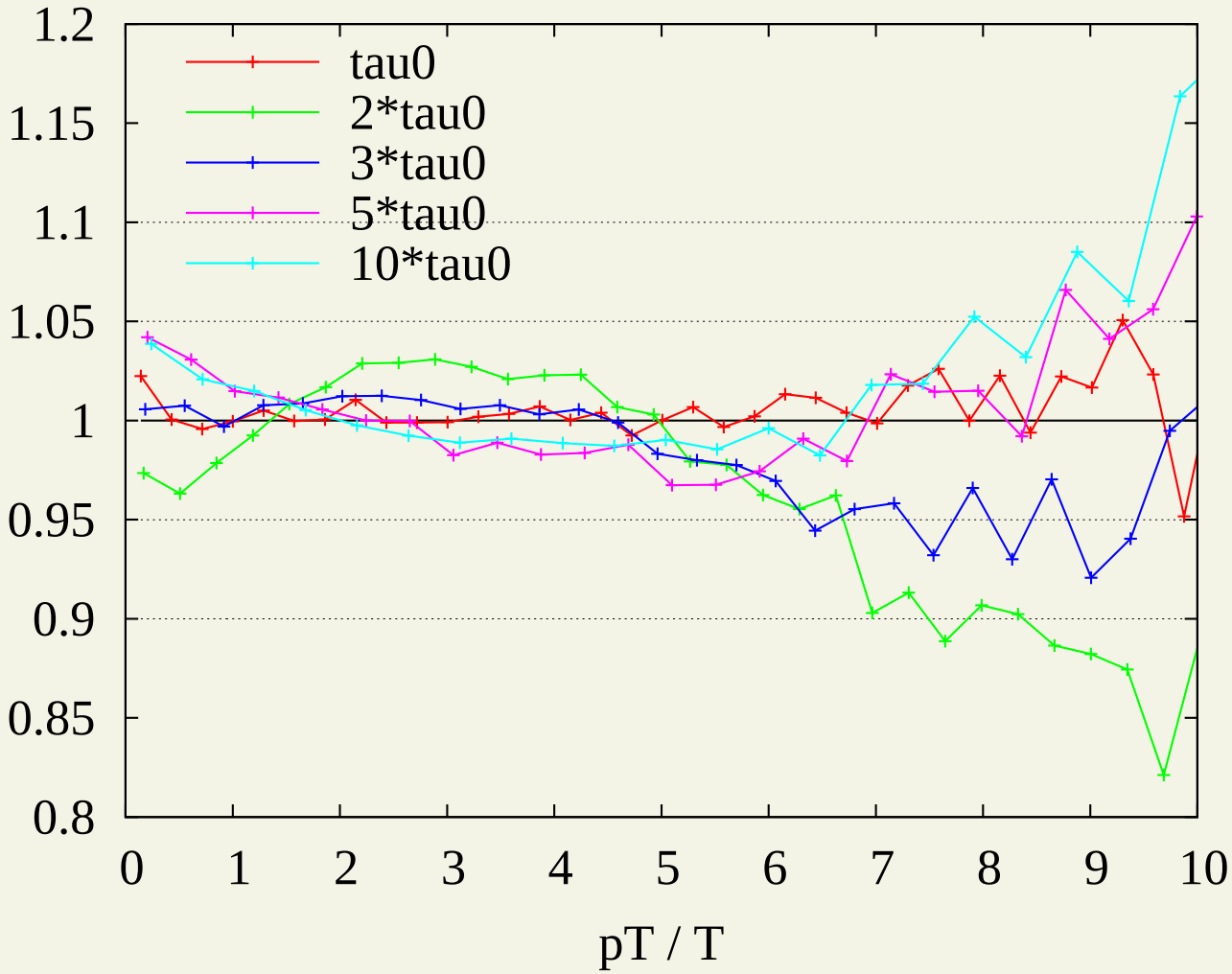
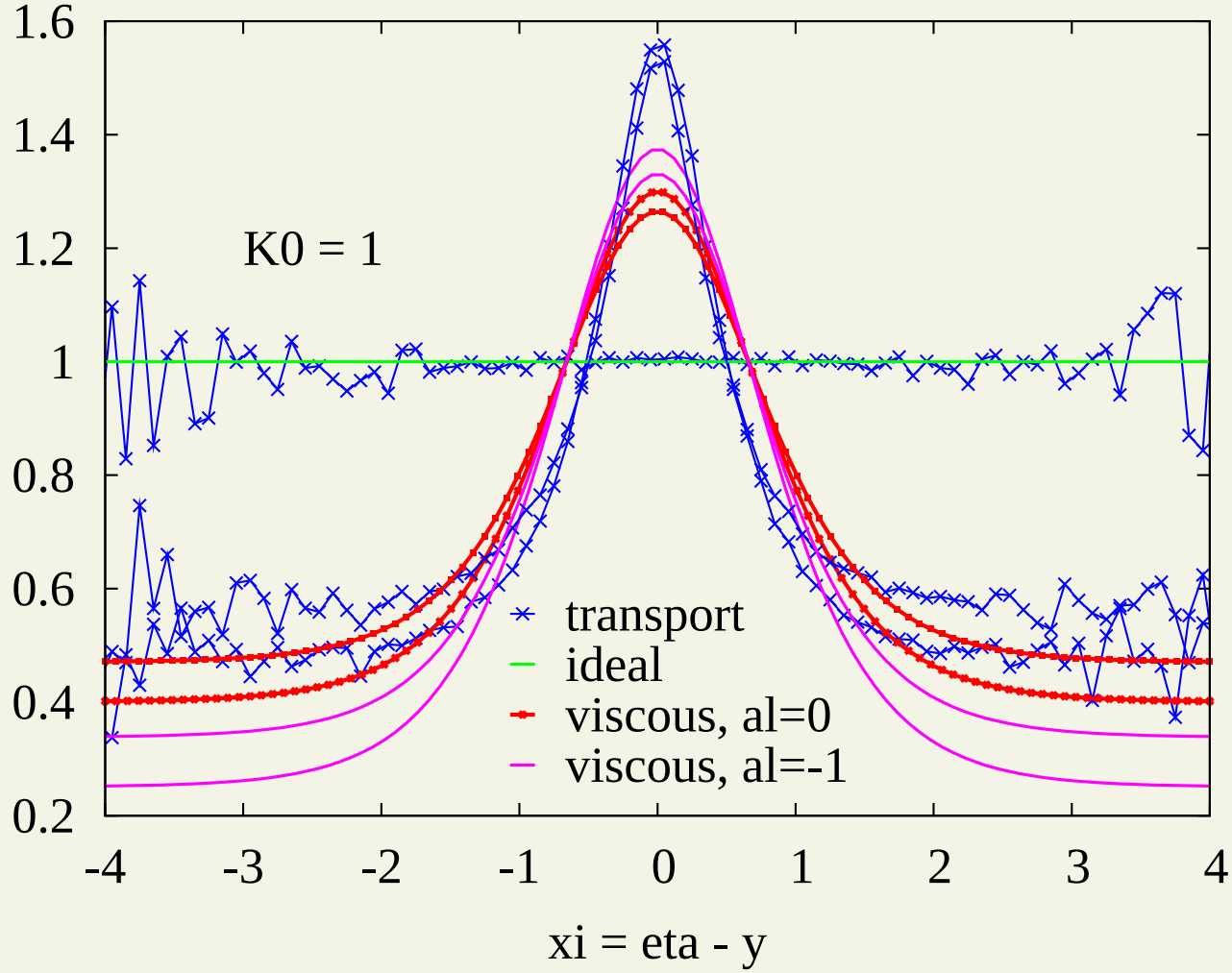


accuracy worsens - should be noticeable in other observables, e.g., v_2

rapidity distributions - ratios relative to IDEAL hydro, $K_0 = 1$ $(\tau/\tau_0 = 1, 2, 5)$

$$\frac{dN/d\xi}{dN^{ideal}/d\xi}, \qquad \xi \equiv \eta - y$$

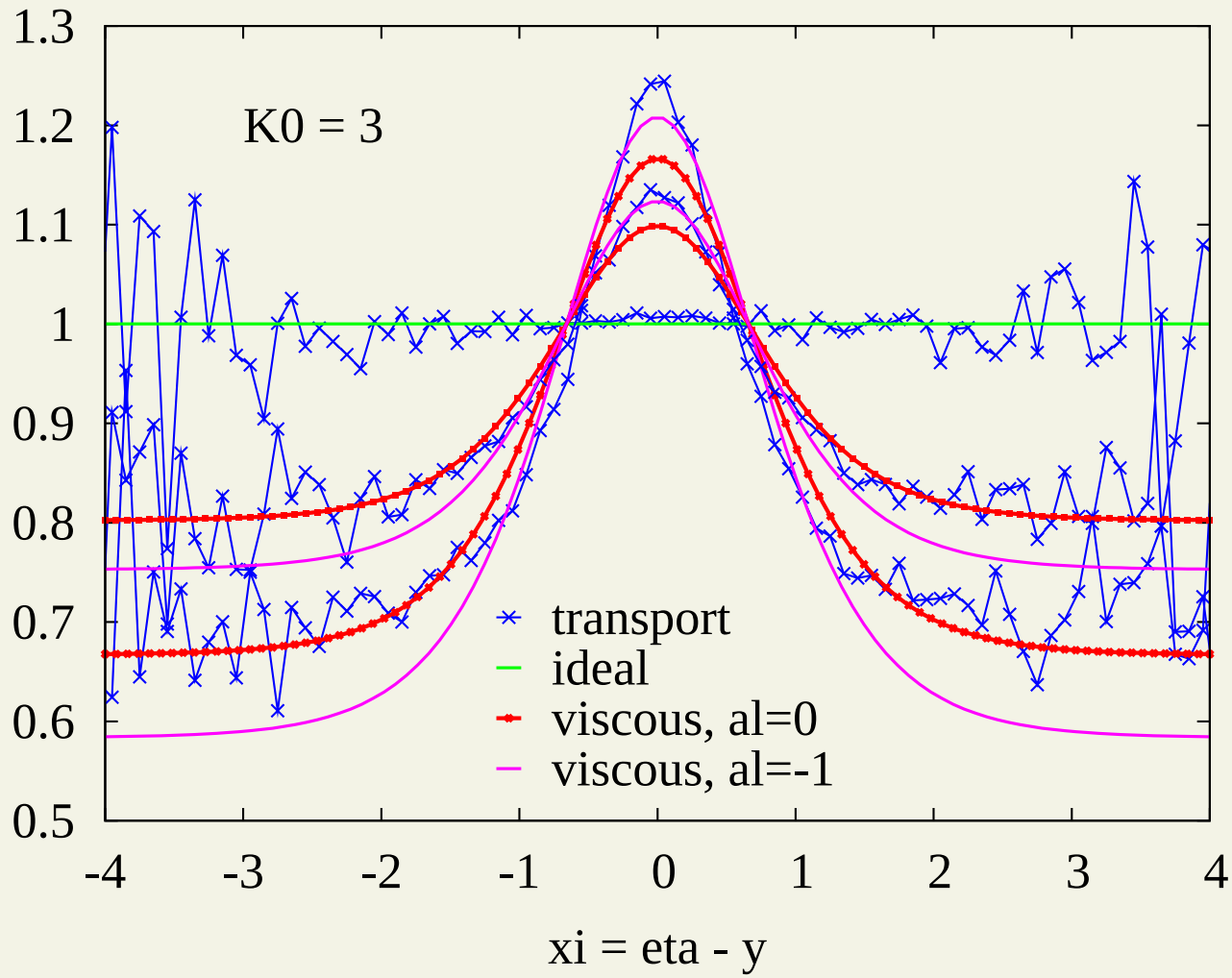
DM ('09):



rapidity distributions - ratios relative to IDEAL hydro, $K_0 = 3$ $(\tau/\tau_0 = 1, 2, 5)$

$$\frac{dN/d\xi}{dN^{ideal}/d\xi}, \quad \xi \equiv \eta - y$$

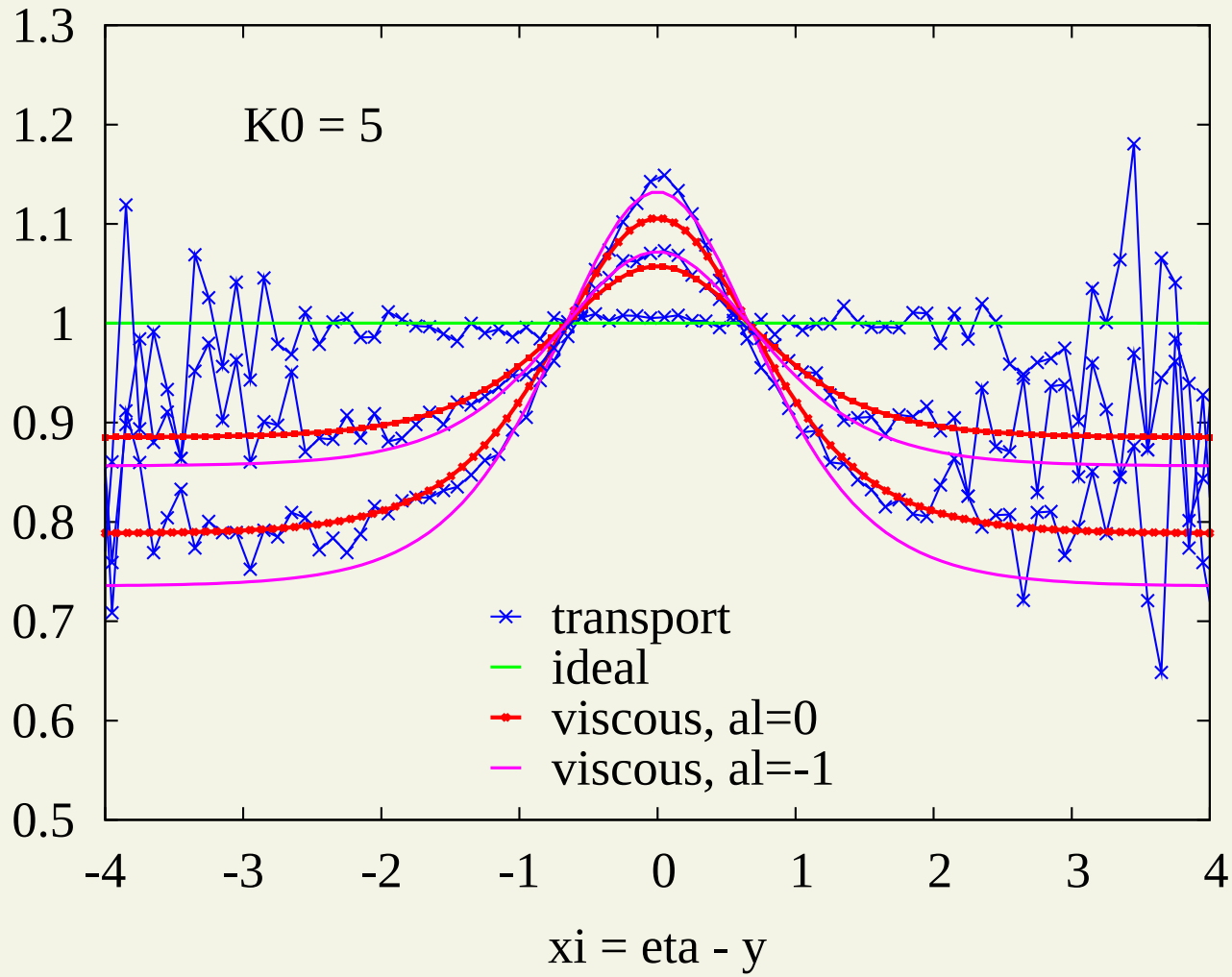
DM ('09):



rapidity distributions - ratios relative to IDEAL hydro, $K_0 = 5$ $(\tau/\tau_0 = 1,\, 2,\, 5)$

$$\frac{dN/d\xi}{dN^{ideal}/d\xi}\ , \qquad \xi \equiv \eta - y$$

DM ('09):

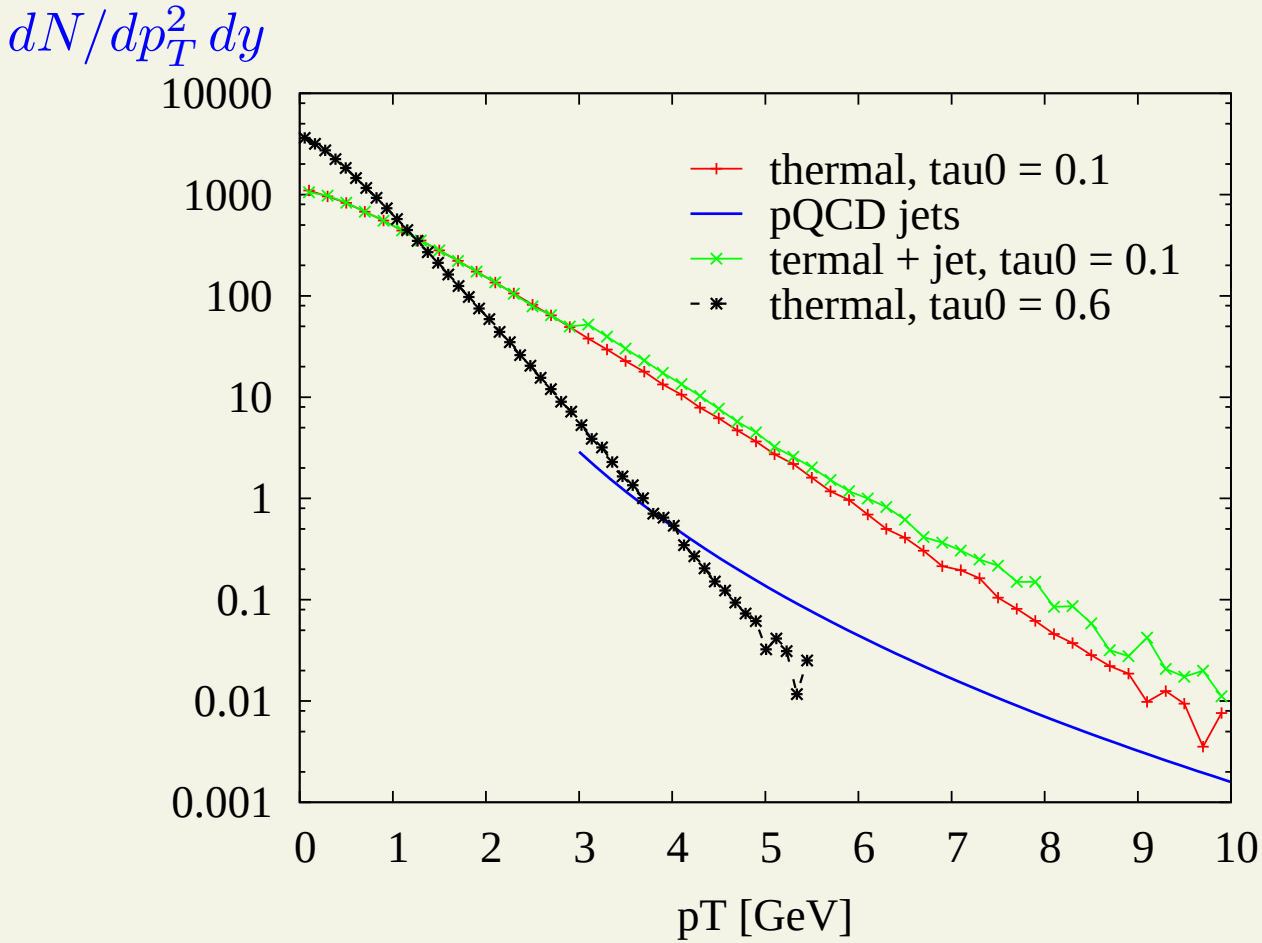


Grad ansatz ($\alpha = 0$) works best, when $K_0 \gtrsim 3$, but it still misses by $20 - 30$% near $\xi = 0$, even for large $K_0 = 5$

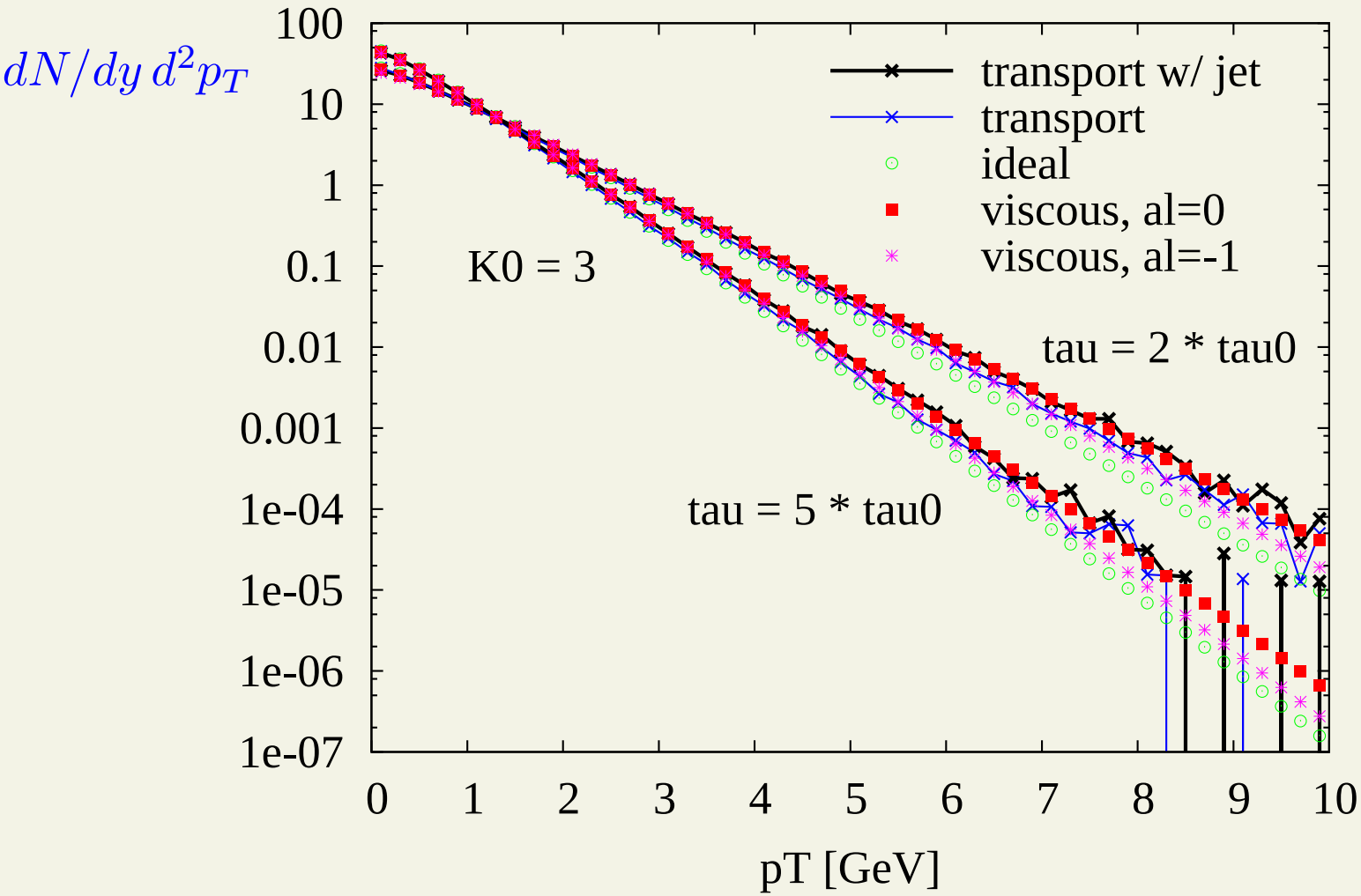
Interplay with pQCD jets

Hydro ansatze must break down at high pT as power-law tails from hard processes take over

how high is high? - depends on what we take for thermalization time



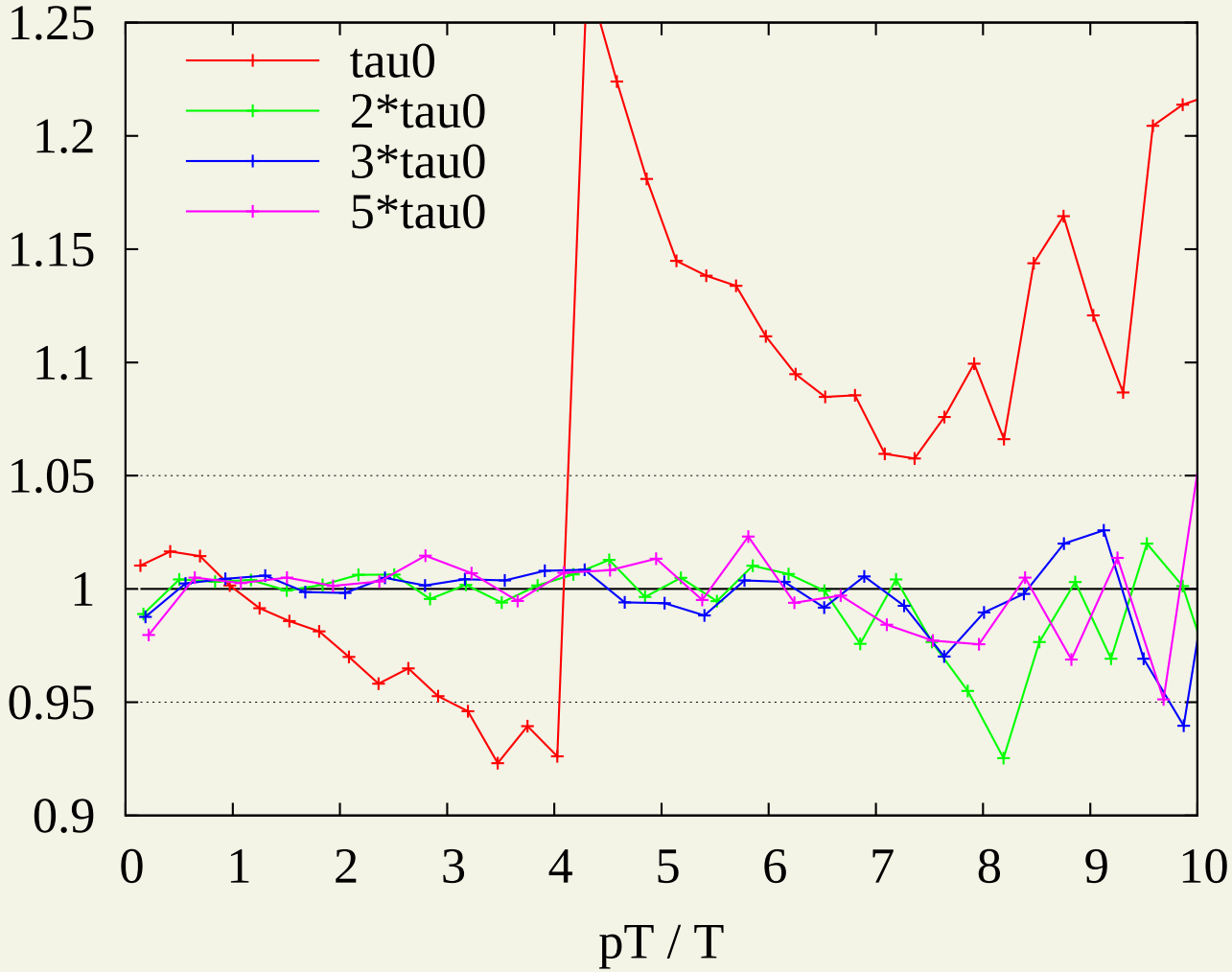
spectra in 0+1D Bjorken scenario at $\tau = 2\tau_0$ and $5\tau_0$, $\eta/s \approx const$, $K_0 = 3$
thermal + jet initconds, MOST OPTIMISTIC $\tau_0 = 0.1$ fm



Grad ansatz still looks good - on a log plot at least

ratio - transport spectra / Grad approximation

DM ('09):



initial deviations get washed out quickly

Summary

We can use covariant transport to study viscous hydrodynamics in full 3+1D. Causality & stability - no problem.

When Israel-Stewart hydrodynamics is justified, Grad's quadratic formula $\delta f \propto f_{eq}\pi^{\mu\nu}p_\mu p_\nu$ turns out to be surprisingly accurate to high $p_T/T \sim 6$. At least in a 0+1D Bjorken scenario and as long as pQCD power-law tails are a modest correction.

some more work ahead:

i) less optimistic (more realistic) jet + hydro initial conditions

ii) radial and elliptic flow (beyond 0+1D)

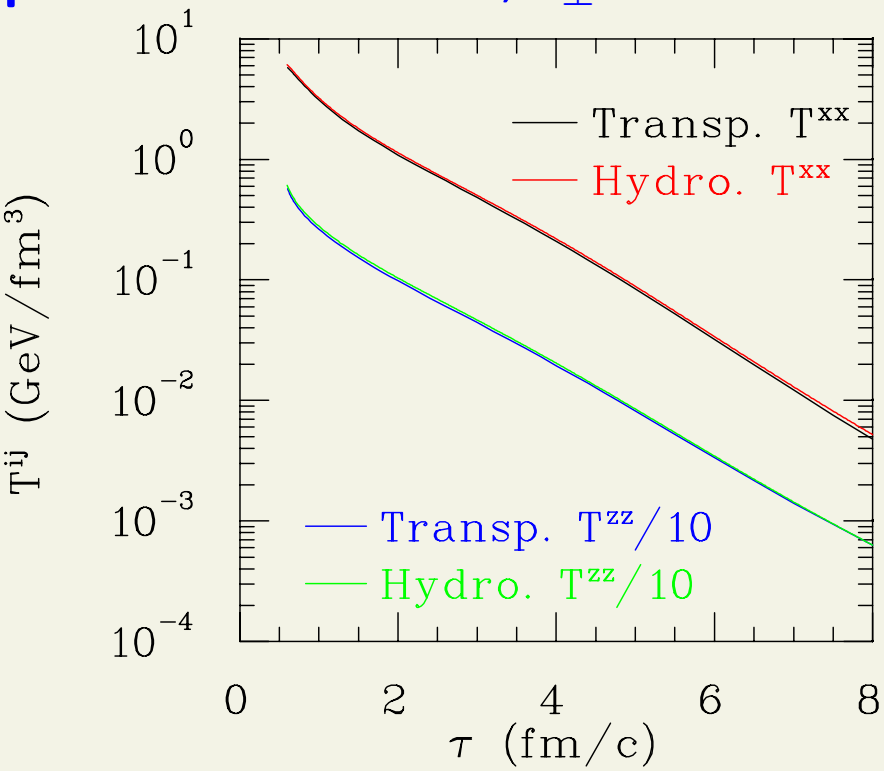
iii) mixtures

Not that bad in 2+1D either

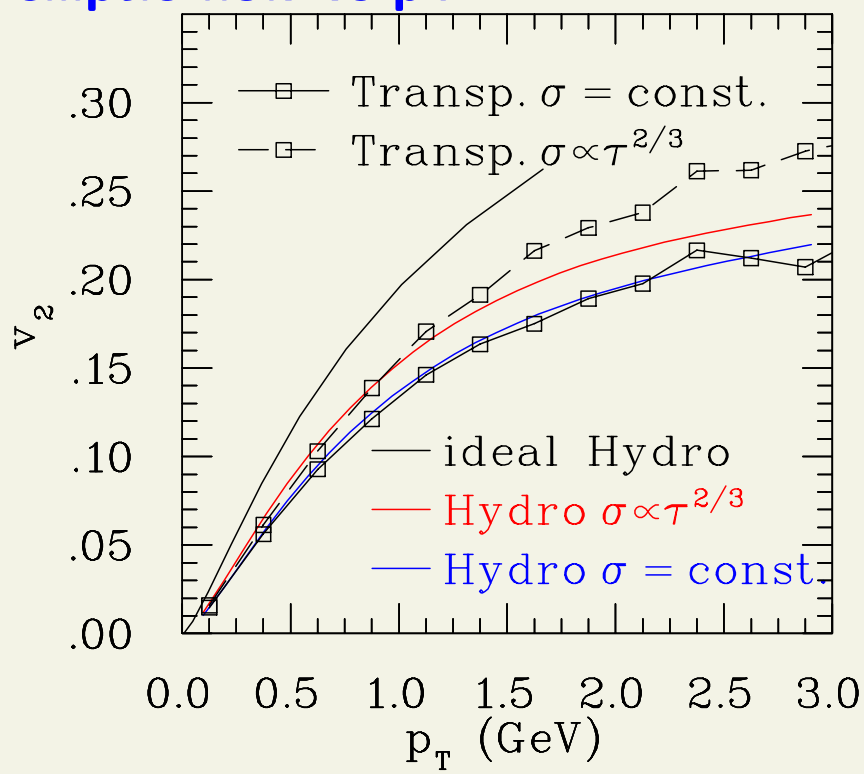
Au+Au at RHIC, $b = 8$ fm

Huovinen & DM ('08)

pressure in the core, $r_\perp < 1$ fm



elliptic flow vs pT



$\boldsymbol{\eta/s \approx 1/(4\pi)}$ $\quad (\sigma \propto \tau^{2/3})$

- $\sigma =$ **const** $\sim 47mb$
- $\eta/s \approx 1/(4\pi)$, **i.e.,** $\sigma \propto \tau^{2/3}$

$\Rightarrow$ **if** $\eta/s \approx 1/(4\pi)$, **or** $\sigma \sim 47mb$, **IS hydro is a good approximation**

and η/s **is close to** $\sim 1/4\pi$ **at RHIC**

Backup slides

Complete set of Israel-Stewart equations of motion

$$
\begin{aligned}
D\Pi &= -\frac{1}{\tau_\Pi}\left(\Pi + \zeta \nabla_\mu u^\mu\right) \qquad (3)\\
&\quad -\frac{1}{2}\Pi\left(\nabla_\mu u^\mu + D\ln\frac{\beta_0}{T}\right)\\
&\quad +\frac{\alpha_0}{\beta_0}\partial_\mu q^\mu - \frac{a_0'}{\beta_0} q^\mu D u_\mu\\
Dq^\mu &= -\frac{1}{\tau_q}\left[q^\mu + \kappa_q \frac{T^2 n}{\varepsilon + p}\nabla^\mu\left(\frac{\mu}{T}\right)\right] - u^\mu q_\nu D u^\nu \qquad (4)\\
&\quad -\frac{1}{2}q^\mu\left(\nabla_\lambda u^\lambda + D\ln\frac{\beta_1}{T}\right) - \omega^{\mu\lambda} q_\lambda\\
&\quad -\frac{\alpha_0}{\beta_1}\nabla^\mu\Pi + \frac{\alpha_1}{\beta_1}(\partial_\lambda \pi^{\lambda\mu} + u^\mu \pi^{\lambda\nu}\partial_\lambda u_\nu) + \frac{a_0}{\beta_1}\Pi D u^\mu - \frac{a_1}{\beta_1}\pi^{\lambda\mu} D u_\lambda\\
D\pi^{\mu\nu} &= -\frac{1}{\tau_\pi}\left(\pi^{\mu\nu} - 2\eta\nabla^{\langle\mu} u^{\nu\rangle}\right) - (\pi^{\lambda\mu}u^\nu + \pi^{\lambda\nu}u^\mu) D u_\lambda \qquad (5)\\
&\quad -\frac{1}{2}\pi^{\mu\nu}\left(\nabla_\lambda u^\lambda + D\ln\frac{\beta_2}{T}\right) - 2\pi_\lambda{}^{\langle\mu}\omega^{\nu\rangle\lambda}\\
&\quad -\frac{\alpha_1}{\beta_2}\nabla^{\langle\mu} q^{\nu\rangle} + \frac{a_1'}{\beta_2} q^{\langle\mu} D u^{\nu\rangle} \ .
\end{aligned}
$$

where $A^{\langle\mu\nu\rangle} \equiv \frac{1}{2}\Delta^{\mu\alpha}\Delta^{\nu\beta}(A_{\alpha\beta}+A_{\beta\alpha}) - \frac{1}{3}\Delta^{\mu\nu}\Delta_{\alpha\beta}A^{\alpha\beta}$, $\quad \omega^{\mu\nu} \equiv \frac{1}{2}\Delta^{\mu\alpha}\Delta^{\nu\beta}(\partial_\beta u_\alpha - \partial_\alpha u_\beta)$