Long Range Rapidity Correlations

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K.D., Francois Gelis, Tuomas Lappi, Raju Venugopalan
arXiv:0911.2720
Color Glass Condensate

- Parton of size $\frac{1}{k_{\perp}^2}$ has cross section $\sigma \sim \alpha_s \frac{1}{k_{\perp}^2}$
- For a hadron of size $S_{\perp}$ geometric overlap occurs when $\frac{dN}{dy} \sigma \sim S_{\perp}$
- Saturation momentum: $k_{\perp,\text{max}}^2 \equiv Q_S^2 \sim \frac{\alpha_s}{S_{\perp}} \frac{dN}{dy}$
Glasma Flux Tubes

- Before

\[ N_{\text{f.t.}} \sim \alpha_s \frac{dN}{dy} \]
\[ \sim 300 \text{ in Au-Au} \]

These flux tubes generate long range 2 particle correlations.

\[ \tau_f = \tau_{\text{f.o.}} \exp \left( -\frac{1}{2} \Delta y \right) \]
Previous Calculations

- Two and three particle correlations in glasma flux tube model yield a geometric interpretation of correlation strength

\[ \overline{C}_2 \sim \frac{\kappa_2}{S_{\perp}Q_s^2} \quad \overline{C}_3 \sim \frac{\kappa_3}{(S_{\perp}Q_s^2)^2} \]

Dumitru, Gelis, McLerran, Venugopalan  
NPA, aX:0804.3858  
\( \kappa_2 \) computed in: Lappi, Srednyak, Venugopalan, aX:0911.2069

KD, D. Fernandez-Fraile, R. Venugopalan  
NPA, aX:0902.4435

- Generalized to \( \overline{C}_N \)  
Gelis, Lappi, McLerran, NPA, aX:0905.3234

- Collimation of signal into the near-side ridge generated by transverse flow  
George Moschelli (NEXT TALK), Sean Gavin

However, all these works have no rapidity dependence.  
Goal of this work: address rapidity dependence.
Multi-particle production at leading log order

- Leading log calculation $\sum_n \alpha_s^n \ln^n \left( \frac{1}{x} \right)$

- Inclusive observables can be expressed in factorized form

$$\langle \mathcal{O} \rangle_{\text{LLog}} = \int \left[ D\Omega_1(\vec{y}, \mathbf{x}_\perp) D\Omega_2(\vec{y}, \mathbf{x}_\perp) \right] W[\Omega_1(\vec{y}, \mathbf{x}_\perp)] W[\Omega_2(\vec{y}, \mathbf{x}_\perp)] \mathcal{O}_{\text{LO}}$$

$$\Omega_{1,2}(\vec{y}, \mathbf{x}_\perp) \equiv \text{P} \exp i g \int_{x_y^\pm}^{x_\pm} \frac{dz^\mp}{\sqrt{z_\perp^2}} \rho_{1,2}(z^\mp, \mathbf{x}_\perp)$$

F. Gelis, T. Lappi and R. Venugopalan,

*High energy factorization and long range rapidity correlations in the Glasma I, II, III*

*PRD, aX:0810.4829 [hep-ph]*
Double Inclusive Spectra

- Factorized form for double inclusive spectra

\[
\left\langle \frac{dN_n}{d^2p_{\perp,1}dy_1 \cdots d^2p_{\perp,n}dy_n} \right\rangle = \int \left[ d\rho_A d\rho_B W_{Y_{\text{beam}}-Y[\rho_A]} W_{Y_{\text{beam}}+Y[\rho_B]} \right] \\
\times \frac{dN_{LO}}{d^2p_{\perp,1}dy_1}(\rho_A, \rho_B) \cdots \frac{dN_{LO}}{d^2p_{\perp,n}dy_n}(\rho_A, \rho_B)
\]
Quantum Evolution

\[
\left\langle \frac{d^2 N}{d^2 p_{\perp,1} dy_1 d^2 p_{\perp,2} dy_2} \right\rangle = \int [d\rho_A d\rho_B]\frac{dN_{LO}}{d^2 p_{\perp,1} dy_1} \frac{dN_{LO}}{d^2 p_{\perp,n} dy_n} W_{y_{beam} - y_1}^A W_{y_{beam} + y_2}^B \frac{dN_{LO}}{d^2 p_{\perp,1} dy_1} \frac{dN_{LO}}{d^2 p_{\perp,n} dy_n}
\]

$$\Delta y \ll 1/\alpha_S$$

\[
\left\langle \frac{d^2 N}{d^2 p_{\perp,1} dy_1 d^2 p_{\perp,2} dy_2} \right\rangle = \int [d\rho_A d\rho_B d\rho_A d\rho_B]\frac{dN_{LO}}{d^2 p_{\perp,1} dy_1} \frac{dN_{LO}}{d^2 p_{\perp,n} dy_n} W_{y_{beam} - y_1}^A W_{y_{beam} + y_2}^B G_{y_1, y_2} G_{y_2, y_1} \frac{dN_{LO}}{d^2 p_{\perp,1} dy_1} \frac{dN_{LO}}{d^2 p_{\perp,n} dy_n}
\]
Leading Order form

- Leading order single inclusive spectra with fixed sources

\[
\frac{dN_1[\rho_1, \rho_2]}{d^2p_\perp dy_p} |_{LO} = \frac{1}{16\pi^3} \lim_{x_0,y_0 \to +\infty} \int d^3x \, d^3y \, e^{ip \cdot (x-y)} (\partial_x^0 - iE_p)(\partial_y^0 + iE_p) \times \sum_{\lambda,a} \epsilon_\lambda^\mu(p) \epsilon_\lambda^\nu(p) A_\mu^a(x)[\rho_1, \rho_2] A_\nu^a(y)[\rho_1, \rho_2].
\]

- at large $k_T$ use perturbative solution to YM eqn.

\[
p^2 A_a^\mu(p) = -if_{abc} g^3 \int \frac{d^2k_\perp}{(2\pi)^2} L^\mu(p, k_\perp) \tilde{\rho}_1^b(k_\perp) \tilde{\rho}_2^c(p_\perp - k_\perp) \frac{k_\perp^2 (p_\perp - k_\perp)^2}{k_\perp^2 (p_\perp - k_\perp)^2}.
\]

- with non-local source correlations

\[
\langle \tilde{\rho}^a(k_\perp) \tilde{\rho}^b(k'_\perp) \rangle = (2\pi)^2 \mu_A^2(y) \delta^{ab} \delta(k_\perp - k'_\perp).
\]
Final Result

- Color sources related to UGD

\[ \phi_A(y, k_\perp) = g^2 \pi (\pi R_A^2) (N_c^2 - 1) \frac{\mu_A^2(y, k_\perp)}{k_{\perp}^2} \]

- Final result is

\[
C(p, q) = \frac{\alpha_s^2}{16\pi^{10}} \frac{N_c^2 (N_c^2 - 1) S_\perp}{d_A^4 p_\perp^2 q_\perp^2} \int d^2 k_{1\perp} \times \\
\left\{ \Phi_{A_1}^2(y_p, k_{1\perp}) \Phi_{A_2}(y_p, p_\perp - k_{1\perp}) \left[ \Phi_{A_2}(y_q, q_\perp + k_{1\perp}) + \Phi_{A_2}(y_q, q_\perp - k_{1\perp}) \right] \right. \\
\left. + \Phi_{A_2}^2(y_q, k_{1\perp}) \Phi_{A_1}(y_p, p_\perp - k_{1\perp}) \left[ \Phi_{A_1}(y_q, q_\perp + k_{1\perp}) + \Phi_{A_1}(y_q, q_\perp - k_{1\perp}) \right] \right\}
\]

- and probes both target and projectile

\[
x_{1p} = \frac{p_\perp}{\sqrt{s}} e^{-y_p} \ ; \ x_{1q} = \frac{q_\perp}{\sqrt{s}} e^{-y_q} \\
x_{2p} = \frac{p_\perp}{\sqrt{s}} e^{+y_p} \ ; \ x_{2q} = \frac{q_\perp}{\sqrt{s}} e^{+y_q}
\]
Nuclear wave-function & evolution

- large $N_c$: adj. UGD written in terms of fund. Wilson lines
  \[
  \Phi_{A_1,2}(x, k_{\perp}) = \frac{\pi N_c k_{\perp}^2}{2 \alpha_s} \int_0^{+\infty} r_{\perp} dr_{\perp} J_0(k_{\perp} r_{\perp}) \left[ 1 - T_{A_1,2}(r_{\perp}, \ln(1/x)) \right]^2
  \]

- Dipole scattering amplitude given from BK equation
  \[
  \frac{\partial T(r, Y)}{\partial Y} = \int dr_1 \mathcal{K}_{\text{LO}}(r, r_1, r_2) \times \\
  \left[ T(r_1, Y) + T(r_2, Y) - T(r, Y) - T(r_1, Y) T(r_2, Y) \right]
  \]

- Leading order Kernal
  \[
  \mathcal{K}_{\text{LO}}(r, r_1, r_2) = \frac{\alpha_s N_c}{2\pi^2} \frac{r^2}{r_1^2 r_2^2}
  \]
Initial Condition

\[ Q_s^2 = c A^{1/3} Q^2 \]

Initial condition consistent with DIS data at \( x \approx 0.01 \)
Evolution

- NLO BK: use “Balitsky prescription”

\[ K_{\text{Bal.}}(r, r_1, r_2) = \frac{\alpha_s(r) N_c}{\pi} \left[ \frac{r^2}{r_1^2 r_2^2} + \frac{1}{r_1^2} \left( \frac{\alpha_s(r_1^2)}{\alpha_s(r_2^2)} - 1 \right) + \frac{1}{r_2^2} \left( \frac{\alpha_s(r_2^2)}{\alpha_s(r_1^2)} - 1 \right) \right] \]


Initial and evolution of dipole-nucleus scattering amplitude consistent with measured nuclear DIS data.
Results for PHOBOS

LRC seen to the kinematic extent of RHIC
LRC up to 10 units in rapidity at LHC.
7 units probe the small x evolution of the nucleon wave-function.
Predictions for LHC II

LRC up to 6 units in rapidity at LHC.
5 units probe the small x evolution of the nucleon wave-function.
Three particle correlation flat in rapidity

- Three particle correlation flat in rapidity
- The same formalism could be used to calculate \( \sigma(\text{Ridge}) \)
Conclusions

- Computed two particle correlation at arbitrary rapidity separations

- Glasma Flux Tube explanation of ridge is becoming Quantitative
  - Have quantitative agreement with rapidity dependence at STAR and PHOBOS

- LRC at the LHC can probe the high energy evolution of the nuclear wave-function
Backup
Rapidity Dependence of Flow

\[
\frac{1}{N_{\text{trig.}}} \frac{dN}{d\Delta \eta} \approx \nu_{ps}^{\text{assoc.}} \cdot F(\Delta \phi_{pq} = 0) \frac{C(p_{\perp}^{\text{trig.}}, p_{\perp}^{\text{assoc.}}, y_{\text{trig.}}, y_{\text{assoc.}} = y_{\text{trig.}} + \Delta \eta, \Delta \phi_{pq} = 0)}{dN_1(p_{\perp}^{\text{trig.}}, y_{\text{trig.}})}
\]

- From a crude fit to BRAHMS data
  \[
  F(\Delta \phi_{pq} = 0) = \cosh(\tanh^{-1} \beta)
  \]
  \[
  \beta(\eta) = 0.72 - 0.04|\eta|
  \]
Scale Dependence

\[ \frac{1}{N_{\text{trig}}} \frac{dN}{d\Delta \eta} \]

- Au+Au 0-30% (PHOBOS)
- p+p (PYTHIA)
- \( p_T^{\text{trig}} = 2.5 \text{ GeV} \)
- \( p_T^{\text{assoc}} = 350 \text{ MeV} \)
- \( y_{\text{trig}} = 0 \)
- \( y_{\text{trig}} = 0.75 \)
- \( y_{\text{trig}} = 1.5 \)
\[ F_2^A(x, Q^2) = \frac{Q^2}{4\pi^2\alpha_{em}} (\sigma_T^A + \sigma_L^A) \]

\[ \sigma_{T,L}^A(x, Q^2) = \int_0^1 dz \int d^2b \ d^2r \ | P \psi_{T,L}(z, Q^2, r) |^2 N_A(b, r, x) \]

\[ N_A(b, r, x) = 2 \mathcal{T}_A(b) N_A(r, x) \quad \sigma_A = 2 \int d^2b \ \mathcal{T}_A(b) \]

\[ \sigma_{T,L}^A(x, Q^2) = \sigma_A \int_0^1 dz \int d^2r \ | \Psi_{T,L}(z, Q^2, r) |^2 N_A(r, x) \]
\[ W_{\Lambda'^+} \left[ \Omega_1'(y, x_\perp) \right] \equiv \left[ 1 + \ln \left( \frac{\Lambda'^+}{\Lambda'^+} \right) \mathcal{H}_{\Lambda'^+} \right] \text{d}y W_{\Lambda'^+} \left[ \Omega_1(y, x_\perp) \right] \]

\[ \langle O_{LO} + \underbrace{O_{NLO}}_{\Lambda'^+ < k^\pm < \Lambda^\pm} \rangle = \langle O_{LO} \rangle' \]