Sum rule approach for quarkonium spectral functions

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In collaboration with Kenji Morita

Update of OPE approach just after Tc

1. Mass, width
2. Observation?

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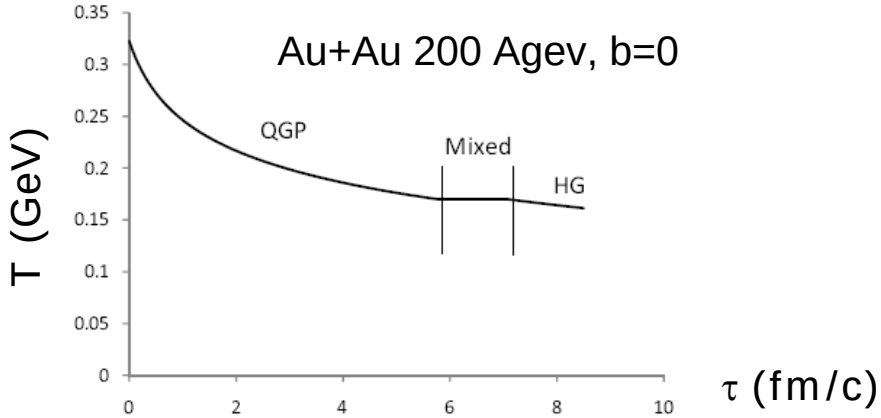
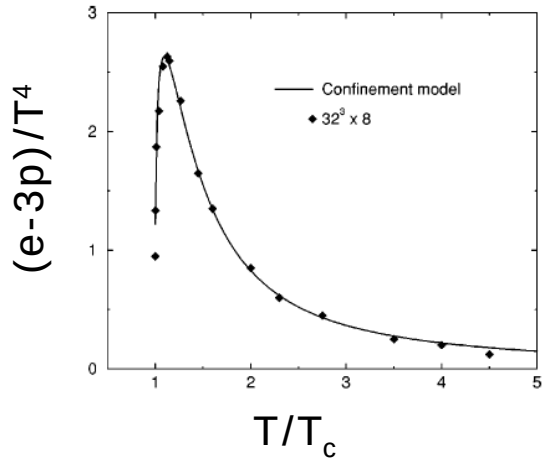
Few things to note about J/ψ near Tc

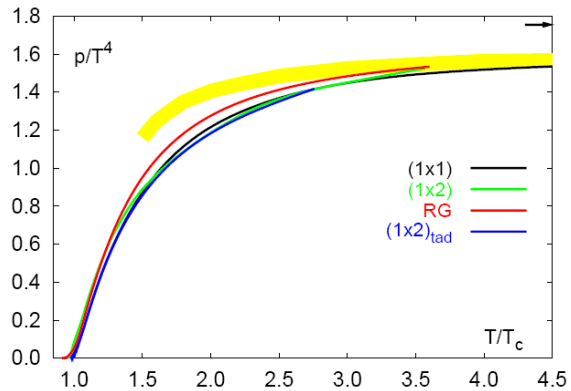
- Tc region is important in HIC



- Large non-perturbative change at Tc



- Resumed perturbation fails

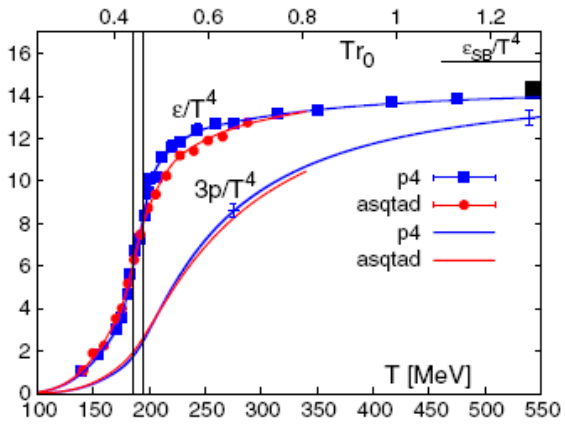


Karsch hep-lat/0106019

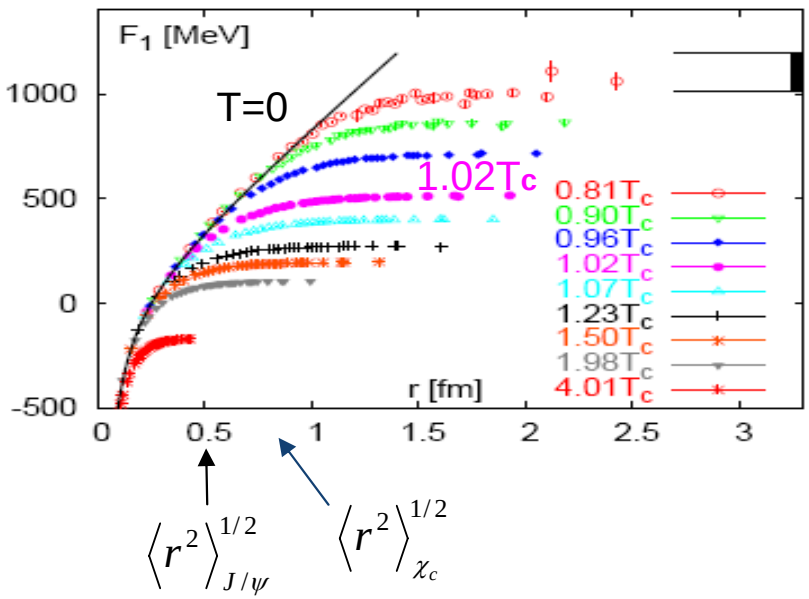
Lattice result near Tc relevant for J/ψ

- Energy density and pressure; Bazavov et al 09

  Local property change

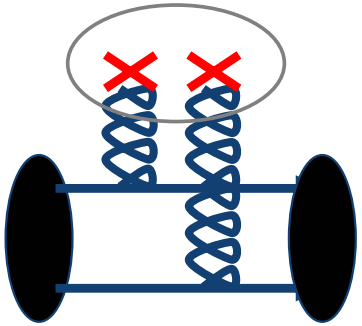


- Singlet potential: Kaczmarek,Zantow 05



Approach based on OPE (K. Morita and S. H. Lee)

- Vacuum



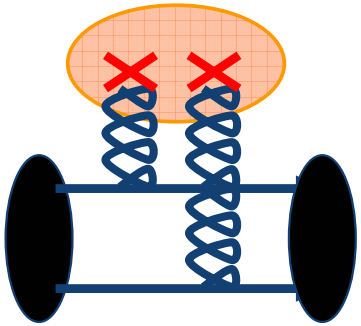
$$\left\langle \frac{\alpha}{\pi} G^2 \right\rangle, \quad \left\langle \frac{\alpha}{\pi} GDDG \right\rangle$$

- Medium corrections

+

Finite temperature

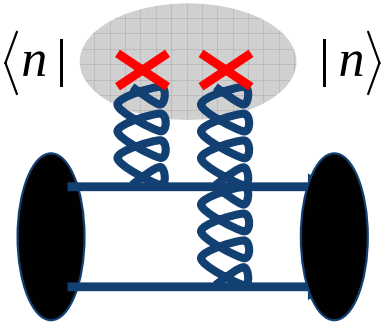
Lattice calculation



Finite density

Nucleon expectation value

$$\langle n | \quad | n \rangle \times \rho_n$$



Operators in medium

- Two independent operators

Gluon condensate $\left\langle \frac{\alpha}{\pi} G^2 \right\rangle = G_0$

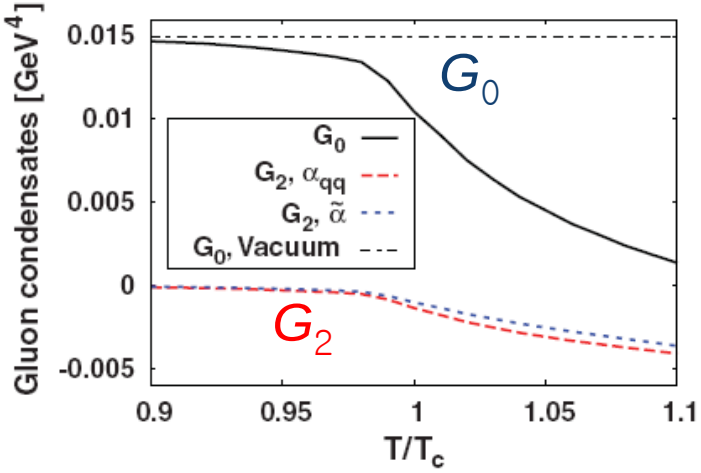
Twist-2 Gluon $\left\langle \frac{\alpha_s}{\pi} G^{\alpha\mu} G^{\beta\mu} \right\rangle = \left(u^\alpha u^\beta - \frac{1}{4} g^{\alpha\beta} \right) G_2$

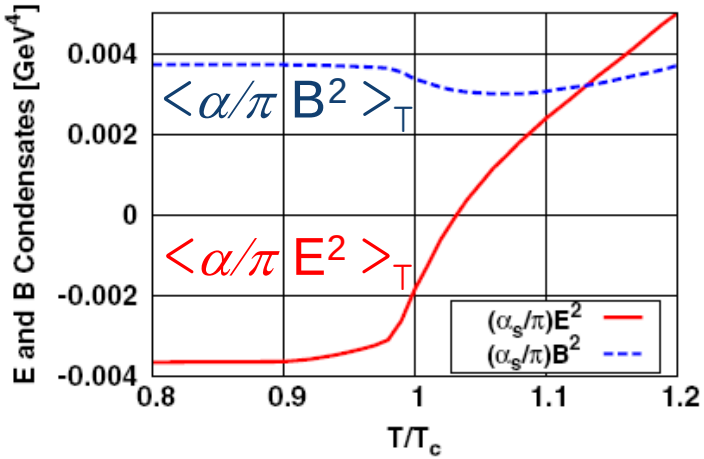
or

$\left\langle \frac{\alpha}{\pi} E^2 \right\rangle$

$\left\langle \frac{\alpha_s}{\pi} B^2 \right\rangle$

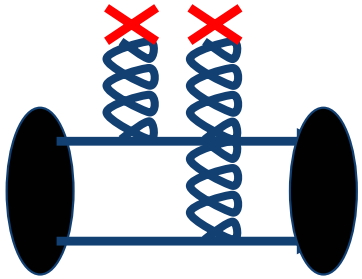
- At finite temperature: from lattice





J/ψ near T_c

QCD vacuum $$\left\langle \frac{\alpha}{\pi} G^2 \right\rangle_0 = 2\left\langle \frac{\alpha}{\pi} B^2 \right\rangle_0 = 2\left\langle \frac{\alpha}{\pi} E^2 \right\rangle_0 = (0.35\,\mathrm{GeV})^4$$



QCD at T_c $$\left\langle \frac{\alpha}{\pi} G^2 \right\rangle_{\mathrm{Tc}} = 0.75 \times \left\langle \frac{\alpha}{\pi} G^2 \right\rangle_0 \qquad \left\langle \frac{\alpha}{\pi} E^2 \right\rangle_{\mathrm{Tc}} = 0.5 \times \left\langle \frac{\alpha}{\pi} E^2 \right\rangle_0$$

$$\left\langle \frac{\alpha}{\pi} B^2 \right\rangle_{\mathrm{Tc}} = \left\langle \frac{\alpha}{\pi} B^2 \right\rangle_0$$

Approaches to Heavy quark system in medium

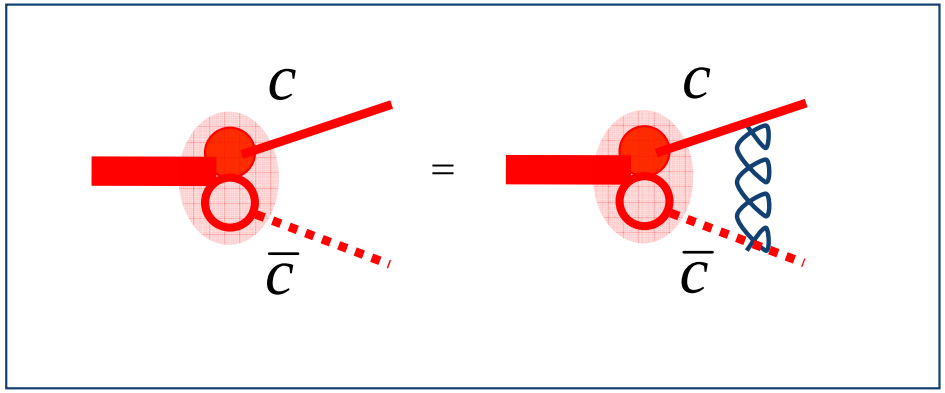
OPE, QCD Stark Effect, and

QCD sum rules

QCD 2nd order Stark Effect : $\varepsilon > \Lambda_{qcd}$

- OPE for bound state: m→ infinity

$$\varepsilon_0 = m\left(N_c g^2/16\pi\right)^2 \;\rightarrow\; O(mg^4), \qquad |\vec{k}| \;\rightarrow\; O(mg^2)$$



$$g^2 \frac{mg^4 (mg^2)^3}{(mg^4)(mg^4)(mg^2)^2} \;\rightarrow\; O(1)$$

- Attractive for ground state

$$\Delta M_i = \sum_n \frac{|\langle i|zE|n\rangle|^2}{E_i - E_n}$$

$$\Delta m_{J/\psi} = -\frac{128}{9\pi^2}\boxed{\frac{a_0^2}{\varepsilon_0}}\int dx \frac{x^{3/2}}{(1+x)^6}\frac{1}{x + a_0^2 \varepsilon m} \times \boxed{\left\langle \frac{\alpha}{\pi} E^2 \right\rangle_{\text{Medium}}}$$

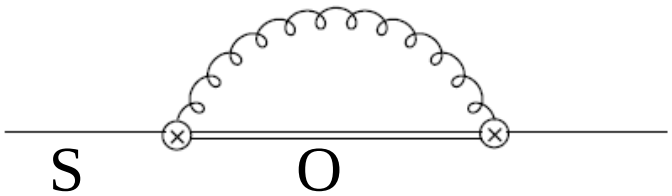
2nd order Stark effect from pNRQCD

- LO Singlet potential from pNRQCD : Brambilla et al.

1/r > Binding > Λ_{QCD},

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{a\,\mu\nu} + \sum_{i=1}^{n_f}\bar{q}_i\, i\not{D} q_i + \int d^3r\ \mathrm{Tr}\left\{\mathrm{S}^\dagger\left[i\partial_0 + C_F\frac{\alpha_{V_s}}{r}\right]\mathrm{S} + \mathrm{O}^\dagger\left[iD_0 - \frac{1}{2N_c}\frac{\alpha_{V_o}}{r}\right]\mathrm{O}\right\}$$
$$+V_A\,\mathrm{Tr}\left\{\mathrm{O}^\dagger\vec{r}\cdot g\vec{E}\,\mathrm{S} + \mathrm{S}^\dagger\vec{r}\cdot g\vec{E}\,\mathrm{O}\right\} + \frac{V_B}{2}\mathrm{Tr}\left\{\mathrm{O}^\dagger\vec{r}\cdot g\vec{E}\,\mathrm{O} + \mathrm{O}^\dagger\mathrm{O}\vec{r}\cdot g\vec{E}\right\} + \ldots\ . \qquad (55)$$

$$[\delta V_S(r)]_{11} = -ig^2\frac{T_F}{N_c}\frac{r^2}{d-1}\int_0^\infty dt\, e^{-it\Delta V}\ \left[\left\langle \vec{E}^a(t)\phi(t,0)_{ab}\vec{E}^b(0)\right\rangle_T\right]_{11},$$



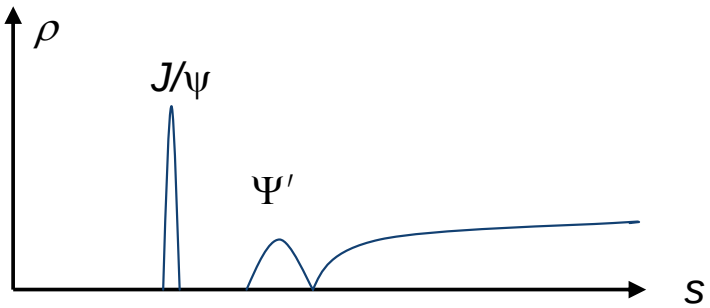
- Derivation
  - Take expectation value $\delta E_{J/\psi} = -ig^2\frac{T_F}{N_c}\frac{-i}{d-1}\int\frac{d^3p}{(2\pi)^3}\frac{r^2}{E_O - E_{J/\psi}}|\psi(p)|^2\left[\left\langle \vec{E}^a(t)\phi(t,0)_{ab}\vec{E}^b(0)\right\rangle_T\right]_{11}$
  - Large Nc limit $\Delta V = \frac{1}{r}\left(\frac{\alpha_{V_O}}{2N_c} + C_F\alpha_{V_S}\right) \approx \frac{N_c\alpha_s}{2r}$
  - Static condensate $\left[\left\langle \vec{E}^a(t)\phi(t,0)_{ab}\vec{E}^b(0)\right\rangle_T\right]_{11} \to \left[\left\langle \vec{E}^a(0)\vec{E}^a(0)\right\rangle_T\right]_{11}$
  - Energy
    $$E_{J/\psi} = 2m_c - \epsilon$$
    $$E_O = 2m_c + p^2/m_c,$$
  - → $= -\frac{1}{18}\int_0^\infty dk^2\left|\frac{\partial\psi(k)}{\partial k}\right|^2\frac{k}{k^2/m_h + \epsilon_0}\left\langle\frac{\alpha_s}{\pi}\Delta E^2\right\rangle_T$

$Q^2 = -q^2 > 0$, QCD sum rules for Heavy quark system

- sum rule at T=0 : can take any $Q^2 >= 0$, $4m^2 + Q^2 >> \langle G \rangle_{\text{vacuum}} = \Lambda^2_{QCD}$

$$M_n = \left(\frac{d}{dQ^2}\right)^n < J(Q), J(0) >= \int ds \frac{\rho(s)}{(s+Q^2)^n}$$

Phenomenological side

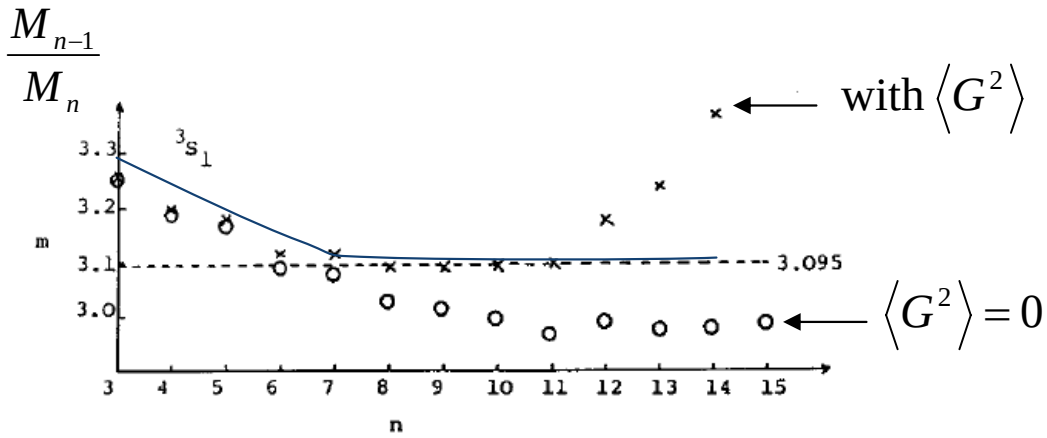


$$M_n = \frac{1}{\left(m^2_{J/\psi}\right)^n}\left(f_{J/\psi} + c\left(\frac{m^2_{J/\psi}}{m^2_{\psi'}}\right)^n ..\right)$$

$$\frac{M_{n-1}}{M_n} = m^2_{J/\psi} + \frac{\left(m^2_{\psi'} - m^2_{J/\psi}\right)}{f_{J/\psi}}\left(\frac{m^2_{J/\psi}}{m^2_{\psi'}}\right)^n$$

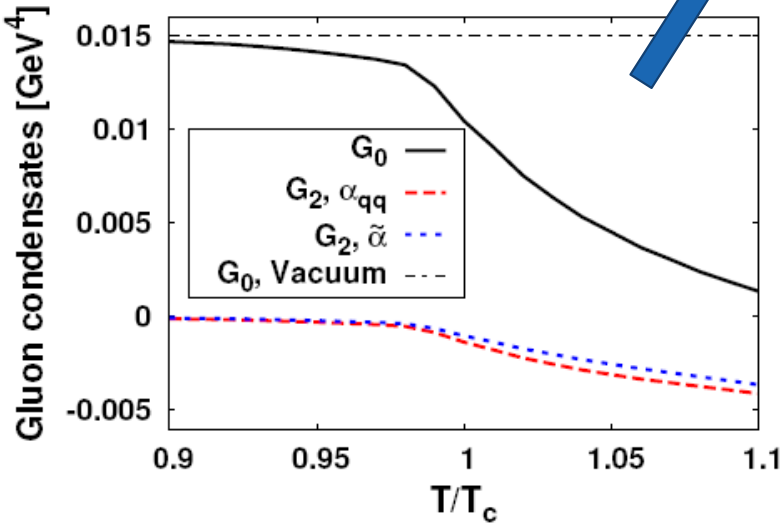
OPE

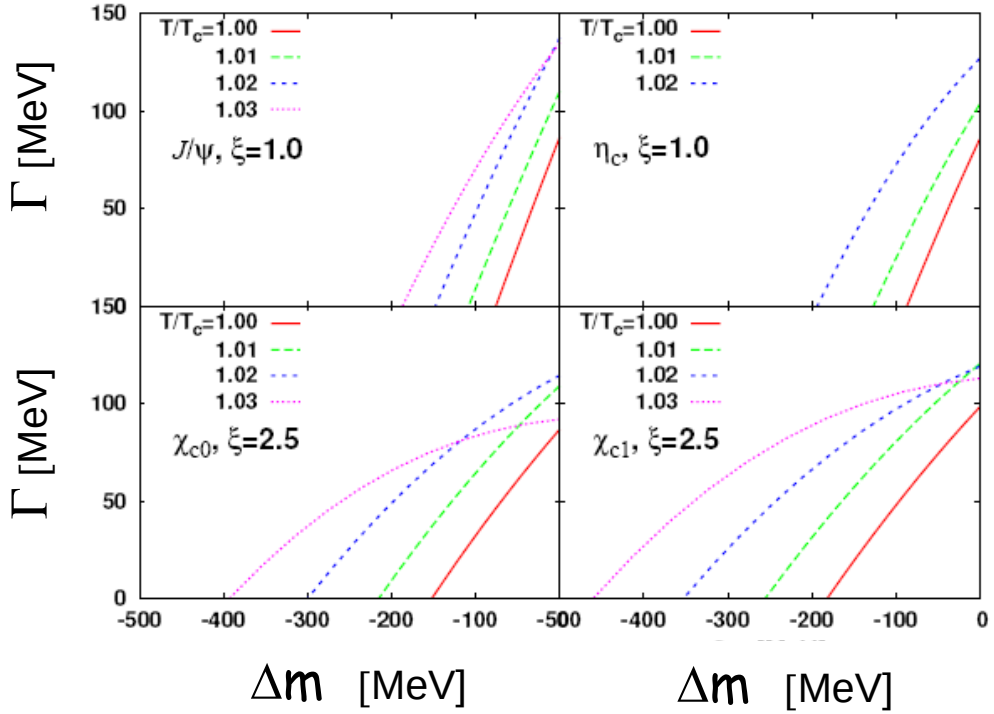
$$M_n = a_n\left(1 + \alpha + \frac{(n+4)!}{n!}\frac{\langle G^2 \rangle}{\left(4m_c^2\right)^2} ..\right)$$



QCD sum rule constraint (Morita, Lee 08)

$$\frac{1}{n!}\left(-\frac{d}{dQ^2}\right)^n \Pi(Q^2) = \int ds \frac{\rho(s)}{(s+Q^2)^{n+1}}$$

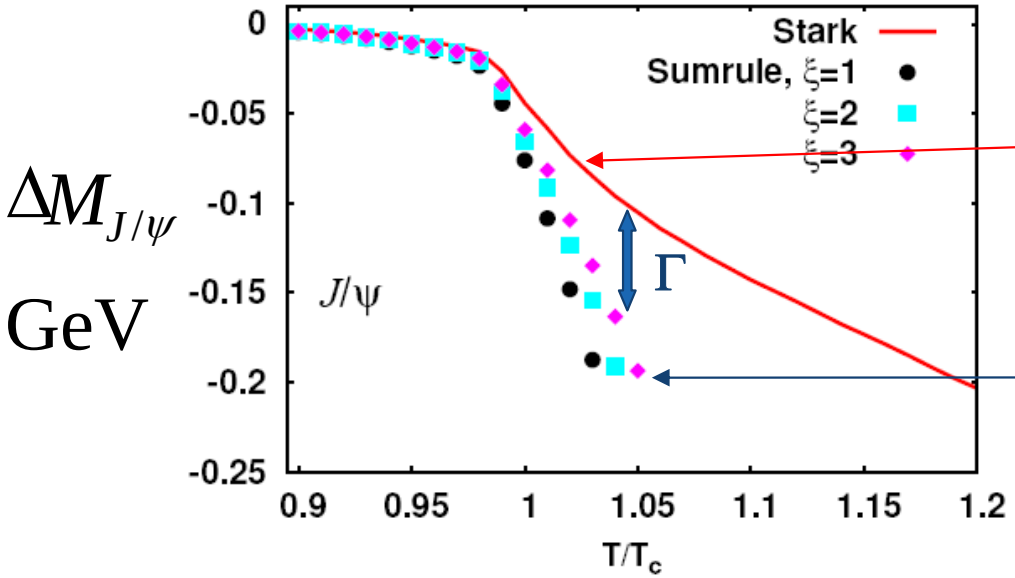




Can also use Borel sum rule

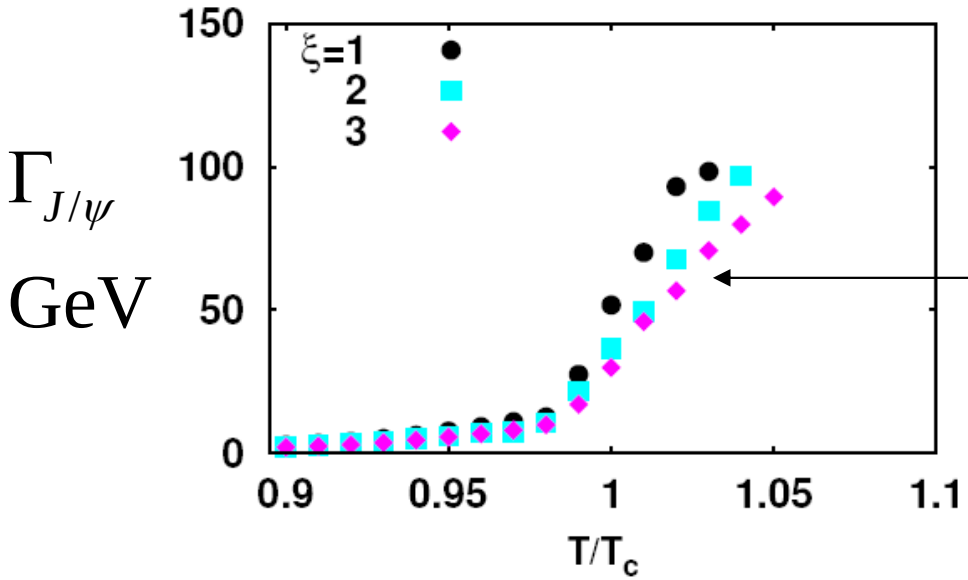
$$B(M) = \int ds \rho(s) \exp(-s/M^2)$$

Mass and width of J/ψ near Tc (Morita, Lee 08)



Δm from QCD Stark Effect

QCD sum rule limit with $\Delta\Gamma = 0$



Γ =constraint-Δm (Stark effect)

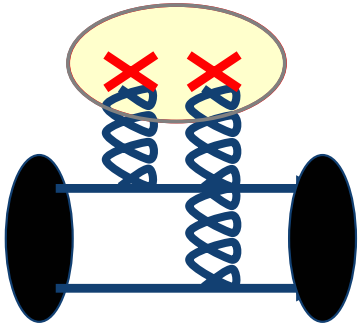
Γ in NLO QCD + confinement model

- Separation of scale in this approach

Non perturbative $\langle G^n \rangle$ $\boxed{+\Delta\langle G^n \rangle_T}$

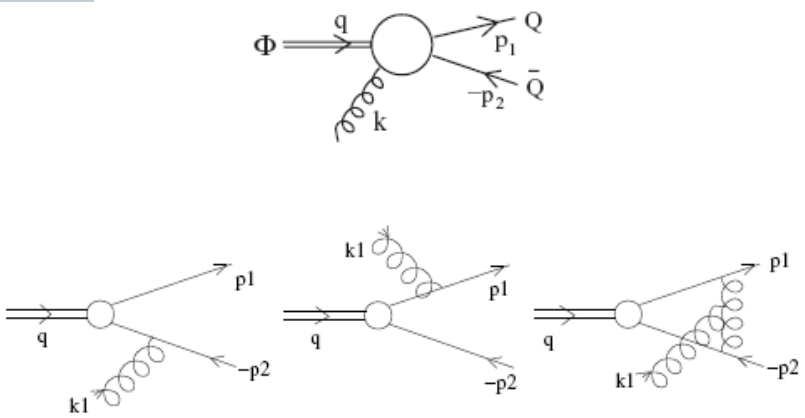
μ

perturbative

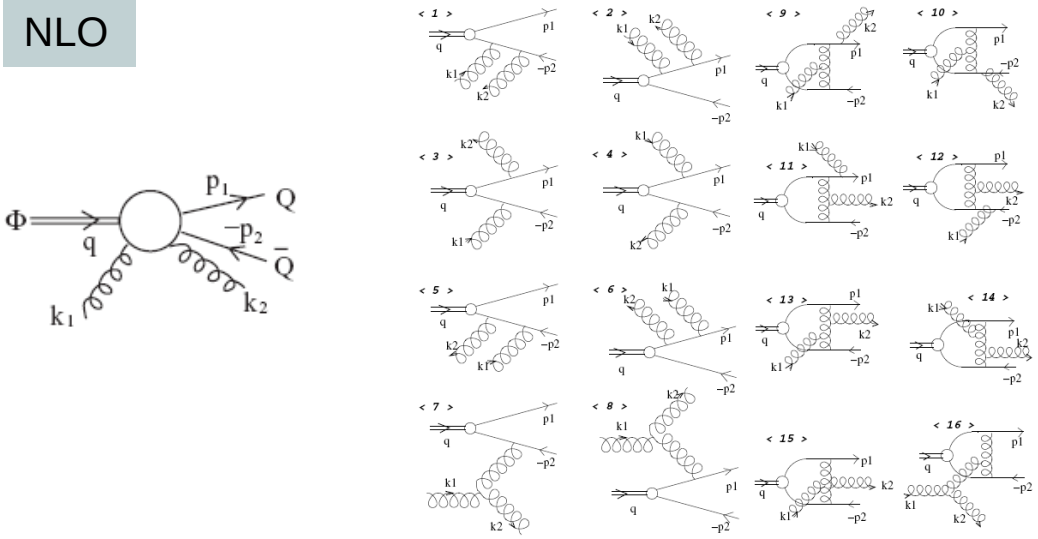


- LO (Peskin + others) and NLO (Song, Lee 07) perturbative Γ

LO



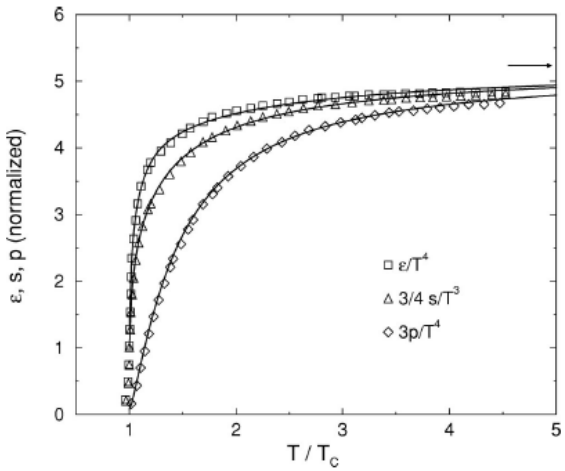
NLO



Schneider, Weise Parametrization

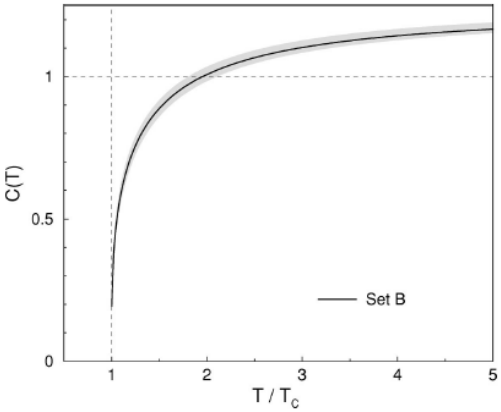
- $$p(T) = \frac{\nu_g}{6\pi^2}\int dk\left[C(T)\, f_B(E_k)\right]\frac{k^4}{E_k} - B(T)$$

  $$\varepsilon(T) = \frac{\nu_g}{2\pi^2}\int dk\left[C(T)\, f_B(E_k)\right]k^2 E_k + B(T)$$

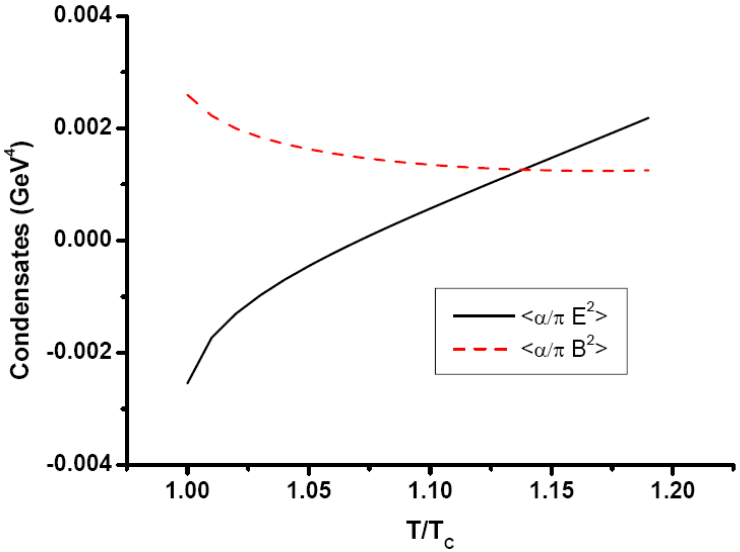


- $$C(T) = C_0\left(\left[1+\delta_c\right] - \frac{T_C}{T}\right)^{\beta_c}$$

  $$m_g(T) = G_0\left(\left[1+\delta\right] - \frac{T_C}{T}\right)^{\beta} T$$

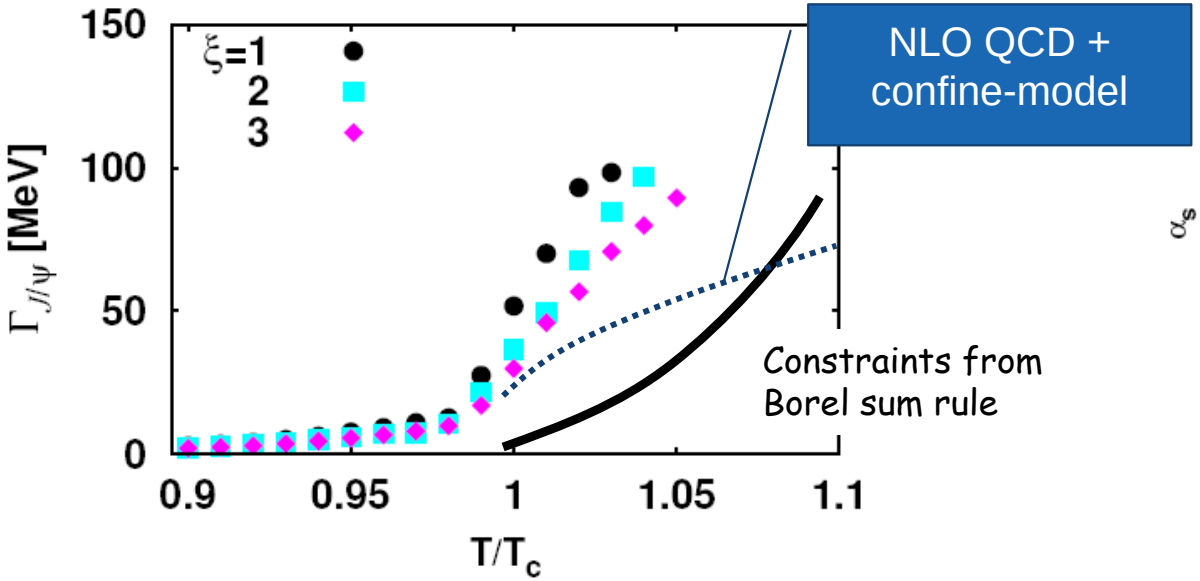


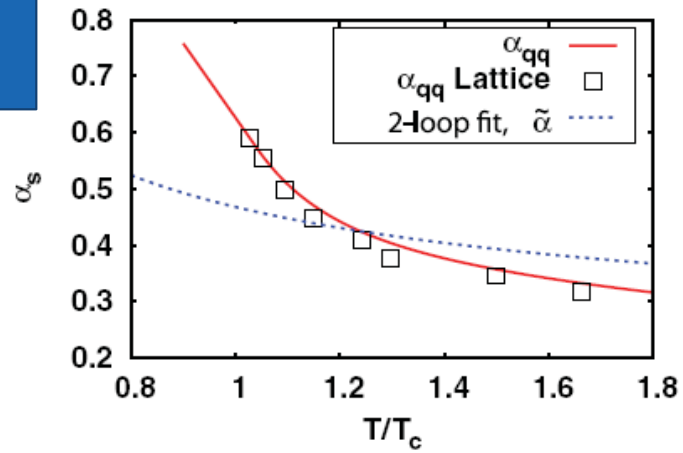
- E,B Condensate



Γ in NLO QCD + confinement model

- Non perturbative part at Tc: confinement model ($m_g(T)$ $C(T)$)





From Tc to 1.05 Tc mass and width seems to rapidly change by 50 MeV ; to probe higher temperature within this region,

1. For mass, need higher dimensional operators
2. For width, Need higher twist contribution

Two places to look

Reconstruction of Imaginary Correlator

and

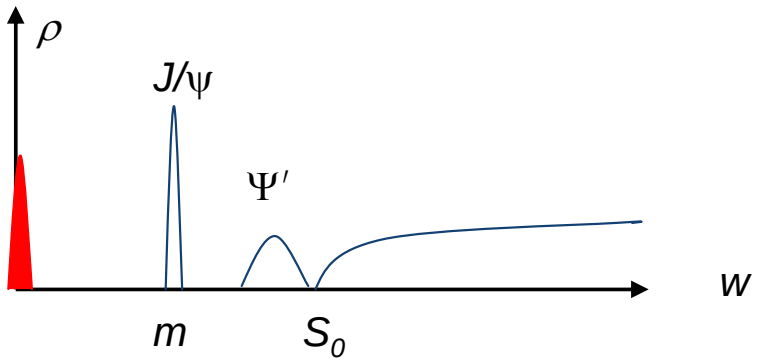
R_{AA} from RHIC

Reconstruction of Imaginary correlator

- Imaginary correlator

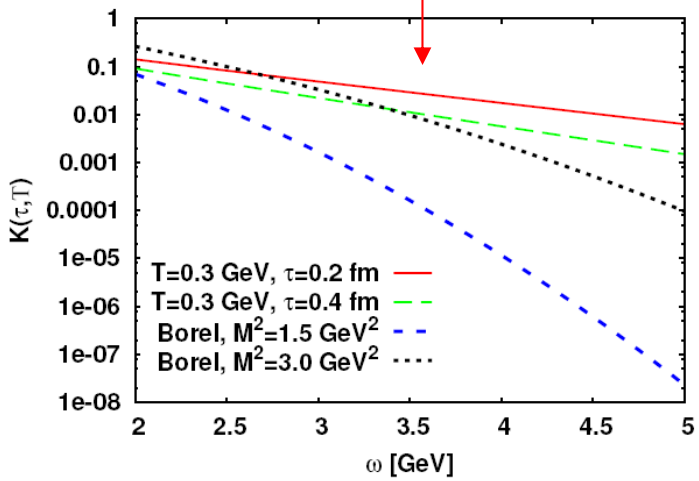
$$G(\tau,T)=\int dwK(w,\tau,T)\rho(w,T)$$

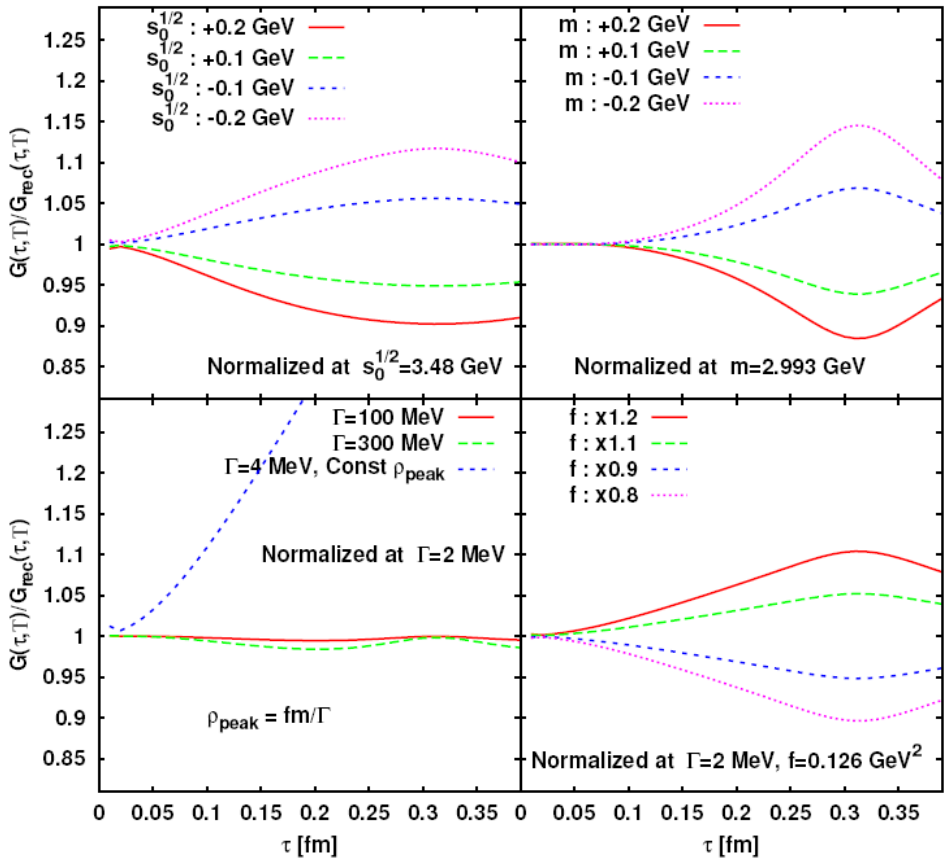
$$K(w,\tau,T)=\frac{\cosh[w(\tau-T/2)]}{\sinh(w/2T)}$$



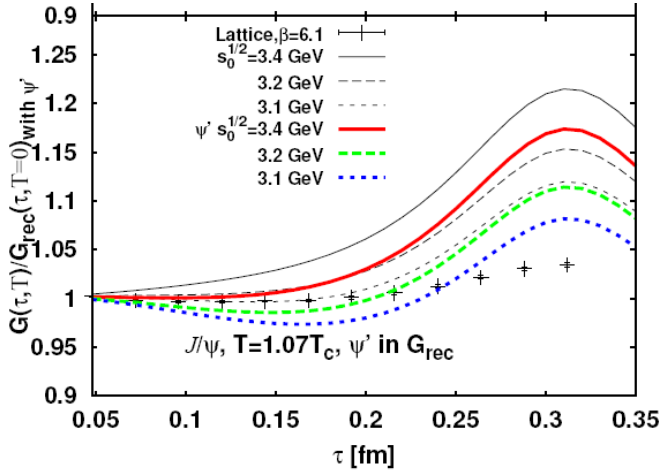
- Reconstructed correlator

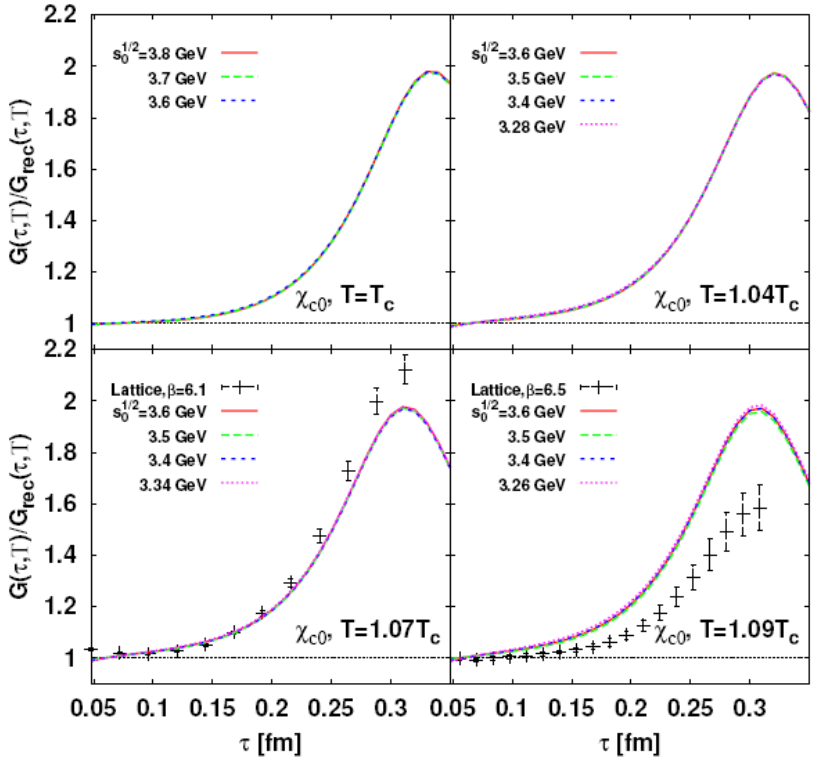
$$G_{rec}(\tau,T)=\int dwK(w,\tau,T)\rho(w,T=0)$$

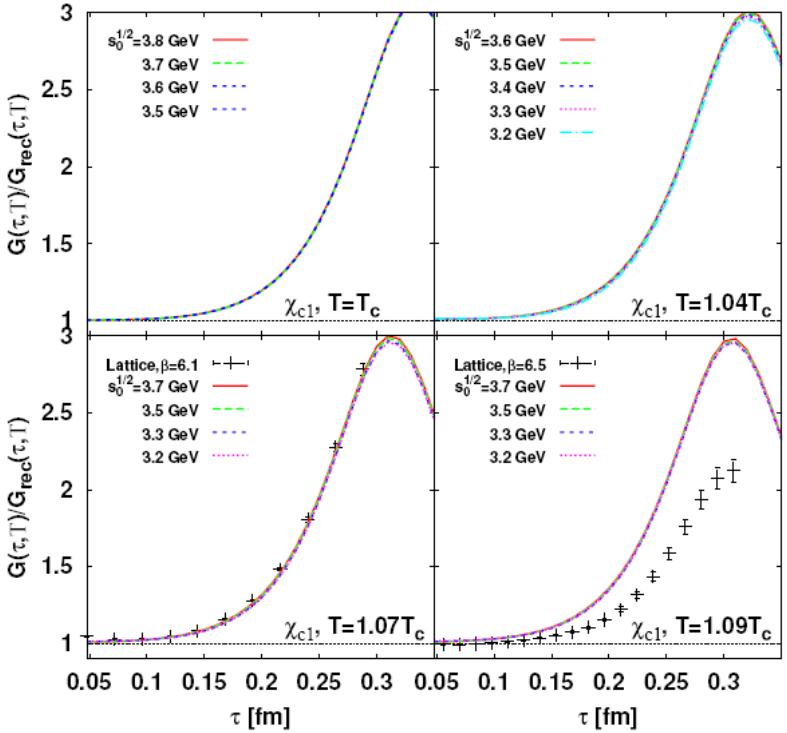


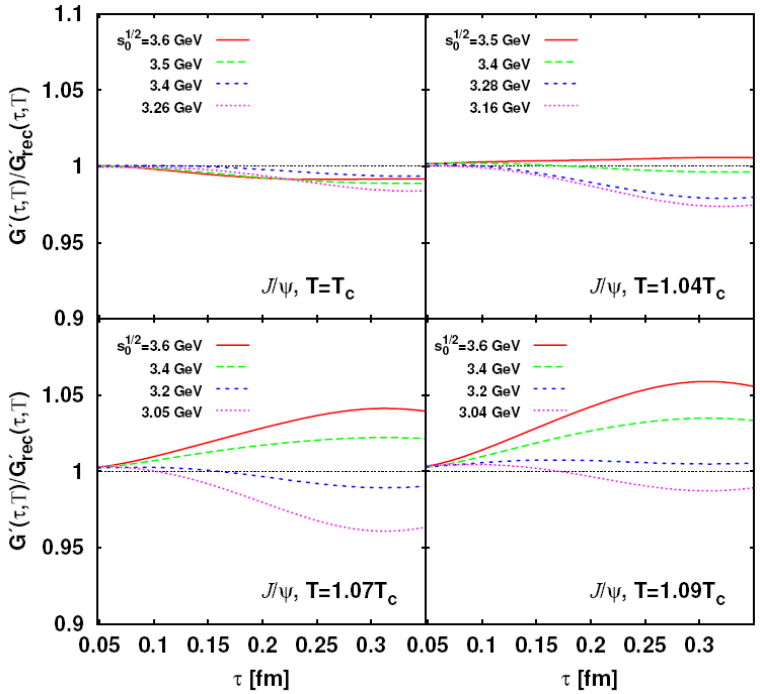


Morita, Lee (09) **arXiv:0908.2856**



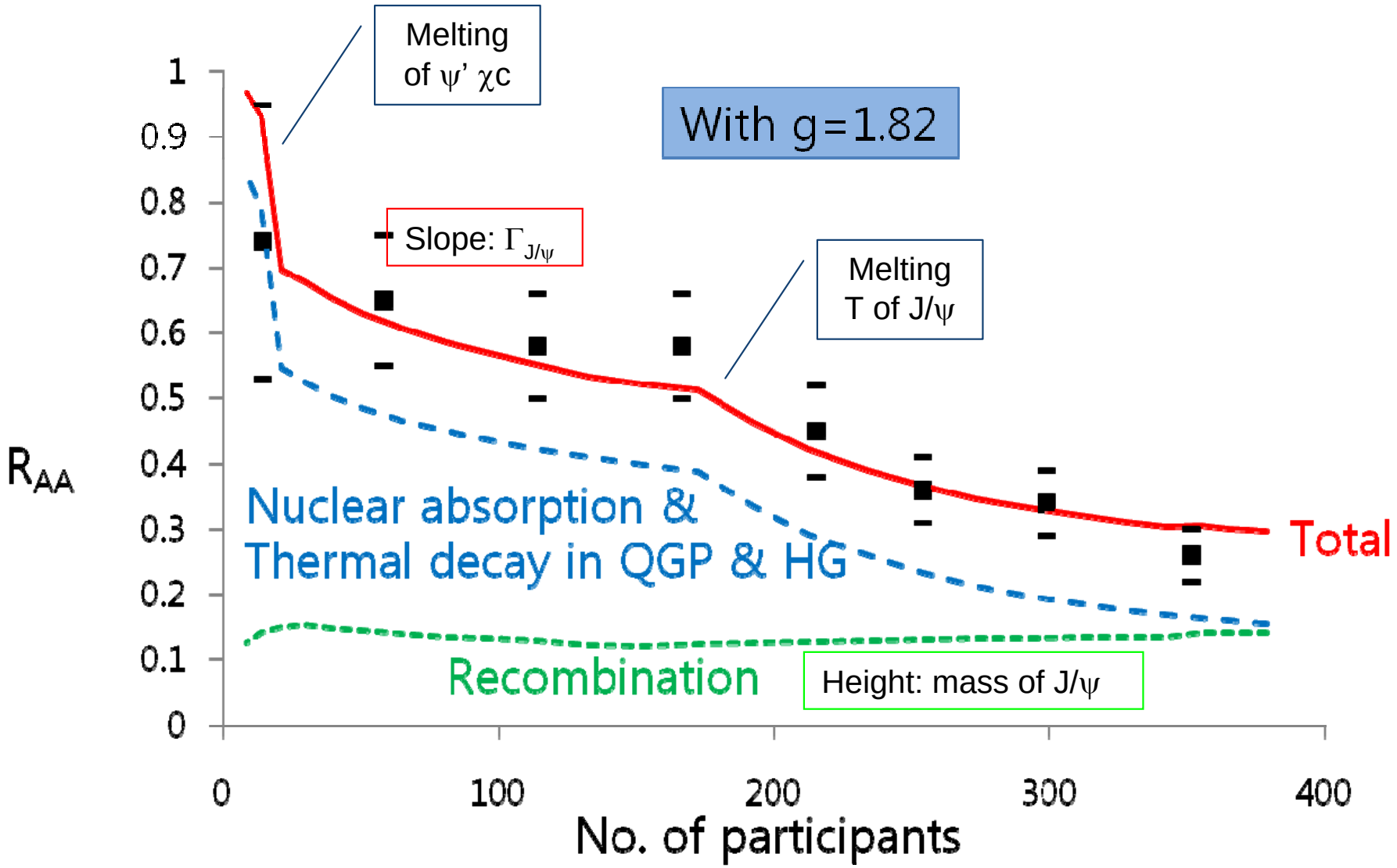






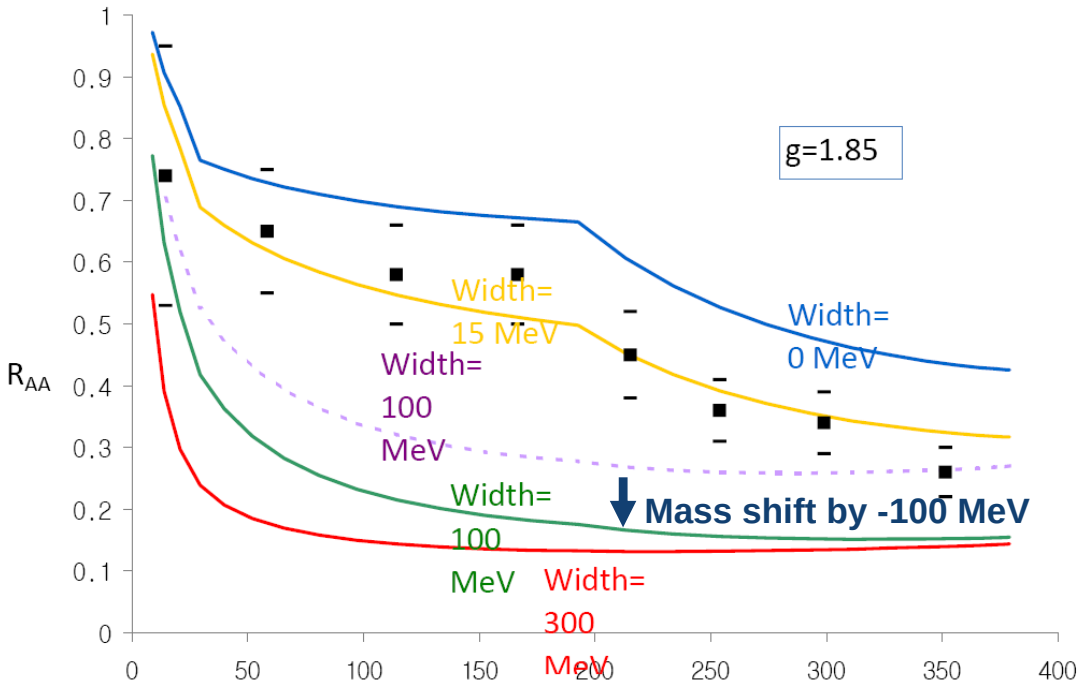
2-comp model (Rapp) Comparison with experimental data of RHIC ($\sqrt{s}$=200 GeV at midrapidity)

T. Song, SHLee (preliminary)

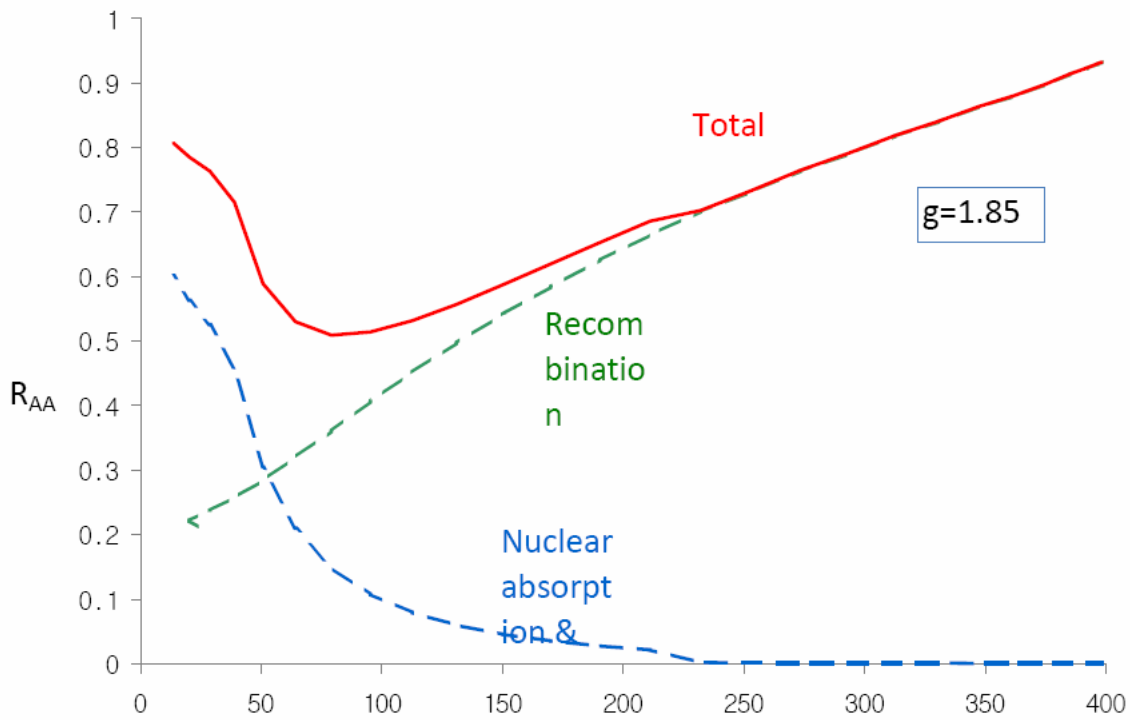


- Effects of width and mass

Assumed recombination effect to be the same



- At LHC



Summary

1. QCD phase transition is characterized by sudden changes in G_0, G_2 → sudden change in mass and width of quarkonium: Stark effect, QCD sum rules

2. Compatible/future comparison with lattice work using reconstructed correlator

3. Such effects affect R_{AA} of J/ψ → smaller uncertainties at RHIC → Looking forward to LHC