Quarkonium spectral functions in potential models (and beyond ...)

F. Riek, R. Rapp

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Cyclotron Institute
Texas A&M University

Outline

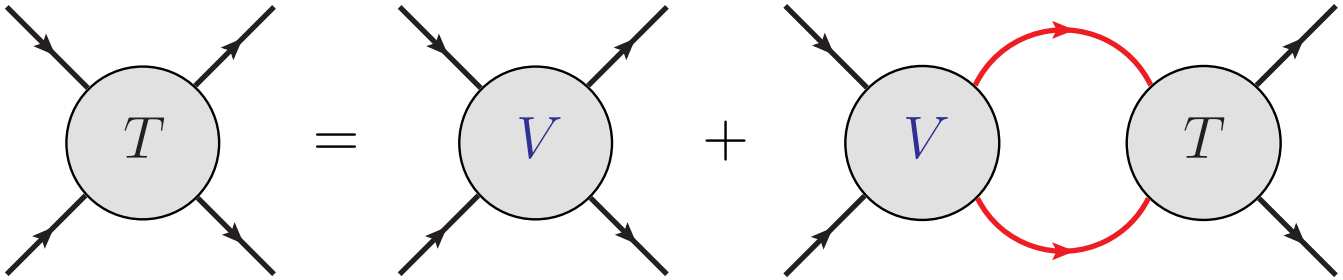
- T-Matrix calculation
- Microscopic model for the potential
- Results for $c\bar{c}$
- Heavy quark transport
- Conclusions / Outlook

Motivation

- Charmonium / bottomonium are probes for the QGP
  - Changes in the spectral functions
  - Transport observables
- Open and hidden charm / bottom in common approach
- Map out systematic uncertainties

T-Matrix formalism

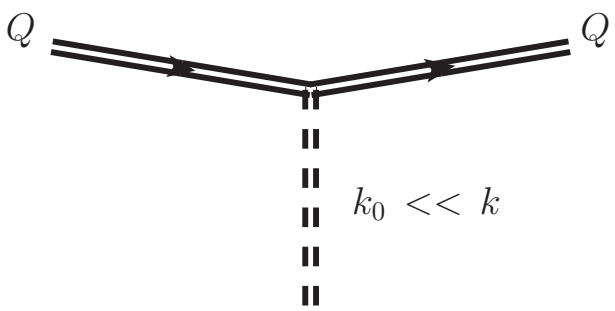
Start with the Bethe-Salpeter equation



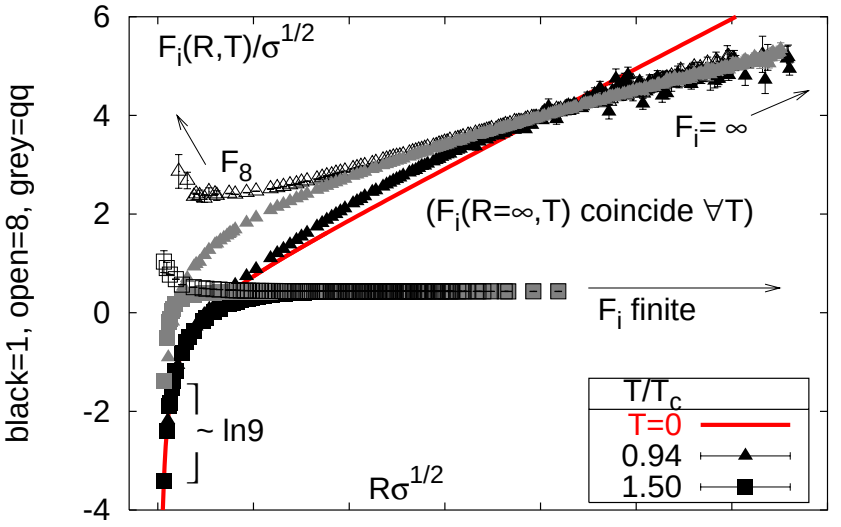
Reduction: $4d \rightarrow 3d$ (Different schemes: BbS, Th, ...)

$$T(E,\vec{q},\vec{k}) = V(\vec{q},\vec{k}) + \int \frac{d^3\vec{p}}{(2\pi)^3} V(\vec{q},\vec{p}) G(E,\vec{p}) T(E,\vec{p},\vec{k})$$

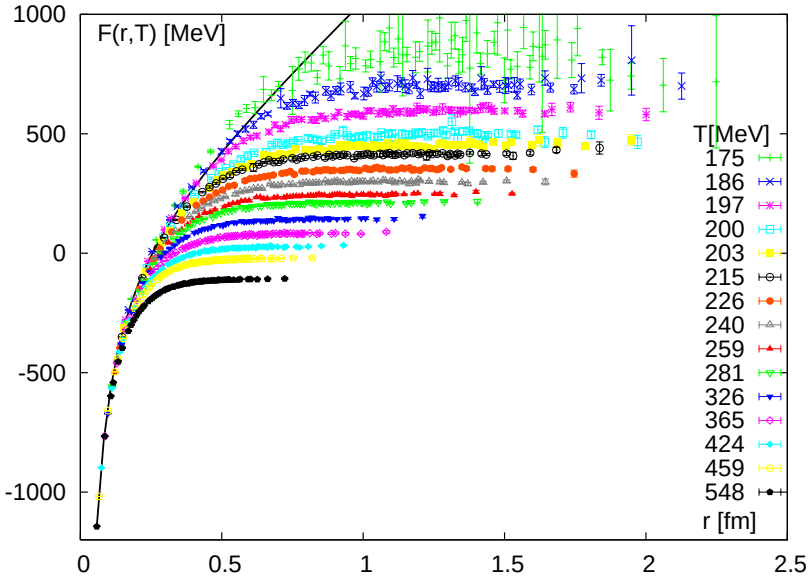
- Heavy quarks: $k_0 \ll k$
  $\rightarrow$ Potential
- One heavy quark determines kinematics
- Non-perturbative interactions
  $\rightarrow$ constrains from lQCD



Potential from Lattice QCD



Zantow et. al. [03]

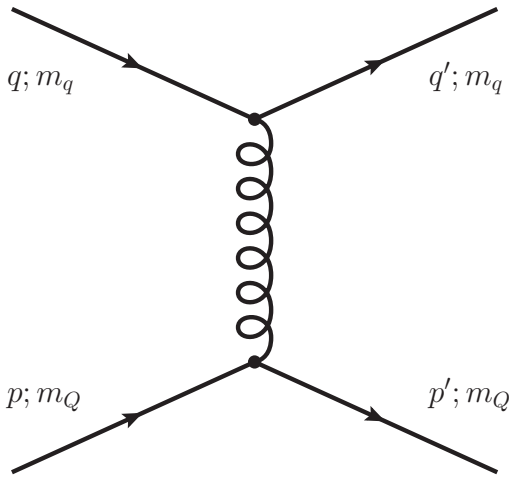


Kaczmarek et. al. [07]

- Potential has Coulomb and String part
- Casimir scaling at short distance → Coulomb part
- Suggests confining part color blind → String part
- Need to distinguish Coulomb and String part
- Need all color channels for transport calculation

Requires model for the potential!

Relativistic factors



$$V \sim \underbrace{\frac{4\pi\alpha}{t - m_D^2}}_{\text{Yukawa}} \underbrace{\bar{u}(q')\,\gamma^\mu\,u(q)\,\bar{u}(p')\,\gamma_\mu\,u(p)}_{\text{Spinor-contraction}}$$

Coulomb interaction:

$$|\bar{u}(q')\,\gamma^\mu\,u(q)\,\bar{u}(p')\,\gamma_\mu\,u(p)|^2 \sim \underbrace{\frac{\omega_q^2\,\omega_Q^2}{16\,m_q^2\,m_Q^2}}_{F_{kin}} \underbrace{(1 - \frac{q^2}{\omega_q\,\omega_Q})^2}_{F_{Breit}}$$

String term:

$$|\bar{u}(q')\;u(q)\,\bar{u}(p')\,u(p)|^2 \sim 4$$

Needed for consistency with high energy limit (pQCD)

Requires model for the potential!

Field theoretical Ansatz for the Potential

Megías, Arriola and Salcedo [05]

In perturbation theory:

$$e^{-F_1(r,T)/T} = e^{(g^2/(2N_c T^2))\,\langle A_{0,a}(x)A_{0,a}(y) - A^2_{0,a}(x)\rangle} + \cdots$$

Ansatz for propagator:

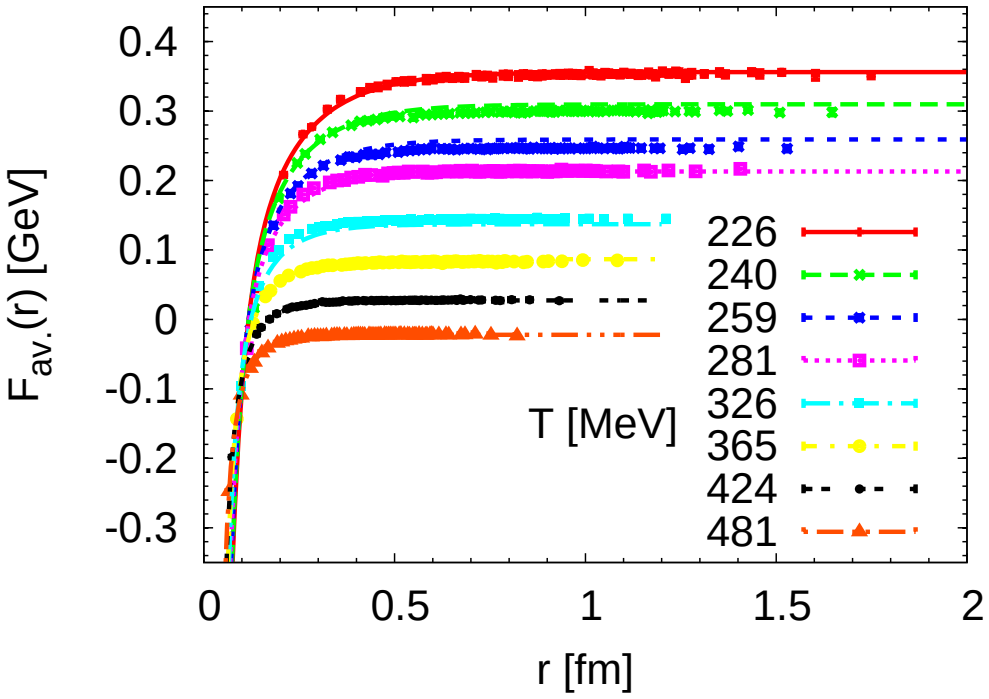
$$D_{00}(\vec{k}\,) = \frac{1}{k^2 + m_D^2} + \frac{m_G^2}{(k^2 + \tilde{m}_D^2)^2}$$

We arrive at:

$$\begin{aligned} F_a(r,T) &= V_a^C(r,T) + V_a^S(r,T) + 2\Sigma_Q(T) \\ &= -\frac{4}{3}\alpha\left(\frac{C_a}{r}e^{-m_D r} + \frac{m_G^2}{2\tilde{m}_D}e^{-\tilde{m}_D r}\right) + \frac{4}{3}\alpha\left(\frac{m_G^2}{2\tilde{m}_D} - m_D\right) \end{aligned}$$

Casimir scaling only for the Coulomb term!

Determination of the parameters



Parameters:
α_s, m_D, $\tilde{m}_D$, m_G

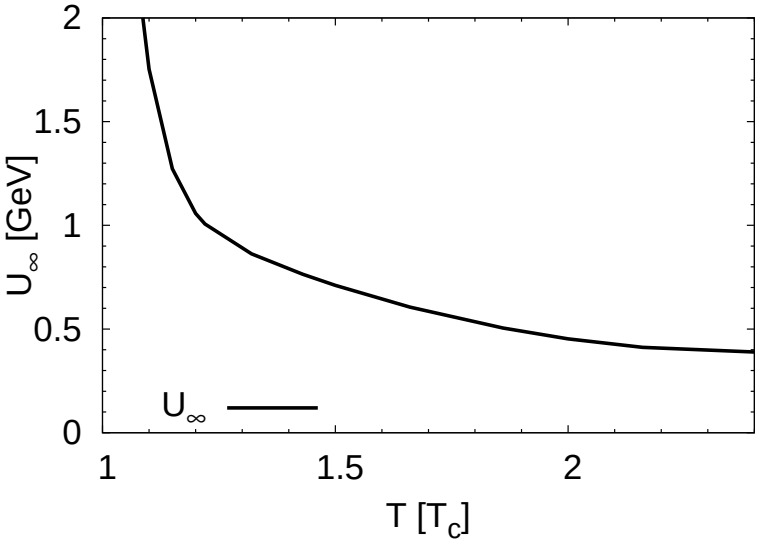
Kaczmarek et. al. [07]

- Fit color averaged free Energy $\rightarrow$ gauge invariant ($\rightarrow$ requires again model)
- Non-perturbative effects (string term) important at finite T

Potential

- What to use: F_1 or U_1?
- U has direct connection to $\langle H \rangle$: static limit: $U_1 = \langle H_{int.} \rangle$
- U does not include Entropy terms
- $\Rightarrow$ use U as potential
- Scheme more general

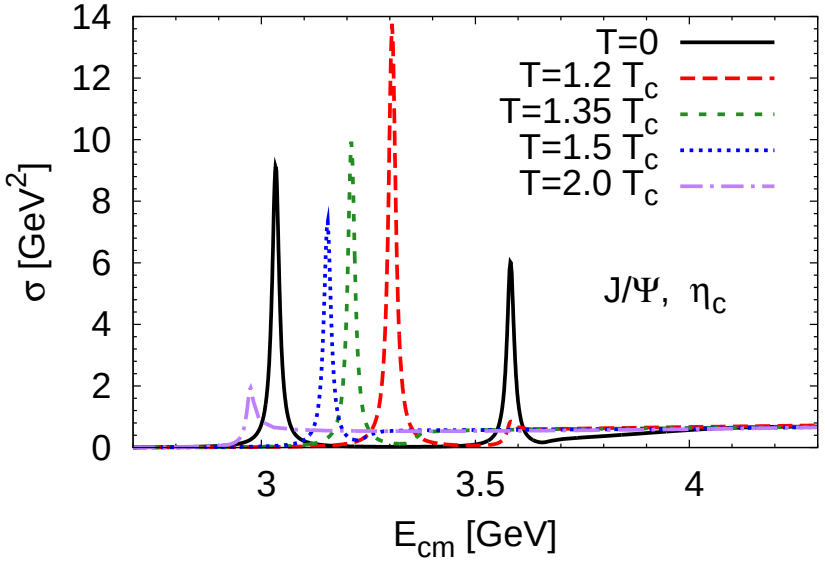
- Quark self energy: $m_{eff} = m_0 + \Sigma_Q(T) = m_0 + \frac{1}{2} U(\infty)$

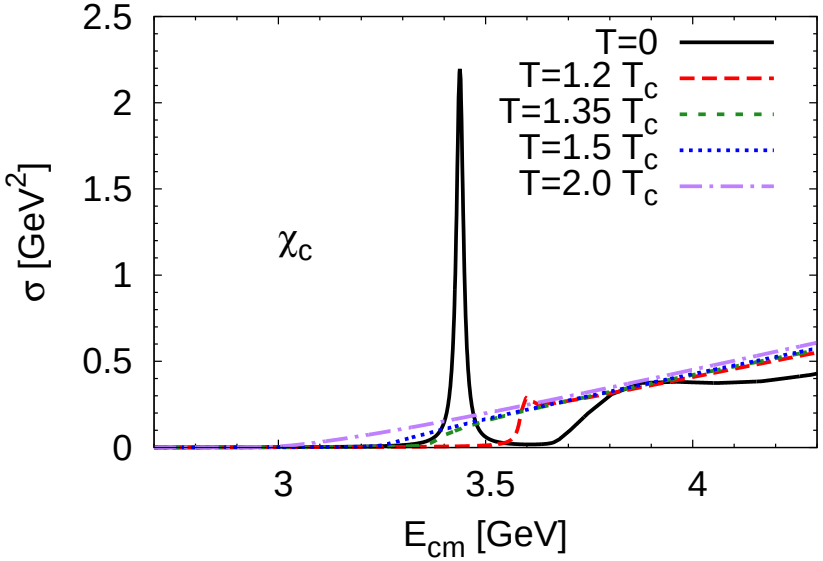


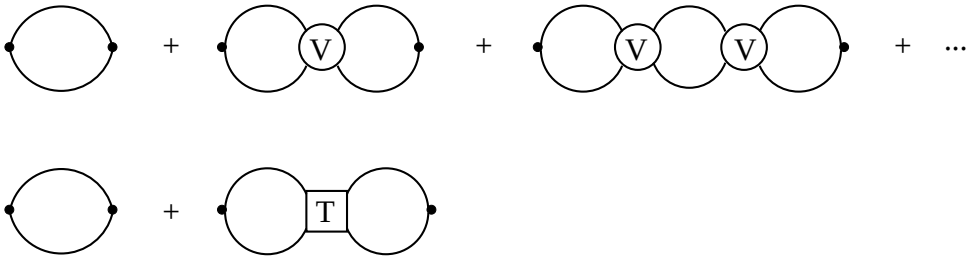
Main Features of the approach

- Non-perturbative method: resummation
  - Large coupling
  - Bound states
- Comprehensive treatment of scattering and bound states
- Microscopic Model to split Coulomb and string terms
  - Relativistic vertex structure
  - Color channels
- Hidden and open charm (bottom) on the same footing.
- Connect charmonium with transport properties (Coefficients in a Fokker-Planck equation)

Results for $c\bar{c}$







$$\sigma(\omega) = \frac{1}{\pi}\Im(\Gamma\, G\, \Gamma + \Gamma\, G\, T\, G\, \Gamma)$$

- Vacuum spectrum OK
- J/Ψ melts at $\sim 2T_c$
- Check with lattice correlators

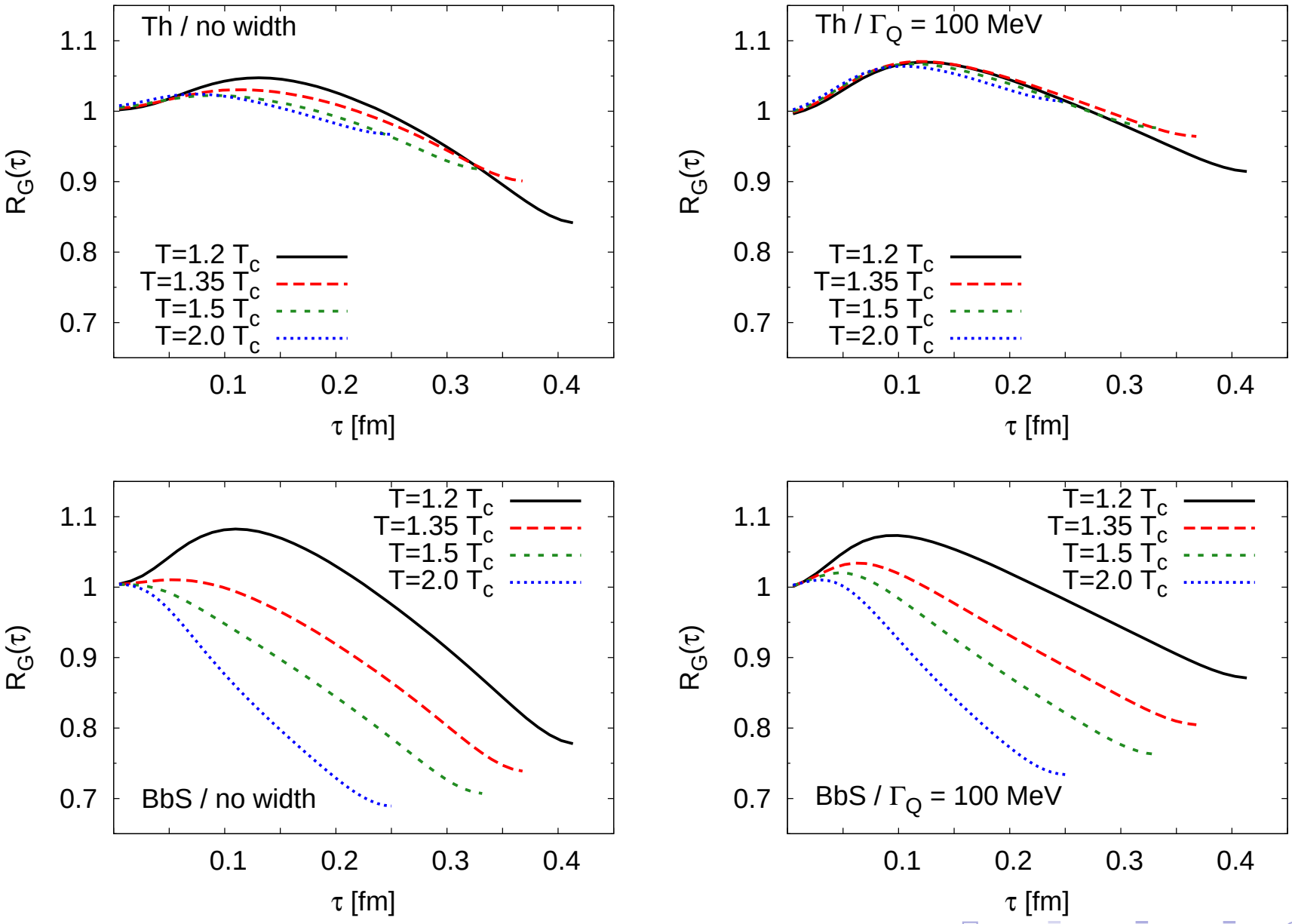
Check with lattice correlators

- Use Correlator ratios

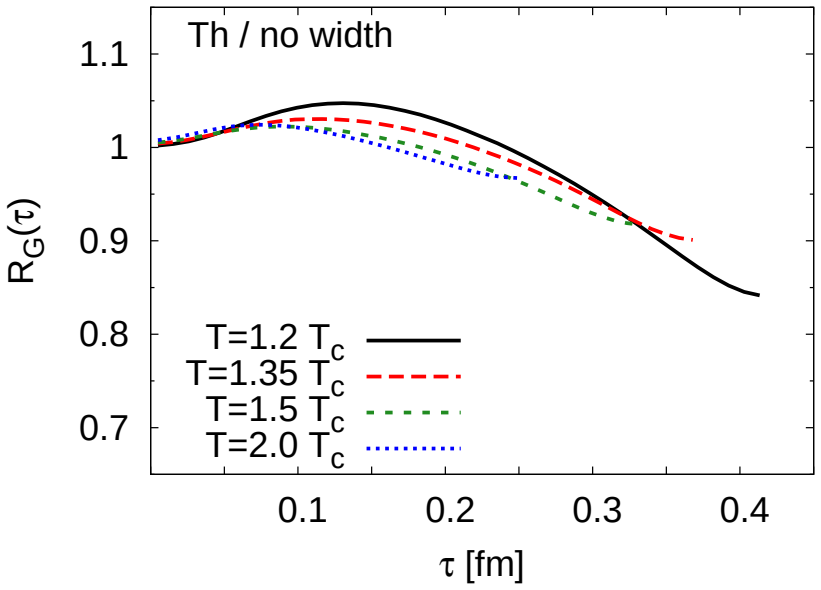
$$R_G(\tau, T) = \frac{\int \sigma(\omega, T)\, K(\omega, \tau, T)\, d\omega}{\int \sigma(\omega, T_{rec.})\, K(\omega, \tau, T)\, d\omega}$$

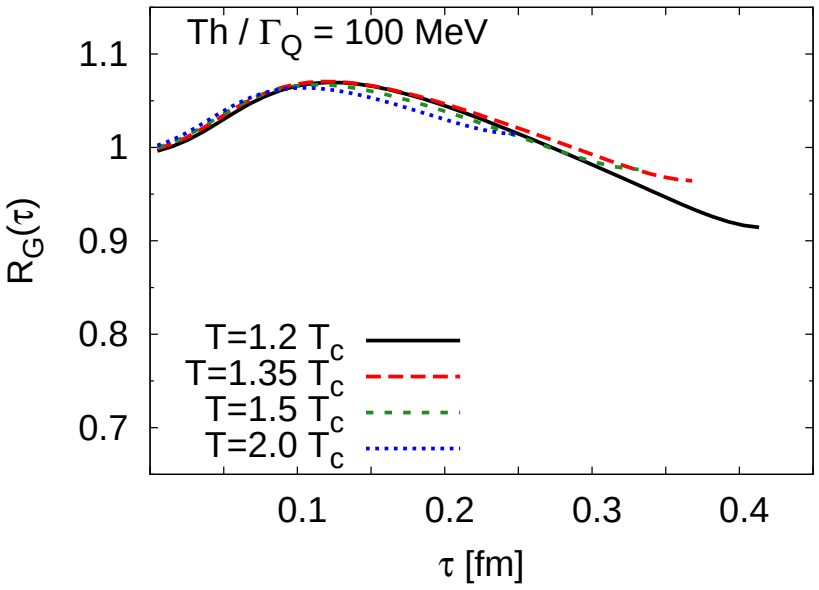
- Use vacuum spectral function for reconstruction
- Concentrate on pseudo-scalar channel
- Lattice suggests ratio $\rightarrow \sim$ 0.9 - 1.0

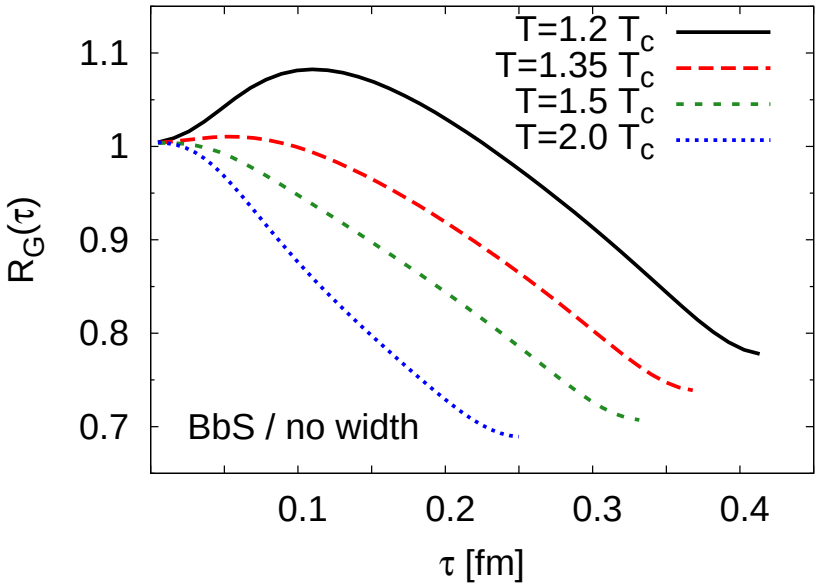
Results for $c\bar{c}$



Results for $c\bar{c}$







$\sim$ 20 % uncertainty due to

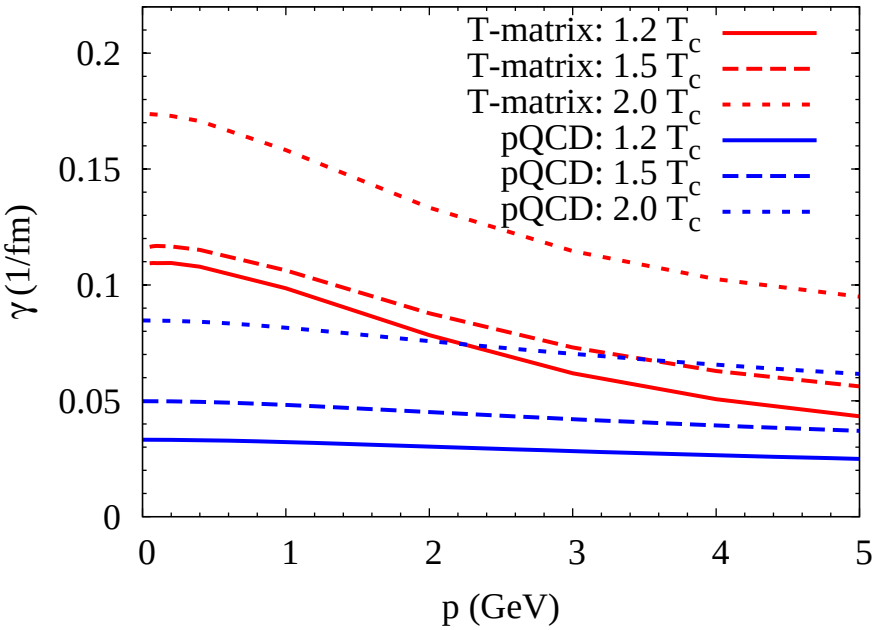
- Many body effects
- Reduction scheme
- Potential

Charm quark transport

Heavy Quarks → Consider Fokker-Planck equation:

$$\frac{\partial f_Q}{\partial t} = \frac{\partial}{\partial p_i}(p_i \gamma f_Q) + \frac{\partial^2}{\partial p_i \partial p_j}(B_{ij} f_Q)$$

$$\gamma(\vec{p}) = \frac{1}{2\,E_p} \int \frac{d^3\vec{q}}{(2\pi)^3 2\,E_q} \int \frac{d^3\vec{q}\,'}{(2\pi)^3 2\,E_{q'}} \int \frac{d^3\vec{p}\,'}{(2\pi)^3 2\,E_{p'}}$$
$$\times \frac{1}{\gamma_c} f_{kin.} \sum |T_{HL}|^2 (2\pi)^4 \delta(q + p - q' - p') f_B(\vec{q})$$



- Calculate heavy-light T-Matrix
- Sum over color channels and s- and p-waves.
- Non perturbative interactions
  ⇒ rate significantly larger than pQCD

Conclusions / Outlook

- Conclusions
  - T-matrix is a consistent thermodynamic approach for scattering and bound states
  - Used microscopic model for the Potential constrained by lQCD
  - Open and hidden charm / bottom described in one approach
  - T-Matrix includes broad range of medium effects (quark self-energies, in-medium potential, ...)
  - Link to transport properties

- Outlook
  - Calculation of correlator ratios also in other channels
  - Coupled channel with $D\bar{D}$
  - Realistic quark spectral functions (zero modes)
  - T-Matrix for Gluons
  - ...