GOETHE UNIVERSITÄT FRANKFURT AM MAIN

The Universality of the Diffusion Wake

Barbara Betz

and Jorge Noronha, Miklos Gyulassy, Giorgio Torrieri, Dirk Rischke, Igor Mishustin

arXiv:0812.4401 [nucl-th]

Mach Cone vs. Diffusion Wake:

Why I believe the latter

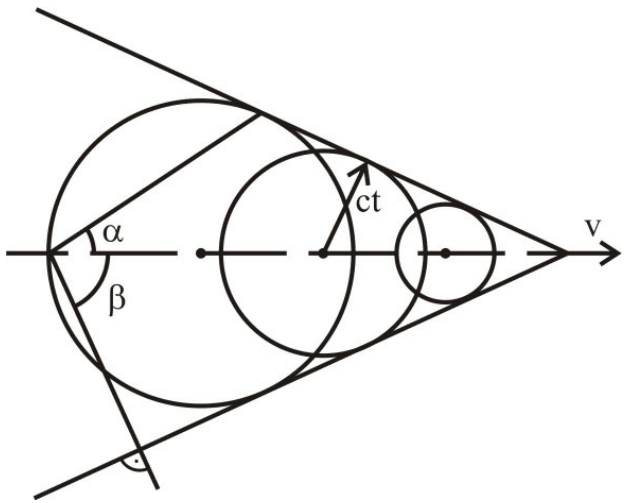
Jet - Studies in HIC I

Can energy lost by jets tell us something about medium properties?

- Medium behaves like a fluid → Data described by hydrodynamics

→ Idea: Propagation of fast parton generates a Mach cone pattern

→ reflect interaction of jet with medium



Mach cone angle sensitive to EoS

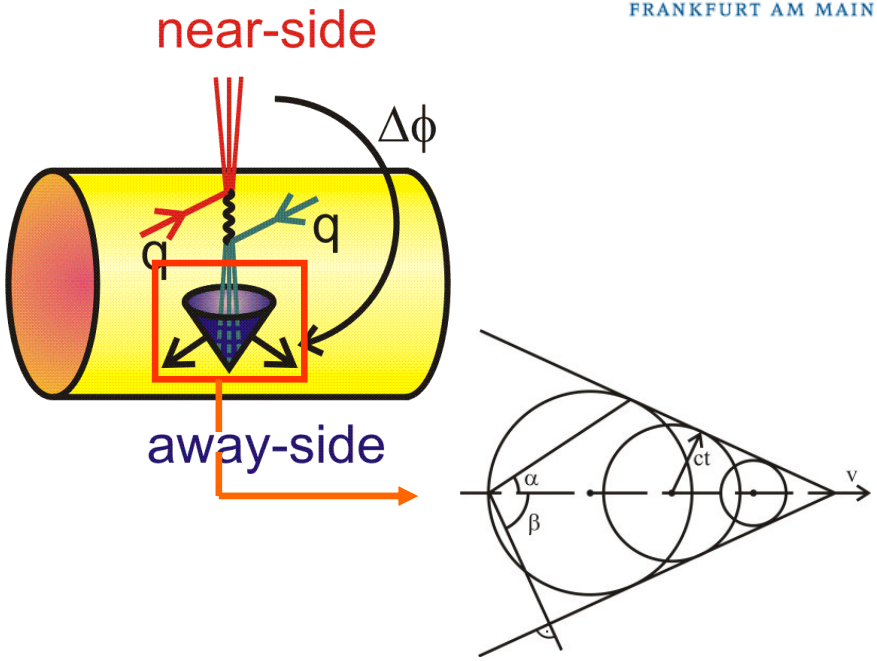
$$\cos \phi_M = c_s / v$$

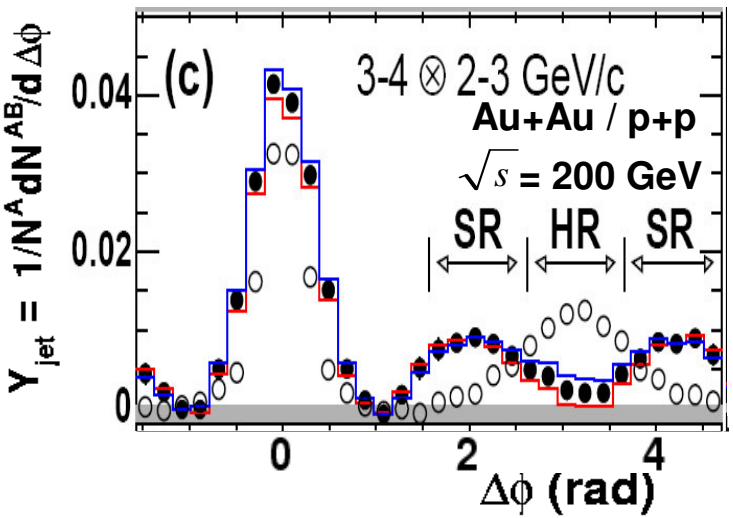
By observation:

- Study EoS of the fluid

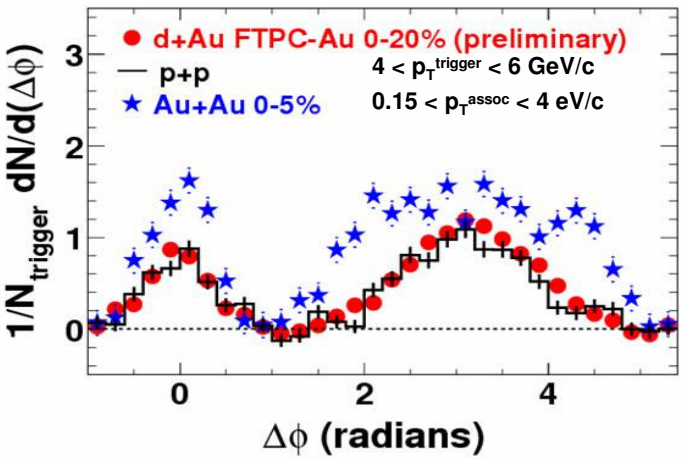
Jet - Studies in HIC II

- Generation of Mach cone pattern
- → Redistribution of energy to lower p_T-particles
- Re-appearance of the away-side for low and intermediate p_T^{assoc}





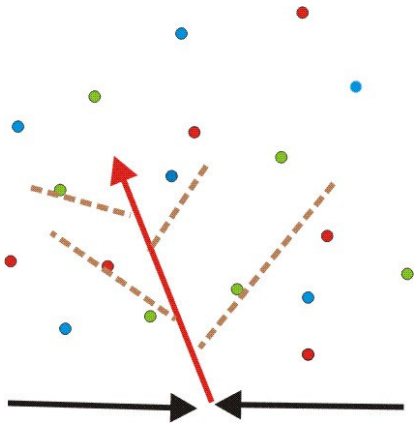
PHENIX, Phys. Rev. C **77**, 011901 (2008)



STAR, Nucl. Phys. A **774**, 129 (2006)

The Theory Part

Modelling of Jets



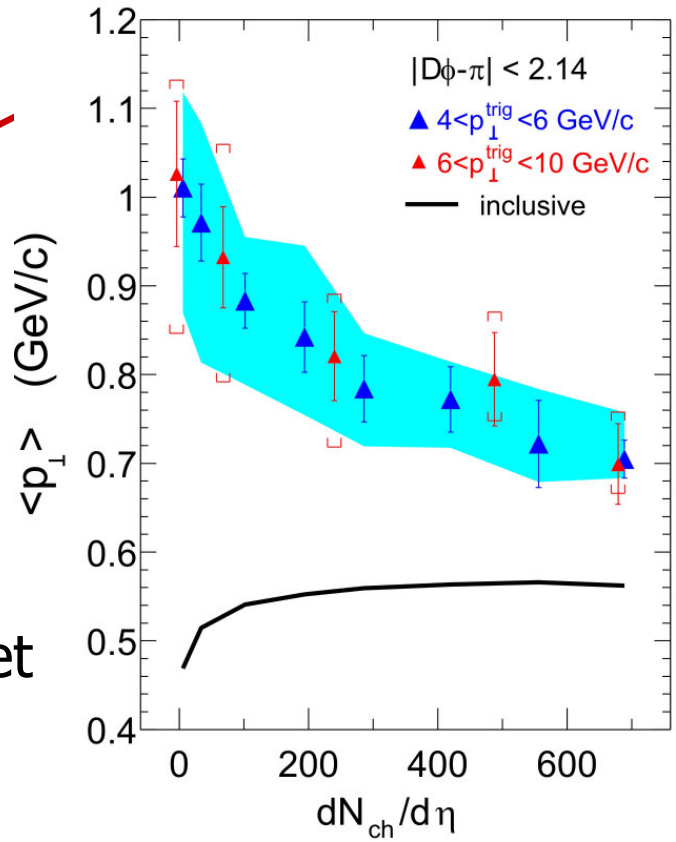
- Jet deposition mechanism unclear
  - GLV, BDMPS, ASW, AMY, Higher Twist

Jets can be modelled using hydrodynamics:

$$\partial_\mu T^{\mu\nu} = S^\nu$$

not the source term of the jet

- residue of energy and momentum given by the jet
  - this residue is unknown



STAR, Phys. Rev. Lett. **95**, 152301 (2005)

Study Jets in (3+1)d Ideal Hydro

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- Hydrodynamics represents (local) conservation of

energy-momentum $\partial_\mu T^{\mu\nu} = 0 \rightarrow \partial_\mu T^{\mu\nu} = S^\nu$

(local) charge $\partial_\mu N^\mu = 0$

- If matter is in local thermodynamical equilibrium

$T^{\mu\nu} = (e+p)u^\mu u^\nu - pg^{\mu\nu}$ $u^\mu = \gamma(1, \vec{v})$ $\gamma = (1-v^2)^{-1/2}$

$N^\mu = nu^\mu$ $g^{\mu\nu} = (+1,-1,-1,-1)$

- EoS $p = p(e) = e/3$

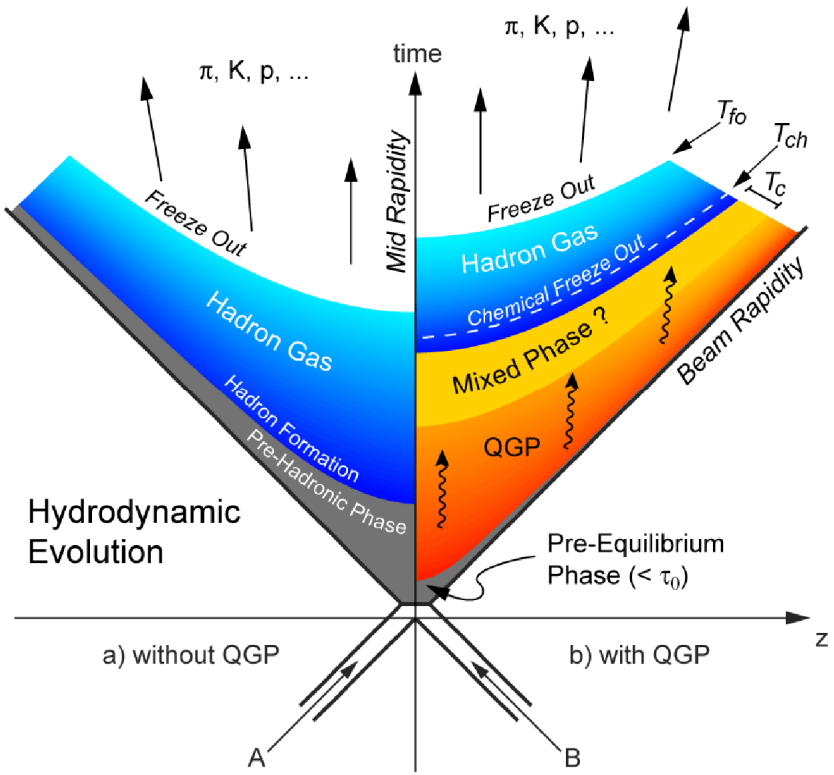
The Caveat: Freeze-out Prescription

Cooper-Frye Freeze-out:

$$\frac{dN}{p_T dp_T d\phi} = C \int dV \exp[-\gamma(E - p_i v^i)/T]$$

$$\sim e^{+\vec{p}\cdot\vec{v}}$$

→ mainly flow driven



- Assumption of isochronous/isothermal freeze-out
- No interaction afterwards

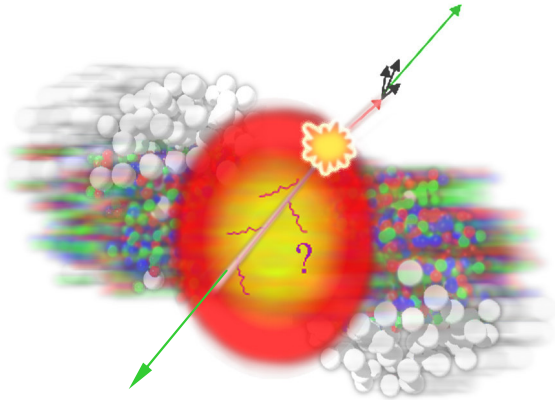
The Punch-Through Jet

Punch – Through Jet I

Applying a static medium and an ideal Gas EoS for massless gluons

→ Maximal fluid response

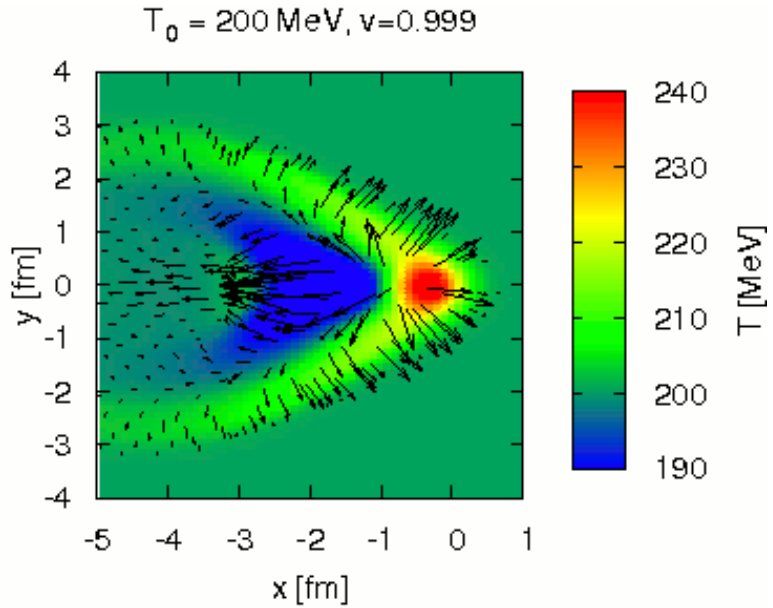
Assume: Near-side jet is not modified by medium



$$S^{\nu} = \int_{\tau_i}^{\tau_f} \frac{dP^{\nu}}{d\tau}\, \delta^{(4)}(x^{\mu} - x^{\mu}(\tau))\, d\tau$$

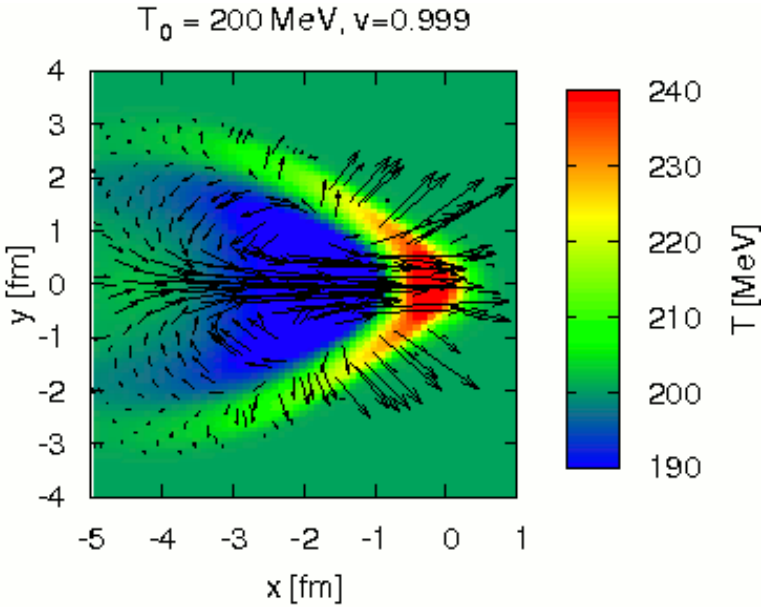
$$P^{\nu} = (E, \vec{M})$$

$$x^{\mu}(\tau) = x_0^{\mu} + u_{jet}^{\mu}\tau$$



$$\frac{dE}{dt} = 1.5\frac{GeV}{fm}, \quad \frac{dM}{dt} = 0\frac{GeV}{fm}$$

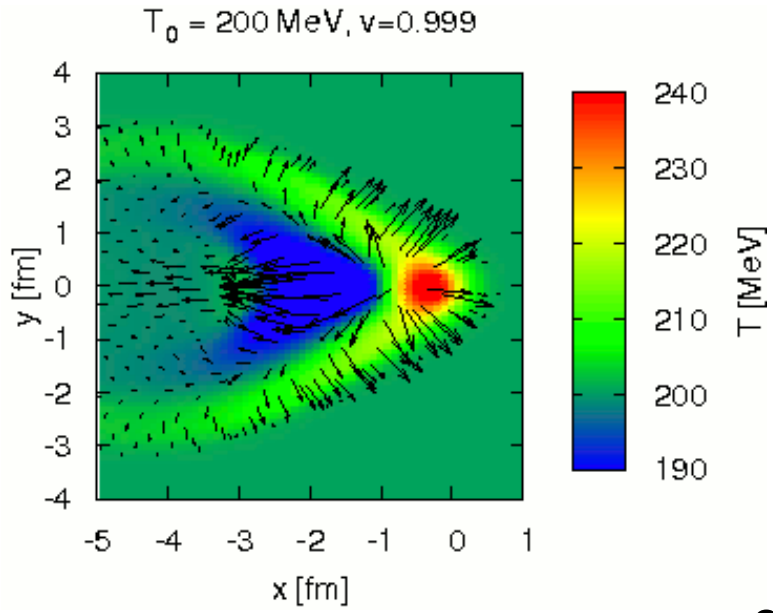
v=0.999



$$\frac{dM}{dt} = \frac{dE}{dt} = 1.5\frac{GeV}{fm}$$

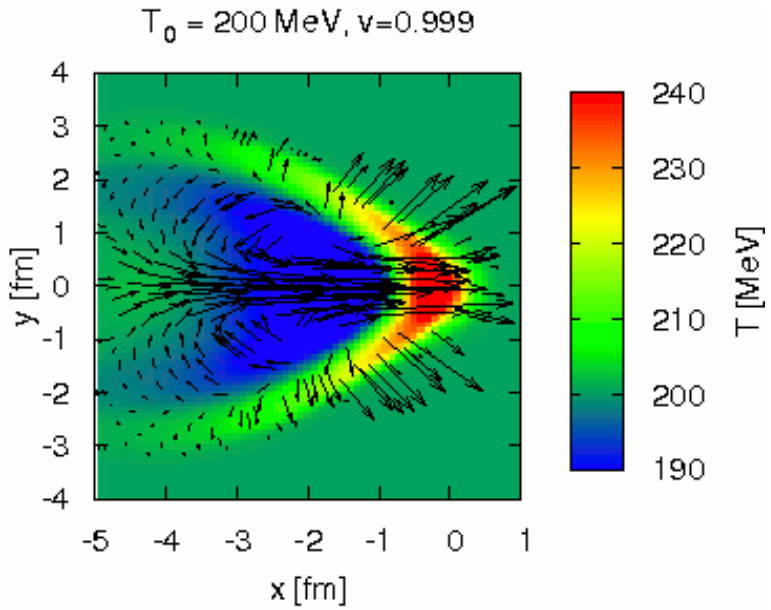
t=4.5/v fm

Punch – Through Jet II



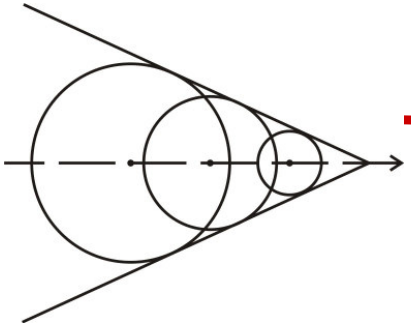
$$\frac{dE}{dt} = 1.5\frac{GeV}{fm}, \quad \frac{dM}{dt} = 0\frac{GeV}{fm}$$

v=0.999

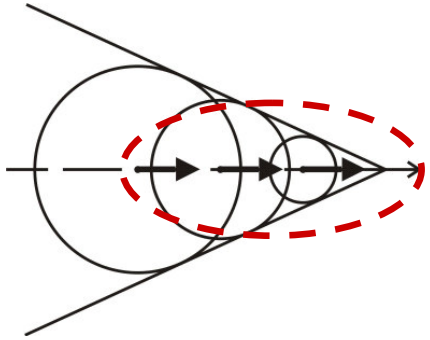


$$\frac{dM}{dt} = \frac{dE}{dt} = 1.5\frac{GeV}{fm}$$

t=4.5/v fm



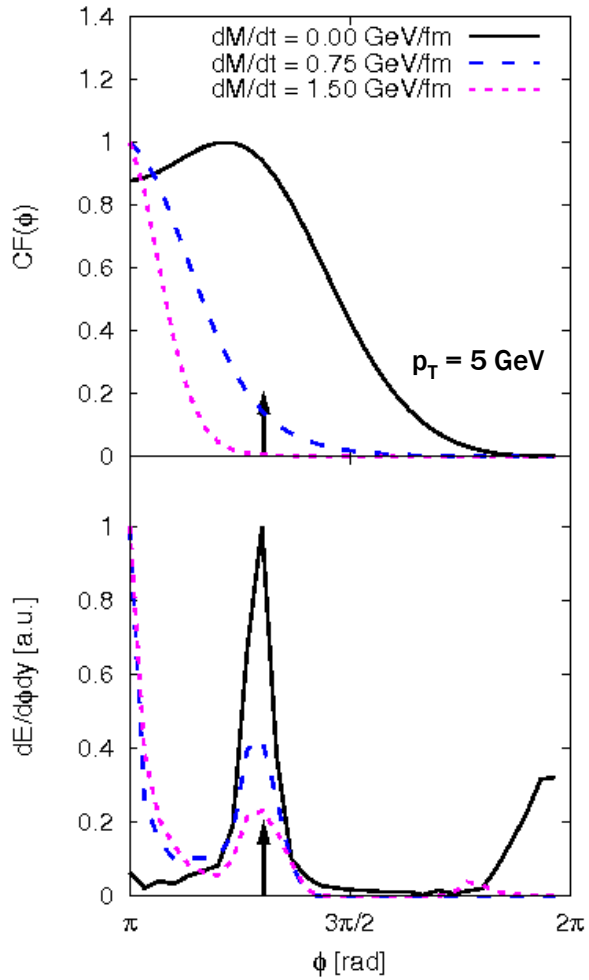
→ Mach cone for sound waves



→ Diffusion wake

Punch – Through Jet III

arXiv:0812.4401 [nucl-th]



Diffusion wake causes peak in jet direction

Normalized, background-subtracted isochronous Cooper-Frye at mid-rapidity

$$\left.\frac{dN}{p_T dp_T dy d\phi}\right|_{y=0} = \int_{\Sigma} d\Sigma_\mu p^\mu [f_{\text{Boltzmann}}(u^\mu, p^\mu, T) - f_{eq}]$$

Energy Flow Distribution

$$\left.\frac{dE}{d\phi dy}\right|_{y=0} = \int d^3\vec{x}\ E(\vec{x})\ \delta(\phi - \Phi(\vec{x}))$$

$$\Phi(\vec{x}) = \tan^{-1}\left(\frac{m_y(\vec{x})}{m_x(\vec{x})}\right)$$

Assuming: Particles in subvolume will be emitted into the same direction

Punch – Through Jet IV

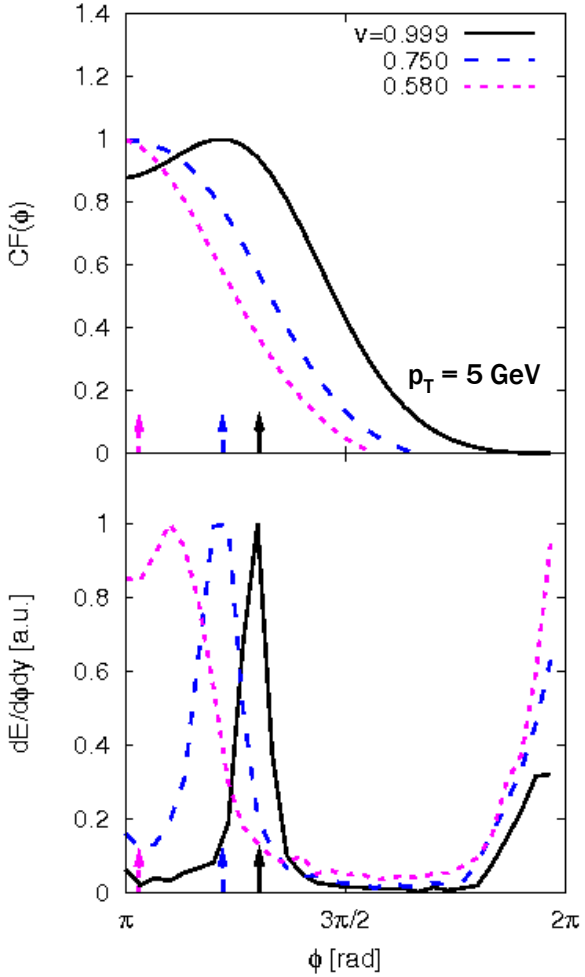
Does the jet-pattern reproduce the features of a Mach cone?

$$\cos \phi_M = c_s / v$$

→ Velocity dependence of the emission angle

Creation of Bow Shock for smaller v strengthens peak in jet direction

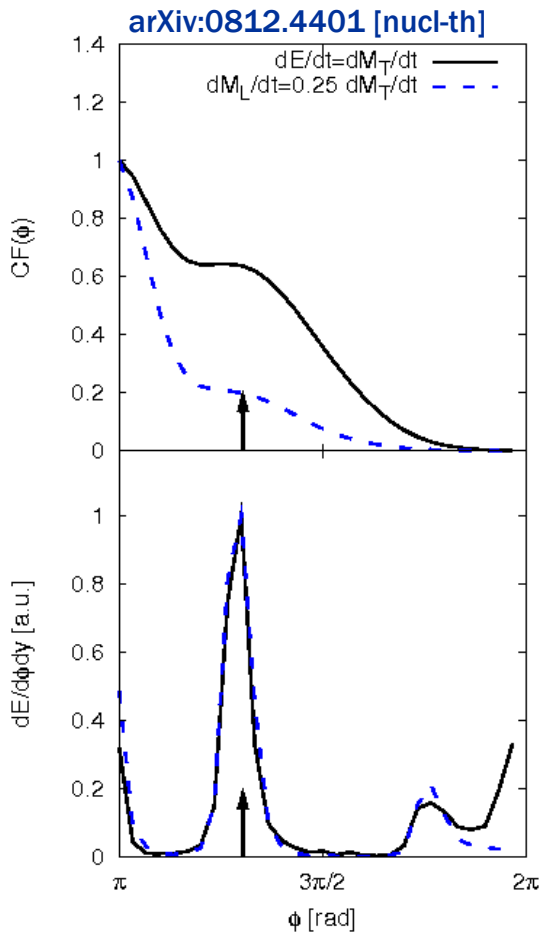
arXiv:0812.4401 [nucl-th]



Punch – Through Jet V

- Transverse momentum deposition:

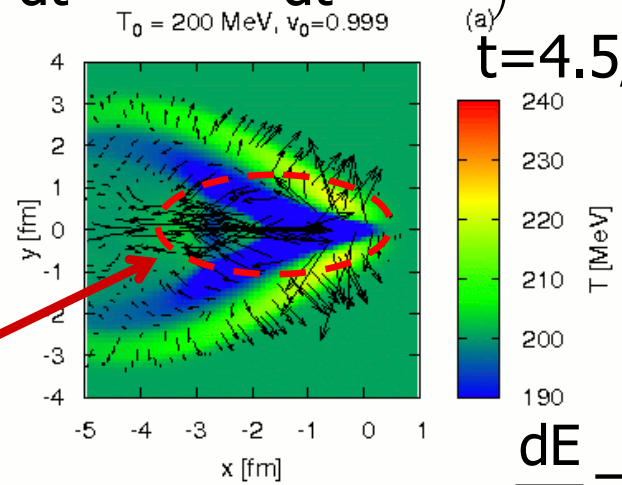
$$S^{\nu} = \frac{1}{(\sqrt{2\pi}\sigma)^3}\exp\left(-\frac{(\vec{r}-\vec{x})^2}{2\sigma^2}\right)\left(\frac{dE}{dt}, \frac{dM_L}{dt}, \frac{dM_T}{dt}\cos\theta, \frac{dM_T}{dt}\sin\theta\right)$$



→ Still influence of diffusion wake

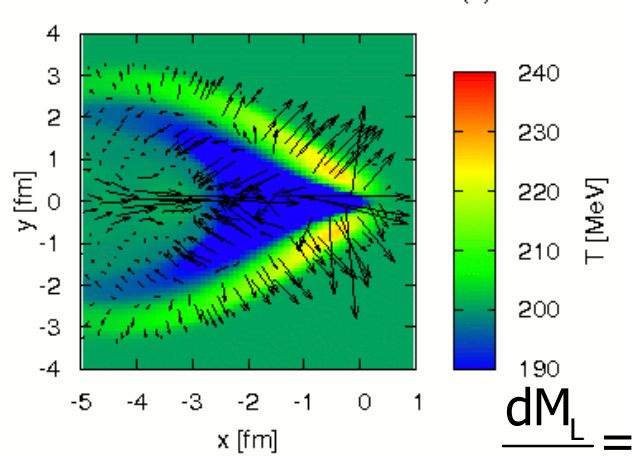
from explosion of matter

→ Vorticity conservation



t=4.5/v fm

$$\frac{dE}{dt} = \frac{dM_T}{dt}$$



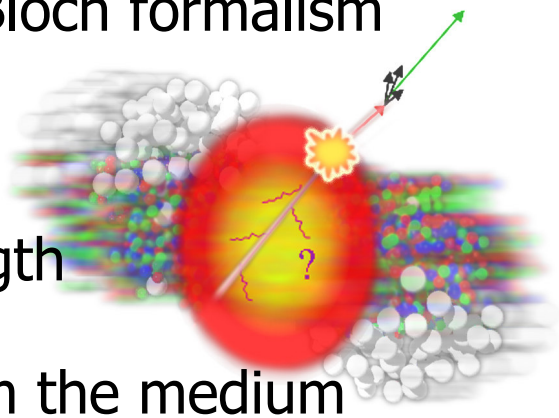
$$\frac{dM_L}{dt} = \frac{1}{4}\frac{dM_T}{dt}$$

The Stopped Jet

Stopped Jet I

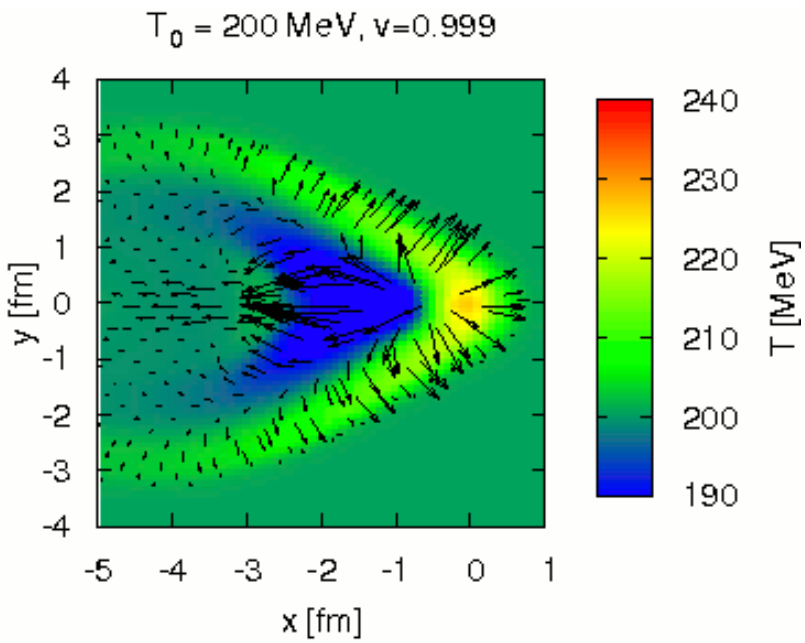
Jet decelerating from v=0.999 according to Bethe-Bloch formalism

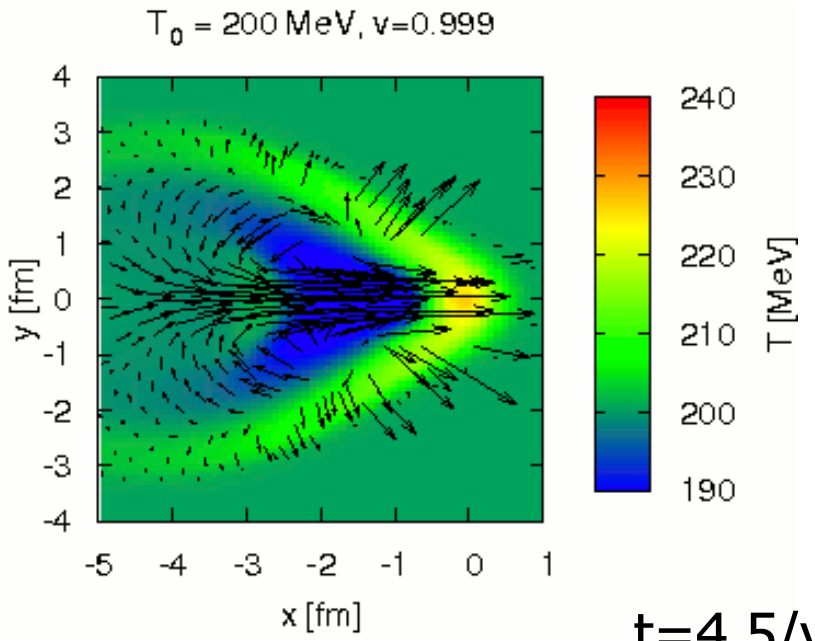
Known and applied in tumor therapy



$$\frac{dE}{dx}(t) = a\frac{1}{v^2(t)}$$ a=-1.36 GeV/fm adjusts path length

Simplest back-reaction from the medium





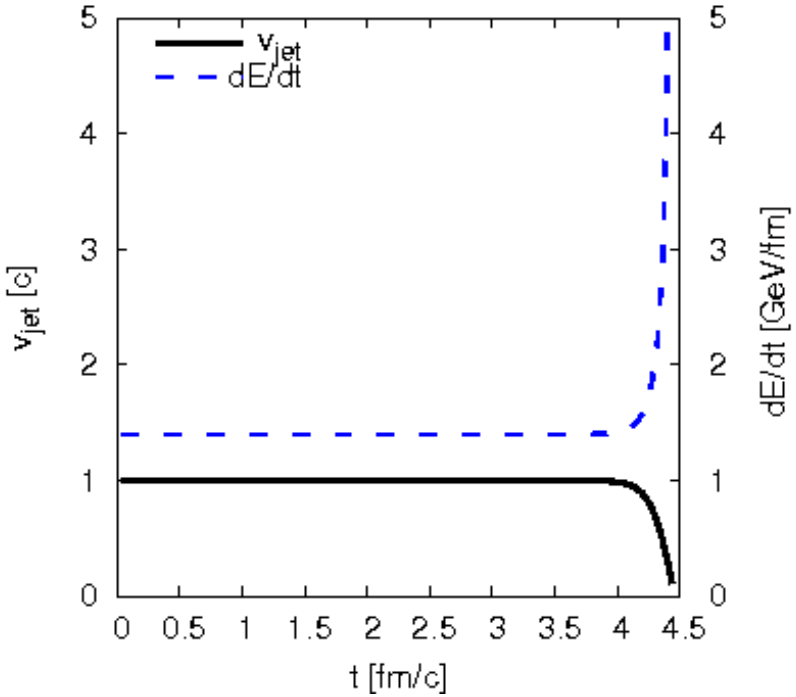
t=4.5/v fm

$$\frac{dE}{dt}(0) = 1.5\frac{GeV}{fm} \quad \frac{dM}{dt}(0) = 0\frac{GeV}{fm}$$

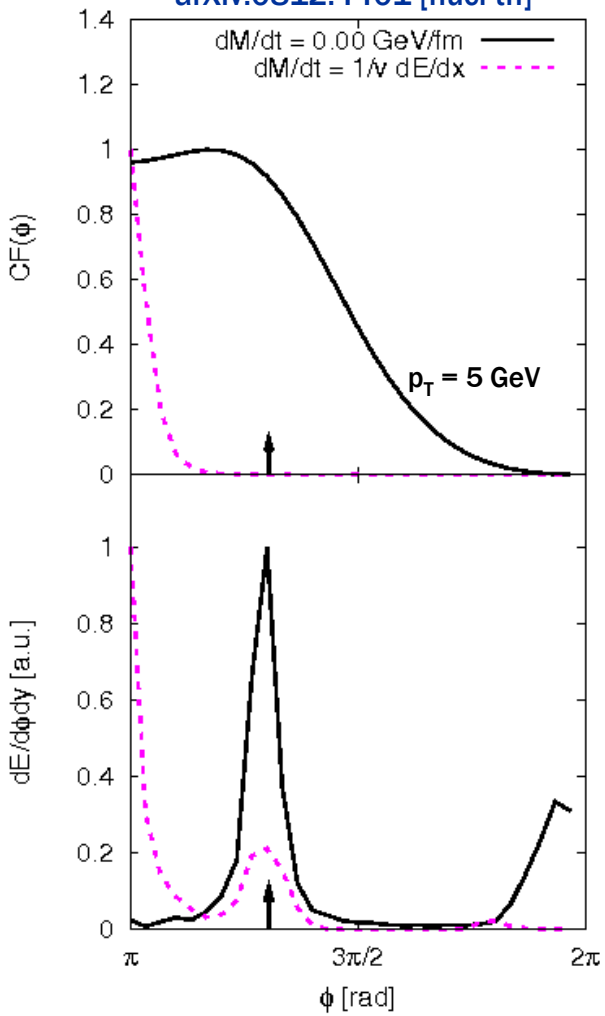
$$\frac{dE}{dt}(0) = v\frac{dM}{dt}(0) = 1.5\frac{GeV}{fm}$$

Stopped Jet II

➔ Bragg Peak



arXiv:0812.4401 [nucl-th]



general: $$\frac{dM}{dt}(t) = \frac{dE}{dx}(t)$$

➔ $$\frac{dE}{dt}(t) = v(t)\,\frac{dM}{dt}(t) \longrightarrow \frac{dM}{dt}(t) > \frac{dE}{dt}(t)$$

Dominance of diffusion wake

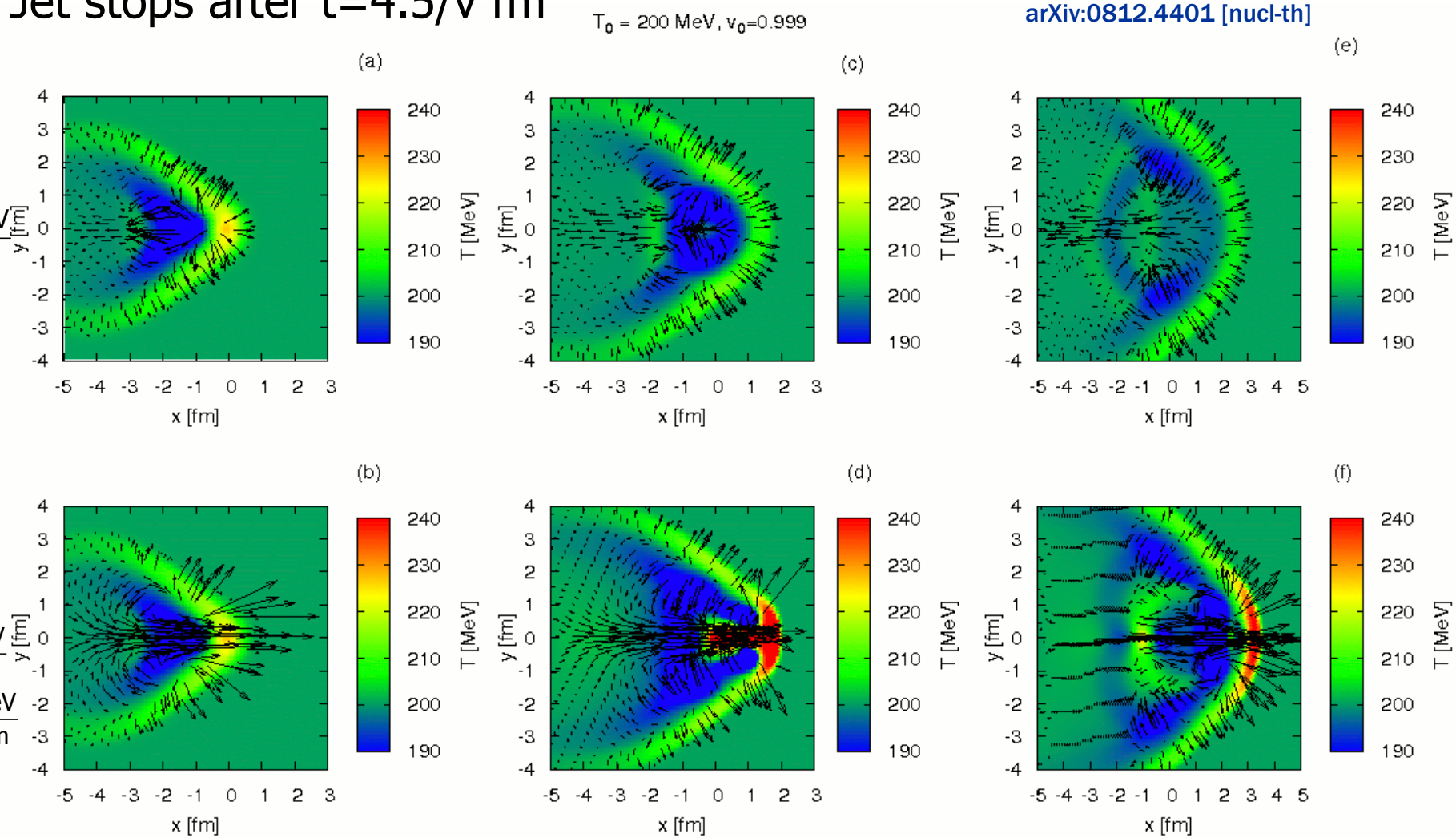
Stopped Jet III

- Jet stops after t=4.5/v fm

arXiv:0812.4401 [nucl-th]

$$\frac{dE}{dt}(0) = 1.5\frac{GeV}{fm}$$
$$\frac{dM}{dt}(0) = 0\frac{GeV}{fm}$$

$$\frac{dE}{dt}(0) = 1.5\frac{GeV}{fm}$$
$$v\frac{dM}{dt}(0) = 1.5\frac{GeV}{fm}$$



t_{FO}=4.5/v fm t_{FO}=6.5/v fm t_{FO}=8.5/v fm

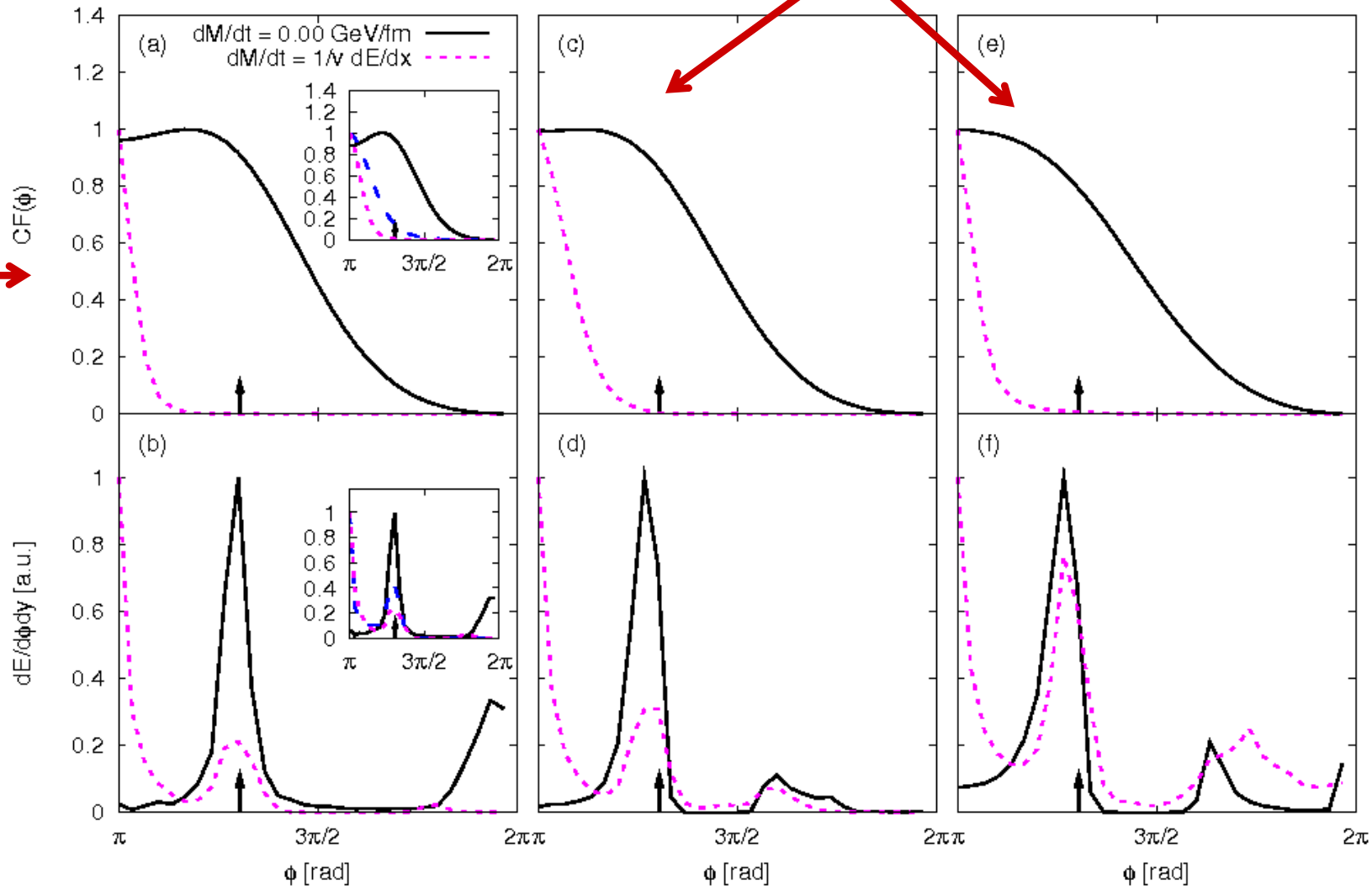
Diffusion wake still present → Vorticity conservation

Stopped Jet IV

Diffusion wake causes peak in jet direction

Larger impact of thermal smearing

arXiv:0812.4401 [nucl-th]

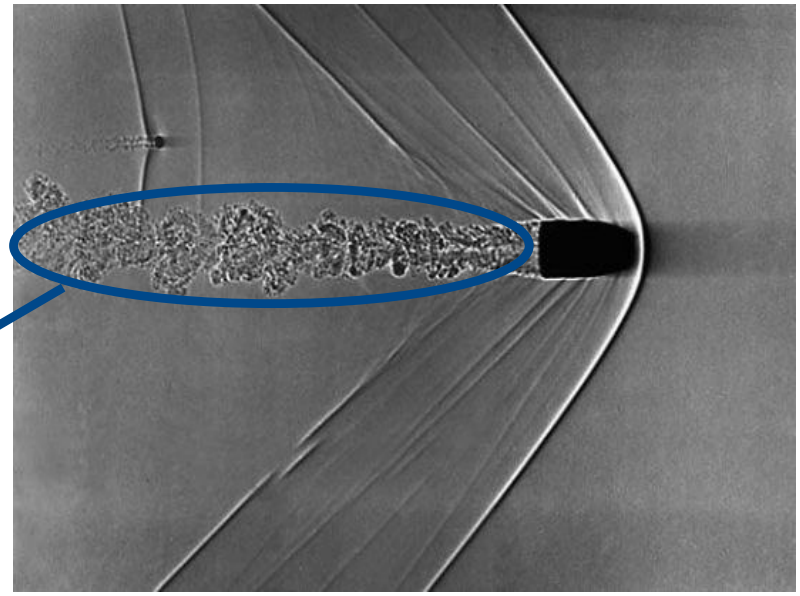


t_{FO}=4.5/v fm t_{FO}=6.5/v fm t_{FO}=8.5/v fm

The Diffusion Wake

The Diffusion Wake

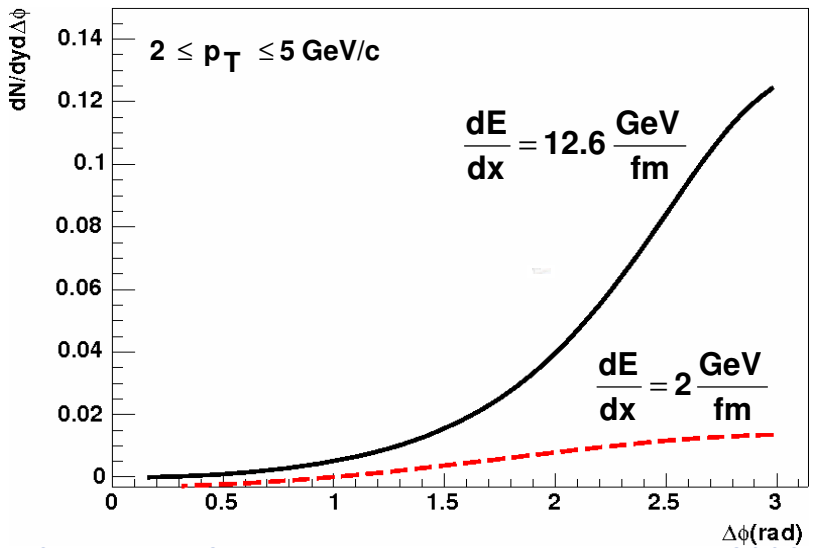
G. Burau, Genua Harbour, September 2008



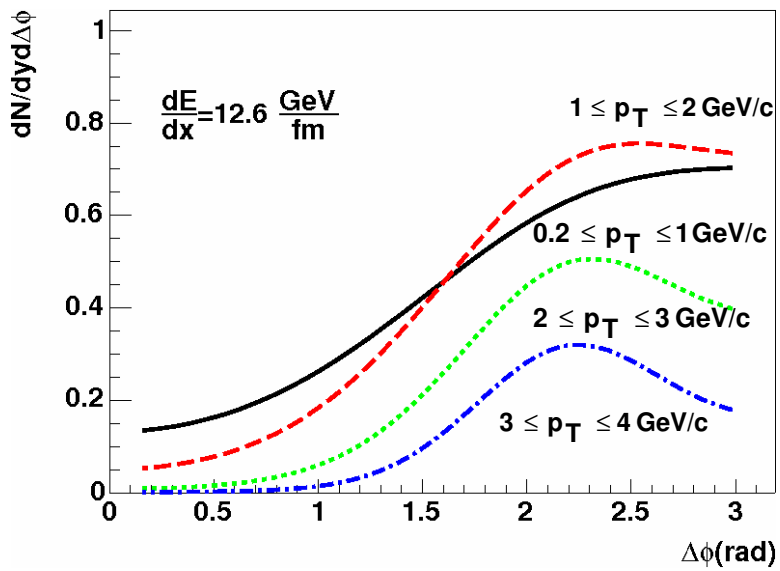
→ The diffusion wake exists!

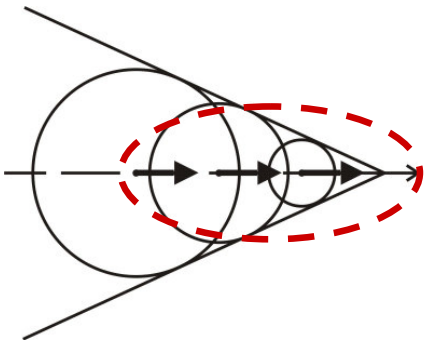
Jet – Energy Loss Studies

- Jet deposits energy and momentum along a trajectory
- Applying linearized hydrodynamics

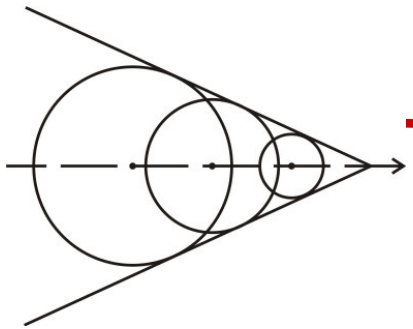


J. Casalderrey-Solana et al., Nucl. Phys. A **774**, 577 (2006)



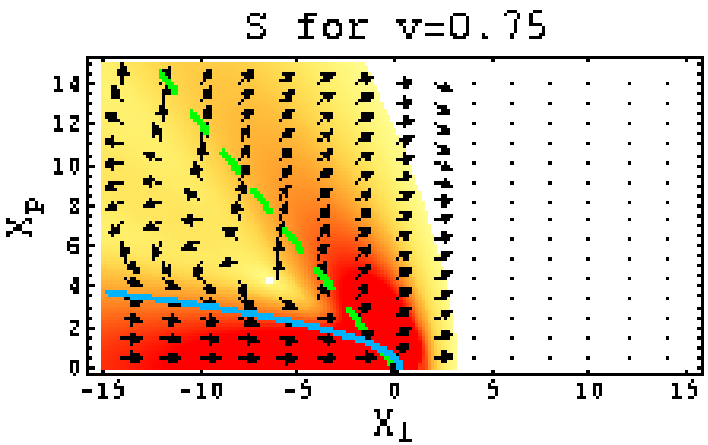


→ Diffusion wake



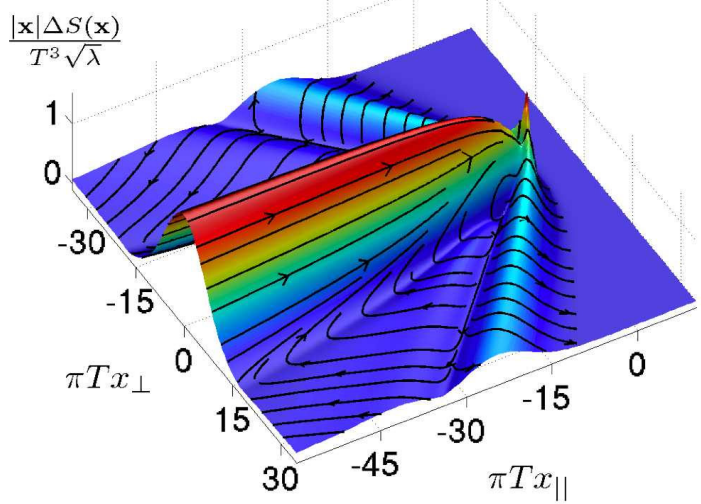
→ Mach cone for sound waves

Jets in AdS/CFT

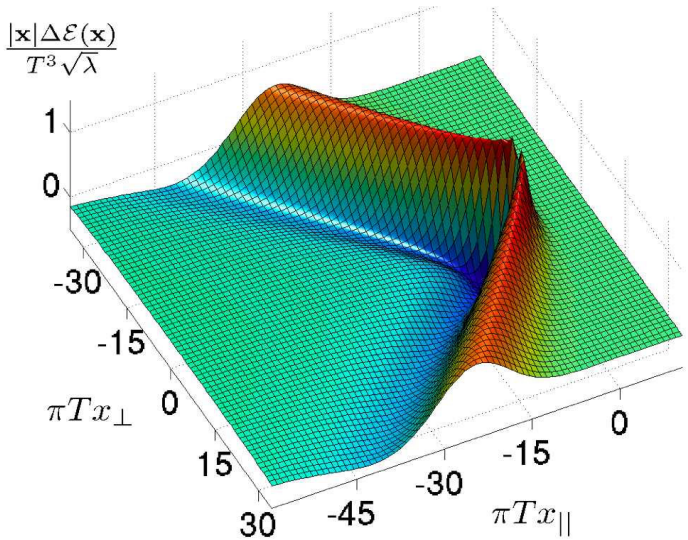


S. Gubser et al., Phys. Rev. Lett. **100**, 012301 (2008)

Pointing vector perturbation



P. Chesler and L. Yaffe, Phys. Rev. D **78**, 045013 (2008)



Energy density perturbation

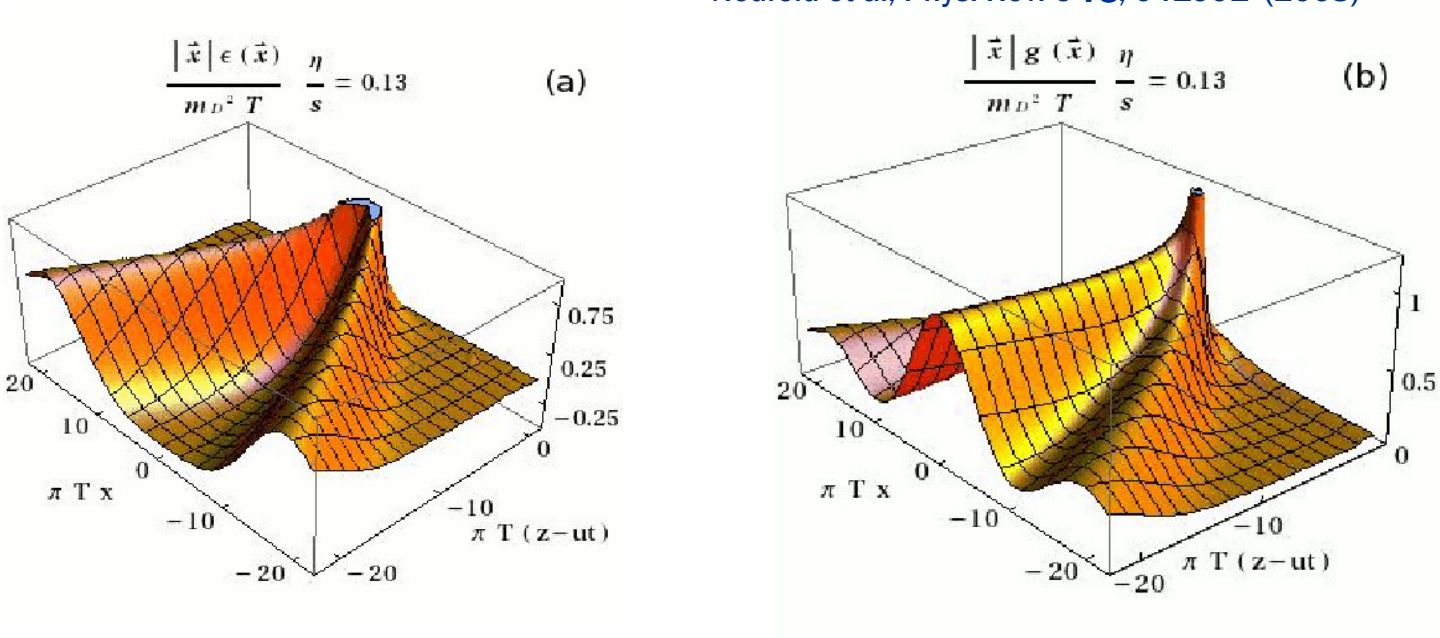
→ Diffusion Wake contribution

→ Attention: No clear Mach cone signal

Jets in pQCD I

Considering a static medium and linearized hydrodynamics for a punch-though jet

Neufeld et al, Phys. Rev. C **78**, 041901 (2008)

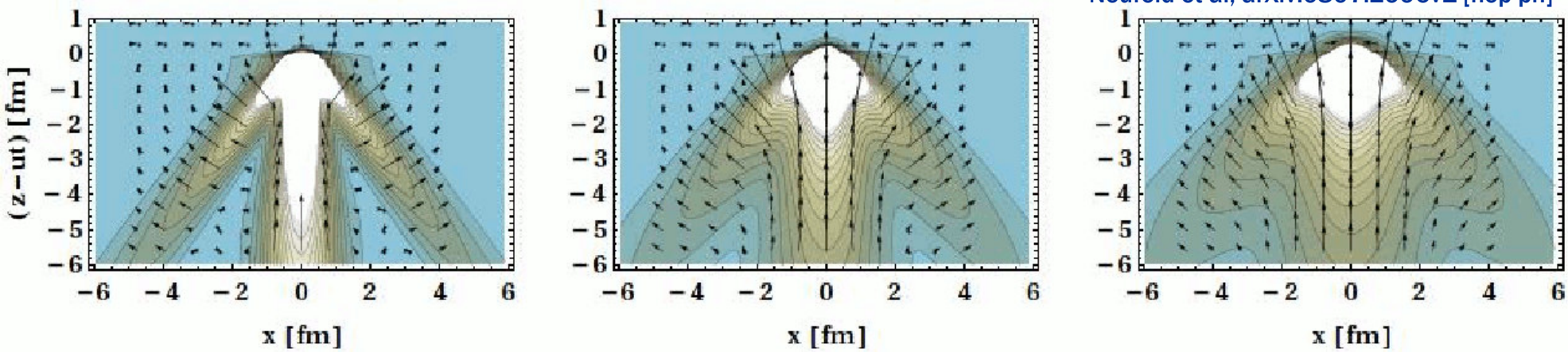


→ Mach cone signal & Diffusion Wake

Jets in pQCD II

Contour plots of magnitude of perturbed momentum density

Neufeld et al, arXiv:0807.2996v1 [hep-ph]



$$\frac{\eta}{s} = \frac{1}{4\pi} \qquad \frac{\eta}{s} = \frac{3}{4\pi} \qquad \frac{\eta}{s} = \frac{6}{4\pi}$$

→ Strong flow in jet-direction → $\frac{dM}{dt} > \frac{dE}{dt}$

Summary

- Diffusion wake is universal and dominates the freezeout distribution

  → always created if dM/dx > threshold

- Freeze-out procedure critically influences correlation pattern

  → Energy flow distribution: peak around Mach cone angle & diffusion wake

- No significant differences between a punch-through and stopped jet

- How will expansion (radial flow) alter the correlation?

Backup

Energy-Momentum Relation

$$\frac{dM}{dt}=\frac{dM}{dx}\frac{dx}{dt}=\beta\frac{dM}{dx}=\beta\left(\frac{dE}{dx}\beta+E\frac{d\beta}{dx}\right)$$

$$E=\gamma m$$

$$M=\beta\gamma m=\beta E$$

$$=\beta^2\frac{dE}{dx}\left(1+\frac{1}{\gamma^2\beta^2}\right)$$

$$=\beta^2\frac{dE}{dx}\left(1+\frac{1}{\gamma^2-1}\right)$$

$$=\beta^2\frac{dE}{dx}\left(\frac{1}{1-1/\gamma^2}\right)$$

$$\beta^2=1-\frac{1}{\gamma^2}$$

$$2\beta\frac{d\beta}{dx}=2\frac{1}{\gamma^3}\frac{d\gamma}{dx}$$

$$\frac{d\beta}{dx}=\frac{1}{\beta\gamma^3}\frac{dE}{dx}\frac{1}{m}$$

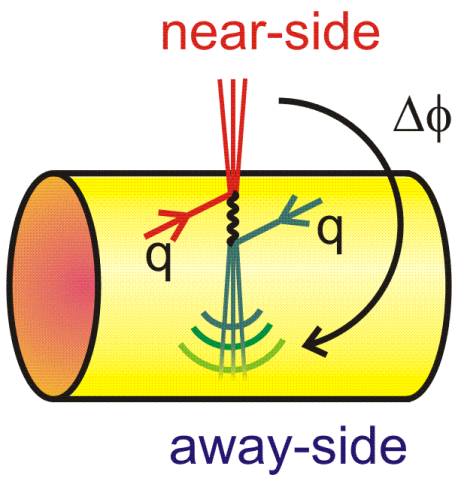
$$\rightarrow \boxed{\frac{dM}{dt}=\frac{dE}{dx}}$$

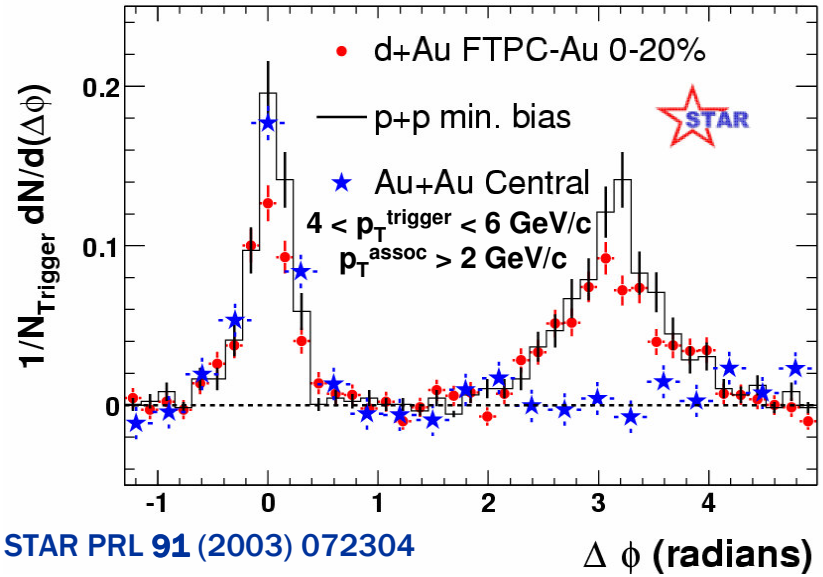
Jet - Studies in HIC

- Jet moving through dense matter, depositing its energy

→ should eventually disappear

- Jet suppression: signal for creation of an opaque matter (Quark-Gluon Plasma)





STAR PRL **91** (2003) 072304

→ Is Quark-Gluon Plasma an (ideal) fluid?

The Caveat

- Assumption :

Correlations from flow anisotropy and jets are uncorrelated

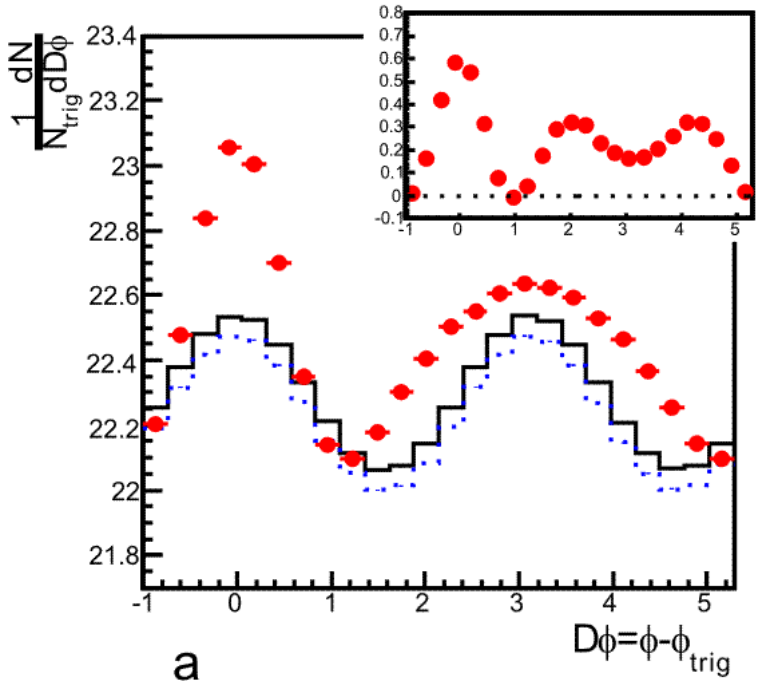
ZYAM (Zero Yield At Minimum)

- Subtraction of:

Estimated elliptic flow modulated background

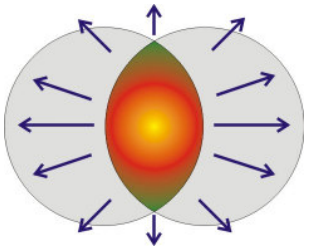
- Leads to:

Double peaked structure



J. Ulery, PoS **LHC07**, 036 (2007)

Background:
Particle correlation from elliptic flow

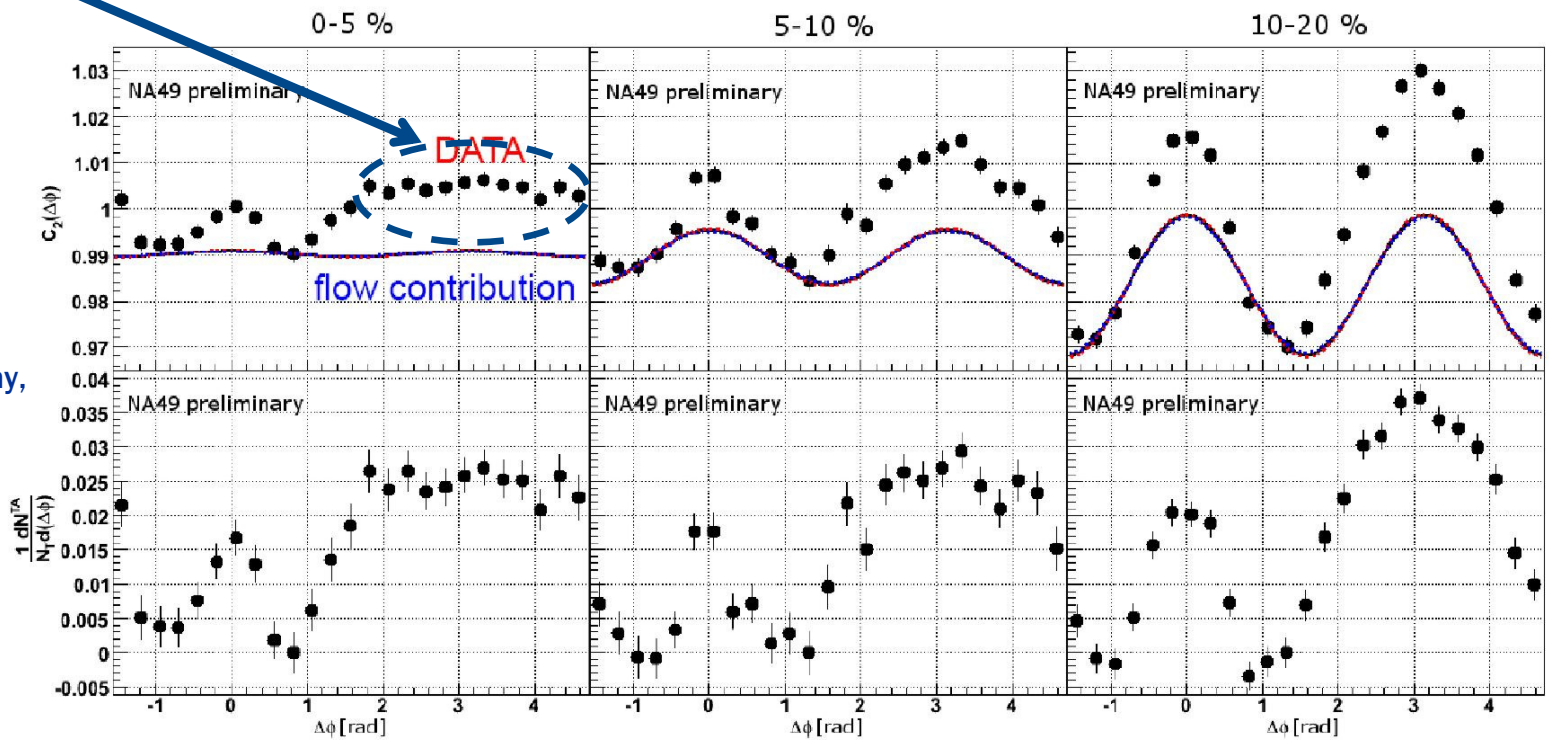


Jet - Studies in HIC

- High-p_T correlations at SPS energies show conical structure?

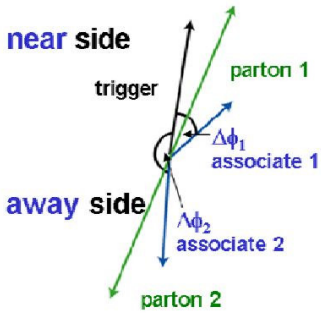
→ Conical structure even before subtracting the effect of elliptic flow - ZYAM

Blume, Talk at Confinement8, Mainz, Germany, 2008



Correlation function:

$$C(\Delta\phi) = \frac{N^{pair}_{same}(\Delta\phi)}{N^{pair}_{mixed}(\Delta\phi)} \times \frac{\int N^{pair}_{mixed}}{\int N^{pair}_{same}}$$



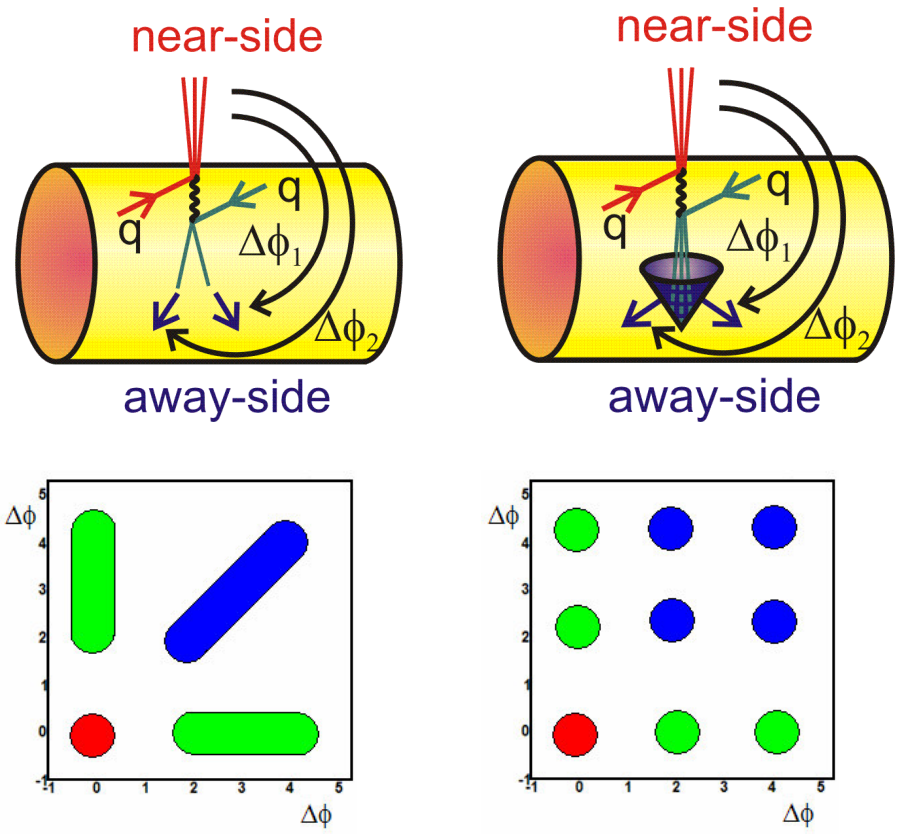
Conditional yield:

$$J_2(\Delta\phi) = \frac{1}{N^T}\frac{dN^{TA}}{d(\Delta\phi)} = \frac{C_2^{jet}(\Delta\phi)}{\int C_2(\Delta\phi')d(\Delta\phi')}\frac{N^{TA}}{N^T}$$

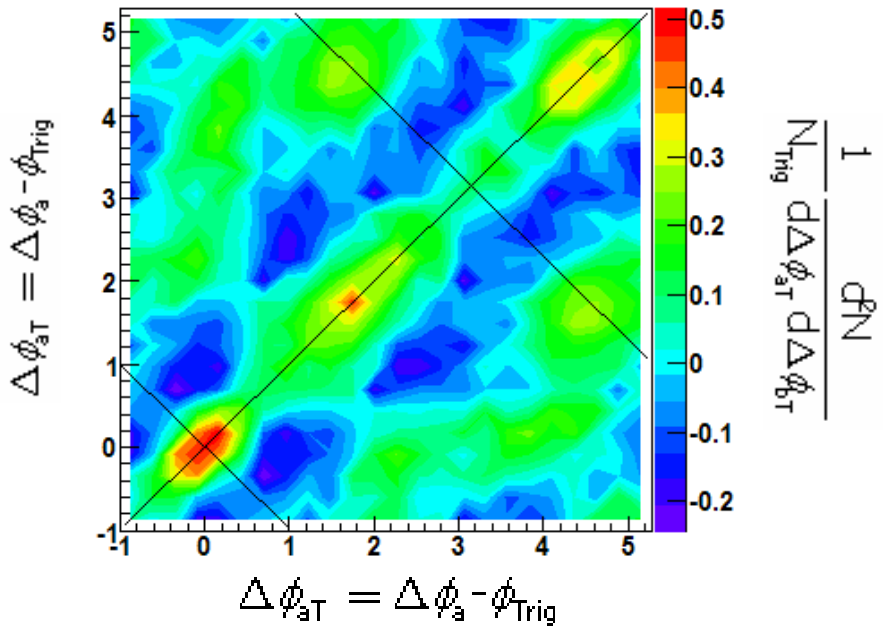
Flow contribution subtracted using ZYAM method

Jet – Studies: 3-Particle Correlations

3-Particle Correlations seem to corroborate the Mach Cone picture



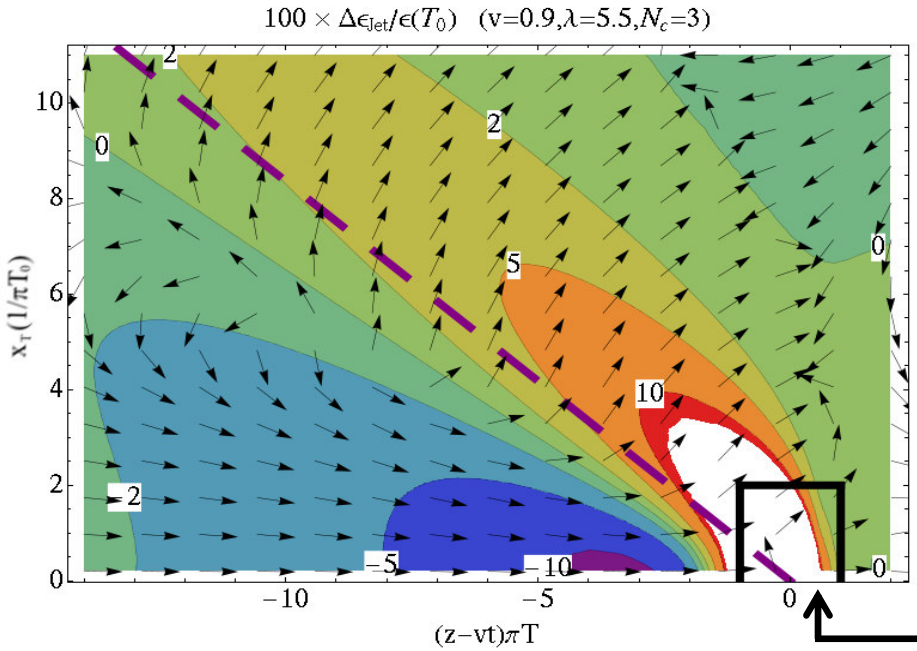
Deflected jet Mach Cone



STAR Int. J. Mod. Phys. E **16** (2007)

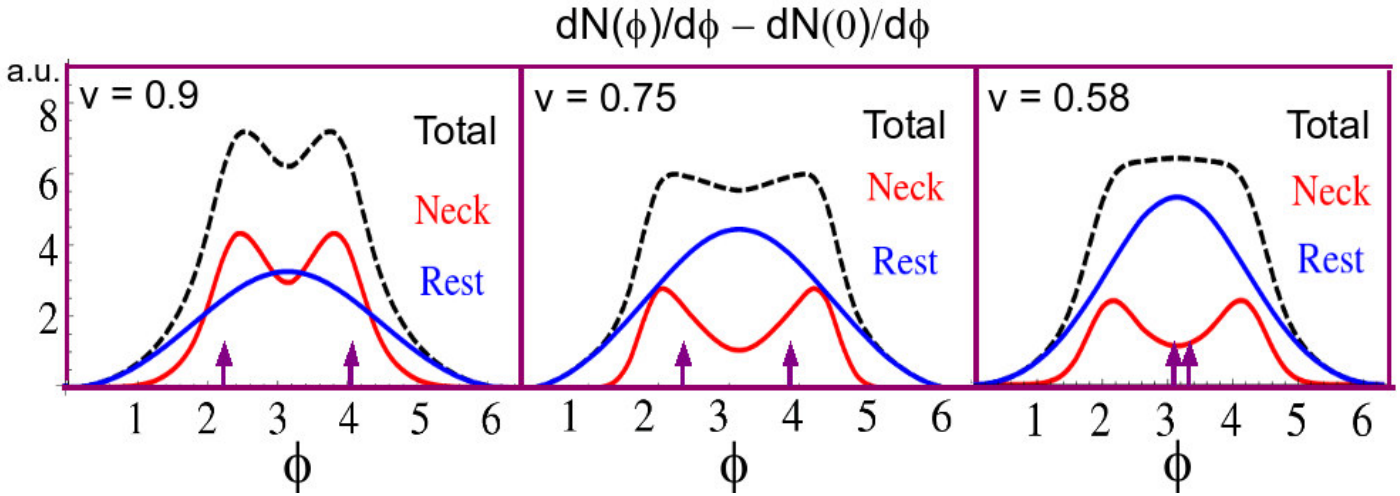
➔ Experimental data show 2 off-diagonal peaks

Jets in AdS/CFT



Noronha et al, arXiv:0807.1038

Non-Mach correlations caused by Neck region



$\cos\phi_M = c_s/v$

Heavy Quark Jets in pQCD vs AdS/CFT

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Idea: Compare weakly and strongly coupled models

Using heavy quark punch-through jet

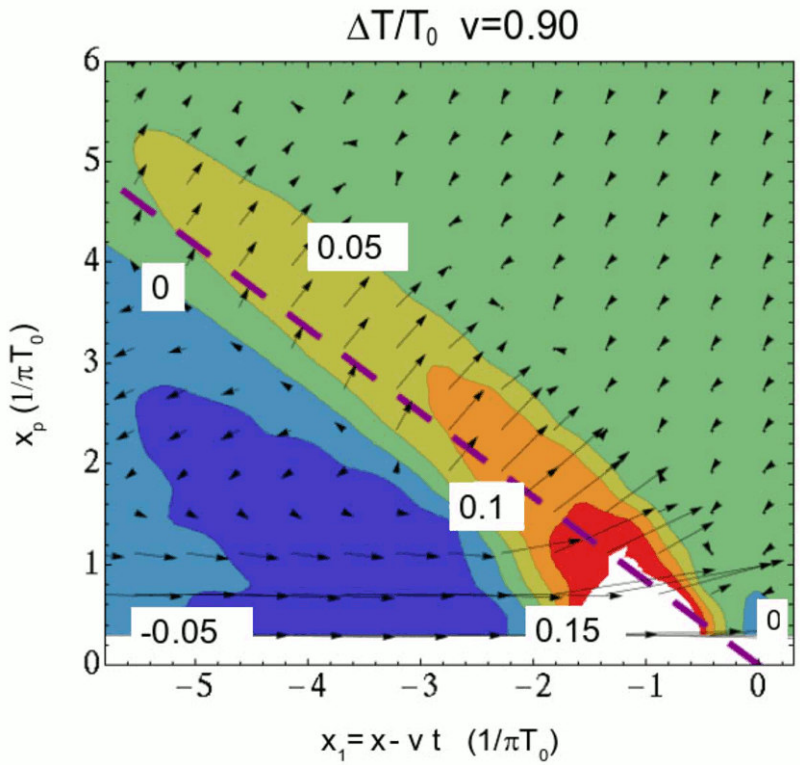
Applying ideal hydrodynamics for a static medium and an ideal Gas EoS of massless gluons

Assume that the near-side jet is not modified by the medium

pQCD: Neufeld et al. source for a heavy quark

AdS/CFT: Stress tables provided by S. Gubser, A. Yarom and S. Pufu with $\eta/s=1/(4\pi)$

t=4.5/v fm



Betz et al, arXiv: 0807.4525 [hep-ph]

pQCD result: No pronounced Neck region

The Neck Zone in pQCD vs AdS/CFT

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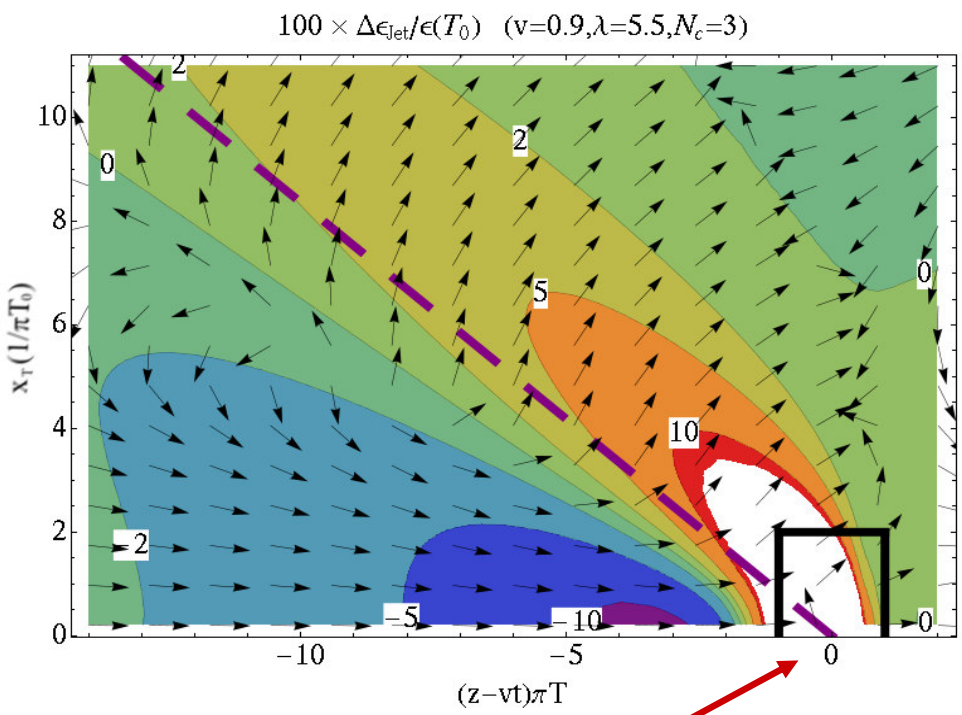
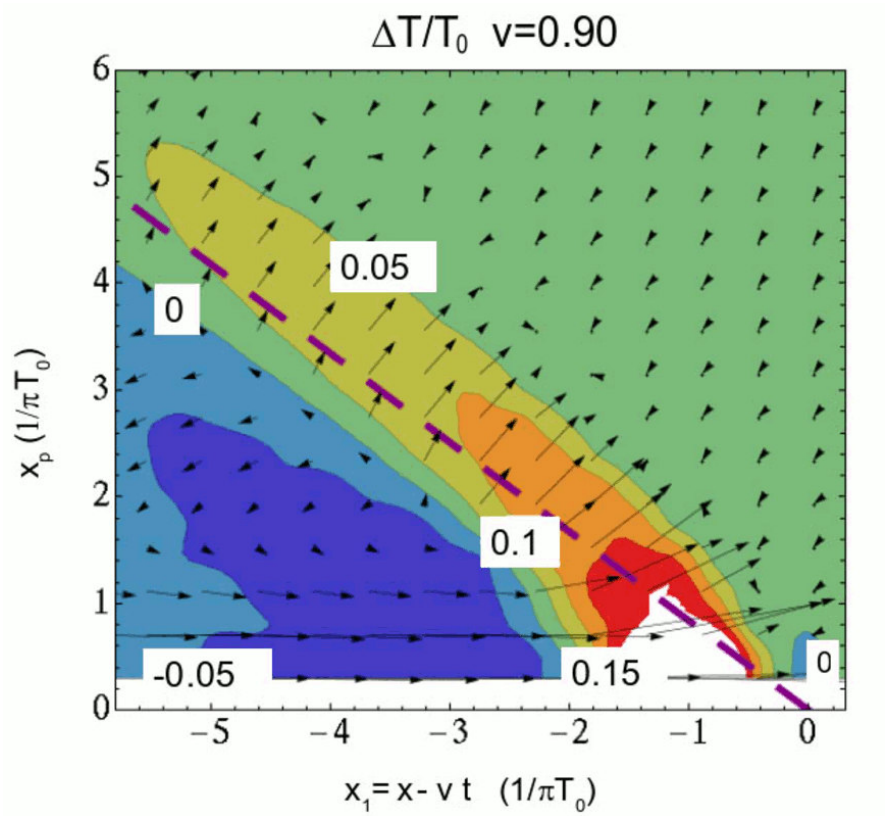
AdS/CFT



Noronha et al, arXiv:0807.1038

Strong transverse flow

pQCD

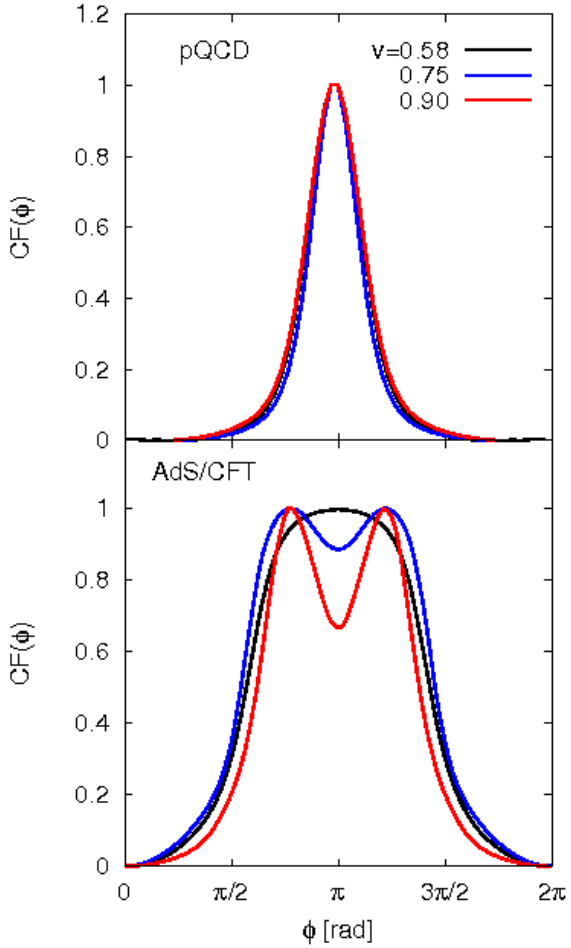


Betz et al, arXiv: 0807.4525 [hep-ph]

No strong transverse flow

Heavy Quark Jets in pQCD vs AdS/CFT

Betz et al, arXiv: 0807.4525 [hep-ph]



p_T = 2.5 GeV

Normalized, background-subtracted isochronous Cooper-Frye at mid-rapidity

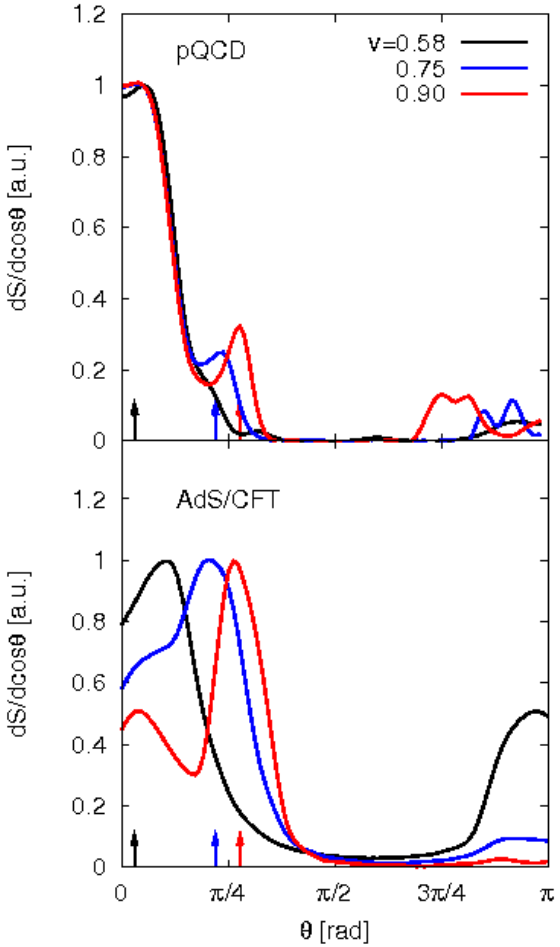
$$\left.\frac{dN}{p_T dp_T dy d\phi}\right|_{y=0} = \int_{\Sigma} d\Sigma_\mu p^\mu [f_{Boltzmann}(u^\mu, p^\mu, T) - f_{eq}]$$

Isochronous freezeout needed to compare pQCD and AdS/CFT

→ No Mach-like peaks

Heavy Quark Jets in pQCD vs AdS/CFT

Betz et al, arXiv: 0807.4525 [hep-ph]



Independent of p_T - cut

Momentum Flow Distribution

$$\frac{dS}{d\cos\theta} = \int d^3\vec{x}\ \left|M(\vec{x})\right| \times \delta(\cos\theta - M_x(\vec{x}) / \left|M(\vec{x})\right|)$$

Assuming: Particles in subvolume will be emitted into the same direction

→ Mach-like peaks &
Strong impact of diffusion wake

An Expanding Medium

- Consequences of expansion?

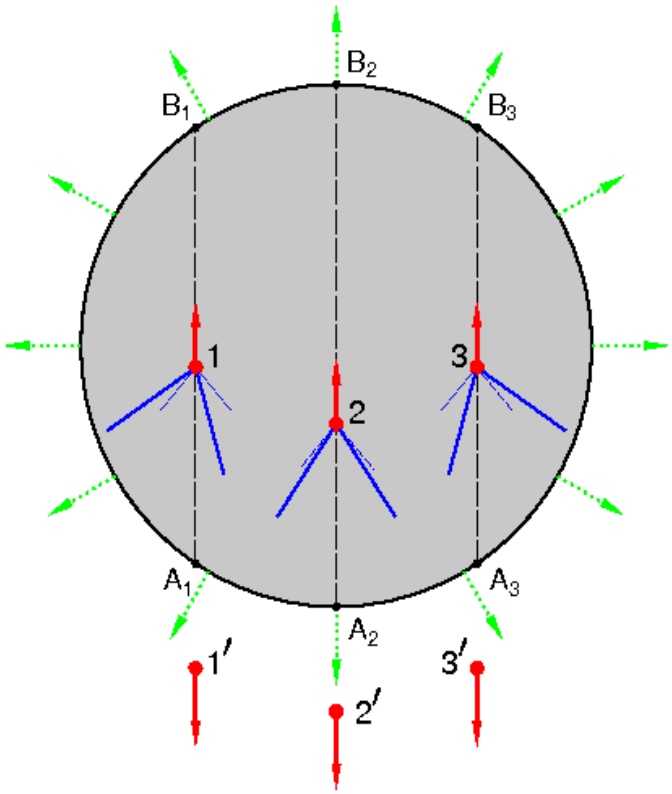
  Radial flow

  Elliptic flow

- Predictions:

  Transverse flow causes disentangling

  Expansion broadens Mach cone angle



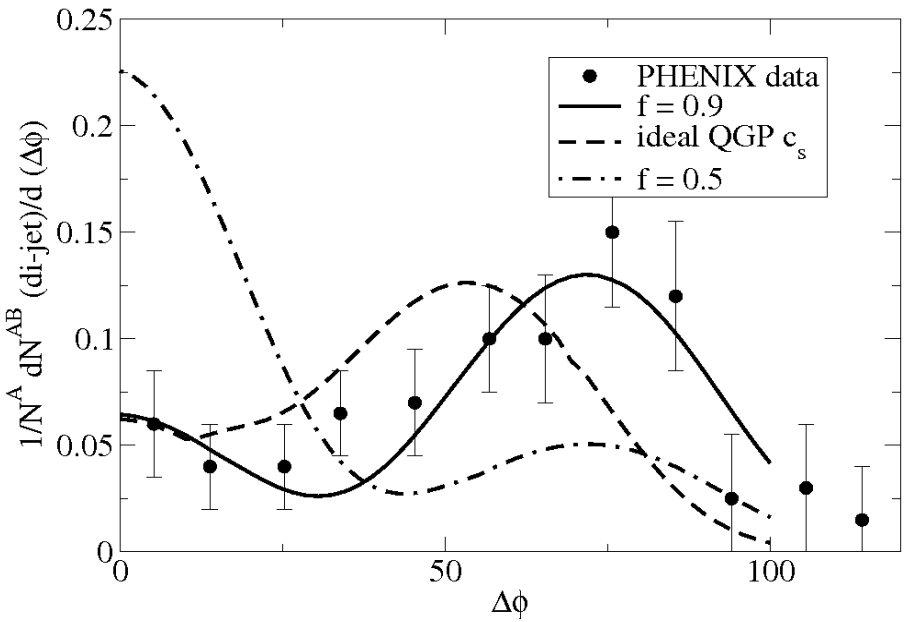
Satarov et al, PLB 627:64 (2005)

Synopsis of different jet energy loss models

Other Jet-Medium Models I

Determines angular correlation pattern

- Fireball model
- Lattice QCD EoS
- BDMPS-like energy loss



Renk and Ruppert, PRC 73:034907 (2006)

f: „fraction ... of energy lost to the medium [that] excites a collective mode“ → sound wave

(1-f): „remaining energy fraction ... [that] in essence heats the medium and leads to some amount of the collective drift along the jet axis...“ → diffusion

→ Mach cones only if dM/dx << dE/dx

Other Jet-Medium Models II

Calculates the azimuthal distribution

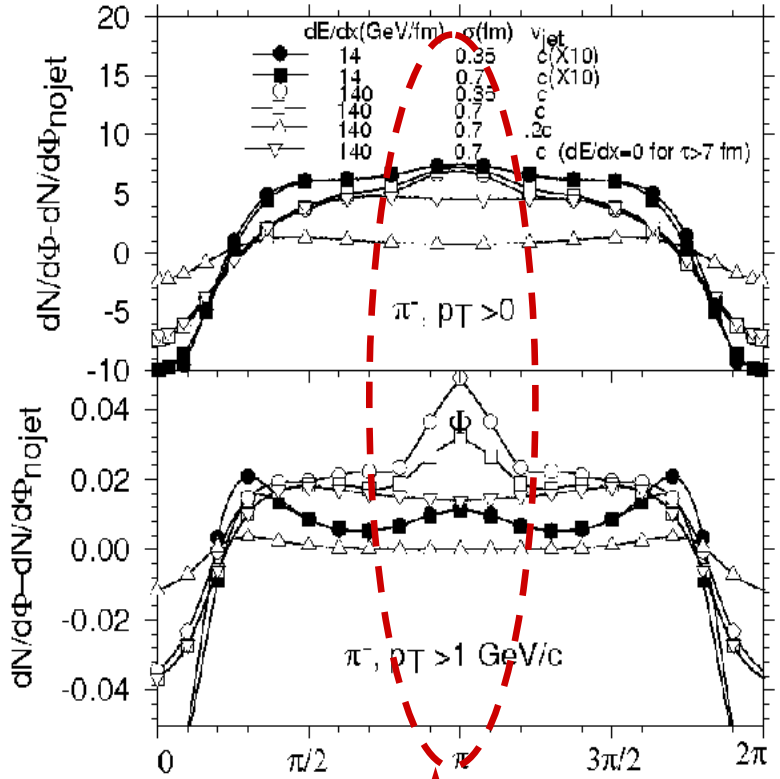
- Expanding (2+1)d hydro (AZHYDRO)
- First-order phase transition
- Isothermal Freeze-out
- Source term:

$$S^{\nu}(x)=S(x)(+1,-1,0,0)$$

$$S(x)=\frac{dE}{dx}(x)\left|\frac{dx_{jet}}{dt}\right|\delta^3(\vec{r}-\vec{r}_{jet}(t))$$

$$\frac{dE}{dx}=\frac{s(x)}{s_0}\frac{dE}{dx}\bigg|_0$$

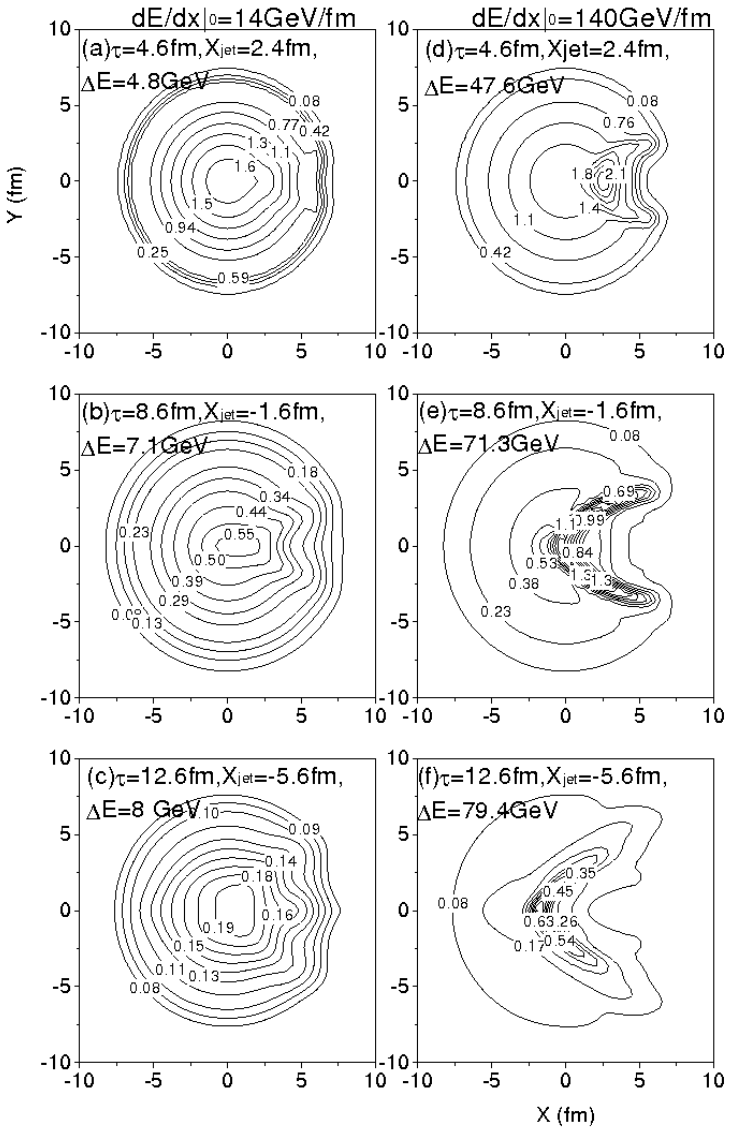
→ Quenched jet



Chaudhuri and Heinz, PRL **97**:062301 (2006)

Diffusion wake

Other Jet-Medium Models II



Chaudhuri and Heinz, PRL **97**:062301 (2006)

Effect from diffusion wake

Hottest region at the head of the jet

→ No Mach cone-like correlation if dE/dx = dM/dx