Jets in quark-gluon plasmas
Turbulent QCD cascade
Bottom-up thermalization

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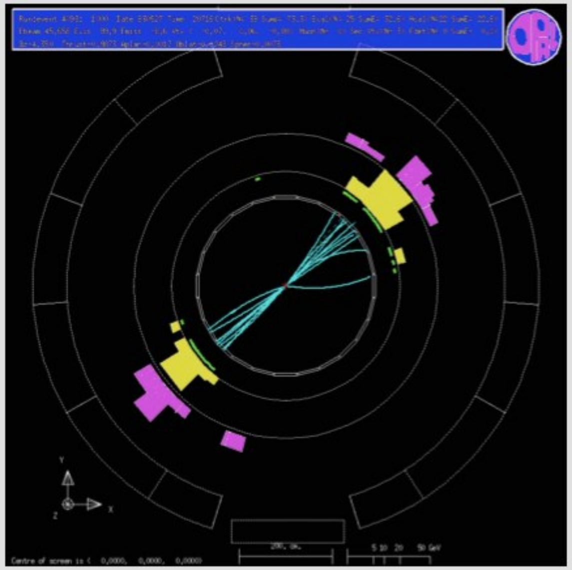
Outline

- Jets in vacuum
- Jets in quark-gluon plasmas: loss of coherence, medium-induced gluon branching
- Turbulent in-medium QCD cascade
- Bottom-up thermalization scenario
- Summary

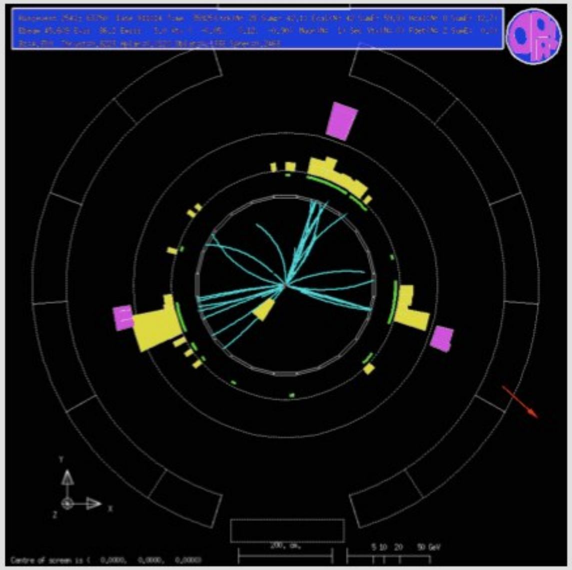
Work done in collaboration with F. Dominguez, E. Iancu and Y. Mehtar-Tani (arXiv:1209.4585, 1301.6102, 1311.5823)

Jets in 'vacuum'

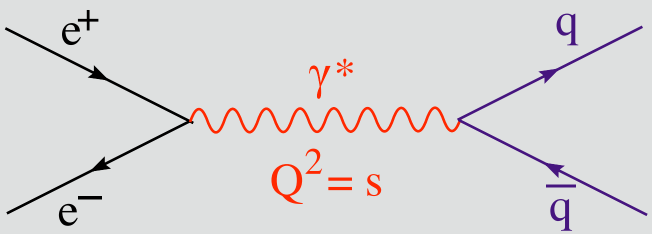
Examples of 2 and 3 jet events at LEP

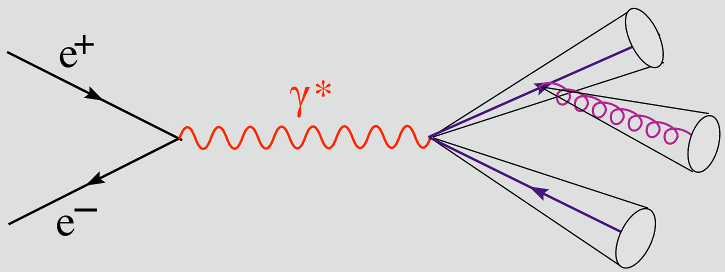


2 jets



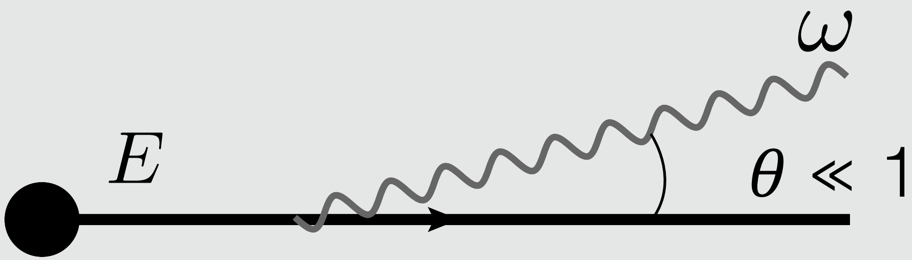
3 jets





Jets in vacuum

- Jets originate from energetic partons that successively branch into additional partons
- Elementary branching process is enhanced in the collinear region $\Longrightarrow$ Collimated jets

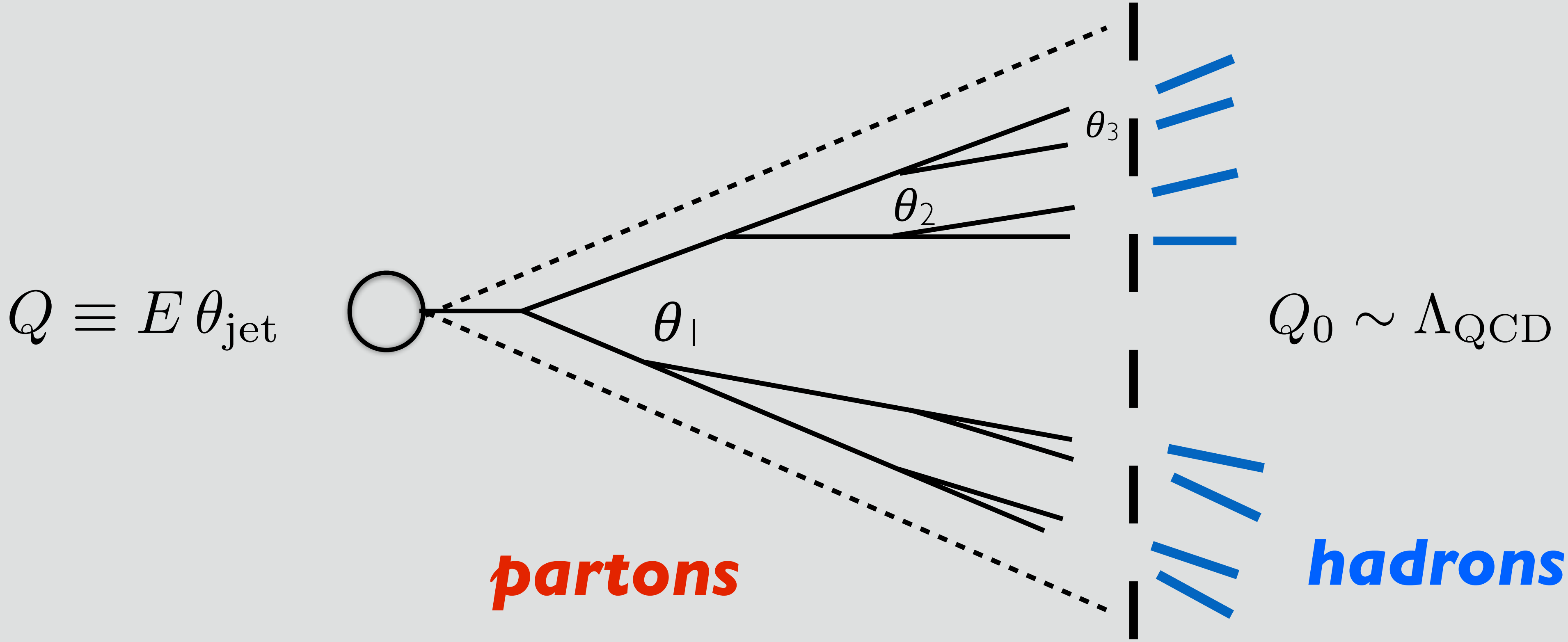


Branching probability

$$dP \sim \alpha_s \, C_R \, \frac{d\theta}{\theta} \frac{d\omega}{\omega}$$

Jets in vacuum

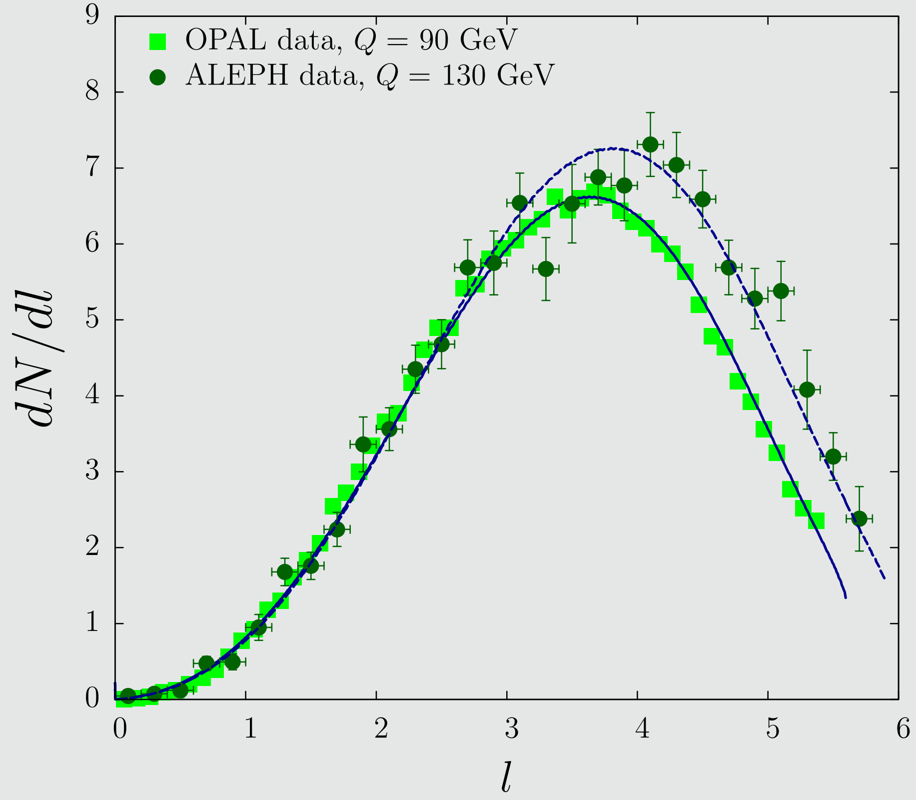
- The jet is a **coherent** object, successive branchings are ordered from larger to smaller angles $\theta_1 > \theta_2 > \ldots > \theta_n$



- large separation of scales (QCD-factorization) $Q \gg \Lambda_{QCD}$

Jets in vacuum

Fragmentation Function

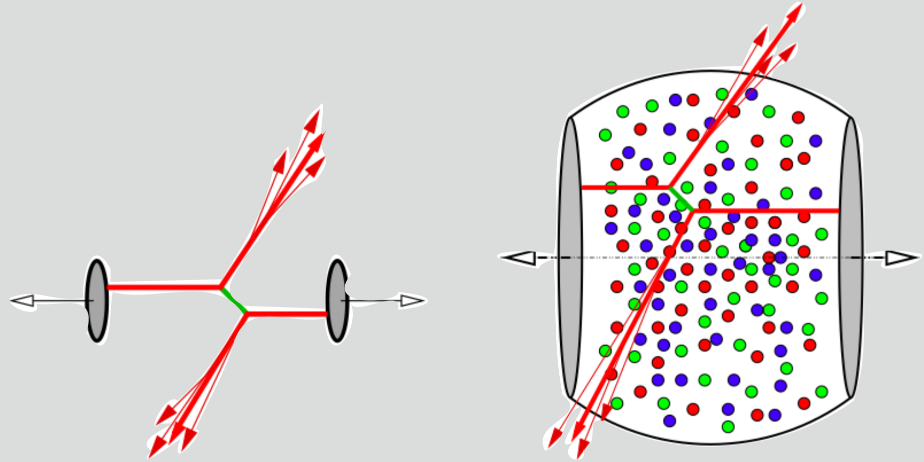


$$l = \ln(E_{\rm jet}/E_{\rm h})$$

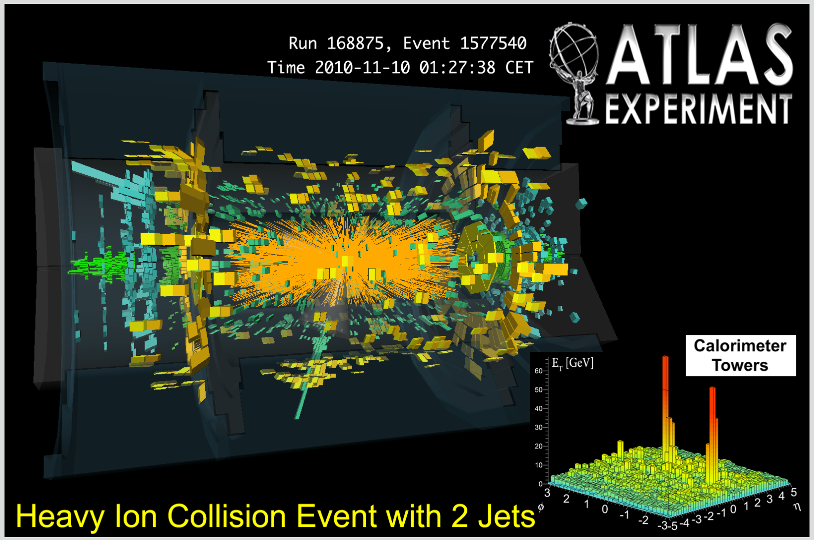
- **Perturbative QCD** prediction for the distribution of hadrons in a jet
- **2 scales:** $Q_0 = \Lambda_{QCD}$ $Q = E\,\theta_{jet}$
- **Angular Ordering:** soft gluon emissions (large l) are suppressed

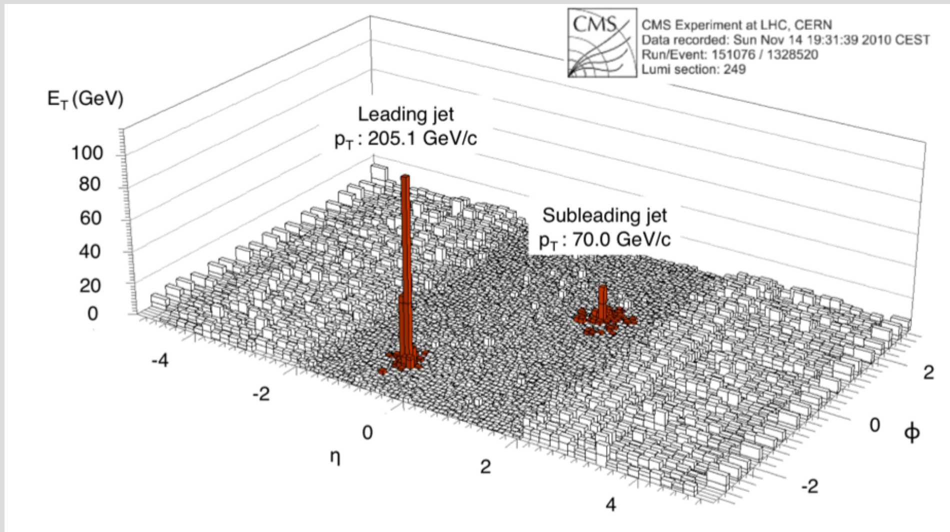
[Dokshitzer, Khoze, Mueller, Troyan, Kuraev, Fong, Webber…80']

Jets in a quark-gluon plasma



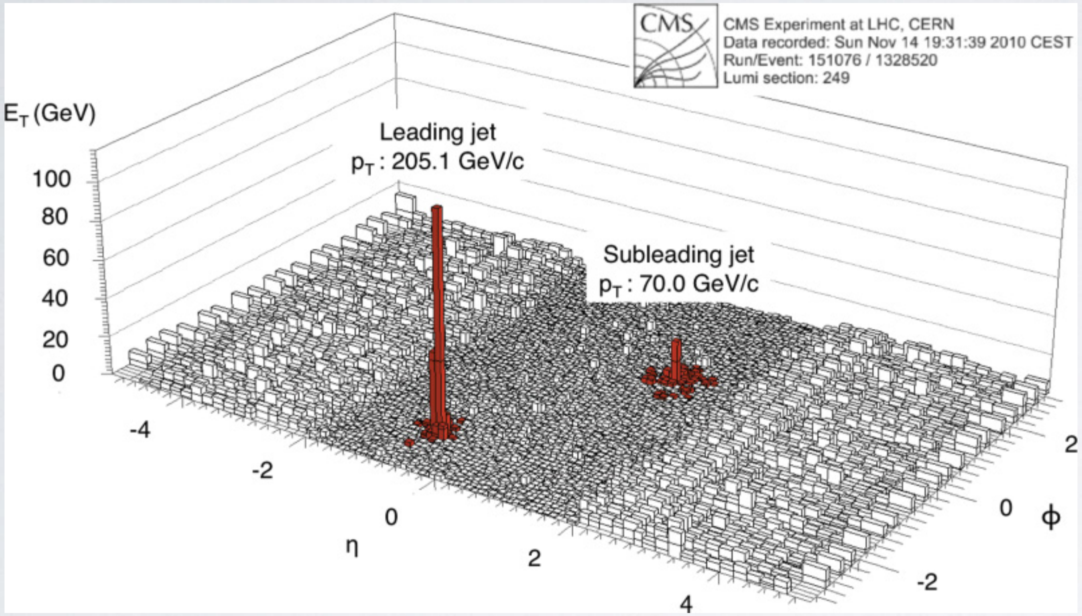
Jets are quenched due to interactions with the QGP

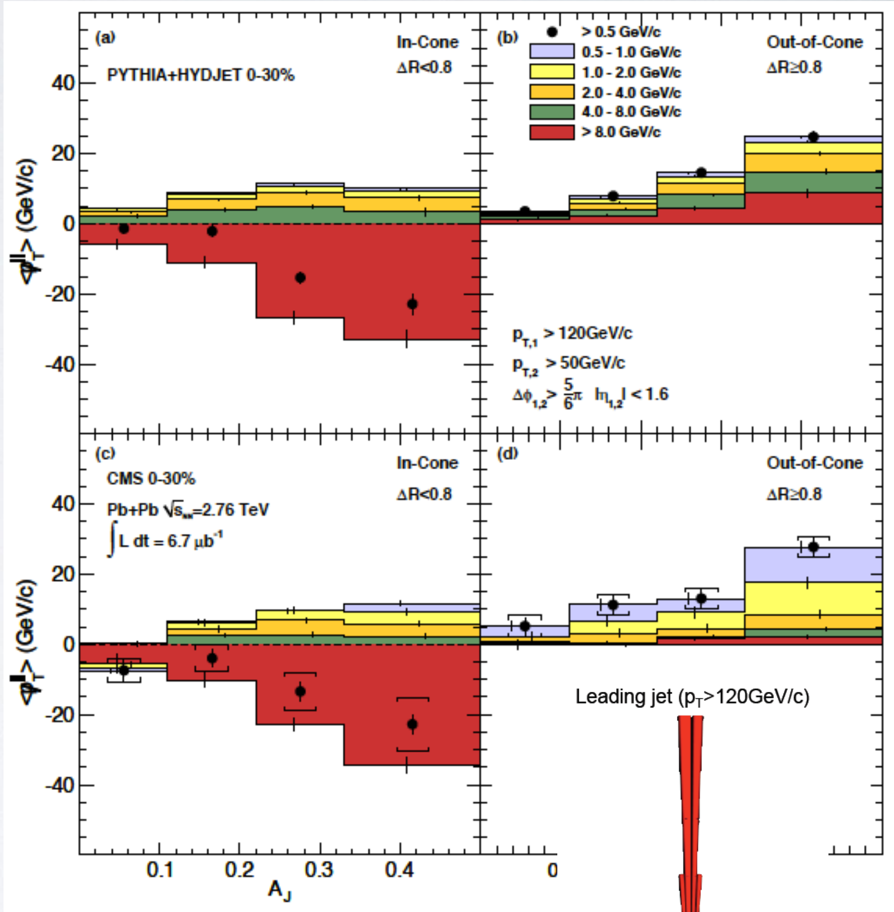


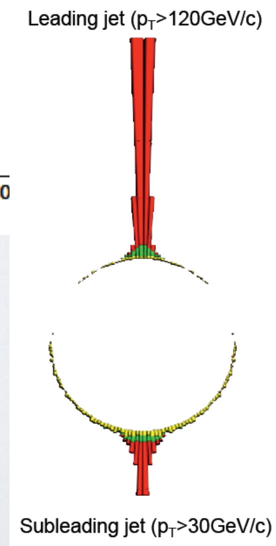


Di-jet asymmetry

there is more to it than just 'jet quenching'...







Missing energy is associated with additional **radiation of many soft quanta at large angles**

$$A_J = \frac{p_{T,1} - p_{T,2}}{p_{T,1} + p_{T,2}}$$

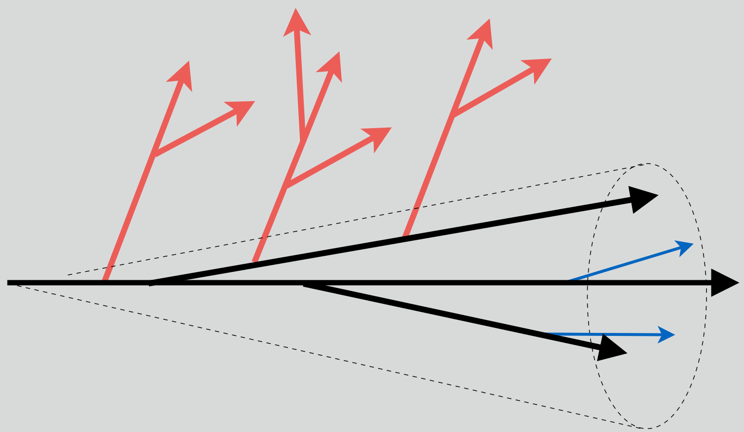
We argue that this reflects a **genuine feature of the in-medium QCD cascade** (JPB, E. Iancu and Y. Mehtar-Tani, arXiv: 1301.6102, PRL)

Jets in a quark-gluon plasma

How does the jet interacts with the medium?

- **Color coherence is altered inside a colored medium**

Y. Mehtar-Tani, K. Tywoniuk, C. A. Salgado, PRL (2011)



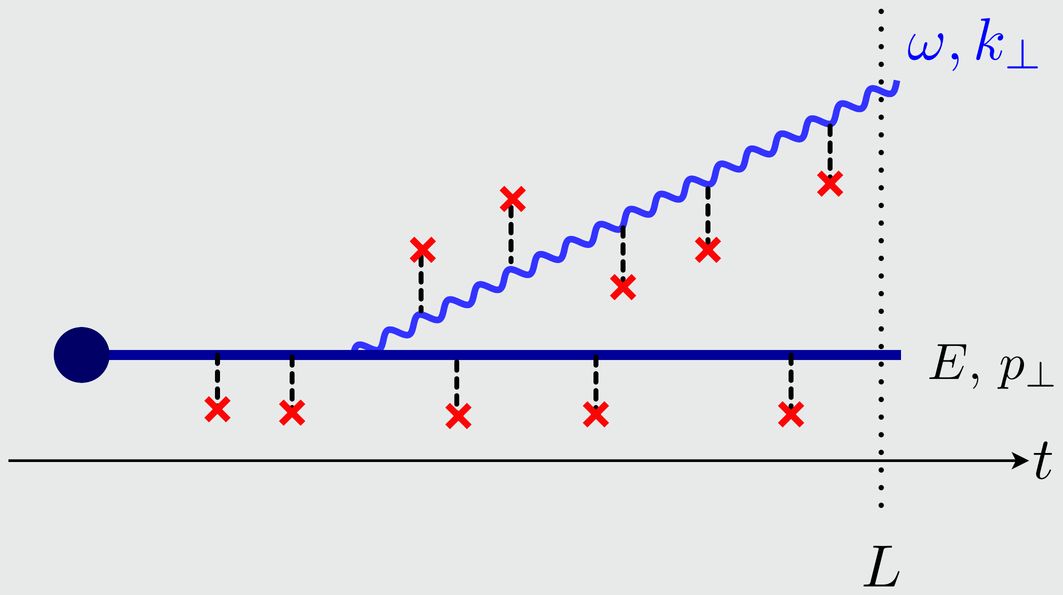
For collimated jets the medium only resolves the total color charge $\langle C_{jet} \rangle \neq 0$

- **Medium-induced radiation off the total jet charge at large angles**

J. -P. B, F. Dominguez, E. Iancu, Y. Mehtar-Tani. (2012-2013)

Medium-induced radiation

- Scatterings with the medium can induce gluon radiation
- The radiation mechanism is linked to transverse momentum broadening



$$\Delta\, k_\perp^2 \;\simeq\; \hat{q}\,\Delta t$$

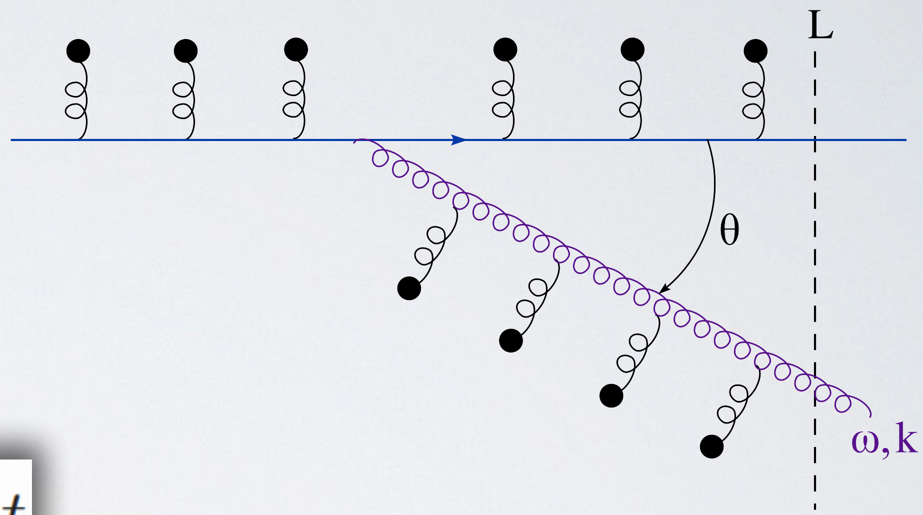
jet quenching parameter

$$\hat{q} \equiv \frac{\Delta k_\perp^2}{\Delta t} \simeq \frac{m_D^2}{\lambda} = \frac{(\text{Debye mass})^2}{\text{mean free path}}$$

[Baier, Dokshitzer, Mueller, Peigné, Schiff (1995-2000) Zakharov (1996)]

The BDMPSZ mechanism

Gluon emission is linked to momentum broadening



$$\frac{1}{\tau_{\rm f}} \sim \frac{k_\perp^2}{2\omega}$$

$$\Delta k_\perp^2 = \hat{q}\Delta t$$

Time scale for the branching process

$$\tau_{\rm br}(\omega) \sim \sqrt{\frac{2\omega}{\hat{q}}}$$

Medium of finite extent

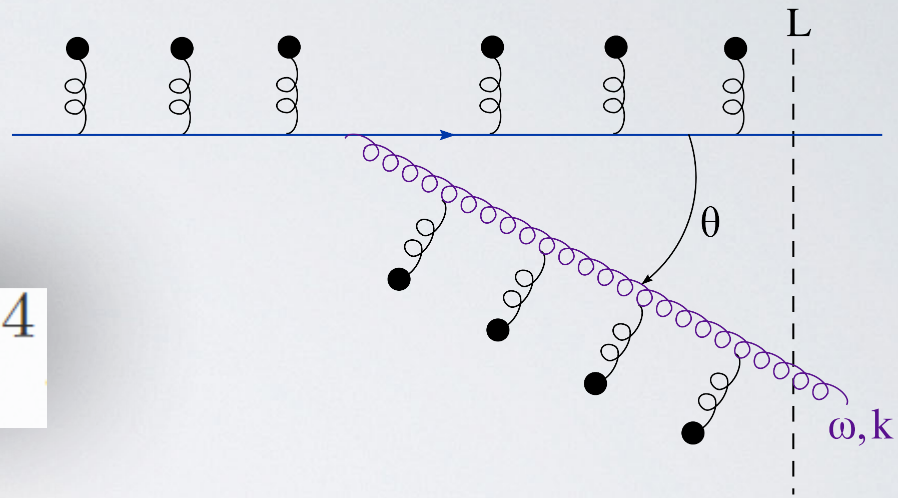
$$\tau_{\rm br} \lesssim L \Rightarrow \omega \lesssim \omega_c \qquad \omega_c \sim \hat{q}L^2$$

Formation time and emission angle

Typical branching kT and angle

$$k_{\rm br}^2 = \hat{q}\tau_{\rm br}$$

$$\theta_{\rm br} \sim k_{\rm br}/\omega \sim \left(\hat{q}/\omega^3\right)^{1/4}$$



Hard gluon: small angle, long time

$$\tau_{\rm br} \lesssim L \qquad \omega \lesssim \omega_c \qquad \theta_{\rm br} \gtrsim \theta_c$$

Soft gluon: large angle, short time

$$\tau_{\rm br} \ll L \qquad \omega \ll \omega_c \qquad \theta_{\rm br} \gg \theta_c$$

BDMPSZ spectrum

$$\omega \frac{\mathrm{d}N}{\mathrm{d}\omega} \simeq \frac{\alpha_s N_c}{\pi} \sqrt{\frac{\omega_c}{\omega}} \equiv \bar{\alpha} \sqrt{\frac{\omega_c}{\omega}} = \bar{\alpha} \frac{L}{\tau_{\mathrm{br}}(\omega)}$$

Hard emissions

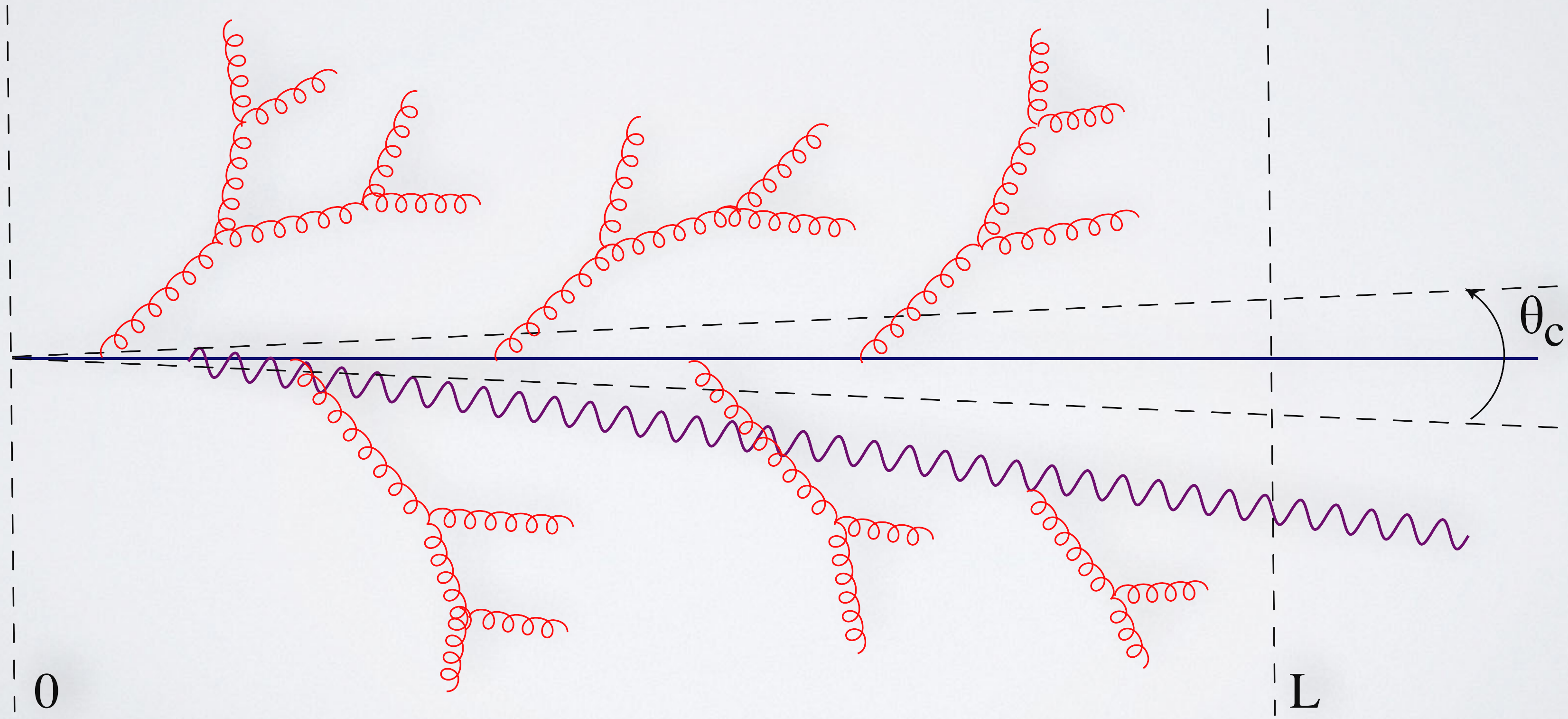
- rare events, with probability $\sim \mathcal{O}(\alpha_s)$
- dominate energy loss: $E_{\mathrm{hard}} \sim \alpha_s \omega_c$
- small angle, not important for di-jet asymmetry

Soft emissions

- frequent, with probability $\sim \mathcal{O}(1)$
- weaker energy loss: $E_{\mathrm{soft}} \sim \alpha_s^2 \omega_c$
- but arbitrary large angles: control di-jet asymmetry

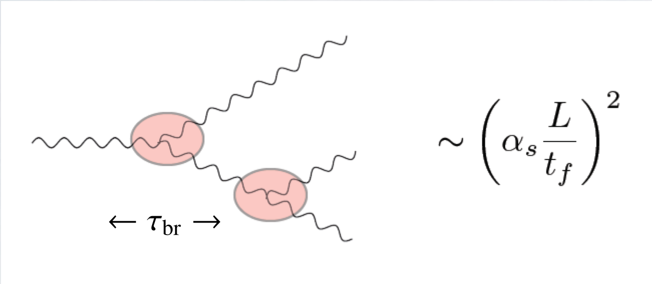
large angles emissions are dominated by soft multiple branchings

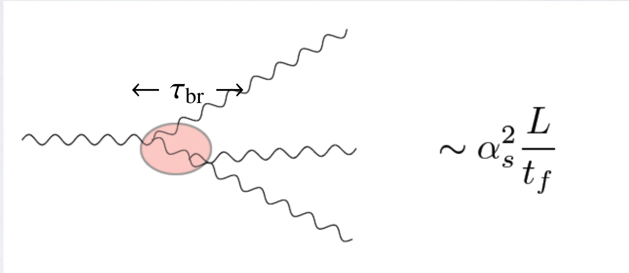
Multiple branchings (de)-coherence in-medium cascade



Multiple emissions

In medium, interference effects are subleading, and independent emissions are enhanced by a factor L/τ_f

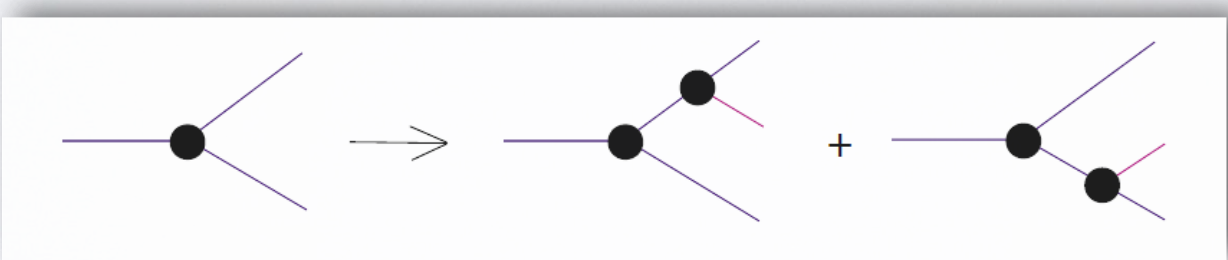




When $\bar{\alpha} L/\tau_{br} \sim 1$ all powers of $\bar{\alpha} L/\tau_{br} \sim 1$ need to be resummed.

Since independent emissions dominate, the leading order resummation is equivalent to a probabilistic cascade, with nearly local branchings

JPB, F. Dominguez, E. Iancu, Y. Mehtar-Tani, arXiv: 1209.4585



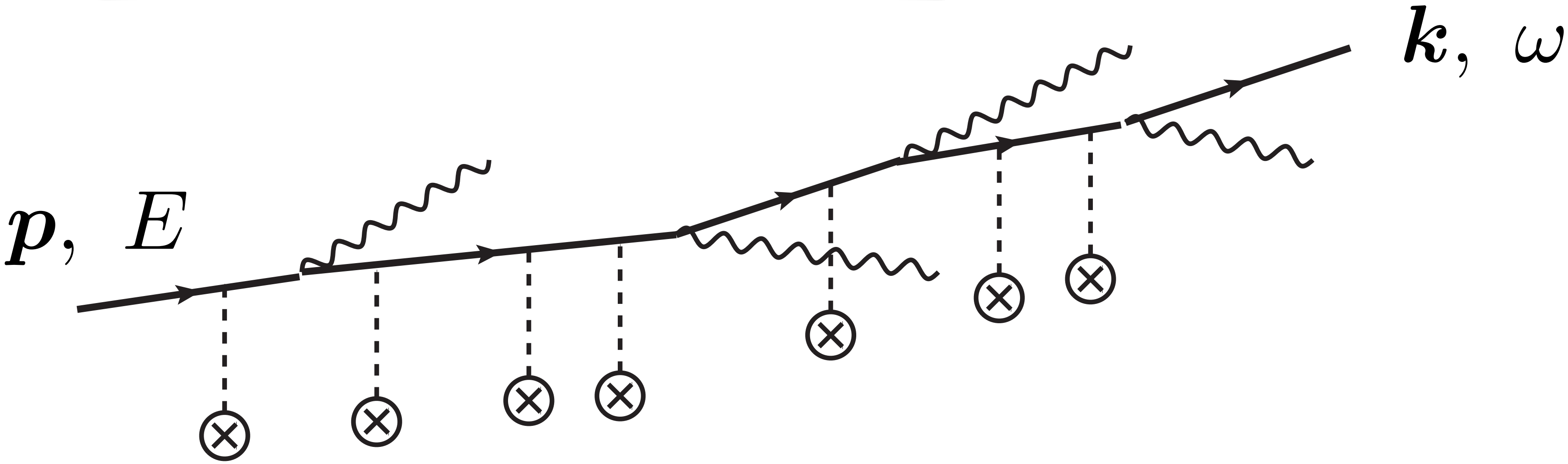
Note: already implemented in some Monte Carlo codes (MARTINI, Q_Pythia, etc)

Inclusive Gluon Distribution

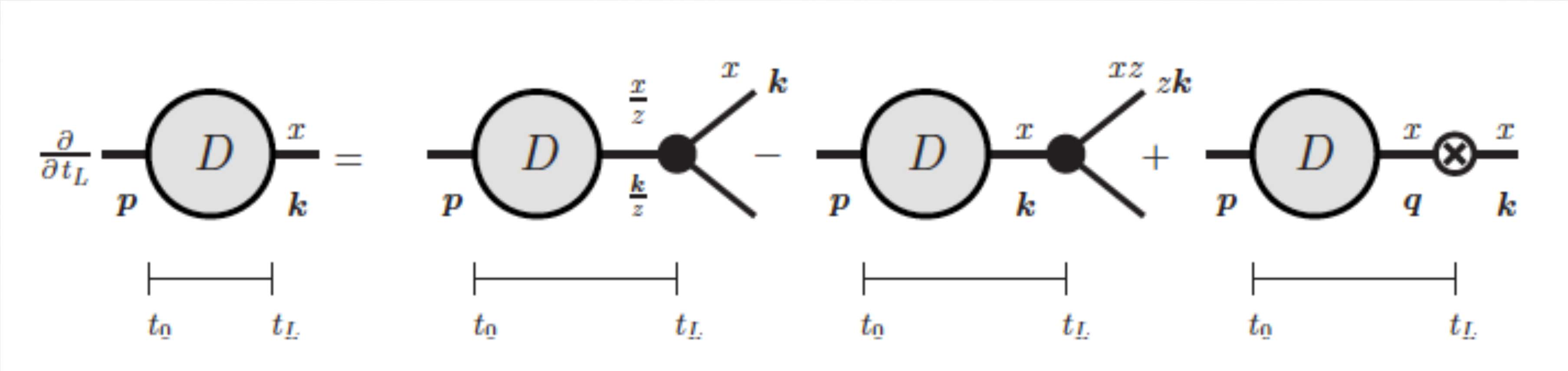
Density of gluons with momentum k inside a parton with momentum p:

$$\boxed{x\,\frac{dN}{dx\,d^2\boldsymbol{k}} \equiv D(x,\boldsymbol{k},t)}$$

$$x = \omega/E$$



Inclusive one-gluon distribution



Leading order equation

$$\frac{\partial}{\partial t} D(x, \boldsymbol{k}, t) = \int_{\boldsymbol{l}} \mathcal{C}(\boldsymbol{l}, t) D(x, \boldsymbol{k} - \boldsymbol{l}, t) + \alpha_s \int_0^1 \mathrm{d}z \left[\frac{2}{z^2} \mathcal{K}\left(z, \frac{x}{z} p_0^+; t\right) D\left(\frac{x}{z}, \frac{\boldsymbol{k}}{z}, t\right) - \mathcal{K}\left(z, x p_0^+; t\right) D(x, \boldsymbol{k}, t) \right]$$

Wave turbulence

S. Nazarenko, 'Wave turbulence' , Springer lecture notes in physics, 2001

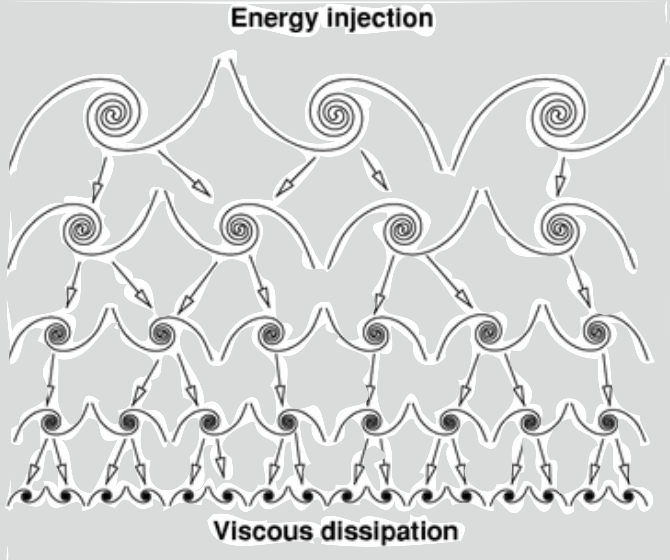
V. Zakharov, V. L'vov, and G. Flakovich,
'Kolmogorov spectra of turbulence, wave turbulence' (Springer-Verlag, 1992)

Di-jet asymmetry and turbulence

soft particles

large angles

Richardson cascade 1921



- gain = loss $\Longrightarrow$ the energy flux is independent of x
- energy flows from large to low frequencies and large angles without accumulating (signature of wave turbulence)

Efficient mechanism for energy transport at large angles

Energy flow through democratic branching

Integrating over transverse momentum yields equation for energy flow

$$\frac{\partial D(x,\tau)}{\partial \tau} = \int \mathrm{d}z\, \mathcal{K}(z) \left[\sqrt{\frac{z}{x}} D\left(\frac{x}{z}, \tau\right) - \frac{z}{\sqrt{x}} D(x,\tau) \right]$$

$$\mathcal{K}(z) = \frac{\bar{\alpha}}{2} \frac{f(z)}{[z(1-z)]^{3/2}}, \qquad f(z) = [1 - z(1-z)]^{5/2}$$

Similar eq. postulated: R. Baier, A. H. Mueller, D. Schiff, D.T. Son (2001) S. Jeon, G. D. Moore(2003)

Formally analogous to DGLAP. But very different kernel... and physics.

$\Longrightarrow$ **A QCD cascade of a new type**

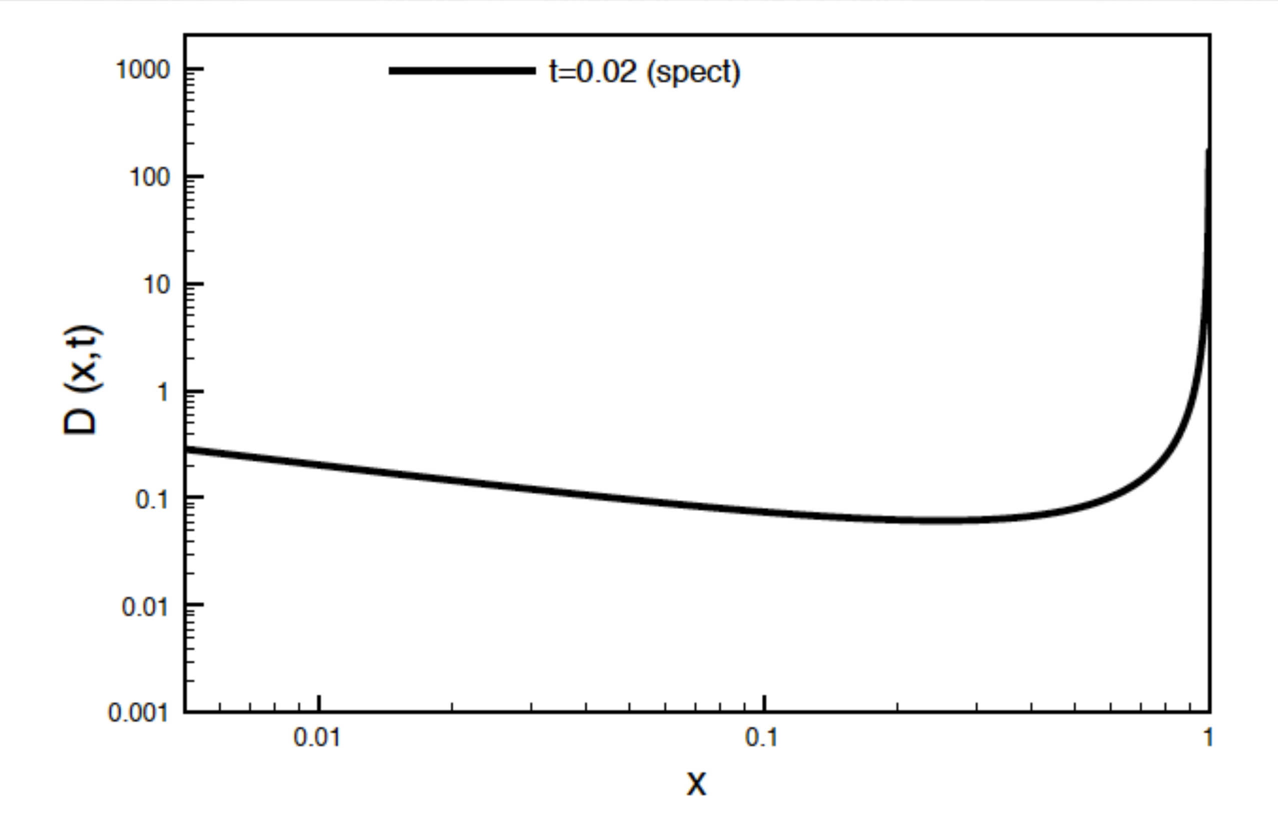
Exhibits wave turbulence

Short times

$$\frac{\partial D(x,\tau)}{\partial \tau} = \int \mathrm{d}z\, \mathcal{K}(z) \left[\sqrt{\frac{z}{x}} D\left(\frac{x}{z}, \tau\right) - \frac{z}{\sqrt{x}} D(x,\tau) \right]$$

At short time, single emission by the leading particle $(D_0(\tau = 0, x) = \delta(x-1))$

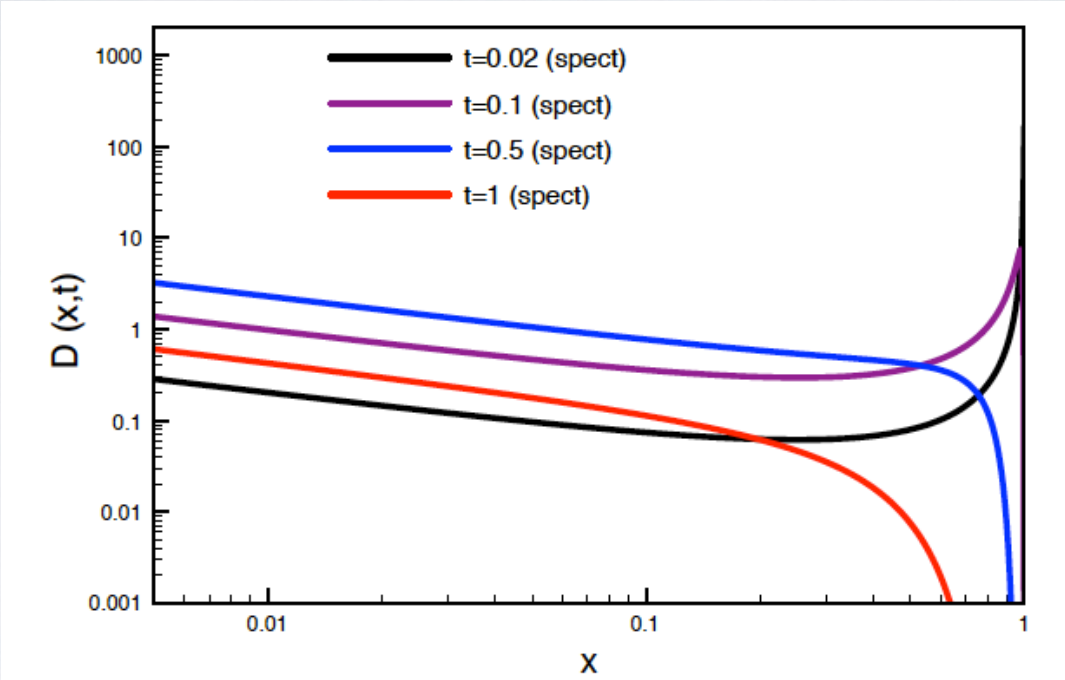
D is the BDMSZ spectrum



How do multiple branchings affect this spectrum ?

One finds (exact result)

$$D(x,t) \simeq \frac{t}{\sqrt{x}}\, \mathrm{e}^{-\pi t^2} \qquad \text{for } x \ll 1$$



Fine (local) cancellations between gain and loss terms

BDMPS spectrum emerges as a fixed point, scaling, spectrum

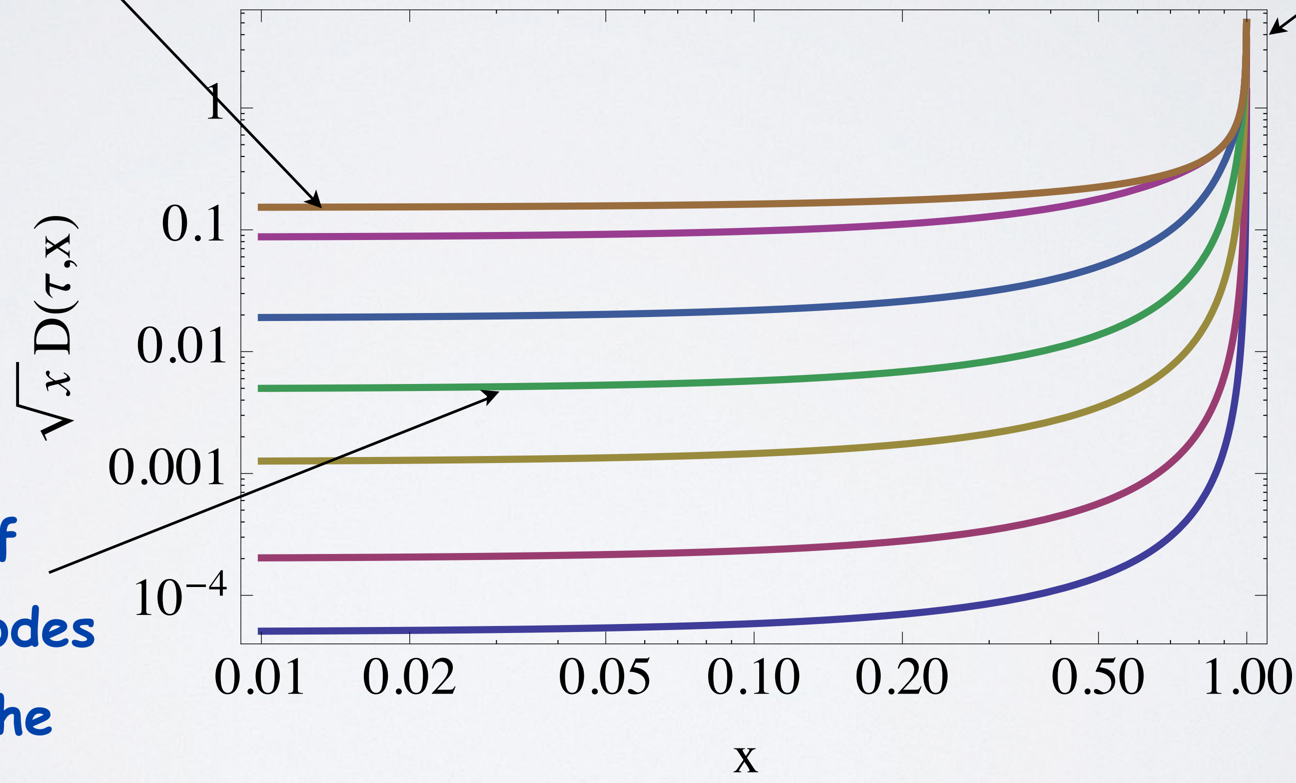
Characteristic features of wave turbulence (Kolmogorov, Zakharov)

Kolmogorov exponent 1/2

Digresssion: source problem

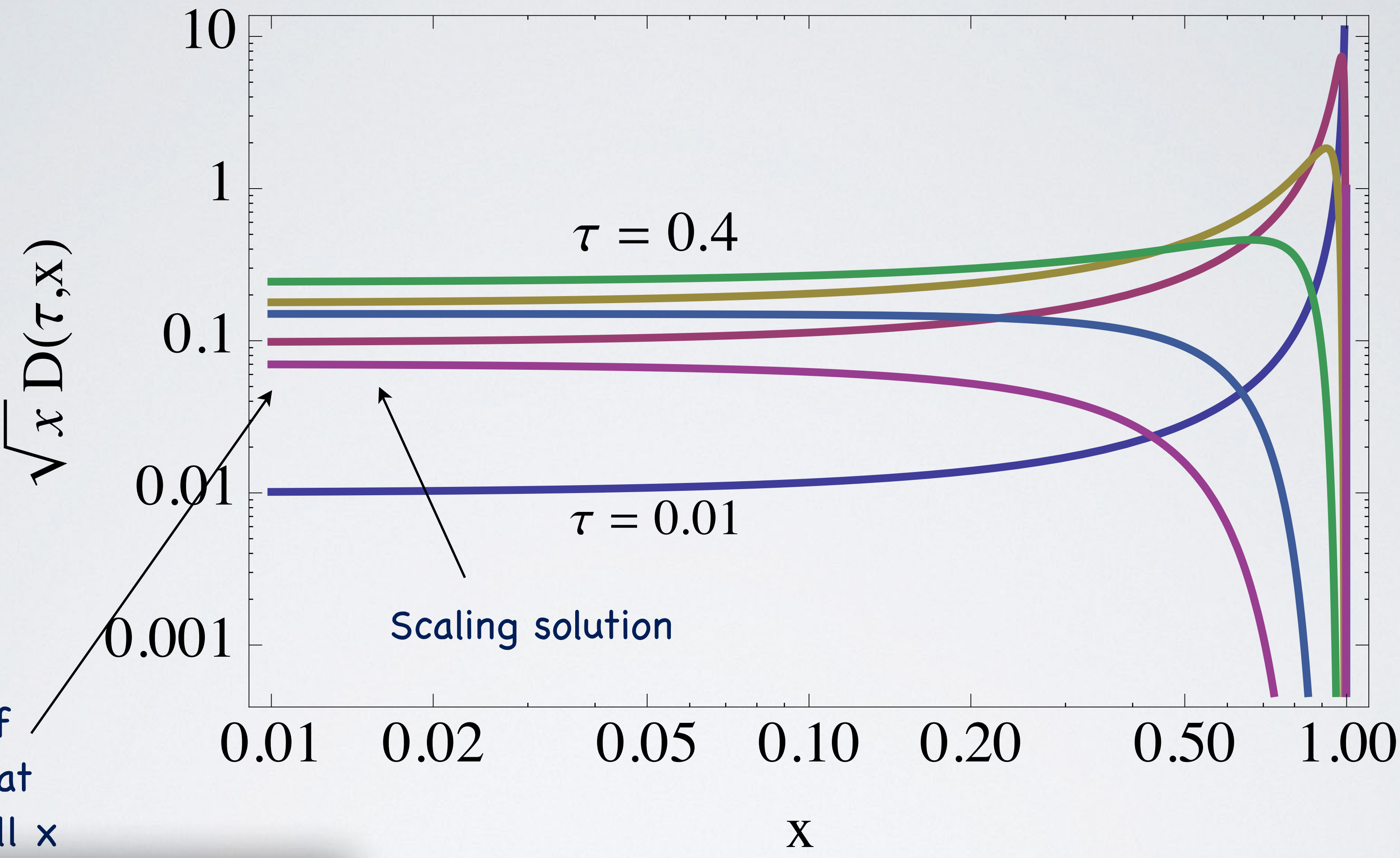
At this (fixed) point ALL the energy flows through the whole system

Energy is injected at x=1, at a constant rate



The population of the various x-modes grows, keeping the shape of the spectrum at small x

Relevance to di-jet asymmetry



Flow of energy at very small x

$$\mathcal{E}_{\rm flow} = E\frac{\upsilon\tau^2}{2} = \frac{\upsilon}{2}\,\bar\alpha^2\omega_c$$

$$\omega_c \equiv \frac{\hat{q}L^2}{2} \qquad \upsilon \simeq 5$$

Estimate $\hat{q} = 1\,\mathrm{GeV}^2/\mathrm{fm}$ $\quad \omega_c \simeq 40$ Gev $\quad \bar\alpha^2 \simeq 0.1$ $\quad \mathcal{E}_{\rm flow} \simeq 15$ GeV

$L = 4$ fm

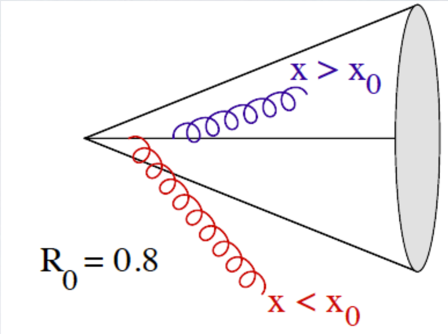
J.-P. B., E. Iancu, Y. Mehtar-Tani, PRL 2013

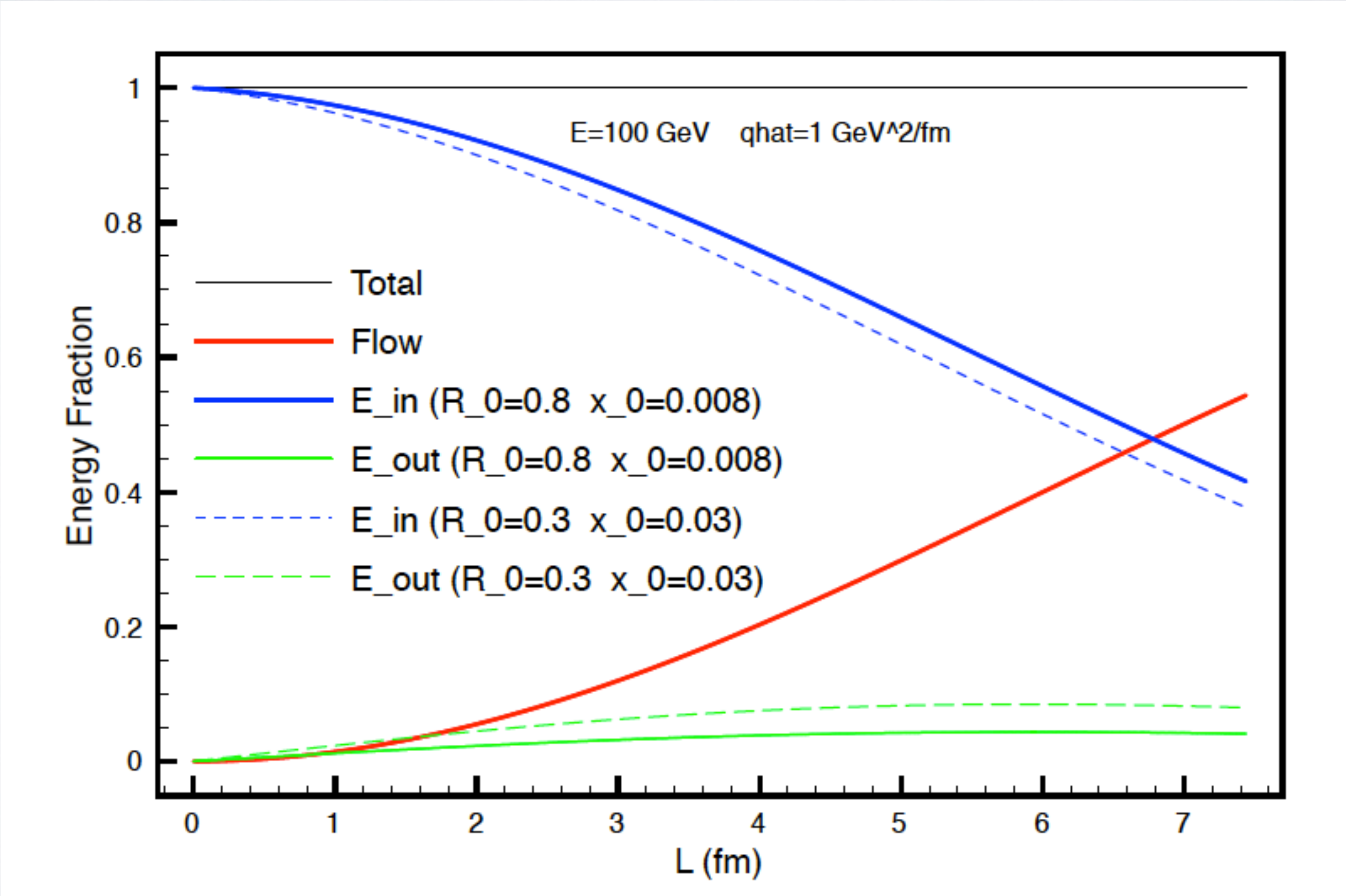
Energy flow at large angle

$E_{\rm in}$ energy in the jet with x>xo

$E_{\rm out}$ energy in the spectrum with x<xo

$E_{\rm out} + E_{\rm flow}$ energy out of the jet cone





Thermalization of the quark-gluon plasma

R. Baier, A. H. Mueller, D. Schiff and D. T. Son, Phys. Lett. B 502, 51 (2001)

J. Berges, K. Bogulavski, S. Schlichting and R. Venugopalan, arXiv: 1303.5650

Bottom-up thermalization

R. Baier, A. H. Mueller, D. Schiff and D. T. Son, Phys. Lett. B 502, 51 (2001)

Based on kinetics after time $\tau \gtrsim Q_s^{-1}$

Assumes initial gluon density of the form $$\frac{\mathrm{d}N}{\mathrm{d}y\mathrm{d}k_\perp^2} = \frac{1}{\alpha} f\left(\frac{k_\perp}{Q_s}\right)$$

1. Fast emission of soft gluons

2. Quick equilibration of the soft gluons which form a thermal bath

3. Hard gluons loose their energy to the soft bath,
till complete thermalization

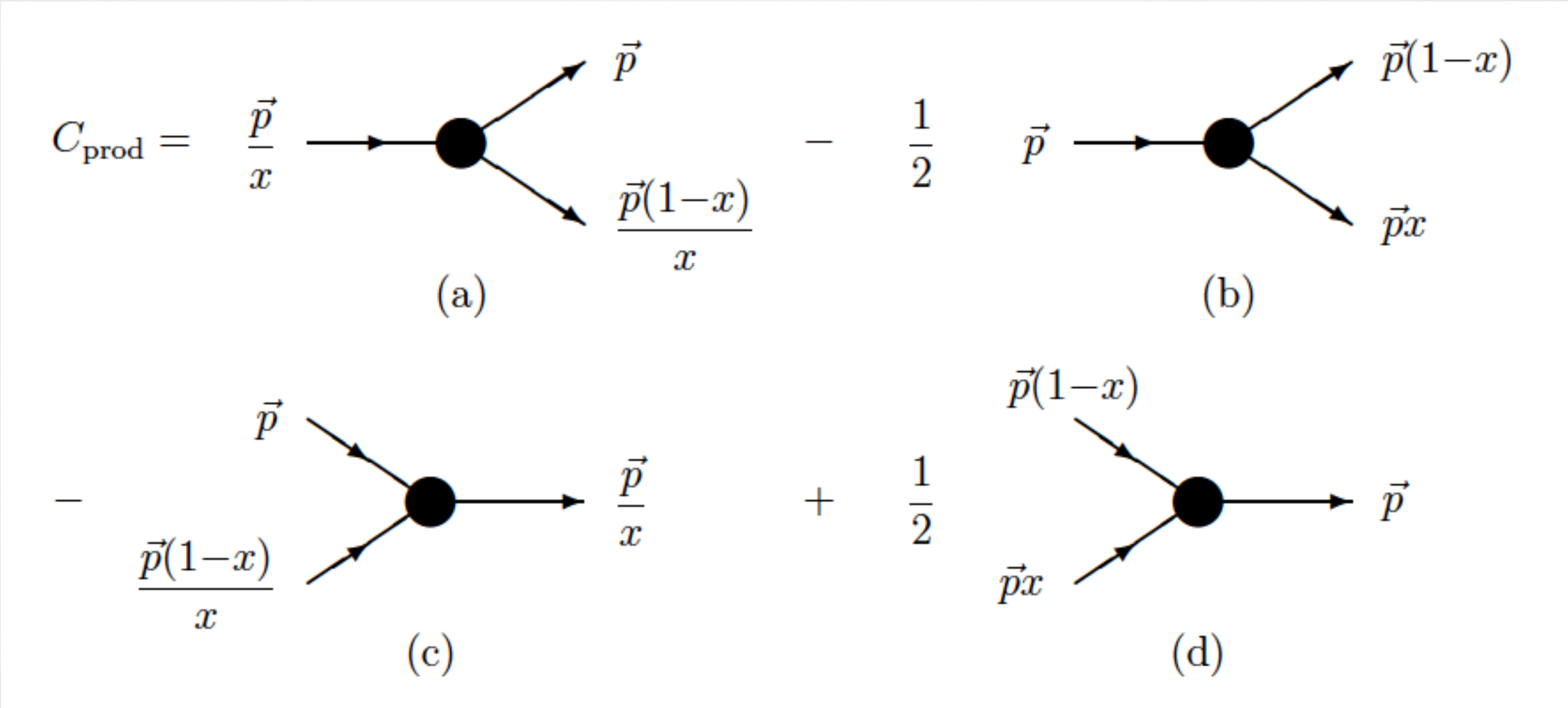
This last phase shares common features with the in-medium QCD cascade described earlier.

Bottom-up last stage

analysis using kinetic theory

$$\left(\frac{\partial}{\partial\tau}-\frac{p_z}{\tau}\frac{\partial}{\partial p_z}\right)f(\vec{p})=C_{\rm el}+C_{\rm prod}$$

inelastic contributions are like gain and loss term in the branching processes



Bottom-up last stage

energy carried by hard gluons $\int dp_\perp \epsilon(p_\perp)$

$$\epsilon(p_\perp) = 2\pi p_\perp^2 \int dp_z \, f(\vec{p})$$

kinetic equation reduces to

$$\frac{1}{\tau}\frac{\partial}{\partial\tau}(\tau\epsilon(p_\perp)) = \frac{\alpha^2 N^{1/2}}{p_\perp^{1/2}} \int dx\, h(x) \left[x^{1/2}\epsilon\left(\frac{p_\perp}{x}\right) - \frac{1}{2}\epsilon(p_\perp)\right]$$

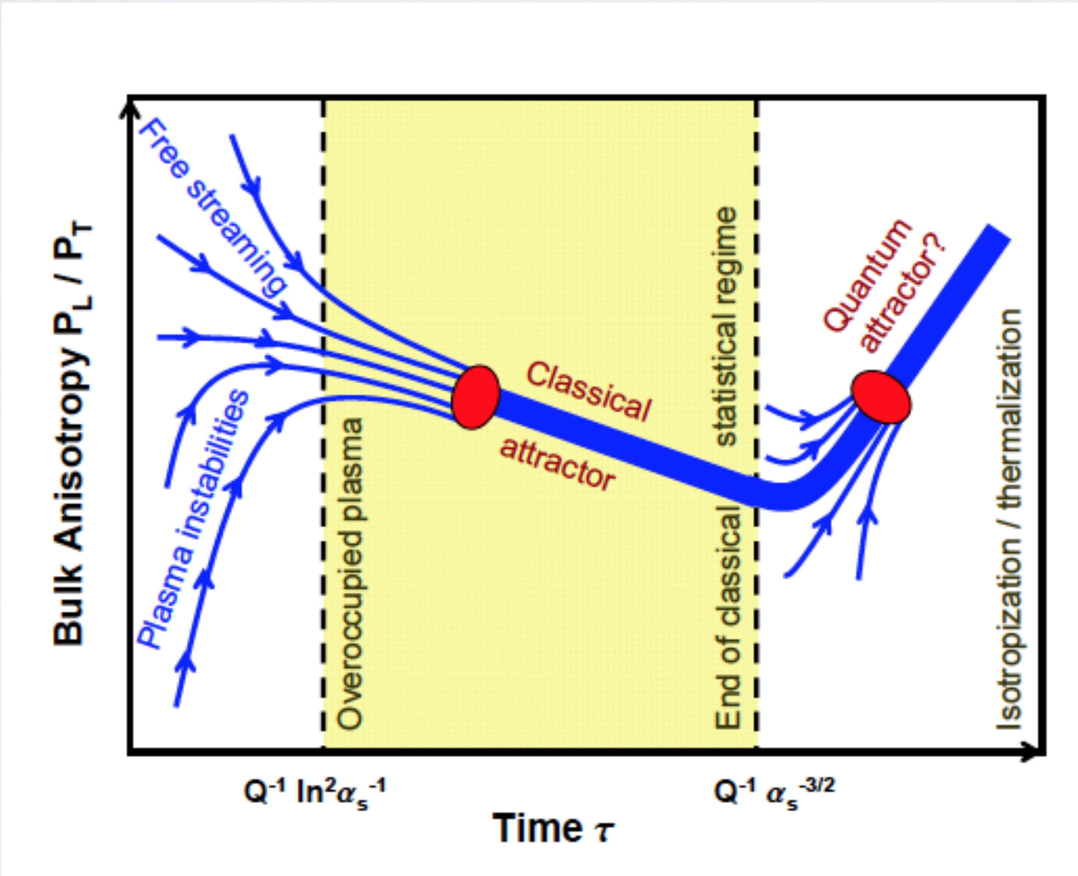
$$h(x) = h_0 \frac{(1-x+x^2)^{5/2}}{(x-x^2)^{3/2}}$$

analogous to the equation controlling the evolution of the spectrum for the in-medium QCD cascade. Same fixed point (stationary solution) with

$$\epsilon(p) \sim 1/\sqrt{p}$$

Classical-statistical simulations

such fixed point solutions appear to control the dynamics in classical-statistical (real-time) simulations



J. Berges, K. Bogulavski, S. Schlichting and R. Venugopalan, arXiv: 1303.5650

Summary

In a medium of large size, the successive branchings can be treated as independent, giving rise to a cascade that is very different from the vacuum cascade (no angular ordering, turbulent flow)

This turbulent cascade provides a simple and natural mechanism for the transfer of jet energy towards very large angles. The mechanism is intrinsic, not related to a specific coupling between the jet and the medium.

This turbulent cascade may play a role in the latest stages of the thermalization of the quark-gluon plasma produced in ultra-relativistic heavy ion collisions