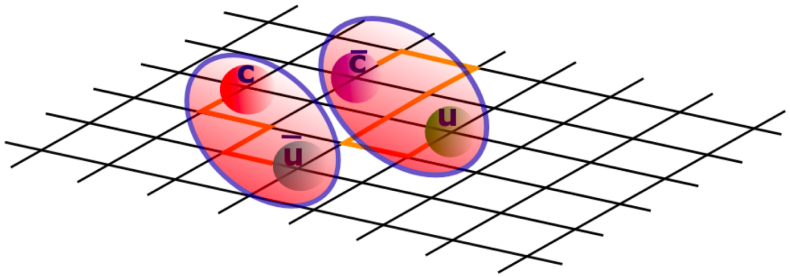
Exotic and conventional hadrons from lattice QCD



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Jefferson Lab, Virginia, USA

colloquium & BNL, NY, 28th July 2015

Outline

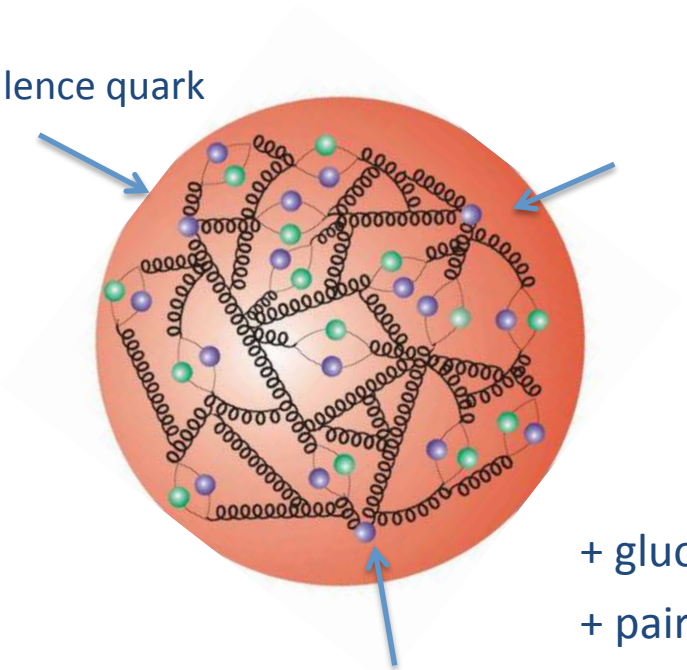
- Experimental appetizer concerning exotic hadrons:
  - very recent discovery of pentaquarks
  - recent discoveries of tetraquarks (just few examples)

- Theoretical approach to study
  conventional and exotic hadrons
  based directly on QCD – lattice QCD

Hadrons

Conventional

valence quark



+ gluons

+ pairs of sea $\underline{q}q$

Exotic

minimal valence content

$\bar{q}\ \bar{q}\ q\ q$ tetraquarks

$\bar{q}\ q\ q\ q\ q$ pentaquarks

.... hybrids

glueballs

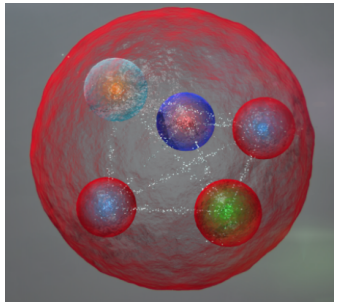
....

Terminology in this talk: tetra(penta) quarks indicate just the number of valence quarks in the state; it is not meant to say anything on how quarks are clustered in them

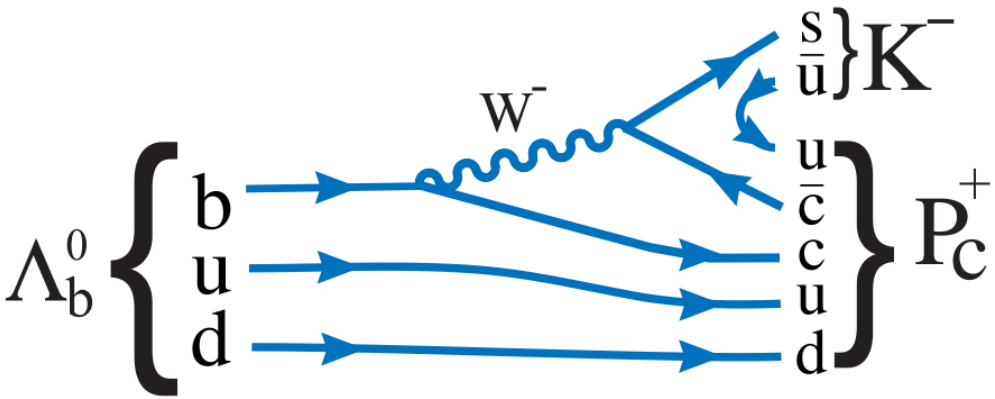
Pentaquarks

LHCb: 1507.03414

14th July 2015

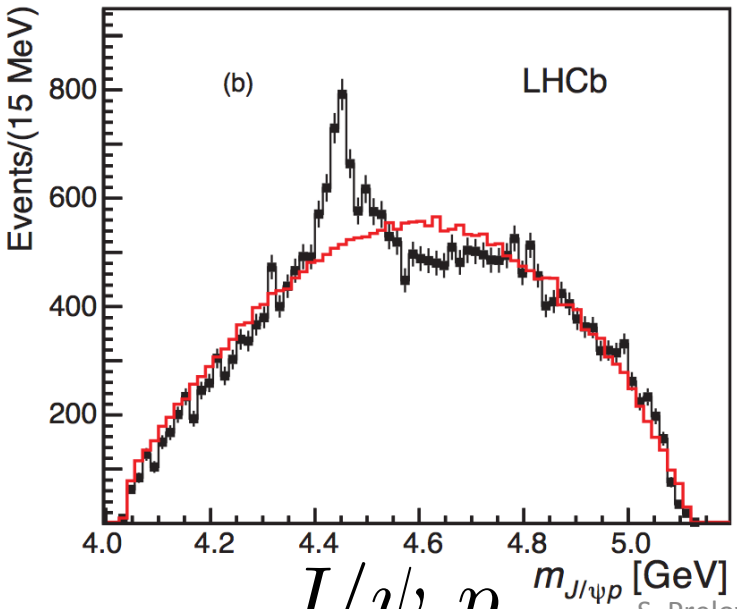


$$\Lambda_b^- \to K^- J/\psi \; p$$

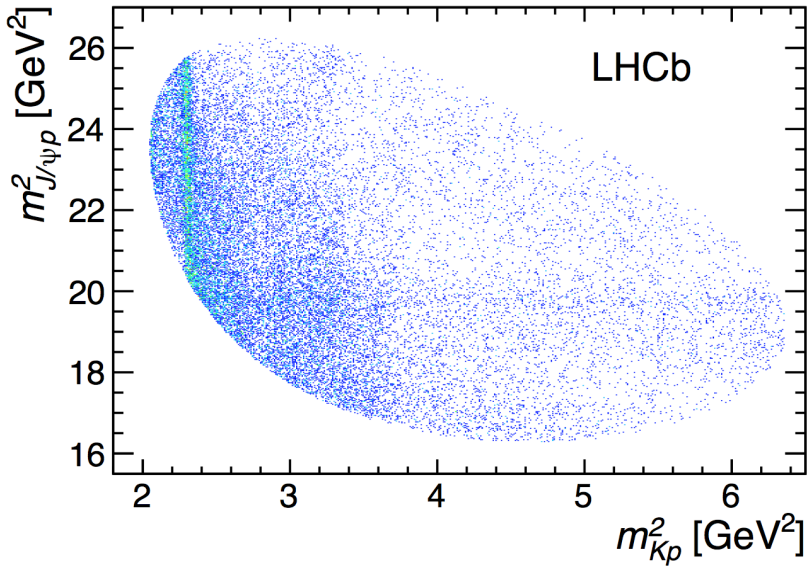


$$P_c^+ \to J/\psi \;\; p$$

$$\bar{c}c \;\; uud$$



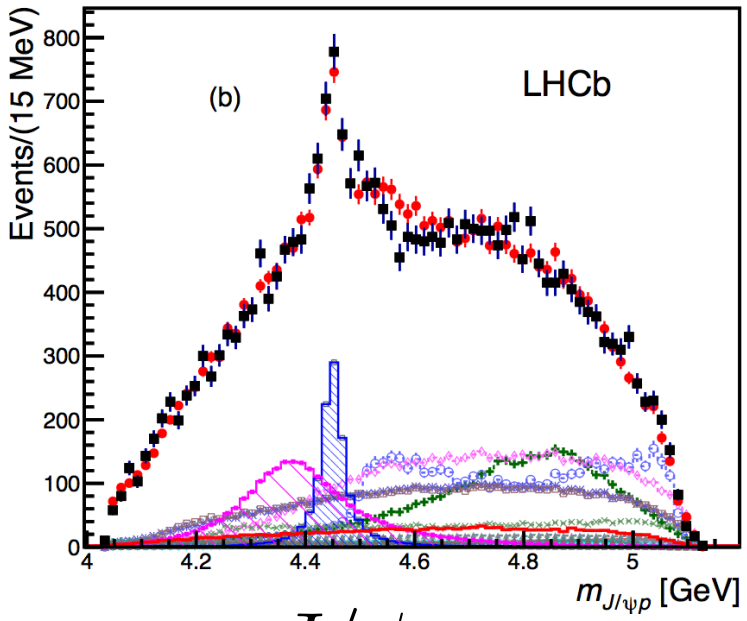
$J/\psi \; p$



$J/\psi \; p$

Two pentaquark candidates from LHCb

LHCb: 1507.03414

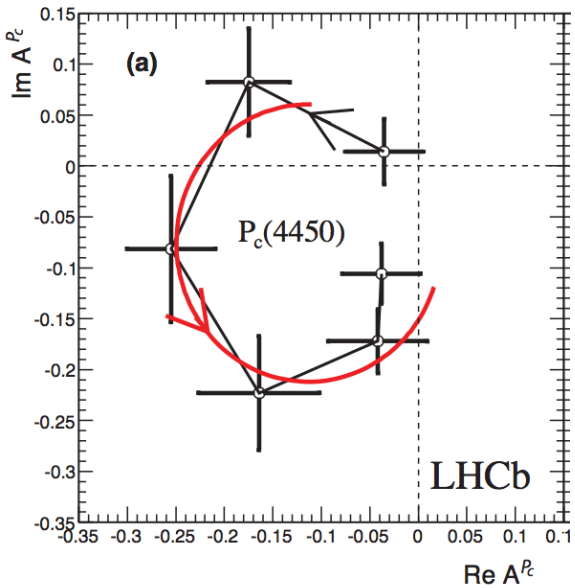


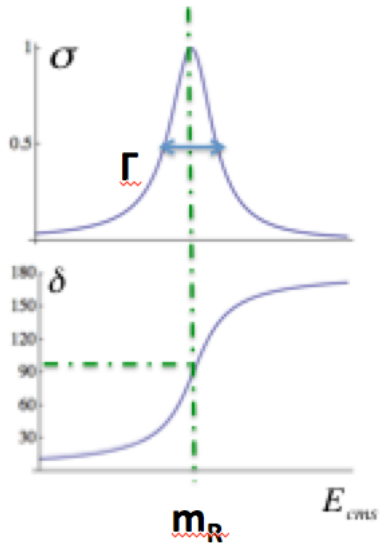
$J/\psi\ p$

scattering matrix T for elastic scattering

$$A \simeq T = \frac{1}{2i}(e^{2i\delta(E)} - 1)$$

$$|T + \frac{1}{2i}| = |\frac{1}{2i}e^{2i\delta(E)}| = \frac{1}{2}$$





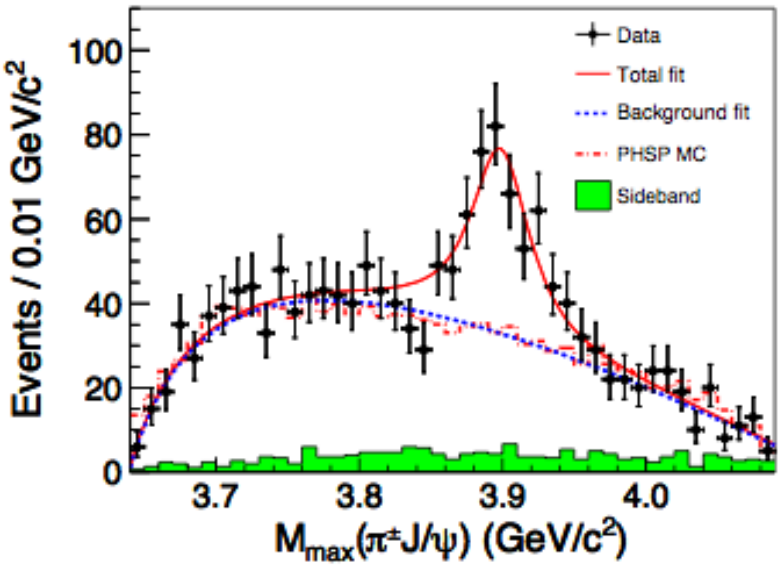
$P_c(4380)$: $M_1 = 4380 \pm 8 \pm 29$ MeV $\Gamma_1 = 205 \pm 18 \pm 86$ MeV $J^P = 3/2^+$ or $5/2^+$

$P_c(4450)$: $M_2 = 4449.8 \pm 1.7 \pm 2.5$ MeV $\Gamma_2 = 39 \pm 5 \pm 19$ MeV $J^P = 5/2^-$ or $3/2^-$

Tetraquarks Z_c

BESIII, Belle, Cleo-c 2013

Example: $Z_c^+(3900)$



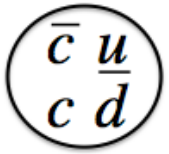
DD* thr.

[BESIII, 2013, 1303.5949, PRL]

state confirmed by Belle, Cleo-c

$$Z_c^+(3900) \to J/\psi\ \pi^+$$

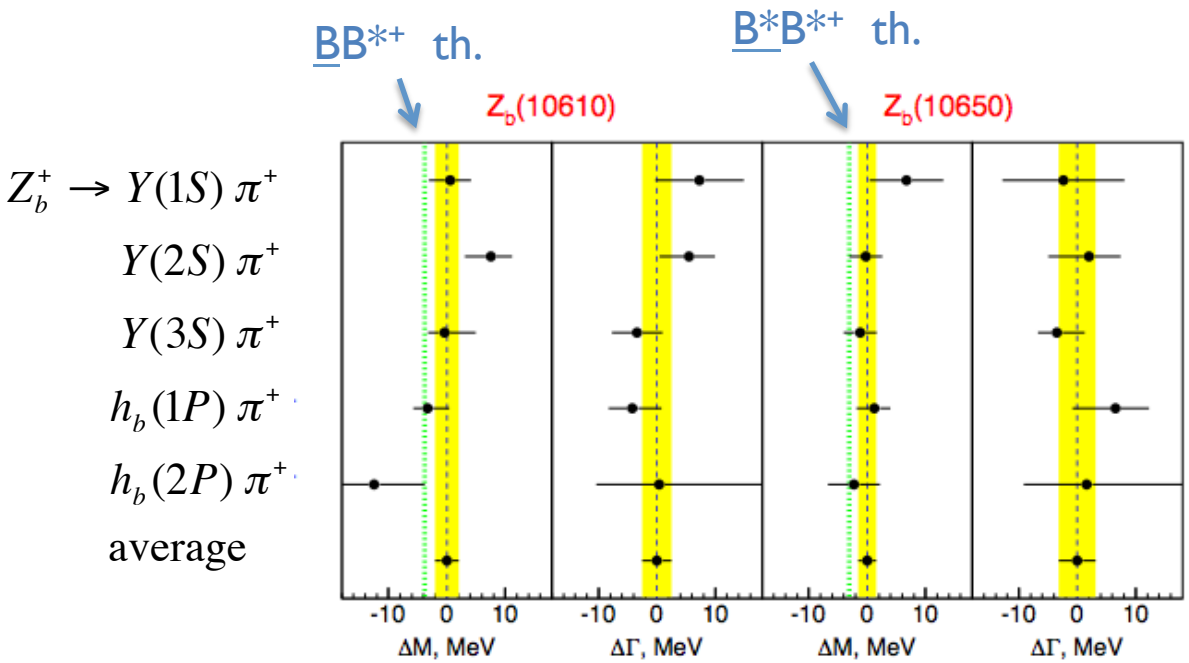
flavor content of the hadron: $\bar{c}c\ \ \bar{d}u$

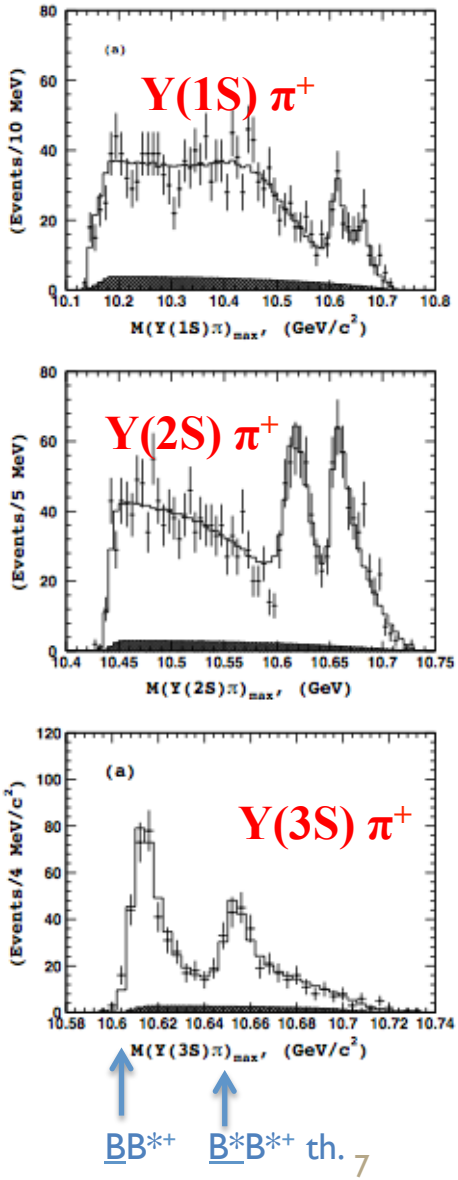


M = 3888 ± 3 MeV , Γ = 35 ± 7 MeV

Tetraquarks Z_b

Belle 2011





- both: $I^G(J^P)=1^+(1^+)$
- both: observed only by Belle 2011 [1105.4583]

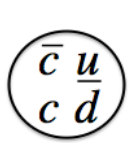
$$Z_b^+ \to \Upsilon(nS)\ \pi^+$$

flavor content of the hadron: $\bar{b}b \quad \bar{d}u$

Challenges for the theory community: quarkonium-*like* states at threshold

TABLE 10: Quarkonium-like states at the open flavor thresholds. For charged states, the C-parity is given for the neutral members of the corresponding isotriplets.

State	M, MeV	Γ, MeV	J^{PC}	Process (mode)	Experiment (#σ)	Year	Status
$X(3872)$	3871.68 ± 0.17	< 1.2	1^{++}	$B \to K(\pi^+\pi^- J/\psi)$	Belle [772, 992] (>10), BaBar [993] (8.6)	2003	Ok
				$p\bar{p} \to (\pi^+\pi^- J/\psi)\ldots$	CDF [994, 995] (11.6), D0 [996] (5.2)	2003	Ok
				$pp \to (\pi^+\pi^- J/\psi)\ldots$	LHCb [997, 998] (np)	2012	Ok
				$B \to K(\pi^+\pi^-\pi^0 J/\psi)$	Belle [999] (4.3), BaBar [1000] (4.0)	2005	Ok
				$B \to K(\gamma\, J/\psi)$	Belle [1001] (5.5), BaBar [1002] (3.5)	2005	Ok
					LHCb [1003] (> 10)		
				$B \to K(\gamma\, \psi(2S))$	BaBar [1002] (3.6), Belle [1001] (0.2)	2008	NC!
					LHCb [1003] (4.4)		
				$B \to K(D\bar{D}^*)$	Belle [1004] (6.4), BaBar [1005] (4.9)	2006	Ok
$Z_c(3885)^+$	3883.9 ± 4.5	25 ± 12	1^{+-}	$Y(4260) \to \pi^-(D\bar{D}^*)^+$	BES III [1006] (np)	2013	NC!
$Z_c(3900)^+$	3891.2 ± 3.3	40 ± 8	$?^{?-}$	$Y(4260) \to \pi^-(\pi^+ J/\psi)$	BES III [1007] (8), Belle [1008] (5.2)	2013	Ok
					T. Xiao *et al.* [CLEO data] [1009] (>5)		
$Z_c(4020)^+$	4022.9 ± 2.8	7.9 ± 3.7	$?^{?-}$	$Y(4260, 4360) \to \pi^-(\pi^+ h_c)$	BES III [1010] (8.9)	2013	NC!
$Z_c(4025)^+$	4026.3 ± 4.5	24.8 ± 9.5	$?^{?-}$	$Y(4260) \to \pi^-(D^*\bar{D}^*)^+$	BES III [1011] (10)	2013	NC!
$Z_b(10610)^+$	10607.2 ± 2.0	18.4 ± 2.4	1^{+-}	$\Upsilon(10860) \to \pi(\pi\Upsilon(1S, 2S, 3S))$	Belle [1012–1014] (>10)	2011	Ok
				$\Upsilon(10860) \to \pi^-(\pi^+ h_b(1P, 2P))$	Belle [1013] (16)	2011	Ok
				$\Upsilon(10860) \to \pi^-(B\bar{B}^*)^+$	Belle [1015] (8)	2012	NC!
$Z_b(10650)^+$	10652.2 ± 1.5	11.5 ± 2.2	1^{+-}	$\Upsilon(10860) \to \pi^-(\pi^+\Upsilon(1S, 2S, 3S))$	Belle [1012, 1013] (>10)	2011	Ok
				$\Upsilon(10860) \to \pi^-(\pi^+ h_b(1P, 2P))$	Belle [1013] (16)	2011	Ok
				$\Upsilon(10860) \to \pi^-(B^*\bar{B}^*)^+$	Belle [1015] (6.8)	2012	NC!



[review: Brambilla et al., 1404.3723]

QCD and strongly coupled gauge theories: challenges and perspectives

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More challenges: quarkonium-*like* states above threshold

TABLE 12: Quarkonium-like states above the corresponding open flavor thresholds. For charged states, the C-parity is given for the neutral members of the corresponding isotriplets.

State	M, MeV	Γ, MeV	J^{PC}	Process (mode)	Experiment (#σ)	Year	Status
$Y(3915)$	3918.4 ± 1.9	20 ± 5	$0/2^{?+}$	$B \to K(\omega J/\psi)$	Belle [1050] (8), BaBar [1000, 1051] (19)	2004	Ok
				$e^+e^- \to e^+e^-(\omega J/\psi)$	Belle [1052] (7.7), BaBar [1053] (7.6)	2009	Ok
$\chi_{c2}(2P)$	3927.2 ± 2.6	24 ± 6	2^{++}	$e^+e^- \to e^+e^-(D\bar{D})$	Belle [1054] (5.3), BaBar [1055] (5.8)	2005	Ok
$X(3940)$	3942^{+9}_{-8}	37^{+27}_{-17}	$?^{?+}$	$e^+e^- \to J/\psi\,(D\bar{D}^*)$	Belle [1048, 1049] (6)	2005	NC!
$Y(4008)$	3891 ± 42	255 ± 42	1^{--}	$e^+e^- \to (\pi^+\pi^- J/\psi)$	Belle [1008, 1056] (7.4)	2007	NC!
$\psi(4040)$	4039 ± 1	80 ± 10	1^{--}	$e^+e^- \to (D^{(*)}\bar{D}^{(*)}(\pi))$	PDG [1]	1978	Ok
				$e^+e^- \to (\eta J/\psi)$	Belle [1057] (6.0)	2013	NC!
$Z(4050)^+$	4051^{+24}_{-43}	82^{+51}_{-55}	$?^{?+}$	$\bar{B}^0 \to K^-(\pi^+\chi_{c1})$	Belle [1058] (5.0), BaBar [1059] (1.1)	2008	NC!
$Y(4140)$	4145.8 ± 2.6	18 ± 8	$?^{?+}$	$B^+ \to K^+(\phi J/\psi)$	CDF [1060] (5.0), Belle [1061] (1.9), LHCb [1062] (1.4), CMS [1063] (>5) D0 [1064] (3.1)	2009	NC!
$\psi(4160)$	4153 ± 3	103 ± 8	1^{--}	$e^+e^- \to (D^{(*)}\bar{D}^{(*)})$	PDG [1]	1978	Ok
				$e^+e^- \to (\eta J/\psi)$	Belle [1057] (6.5)	2013	NC!
$X(4160)$	4156^{+29}_{-25}	139^{+113}_{-65}	$?^{?+}$	$e^+e^- \to J/\psi\,(D^*\bar{D}^*)$	Belle [1049] (5.5)	2007	NC!
$Z(4200)^+$	4196^{+35}_{-30}	370^{+99}_{-110}	1^{+-}	$\bar{B}^0 \to K^-(\pi^+ J/\psi)$	Belle [1065] (7.2)	2014	NC!
$Z(4250)^+$	4248^{+185}_{-45}	177^{+321}_{-72}	$?^{?+}$	$\bar{B}^0 \to K^-(\pi^+\chi_{c1})$	Belle [1058] (5.0), BaBar [1059] (2.0)	2008	NC!
$Y(4260)$	4250 ± 9	108 ± 12	1^{--}	$e^+e^- \to (\pi\pi J/\psi)$	BaBar [1066, 1067] (8), CLEO [1068, 1069] (11) Belle [1008, 1056] (15), BES III [1007] (np)	2005	Ok
				$e^+e^- \to (f_0(980) J/\psi)$	BaBar [1067] (np), Belle [1008] (np)	2012	Ok
				$e^+e^- \to (\pi^- Z_c(3900)^+)$	BES III [1007] (8), Belle [1008] (5.2)	2013	Ok
				$e^+e^- \to (\gamma\, X(3872))$	BES III [1070] (5.3)	2013	NC!
$Y(4274)$	4293 ± 20	35 ± 16	$?^{?+}$	$B^+ \to K^+(\phi J/\psi)$	CDF [1060] (3.1), LHCb [1062] (1.0), CMS [1063] (>3), D0 [1064] (np)	2011	NC!
$X(4350)$	$4350.6^{+4.6}_{-5.1}$	13^{+18}_{-10}	$0/2^{?+}$	$e^+e^- \to e^+e^-(\phi J/\psi)$	Belle [1071] (3.2)	2009	NC!
$Y(4360)$	4354 ± 11	78 ± 16	1^{--}	$e^+e^- \to (\pi^+\pi^-\psi(2S))$	Belle [1072] (8), BaBar [1073] (np)	2007	Ok
$Z(4430)^+$	4458 ± 15	166^{+37}_{-32}	1^{+-}	$\bar{B}^0 \to K^-(\pi^+\psi(2S))$	Belle [1074, 1075] (6.4), BaBar [1076] (2.4) LHCb [1077] (13.9)	2007	Ok
				$\bar{B}^0 \to K^-(\pi^+ J/\psi)$	Belle [1065] (4.0)	2014	NC!
$X(4630)$	4634^{+9}_{-11}	92^{+41}_{-32}	1^{--}	$e^+e^- \to (\Lambda_c^+\bar{\Lambda}_c^-)$	Belle [1078] (8.2)	2007	NC!
$Y(4660)$	4665 ± 10	53 ± 14	1^{--}	$e^+e^- \to (\pi^+\pi^-\psi(2S))$	Belle [1072] (5.8), BaBar [1073] (5)	2007	Ok
$\Upsilon(10860)$	10876 ± 11	55 ± 28	1^{--}	$e^+e^- \to (B_{(s)}^{(*)}\bar{B}_{(s)}^{(*)}(\pi))$	PDG [1]	1985	Ok
				$e^+e^- \to (\pi\pi\Upsilon(1S, 2S, 3S))$	Belle [1013, 1014, 1079] (>10)	2007	Ok
				$e^+e^- \to (f_0(980)\Upsilon(1S))$	Belle [1013, 1014] (>5)	2011	Ok
				$e^+e^- \to (\pi Z_b(10610, 10650))$	Belle [1013, 1014] (>10)	2011	Ok
				$e^+e^- \to (\eta\Upsilon(1S, 2S))$	Belle [948] (10)	2012	Ok
				$e^+e^- \to (\pi^+\pi^-\Upsilon(1D))$	Belle [948] (9)	2012	Ok
$Y_b(10888)$	10888.4 ± 3.0	$30.7^{+8.9}_{-7.7}$	1^{--}	$e^+e^- \to (\pi^+\pi^-\Upsilon(nS))$	Belle [1080] (2.3)	2008	NC!

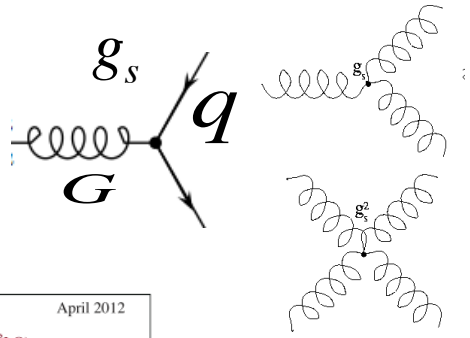
All these believed NOT to be QQ !

[review: Brambilla et al., 1404.3723]

Theoretical problem

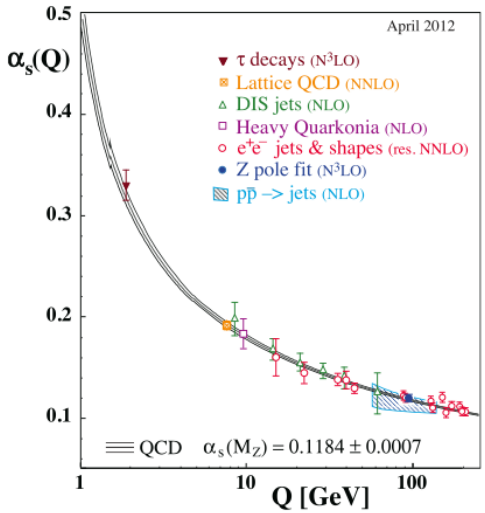
- Fundamental theory of strong interaction between **quarks** and **gluons** is known and reliable: chromodynamics (QCD)

$$L_{QCD} = -\tfrac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + \sum_{q=u,d,s,c,b,t} \bar{q} i\gamma_\mu (\partial^\mu + i g_s G^\mu_a T^a) q - m_q \bar{q} q$$



$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g_s f^{abc} G^\mu_b G^\nu_c$$

- But: $$\alpha_s \equiv \frac{g_s^2}{4\pi} \approx O(1) \quad r \approx 1\,\text{fm}$$

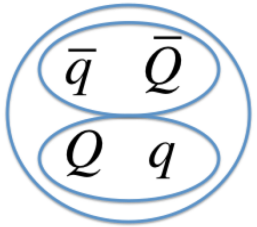


- Calculation of hadron properties (mass, ...) :
  - perturbative expansion in α_s not possible
  - non-perturbative method needed !

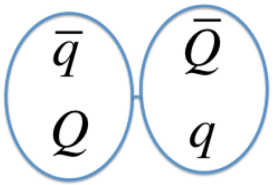
Phenomenological approaches

Approaches that support new exotic hadrons

- compact tetraquark: diquark antidiquark



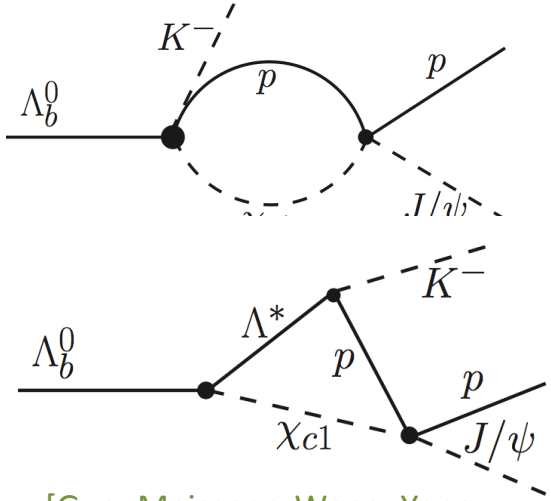
- meson molecules



- ...

Approaches that do not contain new exotic hadrons, and explain bumps in experimental cross sections via

- kinematical effects due to nearby threshold
- coupled channel effects
- kinematics interplay in triangle diagrams
- ...



[Guo, Meissner, Wang, Yang 1507.04950]

I will try to address the question whether observed exotic and conventional hadrons arise directly from QCD.

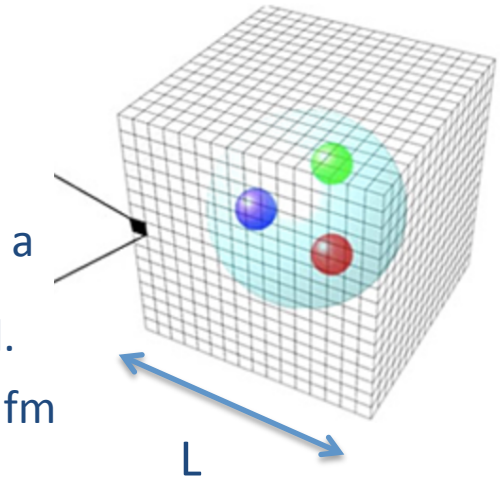
Non-perturbative method: QCD on lattice

$$L_{QCD} = -\tfrac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + \sum_{q=u,d,s,c,b,t} \bar{q} i \gamma_\mu (\partial^\mu + i g_s G^\mu_a T^a) q - m_q \bar{q} q$$

input: g_s , m_q

output: hadron properties

hadron interactions



precision cal.

a~0.05-0.15 fm

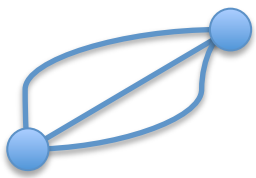
L~ 2-8 fm

Evaluation of Feynman path integrals in discretized space-time

quantum mechanics

$$\int Dx \, e^{i S/\hbar}$$

$$S = \int dt \, L[x(t)]$$



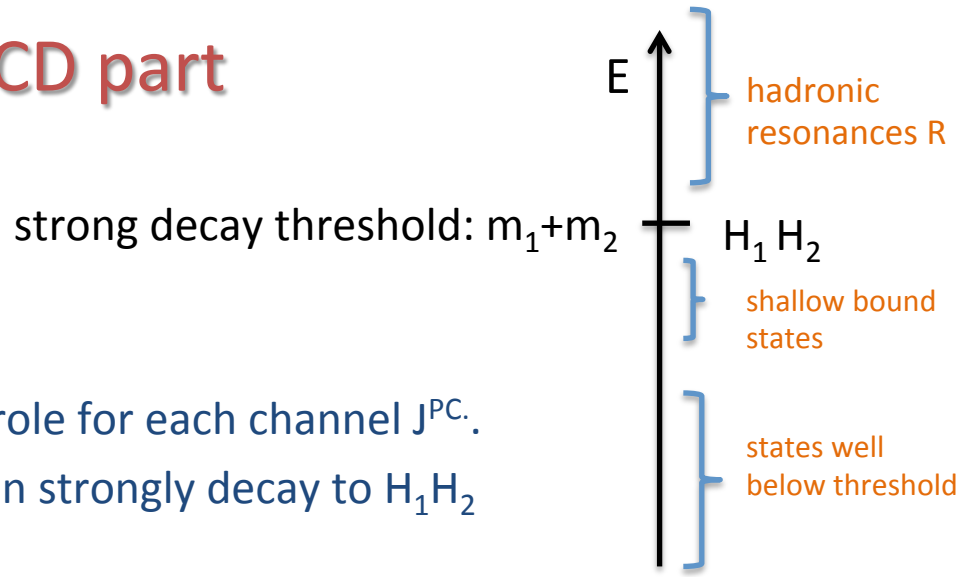
quantum field theory in Euclidian space-time

$$\int DG \, Dq \, D\bar{q} \, e^{-S_{QCD}/\hbar}$$

$$S_{QCD} = \int d^4x \, L_{QCD}[G(x), q(x), \bar{q}(x)]$$

x,t (Minkovsky) → x, i t (Euclidean)

Outline for lattice QCD part



The location of the threshold pays a major role for each channel J^{PC}.
If the hadron mass is above threshold, it can strongly decay to H_1H_2 as long as quantum numbers allow decay.

- <u>Hadrons well bellow threshold:</u> *"easy"*
  Unfortunately none of exotic experimental candidates is found well below threshold.

- <u>Hadron resonances</u> above threshold and <u>states slightly below threshold</u> : *challenging*
  - until recently: so-called single-hadron approximation
  - now: rigorous treatment by determining scattering matrix for two hadrons H_1 H_2

- lattice searches for <u>manifestly exotic hadrons</u> (usually above several thresholds): *very challenging*

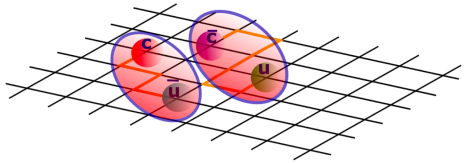
Discrete energies of eigenstates: E_n

mesons

J^{PC} $\quad \mathcal{O} = \bar{q}\Gamma q, \quad (\bar{q}\Gamma_1 q)_{\vec{p}_1}(\bar{q}\Gamma_2 q)_{\vec{p}_2}, \quad [\bar{q}\Gamma_3\bar{q}][q\Gamma_4 q], \ldots$

$\rho(770),\ 1^{--}: \quad \bar{d}u, \quad (\bar{d}d)(\bar{d}u) = \pi\pi$

$X(3872),\ 1^{++}: \quad \bar{c}c, \quad (\bar{c}u)(\bar{u}c) = D\bar{D}^*, \quad [\bar{c}u][cu]$



baryon

$p \quad 1/2^+: \quad uud, \quad (\mathrm{uud})(\bar{u}c) = p\pi$

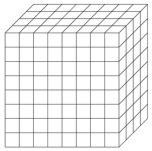
$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t)\ \mathcal{O}_j^+(0) | 0 \rangle = \sum_n \langle 0 | \mathcal{O}_i | n \rangle\ e^{-E_n t} \langle n | \mathcal{O}_j^+ | 0 \rangle = \sum_n Z_i^n\ Z_j^{n*}\ e^{-E_n t}$$

All eigenstates with given J^{PC} appear in principle (example: proton channel ½⁺)

- <u>single hadron states</u> p=proton $\quad m_p = E_1$ for P=0 (after extrapolation)
- <u>two-hadron states</u> p π $\quad E_n$ rigorously render two-hadron scattering matrix (for example pπ scattering matrix)

States well below threshold: "easy" precision spectrum

- $m=E_n$ for P=0 $a\to 0$, $L\to\infty$, $m_q\to m_q^{phy}$



- Available from a number of lattice QCD collaborations for a number of years
- Only three examples shown

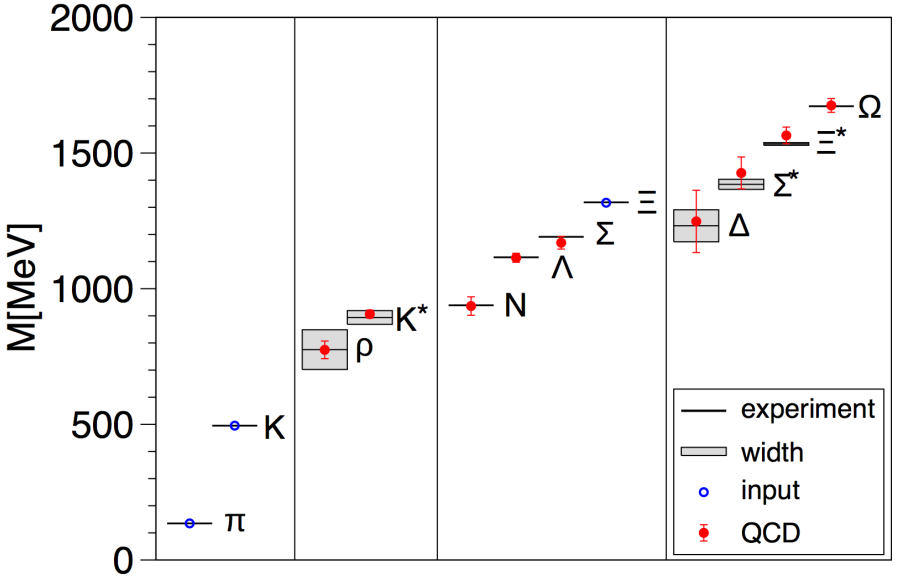
Proton and neutron

constitute more than 99%
of the bright side of universe

$m_u c^2 \approx 2\ \text{MeV}$ $\quad m_d c^2 \approx 5\ \text{MeV}$

$m_p c^2 \approx 938.3\ \text{MeV}$ $\quad m_n c^2 \approx 939.5\ \text{MeV}$

Higgs mechanism provides tiny contribution, the rest of visible mass in universe is due to the strong interaction in hadrons

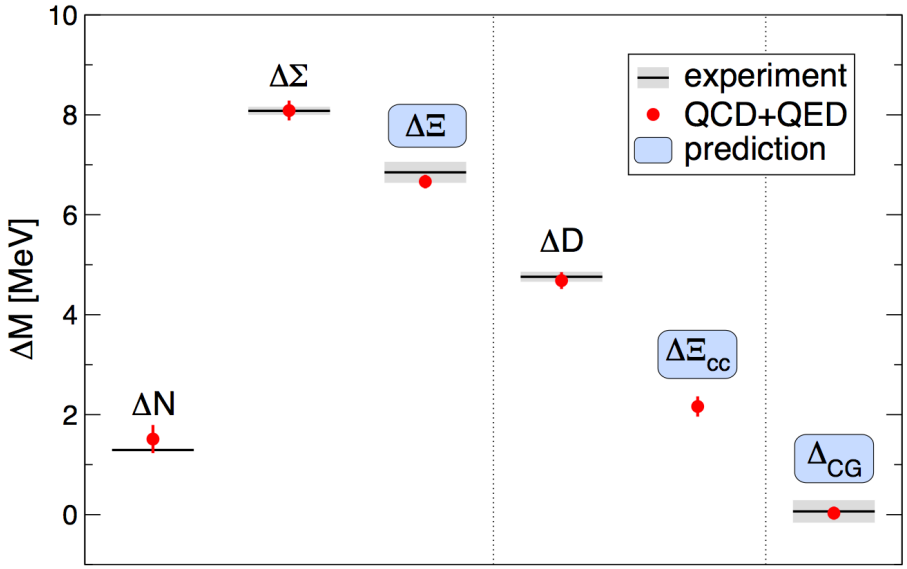


BMW collaboration
Science 322,2008

QCD

$m_u = m_d$

m_n , m_p



m_n - m_p

BMW collaboration
Science 347,2015

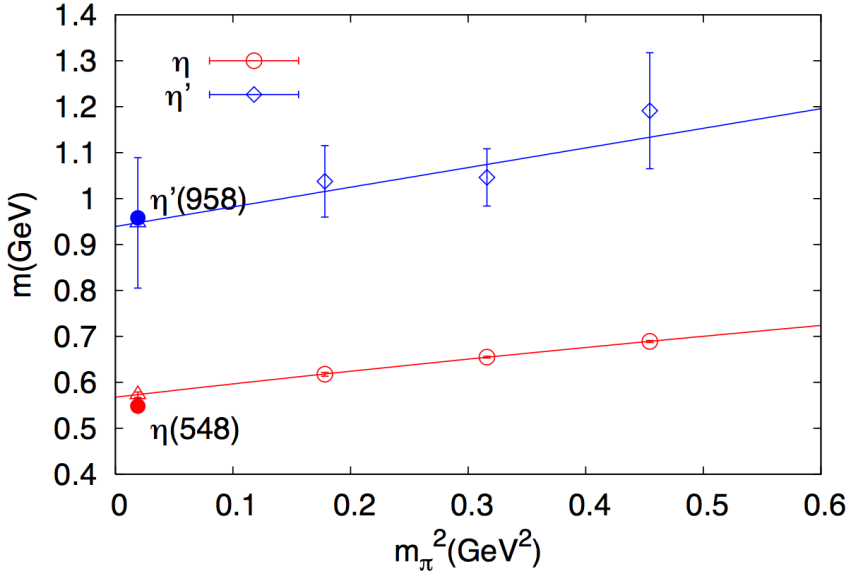
QCD + QED

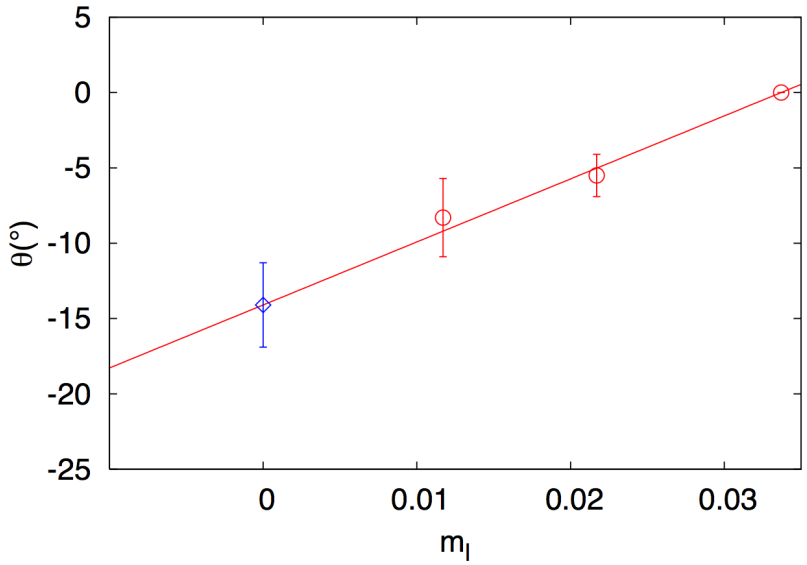
$m_u \neq m_d$

η and η' Mesons from Lattice QCD

N. H. Christ,[1] C. Dawson,[2] T. Izubuchi,[3,4] C. Jung,[3] Q. Liu,[1] R. D. Mawhinney,[1] C. T. Sachrajda,[5] A. Soni,[3] and R. Zhou[6]

(RBC and UKQCD Collaborations)





$J^{PC}=0^{-+}$, I=0

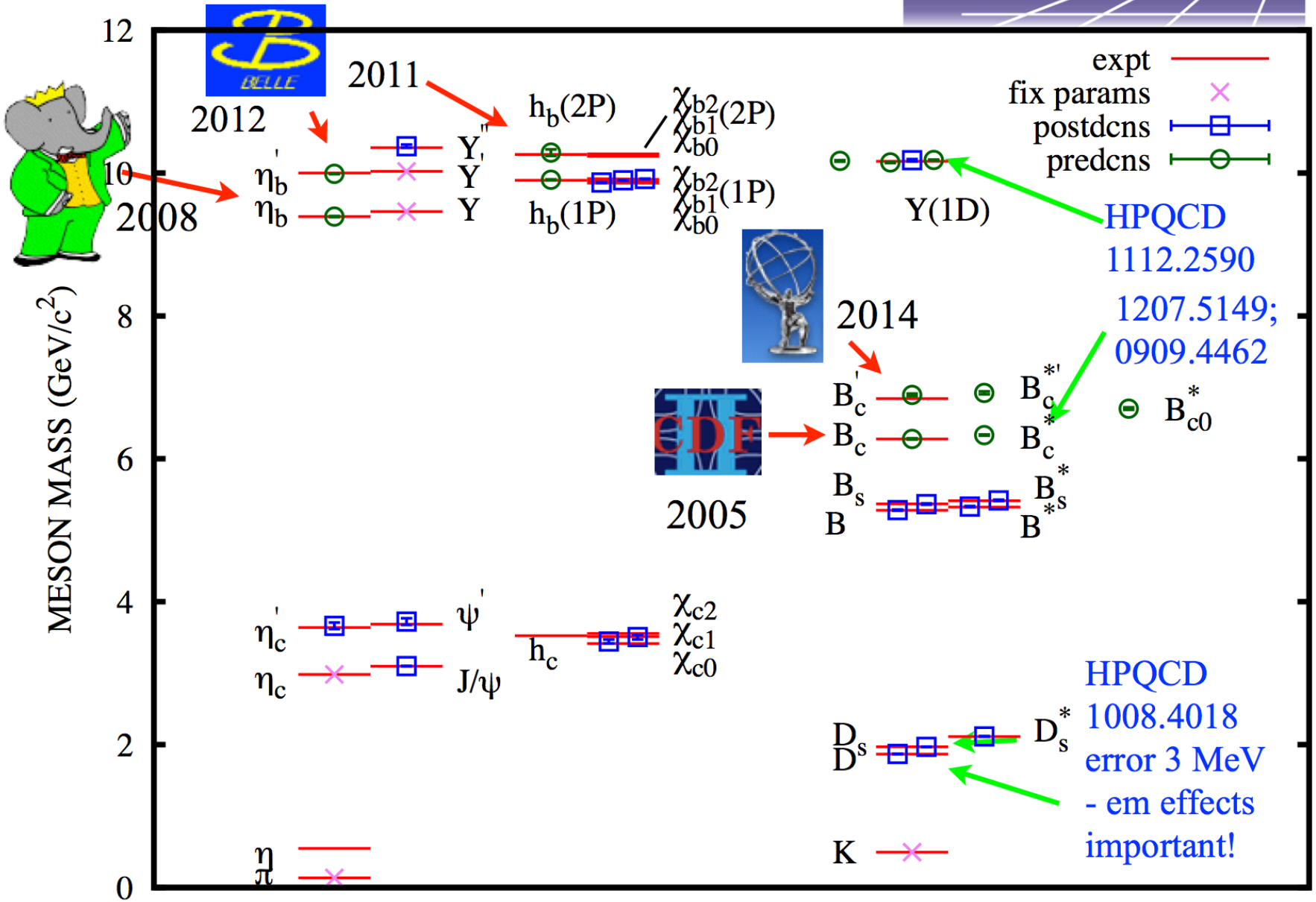
$$\begin{pmatrix} |\eta\rangle \\ |\eta'\rangle \end{pmatrix} = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix} \begin{pmatrix} |8\rangle_{\rm sym} \\ |1\rangle_{\rm sym} \end{pmatrix}$$

$$|8\rangle = \frac{1}{\sqrt{6}}(\bar{u}u + \bar{d}d - 2\bar{s}s)$$

$$|1\rangle = \frac{1}{\sqrt{3}}(\bar{u}u + \bar{d}d + \bar{s}s)$$

High Precision QCD

$m=E_n$ The gold-plated meson spectrum



Hadrons near or above strong decay threshold challenging

Single–hadron approximation used until few years ago:

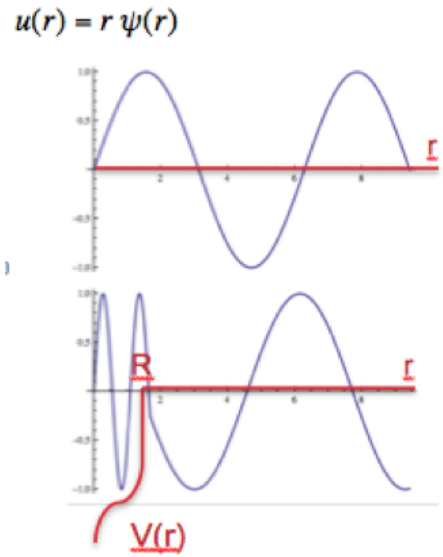
ignores strong decays of resonances and effects of thresholds

I will concentrate on "Rigorous approach" which addresses

scattering of two hadrons ..... in experiment or theory

Two-hadron scattering

Scattering phase shift δ



QM interpretation:

$$\psi(r) \propto \frac{\sin(pr)}{r}$$

$$\psi(r) \propto \frac{\sin(pr+\delta)}{r} \qquad r > R$$

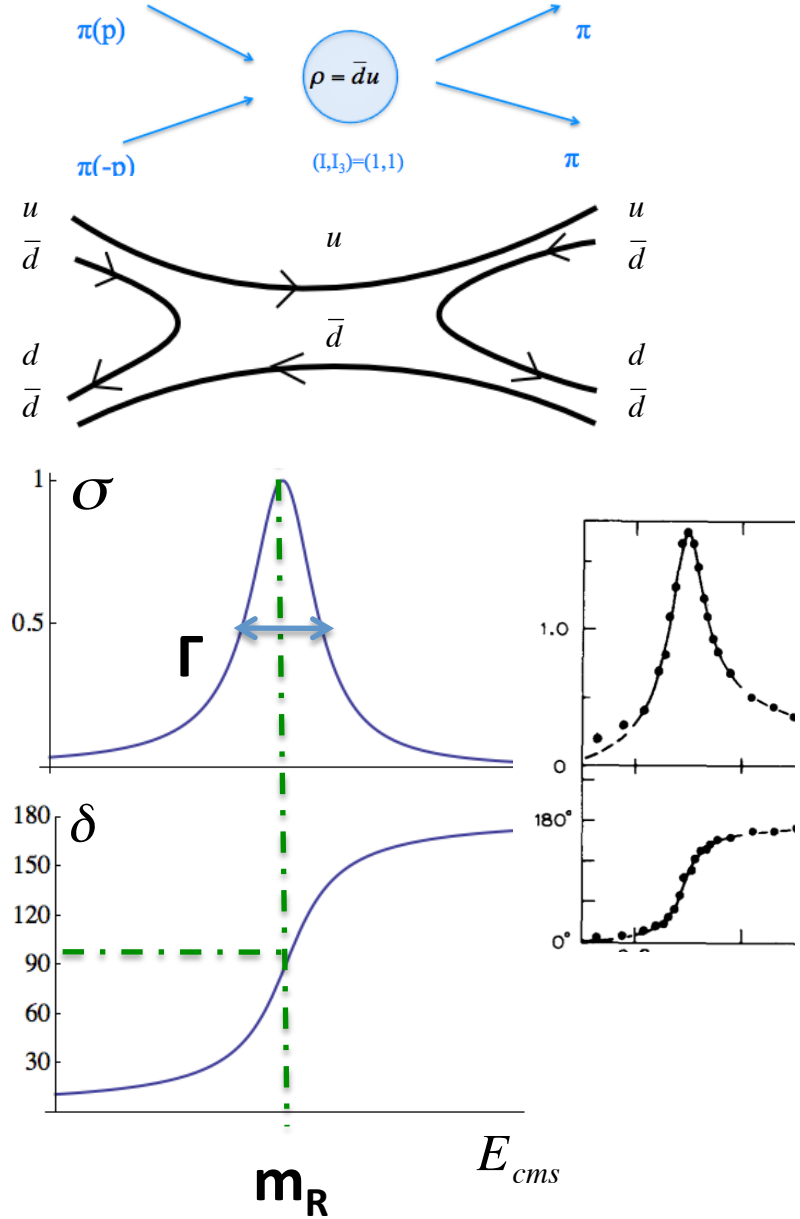
$$\text{unknown} \qquad r < R$$

scattering matrix T(E) from δ(E) for elastic scat:

$$S(E) = 1 + 2\, i\, T(E) = e^{2\, i\, \delta(E)}$$

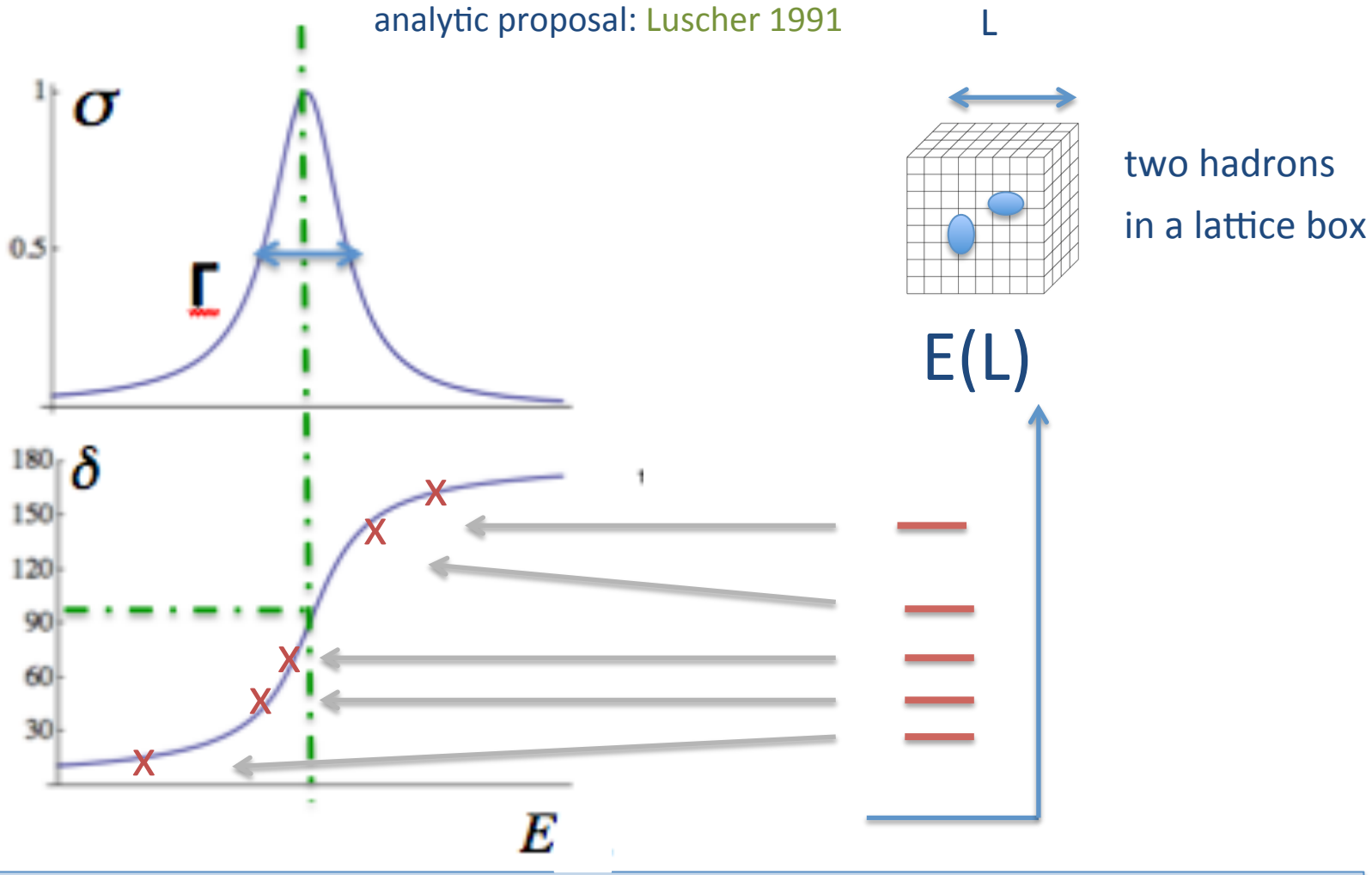
$$T = \frac{e^{2i\delta} - 1}{2i} = \sin\delta\; e^{i\delta} = \frac{1}{\cot\delta - i}$$

$$\sigma \propto |T|^2 = \sin^2\delta$$



Rigorous treatment: scattering info from spectrum

analytic proposal: Luscher 1991



scattering phase shifts at infinite volume $\delta(E)$ ← $E(L)$ energies from lattice with spatial extent L

Relation between E and δ(E)

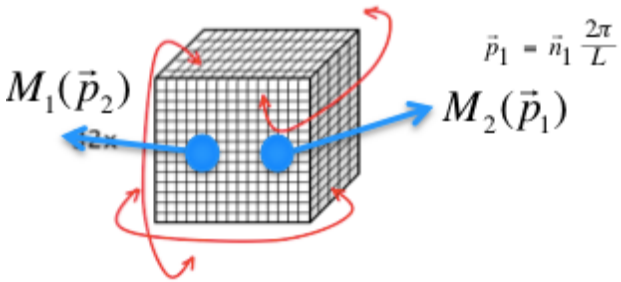
analytic proposal: Luscher 1991

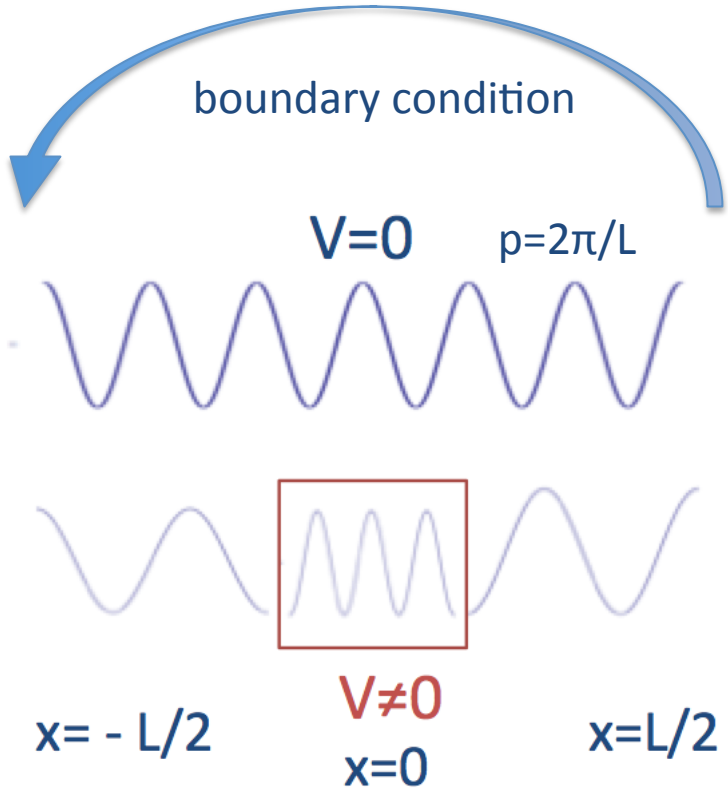
Energies of two hadrons in a box:

$$E(L) = \sqrt{m_1^2 + \vec{p}_1^{\,2}} + \sqrt{m_2^2 + \vec{p}_2^{\,2}} + \Delta E$$

- due to strong interaction
- gives rigorous info on δ

$$\vec{p}_{1,2} = \frac{2\pi}{L}\vec{n}$$

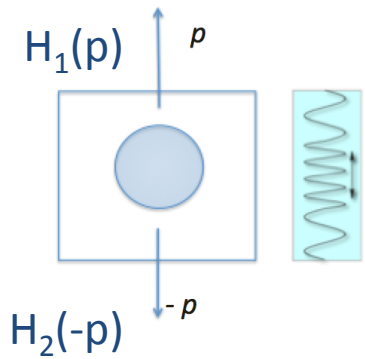




Scattering of two hadrons

elastic scattering with total momentum P=0: $E=E_{cm}$

$E_n(L) \xrightarrow{\text{Luscher's eq.}} \delta(E)$



Scattering matrix for partial wave l:

$$S(E) = e^{2i\delta(E)}, \quad S(E) = 1 + 2iT(E), \quad T(E) = \frac{1}{\cot\delta(E) - i}$$

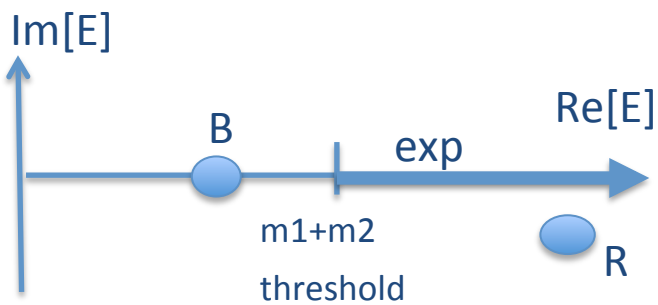
Bound state (B):

$\cot[\delta(E_B)] = i$, $E_B < m_1+m_2$

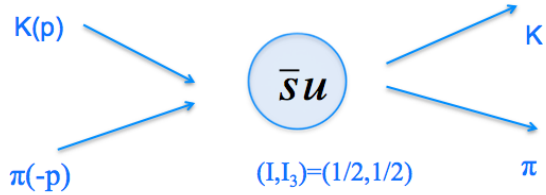
Resonance (R) (of Breit-Wigner type):

$$T(E) = \frac{-E\,\Gamma}{E^2 - m_R^2 + i\,E\,\Gamma}, \quad \Gamma(E) = g^2\,\frac{p^{2l+1}}{E^2}$$

Locations of poles of T(E)
for res. and bound st.



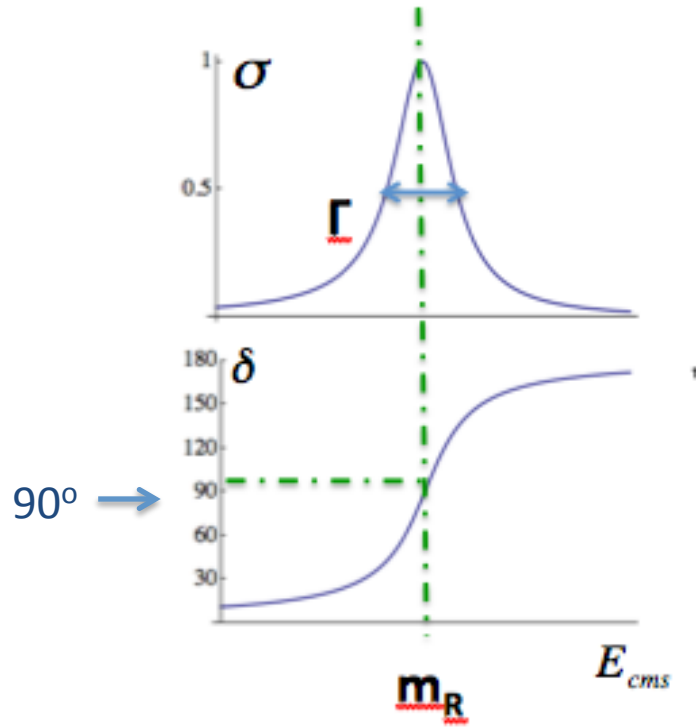
Resonances above threshold



Decay quickly through strong interaction

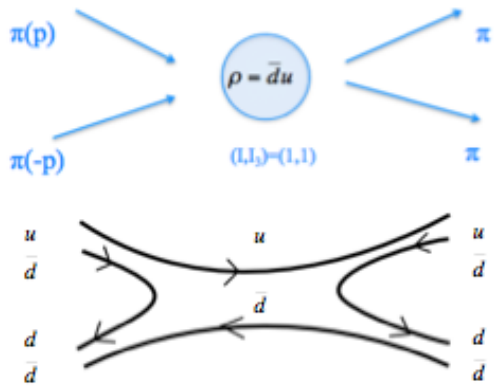
$$\tau = \hbar / \Gamma$$

Γ=10–300 MeV



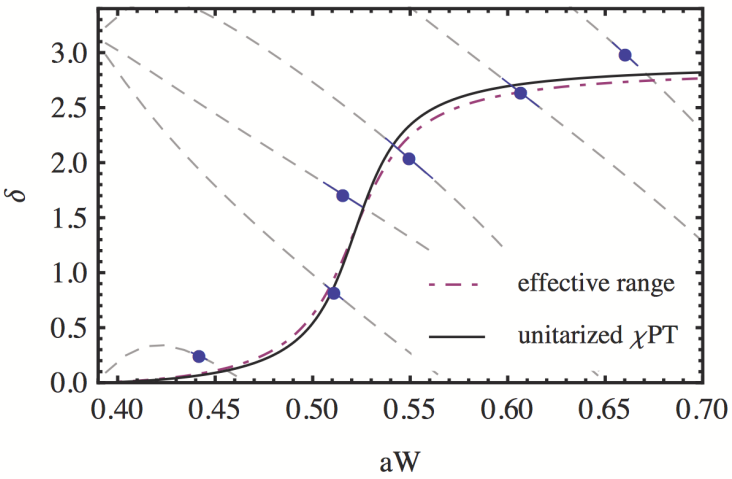
ρ resonance

(the only "mature" hadronic resonance from lattice)

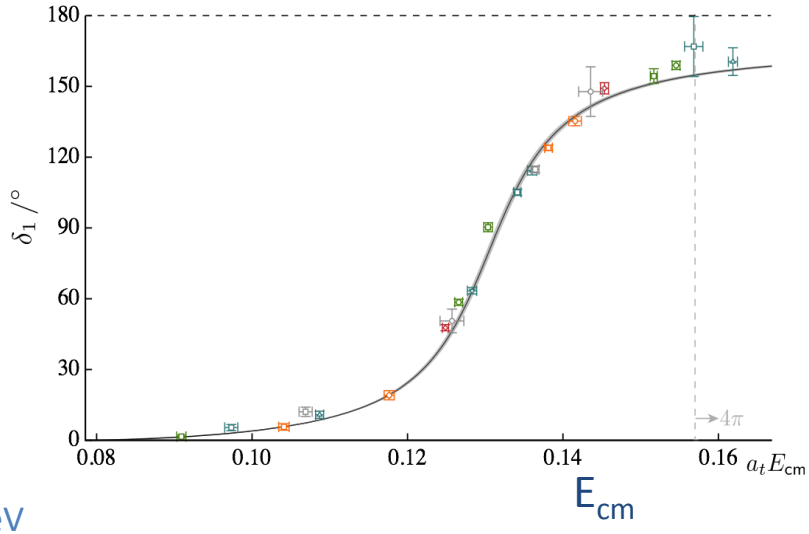


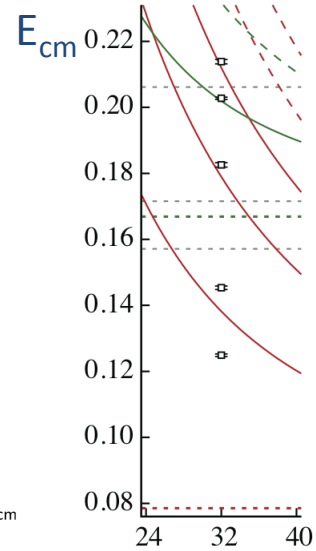
$$\mathcal{O}: \ \bar{u}d, \ \pi\pi$$

$E_{cm} \rightarrow \delta(E_{cm})$



[Pelissier, Alexandru,PRD 2013], $m_\pi \approx 300$ MeV





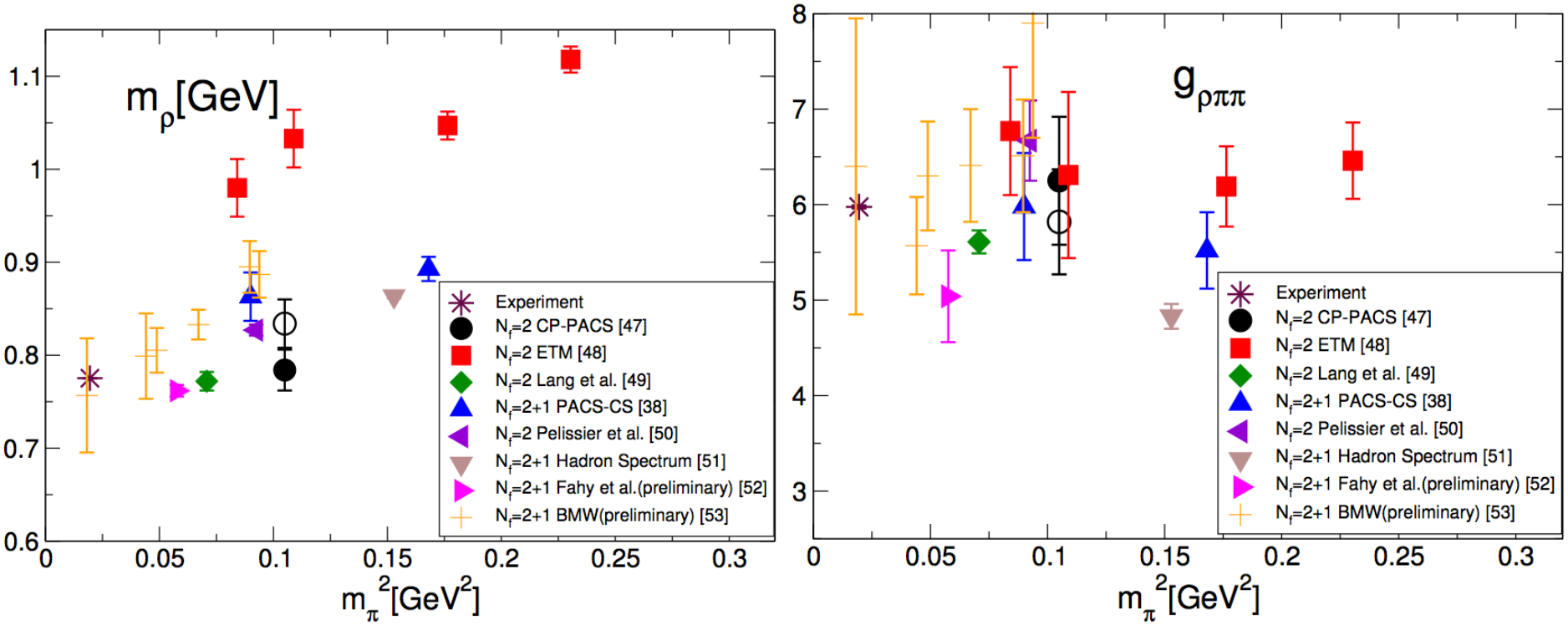
[HSC, PRD 1507.02599] $m_\pi \approx 236$ MeV

$$\text{BW}: \quad \delta = \text{acot}\frac{m_R^2 - E_{cms}^2}{m_R\,\Gamma} \quad \rightarrow \quad m_R, \quad \Gamma \text{ or } g_{\rho\pi\pi}$$

ρ resonance: lattice results

from T. Yamazaki [lat14 plenary, 1503.0867]

$$\Gamma = \frac{g^2_{\rho\pi\pi}}{6\pi}\frac{p^3}{E^2}$$



Other resonances simulated on the lattice

$\bar{c}c$	$\bar{u}u$	$\bar{s}u$	$\bar{c}u$
$\eta_c(1S)$	$\pi^\pm$	$K^\pm$	$D^\pm$
$J/\psi(1S)$	π^0	K^0	D^0
$\chi_{c0}(1P)$	η	$K_S{}^0$	$D^*(2007)^0$
$\chi_{c1}(1P)$	$f_0(500)$ or σ was $f_0(60$	$K_L{}^0$	$D^*(2010)^\pm$
$h_c(1P)$	$\rho(770)$	$K_0^*(800)$ or κ	$D_0^*(2400)^0$
$\chi_{c2}(1P)$	$\omega(782)$	$K^*(892)$	$D_0^*(2400)^\pm$
$\eta_c(2S)$	$\eta'(958)$	$K_1(1270)$	$D_1(2420)^0$
$\psi(2S)$	$f_0(980)$	$K_1(1400)$	$D_1(2420)^\pm$
$\psi(3770)$	$a_0(980)$	$K^*(1410)$	$D_1(2430)^0$
$X(3872)$	$\phi(1020)$	$K_0^*(1430)$	$D_2^*(2460)^0$
$\chi_{c0}(2P)$wa	$h_1(1170)$	$K_2^*(1430)$	$D_2^*(2460)^\pm$
$\chi_{c2}(2P)$	$b_1(1235)$	$K(1460)$	$D(2550)^0$
$X(3940)$	$a_1(1260)$	$K_2(1580)$	$D(2600)$
$\psi(4040)$	$f_2(1270)$	$K(1630)$	$D^*(2640)^\pm$
$X(4050)^\pm$	$f_1(1285)$	$K_1(1650)$	$D(2750)$
$X(4140)$	$\eta(1295)$	$K^*(1680)$	
$\psi(4160)$	$\pi(1300)$	$K_2(1770)$	
$X(4160)$	$a_2(1320)$	$K_3^*(1780)$	
$X(4250)^\pm$	$f_0(1370)$	$K_2(1820)$	
	$h_1(1380)$	$K(1830)$	
	$\pi_1(1400)$		
	$\eta(1405)$		
	$f_1(1420)$		
	$\omega(1420)$		
	$f_2(1430)$		
	$a_0(1450)$		
	$\rho(1450)$		

well below open charm decay th.

well below strong decay th.

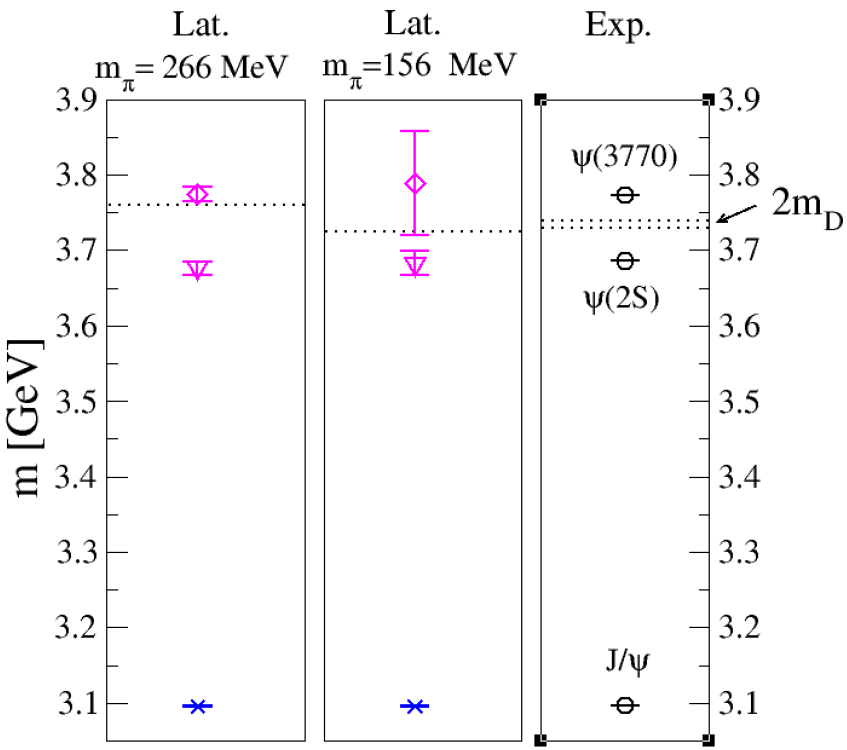
pioneering simulations of resonances with rigorous treatment 2012-2015

cu [Mohler, S.P. , Woloshyn: PRD 2013] su [S.P., Lang, Leskovec, Mohler, PRD 2013; Wilson, Dudek, Edwards, Thomas, PRL 20114, PRD 2015] cc [Lang, Leskovec, Mohler, S.P., 1503.05363] a1,b1 [Lang, Leskovec, Mohler, S.P., JHEP 2014]

General result: m_R and Γ (or coupling) close to experimental values.

Exception: $\chi_{c0}(2P)$ - puzzling state

Resonance ψ(3770) and bound st. ψ(2S) from DD scattering in p-wave



$\mathcal{O}$: $\bar{c}\,c$, $D\bar{D}$, $J^{PC} = 1^{--}$

DD scat. in p-wave is simulated

T-matrix is determined from E_n

Fit of T-matrix gives:

BW resonance ψ(3770):

m_R (magenta diamonds)

Γ (given below)

Bound state ψ(2S) from pole in T:

m_B (magetna triangles)

$\eta_c(1S)$
$J/\psi(1S)$
$\chi_{c0}(1P)$
$\chi_{c1}(1P)$
$h_c(1P)$
$\chi_{c2}(1P)$
$\eta_c(2S)$
$\psi(2S)$
$2m_D$
$\psi(3770)$
$X(3872)$
$\chi_{c0}(2P)$wa
$\chi_{c2}(2P)$
$X(3940)$
$\psi(4040)$
$X(4050)^{\pm}$
$X(4140)$
$\psi(4160)$
$X(4160)$
$X(4250)^{\pm}$

ψ(3770)	Mass [MeV]	g (no unit)
Lat (m_π=266 MeV)	3774 ±6±10	19.7 ±1.4
Lat (m_π=156 MeV)	3789 ±68±10	28 ± 21
Exp.	3773.15± 0.33	18.7 ± 1.4

$$\Gamma = \frac{g^2}{6\pi}\frac{p^3}{s}$$

Lang, Leskovec, Mohler, S.P., 1503.05363

States slightly below threshold

$p\eta_c$ pentaquark

D_{s0}^*, D_{s1}, B_{s0}^*, B_{s1}, X(3872)

deuterium-like systems

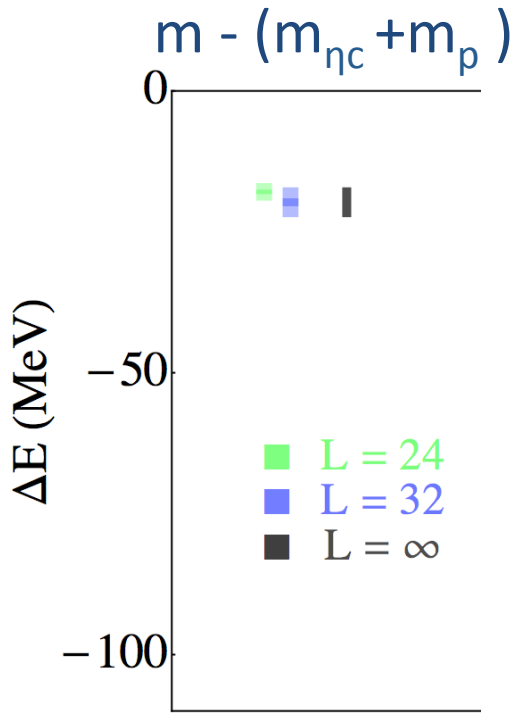
Bound state of a η_c and p from lattice

[NPLQCD, 1410.7069, PRD, m_π~800 MeV]

$\eta_c \; p$

$\bar{c}c \; uud$

~ 20 MeV below th. $m_{\eta c}+m_p$



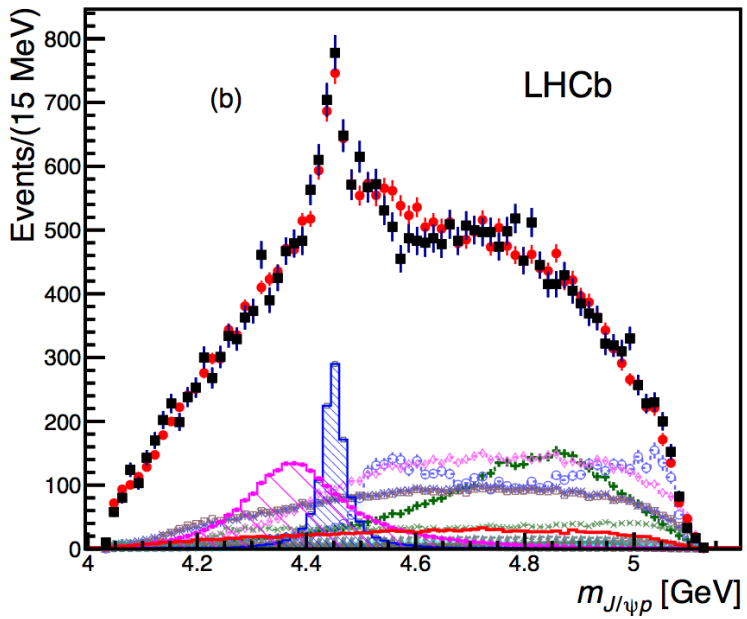
Two pentaquark resonances in J/ψ and p from exp

LHCb: 1507.03414

$J/\psi \; p$

$\bar{c}c \; uud$

~ 400 MeV above th. $m_{J/\psi}+m_p$



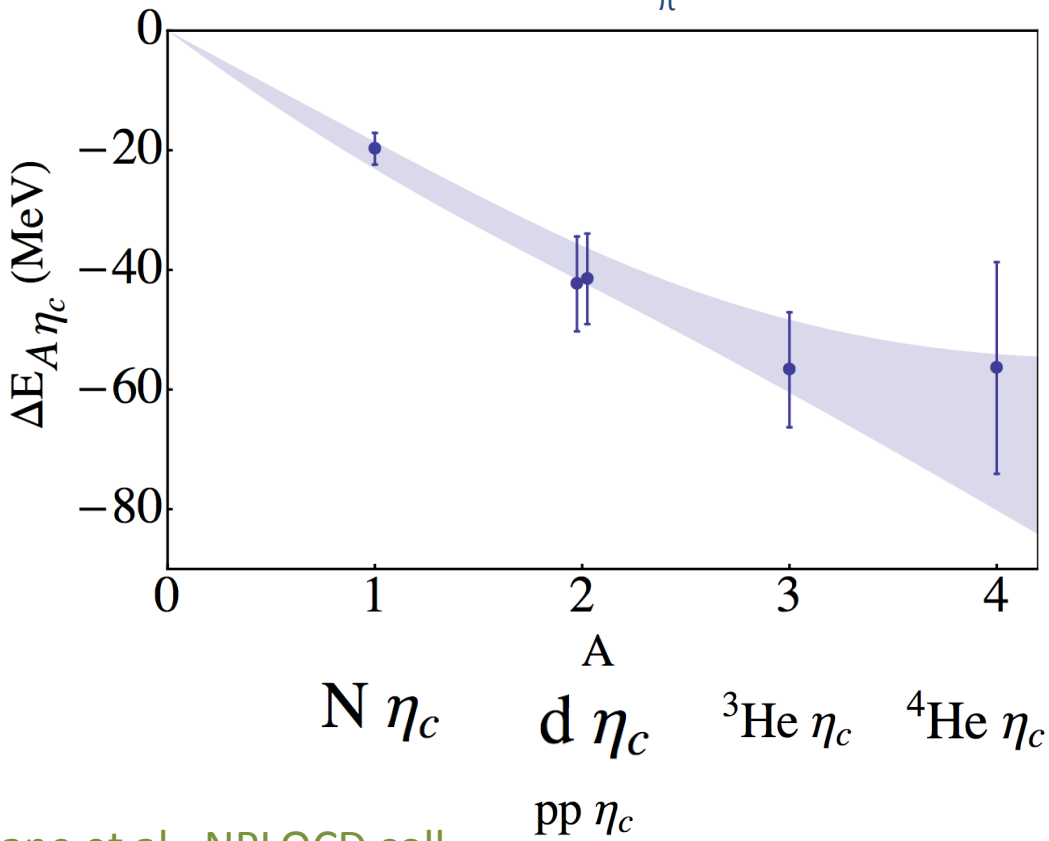
Bound states of a charmonium and a nucleus

Exp: no reliable candidates for such states yet

Lat: SU(3) symmetric point, $m_u=m_d=m_s$, m_π~800 MeV, L=3.4, 4.5, 6.7 fm

Bound states found at this m_π. Not known yet whether these bound states survive at m_π^{phys}

m_π~800 MeV



Binding energies

for a system composed of

η_c + nucleus (A)

number of nucleons in nucleus

[Beane et al., NPLQCD coll. , 1410.7069]

Comment on the burried pentaquark $\Theta^+(1540)$

$$\Theta^+(1540) \to K^+ \quad n$$
$$\underline{s}u \quad udd$$

Exp:

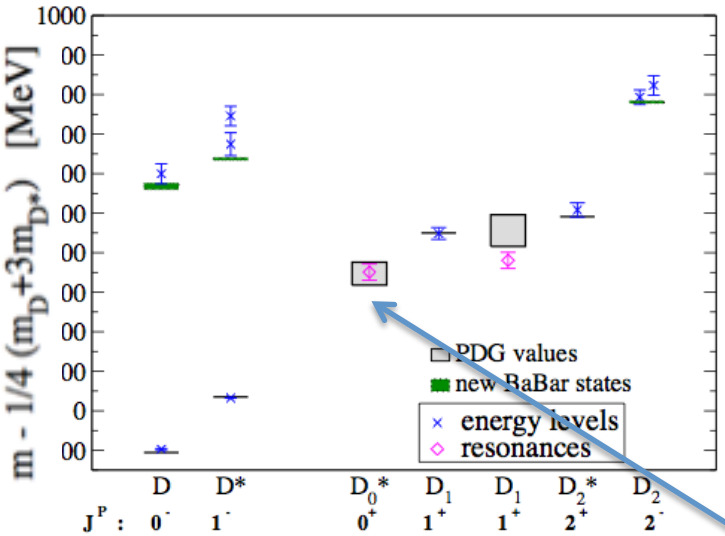
- Signal was found experimentally about 100 MeV above Kn threshold
- Not confirmed by more precise studies

Lattice

- Initial simulations gave some indication in favor of its existence
- Not confirmed by more precise studies

... and all references are missing here, sorry

D_s and D scalar meson puzzle



D mesons (resonances)

scalar D resonance in Dπ

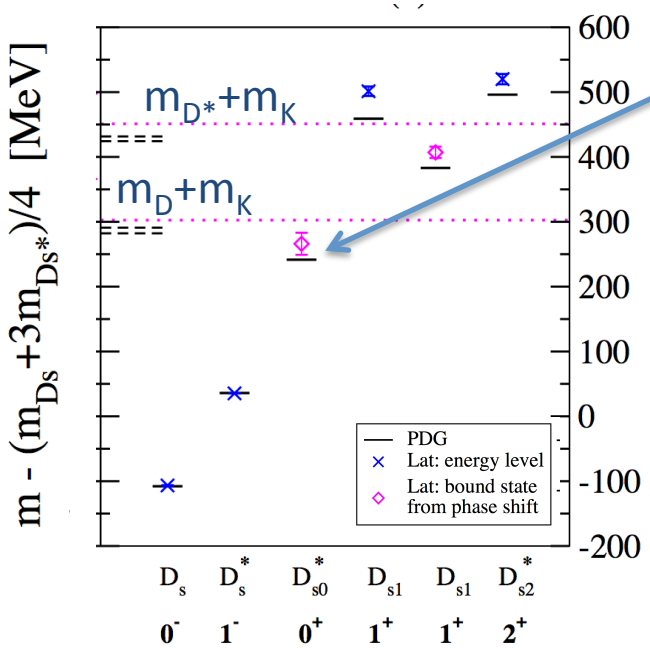
[D. Mohler, S.P. , R. Woloshyn: PRD 2013]

- why do these scalar partners have mass so close ?

$$D_0^*(2400): \quad M \approx 2318 \text{ MeV} \quad \Gamma \approx 267 \text{ MeV} \quad \bar{c}u \text{ or } \bar{c}u\bar{s}s \,?$$

$$D_{s0}(2317): \quad M \approx 2318 \text{ MeV} \quad \Gamma \approx 0 \text{ MeV} \quad \bar{c}s \text{ or } \bar{c}s[\bar{u}u + \bar{d}d] \,?$$

1) is D_0^* mass pushed up : valence ss pair ?? ✖
2) is D_{s0}^* mass pushed down : effect of DK threshold ?? ✔

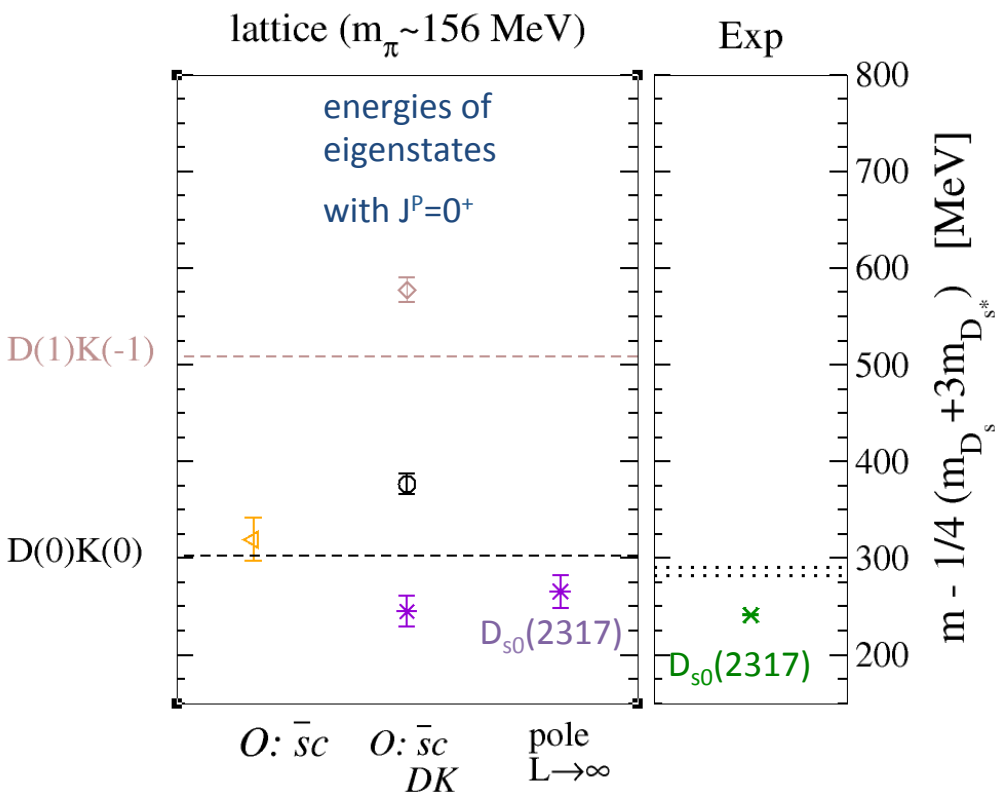


D_s mesons (near-threshold)

[C. Lang, L. Leskovec, D. Mohler, S.P. , R. Woloshyn: PRL 2013, PRD 2014]

Scalar meson D_{s0} below DK threshold

$$\mathcal{O} = \bar{s}c, \;\; DK$$



[D. Mohler, C. Lang, L. Leskovec, S.P. , R. Woloshyn, 1308.3175, Phys. Rev. Lett.]

- phase shift for DK scattering
  E → δ(E)
- interpolation of δ(E) near th. using effective range expansion
- This renders scattering matrix in all region near threshold
- pole in T(E) found slightly below th.

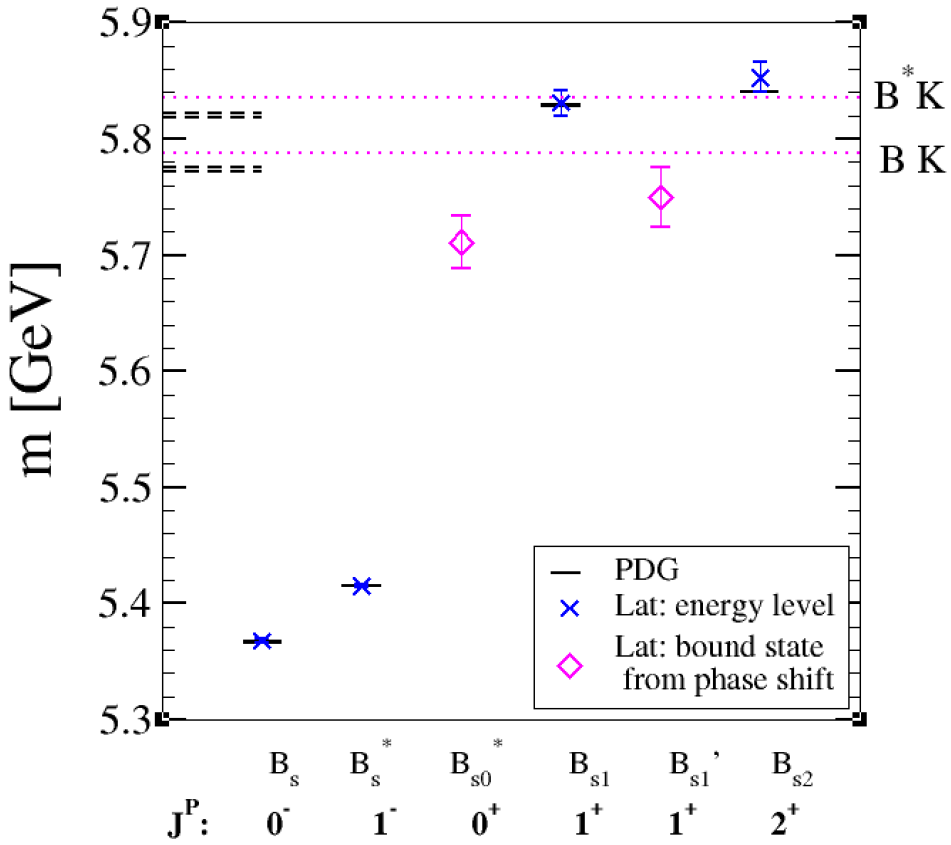
$$T \propto \frac{1}{\cot\delta - i} = \infty$$

- attributed to D_{s0}(2317)

Mass prediction for missing B_{s0} and B_{s1}

$$\mathcal{O} = \bar{s}b, \;\; BK$$

m_π=156 MeV



Quantities shown:

for two bound states :

$$m_B = (m_B - E_{th})^{lat} + E_{th}^{exp}$$

for other states :

$$m = (m - \bar{m})^{lat} + \bar{m}^{lat}$$

for dotted lattice thresholds :

$$E_{th} = (E_{th} - \bar{m})^{lat} + \bar{m}^{lat}$$

$$\bar{m} \equiv \tfrac{1}{4}(m_{Bs} + 3m_{Bs^*})$$

- B_{s1}' and B_{s2} agree well with exp

Predictions:

- B_{s0} bound state below BK th.
- B_{s1} bound state below B*K th.

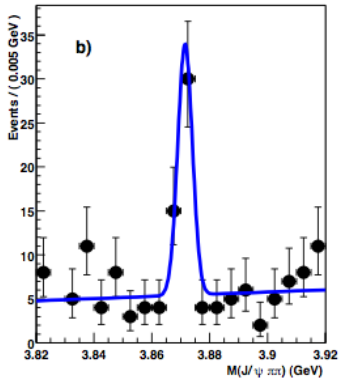
[C. Lang, D. Mohler, S.P. , R. Woloshyn: 1501.0164]

X(3872) , $J^{PC}=1^{++}$, charmonium-like

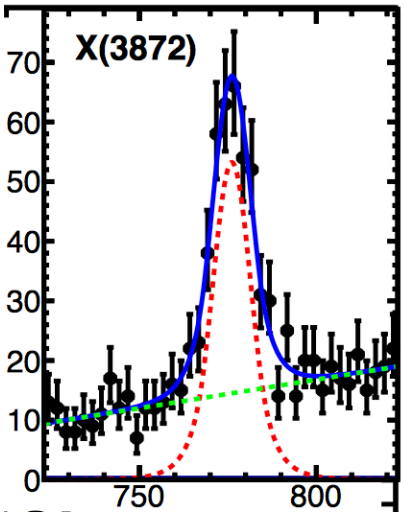
- First charmonium-like state discovered [Belle, PRL, 2003]
- sits within 1 MeV of $D^0\underline{D}^{0*}$ threshold
  8 MeV below D^+D^{*-} threshold
  } isospin breaking effects my be important
- believed to have a large molecular $D^0\underline{D}^{0*}$ Fock component
- $\Gamma < 1.2$ MeV
- decays to I=0, 1 equally important

  X(3872) → J/Ψ ω (I=0)

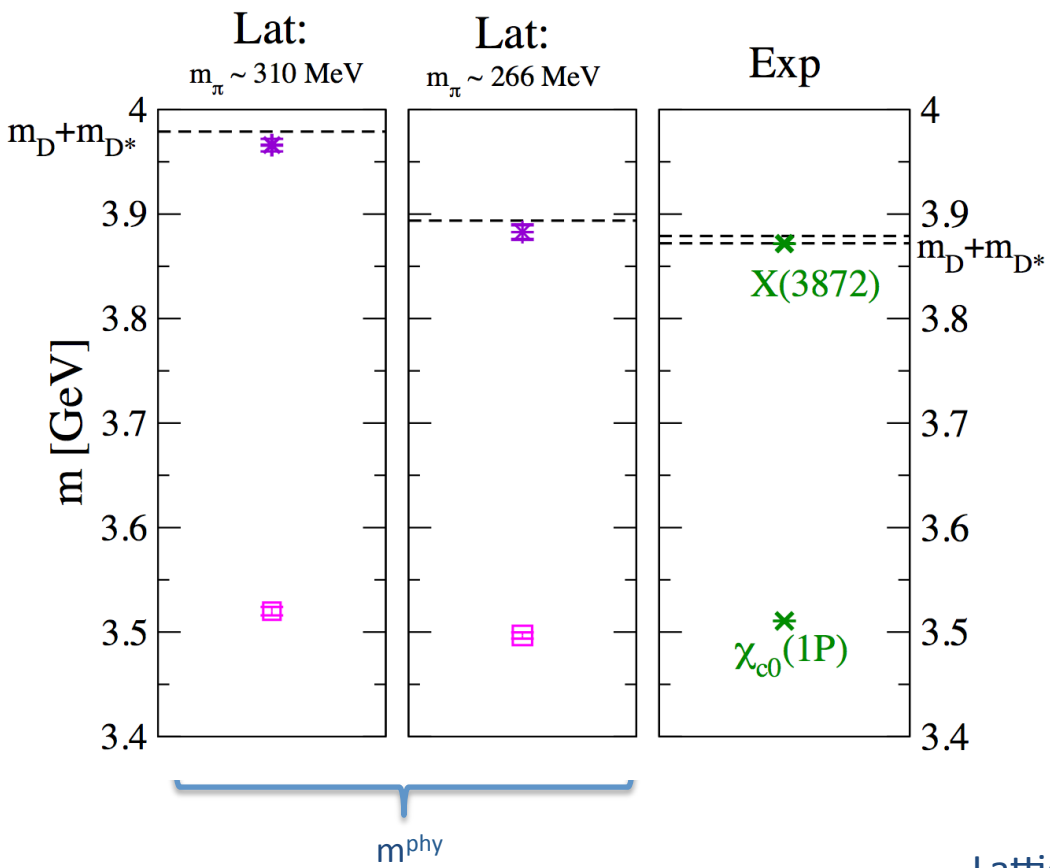
  X(3872) → J/Ψ ρ (I=1)



[LHCb, PRL 2013]



X(3872) as bound state from DD* scattering, $J^{PC}=1^{++}$, I=0



$\mathcal{O}: \bar{c}\, c,\ D\bar{D}^*$

- ground state: $\chi_{c1}(1P)$
- DD* scattering matrix near th. determined
- A pole of $T \propto \frac{1}{\cot\delta - i} = \infty$ found just below th. (violet star)
- The pole attributed to X(3872), which is a shallow bound state in both simulations
- Position of DD* threshold depends on $m_{u/d}$, and may be affected by discretization effects related to charm quark

X(3872)	$m - (m_{D0}+m_{D0*})$
lat (m_π=310 MeV)	- 13 ± 6 MeV
lat (m_π=266 MeV)	- 11 ± 7 MeV
exp	- 0.14 ± 0.22 MeV

Lattice evidence for X(3872):

- $m_\pi \approx 266$ MeV, a=0.124 fm, L= 2 fm

[S.P. and Leskovec: 1307.5172, PRL 2013]

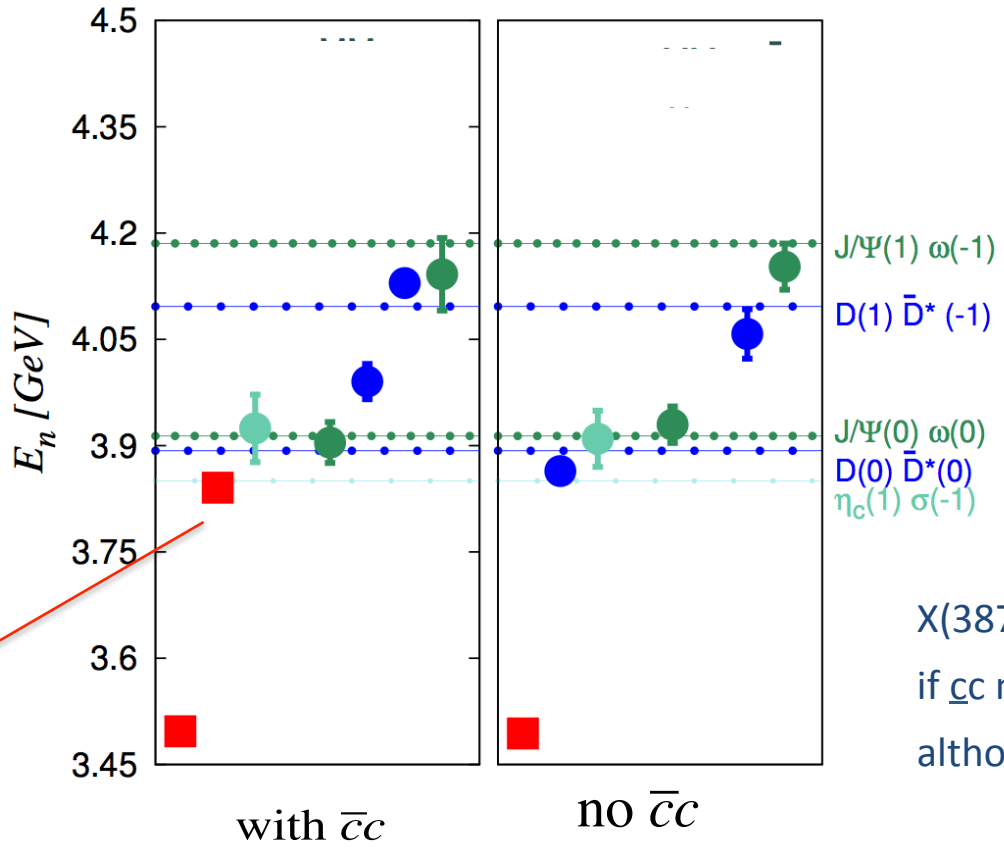
- $m_\pi \approx 310$ MeV, a=0.15 fm, L=2.4 fm , HISQ

[Lee, DeTar, Na, Mohler , update of proc 1411.1389]

Which Fock components are essential for X(3872) with I=0 ?

$J^{PC}=1^{++}$ $\quad \mathcal{O}: \underbrace{\bar{c}\,c, \quad D\bar{D}^*}_{\text{essential}}, \; J/\psi\,\omega, \; \chi_{c1}\eta, \; \eta_c\sigma, \; \underbrace{[\bar{c}u]_{3c}[cu]_{3c}, \; [\bar{c}u]_{6c}[cu]_{6c}}_{\text{do no seem not essential}}$

energies of eigenstates



X(3872) found only if cc in the basis

X(3872) not found if cc not in the basis, although [cu][cu] in the basis

[M. Padmanath, C.B. Lang, S.P., 1503.03257]

Searches for hadrons with exotic flavor

very challenging

$$Z_c = \bar{c}c\bar{d}u, \quad Z_b = \bar{b}b\bar{d}u, \quad P_c = \bar{c}cuud$$

few searches — to challenging — to recent

Even more challenging since most of experimental exotic states

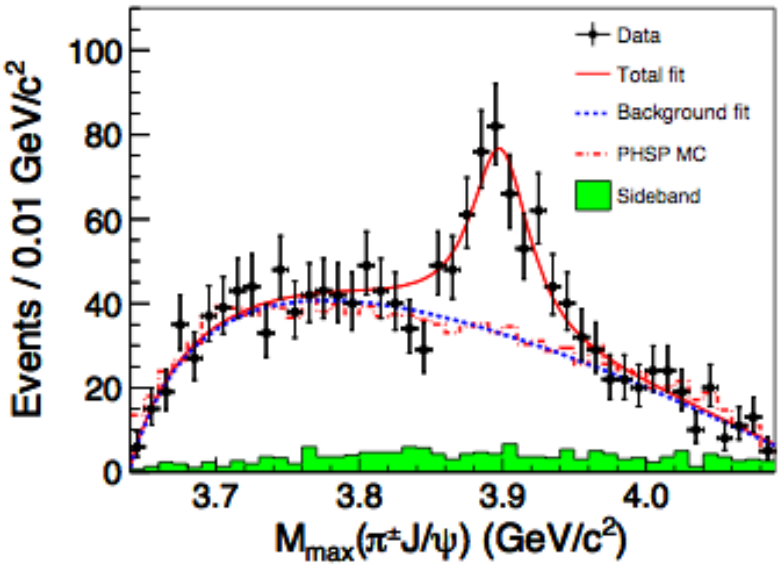
- are above several thresholds and decay to several two-meson final states
- require simulation of coupled-channels

Coupled-channel scattering matrix extracted on lattice so far only in

- a-la Luscher: Kπ, Kη system [Willson, Dudek, Edward, Thomas, HSC, PRL 2014]

Let's refresh on tetraquarks Z_c from experiment

Example: $Z_c^+(3900)$



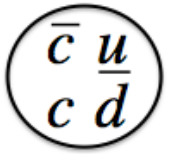
DD* thr.

[BESIII, 2013, 1303.5949, PRL]

state confirmed by BeSIII, Belle, Cleo-c

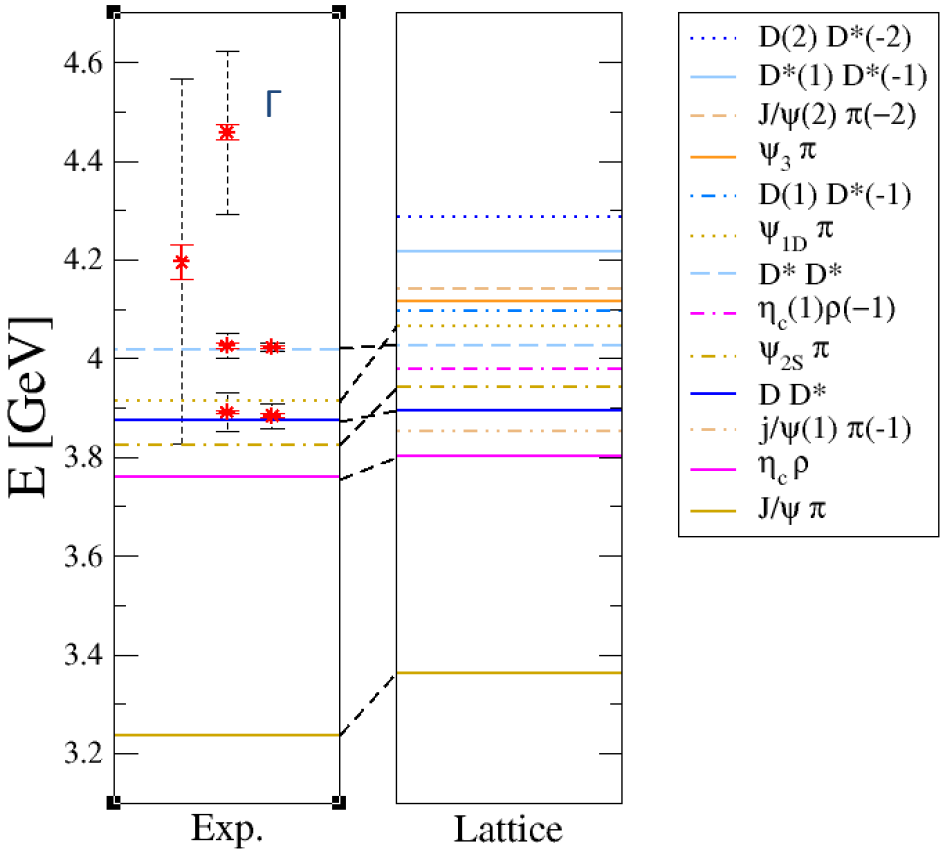
$$Z_c^+(3900) \to J/\psi\ \pi^+$$

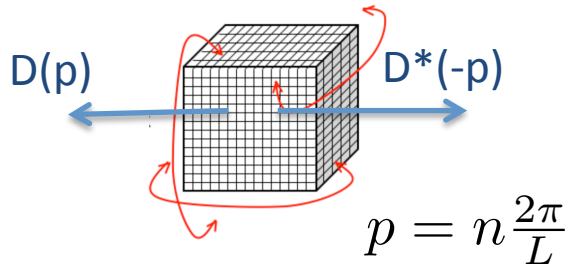
flavor content of the hadron: $\bar{c}c\ \ \bar{d}u$



M = 3888 ± 3 MeV , Γ = 35 ± 7 MeV

Z_c^+ channel : $I^G=1^+$, $J^{PC}=1^{+-}$





Lattice:

Horizontal lines represent energies of 13 two-meson states in non-interacting case

$E = E[\, M_1(p_1)\,] + E[\, M_2(p_2)\,]$

Extracting 13 two-meson states is a challenge

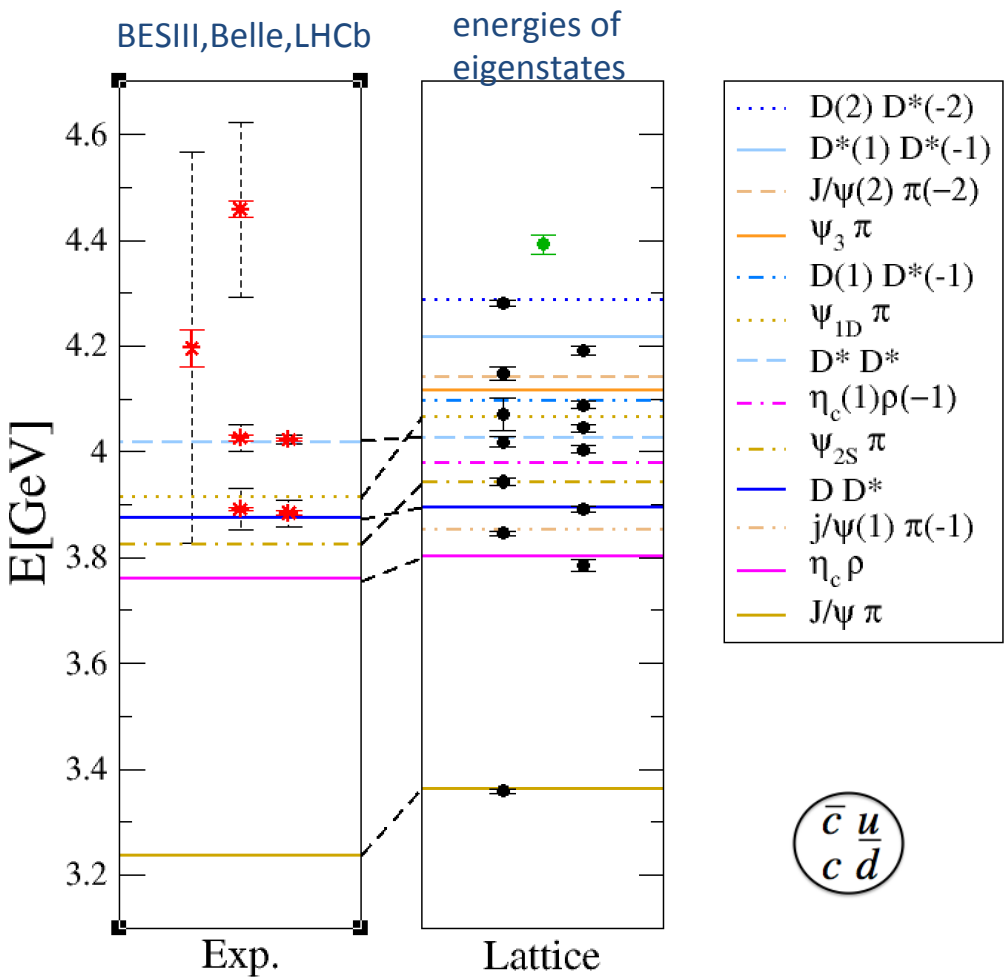
$$\mathcal{O} : (\bar{c}u)(\bar{d}c),\ \ (\bar{c}c)(\bar{d}u),\ \ [\bar{c}\bar{d}][cu]$$

[S.P., Lang, Leskovec, Mohler, 1405.7612, PRD 2015]

Ensemble (2), $m_\pi \approx 266$ MeV, $L \approx 2$ fm, Nf=2

Z_c^+ channel : $I^G=1^+, J^{PC}=1^{+-}$

$$\mathcal{O} : (\bar{c}u)(\bar{d}c),\ (\bar{c}c)(\bar{d}u),\ [\bar{c}\bar{d}][cu]$$



[S.P., Lang, Leskovec, Mohler, 1405.7612, PRD 2015]

similar conclusion [S.-H. Lee, C. DeTar, H. Na, 1411.1389]

Lattice results:

- 13 expected two-meson eigenstates found as expected (black circles)
- no additional eigenstate below 4.2 GeV
- no candidate for Z_c^+ below 4.2 GeV

Exp:

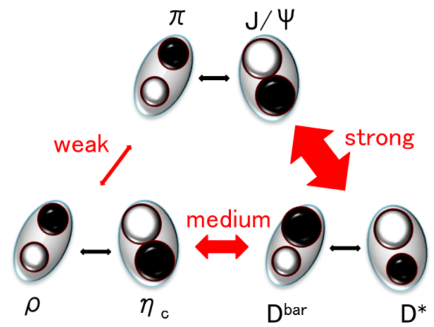
- $Z_c^+(3900)$ confirmed by three exp

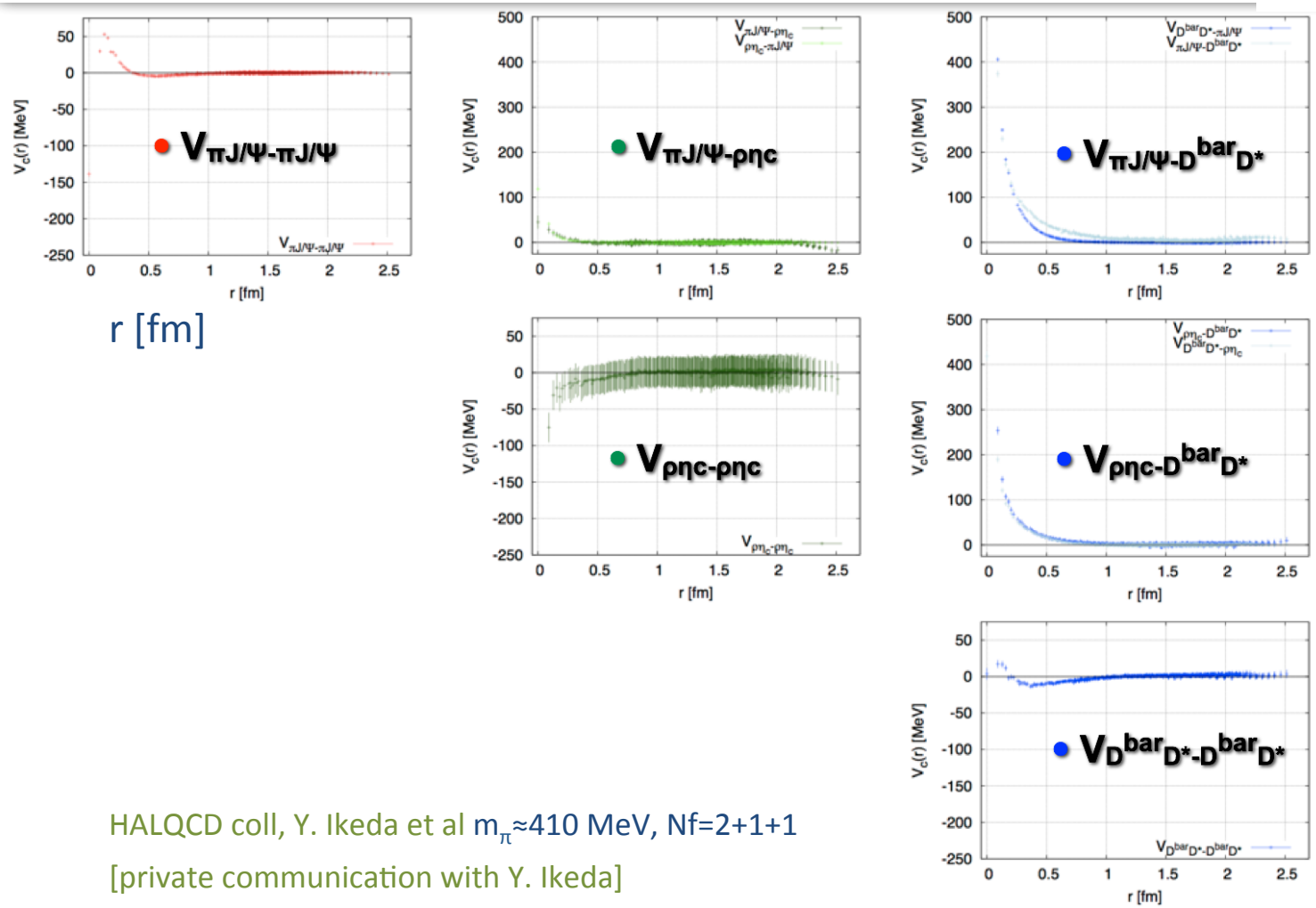
Puzzle:

- why no eigenstate for Zc(3900) ?
- it would be naively expected if Zc(3900) related to a resonance pole
- is Zc(3900) of a different origin?
- perhaps coupled channel effect?

Z_c^+ channel : $I^G=1^+$, $J^{PC}=1^{+-}$

- HALQCD method [application on H-dibaryon: HALQCD, 1504.01717]





r [fm]

indication for coupled channel eff.

HALQCD coll, Y. Ikeda et al $m_\pi \approx 410$ MeV, Nf=2+1+1

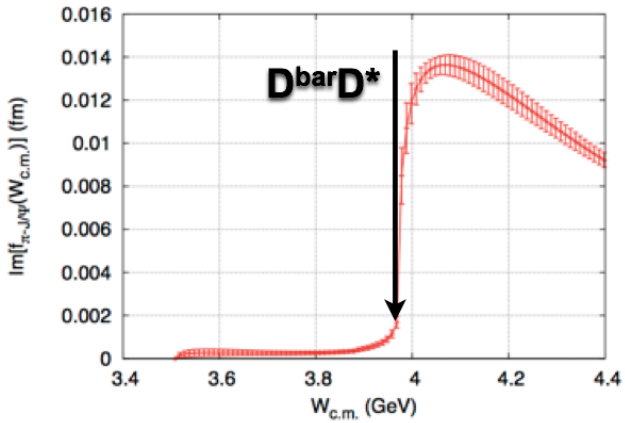
[private communication with Y. Ikeda]

Z_c^+ channel : $I^G=1^+$, $J^{PC}=1^{+-}$

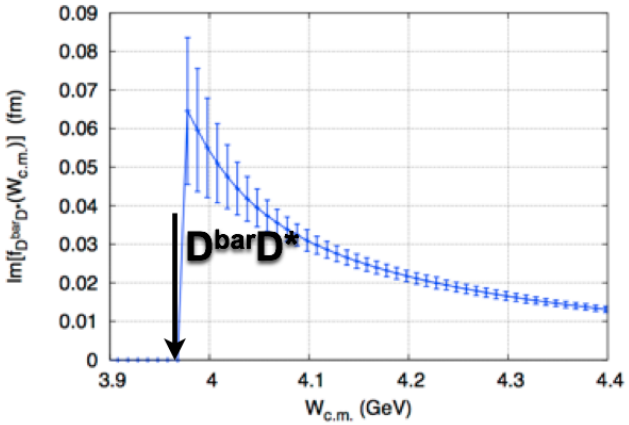
Lattice quantity related to cross-section

Experimental cross-section related to $Z_c^+(3900)$

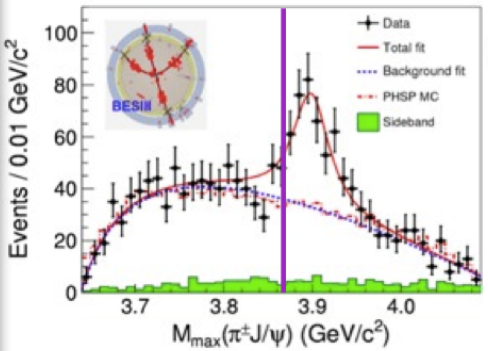
- **πJ/Ψ invariant mass (m_π=410MeV)**



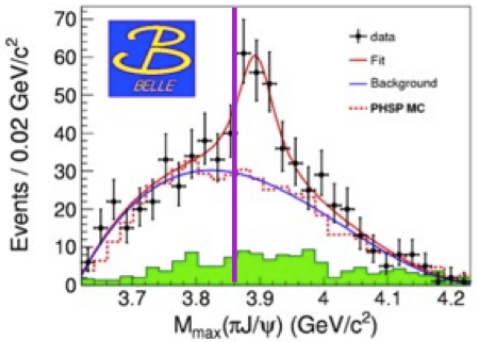
- **$D^{bar}D^*$ invariant mass (m_π=410MeV)**



- **e^+e^- --> π(πJ/Ψ) @ 4.26GeV**

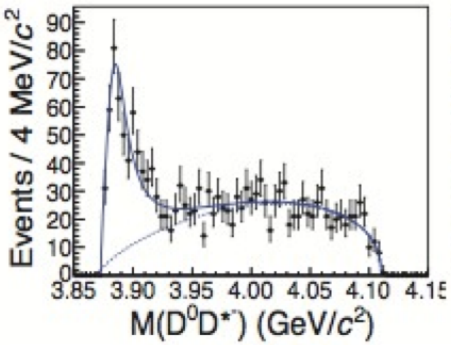


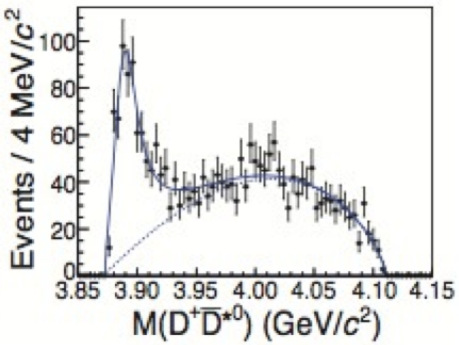
BESIII Coll., PRL110 (2013).



Belle Coll., PRL110 (2013).

- **e^+e^- --> $\pi^{+/-}$ $(D^{bar}D^*)^{-/+}$**





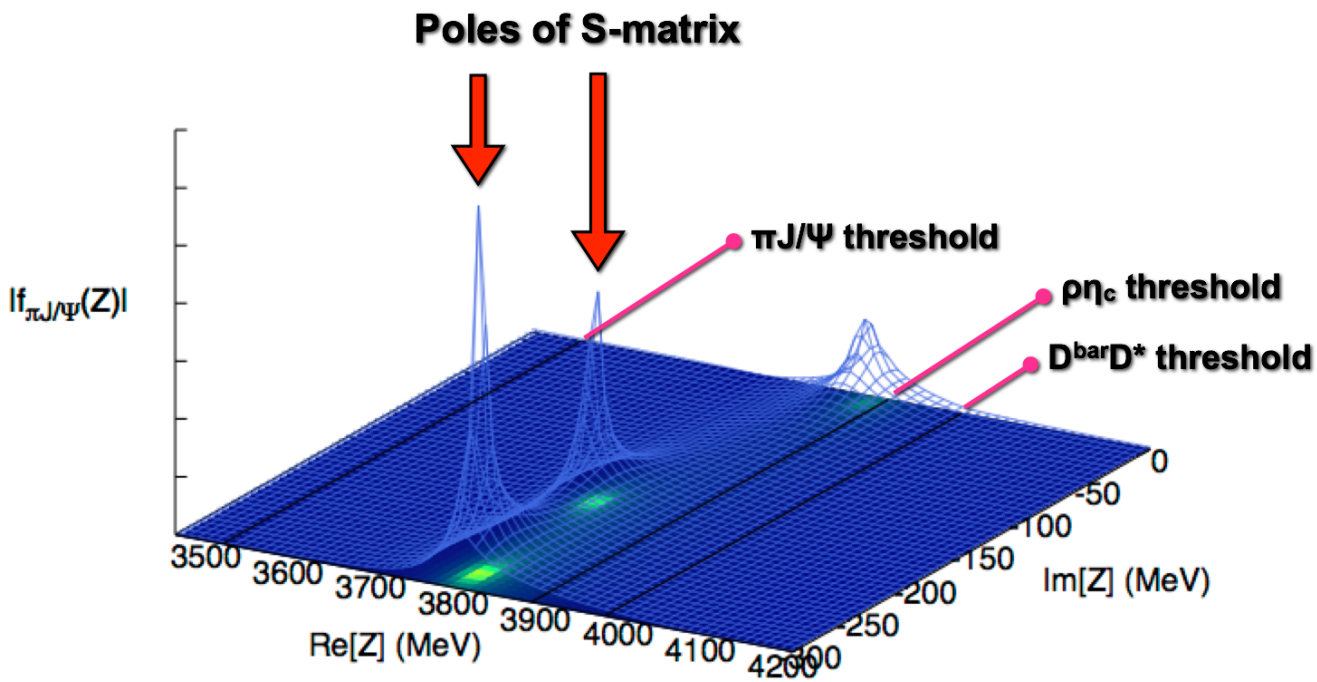
BESIII Coll., PRL112 (2014).

HALQCD coll, Ikeda et al m_π≈410 MeV, Nf=2+1+1

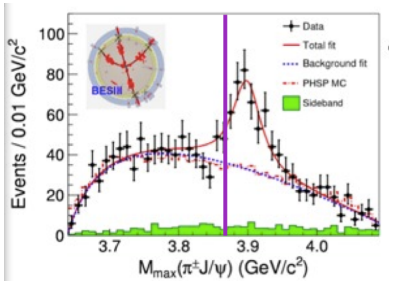
[private communication with Y. Ikeda]

Lineshapes resemble experimental Zc(3900).

Z_c^+ channel : $I^G=1^+$, $J^{PC}=1^{+-}$



DD*



- exp: $Z_c^+(3900)$ peak appears above DD* th.
- HALQCD: poles found BELOW DD* th.

  pole NOT interpreted as a resonance (such a pole would be expected above DD*)
- Remains to be seen if HALQCD result is consistent with absence of Zc eigenstate in S.P. et al, PRD 2015
- Scattering matrix has not yet been extracted from Luscher approach

HALQCD coll, Ikeda et al $m_\pi \approx 410$ MeV, Nf=2+1+1

[private communication with Y. Ikeda]

Conclusions

- **Experiment** (in brief): exciting tetraquark and pentaquark candidates

- **Phenomenology:** many many interpretations of exotic experimental "bumps", some of them do not need to introduce exotic hadrons to explain experimental data

- **Lattice QCD** (in brief):

❑ Evidence found for many hadrons with **non-exotic** flavor:

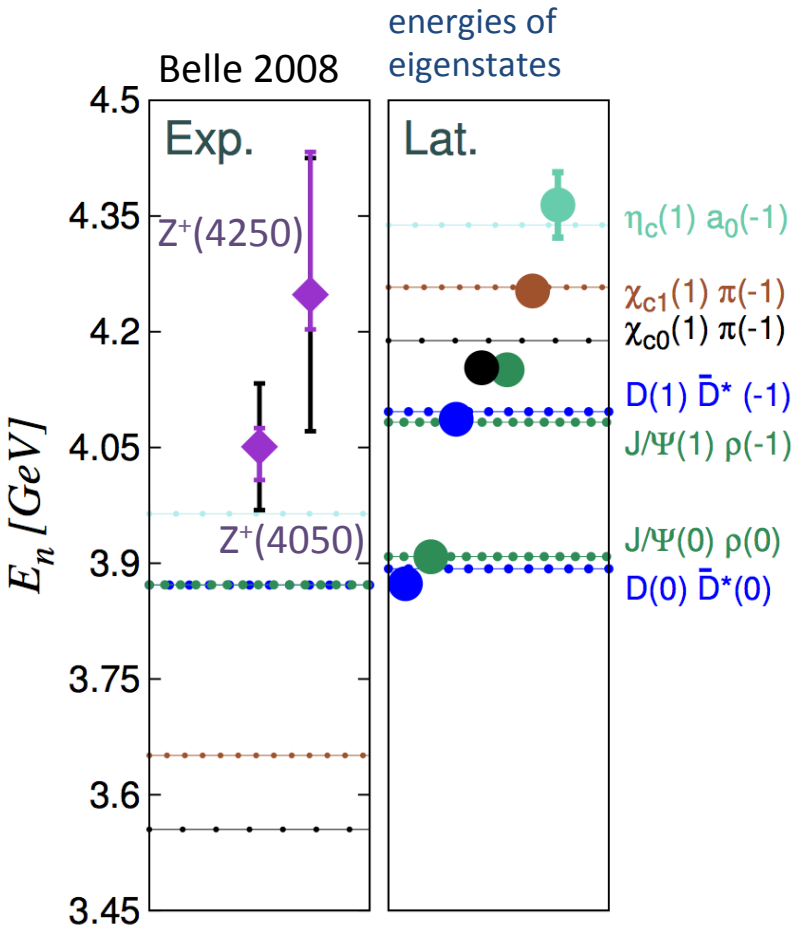
- states well below th. : charmonium , D, π, K ... and all the others
- shallow bound states : D_{s0}, D_{s1}, B_{s0}, B_{s1} , X(3872) with I=0
- resonances via BW : ρ, K*, $K_0^*(1430)$, K_2, D_0^*, D_1, a_1, b_1 , Ψ(3770)

❑ Hadrons with **exotic** flavor

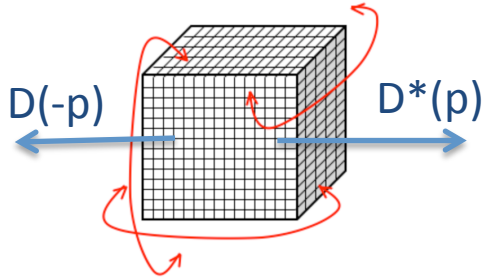
- if exotic hadrons were below strong decay threshold, they would be easy to (dis)prove in LQCD
- unfortunately, most of exotic hadrons can decay into several channels via strong interaction
- therefore lattice has not given yet a final answer which (if any) exotic hadrons arise from QCD
- I have (hopefully) given you some flavor in which direction progress is going
- this is an exciting topic at present and I am looking forward to face further challenges it poses ...

Backup slides

charged partner of X(3872); channel $I^G=1^-$, $J^{PC}=1^{++}$, $\underline{c}\underline{c}\underline{d}u$



[M. Padmanath, C.B. Lang, S.P., 1503.03257]



$$p = n\frac{2\pi}{L}$$

$$\mathcal{O} : (\bar{c}u)(\bar{d}c),\ \ (\bar{c}c)(\bar{d}u),\ \ [\bar{c}\bar{d}][cu]$$

- Horizontal lines: energies of expected two-meson states in limit of no interaction: $E = E[\ M_1(p_1)\ ] + E[\ M_2(p_2)\ ]$
- Circles: energies of eigenstates from latt
- Only expected two-meson states observed.
- No lattice candidate for charged X(3872). In agreement with absence of such state in exp.
- No lattice candidate for other charged state. Two Belle 2008 states are exp. unconfirmed.

Y(4140), $J^{PC}=?^{?+}$, $\underline{c}c\underline{s}s$

$$Y(4140) \to J/\psi\ \phi$$
$$\bar{c}c \quad \bar{s}s$$

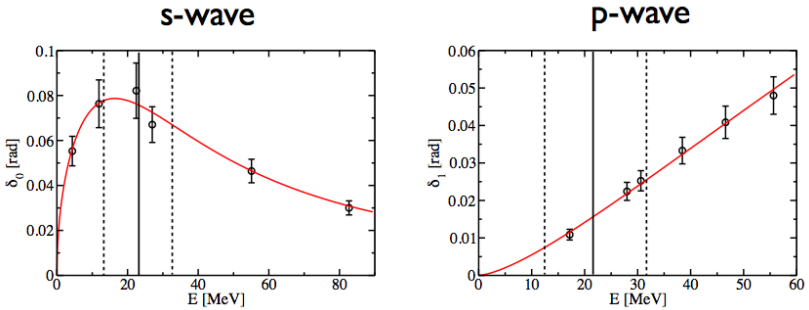
Experiment:

peak in J/ψ Φ just above J/ψ Φ threshold

found: CDF 2009, CMS 2012, D0 2013, Babar 2015

not found: Belle 2010, LHCb 2012

J/Ψ Φ scattering phase shift [rad]



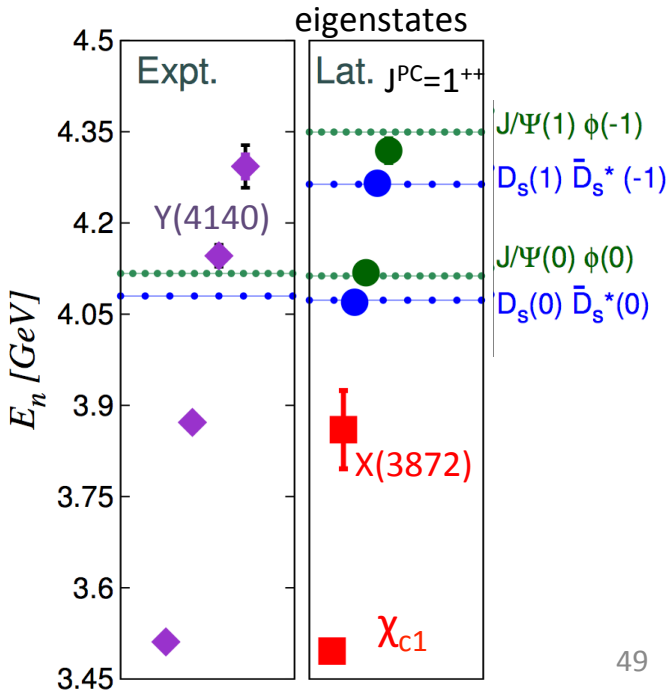
Lattice:

- S. Ozaki and S. Sasaki, 1211.5512, PRD
  strange quark annihilation neglected
  no resonant structure found for

- M. Padmanath, C.B. Lang, S.P., 1503.03257

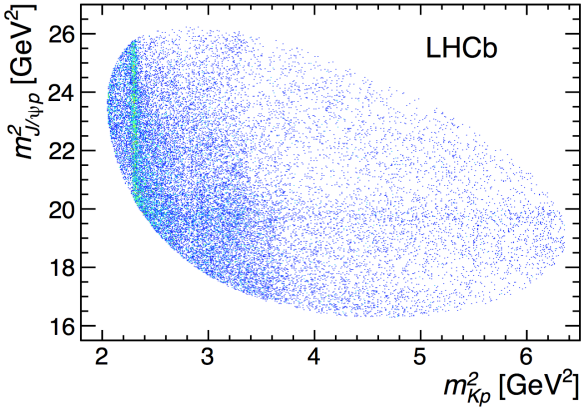
$$\mathcal{O}: \ \bar{c}c, \ (\bar{c}s)(\bar{s}c), \ (\bar{c}c)(\bar{s}s), \ [\bar{c}\bar{s}][cs]$$

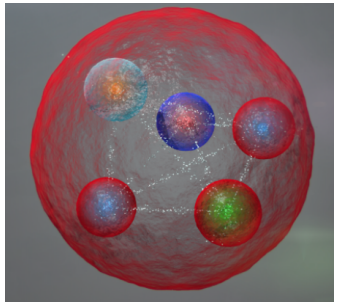
channel $J^P=1^+$ considered only: expected two-particle eigenstates found and χ_{c1}, X(3872) but not Y(4140)



Pentaquarks

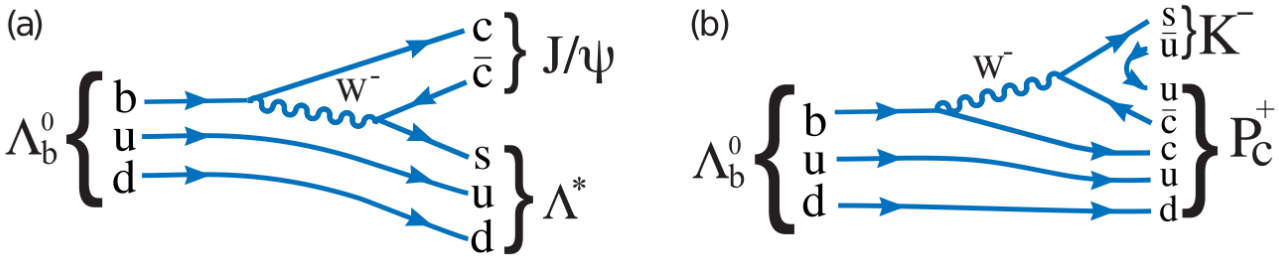
$$\Lambda_b^- \to K^- J/\psi \; p$$





LHCb: 1507.03414

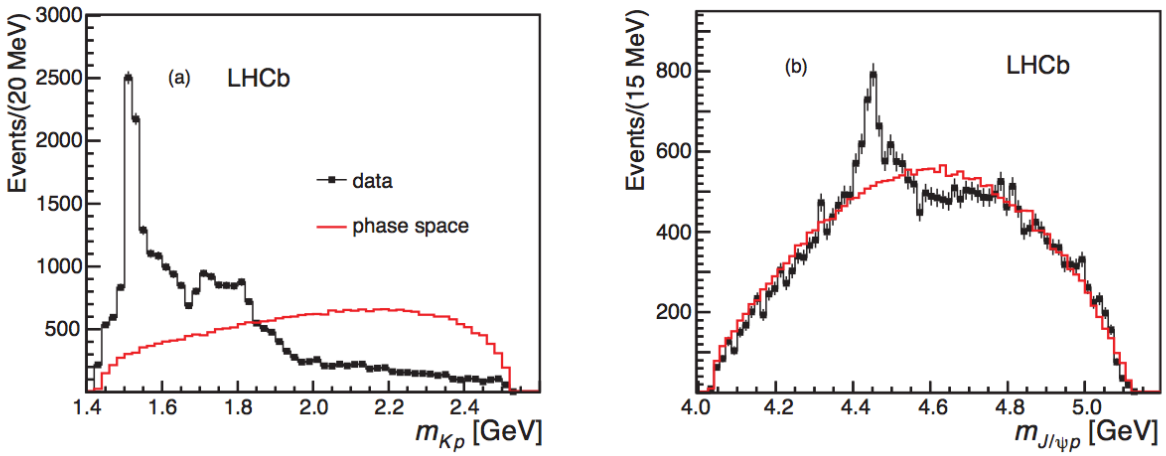
14th July 2015



$$P_c^+ \to J/\psi \;\; p$$

$$\bar{c}c \;\; uud$$

Figure 1: Feynman diagrams for (a) $\Lambda_b^0 \to J/\psi\,\Lambda^*$ and (b) $\Lambda_b^0 \to P_c^+ K^-$ decay.

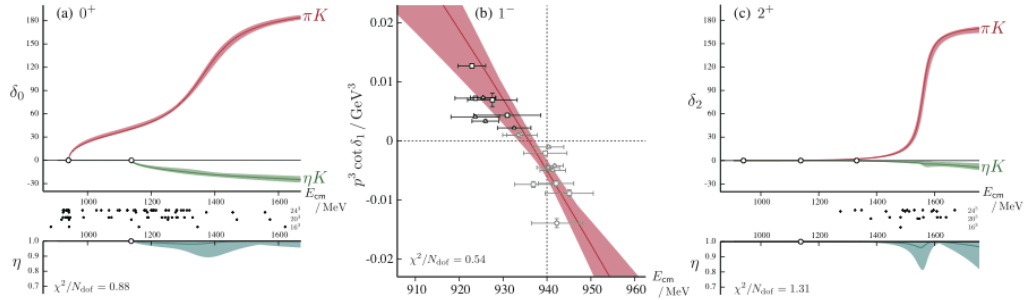


State	J^P	M_0 (MeV)	Γ_0 (MeV)
$\Lambda(1405)$	$1/2^-$	$1405.1^{+1.3}_{-1.0}$	50.5 ± 2.0
$\Lambda(1520)$	$3/2^-$	1519.5 ± 1.0	15.6 ± 1.0
$\Lambda(1600)$	$1/2^+$	1600	150
$\Lambda(1670)$	$1/2^-$	1670	35
$\Lambda(1690)$	$3/2^-$	1690	60
$\Lambda(1800)$	$1/2^-$	1800	300
$\Lambda(1810)$	$1/2^+$	1810	150
$\Lambda(1820)$	$5/2^+$	1820	80
$\Lambda(1830)$	$5/2^-$	1830	95
$\Lambda(1890)$	$3/2^+$	1890	100
$\Lambda(2100)$	$7/2^-$	2100	200
$\Lambda(2110)$	$5/2^+$	2110	200
$\Lambda(2350)$	$9/2^+$	2350	150
$\Lambda(2585)$	?	$\approx$2585	200

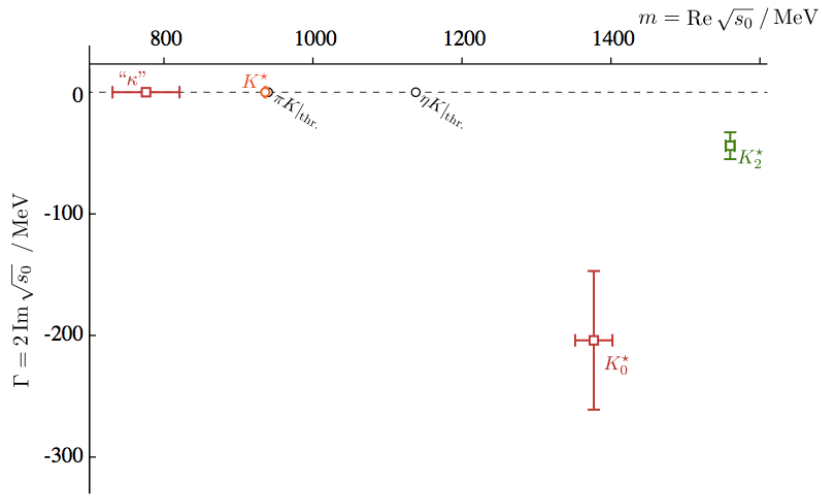
Resonances in Kπ, Kη coupled channels

- $\bar{q}q$, Kπ, Kη interpolators
- a number of different 0<P≤2
- for each E_n: one determinant equation for many unknowns
- T-matrix parametrized to get around this problem
- the location of poles of T-matrix in complex given below
- K*(892) and κ are below threshold for this m
- K_0^* , K_2^* are resonances
- m_π=391 MeV, N_L=16, 20, 24

[Dudek, Edwards, Thomas, Wilson, HSC, 1406.4158, PRL 2014]



$$t_{ii} = \frac{(\eta e^{2i\delta_i} - 1)}{2i\rho_i}, \; t_{ij} = \frac{\sqrt{1-\eta^2}\, e^{i(\delta_i+\delta_j)}}{2\sqrt{\rho_i\,\rho_j}}$$

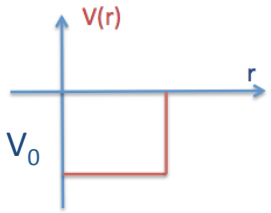


$$\det\left[\delta_{ij}\delta_{JJ'} + i\rho_i\, t_{ij}^{(J)}(E_{\mathrm{cm}})\left(\delta_{JJ'} + i\mathcal{M}_{JJ'}^{\vec{P}\Lambda}(p_i L)\right)\right] = 0,$$

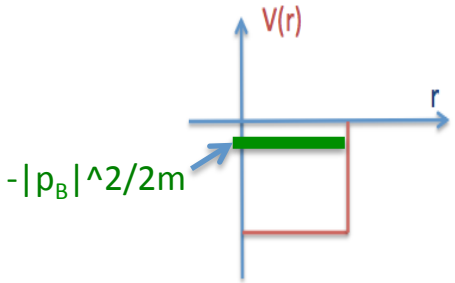
location of poles in T matrix in complex plane

How to identify a shallow bound state? (continued)

V0 that provides no bound st.

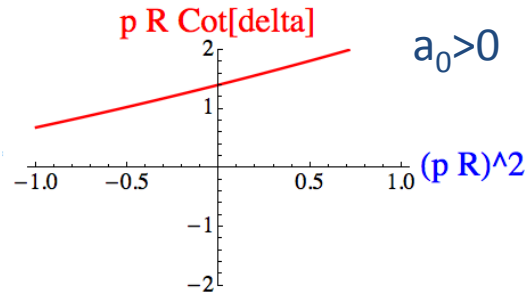


V0 that provides one shallow bound st.

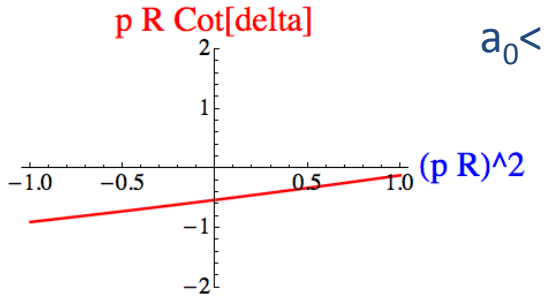


$$\lim_{p\to 0} p \cot\delta(p) = \frac{1}{a_0}$$

a_0=scattering length

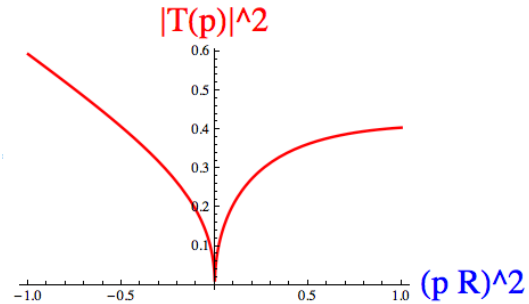


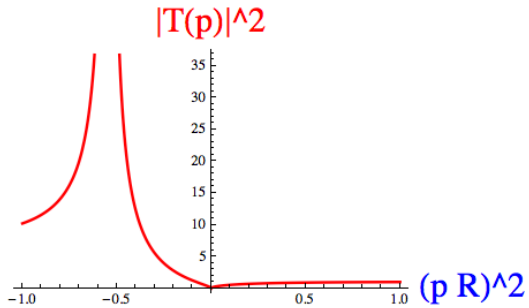
$a_0>0$



$a_0<0$

$$|T(p)|^2 \propto \left|\frac{1}{\cot\delta - i}\right|^2$$

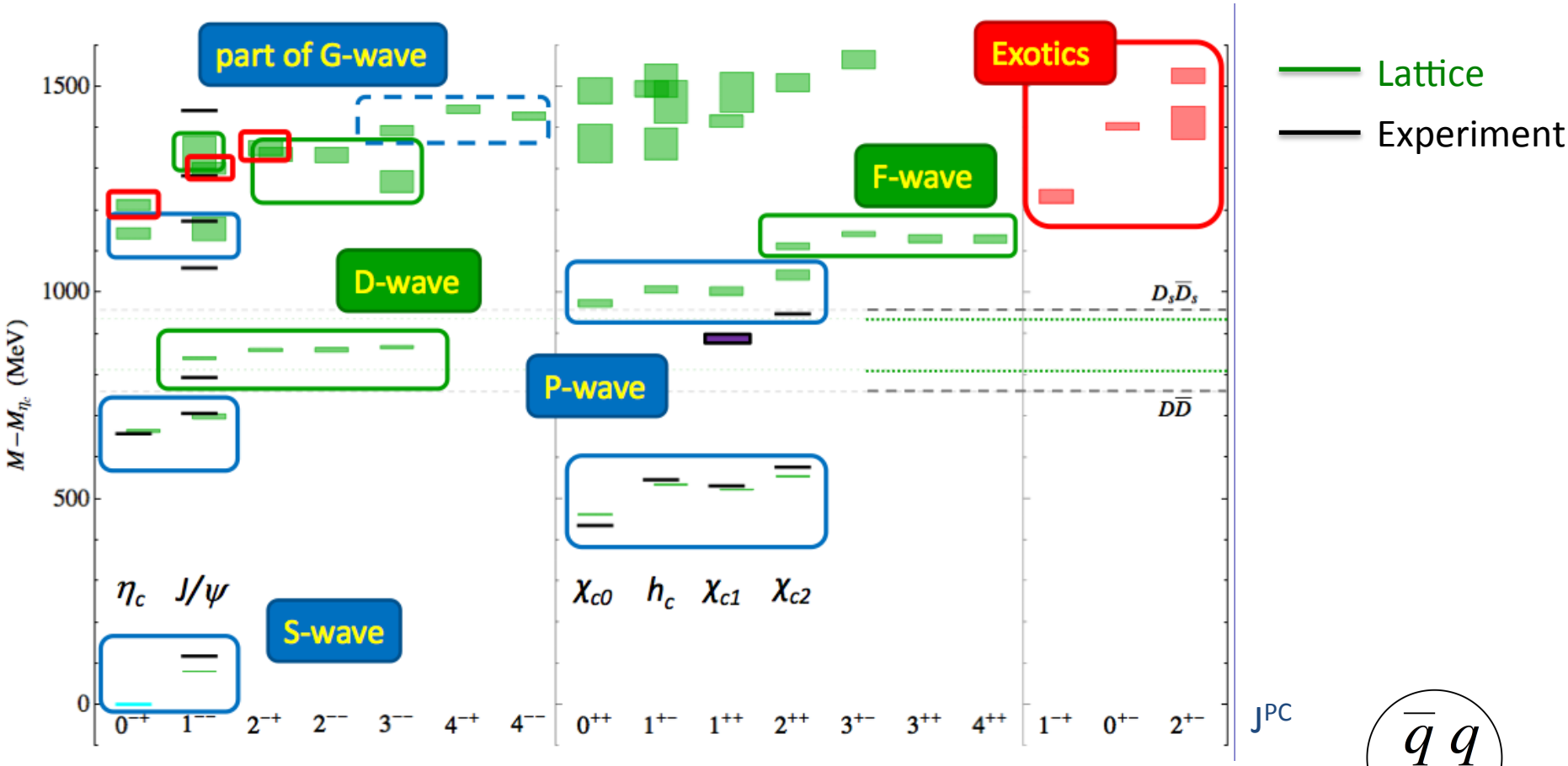


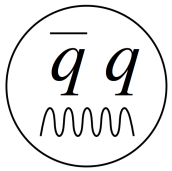


$-(|p_B|\,R)$^2

scattering matrix
has a pole

$\underline{c}c$ spectrum: single-hadron approximation





[HSC , L. Liu et al: 1204.5425, JHEP]

- $m_\pi \approx 400$ MeV, $L \approx 2.9$ fm, Nf=2+1
- identification with $n\ ^{2S+1}L_J$ multiplets using $\langle O|n\rangle$
- green: lat, black: exp

Hybrids:

some of them have exotic J^{PC}

large overlap with $O = \underline{q}\ F_{ij}\ q$

Hadrons near or above threshold: single-hadron approximation

- only interpolating fields $\mathcal{O} \approx \bar{q}\, q$ or $\mathcal{O} \approx q\, q\, q$
- assumptions: all energy levels correspond to "one-hadron" states

  no two-hadron state is seen

  m=E (for P=0)

  these are strong assumptions ...

  but results still present valuable reference point