Low-energy Precision Physics and the Role of Lattice QCD

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HELMHOLTZ ASSOCIATION
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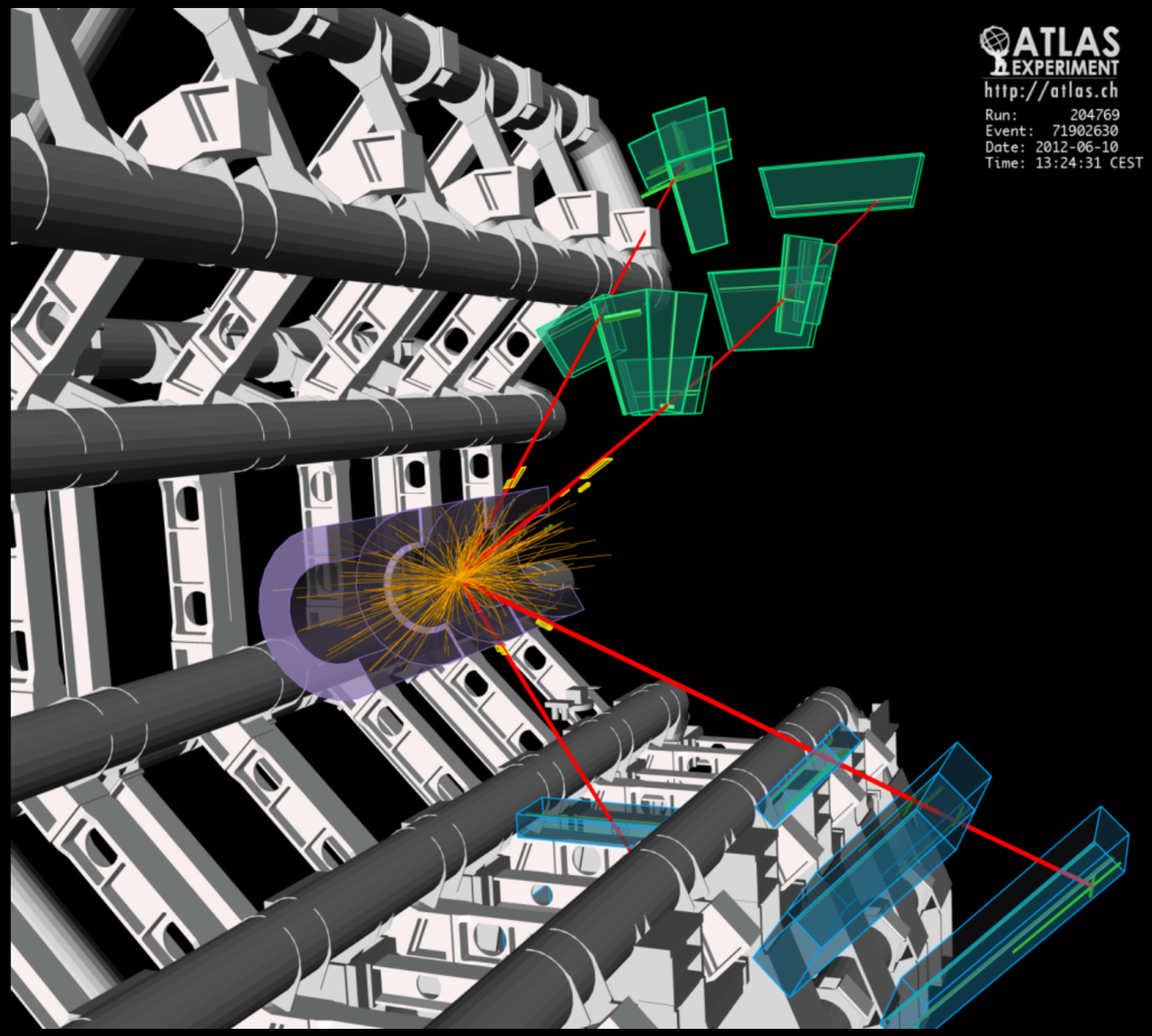
SFB 1044
THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

The Standard Model after the Higgs discovery



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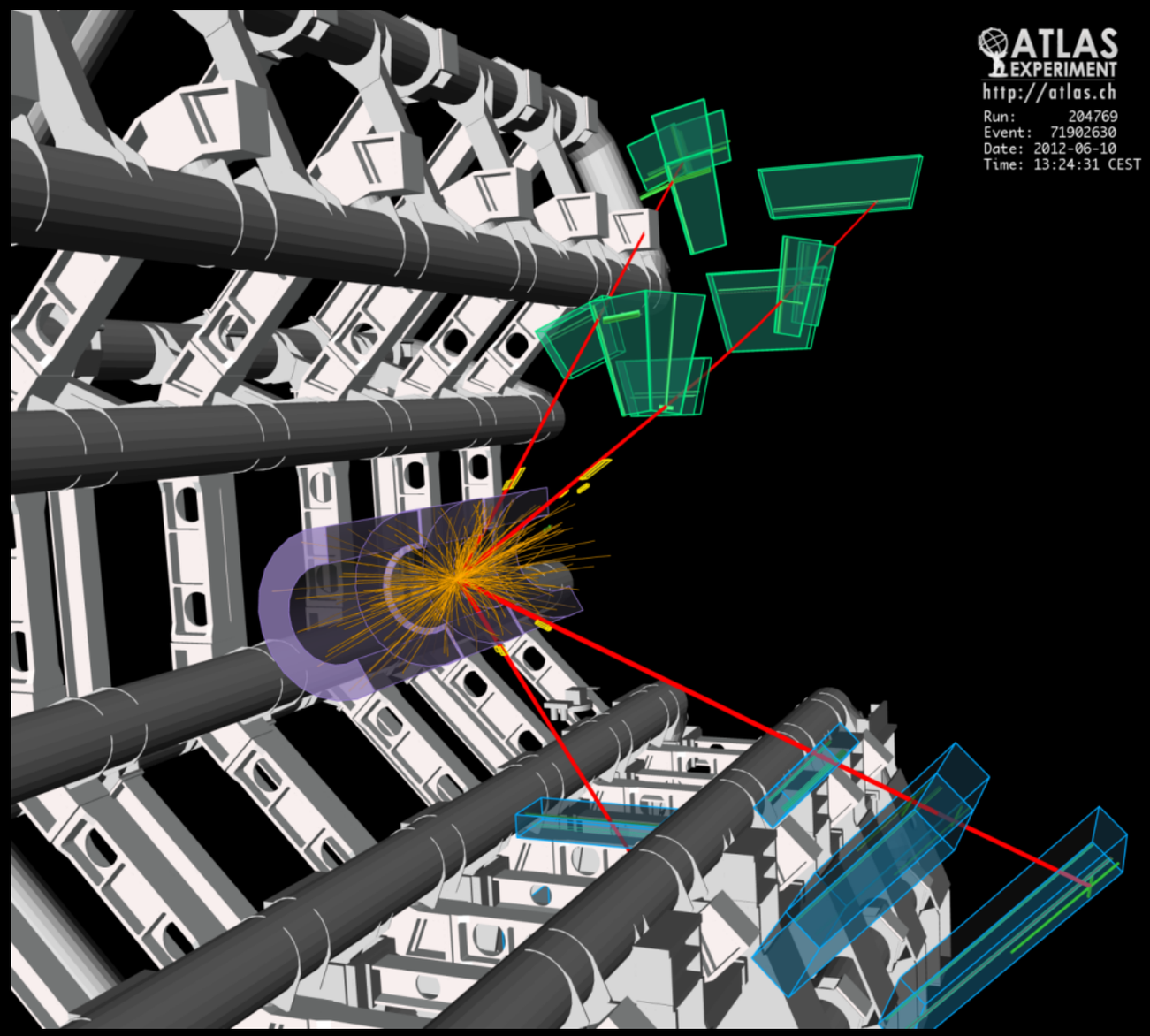

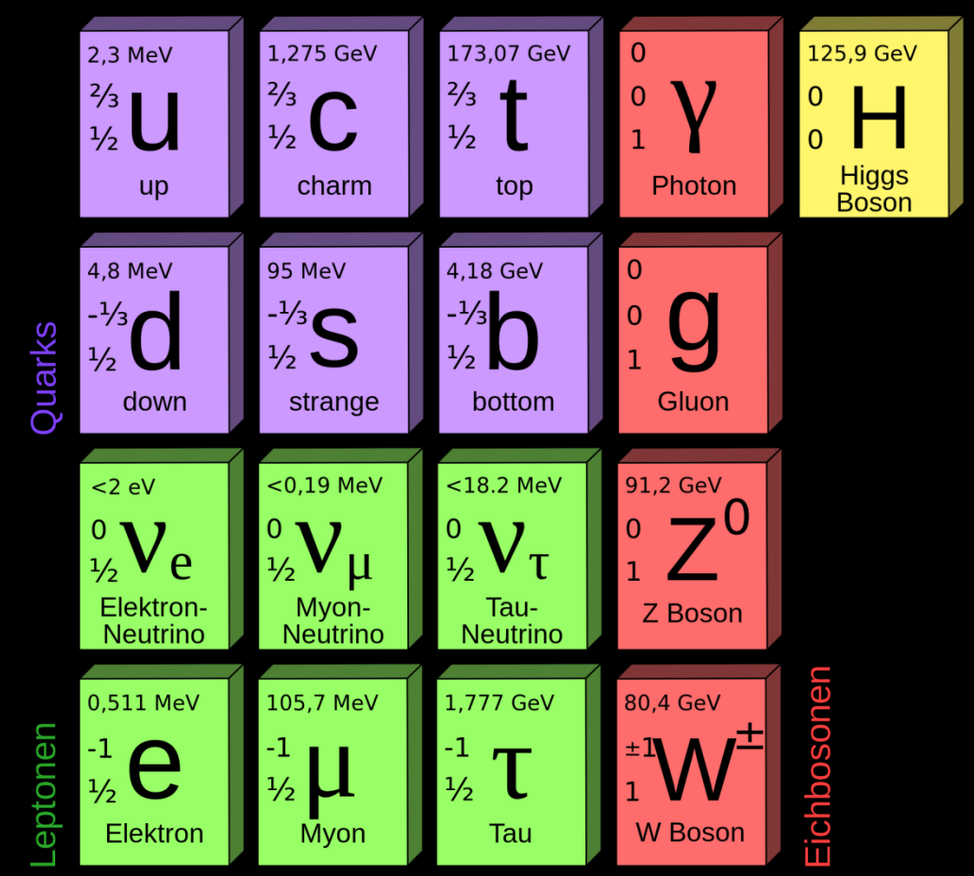

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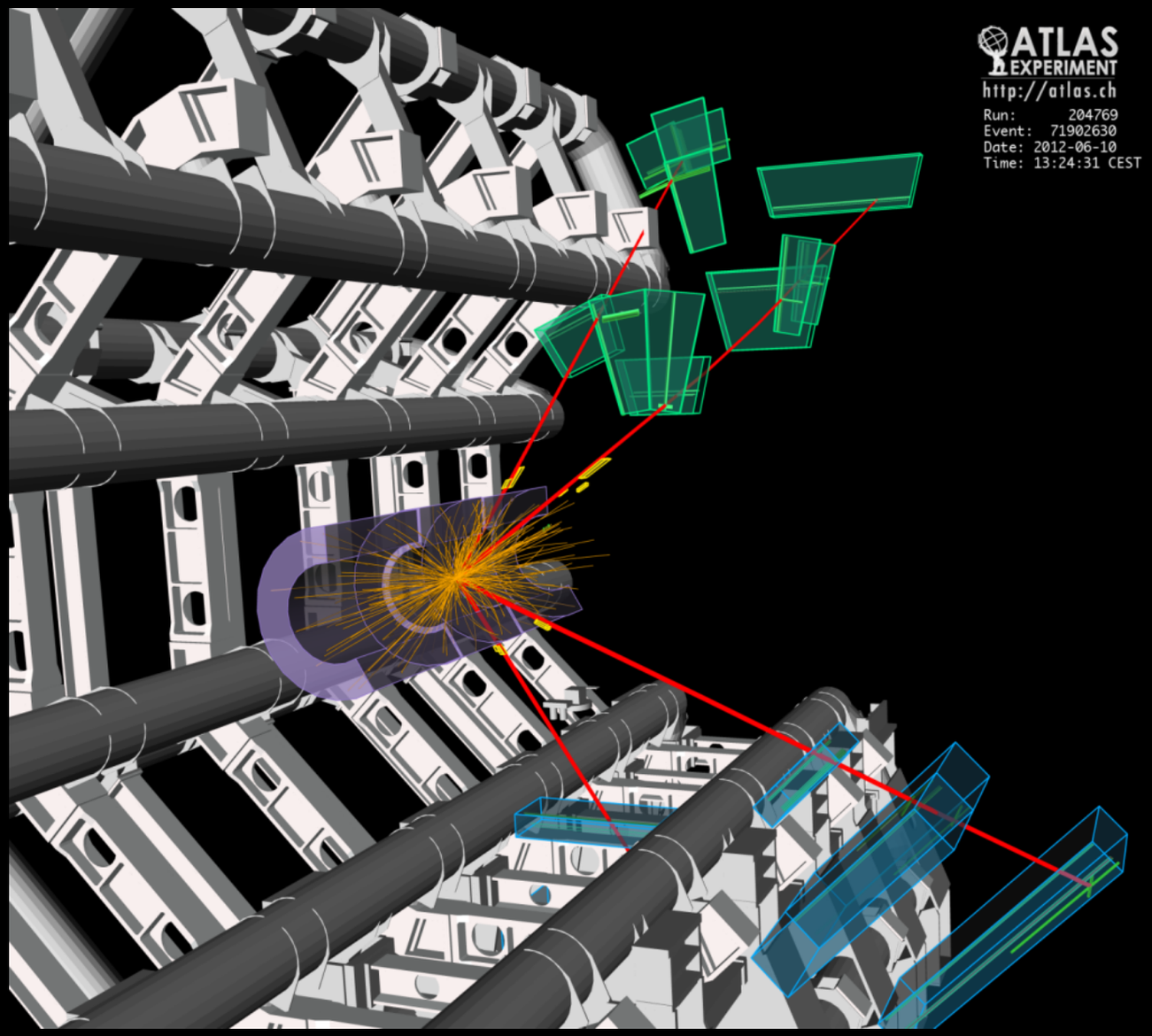

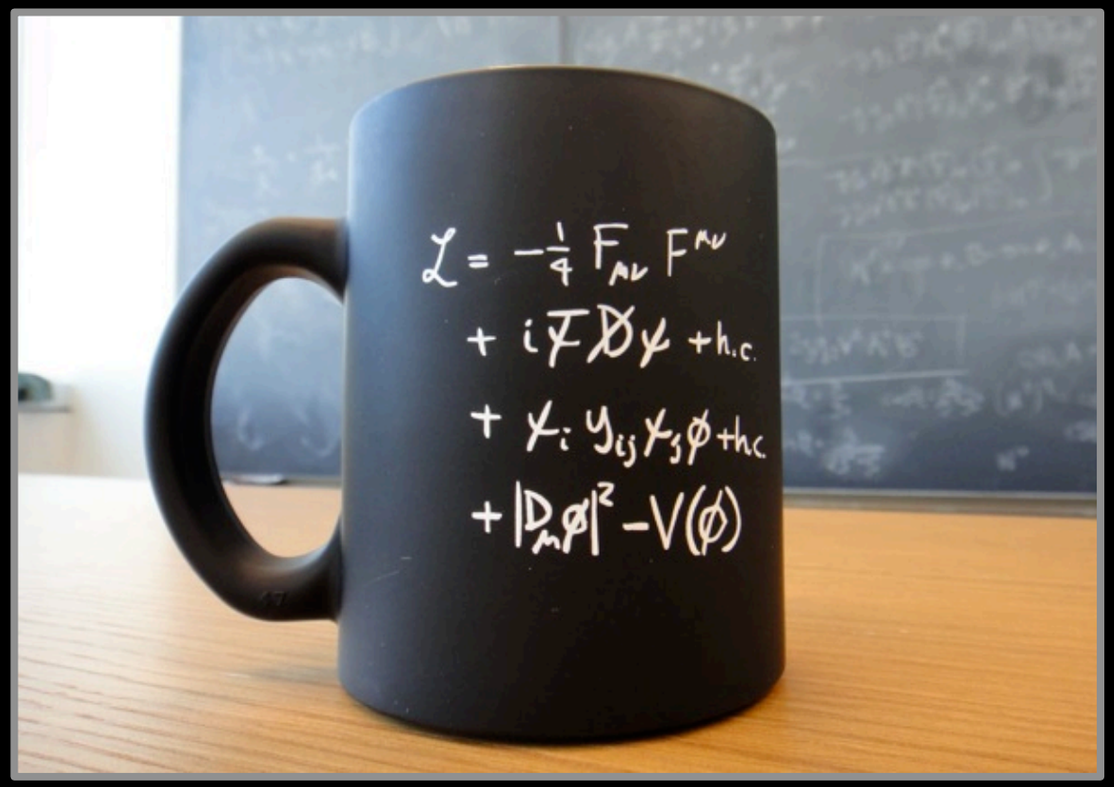

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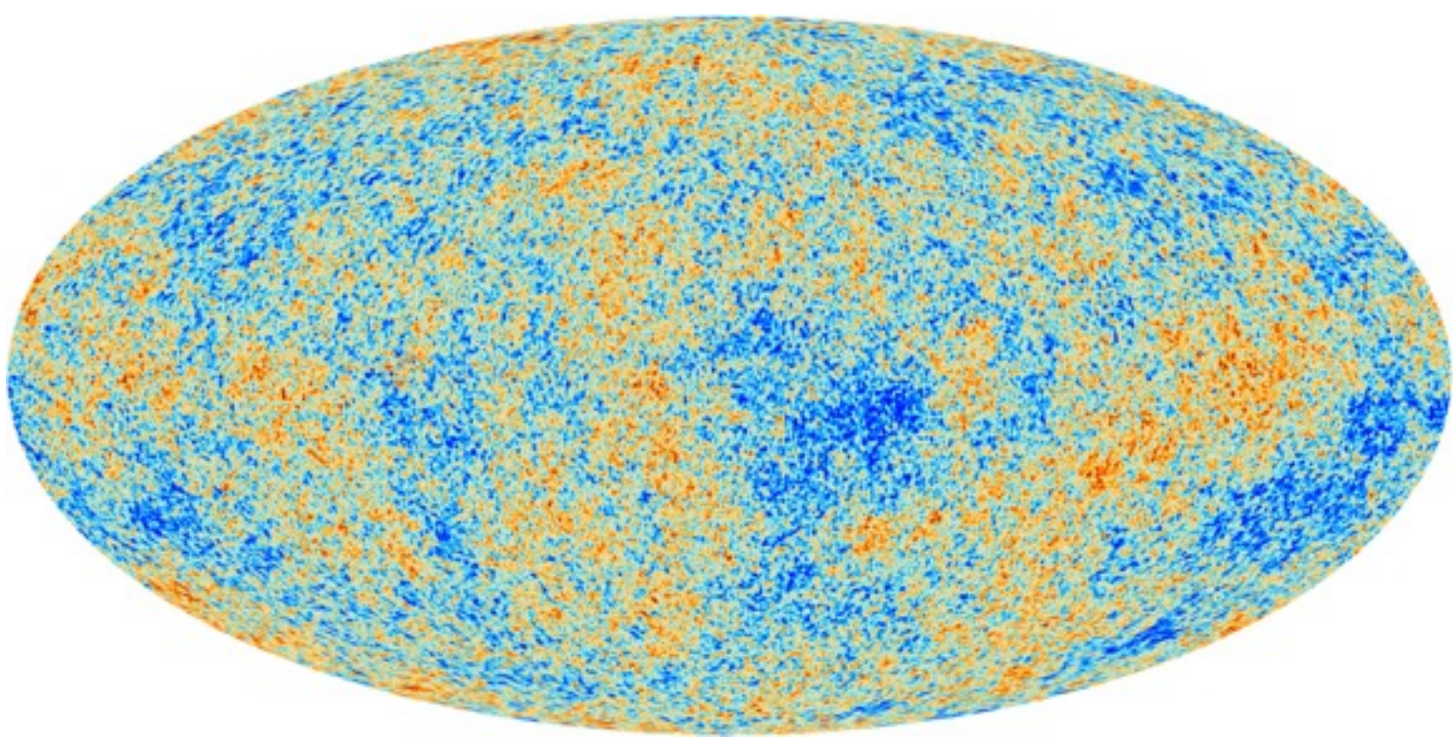
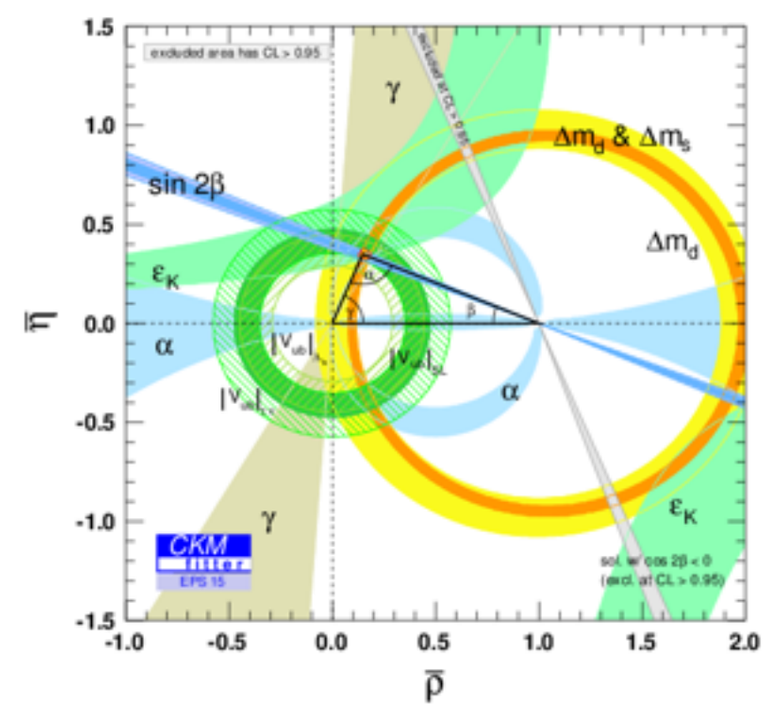
  - Mass and scale hierarchies:

$$m_{\rm top}/m_{\nu_e} > 10^{11} \qquad\qquad m_{\rm Higgs} \ll m_{\rm Planck}$$

  - Dark matter and dark energy
  - Amount of CP violation to sustain matter/antimatter asymmetry





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  - Search for new particles and phenomena at higher energy
  - Search for enhancement of rare phenomena
  - Compare precision measurements to SM predictions

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    Control over **hadronic uncertainties**

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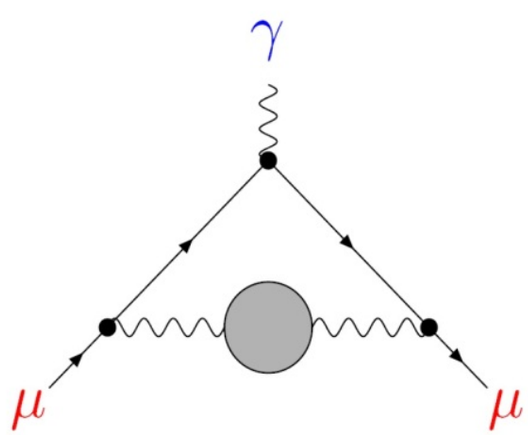
Precision Tests of the Standard Model

Anomalous magnetic moment of the muon:

$$a_\mu \equiv \tfrac{1}{2}(g-2)_\mu = \begin{cases} 116\,592\,080(54)(33)\cdot 10^{-11} & \text{E821 @ BNL} \\ 116\,591\,828(43)(26)(2)\cdot 10^{-11} & \text{SM prediction} \end{cases}$$

Hadronic vacuum polarisation:

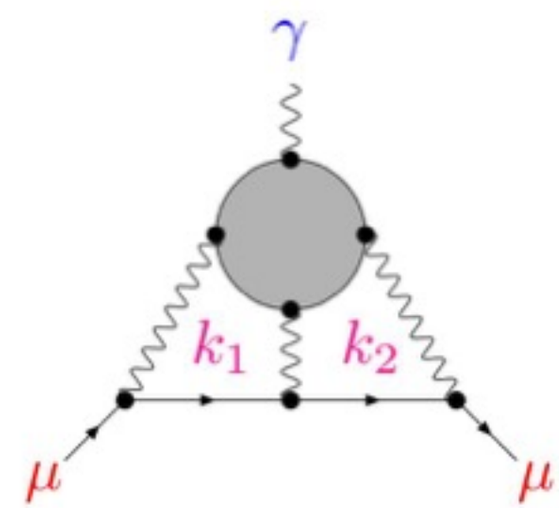


Dispersion theory:

$$a_\mu^{\rm hvp} = (6949 \pm 43)\cdot 10^{-11}$$

(combined e^+e^- data)

Hadronic light-by-light scattering:



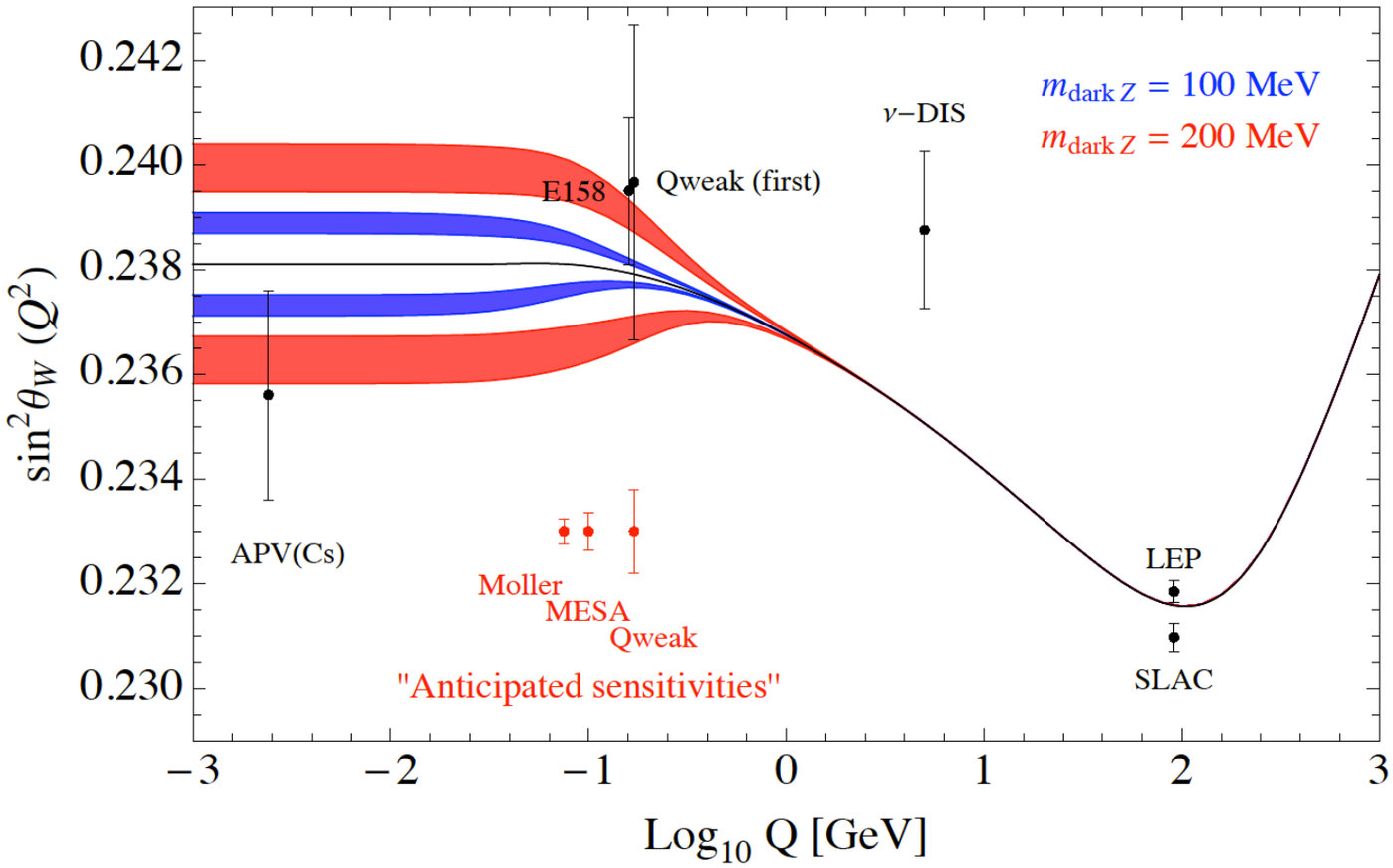
Model estimates:

$$a_\mu^{\rm hlbl} = (105 \pm 26)\cdot 10^{-11}$$

"Glasgow consensus"

Precision Tests of the Standard Model

Running of electroweak mixing angle: $\sin^2\theta_W$

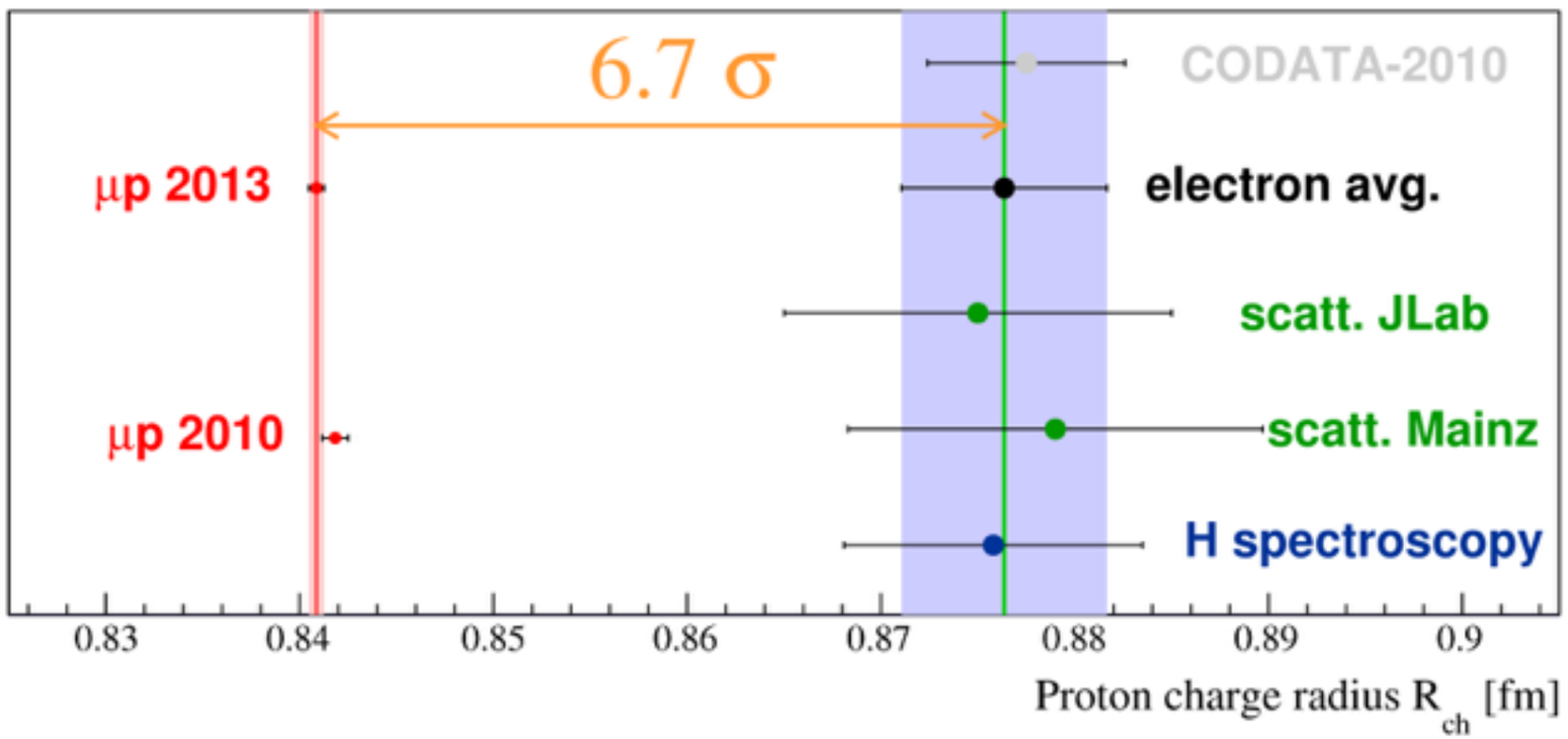


- Running of $\sin^2\theta_W$ at low energies discriminates between "New Physics" scenarios
- Challenge for theory: hadronic contributions

Precision Tests of the Standard Model

Proton Radius Puzzle

[Arrington 2015]



Muonic Hydrogen: $r_E = 0.8409 \pm 0.0004\,\mathrm{fm}$ *[Antognini et al. 2013]*

Electronic systems: $r_E = 0.8775 \pm 0.0051\,\mathrm{fm}$ *[CODATA 2012]*

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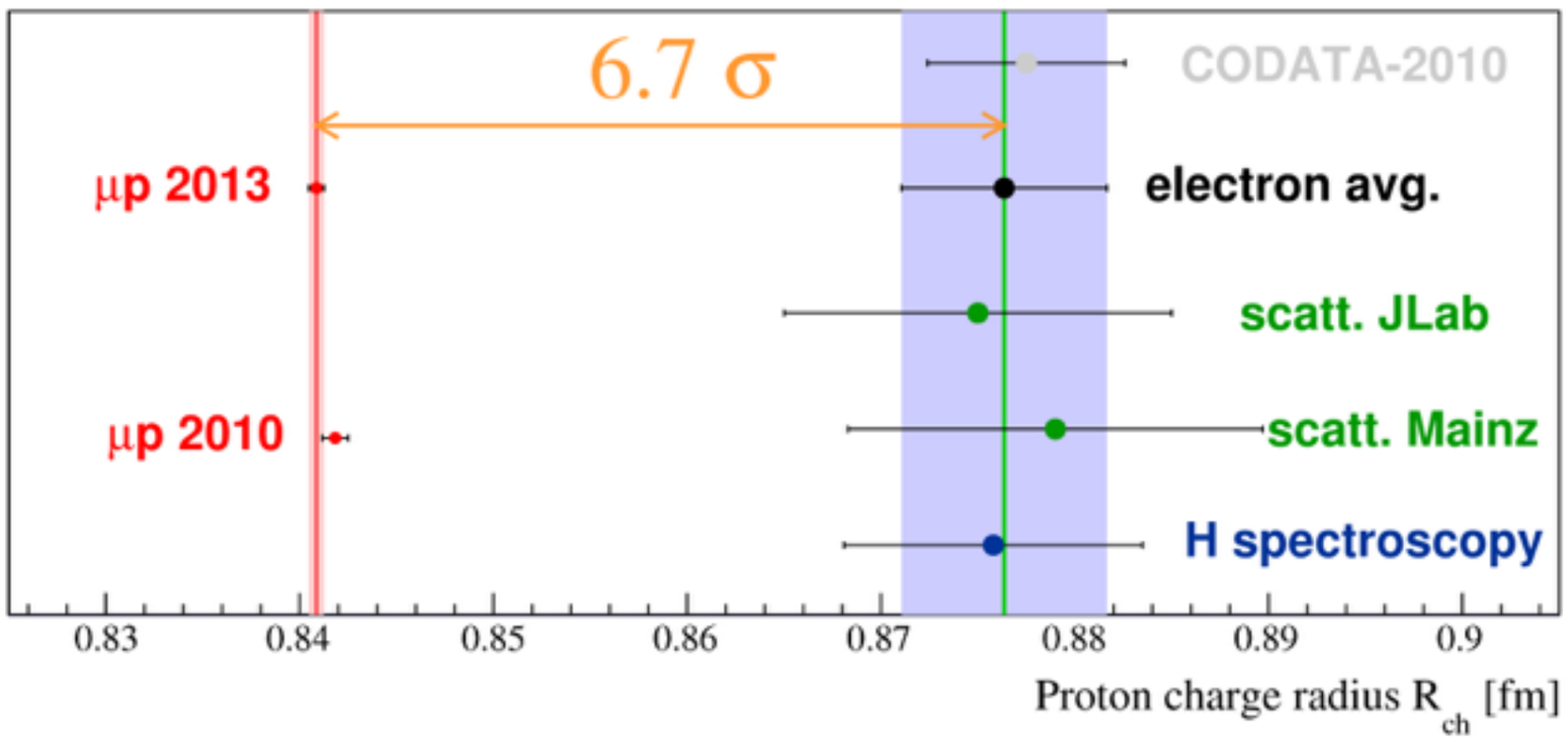

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NUCLEAR STRUCTURE

Proton-radius puzzle deepens

Precision Tests of the Standard Model

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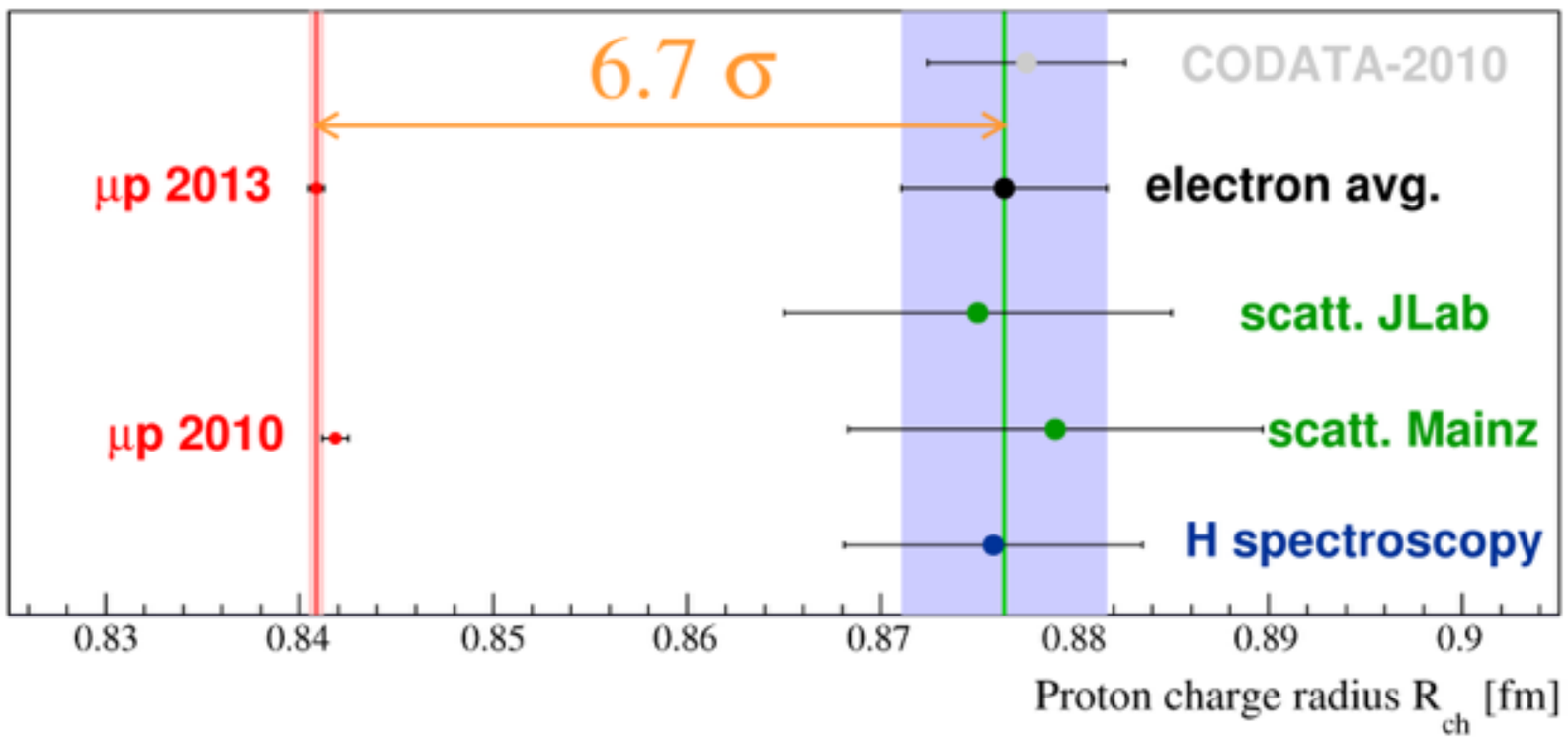

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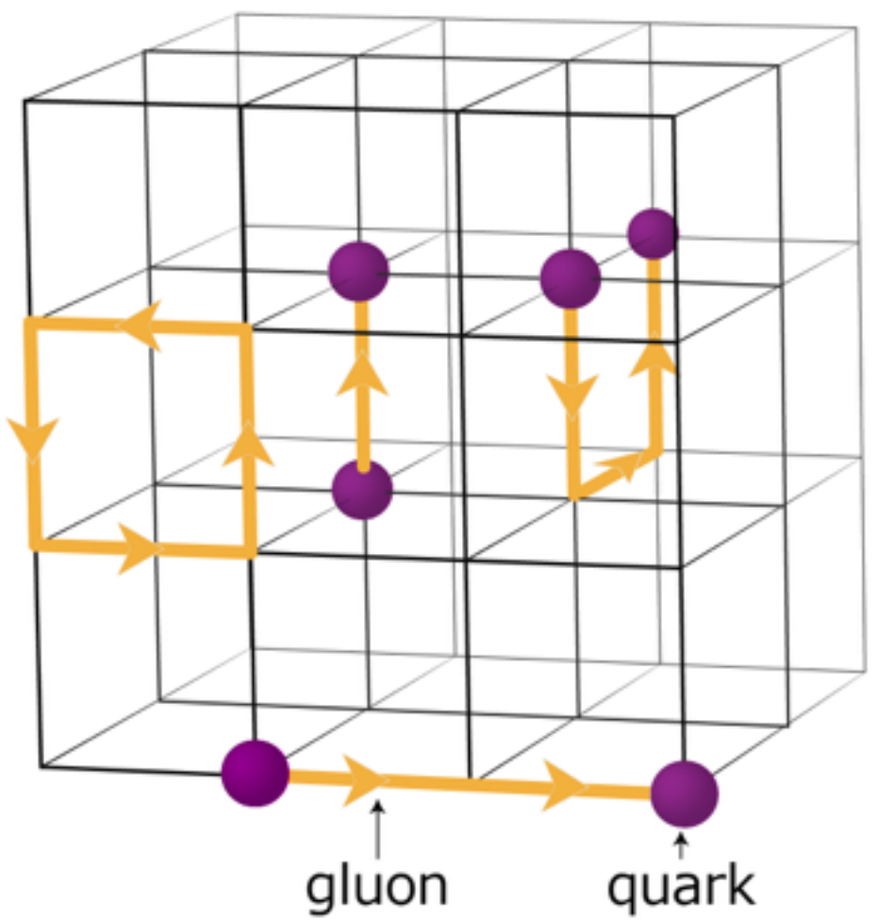
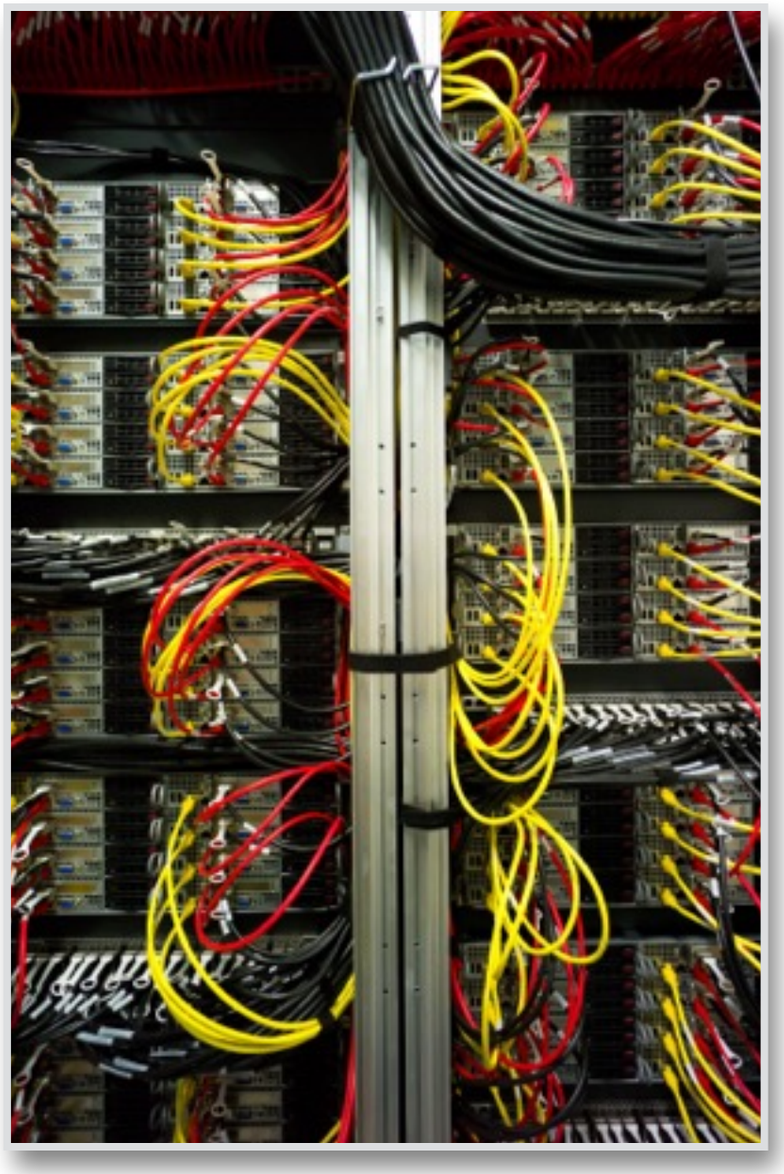
* Signal for "New Physics" or poorly understood hadronic effects?

Precision Tests of the Standard Model

* Accuracy of Standard Model tests limited by hadronic contributions
* Employ “ab initio” approach: Lattice QCD





“Clover” @ Mainz

Outline

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V. Summary & Outlook

Low-energy precision experiments at Mainz

Mainz Microtron
MAMI

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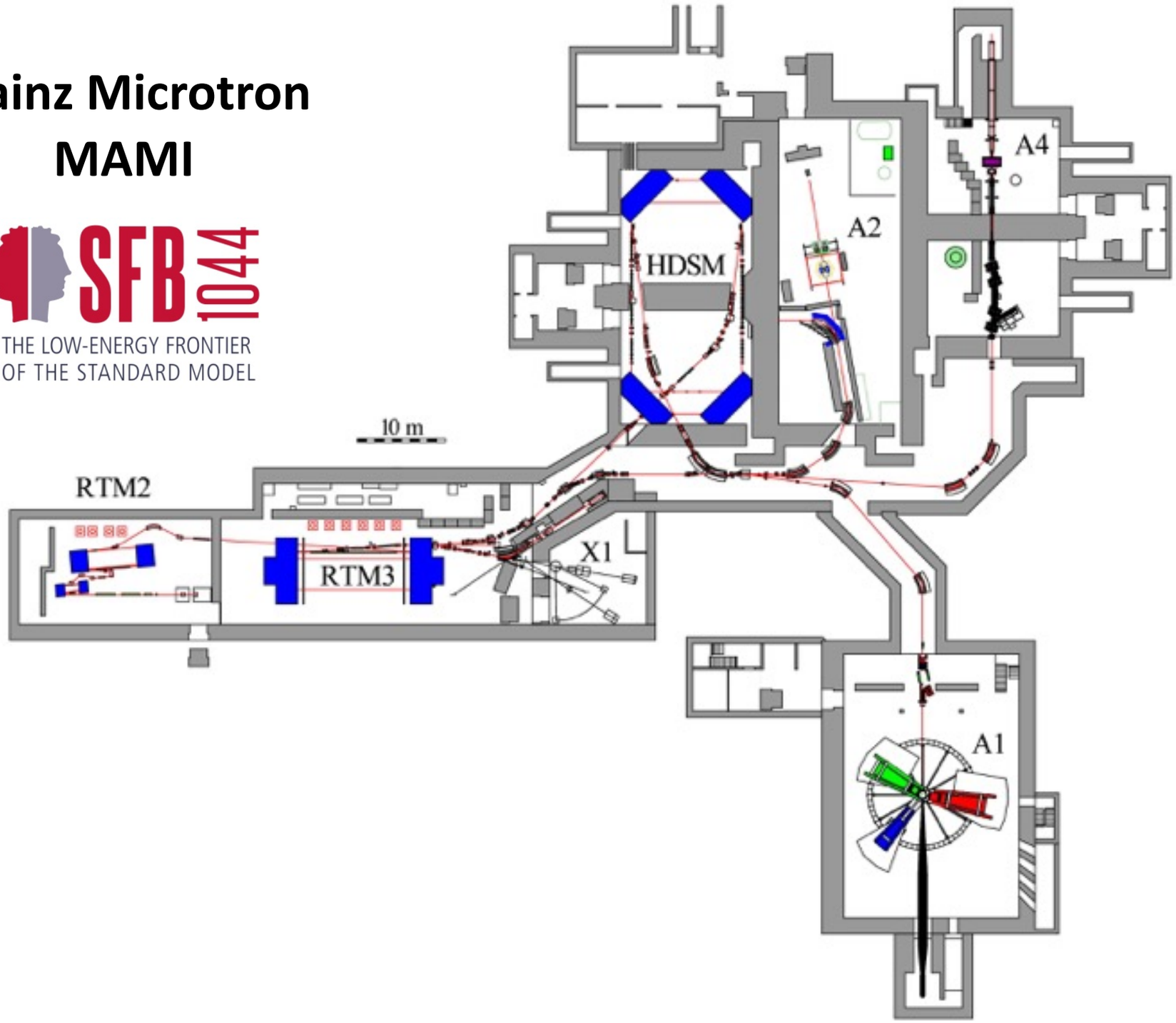

Low-energy precision experiments at Mainz

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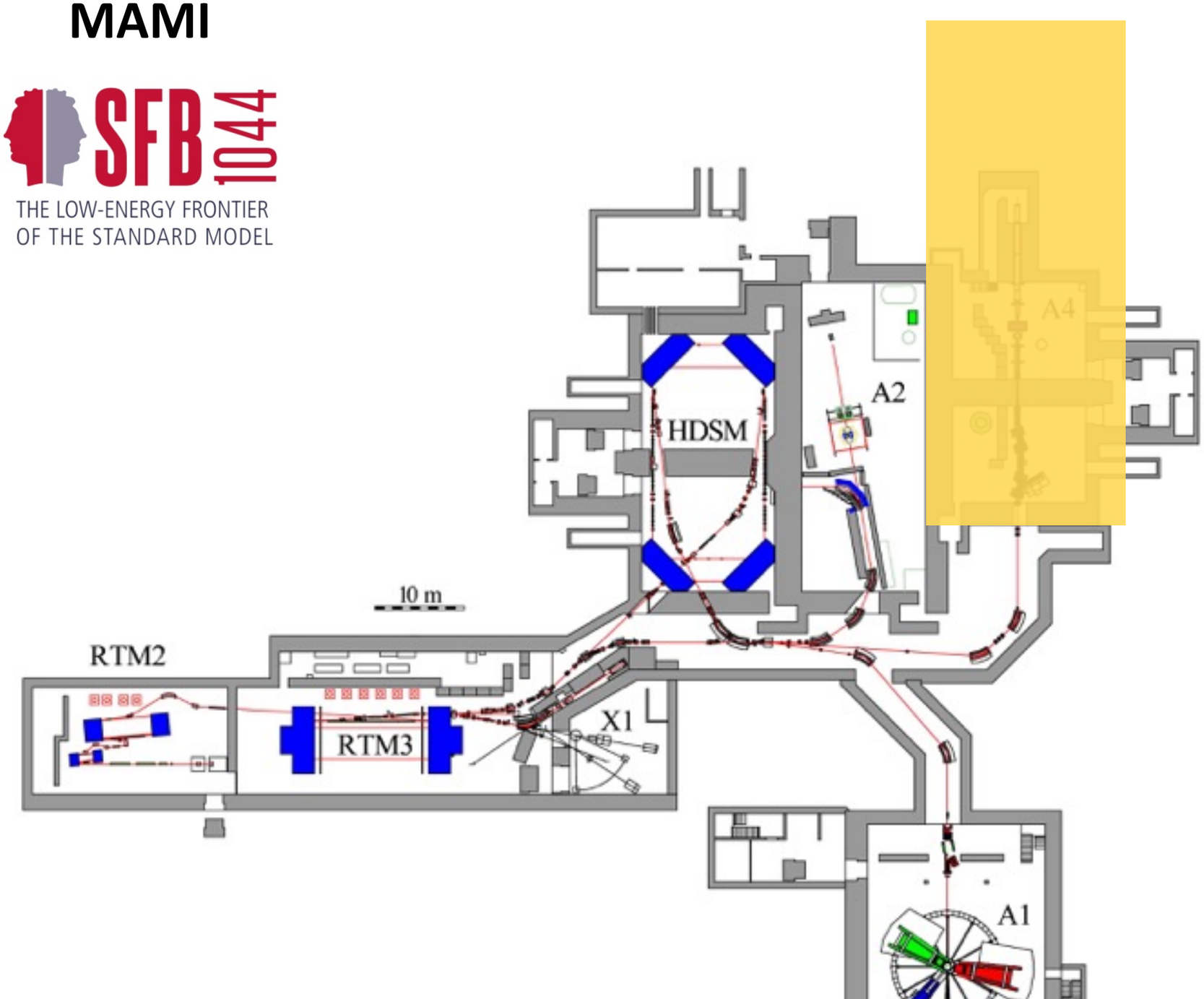
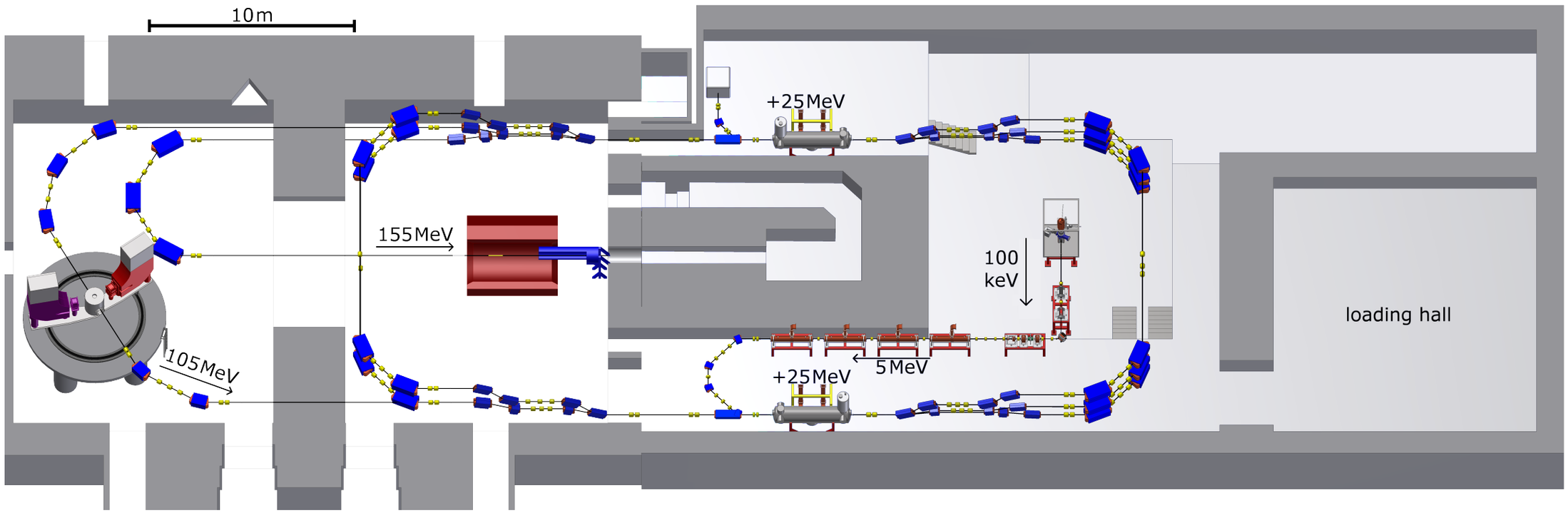
MESA
Accelerator



Low-energy precision experiments at Mainz

* MESA — "**M**ainz **E**nergy-Recovering **S**uperconducting **A**ccelerator



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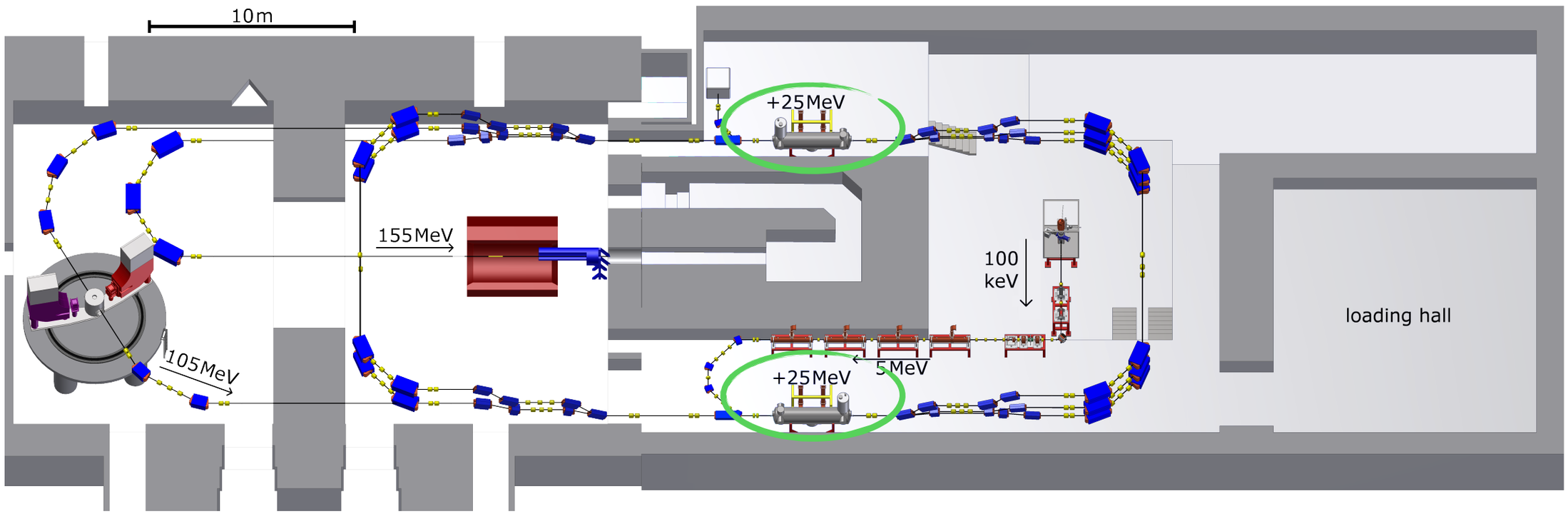

Superconducting cavities

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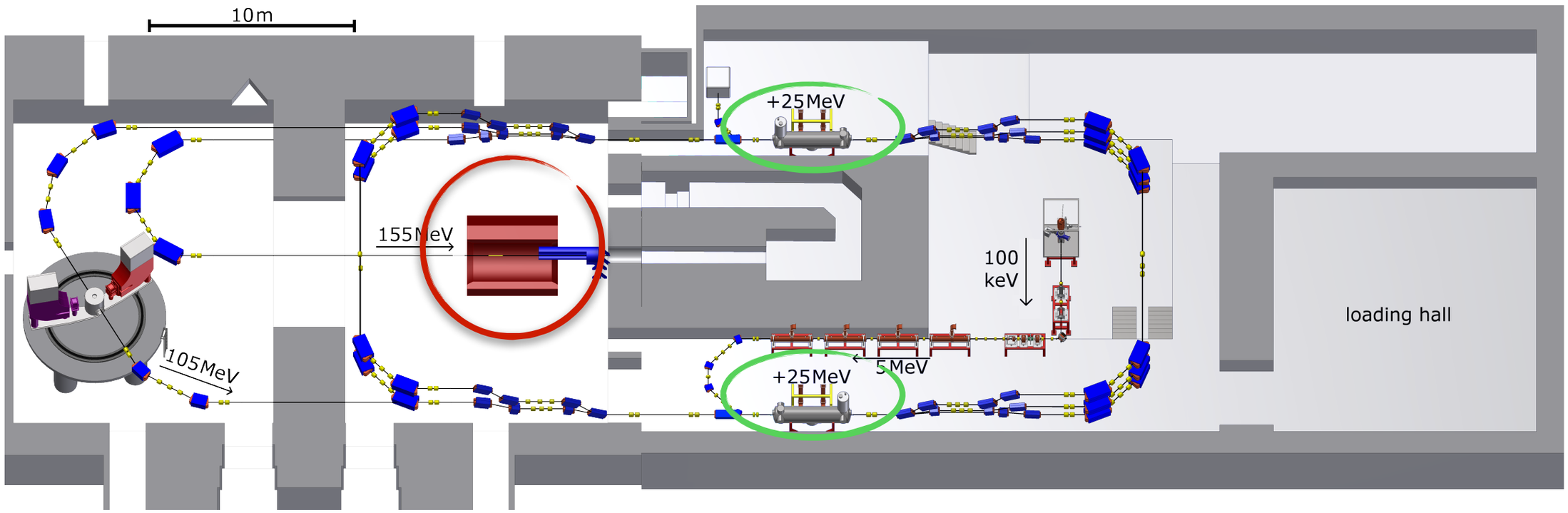

P2 **Superconducting cavities**

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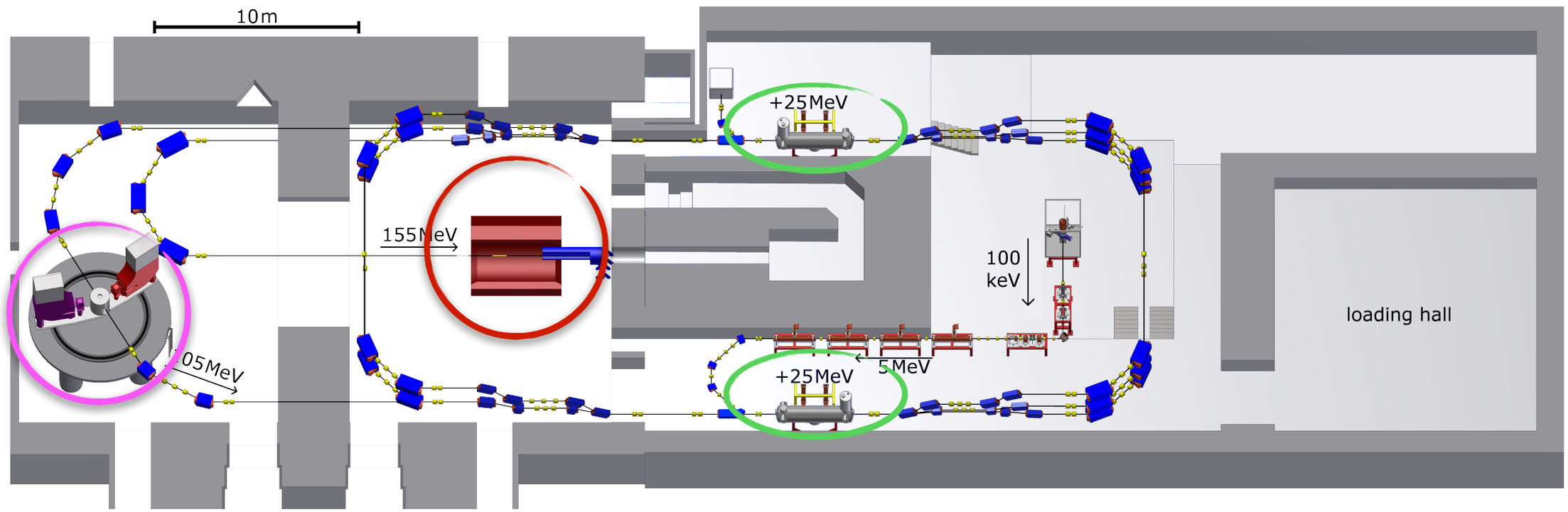

MAGIX P2 Superconducting cavities

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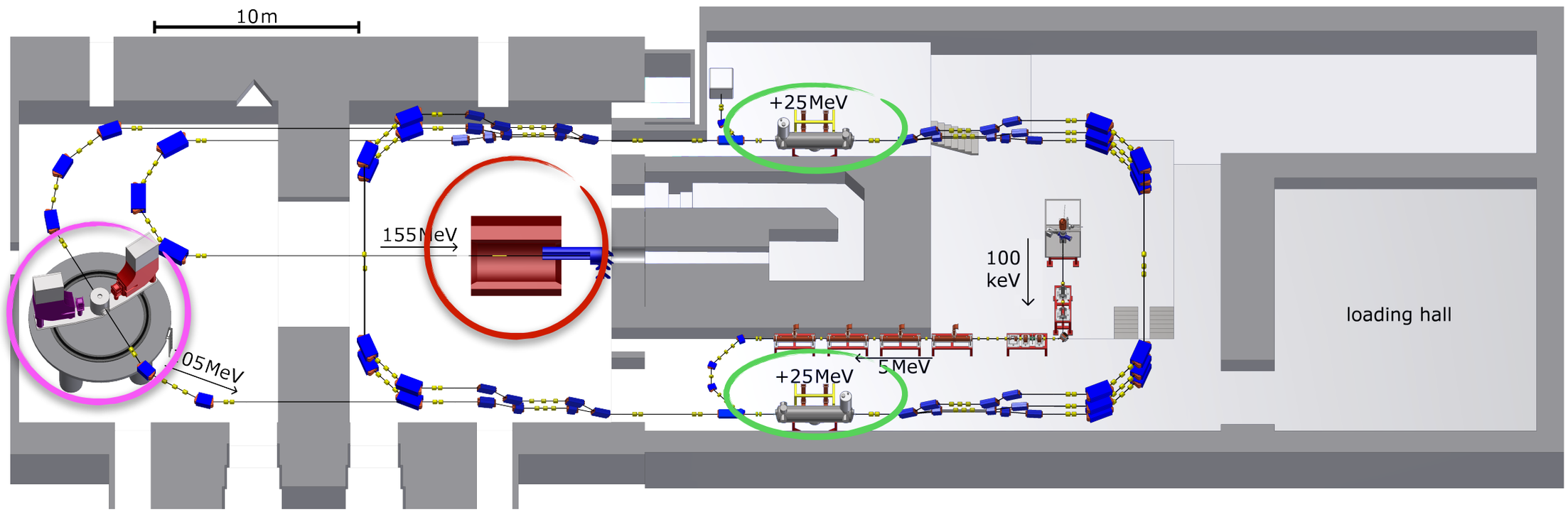

MAGIX P2 Superconducting cavities

“Energy Recovery” vs. “Extracted Beam” modes

PRiSMA

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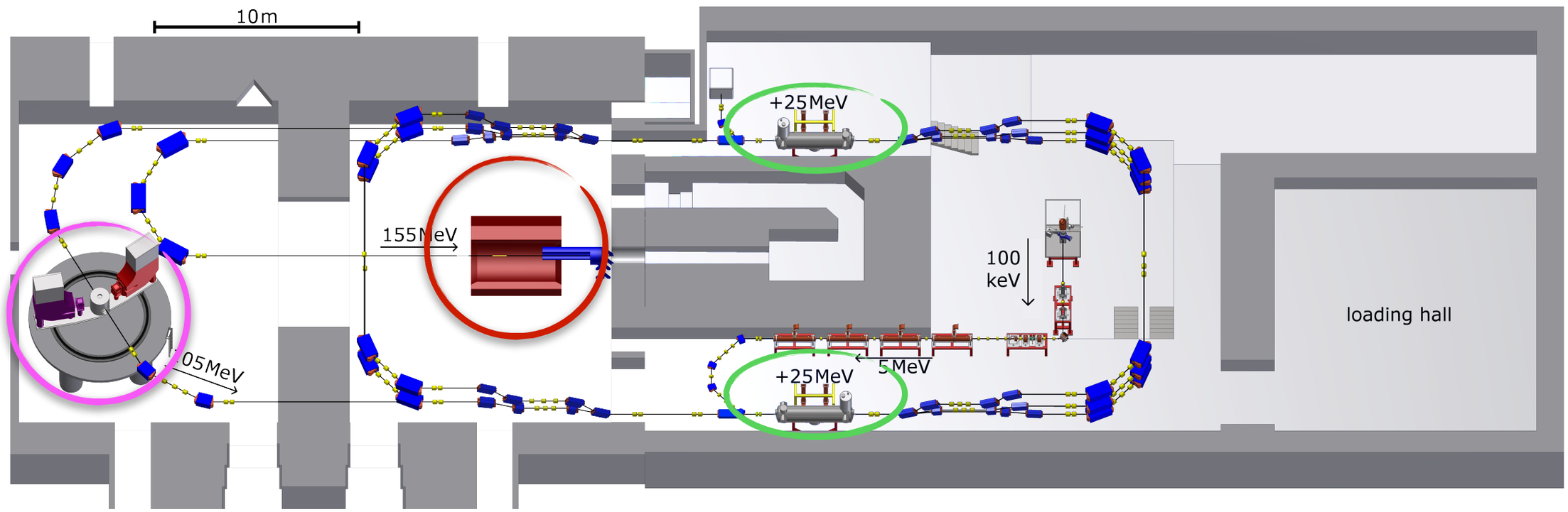

MAGIX P2 Superconducting cavities

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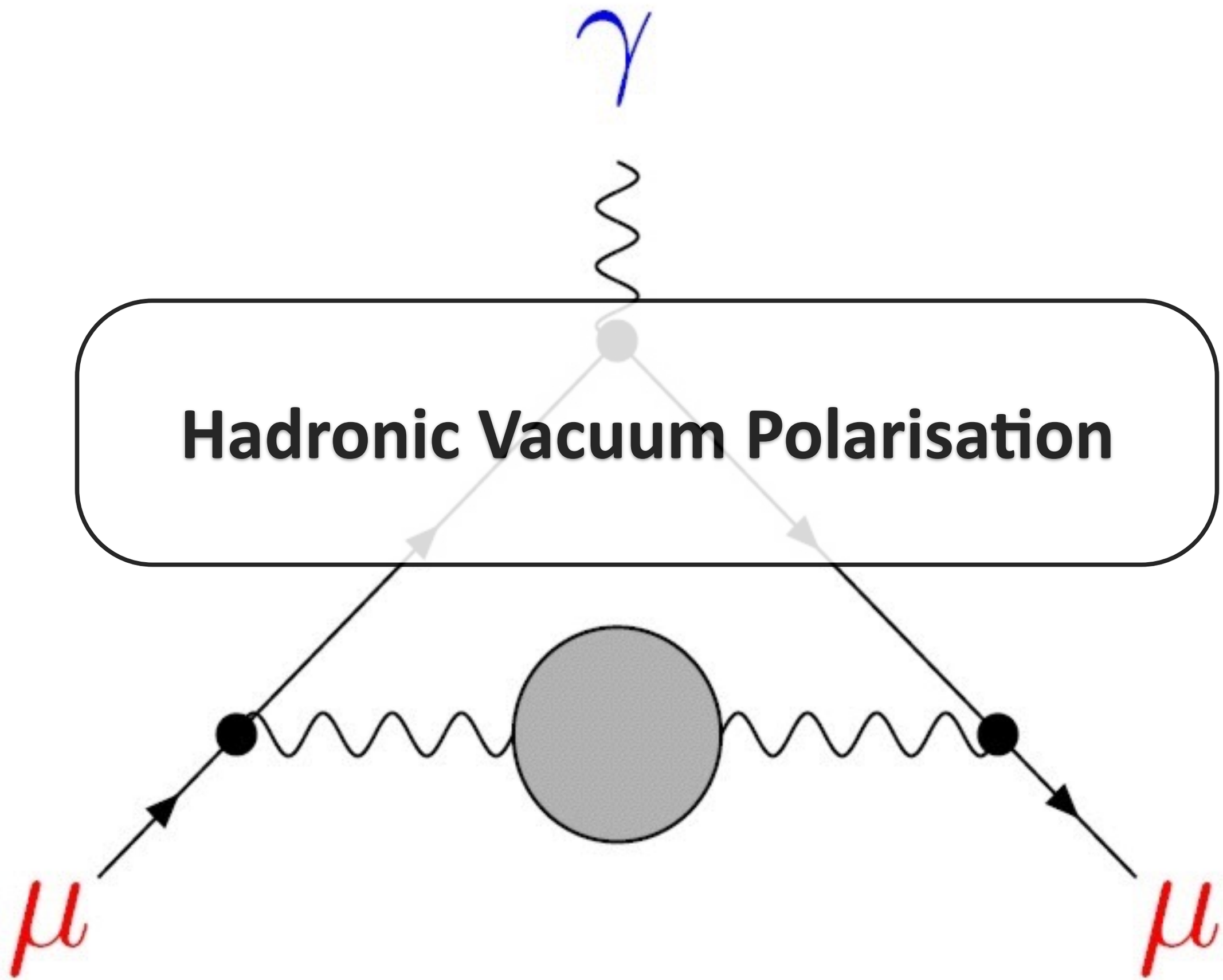
“Energy Recovery” vs. “Extracted Beam” modes

Beam energy: 105 MeV / 155 MeV Current: 1–2 mA

Luminosity: up to 10^{39} $cm^{-2}s^{-1}$

Hadronic Vacuum Polarisation



Anomalous magnetic moment of the muon

Motivation for first-principles approach:

* No reliance on experimental data
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  — improve direct determination of a_μ by a factor four
* Can lattice QCD deliver estimates with sufficient accuracy in the coming years?

$$\delta a_\mu^{\rm hvp}/a_\mu^{\rm hvp} < 0.5\%, \qquad \delta a_\mu^{\rm hlbl}/a_\mu^{\rm hlbl} \lesssim 10\%$$

The Mainz $(g-2)_\mu$ project

Collaborators:

N. Asmussen, A. Gérardin, J. Green, O. Gryniuk, G. von Hippel, H. Horch, H. Meyer, A. Nyffeler, V. Pascalutsa, A. Risch, HW

M. Della Morte, A. Francis, B. Jäger, V. Gülpers, G. Herdoíza

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Topics:

* Hadronic vacuum polarisation
* Light-by-light scattering
* Running of α_{em} and $\sin^2\theta_{\mathrm{W}}$
* Determination of α_{s} from vacuum polarisation function

Lattice QCD approach to HVP

* Convolution integral over Euclidean momenta: *[Lautrup & de Rafael; Blum]*

$$a_\mu^{\rm hvp} = 4\alpha^2 \int_0^\infty dQ^2\, f(Q^2)\left\{\Pi(Q^2) - \Pi(0)\right\}$$

$$\Pi_{\mu\nu}(Q) = \int d^4x\, {\rm e}^{iQ\cdot x}\, \left\langle J_\mu(x) J_\nu(0)\right\rangle \equiv (Q_\mu Q_\nu - \delta_{\mu\nu} Q^2)\Pi(Q^2)$$

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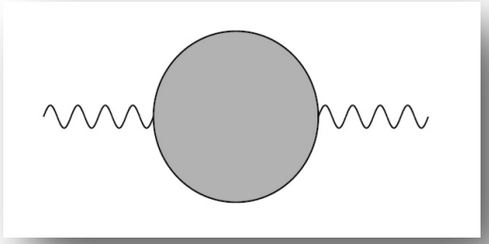

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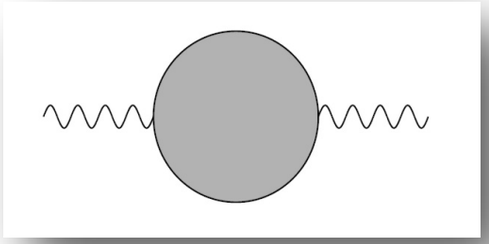

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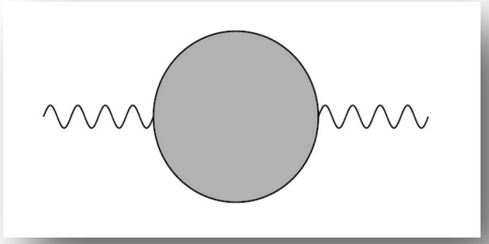


* Determine VPF $\Pi(Q^2)$ and additive renormalisation $\Pi(0)$
* Integrand peaked near $Q^2 \approx (\sqrt{5} - 2)m_\mu^2$
* Lattice momenta are quantised: $Q_\mu = \dfrac{2\pi}{L_\mu}$
* Statistical accuracy of $\Pi(Q^2)$ deteriorates as $Q \longrightarrow 0$

Lattice QCD approach to HVP

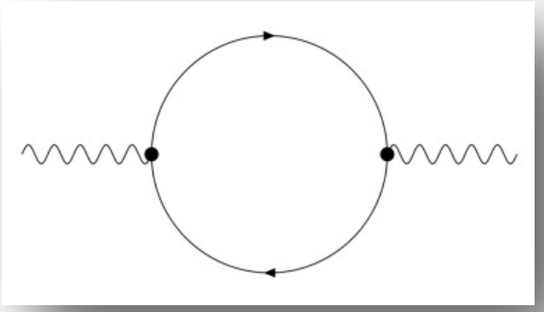
Challenges:

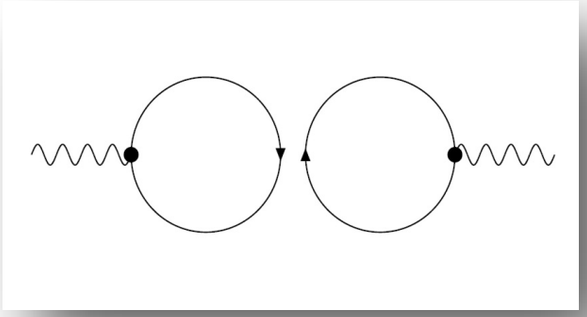
* Statistical accuracy at the sub-percent level required

* Reduce systematic uncertainty associated with region of small Q^2
  $\Leftrightarrow$ accurate determination of $\Pi(0)$

* Perform comprehensive study of finite-volume effects

* Include quark-disconnected diagrams



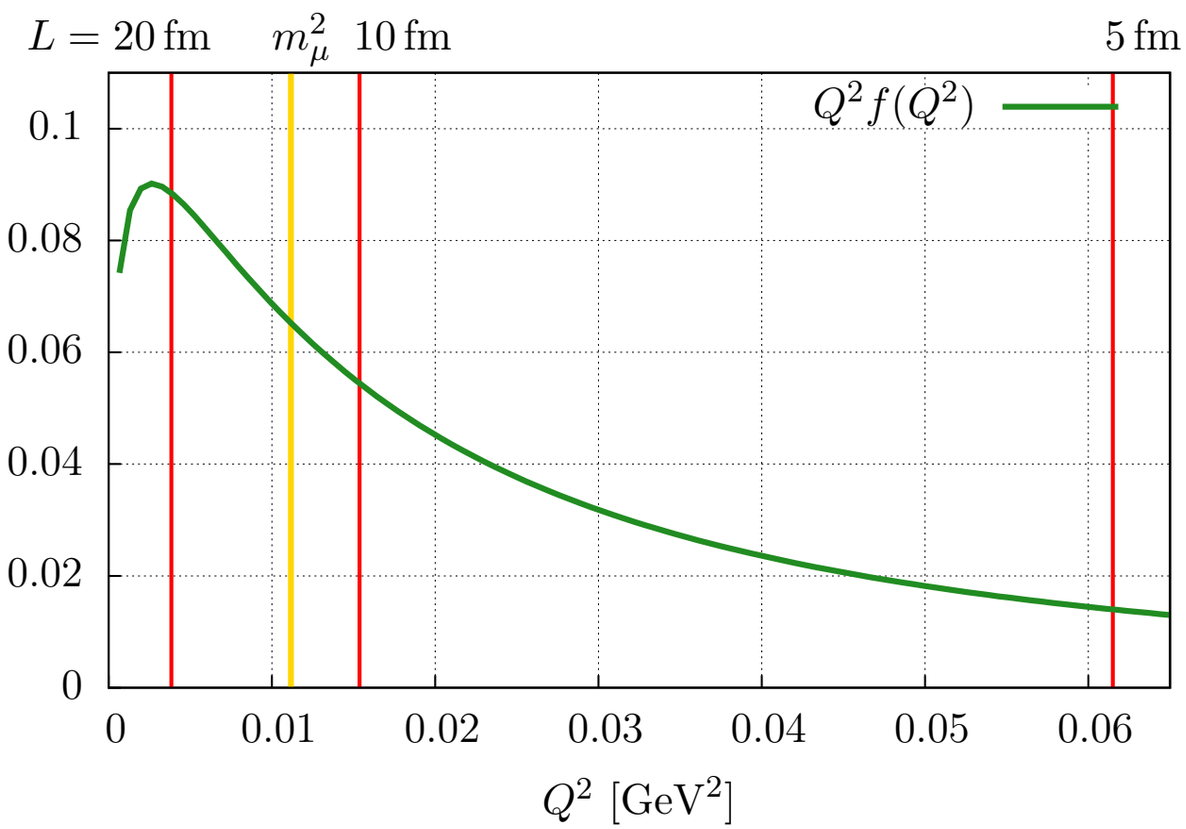


* Include isospin breaking: $m_u \neq m_d$, QED corrections

Low-momentum region

* Convolution integral revisited:

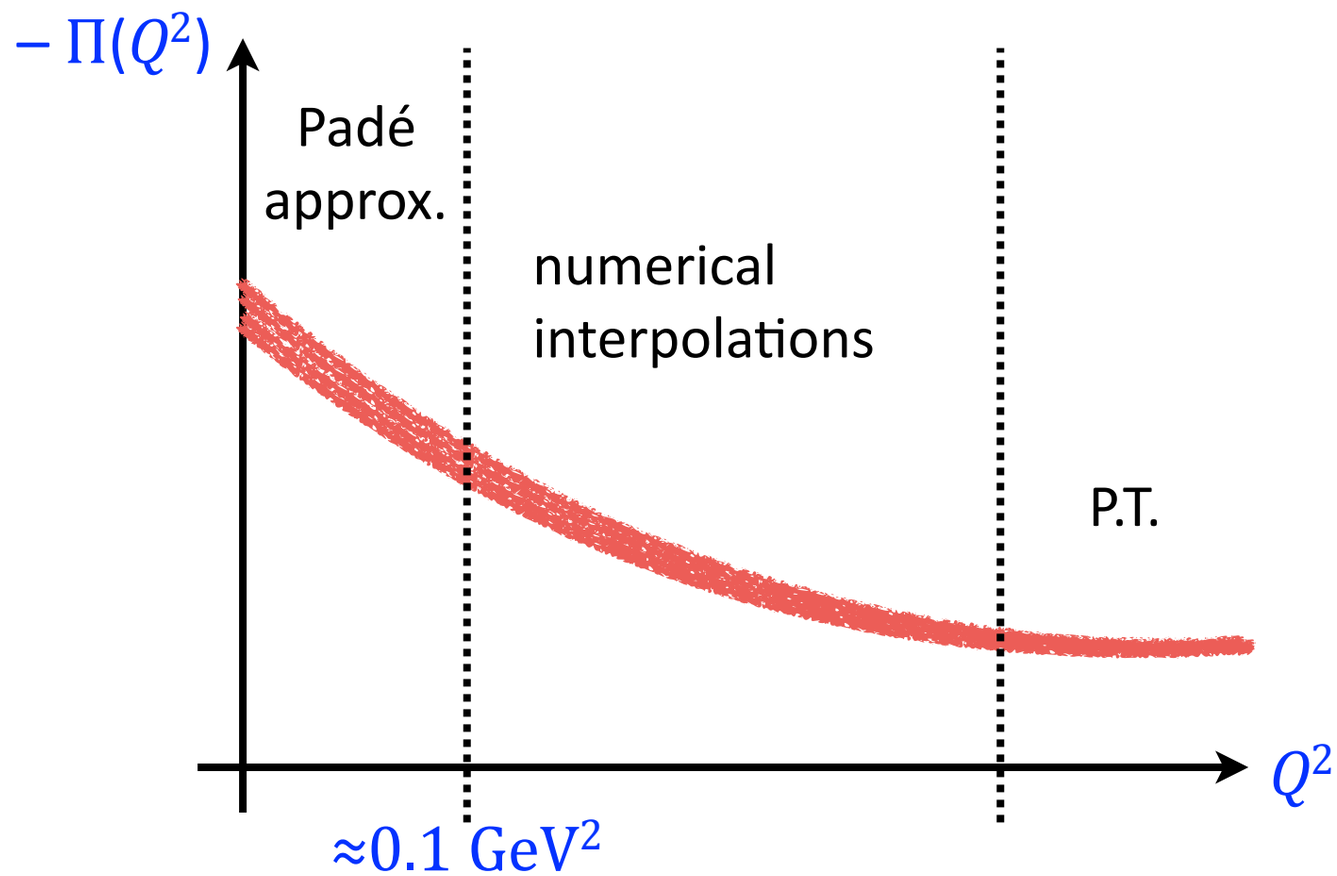
$$a_\mu^{\mathrm{hvp}} = 4\alpha^2 \int_0^\infty dQ^2\, f(Q^2) \left\{ \Pi(Q^2) - \Pi(0) \right\}$$



* Accurate determination requires large statistics on large volumes

Low-momentum region: "Hybrid method"

* Minimise model dependence: *[Golterman, Maltman & Peris, Phys Rev D90 (2014) 074508]*



* Determine $\Pi(0)$ and $\Pi(Q^2)$ from models in small-momentum region:
  Padé approximants, conformal polynomials, time moments

Low-momentum region: Time moments

* Expansion of VPF at low-Q^2: $$\Pi(Q^2) = \Pi_0 + \sum_{j=1}^{\infty} Q^{2j}\Pi_j$$

* Vacuum polarisation for $Q = (\omega, \vec{0})$:

$$\Pi_{kk}(\omega) = a^4 \sum_{x_0} \mathrm{e}^{i\omega x_0} \sum_{\vec{x}} \langle J_k(x) J_k(0) \rangle$$

* Spatially summed vector correlator: $$G(x_0) = -a^3 \sum_{\vec{x}} \langle J_k(x) J_k(0) \rangle$$

* Time moments: *[Chakraborty et al., Phys Rev D89 (2014) 114501]*

$$G_{2n} \equiv a \sum_{x_0} x_0^{2n} G(x_0) = (-1)^n \frac{\partial^{2n}}{\partial \omega^{2n}} \left\{ \omega^2 \hat{\Pi}(\omega^2) \right\}_{\omega^2=0}$$

* Expansion coefficients: $$\Pi(0) \equiv \Pi_0 = \frac{1}{2} G_2, \quad \Pi_j = (-1)^{j+1} \frac{G_{2j+2}}{(2j+2)!}$$

Time-Momentum Representation

* Integral representation of subtracted VPF $\hat{\Pi}(Q^2) \equiv \Pi(Q^2) - \Pi(0)$

$$\Pi(Q^2) - \Pi(0) = \frac{1}{Q^2} \int_0^\infty dx_0\, G(x_0) \left[Q^2 x_0^2 - 4 \sin^2\left(\tfrac{1}{2} Q x_0\right) \right]$$

$$G(x_0) = -a^3 \sum_{\vec{x}} \langle J_k(x) J_k(0) \rangle$$

[Bernecker & Meyer, Eur Phys J A47 (2011) 148, Francis et al. Phys Rev D88 (2013) 054502, Feng et al., Phys Rev D88 (2013) 034505]

* Q^2 is a tuneable parameter

* No extrapolation to $Q^2 = 0$ required

* Must determine $I = 1$ vector correlator $G(x_0)$ at all distances

  $\longrightarrow$ $G(x_0)$ dominated by two-pion state for $x_0 \longrightarrow \infty$

  $\longrightarrow$ Include multi-particle states to capture long-distance behaviour

TMR: model dependence

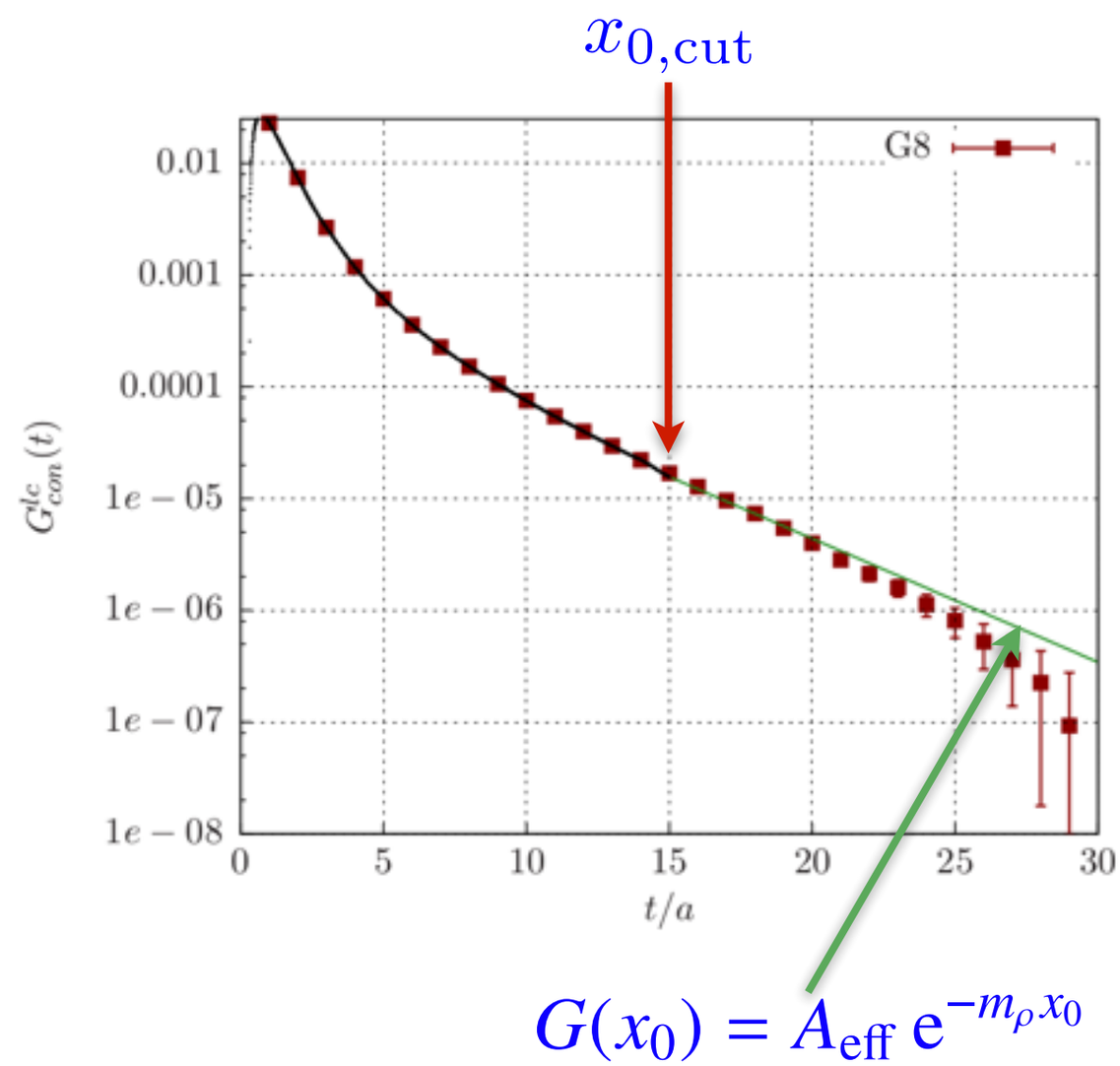
* Determine long-distance contribution to vector correlator:

$$G(x_0) = \begin{cases} G(x_0)^{\text{data}}, & x_0 \leq x_{0,\text{cut}} \\ G(x_0)^{\text{fit}}, & x_0 > x_{0,\text{cut}} \end{cases}$$

* Single-exponential fit for $x > x_{0,\text{cut}}$

* $G(x_0)$ dominated by two-pion state at long distances

⇒ Include multi-particle states to eliminate model dependence when $x_0 \longrightarrow \infty$



Current data sets and statistics

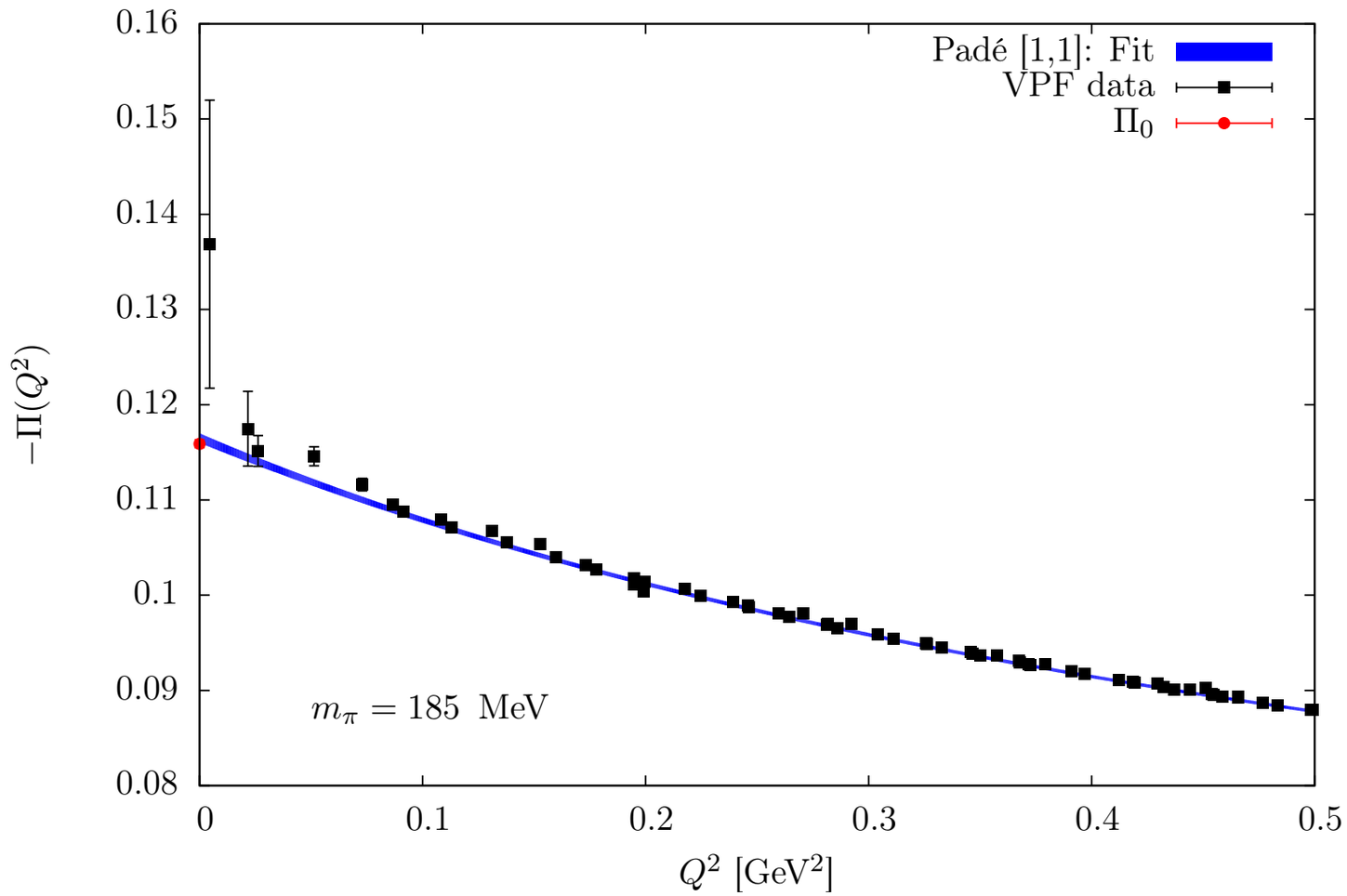
* $N_\mathrm{f} = 2$ flavours of $\mathrm{O}(a)$ improved Wilson fermions
* Three values of the lattice spacing: $a = 0.076, 0.066, 0.049\ \mathrm{fm}$
* Pion masses and volumes: $m_\pi^{\min} = 185\,\mathrm{MeV}, \quad m_\pi L > 4$
* 2000–4000 measurements per ensemble

To be processed:

* $N_\mathrm{f} = 2{+}1$ flavours of $\mathrm{O}(a)$ improved Wilson fermions; tree-level Symanzik gauge action; open boundary conditions
* Five values of the lattice spacing; physical pion mass

Comparison: Fits versus Time moments

* Construct Padé approximants either from fits or time moments



Fit Padé [1,1] for

$$Q^2_{\rm low} \lesssim 0.5\,{\rm GeV}^2$$

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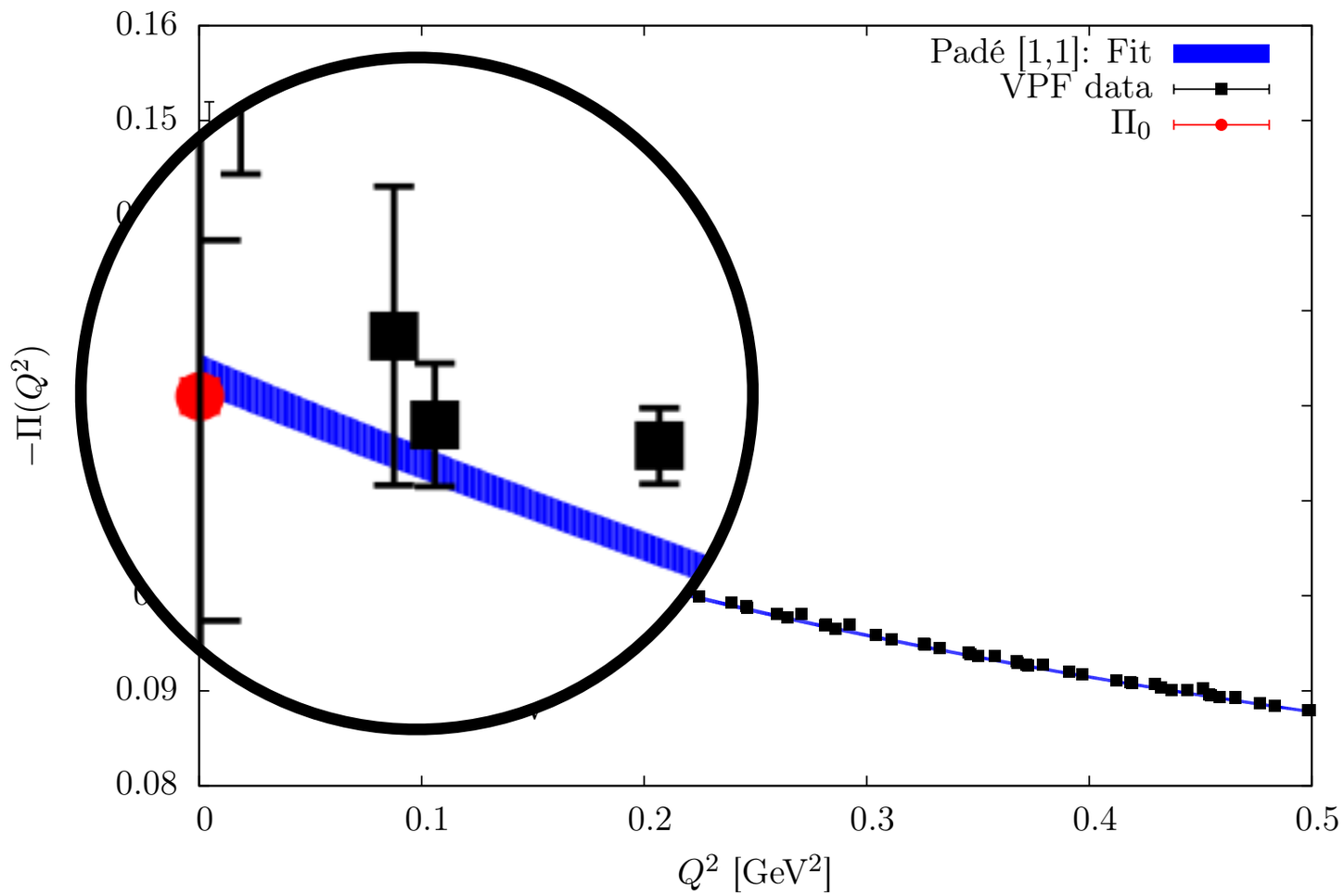

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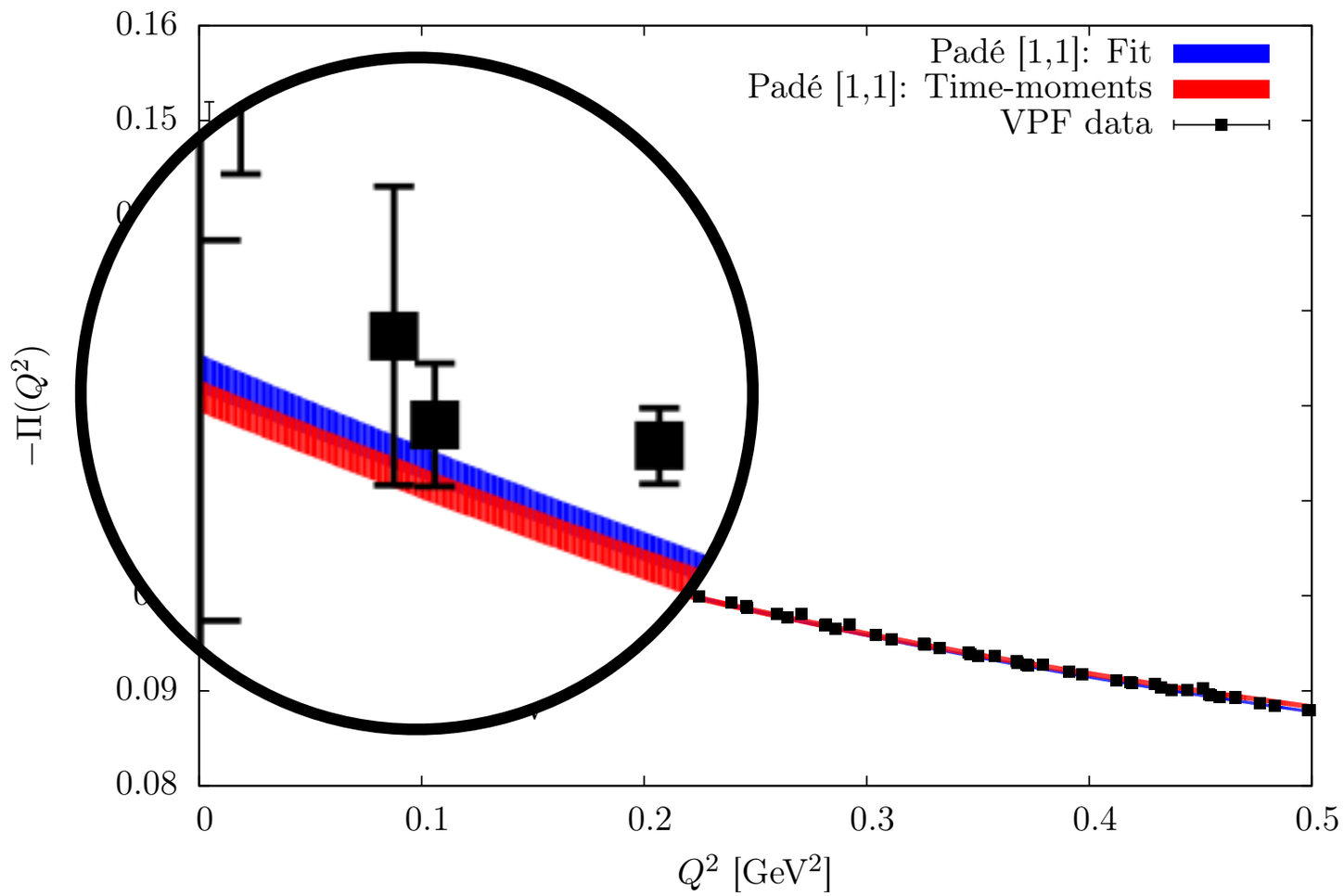

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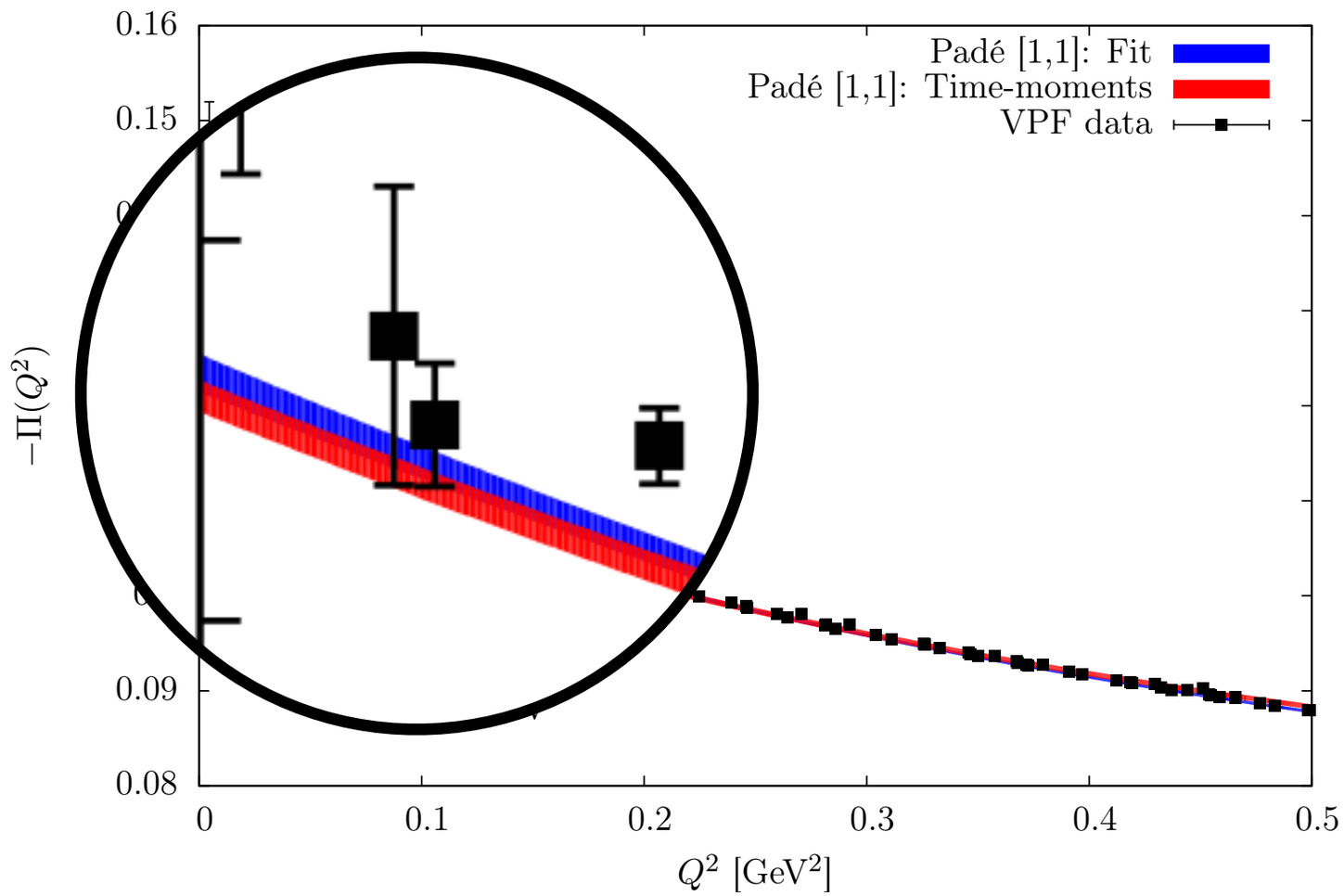


Fit Padé [1,1] for

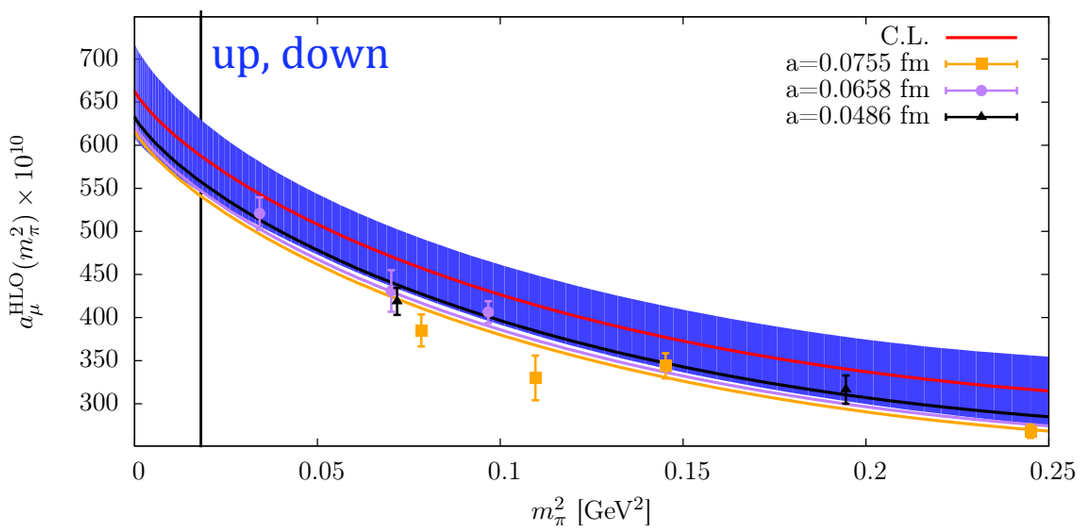
$Q^2_{\rm low} \lesssim 0.5\,{\rm GeV}^2$

* Low-order Padé approximants consistent for $Q^2 < 0.5$ GeV^2

* Apply trapezoidal rule to evaluate convolution integral for $Q^2 \geq 0.5$ GeV^2

Chiral and continuum extrapolations

* Use collection of different functional forms, e.g.



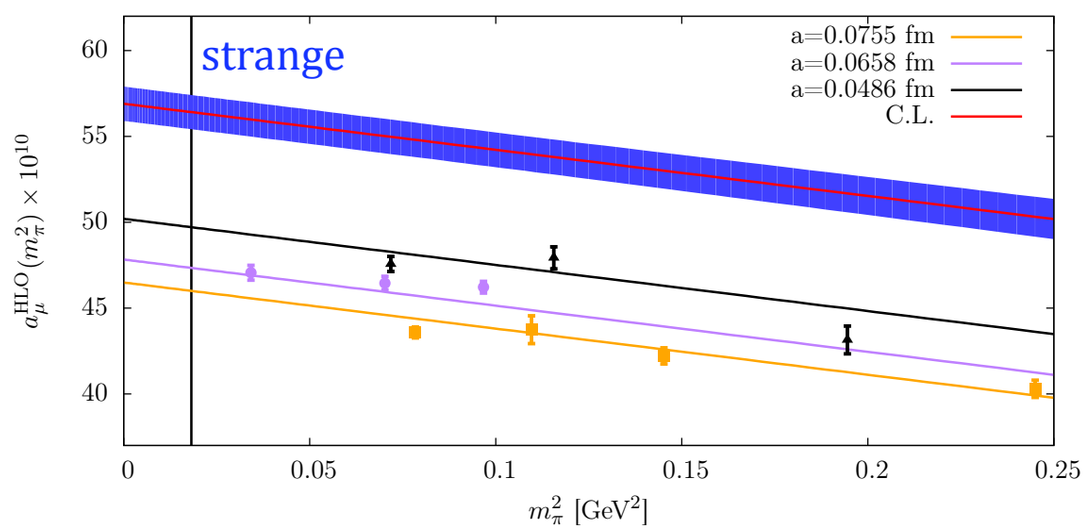
Fit A:

$$b_0 + b_1 m_\pi^2 + b_2 m_\pi^2 \ln(m_\pi^2) + b_3 a$$

Fit B:

$$b_0 + b_1 m_\pi^2 + b_2 m_\pi^4 + b_3 a$$

.....



* Perform cuts in pion mass and lattice spacing

* Lattice spacing effects clearly resolved for larger quark masses

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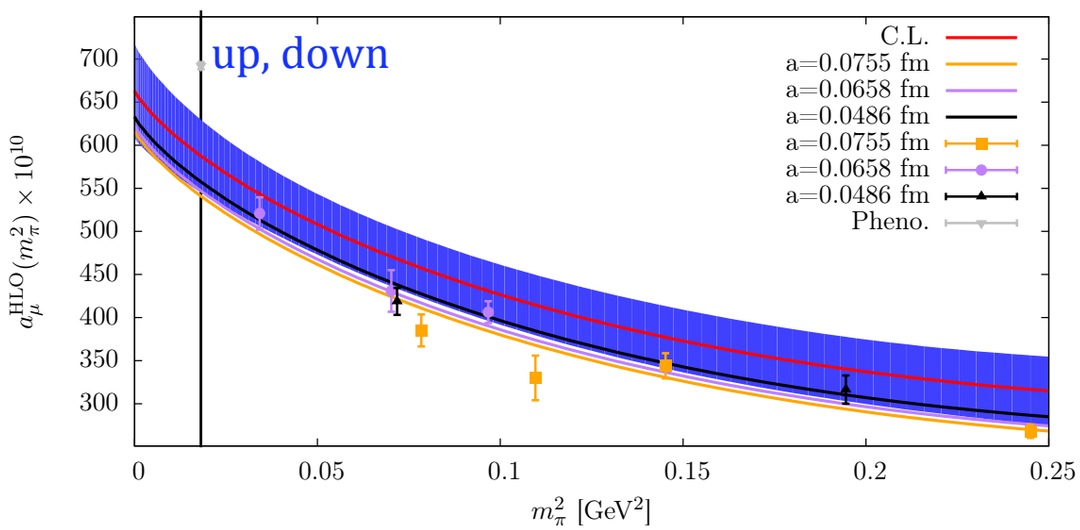

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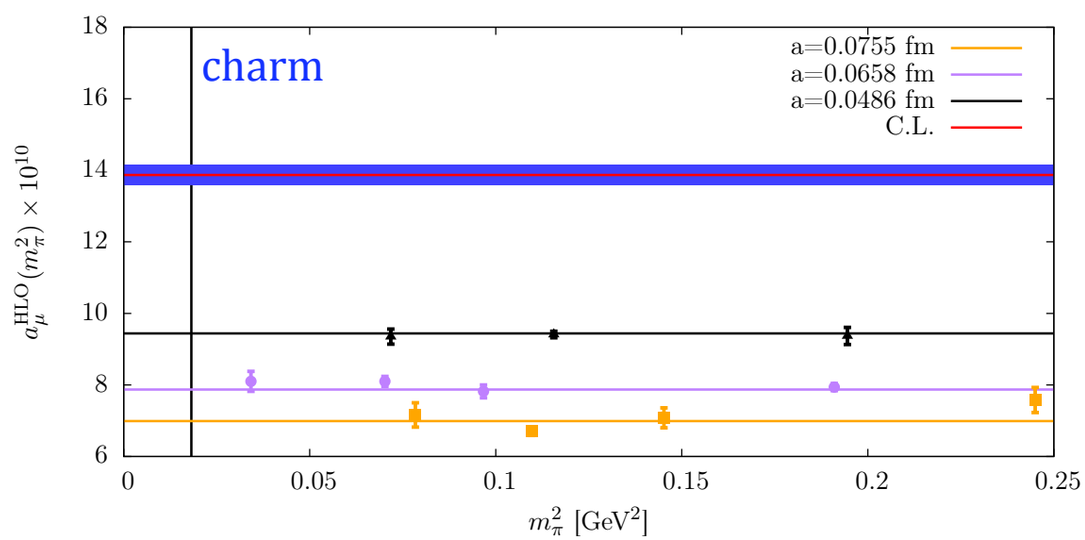
.....



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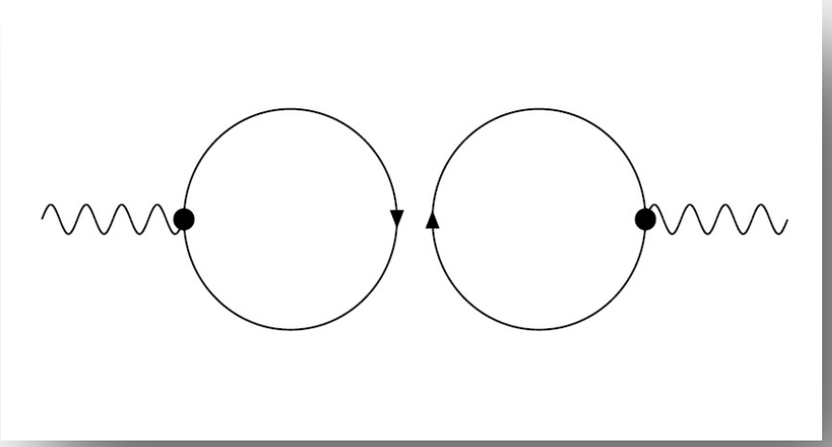
Disconnected Contributions

* Current-current correlator contains quark-disconnected contributions

$$\left\langle \mathrm{Tr}\,(\gamma_\mu S^f(x,x))\,\mathrm{Tr}\,(\gamma_\nu S^{f'}(y,y))\right\rangle$$

stochastic evaluation



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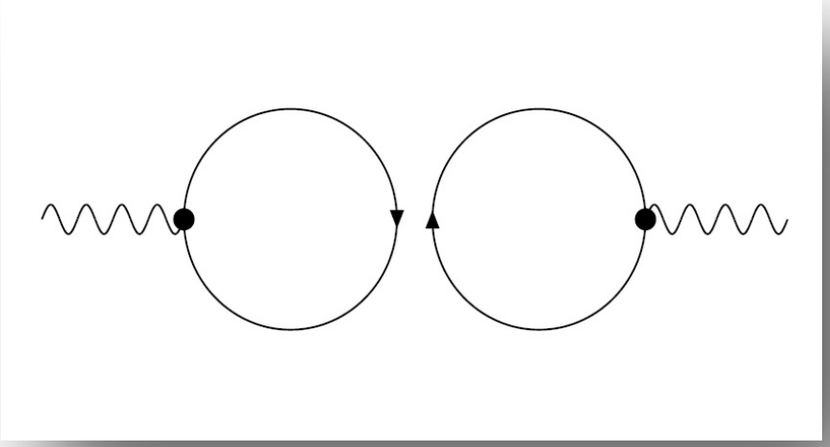

* Electromagnetic current correlator with *u, d, s* quarks:

$$G(x_0) := -a^3 \sum_{\vec{x}} \langle J_k(x) J_k(0)\rangle, \quad J_k = \tfrac{2}{3}\bar{u}\gamma_k u - \tfrac{1}{3}\bar{d}\gamma_k d - \tfrac{1}{3}\bar{s}\gamma_k s$$

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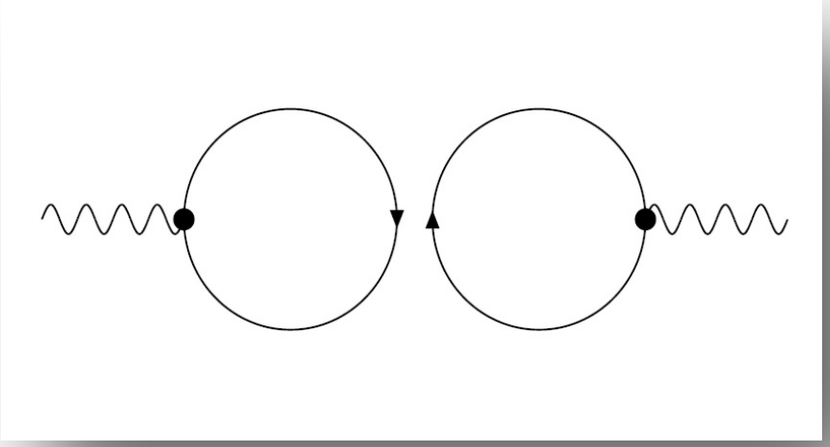

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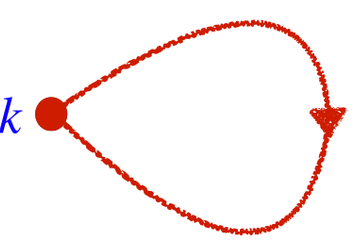
* $G(x_0)$ splits into connected and disconnected parts:

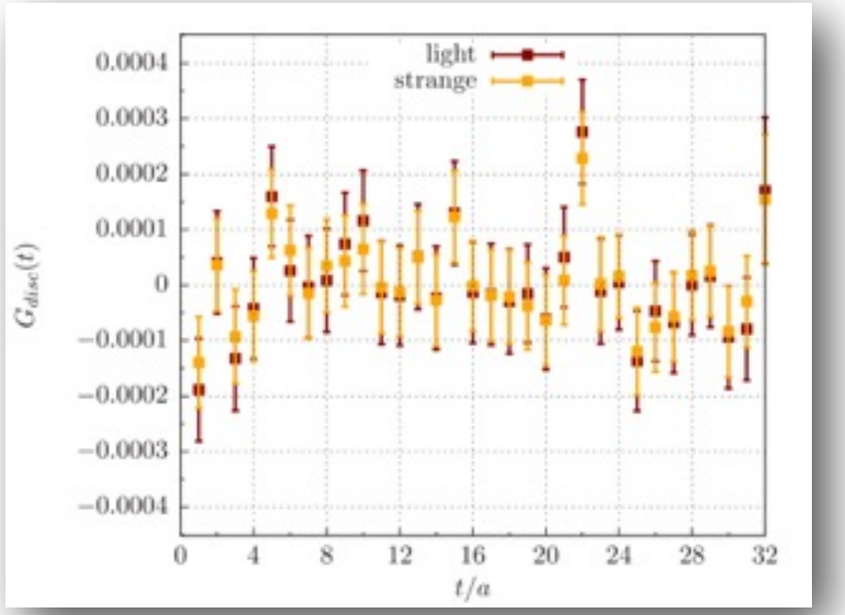
$$G(x_0) = \tfrac{5}{9}C^{\ell\ell}(x_0) + \tfrac{1}{9}C^{ss}(x_0) - \tfrac{1}{9}D^{\ell s}(x_0), \qquad (m_u = m_d = m_\ell)$$

$$D^{\ell s}(x_0) = \underbrace{\left(\Delta^\ell(x_0) - \Delta^s(x_0)\right)}_{\text{noise cancellation}}\left(\Delta^\ell(0) - \Delta^s(0)\right) \qquad \Delta^f(x_0) \sim \gamma_k$$



Disconnected Contributions

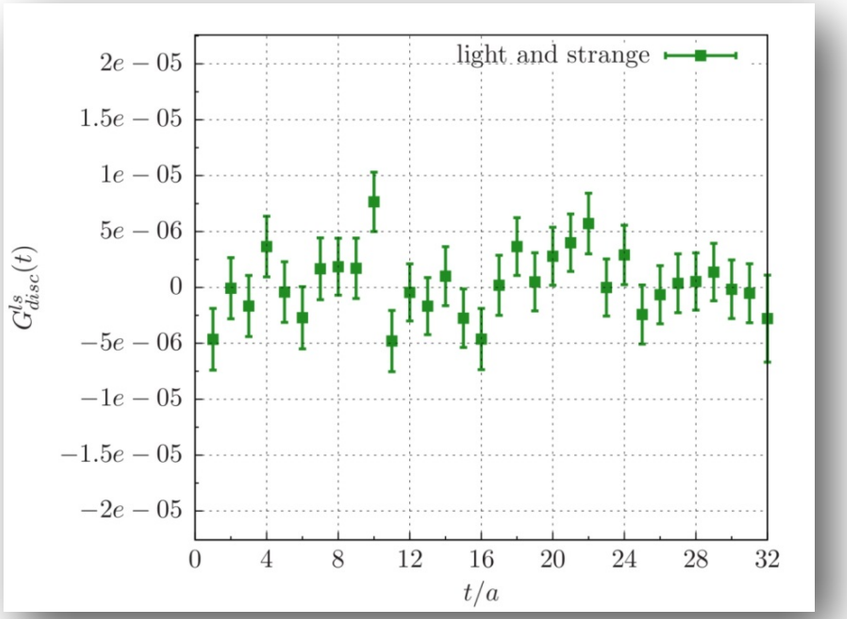
* Stochastic noise cancellation in $\Delta^{\ell}(x_0) - \Delta^{s}(x_0)$ *[Gülpers et al., arXiv:1411.7592]*



Disconnected contribution for $x_0 \longrightarrow \infty$:

$$-\frac{2}{9}\frac{D^{\ell s}}{C^{\ell\ell}} \xrightarrow{x_0\to\infty} -\frac{1}{9}$$

$$D^{\ell s} = \begin{cases} 0, & x_0 \leq 1.6\,\mathrm{fm} \\ -1/9, & x_0 > 1.6\,\mathrm{fm} \end{cases}$$



* Upper bound on systematic error in a_μ^{hvp}

Disconnected Contributions

* Stochastic noise cancellation in $\Delta^{\ell}(x_0) - \Delta^{s}(x_0)$ *[Gülpers et al., arXiv:1411.7592]*

* $D^{ls}(x_0)$ dominates statistical uncertainty for $x_0 > 1.6$ fm

  Disconnected contribution for $x_0 \longrightarrow \infty$:

$$-\frac{2}{9}\frac{D^{\ell s}}{C^{\ell\ell}} \xrightarrow{x_0\to\infty} -\frac{1}{9}$$

* Data compatible with

$$D^{\ell s} = \begin{cases} 0, & x_0 \leq 1.6\,\text{fm} \\ -1/9, & x_0 > 1.6\,\text{fm} \end{cases}$$

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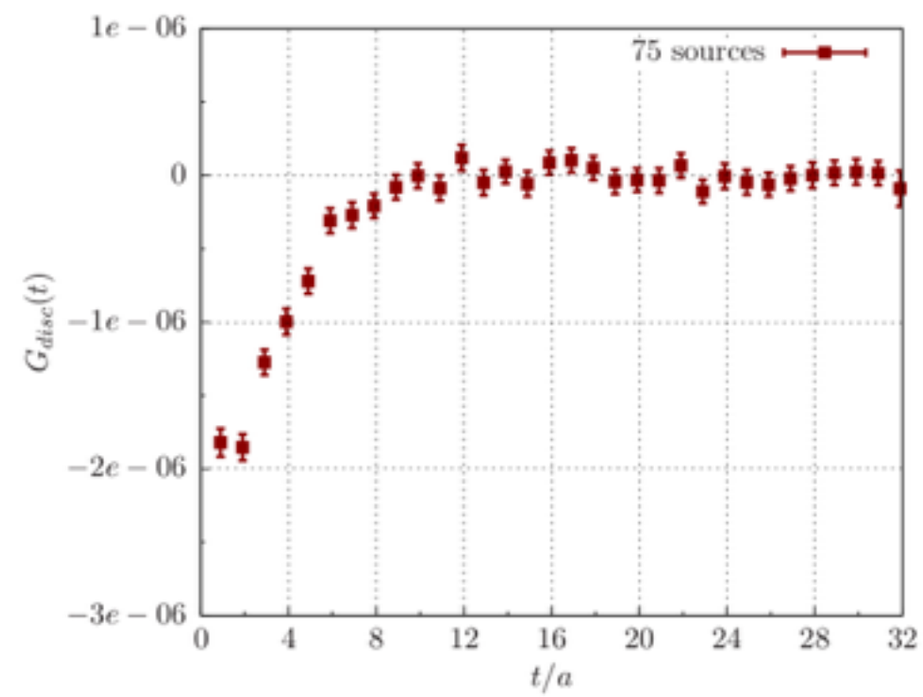
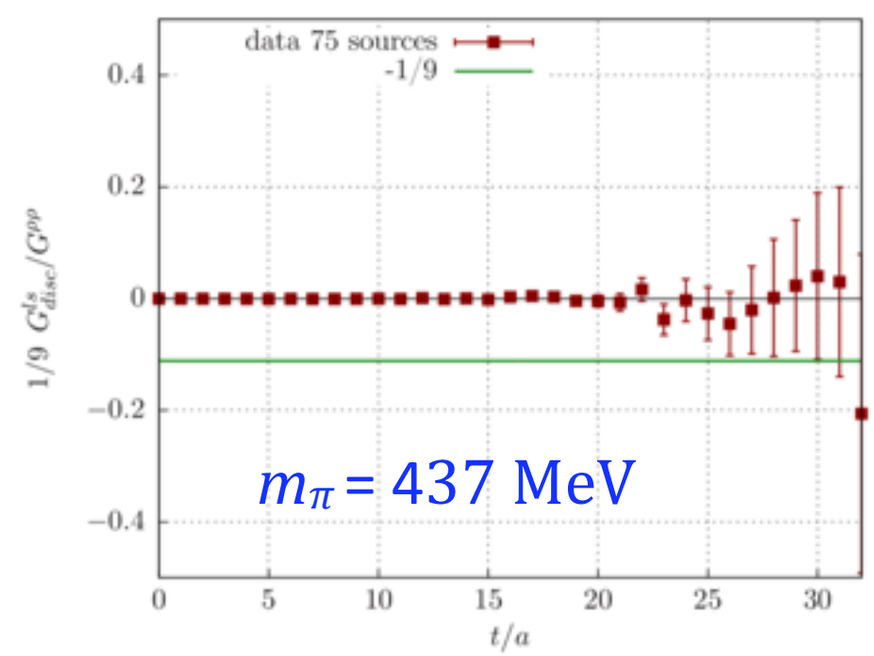




Disconnected Contributions: Summary

* Non-zero disconnected contribution can be resolved

HPQCD: Anisotropic Clover action; $m_\pi = 391$ MeV; $a_s \approx 0.12$ fm; Distillation

$$(a_\mu^{\rm hvp})_{\rm disc}/(a_\mu^{\rm hvp})_{\rm con}^{(\ell\ell)} = -0.14(5)\%, \quad (a_\mu^{\rm hvp})_{\rm disc} \approx -0.84 \cdot 10^{-10}$$

RBC/UKQCD: Domain wall fermions; physical pion mass; $a \approx 0.11$ fm, $m_\pi L \approx 3.9$;

$$(a_\mu^{\rm hvp})_{\rm disc}/(a_\mu^{\rm hvp})_{\rm con}^{(\ell\ell)} = -1.6(7)\%, \quad (a_\mu^{\rm hvp})_{\rm disc} = -(9.6 \pm 3.3 \pm 2.3) \cdot 10^{-10}$$

CLS/Mainz: $N_{\rm f} = 2$ Clover fermions; $m_\pi = 311, 437$ MeV; $a = 0.063$ fm;

$$(a_\mu^{\rm hvp})_{\rm disc}/(a_\mu^{\rm hvp})_{\rm con}^{(\ell\ell)} < -1\%$$

Bali & Endrődi: Rooted staggered fermions; physical pion mass; $a = 0.1 - 0.29$ fm;

$$\Pi^{\rm disc}/\Pi^{\rm con} = -(3.6 \pm 4.5) \cdot 10^{-4} \quad \text{at} \quad Q^2 = 0.03\ {\rm GeV}^2$$

Finite-volume effects

* Starting point: $a_\mu^{\rm hvp}(L) = \int_0^\infty dx_0\, G(x_0, L)\, w(x_0)$

* Finite-volume shift determined by $G(x_0, \infty) - G(x_0, L)$

* Infinite volume:

$$G(x_0, \infty) = \int_0^\infty d\omega\, \omega^2 \rho(\omega^2) {\rm e}^{-\omega|x_0|} = \frac{1}{48\pi^2} \int_0^\infty d\omega\, \omega^2 (1 - 4m_\pi^2/\omega^2)^{3/2} |F_\pi(\omega)|^2 {\rm e}^{-\omega x_0}$$

* Finite volume:

$$G(x_0, L) = \sum_n |A_n|^2\, {\rm e}^{-\omega_n x_0}, \quad \omega_n = 2\sqrt{m_\pi^2 + k_n^2}$$

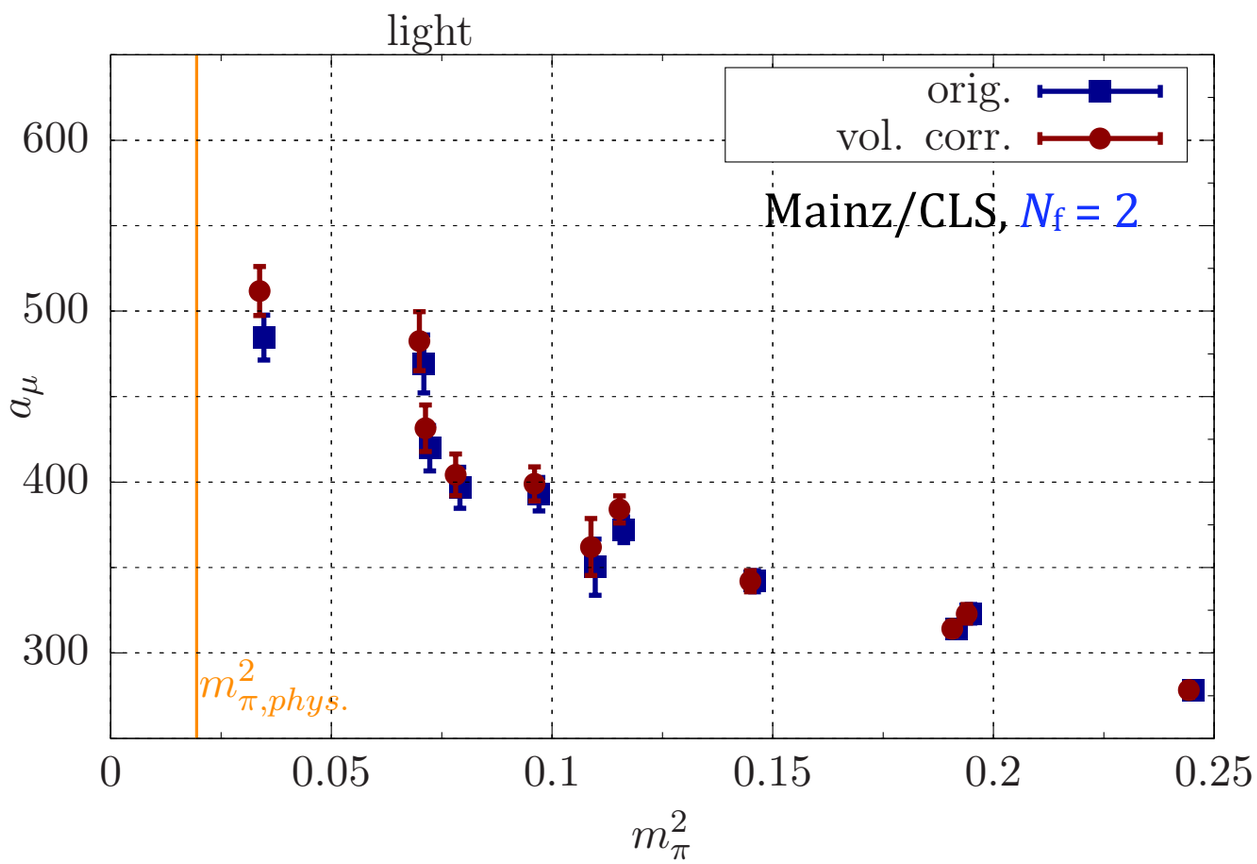
* $k_{\rm n}$ and $\omega_{\rm n}$ determined by two-particle spectrum in a box; $A_{\rm n}$ related to timeline pion form factor

[M. Lüscher 1991; [H.B. Meyer, PRL 107 (2011) 072002]

Finite-volume effects: TMR analysis

* Input quantity: timelike pion form factor $F_\pi(\omega) = |F_\pi(\omega)|\, e^{i\delta_{11}(k)}$

* Use Gounaris-Sakurai parameterisation and evaluate $|F_\pi(\omega)|$, $\delta_{11}(k)$ for given (m_π, m_ρ) of a given gauge ensemble:

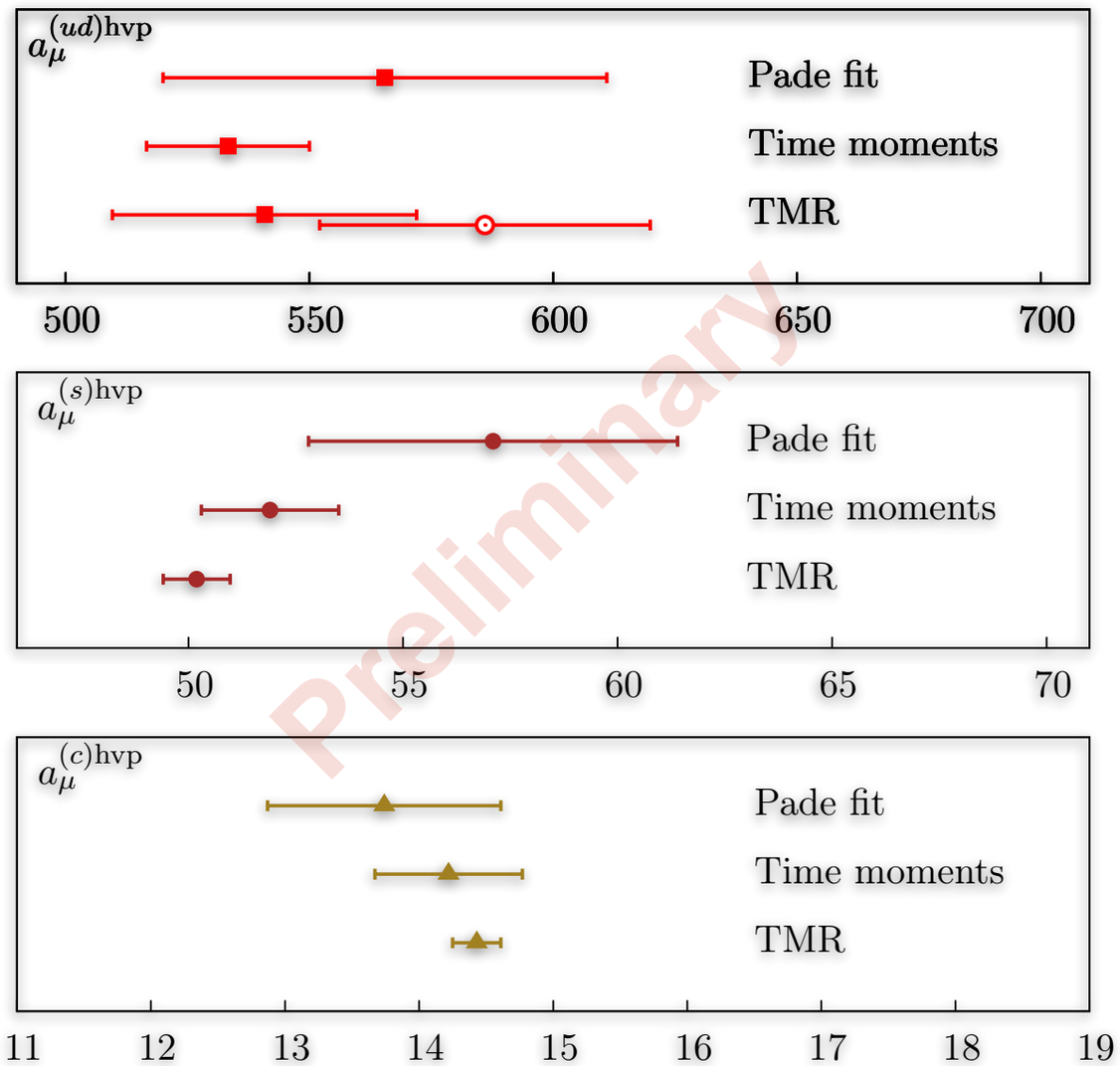


[H.B. Meyer et al., in preparation]

* Example: $m_\pi = 190$ MeV, $L = 4.0$ fm: $a_\mu^{\rm hvp}(\infty) - a_\mu^{\rm hvp}(L) = 5.2\%$

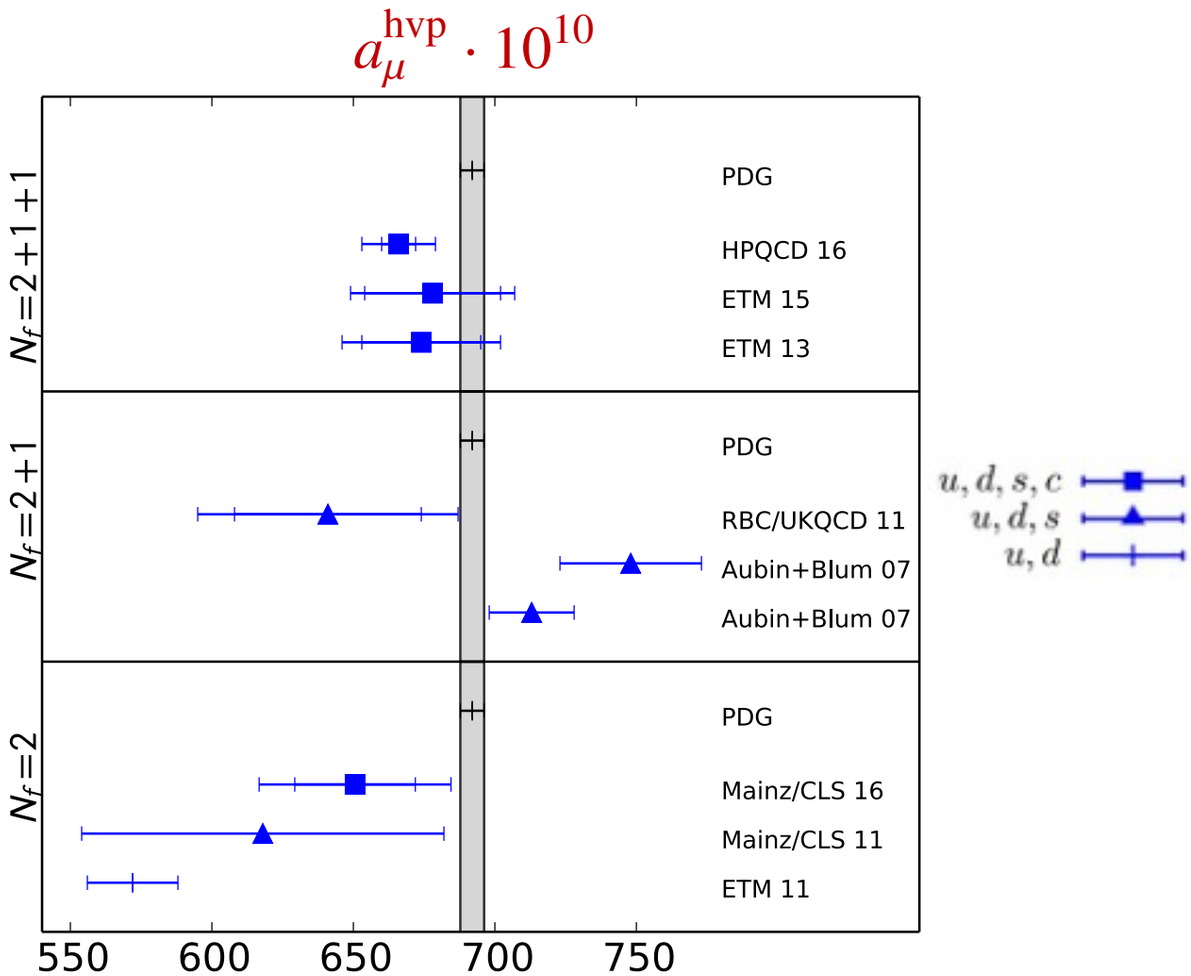
a_μ^{hvp} at the physical point

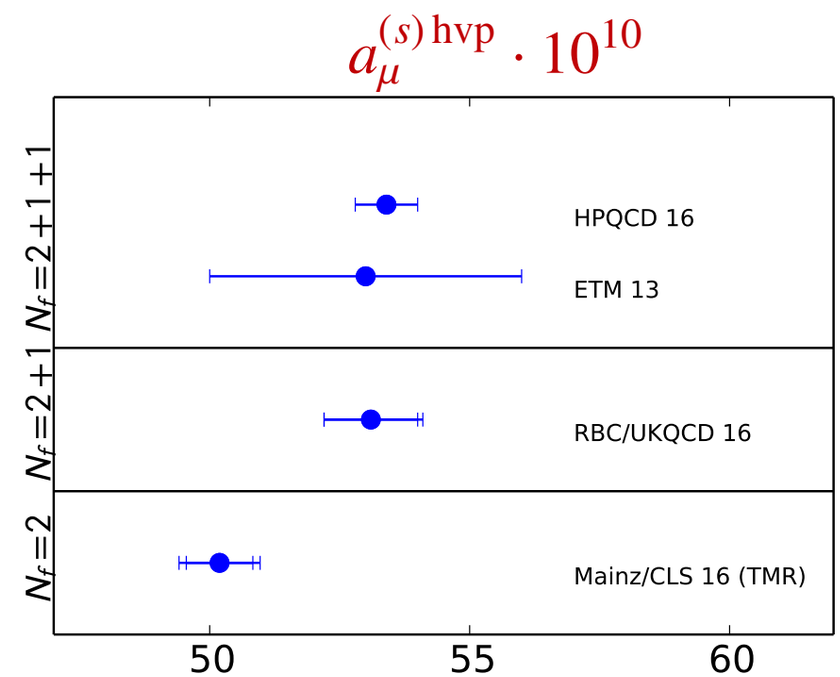
* Determine contributions from individual quark flavours: (u,d), s, c

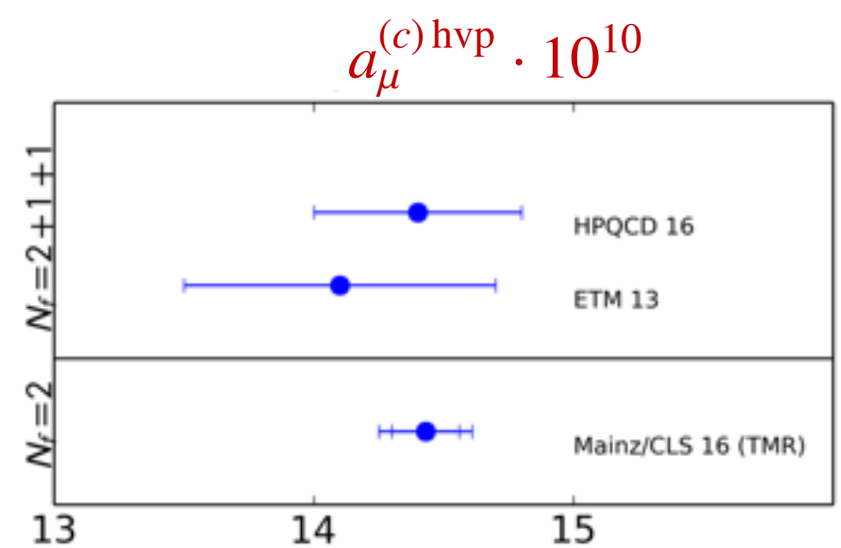


* Different methods consistent at the level of 1σ
* Significant shift due to finite-volume effects
* Overall accuracy dominated by u, d contribution
* Contributions from disconnected diagrams below 1%

Summary on a_μ^{hvp}



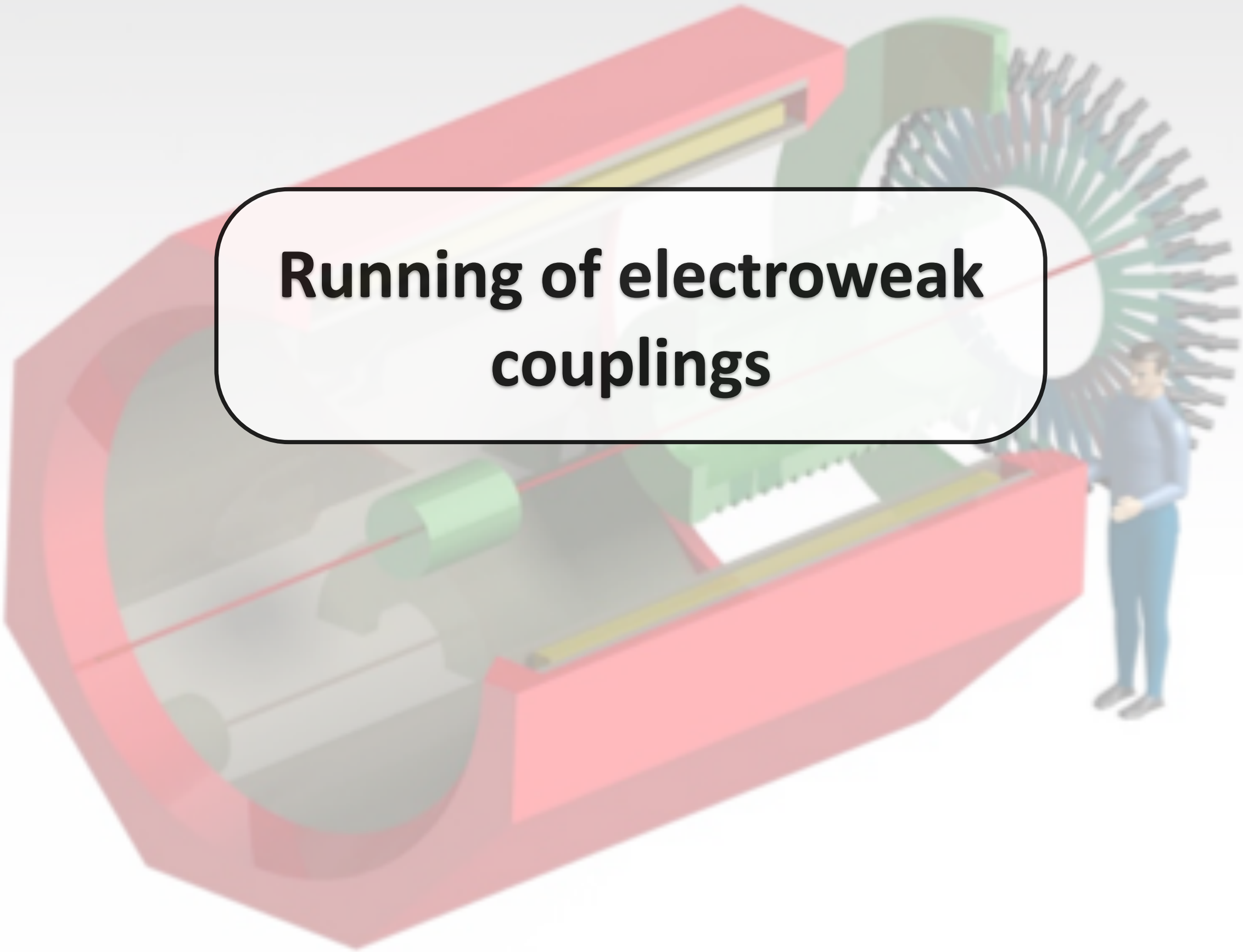




* Individual flavour contributions:

light (u,d)	$\approx 90\%$
strange (s)	$\approx 8\%$
charm (c)	$\approx 2\%$

Running of electroweak couplings

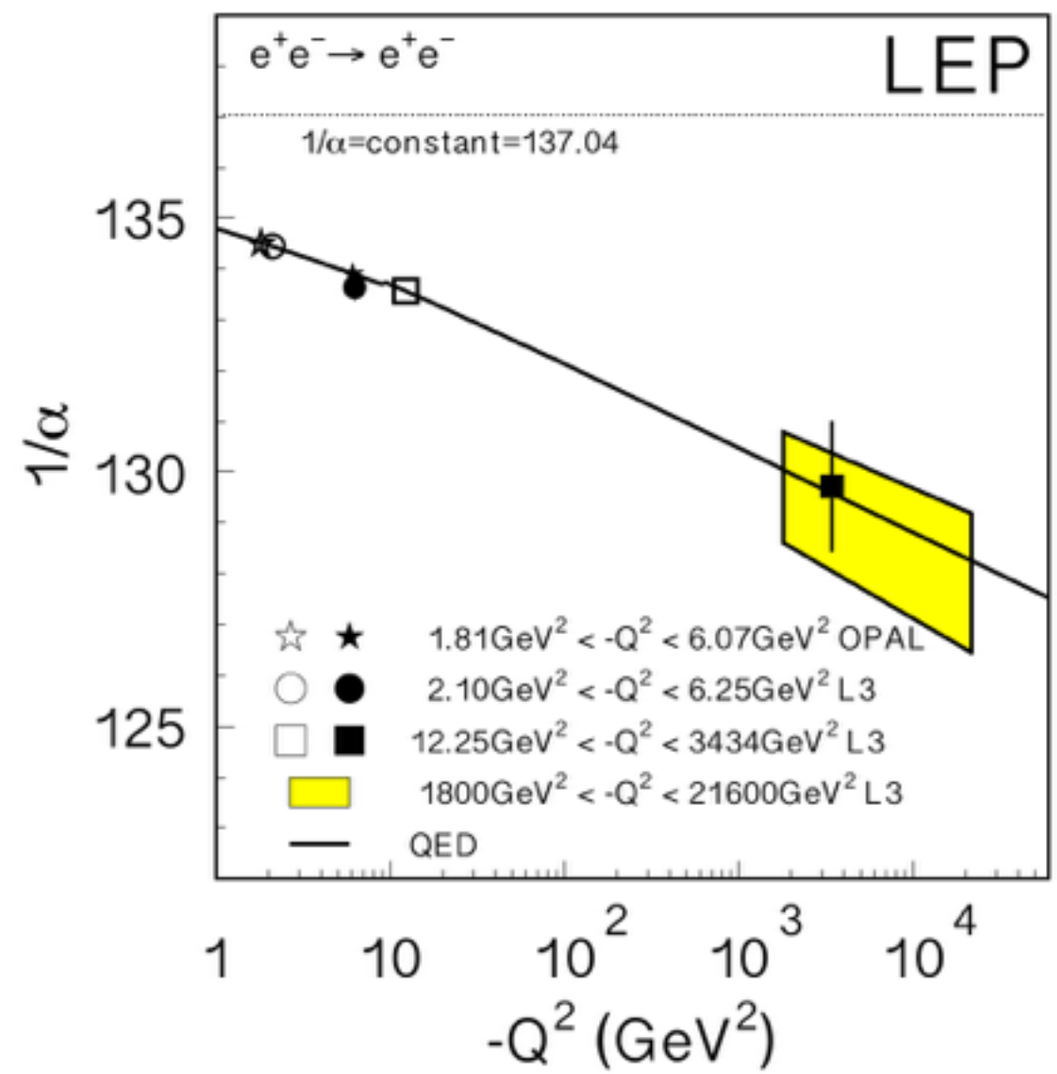


Running of α and $\sin^2\theta_W$

* Fine structure constant: $$\alpha(Q^2) = \frac{\alpha}{1 - \Delta\alpha(Q^2)}$$

* Hadronic contributions:

$$\Delta\alpha_{\rm had}(Q^2) = -\frac{\alpha Q^2}{3\pi}\int_{4m_\pi^2}^{\infty} ds\,\frac{R_{\rm had}(s)}{s(s-Q^2)}$$



* Dispersion theory:

$$\Delta\alpha^{(5)}_{\rm had}(M_Z^2) = (276.26 \pm 1.38)\cdot 10^{-4}$$

Running of α and $\sin^2\theta_W$

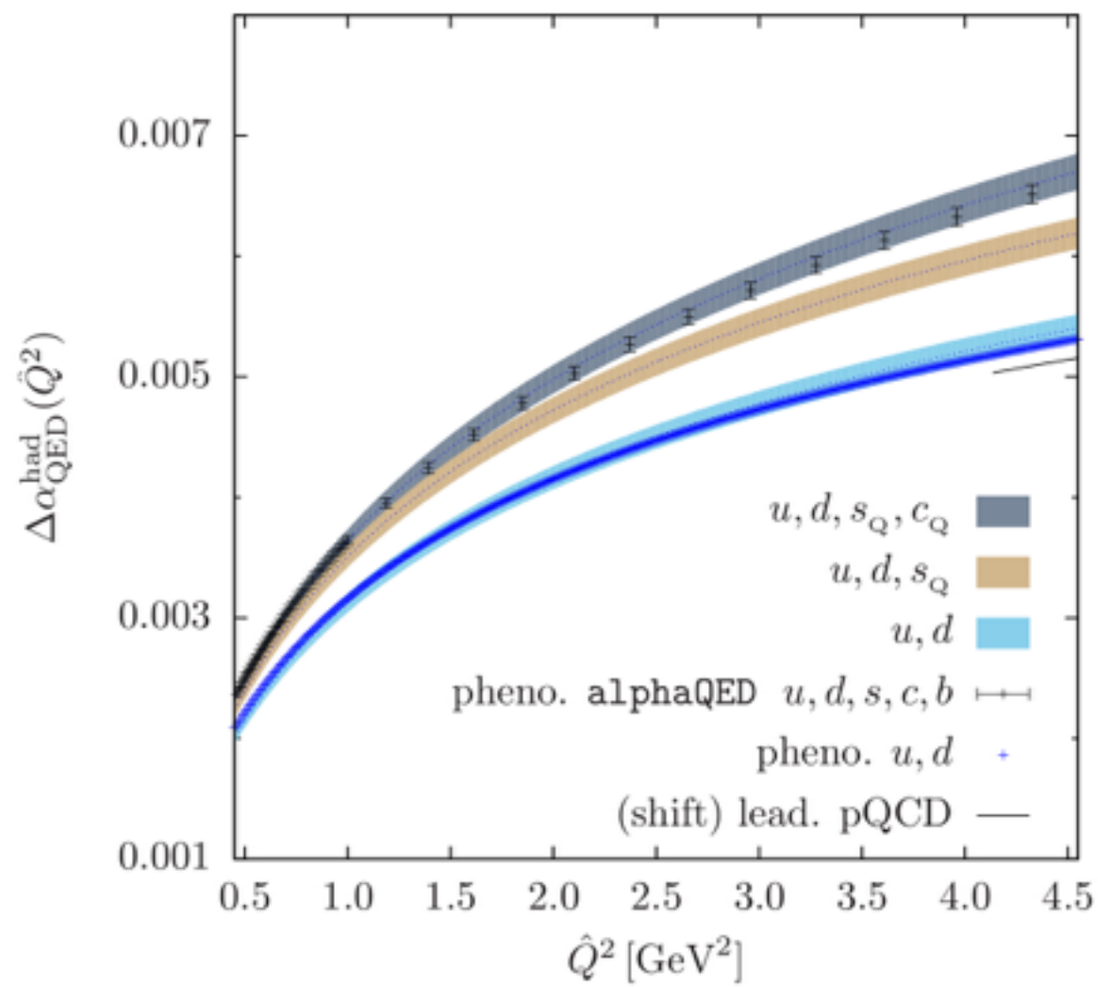
* Fine structure constant: $$\alpha(Q^2) = \frac{\alpha}{1 - \Delta\alpha(Q^2)}$$

* Adler function: $$D(Q^2) = -12\pi^2 \frac{d\Pi(Q^2)}{d\ln Q^2} = \frac{3\pi}{\alpha}\frac{d\Pi(Q^2)}{d\ln Q^2}\Delta\alpha_{\rm had}(Q^2)$$

* $\Delta\alpha_{\rm had}(Q^2)$ accessible in lattice QCD via vacuum polarisation

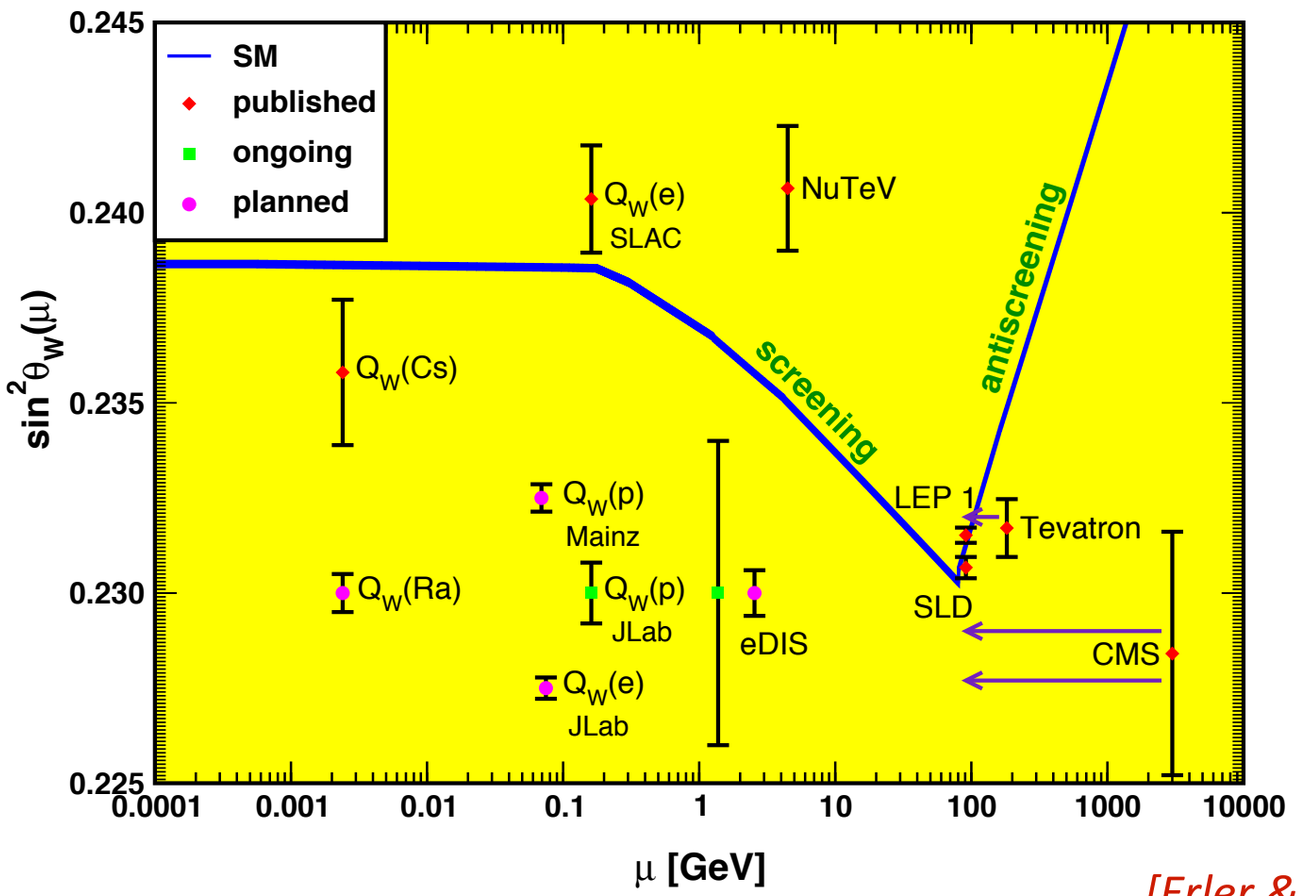
* Similar accuracy as phenomenological approach

[H. Horch, G. Herdoíza @ Lattice 2015]



Running of α and $\sin^2\theta_W$

* Definition: $$\sin^2\theta_W(Q^2) = \underbrace{\sin^2\theta(0)}_{0.23864}\left(1 - \Delta\sin^2\theta_W(Q^2)\right)$$



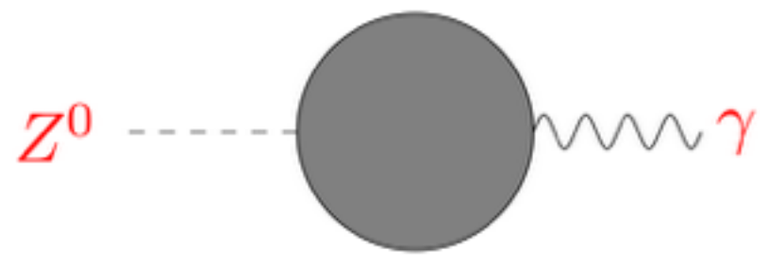
[Erler & Su, PPNP 71 (2013) 119]

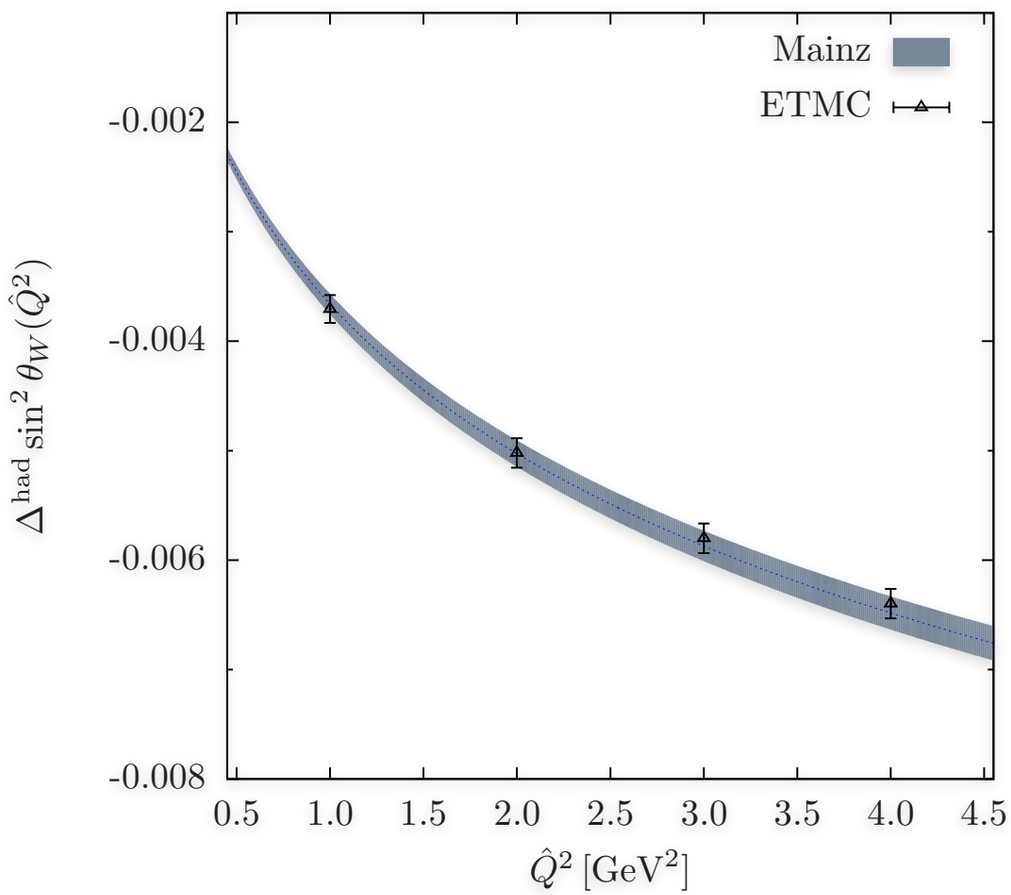
* Dispersive approach requires separation of contributions from up/down-type quarks

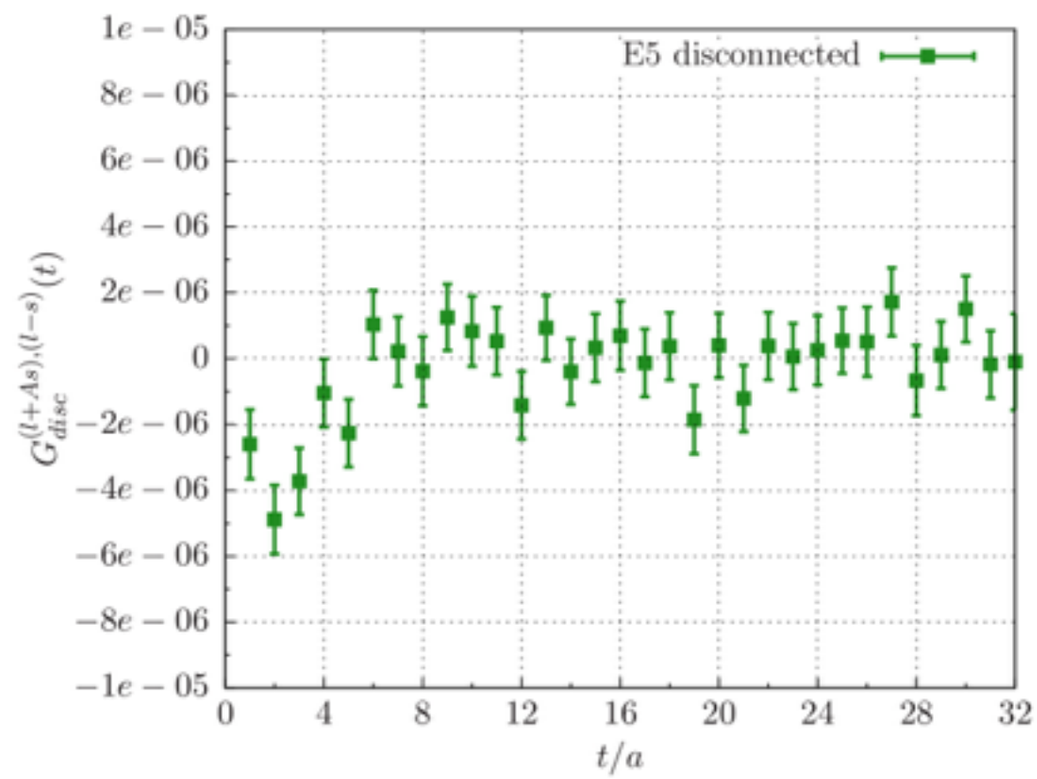
Running of $\sin^2\theta_W$

* Euclidean approach:

$$\Pi^{\gamma Z}_{\mu\nu}(Q) = \int d^4x\, e^{iQ\cdot x} \left\langle V^Z_\mu(x) J_\nu(0) \right\rangle$$

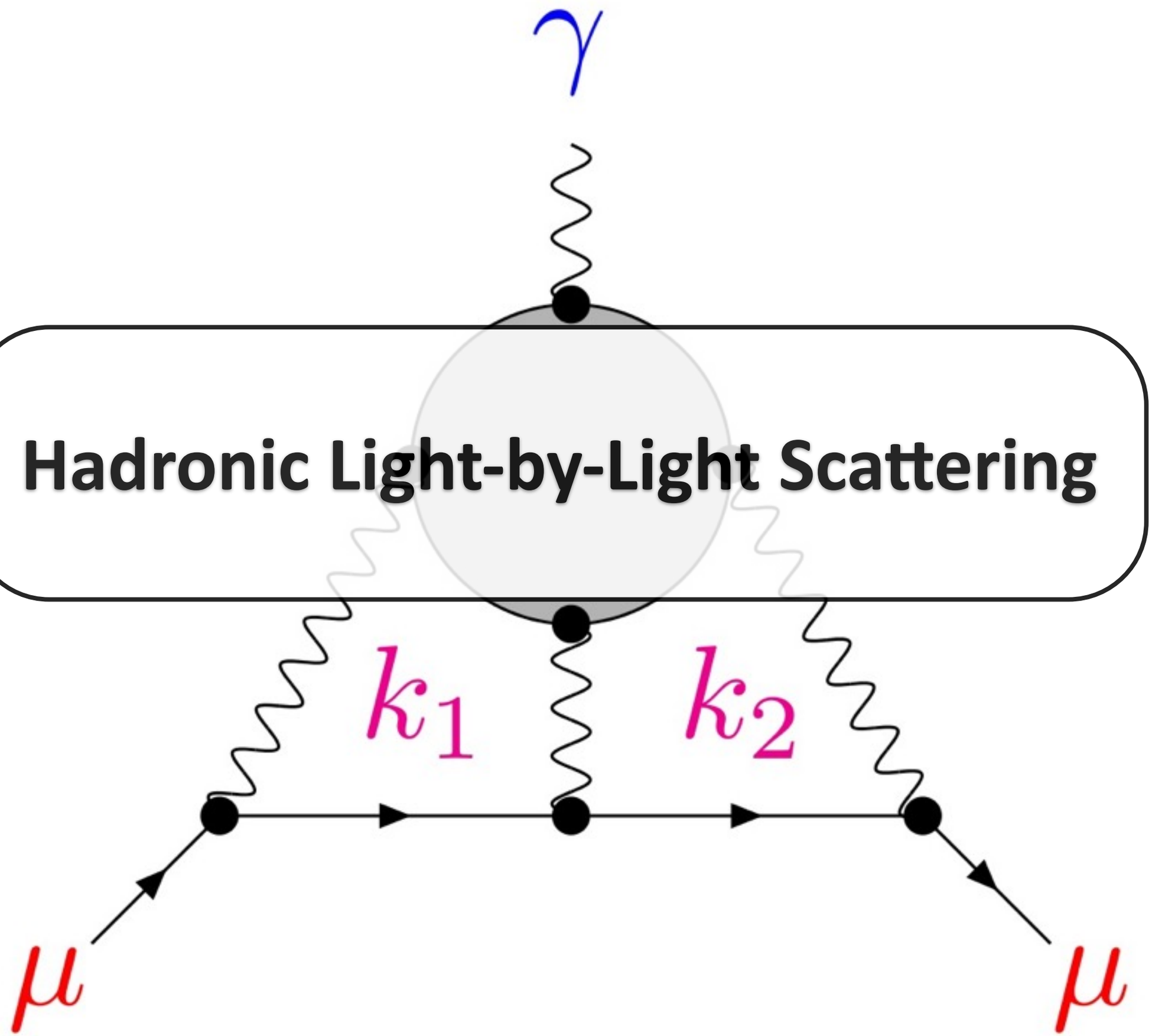






[H. Horch, G. Herdoíza, V. Gülpers @ Lattice 2015]

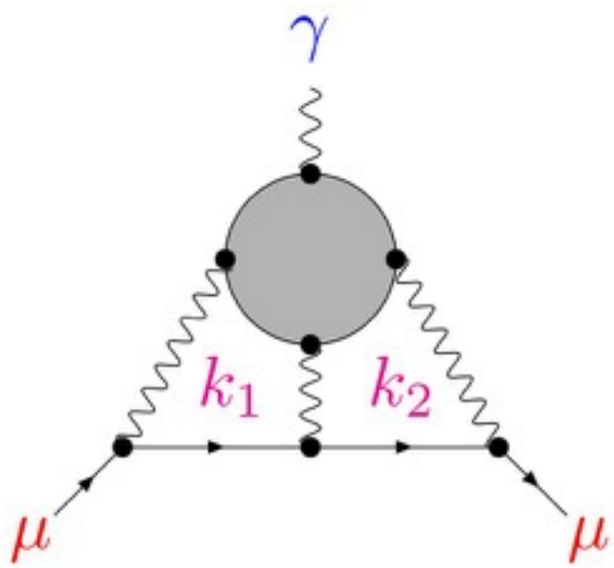
Hadronic Light-by-Light Scattering



Lattice QCD approaches to HLbL scattering

* Numerically very demanding:
  - Compute 4pt correlation function for two independent momenta, k_1, k_2

    ⇒ Cost scales proportional to $(\text{volume})^2$



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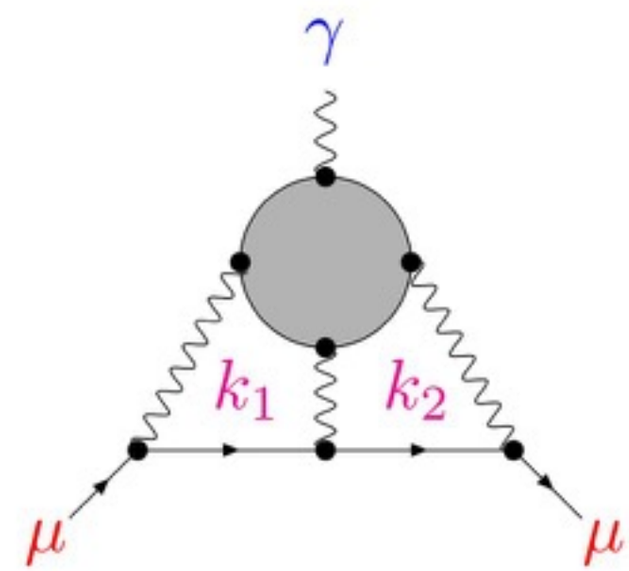

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Proposed techniques:

* QCD + QED simulations

  [Hayakawa et al., PoS LAT2005 (2006) 353, Blum et al., Phys Rev Lett 114 (2015) 012001]

* QCD + stochastic QED

  [Blum et al., Phys Rev D93 (2016) 014503; Blum et al., arXiv:1610:04603]

* Light-by-light four-point function + exact QED kernel

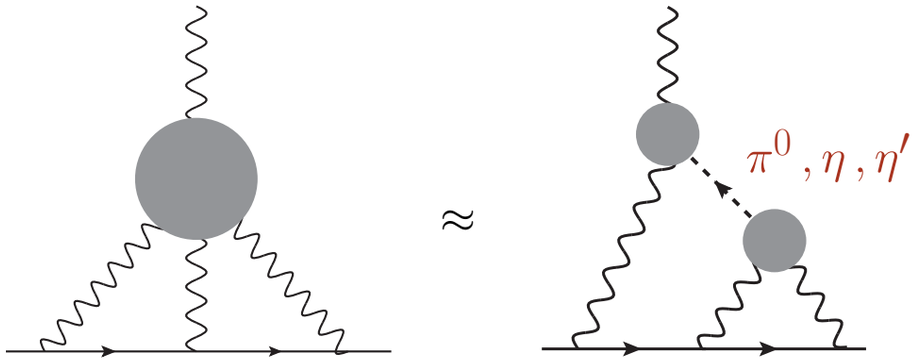
  [Green, Gryniuk, von Hippel, Meyer, Pascalutsa, Phys Rev Lett 115 (2015) 222003]

* Lattice calculations of dominant sub-processes

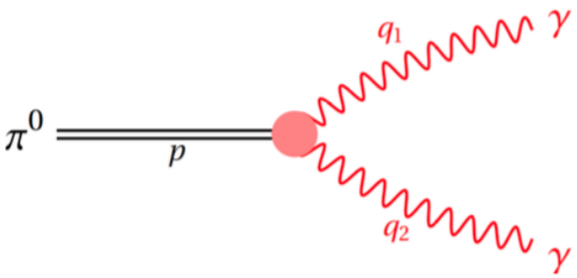
  [Feng et al., Phys Rev Lett 109 (2012) 182001, Gérardin, Meyer, Nyffeler, Phys Rev D94 (2016) 074507]

HLbL from transition $\pi^0 \longrightarrow \gamma^*\gamma^*$

* Pseudoscalar meson exchange expected to dominate LbL scattering:
  [Nyffeler, EPJ Web Conf 118 (2016) 01024, arXiv:1602.03737]



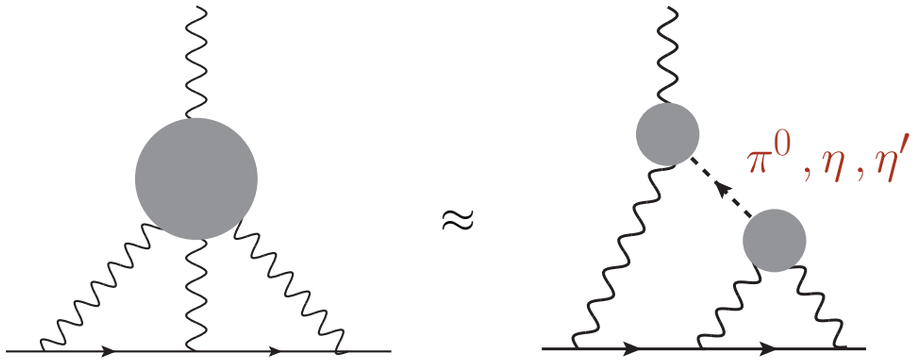
* Compute transition form factor between π^0 and two off-shell photons:



$$\epsilon_{\mu\nu\alpha\beta}\, q_1^\alpha q_2^\beta\, \mathcal{F}_{\pi^0\gamma^*\gamma^*}(m_\pi^2; q_1^2, q_2^2) \equiv M_{\mu\nu}$$

HLbL from transition $\pi^0 \longrightarrow \gamma^*\gamma^*$

* Pseudoscalar meson exchange expected to dominate LbL scattering:
  [Nyffeler, EPJ Web Conf 118 (2016) 01024, arXiv:1602.03737]



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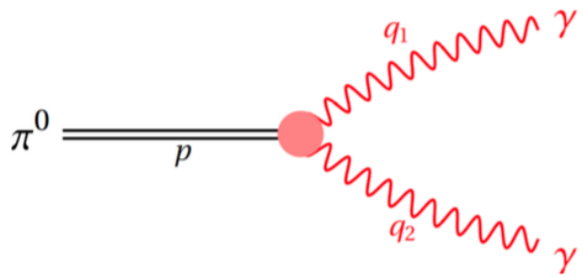

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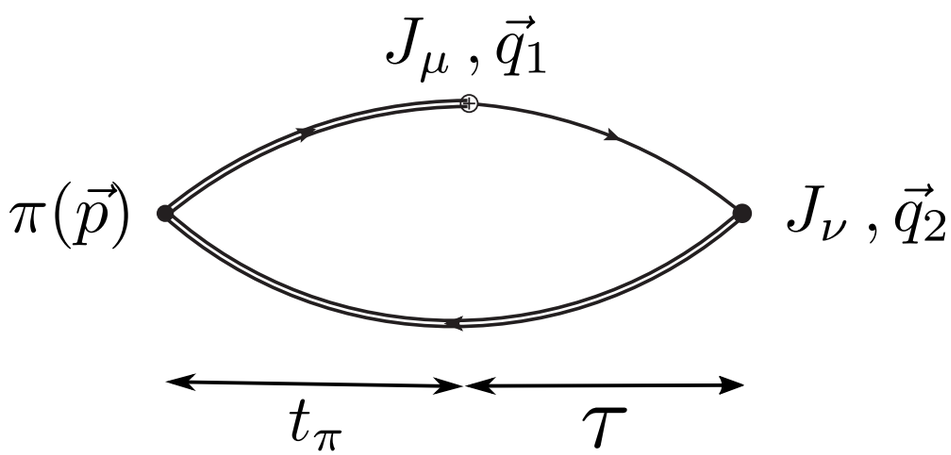

$$M_{\mu\nu} \sim C^{(3)}_{\mu\nu}(\tau, t_\pi; \vec{p}, \vec{q}_1, \vec{q}_2) =$$

$$\sum_{\vec{x},\vec{z}} \left\langle T\left\{J_\nu(\vec{0}, \tau + t_\pi) J_\mu(\vec{z}, t_\pi) P(\vec{x}, 0)\right\}\right\rangle \mathrm{e}^{i\vec{p}\cdot\vec{x}} \mathrm{e}^{-i\vec{q}_1\cdot\vec{z}}$$

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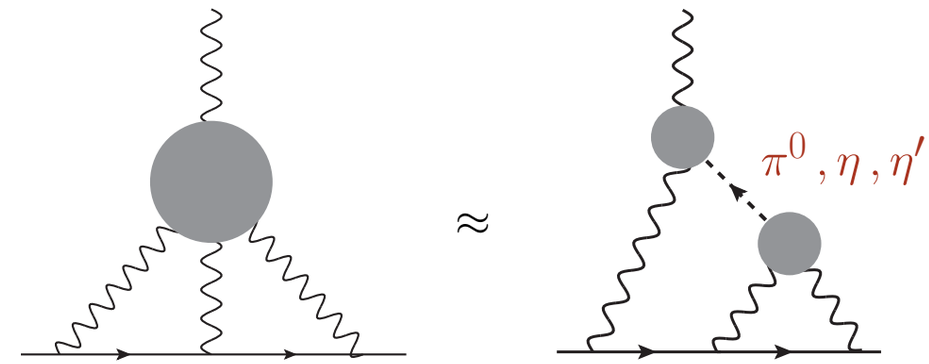

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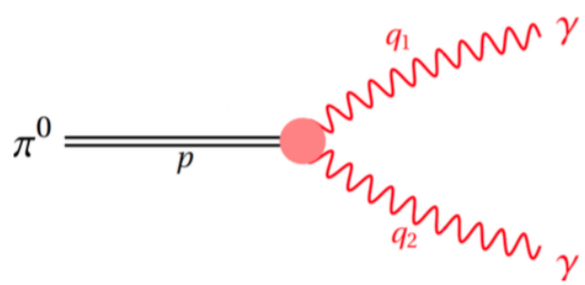

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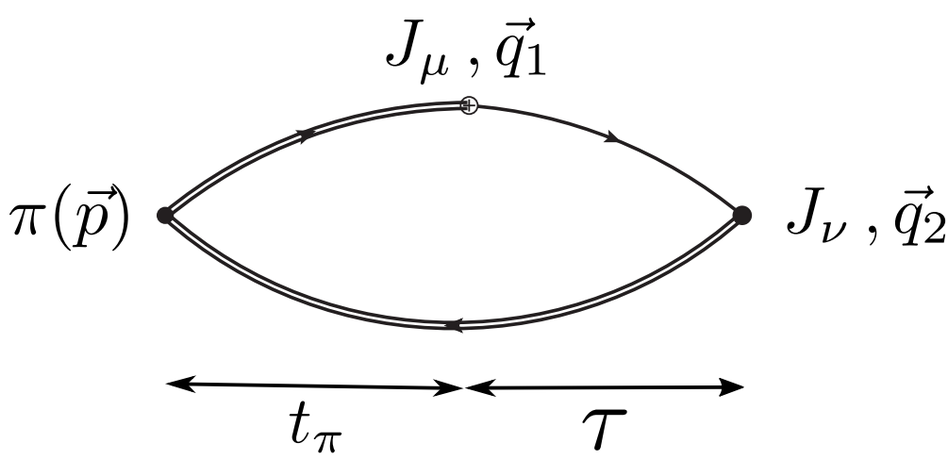

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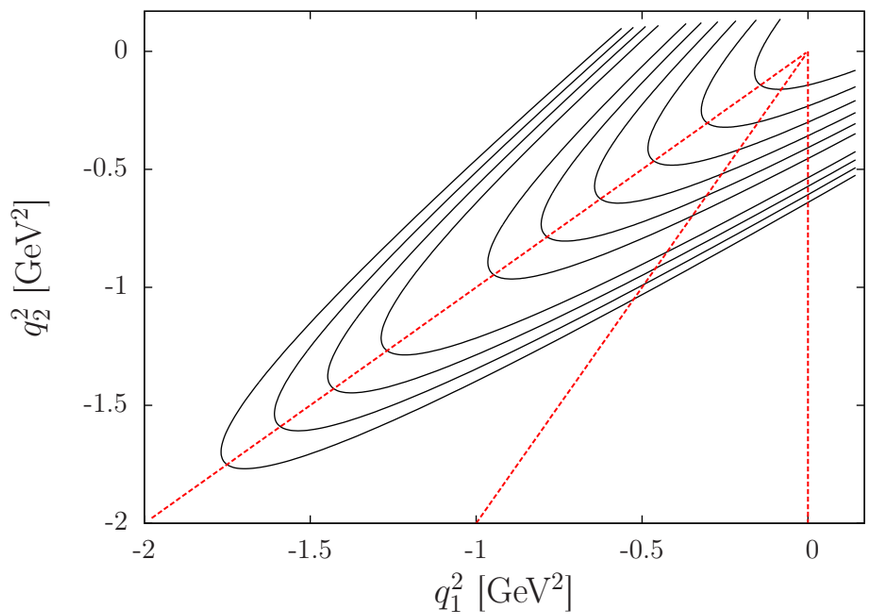
* Compute connected and disconnected diagrams

Transition form factor $\pi^0 \longrightarrow \gamma^*\gamma^*$

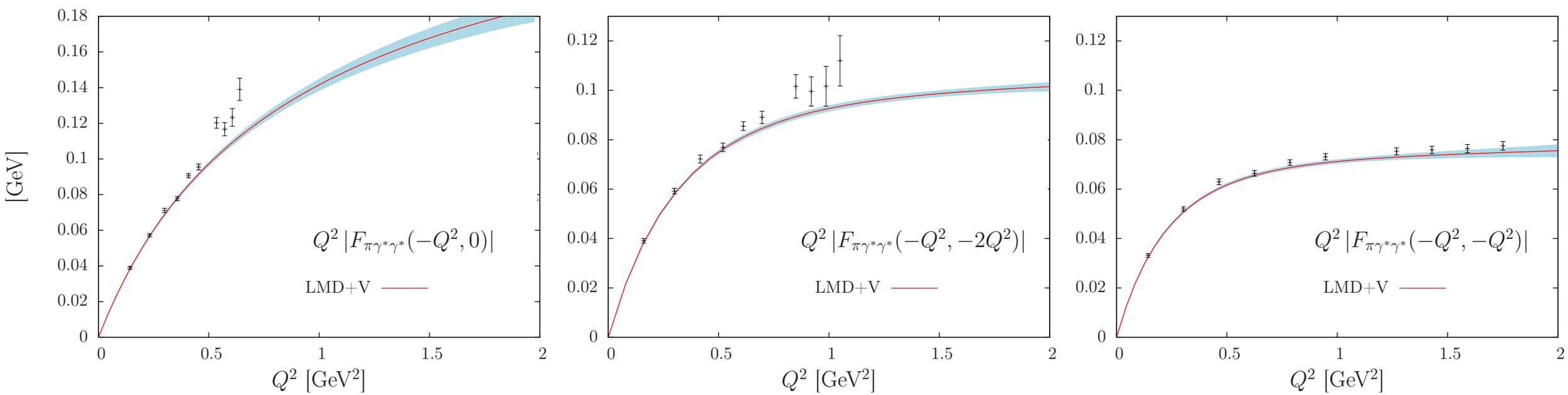
* Kinematics: $\vec{p} = 0, \quad q_1^2 = \omega_1^2 - |\vec{q}_1|^2, \quad q_2^2 = (m_\pi - \omega_1)^2 - |\vec{q}_1|^2$



Lowest meson dominance (LMD) model:

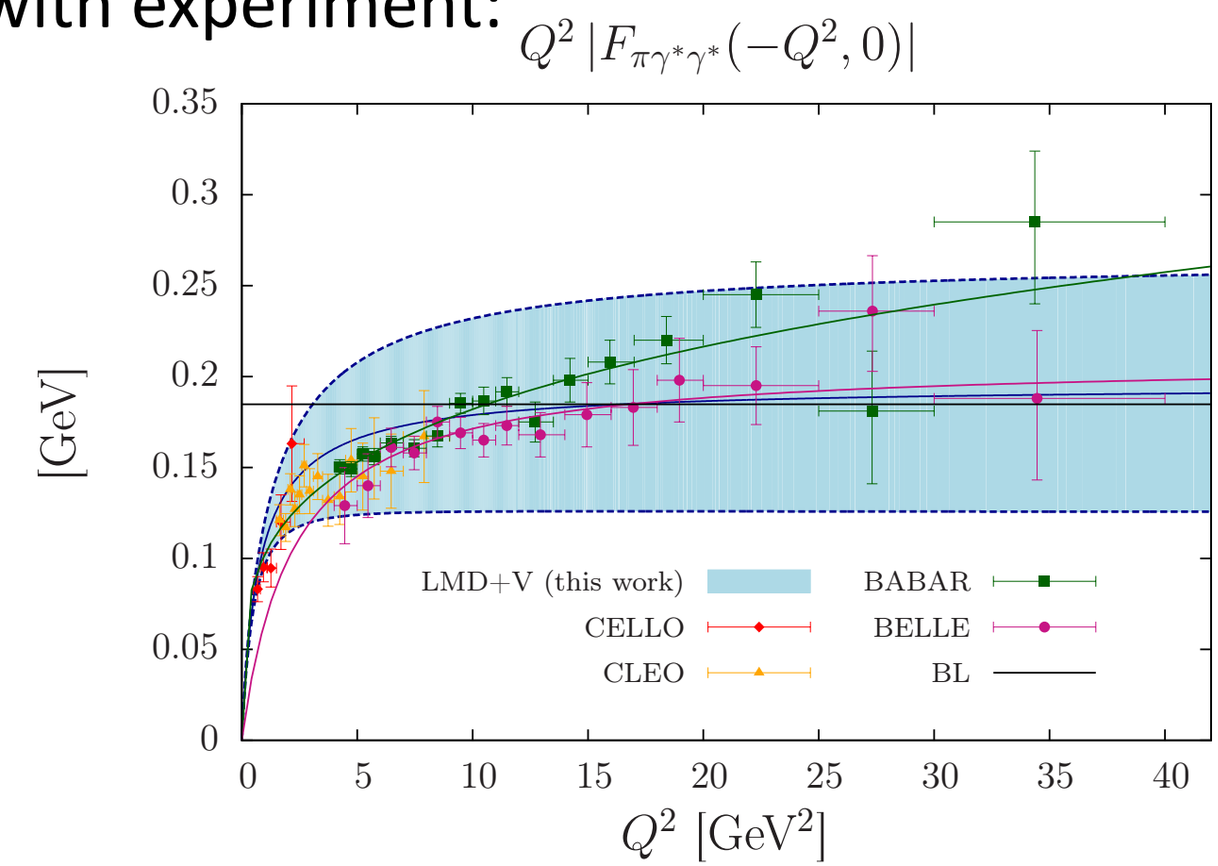
$$\mathcal{F}^{\mathrm{LMD}}_{\pi^0\gamma^*\gamma^*} = \frac{\alpha M_{\mathrm{V}}^4 + \beta(q_1^2 + q_2^2)}{(M_{\mathrm{V}}^2 - q_1^2)(M_{\mathrm{V}}^2 - q_2^2)}$$

(refinement: LMD-V model)



HLbL contribution from $\pi^0 \longrightarrow \gamma^*\gamma^*$

* Comparison with experiment:



[Gérardin, Meyer, Nyffeler, Phys Rev D94 (2016) 074507]

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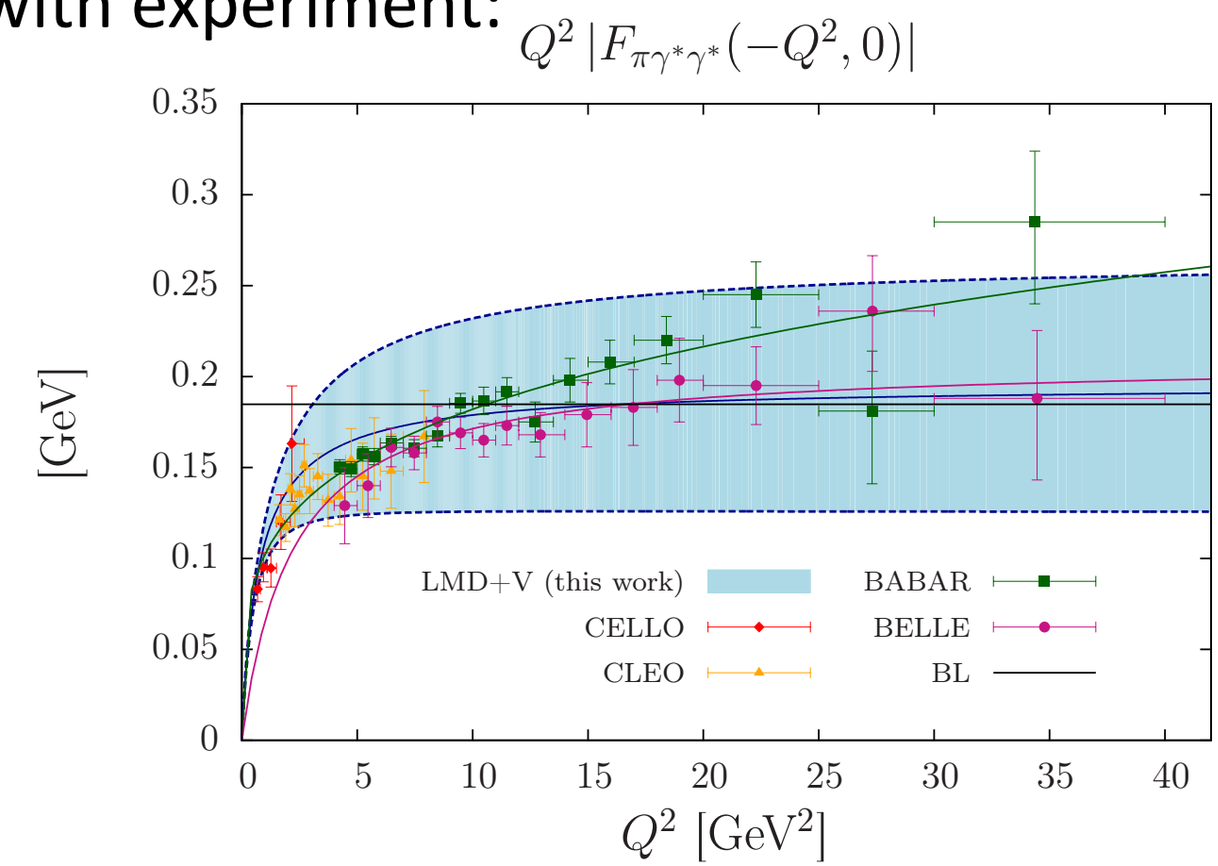

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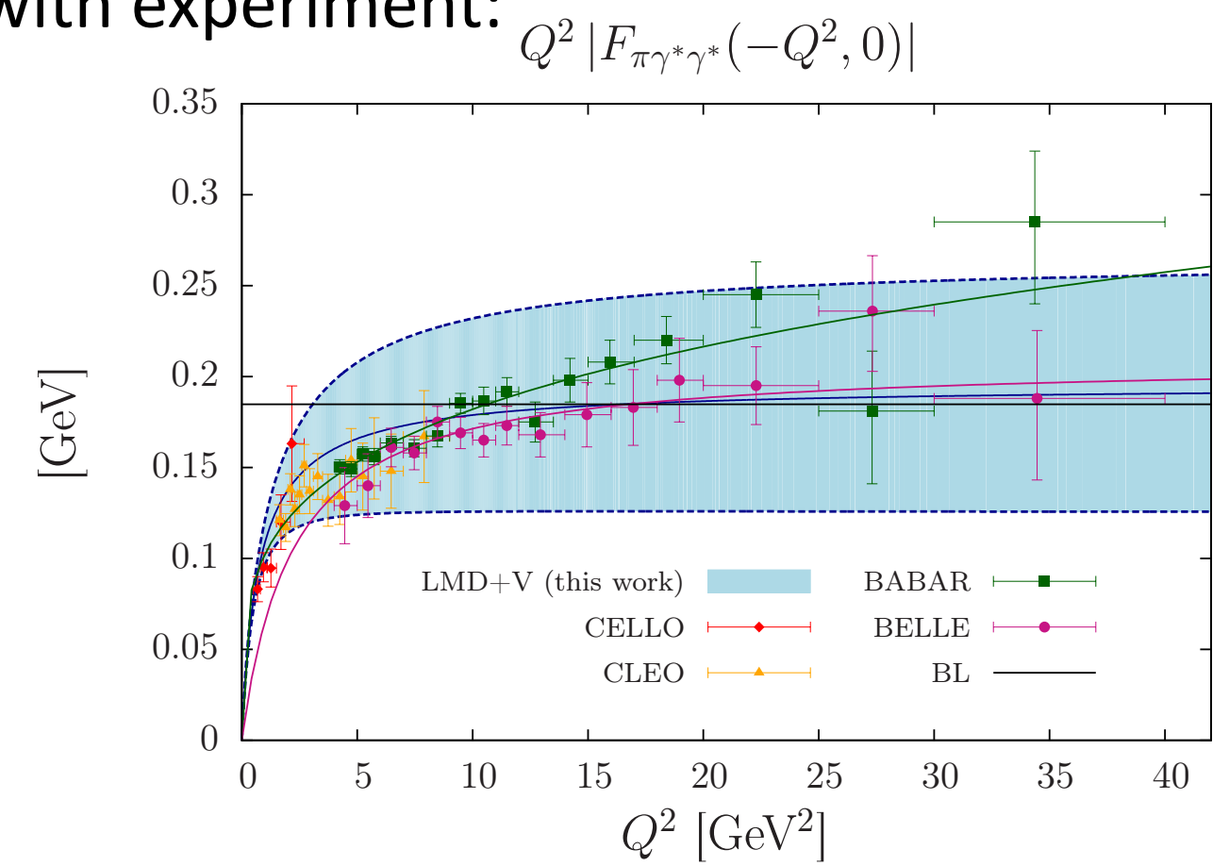


Brodsky-Lepage

* Results for π^0 contribution to hadronic light-by-light scattering:

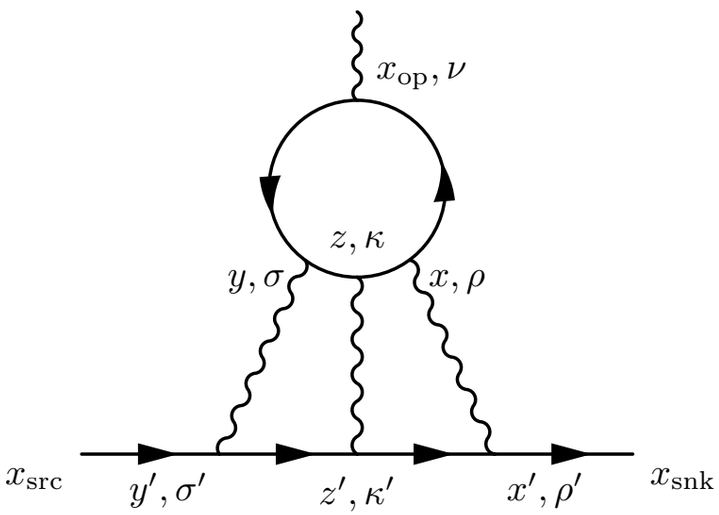
$$(a_\mu^{\rm hlbl})_{\pi^0} = (65.0 \pm 8.3) \cdot 10^{-11} \quad \text{(LMD+V)}$$

⇒ agrees well with phenomenological studies

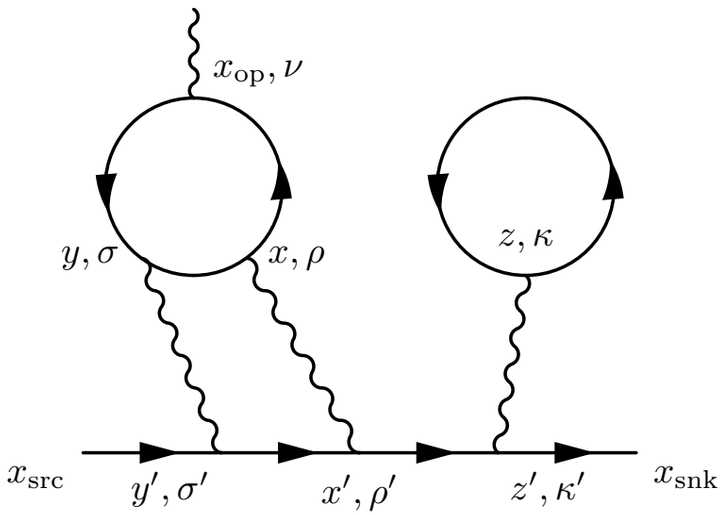
[Gérardin, Meyer, Nyffeler, Phys Rev D94 (2016) 074507]

Direct calculation of HLbL scattering

* Compute HLbL scattering contribution from QCD + stochastic QED



connected



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[Blum et al. (RBC-UKQCD), arXiv:1610.04603]

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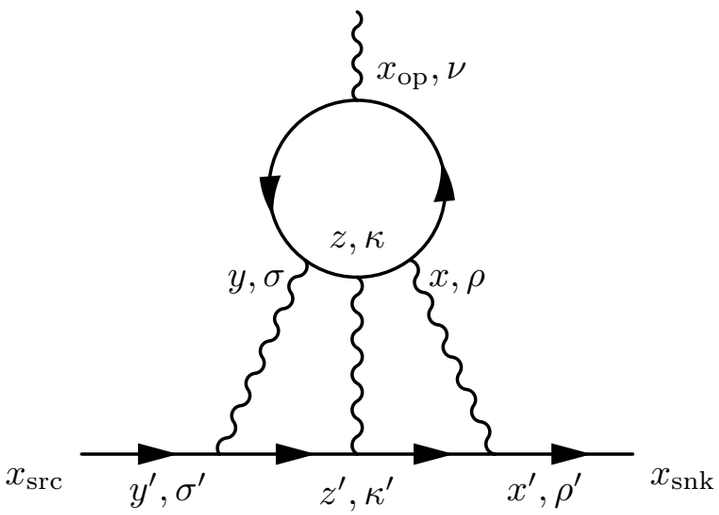

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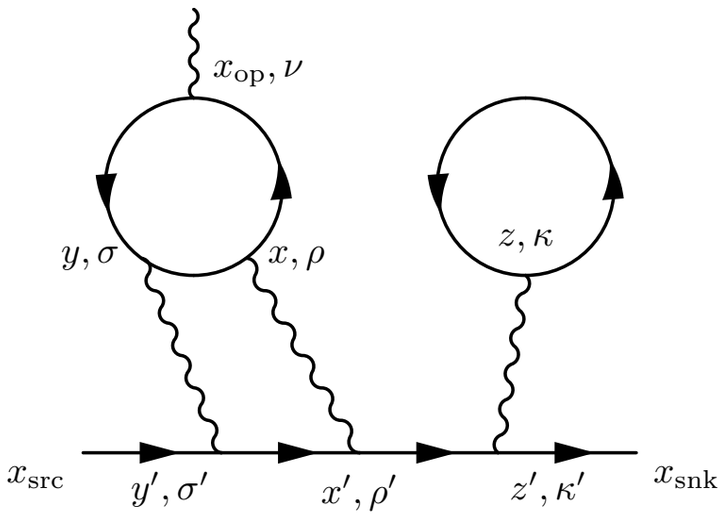

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* Results at the physical pion mass:

$$(a_\mu^{\rm hlbl})_{\rm con} = (116.0 \pm 9.6) \cdot 10^{-11}$$

$$(a_\mu^{\rm hlbl})_{\rm disc}^{\rm LO} = (-62.5 \pm 8.0) \cdot 10^{-11}$$

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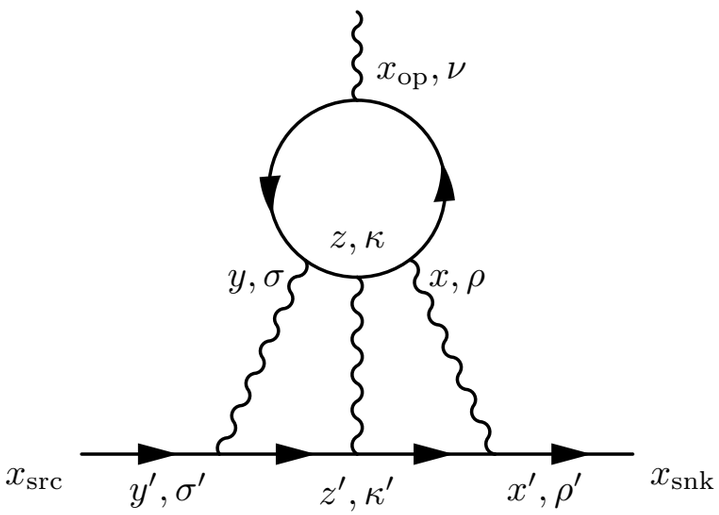

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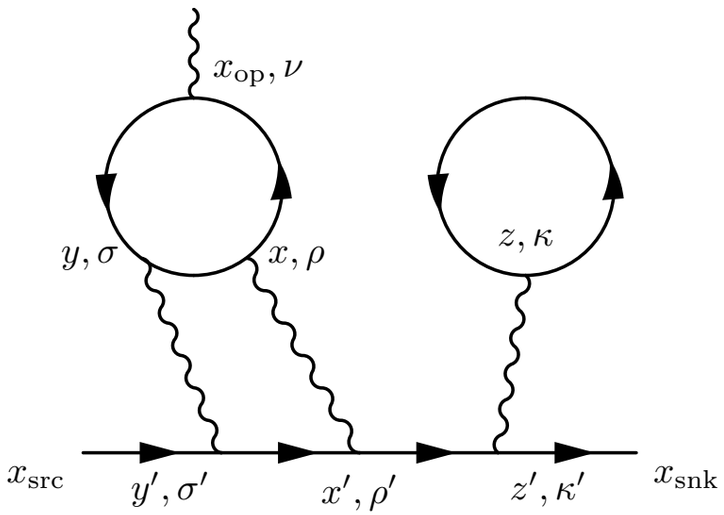

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* Must account for other systematic effects: finite-volume,…

[Blum et al. (RBC-UKQCD), arXiv:1610.04603]

EMMERT

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Can lattice QCD deliver estimates with sufficient accuracy?

$$\delta a_\mu^{\rm hvp}/a_\mu^{\rm hvp} < 0.5\%, \qquad \delta a_\mu^{\rm hlbl}/a_\mu^{\rm hlbl} \lesssim 10\%$$

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- Statistical accuracy limited by disconnected diagrams
- Disconnected contributions: $\lesssim 1\%$
- Finite-volume effects: $3-7\%$ for $m_\pi \simeq 140$ MeV, $m_\pi L \sim 4$
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Hadronic Light-by-Light Scattering:

- Statistical accuracy: $\approx 10\%$ (connected)
- Disconnected contributions can be resolved
- Phenomenological models can be verified