

# Waves of Anomaly in QGP : Chiral Magnetic Wave and Chiral Shear Wave

**Ho-Ung Yee**

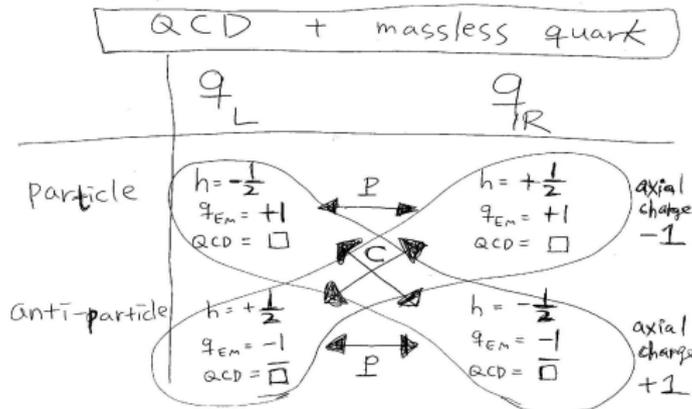
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References : [CMW](#) : A work in progress with Dima Kharzeev  
[CSW](#) : arXiv:0910.5915 [hep-th](with B.Sahoo)

# P, C, and anomaly basics



- P and C are good symmetries of QCD with massless quarks
- $U(1)_A$  suffers from anomaly in QCD : it is **not** a true symmetry

$$\partial_\mu J_A^\mu \sim \epsilon^{\mu\nu\alpha\beta} \text{Tr} (G_{\mu\nu} G_{\alpha\beta}) \quad (1)$$

One fact : this anomaly is  $\frac{1}{N_c^2}$ -suppressed in large  $N_c$  limit

- But,  $SU(N_F)_A$  for multiple flavors are good global symmetries free of QCD anomaly
- There are also **triangle anomaly** of AVV and AAA

# Why triangle anomalies can be interesting

- It is often said that results from anomalies are robust, independent of dynamics such as coupling constant. This is both good and bad.
- But, the important assumption in this statement is that we are considering **low energy limits** which results in various **low energy theorems**. For finite frequency or momentum  $\omega, k \neq 0$ , there are in general no such theorems, and we DO expect the results to depend on dynamics such as temperature, coupling constants, etc.
- For strongly coupled QGP with a lot of time/space dependent dynamics going on, it will be **non-trivial** to compute such effects

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# A table of recent interesting discoveries

	$T < T_c$	$T > T_c$ (RHIC,LHC)
$B = 0$	<ul style="list-style-type: none"> <li>- gauged WZW term <math>\pi^0 \rightarrow 2\gamma</math>, etc</li> <li>- <math>\mu \neq 0</math>, Quarkyonic spiral</li> </ul>	<ul style="list-style-type: none"> <li>- chiral vortex current (<math>\mu \neq 0</math>) <math>J^\mu \sim \epsilon^{\mu\nu\alpha\beta} u_\nu \partial_\alpha u_\beta</math></li> <li>- <b>chiral shear wave</b> (<math>\mu \neq 0</math>)</li> </ul>
$B \neq 0$	<ul style="list-style-type: none"> <li>- meson(pion) supercurrent (<math>\mu \neq 0</math>) <math>\partial\pi \neq 0</math> "chiral spiral"</li> <li>- chiral magnetic spiral (<math>\mu \neq 0</math>) <math>J^1 + iJ^2 \neq 0</math></li> </ul>	<ul style="list-style-type: none"> <li>- chiral magnetic effect (<math>\mu \neq 0</math>) <math>\vec{J} \sim \vec{B}\mu</math></li> <li>- <b>chiral magnetic wave</b> (<math>\mu = 0</math>)</li> </ul>

Important difference between chiral magnetic effect and **chiral magnetic wave** will be that CMW exists even in the average **neutral** plasmas

# Chiral Magnetic Wave: Derivation

- Consider general deconfined QCD plasma with applied magnetic field  $\vec{B} = B\hat{x}^1$ . The plasma can be **neutral** in general.
- We will treat electromagnetism as **non-dynamical** external environment
- Let's start from the basic **chiral magnetic effects**

$$\vec{j}_V = \frac{N_c e \vec{B}}{2\pi^2} \mu_A \quad , \quad \vec{j}_A = \frac{N_c e \vec{B}}{2\pi^2} \mu_V \quad , \quad (2)$$

$$\begin{pmatrix} \vec{j}_V \\ \vec{j}_A \end{pmatrix} = \frac{N_c e \vec{B}}{2\pi^2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \mu_V \\ \mu_A \end{pmatrix} \quad . \quad (3)$$

- We are interested in small fluctuations out of neutral plasma, so let's expand  $\mu_{V,A}$  in terms of small charge densities  $j_{V,A}^0$  linearly

$$\begin{pmatrix} \mu_V \\ \mu_A \end{pmatrix} = \begin{pmatrix} \frac{\partial \mu_V}{\partial j_V^0} & \frac{\partial \mu_V}{\partial j_A^0} \\ \frac{\partial \mu_A}{\partial j_V^0} & \frac{\partial \mu_A}{\partial j_A^0} \end{pmatrix} \begin{pmatrix} j_V^0 \\ j_A^0 \end{pmatrix} \equiv \begin{pmatrix} \alpha_{VV} & \alpha_{VA} \\ \alpha_{AV} & \alpha_{AA} \end{pmatrix} \begin{pmatrix} j_V^0 \\ j_A^0 \end{pmatrix} \quad . \quad (4)$$

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# CMW Derivation continued

- Recall that chemical potentials are defined as  $\mu_i = \frac{\partial \mathcal{F}}{\partial j_i^0}$  ,  $i = V, A$  where  $\mathcal{F}$  is free energy, so

$$\alpha_{ij} = \frac{\partial^2 \mathcal{F}}{\partial j_i^0 \partial j_j^0} . \quad (5)$$

are in fact **susceptibilities**

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- Diagonalize in terms of chiral basis

$$j_L^\mu \equiv \frac{1}{2} (j_V^\mu - j_A^\mu) \quad , \quad j_R^\mu \equiv \frac{1}{2} (j_V^\mu + j_A^\mu) . \quad (7)$$

One can also rewrite

$$\alpha = \frac{1}{2} \left( \frac{\partial \mu_L}{\partial j_L^0} \right) = \frac{1}{2} \left( \frac{\partial^2 \mathcal{F}}{\partial j_L^0 \partial j_L^0} \right) = \frac{1}{2} \left( \frac{\partial \mu_R}{\partial j_R^0} \right) = \frac{1}{2} \left( \frac{\partial^2 \mathcal{F}}{\partial j_R^0 \partial j_R^0} \right) . \quad (8)$$

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The semi-final result is

$$\vec{j}_{L,R} = \mp \left( \frac{N_c e \vec{B} \alpha}{2\pi^2} \right) j_{L,R}^0 - D_L \frac{\vec{B}(\vec{B} \cdot \vec{\nabla})}{B^2} j_{L,R}^0 + \dots \quad , \quad (9)$$

with a longitudinal diffusion constant  $D_L$ . This should be taken as a hydrodynamic constitutive equation in [long wave-length expansion](#)  
A few comments are

- What we are claiming is that the above result is valid for **arbitrarily large** magnetic field  $\mathbf{B}$ . Several previous literature considered only linear in  $\mathbf{B}$
- Note that  $\alpha$  will be a non-trivial, non-linear function of  $\mathbf{B}$  as well as temperature, coupling constant, etc. We will see this explicitly later for infinitely large  $\mathbf{B}$  case and holographic QCD computation
- It might also be interesting to consider [transverse](#) diffusion coefficient  $D_T$

# New propagating modes out of this

Now, plug the above in the conservation laws  $\partial_\mu j_{L,R}^\mu = 0$ , and consider only longitudinal derivative  $\partial_1$ , which results in

$$\left( \partial_0 \mp \frac{N_c e B \alpha}{2\pi^2} \partial_1 - D_L \partial_1^2 \right) j_{L,R}^0 = 0 \quad . \quad (10)$$

- This is a **directional (chiral) wave** with velocity

$$v_\chi = \frac{N_c e B \alpha}{2\pi^2} = \frac{N_c e B}{4\pi^2} \left( \frac{\partial \mu_L}{\partial j_L^0} \right) = \frac{N_c e B}{4\pi^2} \left( \frac{\partial \mu_R}{\partial j_R^0} \right) \quad . \quad (11)$$

- The momentum space dispersion relation looks as

$$\omega = \mp v_\chi k - i D_L k^2 + \dots \quad , \quad (12)$$

The propagating leading part would be absent if there was no anomaly or magnetic field  $B$ , and it is essentially important to have them to find the propagating behavior

# Large $B$ limit and 1+1 dimensional reduction

We will consider one particular weak coupling limit,  $eB \rightarrow \infty$

- At first, the expression

$$v_\chi = \frac{N_c eB}{4\pi^2} \left( \frac{\partial \mu_L}{\partial j_L^0} \right) \quad (13)$$

looks worrisome in view of causality

- It is natural to expect that the quarks are sitting in the lowest Landau Levels, as the gap  $\Delta = \sqrt{eB} \gg T$ . We have an **effective 1+1 dimensional reduction**.
- For a chemical potential  $\mu_L$ , 1+1 dimensional free fermion density is simply  $\frac{\mu_L}{(2\pi)}$ . We also have transverse space density of state of  $\frac{eB}{(2\pi)}$ , so that the net 3-dimensional density  $j_L^0$  is

$$j_L^0 = \left( \frac{\mu_L}{(2\pi)} \right) \left( \frac{eB}{(2\pi)} \right) = \frac{eB}{4\pi^2} \mu_L \quad (14)$$

or  $\left( \frac{\partial \mu_L}{\partial j_L^0} \right) = \frac{4\pi^2}{eB}$  in the limit of  $eB \rightarrow \infty$ . This gives

$$v_\chi \rightarrow 1 \quad , \quad eB \rightarrow \infty \quad (15)$$

# 1+1 dimensional bosonization

## Let's pursue further aspects of this 1+1 dimensional reduction

- From  $q_L$  one gets 1+1 dimensional chiral fermion in LLL,  $\psi_L$ . Vice Versa from  $q_R$  to get  $\psi_R$ . Together they form a single 1+1 dimensional **massless** Dirac fermion
- One has  $U(1)_V \times U(1)_A$  symmetries with currents  $j_V^\mu$  and  $j_A^\mu$ . 1+1 dimensional  $\gamma$ -matrices imply that

$$j_V^\mu = \epsilon^{\mu\nu} j_{A\nu} \quad (16)$$

This can be written in terms of a real scalar field  $\phi$  by

$$j_V^\mu = \epsilon^{\mu\nu} \partial_\nu \phi \quad , \quad j_A^\mu = \partial^\mu \phi \quad (17)$$

where conservation of  $j_V^\mu$  is automatic.

- Then, the conservation of axial current  $j_A^\mu$  implies

$$\partial_\mu j_A^\mu = \partial_\mu \partial^\mu \phi = 0 \quad (18)$$

so  $\phi$  is a massless scalar boson

# Bosonization continued...

- Bosonization says that the two **quantum** theories are in fact **equivalent** with a dictionary

$$\psi_L \sim e^{i\phi_L} \quad , \quad \psi_R \sim e^{-i\phi_R} \quad (19)$$

where  $\phi = \phi_L + \phi_R$ , or

$$\bar{\psi}_R \psi_L \sim e^{i\phi} \quad (20)$$

- The mass term will be  $\cos(\phi)$  under this, which gives us sine-Gordon model

**Our basic identification is :** Chiral magnetic wave is a bosonized wave of  $\phi$

# Holographic computation

- One can study the same phenomena in the framework of holographic QCD for strongly coupled regime. We choose Sakai-Sugimoto model as it contains all the necessary ingredients of chiral symmetry and its triangle anomaly
- Triangle anomaly is encoded as a 5D Chern-Simons term in the bulk
- One can study  $v_\chi$  as well as  $D_L$  systematically in the model

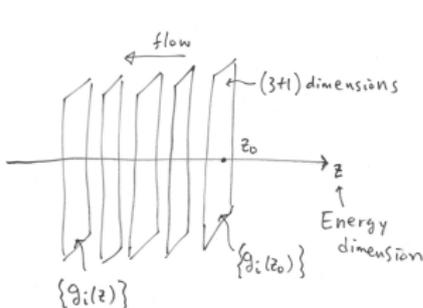
# Holography or AdS/CFT approach

## Very brief words about AdS/CFT ideas

- We have an **effective theory** for large  $N_c$  and strong t'Hooft coupling limit of a gauge theory living in **five dimensions**
- The additional dimension is roughly energy-scale of the theory. Recall RG still works in large  $N_c$  limit. 5D theory involves **gravity** :

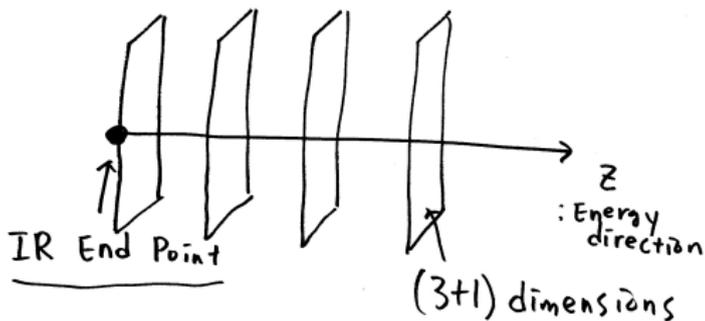
**Quantum** RG invariance  $\longleftrightarrow$  **Classical** general covariance

- Large  $N_c$  factorization implies **classical or statistical** description of **master variables** while RG still makes sense : **holography is one way of combining these two**



# Finite temperature plasma is described by Black Hole

In the 5D gravity description, finite temperature plasma is described by **Black Hole** or more precisely, **Black Brane**



- There is no rigorous understanding of this
- It has been more like **“empirical”**
- Hawking temperature is identified with plasma temperature naturally

# How to study physics in AdS/CFT ?

## Just two facts we need from AdS/CFT dictionary

- Global symmetry in gauge theory  $\longleftrightarrow$  Gauge symmetry in 5D

How it works?

$$A_\mu(x^\mu, Z) \sim A_\mu^{(0)} + \sum_n f_n(Z) A_\mu^{(n)} \quad (21)$$

where  $A_\mu^{(0)}$  is interpreted as a source for the current  $J^\mu$  in gauge theory, and  $A_\mu^{(n)}$  are massive vector mesons that can be created by  $J^\mu$

It is a **unified** description of external sources and dynamical excitations

- An important ingredient for our purposes

Triangle anomaly in 4D  $\longleftrightarrow$  5D Chern-Simons term  $A \wedge F \wedge F$

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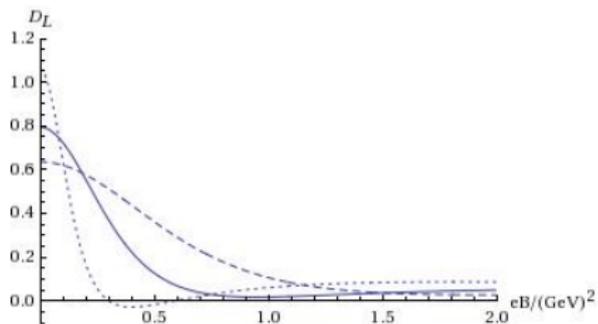
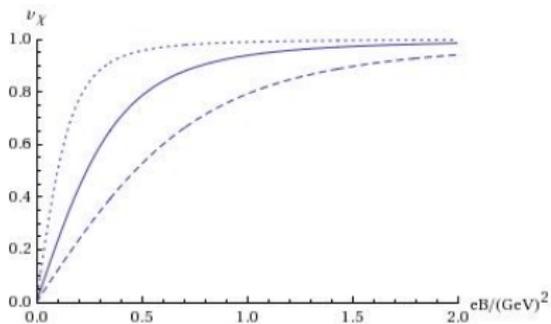
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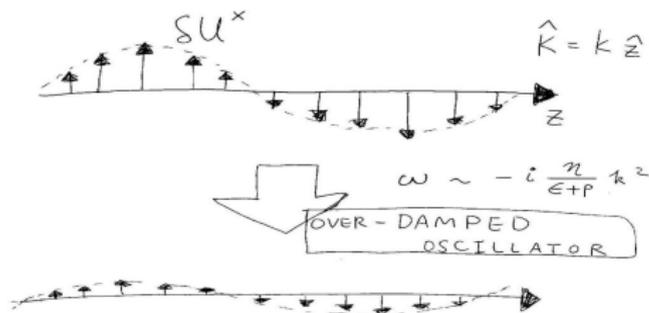
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# Results from Sakai-Sugimoto model



# Our next subject : Chiral Shear Wave

Some pictorial **cartoon** of **damping shear waves**



- **Shear modes** are waves of transverse velocity field  $\delta u^i$  where  $i = x$  or  $i = y$  when the wave-vector is  $\vec{k} = k\hat{z}$
- It is **not** a propagating mode, but exponentially damping
- From the constitutive relation of  $T^{\mu\nu}$ , the **leading order** dispersion relation starts as

$$\omega \approx -\frac{i\eta}{\epsilon + \rho} k^2 + \dots \quad (22)$$

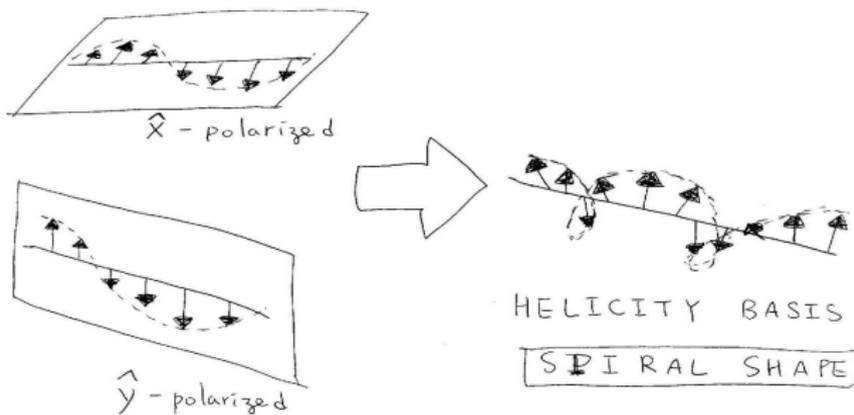
where  $\eta$  is the shear viscosity

# Why anomaly affects this and how ?

**Q** : How come anomaly could affect these transverse shear modes ?

**The basic reason** : In the **charged** plasma, transverse velocity fluctuations  $\delta u^i$  induce **charge current flow** too, i.e. they are coupled with each other dynamically

**Net effect** :  $\delta u^x$  and  $\delta u^y$  mix with each other, and the correct eigen modes are **definite helicity modes**



The two opposite helicity modes turn out to have different **sub-leading** dispersion relations from **anomaly**

# AdS/CFT computation of chiral shear waves

- As a model theory, we take

$$(16\pi G_5)\mathcal{L} = R + 12 - \frac{1}{4}F_{MN}F^{MN} - \frac{\kappa}{4\sqrt{-g_5}}\epsilon^{MNPQR}A_MF_{NP}F_{QR} \quad , \quad (23)$$

and consider a charged black-hole plasma background

- Study linear fluctuations, especially helicity  $\pm 1$  transverse shear modes. One finds that Chern-Simons effect ( anomaly effect) appears only on these modes
- From analyzing these modes in AdS/CFT, one can obtain their dispersion relations. The result depends on **the sign of helicity**, which maybe called **chiral shear waves**
- Our result is

$$\omega \approx -i\frac{\eta}{\epsilon + \rho}k^2 \pm i\frac{\kappa Q^3}{8m^2 r_H^3}k^3 + \mathcal{O}(k^4) \quad : \quad \text{helicity } \pm 1 \quad , \quad (24)$$

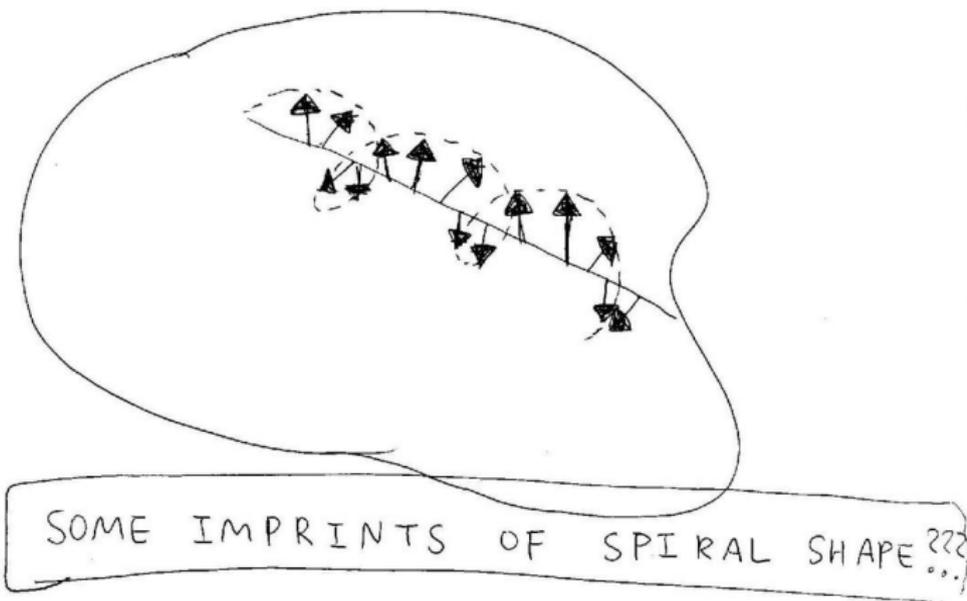
where  $\mathcal{O}(k^3)$  is the chiral term from anomaly. In fact, any term with **odd** powers of  $k$  comes from anomaly

- Domokos-Harvey and Nakamura-Ooguri-Park observed that for a sufficiently large  $\kappa$ , the anomaly-induced term can be big enough to overcome the leading piece, to induce an **instability** toward forming chiral shear waves

# How to observe experimentally ?

- One helicity mode has a larger imaginary frequency than the other, so it would decay faster than the other
- After some time, only one helicity modes should dominate.

**It will look like a SPIRAL SHAPE transverse fluctuation**



# Thank you very much

**Thank you very much for listening**