Calculating the incoherent and total cross-sections in diffractive exclusive vector meson production (and DVCS) in eA

Tobias Toll with T. Ullrich
RIKEN Lunch Seminar
10/27/11

Sartre - A Monte Carlo event generator for diffraction in eA

Calculating the incoherent and total cross-sections in diffractive exclusive vector meson production (and DVCS) in eA

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What we want:
To build a Monte Carlo event generator for an EIC

What exists for eA:
DPM-JetIII - not maintained, no diffraction

Diffraction will play a big role in the EIC eA programme.
No existing MC event generator for this physics.
### e+A Physics Program: Science Matrix

Result of INT workshop in Seattle in fall ’10 (arXiv: 1108.1713)

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<th>Observables</th>
<th>What we learn</th>
<th>Phase-I</th>
<th>Phase-II</th>
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<td>integrated gluon distributions</td>
<td>$F_{2,L}$</td>
<td>nuclear wave function; saturation, $Q_s$</td>
<td>gluons at $10^{-3} &lt; x &lt; 1$</td>
<td>saturation regime</td>
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<tr>
<td>$k_T$ dependent gluons; gluon correlations</td>
<td>di-hadron correlations</td>
<td>non-linear QCD evolution / universality</td>
<td>onset of saturation</td>
<td>measure $Q_s$</td>
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<td>transport coefficients in cold matter</td>
<td>large-x SIDIS; jets</td>
<td>parton energy loss, shower evolution; energy loss mechanisms</td>
<td>light flavors and charm; jets</td>
<td>rare probes and bottom; large-x gluons</td>
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<tr>
<td>$b$ dependence of gluon distribution and correlations</td>
<td>Diffractive VM production and DVCS, coherent and incoherent parts</td>
<td>Interplay between small-x evolution and confinement</td>
<td>Moderate $x$ with light and heavy nuclei</td>
<td>Extend to low-$x$ range (saturation region)</td>
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</tbody>
</table>
Probing the Nucleus at small $x$

At large $x$: large $p^+$, short wavelength in $x^-$, individual nucleons can be resolved.
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At smaller $x$, coherently probe larger area.
Probing the Nucleus at small $x$

At large $x$: large $p^+$, short wavelength in $x^-$, individual nucleons can be resolved.

At smaller $x$, coherently probe larger area.

At $x \ll \frac{A^{-1/3}}{M_N R_p}$ coherently probing the whole nucleus.

Challenge for MC, can not just use “A x Pythia”!!
Measuring the $b$-dependence of gluons in a nucleus

$$t = (p - p')^2 \text{ or } t = (p - Y)^2$$

To be able to measure $t$ with high precision, the whole $X$ system needs to be measured.

Can’t measure $Y$ well, or $p'$ at all.

Need exclusive diffractive processes:
Vector Mesons and DVCS
Measuring the b-dependence of gluons in a nucleus

Gluons - small $x$!

The $b$-dependence is the Fourier conjugate of

$$\Delta = \sqrt{-t}$$

need to measure the $t$-distribution!
Start with $\mathcal{E}$
The Dipole Model

Elastic photon-proton scattering

\[ A_{\gamma^* p}(x, Q, \Delta) = \sum_f \sum_{h, \bar{h}} \int d^2 r \int_0^1 \frac{dz}{4\pi} \Psi^{*}_{h\bar{h}}(r, z, Q) A_{q\bar{q}}(x, r, \Delta) \Psi_{h\bar{h}}(r, z, Q) \]

\[ \Delta \equiv (p'^\mu - p^\mu)_\perp \]

The Dipole Model

\[ \mathcal{A}^{\gamma^*p}(x, Q, \Delta) = \sum_f \sum_{h, \bar{h}} \int d^2r \int_0^1 \frac{dz}{4\pi} \Psi^*_h(r, z, Q) A_{q\bar{q}}(x, r, \Delta) \Psi_{\bar{h}h}(r, z, Q) \]

Use:

Optical theorem:

\[ A_{q\bar{q}}(x, r, \Delta) = \int d^2b \ e^{-ib \cdot \Delta} A_{q\bar{q}}(x, r, b) = i \ \int d^2b \ e^{-ib \cdot \Delta} 2 \left[ 1 - S(x, r, b) \right] . \]

Real Part of S-matrix:

\[ \sigma_{q\bar{q}}(x, r) = \text{Im} A_{q\bar{q}}(x, r, \Delta = 0) = \int d^2b \ 2[1 - \text{Re} S(x, r, b)] \]

Define dipole cross-section:

\[ \frac{d\sigma_{q\bar{q}}}{d^2b} = 2\mathcal{N}(x, r, b) \]
Vector Meson Production

\[ A_{T,L}^{\gamma* p \rightarrow V p} (x, Q, \Delta) = i \int d^2r \int_0^1 \frac{dz}{4\pi} \int d^2b (\Psi^*_V \Psi)_{T,L} e^{-i([1-z]r+b) \cdot \Delta} \frac{d\sigma_{q\bar{q}}}{d^2b} \]

\[ \Delta \equiv (p'^\mu - p^\mu)_\perp \]

“Known from QED”

Needs to be modeled
Vector Meson Production

\[ A_{T,L}^{\gamma^* p \rightarrow V p}(x, Q, \Delta) = \]
\[ i \int d^2 r \int_0^1 \frac{dz}{4\pi} \int d^2 b (\Psi^*_V \Psi)_{T,L} e^{-i([1-z]r+b) \cdot \Delta} \frac{d\sigma_{q\bar{q}}}{d^2 b} \]

“Known from QED”

Needs to be modeled
The b-Sat Model

A model with multiple scatterings.
No gluon-gluon recombinations!

\[ \frac{d\sigma_{q\bar{q}}}{d^2b} = 2 \left[ 1 - \exp \left( -\frac{\pi^2}{2N_c} r^2 \alpha_s(\mu^2) x g(x, \mu^2) T(b) \right) \right] \]
The b-Sat Model

\[ \frac{d \sigma_{q\bar{q}}}{d^2 b} = 2 \left[ 1 - \exp \left( -\frac{\pi^2}{2N_c} r^2 \alpha_s(\mu^2) x g(x, \mu^2) T(b) \right) \right] \]

\[ \mu^2 = \frac{4}{r^2} + \mu_0^2 \]
The b-Sat Model

\[ T_G(b) = \frac{1}{2\pi B_G} e^{-\frac{b^2}{2B_G}} \]

\[ \frac{d\sigma_{q\bar{q}}}{d^2b} = 2 \left[ 1 - \exp \left( -\frac{\pi^2}{2N_c} r^2 \alpha_s(\mu^2) x g(x, \mu^2) T(b) \right) \right] \]
First comparison with data

Exclusive electroproduction of J/Psi mesons at HERA Nuc. Phys. B695

Black Curve: XDVMP b-CGC
Red Curve: Black Curve x 1.5

Something is missing!!

Plots produced by Ramiro Debbe
Real Amplitude Corrections

So far the amplitude has been assumed to be purely imaginary.

To take the Real part of the amplitude into account it can be multiplied by a factor \((1 + \beta^2)\).

\(\beta\) is the ratio Real/Imaginary parts of the Amplitude:

\[
\beta = \tan \left( \lambda \frac{\pi}{2} \right)
\]

\[
\lambda \equiv \frac{\partial \ln \left( \mathcal{A}_{T,L}^{\gamma^* p \rightarrow E_p} \right)}{\partial \ln(1/x)}
\]

This goes bad for large \(x \sim 10^{-2}\).
Real Amplitude Corrections

\[ \beta = \tan \left( \lambda \frac{\pi}{2} \right) \]

\[ \lambda \equiv \frac{\partial \ln \left( A_{T,L}^{\gamma^*p \rightarrow Ep} \right)}{\partial \ln (1/x)} \]
Skewedness Corrections

The two gluons carry different momentum fractions

This is the Skewed effect

In leading $\ln(1/x)$ this effect disappears

It can be accounted for by a factor $R_g$

$$ R_g(\lambda) = \frac{2^{2\lambda+3}}{\sqrt{\pi}} \frac{\Gamma(\lambda + 5/2)}{\Gamma(\lambda + 4)} $$

Again, this goes bad for large $x \sim 10^{-2}$!

Implemented with exponential damping to control this.
Some $e\pi$ results using tables

Exclusive electroproduction of $J/\psi$ mesons at HERA

ZEUS Collaboration

$\chi^2/\text{ndf} = 0.62$
Some $\bar{e}p$ results using tables

Measurement of Deeply Virtual Compton Scattering and its $t$-dependence at HERA

$\chi^2/\text{ndf} = 5.36$

$\chi^2/\text{ndf} = 1.81$
Going from $e^p$ to $e^A$
Going from $e\bar{p}$ to $eA$

**$e\bar{p}$:**
\[
\text{Re}(S) = 1 - \mathcal{N}^{(p)}(x, r, b) = 1 - \frac{1}{2} \frac{d\sigma^{(p)}_{q\bar{q}}(x, r, b)}{d^2b}
\]

**$eA$:** Independent scattering approximation

\[
1 - \mathcal{N}^{(A)} = \prod_{i=1}^{A} \left(1 - \mathcal{N}^{(p)}(x, r, |b - b_i|)\right)
\]

Assume the Woods-Saxon distribution

**$b\text{Sat}$:**
\[
\frac{d\sigma_{q\bar{q}}^A}{d^2b} = 2 \left[ 1 - \exp \left( -\frac{\pi^2}{2N_c} r^2 \alpha_s(\mu^2) x g(x, \mu^2) \sum_{i=1}^{A} T_p(b - b_i) \right) \right]
\]
Generating a Nucleus

Generate radii according to the Woods-Saxon distribution

\[ \rho(r) = \frac{\rho_0}{1 + e^{\frac{r - R_0}{a}}} \]

\[ \frac{d^3 N}{d^3 r} \]

First generate according to r:

\[ \frac{dN}{dr} = 4\pi r^2 \rho(r) \]

Then generate angular distributions uniform in \( \phi \) and \( \cos(\theta) \)

This is done with a condition that two nucleons can not be within a core distance of \( \sim 0.8\text{fm} \).

If they are: regenerate angles (not radius!)
Generating a Nucleus

Lead 208 in the r-phi plane, each nucleon is supplemented with a Gaussian width (bSat).
Going from $ep$ to $eA$

Another difference in $eA$:
The Nucleus can break up into colour neutral fragments!

When the nucleus breaks up, the scattering is called incoherent

When the nucleus stays intact, the scattering is called coherent

Total cross-section = $\text{incoherent} + \text{coherent}$
Incoherent Scattering

Nucleus dissociates \((f \neq i)\):

\[
\sigma_{\text{incoherent}} \propto \sum_{f \neq i} \langle i | A | f \rangle^\dagger \langle f | A | i \rangle
\]

\[
= \sum_{f} \langle i | A | f \rangle^\dagger \langle f | A | i \rangle - \langle i | A | i \rangle^\dagger \langle i | A | i \rangle
\]

\[
= \langle i | |A|^2 | i \rangle - |\langle i | A | i \rangle|^2 = \langle |A|^2 \rangle - |\langle A \rangle|^2
\]

The incoherent CS is the variance of the amplitude!!

\[
\frac{d\sigma_{\text{total}}}{dt} = \frac{1}{16\pi} \langle |A|^2 \rangle
\]

\[
\frac{d\sigma_{\text{coherent}}}{dt} = \frac{1}{16\pi} |\langle A \rangle|^2
\]
Defining the average

\[
\frac{d\sigma_{\text{total}}}{dt} = \frac{1}{16\pi} \left| \langle |A|^2 \rangle \right|_\Omega
\]

\[
\frac{d\sigma_{\text{coherent}}}{dt} = \frac{1}{16\pi} |\langle A \rangle_\Omega|^2
\]

Define average:

\[
\langle \mathcal{O} \rangle_\Omega \approx \frac{1}{C_{\text{max}}} \sum_{j=1}^{C_{\text{max}}} \mathcal{O}(\Omega_j)
\]

\[
\mathcal{A}(\Omega_j) = \int dr \frac{dz}{4\pi} d^2b (\Psi^*_V \Psi)(r, z) 2\pi r b J_0([1 - z]r \Delta) e^{-ib \cdot \Delta} \frac{d\sigma_{q\bar{q}}}{d^2b}(x, r, b, \Omega_j)
\]

4 four-dimensional integrations for each phase-space point and configuration

How many configurations???
Convergence of sum:

Need ~1000 configurations to describe 5th minimum!!

\[ Q^2 = 10^{-4} \]
\[ x_{IP} \approx 0.006 \]
Problem with convergence of distribution at large $|t|$: Average (coherent) $\ll\ll\ll\ll\ll\ll$

Variance (incoherent)

Or: At large $|t|$ the nucleus is probed at a smaller scale. $\Delta = \sqrt{-t}$ is the Fourier conjugate of $b$. 

$Q^2 = 10$ GeV$^2$

$x_{IP} = 5 \times 10^{-3}$
Problem with convergence of distribution at large $|t|$:  

Average (coherent)  

Variance (incoherent)  

Solution  

Calculate the average from:  

$$  \left\langle \frac{d\sigma_{q\bar{q}}}{d^2b} \right\rangle_\Omega = 2 \left[ 1 - \left(1 - \frac{T_A(b)}{2} \sigma_{qq}^{(p)} \right)^A \right] $$

An Impact parameter dipole saturation model - Kowalski, Henri & Derek Teaney  
Some eA results w/o tables

Note: the b-distribution one gets by Fourier transform of the coherent $t$-distribution.

The incoherent distribution contains all nucleon correlations in the nucleus - very interesting in itself!!
Generating events
How Sartre works

4 four-dimensional integrations for each phase-space point and configuration

$\sim 1600$ 4D integrals/point

Use 3D lookup tables in $Q^2, W^2, t$ independent of $s$ and use the Open Science Grid to produce the tables.

Four tables to create a cross-section point:

$$
\langle |A_T|^2 \rangle, \langle A_T \rangle, \langle |A_L|^2 \rangle, \langle A_L \rangle
$$

$$
\frac{d^3 \sigma}{dQ^2 dW^2 dt} = f_T^\gamma \langle |A_T|^2 \rangle + f_L^\gamma \langle |A_L|^2 \rangle
$$

Transverse if:

$$
\frac{f_T^\gamma \langle |A_T| \rangle}{f_T^\gamma \langle |A_T| \rangle + f_L^\gamma \langle |A_L| \rangle} > R
$$

Breakup if:

$$
\frac{|\langle A_T \rangle|^2 - \langle |A_T|^2 \rangle}{|\langle A_T \rangle|^2} > R
$$
How Sartre works

Table Generator

User Settings
(s, A, table range, beam, ...)

Model Parameter

Nucleus Model

Dipole Model

Numerics

Tables for:
\langle T \rangle, \langle T^2 \rangle, \langle L \rangle, \langle L^2 \rangle \nu 3D: t, Q^2, W^2

Thursday, October 27, 2011
How Sartre works

Table Generator

User Settings
(s, A, table range, beam, ...)

Model Parameter

Nucleus Model

Dipole Model

Numerics

Tables for:
<T>, ⟨T²⟩, ⟨L⟩, ⟨L²⟩ ν 3D: t, Q², W²

Event Generator

User Settings
(A, kinematic range, beam, # of events, dipole model, ...)

Cross-Section Calculation

\[ \frac{d^3\sigma}{dt\,dQ^2\,dW^2} \]

PDF

UNU.RAN
3D Random Generator

t, Q², W²

Final State Generator

Event Record

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Detecting Nuclear Breakup

- Detecting all fragments $p_{A'} = \sum p_n + \sum p_p + \sum p_d + \sum p_\alpha \ldots$ not possible
- Focus on n emission
  - Zero-Degree Calorimeter
  - Requires careful design of IR

Additional measurements:
- Fragments via Roman Pots
- $\gamma$ via EMC

Traditional modeling done in pA:

Intra-Nuclear Cascade
- Particle production
- Remnant Nucleus (A, Z, E*, ...)
- ISABEL, INCL4

De-Excitation
- Evaporation
- Fission
- Residual Nuclei
- Gemini++, SMM, ABLA (all no $\gamma$)
Experimental Reality

Here eRHIC IR layout:

Need ±X mrad opening through triplet for \( n \) and room for ZDC

Big questions:
- Excitation energy \( E^* \)?
- \( \text{ep: } d\sigma/M_\gamma \sim 1/M_\gamma^2 \)
- \( eA? \) Assume \( \text{ep} \) and use \( E^* = M_\gamma - m_p \) as lower limit
Final notes:

When presented like this, things seem quite straightforward and simple,

BUT, don’t forget:
Final notes:

The First Commandment of Event Generation

Thou shalt never believe event generation is easy
Final notes:

We’ve had (and still have) a plethora of technical and numerical problems:

Real and skewedness corrections can be tweaked to better describe the cross-sections

\[(1 + \beta^2)R_g\] 

\[(1 + \beta^2)\]
Final notes:
We’ve had (and still have) a plethora of technical and numerical problems:

Linear interpolation $->$ a bias to small values, switched to a polynomial interpolation, need to adjust the parameters thoroughly.

Linear

Polynomial

(true-interpolation)/true
Final notes:

We’ve had (and still have) a plethora of technical and numerical problems:

Using UNU.RAN to generate events from the distribution. This has to be set-up with the maximum value in the distribution. It’s been a lot of cooking and trial and error to find a reliable method for this.
Final notes:

We’ve had (and still have) a plethora of technical and numerical problems:

Spikes in the distribution!!
Each phase-space point is the result of 1600 4d integrals.
In a few % of the points, there is a spike.
This will ruin the MC-generation, unless controlled!

These spikes comes from only a few integrals. I may have just found a way to identify the rotten eggs and exclude them.
Summary and outlook

We have developed a method to calculate exclusive diffractive vector meson production and DVCS in eA collisions.

We are currently implementing it in a Monte Carlo event generator called Sartre. So far it has only been tested for ep, and describes the data well.

Sartre can also be extended to the general diffractive process:

\[ e + A \rightarrow e' + X + A' \]
Summary and outlook

We have developed a method to calculate exclusive diffractive vector meson production and DVCS in $eA$ collisions.

We are currently implementing it in a Monte Carlo event generator called Sartre. So far it has only been tested for $ep$, and describes the data well.

Sartre can also be extended to the general diffractive process:

$$e + A \rightarrow e' + X + A'$$

Thank you!
BACKUP
The ten commandments of event generation:

1. Thou shalt never believe event generation is easy
2. Thou shalt always cover the whole of phase space
3. Thou shalt never assume that a jet is a parton or a jet
4. Thou shalt never double-count emissions
5. Thou shalt always remember that an NLO generator does not always produce NLO results
6. Thou shalt always be independent of Lorentz frame
7. Thou shalt always conserve energy and momentum
8. Thou shalt always resum when NLO corrections are large
9. Thou shalt not be afraid of parameters
10. Thou shalt only have nine commandments of event generation

By Leif Lönnblad
(1 + \beta^2) R_g

\text{skewedness x real amplitude}

\lambda

Thursday, October 27, 2011