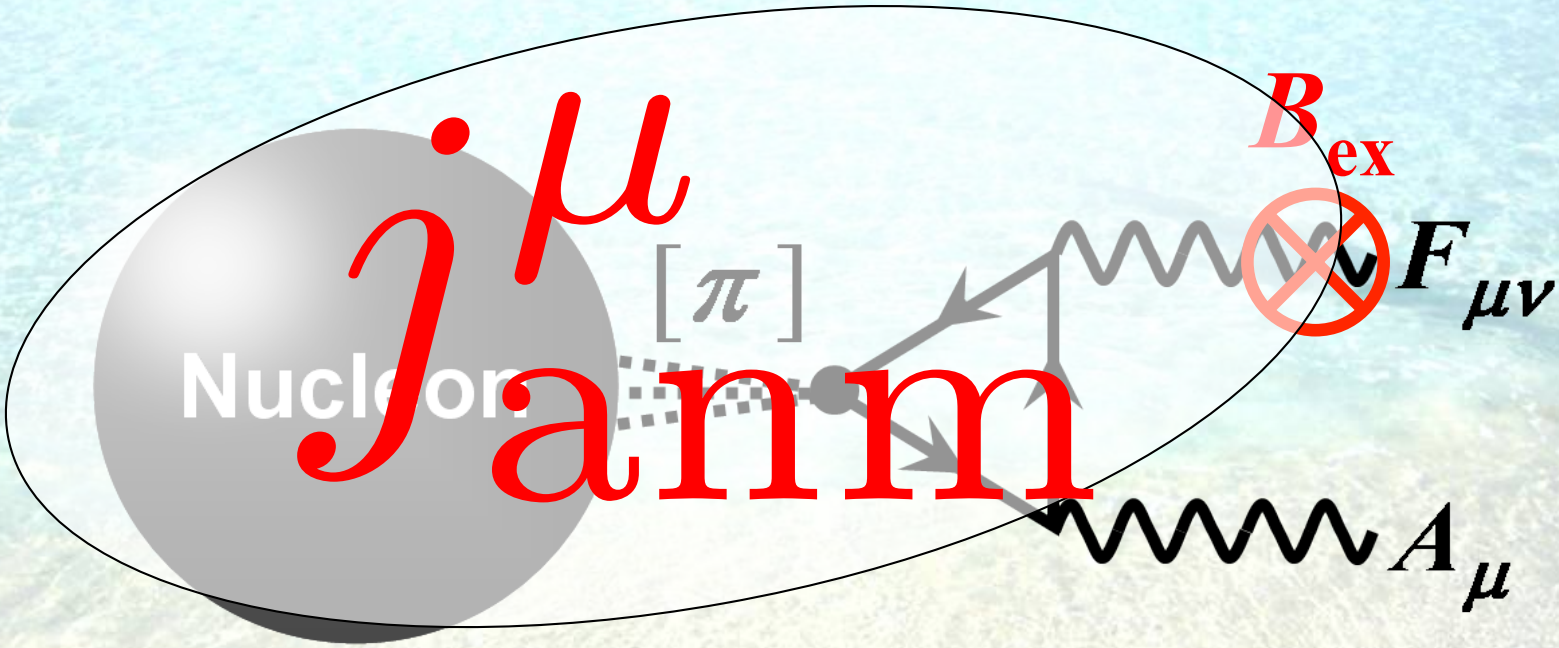
Anomaly-induced charges in nucleons

Yu Maezawa (BNL)



Skyrme model + **WZW** action

→ **Electric charge** and quadrupole moment induced under **external magnetic field**

Eto, Hashimoto, Iida, Ishii, Y.M. arXiv: 1103.5443, 1109.0020

Anomaly-ind… …ucleons

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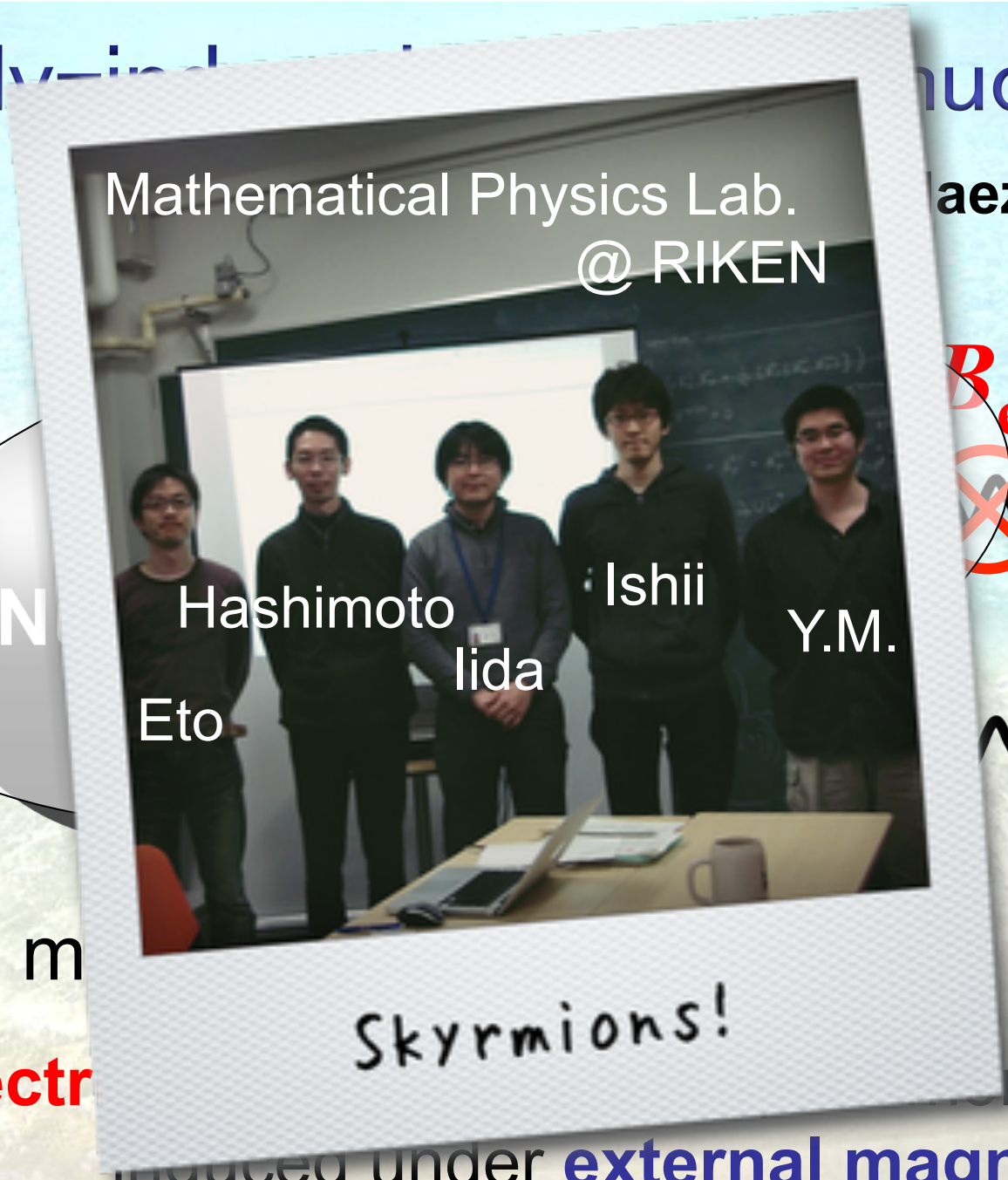


B_{ex}

$\sim F_{\mu\nu}$

N

$\sim A_\mu$

Skyrme m…

→ **Electr**…ment

…induced under **external magnetic field**

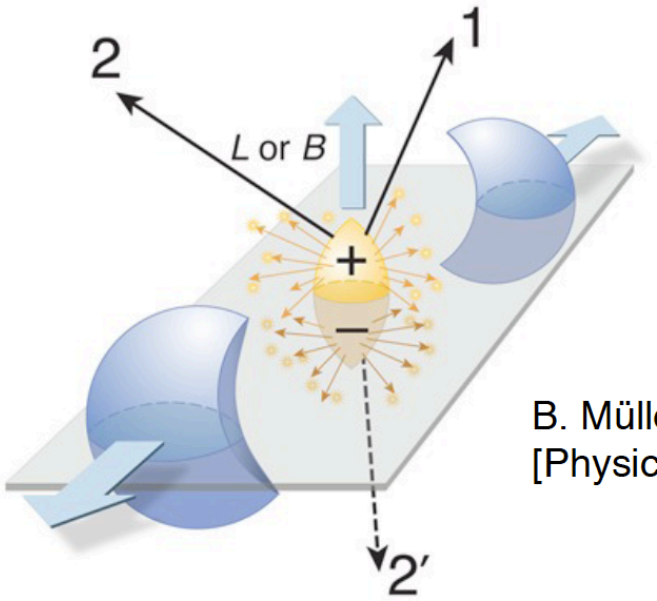
Eto, Hashimoto, Iida, Ishii, Y.M. arXiv: 1103.5443, 1109.0020

Contents

- **Introduction**
- **Skyrm & Wess-Zumino-Witten action**
  - ➡ EM current (gauge inv. & conserved)
- **Induced charge by external magnetic field**
- **Where the charge comes from?**
  - ➡ Pion cloud effect cf) Kharzeev et al., PRD84(2011)037503
- **Discussion**
  - ✓ Relation to other phenomena?
  - ✓ Artificial problems?

Introduction

Chiral Magnetic Effect



B. Müller
[Physics 2, 104, 2009]

- Parity violation in heavy ion collision: $\boldsymbol{E}\cdot\boldsymbol{B}$
- Very strong magnetic field: $B \sim 10^{14-15}[G]$

Chiral anomaly (WZW term) with B_{ex}

+ Nucleon (Skyrm model)

Skyrme model ($N_f = 2$)

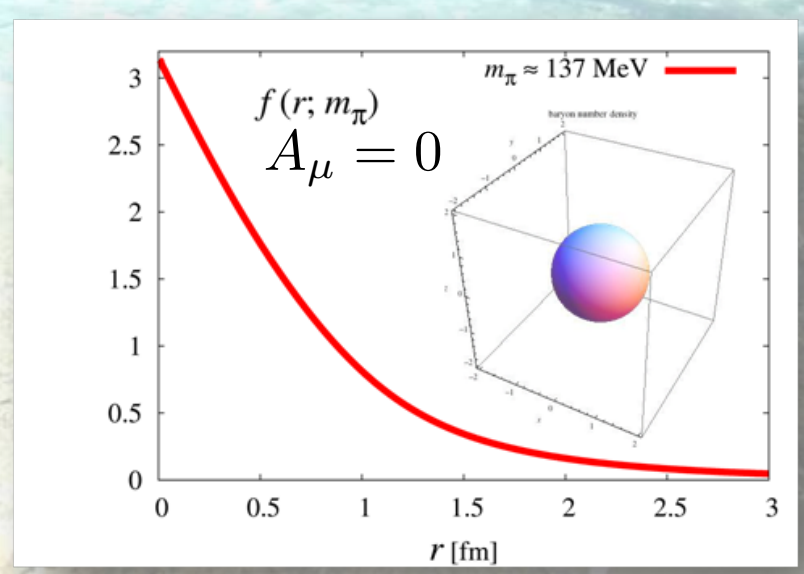
Nucleon (Skyrmion) = Topological soliton of pions

$$S_{\rm SKM} = \int d^4x (\mathcal{L}_{\rm kin} + \mathcal{L}_{\rm mass} - \tfrac{1}{4} F_{\mu\nu} F^{\mu\nu})$$

$$R_\mu = D_\mu U U^\dagger$$

$$\mathcal{L}_{\rm kin} = -\frac{F_\pi^2}{16} {\rm tr}(R_\mu R^\mu) + \frac{1}{32 e_s^2} {\rm tr}([R_\mu, R_\nu][R^\mu, R^\nu])$$

$$\mathcal{L}_{\rm mass} = \frac{m_\pi^2 F_\pi^2}{16} {\rm tr}(U + U^\dagger - 2)$$



Pion field (hedgehog ansatz)

$$U = e^{2i\pi\cdot\tau/F_\pi} = e^{if(r)\hat{\mathbf{x}}\cdot\tau}$$

Reproduce several quantities (F_π, g_A, …) within 30% errors

[Adkins-Nappi-Witten, 1983]

Wess-Zumino-Witten action (Nf = 2)

$$S_{\rm WZW} = -\int d^4x \frac{e}{2} j_B^\mu A_\mu$$

$$j_B^\mu = \frac{1}{24\pi^2}\epsilon^{\mu\nu\rho\sigma}{\rm tr}[R_\nu R_\rho R_\sigma] - \frac{e}{16\pi^2}\epsilon^{\mu\nu\rho\sigma}\partial_\nu(A_\rho P_\sigma)$$

$$P_\sigma \equiv i{\rm tr}[\tau_3(U^\dagger D_\sigma U + D_\sigma U U^\dagger)]$$

- **First term: baryon number**

$$N_B = \int d^3x \frac{1}{24\pi^2}\epsilon^{ijk}{\rm tr}[R_i R_j R_k]$$

- **Second term: chiral anomaly of QCD**

$$-\frac{e^2}{16\pi^2 F_\pi}\epsilon^{\mu\nu\rho\sigma}A_\mu F_{\nu\rho}\partial_\sigma\pi^0 + \mathcal{O}(F_\pi^{-3}) = -\frac{e^2}{4\pi^2 F_\pi}\pi^0 \vec{E}\cdot\vec{B} + \mathcal{O}(F_\pi^{-3})$$

$\pi_0 \to 2\gamma$

✸ j^μ_B **is gauge invariant**

Anomalous current and total charge

- Decomposition: $A_\mu = \bar{A}_\mu + a_\mu$ (background, fluctuation)

EM current : gauge invariant & conserved

$$j^\mu_{\mathrm{em,WZW}} = \delta S_{\mathrm{WZW}}/\delta a_\mu$$

$$= \epsilon^{\mu\nu\rho\sigma}\left[\frac{e}{48\pi^2}\mathrm{tr}[R_\nu R_\rho R_\sigma] + \frac{e^2}{32\pi^2}\partial_\sigma(\bar{A}_\rho P_\nu)\right] + j^\mu_{\mathrm{anm}}$$

$$j^\mu_{\mathrm{anm}} = -\frac{e^2}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}\bar{F}_{\nu\rho}P_\sigma$$ **: anomaly current**

Total charge

$$Q_e = \int d^3x\,(j^0_V + j^0_{\mathrm{em,WZW}}) = e(I_3 + \tfrac{1}{2}N_B) + Q_{\mathrm{anm}}$$

"Gell-Mann-Nishijima formula"

Anomalous current with Skyrmion

Perturbative expansion of e,

$$U = \exp\left(i\vec{\tau}\cdot(\vec{f}_0 + \vec{f}_1 + \cdots)\right),$$

$$A_\mu = A_\mu^{(0)} + A_\mu^{(1)} + \cdots .$$

$$P_\sigma \equiv i\mathrm{tr}[\tau_3(U^\dagger\partial_\sigma U + \partial_\sigma U U^\dagger)]$$

Leading: Static solution w/o EM interaction

$$U = e^{if(r)\hat{\mathbf{x}}\cdot\tau} \quad \Rightarrow \quad j^\mu_{\mathrm{anm}} = -\frac{e^2}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}\bar{F}_{\nu\rho}P_\sigma$$

Soliton quantization: "slowly rotating" in moduli space

$$U(x) \to g(t)U(x)g^\dagger(T), \quad g(t) \in \mathrm{SU}(2_{\mathrm{F}}) \simeq S^3$$

Anomaly-current in external magnetic field ($2\boldsymbol{B}^i = \boldsymbol{\varepsilon}^{ijk}\boldsymbol{F}_{jk}$)

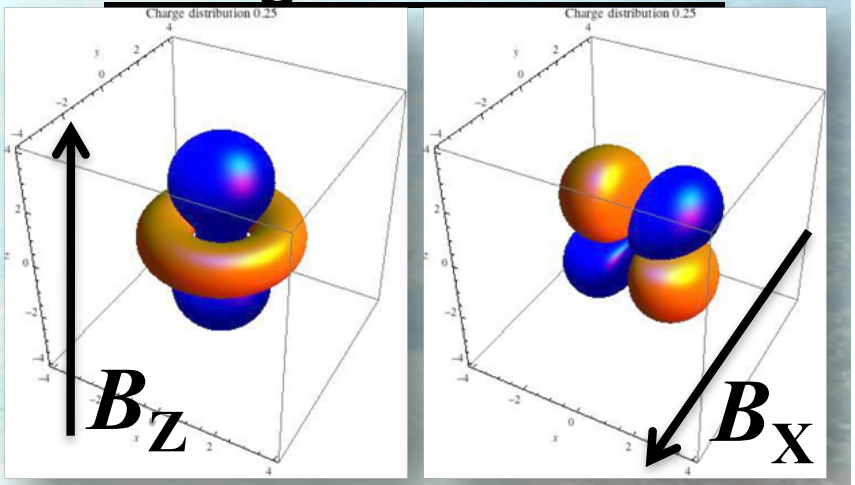
$$\langle j^0_{\mathrm{anm}}\rangle_{I_3,S_3} = \frac{i}{32\pi^2}e^2 B_i\langle P_i\rangle_{I_3,S_3}, \quad \langle j^i_{\mathrm{anm}}\rangle = 0$$

$$\langle P_0\rangle_{I_3,S_3} = 0, \quad \langle P_i\rangle_{I_3,S_3} = \tfrac{16i}{3}(e_s F_\pi)I_3 S_3\left[\left(f' - \frac{\sin(2f)}{2r}\right)\hat{x}_i x_3 + \frac{\sin(2f)}{2r}\delta_{i3}\right]$$

Induced charge by external magnetic field

$$\langle j^0_{\rm anm}\rangle_B \neq 0, \quad \langle j^i_{\rm anm}\rangle_B = 0$$

Charge distribution



Anomalous charge

$$Q_{\rm anm} = \int d^3x \langle j^0_{\rm anm}\rangle$$

$$= e\frac{4}{9\pi} I_3 S_3 \frac{eB_Z}{(e_s F_\pi)^2} c_0$$

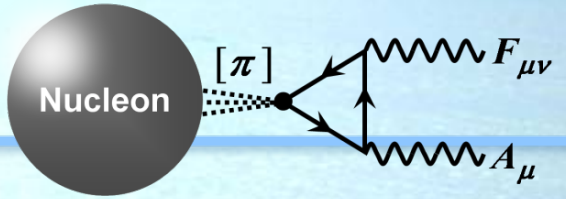
Numerical coefficient : $c_0 = \int_0^\infty dr \left[r^2 f' + \sin(2f)\right] \sim -10.2$

Dipole moment = 0, Quadrupole mom. ≠ 0

Magnitude $Q_{\rm anm}/e \sim 10^{-20}[{\rm G}^{-1}] \times I_3 S_3 \times B\,[{\rm G}]$

Magnetor (nutron star): B = 10^{15} [G], Heavy-ion collision: B = 10^{17} [G]

Where the charge comes from?



Pion expansion

$$U = e^{2i\pi\cdot\tau/F_\pi} = e^{if(r)\hat{\mathbf{x}}\cdot\tau}$$

$$j^\mu_{\rm anm} A_\mu \sim {\rm tr}[\tau_3 U^\dagger \partial U] AF \sim \underline{\partial\pi_0 AF} + \underline{\pi\pi\partial\pi AF} + \cdots$$

$\pi_0 \to 2\gamma$ **Multi-pion effect (pion cloud)**

Contribution from $\pi_0 \to 2\gamma$ to $Q_{\rm anm}$: $\pi_0 \sim f(r)\hat{x}_3$

$$\int d^3x \partial\pi_0 = \frac{4\pi}{3}\int_0^\infty dr[r^2 f' + 2rf] = \frac{4\pi}{3}\left[r^2 f\right]_0^\infty = 0$$ **: vanish**

Induced charge carried by ***pion cloud*** surrounding nucleons

c.f.) Kharzeev-Yee-Zahed, PRD84(2011)037503

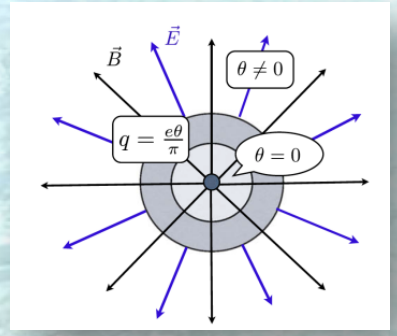
$\pi_0 \to 2\gamma$ Lagrangian: calculation $j_{\rm anm}$

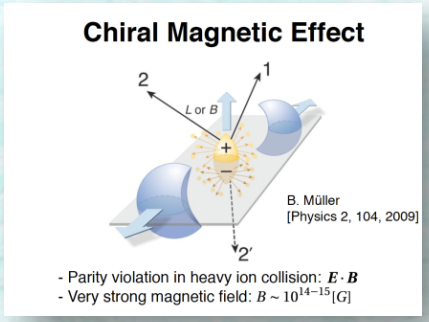
$$\mathcal{L}_{\rm anm} \sim e^2 N_c \pi^0 \vec{E}\cdot\vec{B}$$

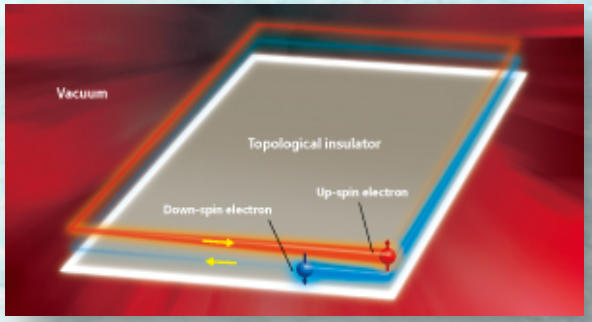
→ Induced charge = 0
Quadrupole ≠ 0

Relation to other phenomena?

Witten effect, chiral magnetic effect, topological insulator…







…described by $\theta F\tilde{F}$

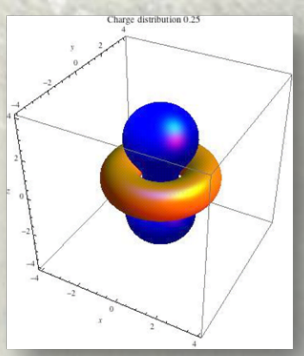
e.g.) CME: $\theta F_{\mu\nu}\tilde{F}^{\mu\nu} \longrightarrow -\epsilon^{\mu\nu\rho\sigma}(\partial_\mu\theta)A_\nu F_{\rho\sigma}$

$$j^\mu = -\epsilon^{\mu\nu\rho\sigma}(\partial_\nu\theta)F_{\rho\sigma} \xrightarrow{\theta=\theta(t)} j^i = -(\partial_0\theta)B^i$$

<u>Anomaly-induced charge</u>

$$j^\mu_{\rm anm} = -\frac{e^2}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}\bar{F}_{\nu\rho}P_\sigma$$

$$\Rightarrow \langle j^0_{\rm anm}\rangle_B \neq 0, \quad \langle j^i_{\rm anm}\rangle_B = 0$$



Anomaly-induced charges by external mag. filed

Can you believe?? artificial problems

- Uniform background magnetic field?

  Magnetic field should close — Induced by circular current

- Cancelation due to deformation of Skyrmion?

  Leading-order calc. up to O(eB) — No cancelation in pertavative discussion

  … back reaction (future)

- Skyrme model and WZW action?

  Reproduce chiral anm. and nucleons — e.g.) Nucleon in Lattice QCD with mag. field (future)

- Charge conservation violated?

  Gauge inv. j_{anm} used in static manner

  What's happen when ex. field is turned on?

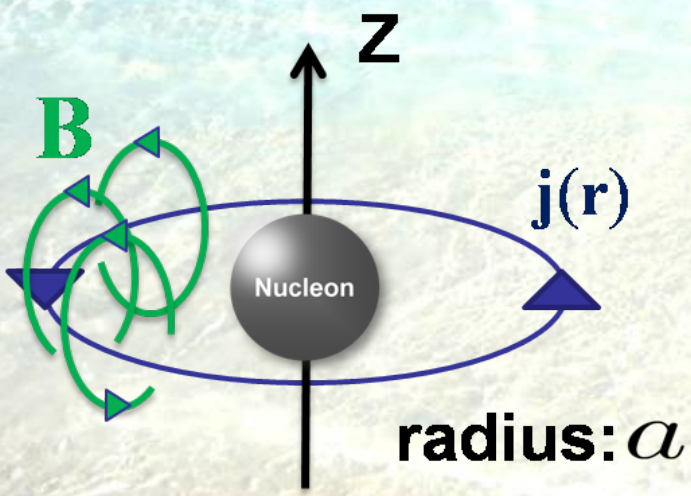
  Time evolution of quantized Skyrmion with gauge field…

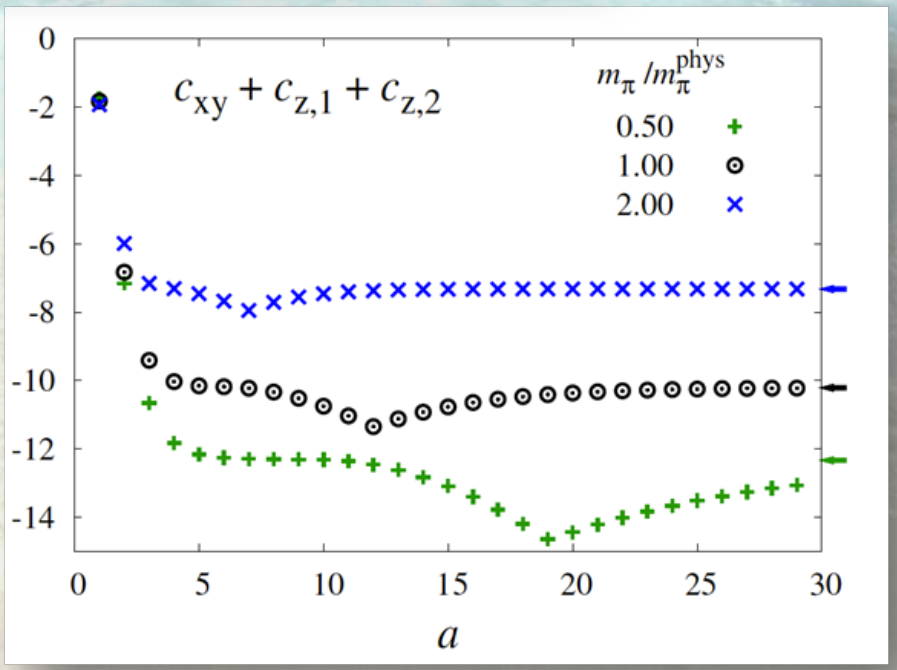
Uniform magnetic field … ill defined?

Induced charge in circular electric current

$$\mathbf{j}(\mathbf{r}) \equiv \frac{j_0 a}{2\pi}\delta(z)\delta(\sqrt{x^2+y^2}-a)(-\sin\zeta, \cos\zeta, 0)$$

$$Q_{\rm anm} = \frac{4eN_c}{27\pi}(I_3 S_3)\frac{ej_0}{(e_s F_\pi)^2}(c_{xy}+c_{z,1}+c_{z,2})$$





… Q_{anm} induced for circular current

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Cancelation due to deformation of Skyrmion?

$$Q = e\left(I_3 + \tfrac{1}{2}N_B\right) + Q_{\rm anm}, \quad Q_{\rm anm} = \mathcal{O}(e^2 B)$$

- Two possible sources of cancellation due to deformation
  1. I_3
  2. N_B : topological charge ⟶ no correction by $B_{\rm ex}$

- Modification of wave functions

$$|l = 1/2\rangle \;=\; |l = 1/2\rangle_0 + eB_3 \sum_{n=1}^{\infty} \frac{V_{l=1/2,\, l=n+1/2}}{E_{l=1/2} - E_{l=n+1/2}} |l = n + 1/2\rangle_0 + \mathcal{O}((eB)^2).$$

Thus $_0\langle n + 1/2|I_3|1/2\rangle_0 = 0$ ⟹ $\langle I_3\rangle = \langle I_3\rangle_0 + \mathcal{O}((eB)^2)$

…can not cancel $Q_{\rm anm}$

Justification: back reaction…(future)

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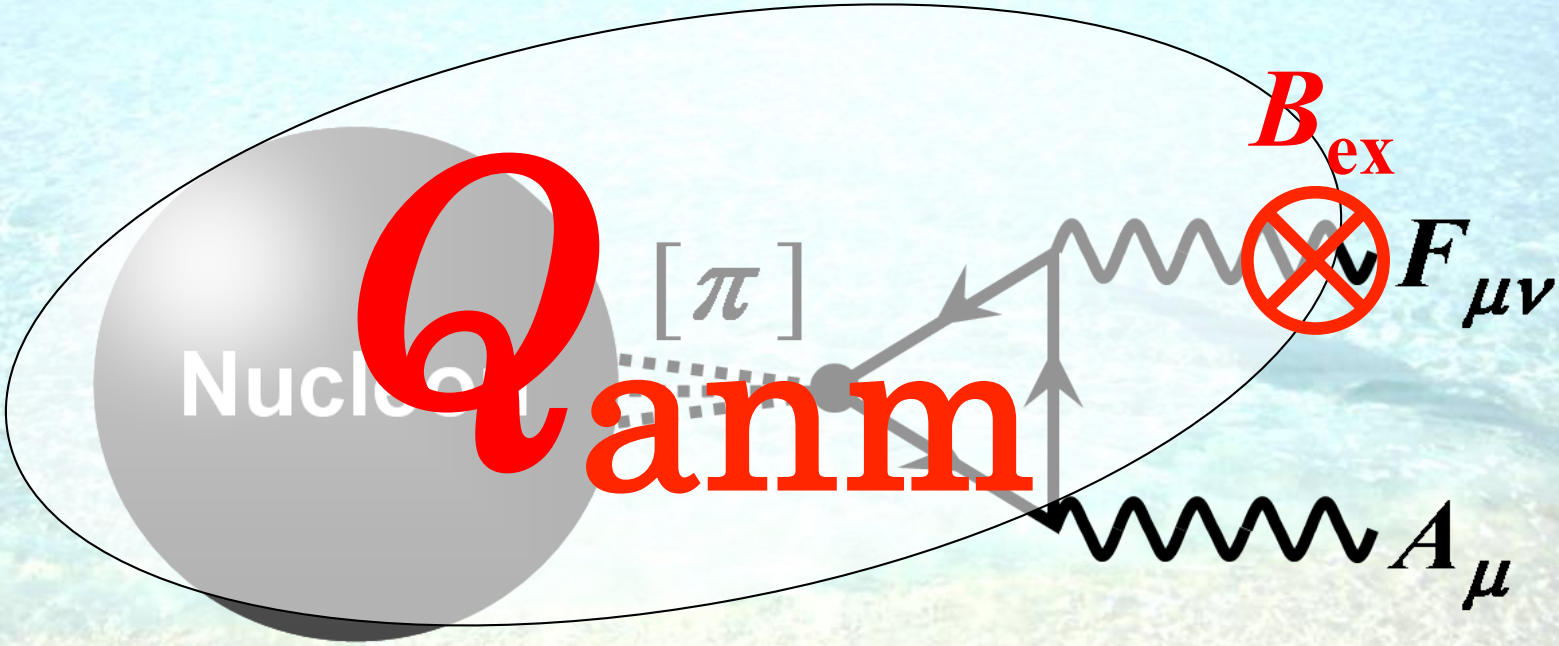
Anomalous charge induced!



THANK YOU
FOR YOUR ATTENTION!