The Spin of Holographic Electrons at Nonzero Temperature and Density

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Plan for the Talk

- General motivation, quantum phase transitions, and strange metals.
- Holographic electrons and the role of spin.
  - The importance of spin-orbit coupling.
  - Spin, damping, and black hole quasinormal modes.

work with my student Jie Ren

Quantum Phase Transition:
a phase transition between different quantum phases (phases of matter at $T = 0$). Quantum phase transitions can only be accessed by varying a physical parameter — such as magnetic field or pressure — at $T = 0$.

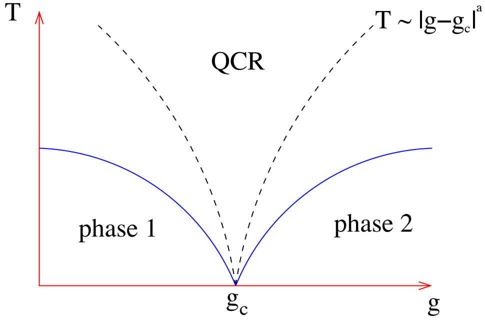


Figure: Phase diagram paradigm

Experimental relevance

Many important physical systems may have quantum critical points (QCPs). The QCP has an effective field theory description which continues to be valid at small "distances" away from the QCP. This quantum critical region may be in an experimentally accessible regime.

Examples:

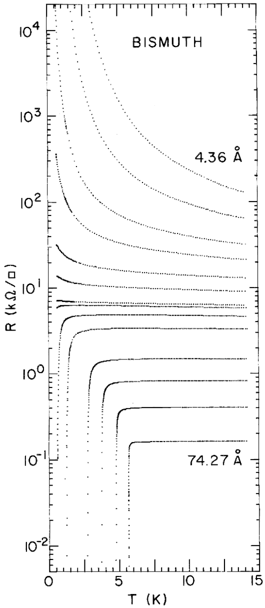
- superfluid-insulator transition in thin films
- heavy fermion compounds
- high temperature, under-doped superconductors at $T > T_c$
- quark-gluon plasma

Systems for which T is the dominant scale.

Thin Films

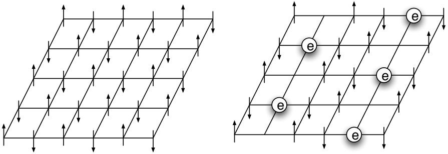
Conductivity σ
$\sigma_{\mathrm{thick}}(T \to 0) = \infty$
$\sigma_{\mathrm{thin}}(T \to 0) = 0$

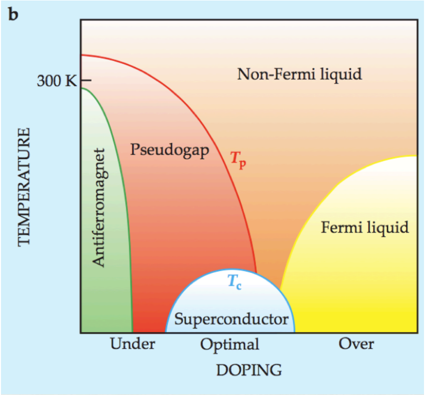
Haviland, Liu, and Goldman, *Phys. Rev. Lett.*, **62**, 2180 (1989)



High T_c Superconductors

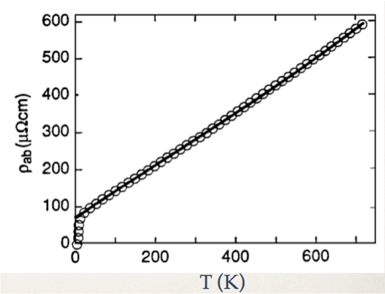
- La_2CuO_4 is an antiferromagnetic insulator
- 2d physics: The Cu atoms arrange themselves into a square lattice on separated sheets.
- Hole doping: substitute some of the La with Sr, $La_{2-x}Sr_xCuO_4$
- The over doped region is weakly interacting. The under doped region is strongly interacting.





Linear Resistivity

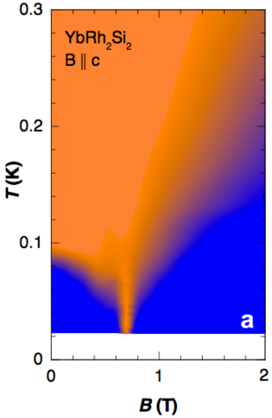
- High T_c superconductors exhibit a linear rise in the resistivity for $T > T_c$ in the strange metal regime that has eluded explanation.
- A Fermi liquid, i.e. a weakly interacting system of electrons, would not yield this behavior.
- On the right is a plot for the high T_c compound Bi-2201.



Martin, Fiory, Fleming, et al., *Phys. Rev.* **B41** 846 (1990).

Heavy Fermions

- Heavy fermion compounds contain rare earth elements that form a lattice of localized magnetic moments.
- A lattice version of the Kondo effect gives the electrons a very large effective mass.
- By tuning pressure, magnetic fields or doping, one can produce superconductivity, typically at very low T.
- There is also a strange metal region of the phase diagram.



The orange region has linear in T resistivity. The blue region has T^2.
Custers, Gegenwart, Wilhelm et al., *Nature* **424**, 524 (2003)

Comments about Scale Invariance

At the quantum critical point, the system is invariant under

$$t \to \lambda^z t \qquad \text{and} \qquad x \to \lambda x \ .$$

where z is the dynamical critical exponent.

- The Lorentzian case $z = 1$:
  - insulating quantum antiferromagnets (relevant for high T_c)
  - Bose Hubbard-like models at p/q filling (optical lattices)
- The case $z = 2$ is more common (Galilean, Schrödinger, and Lifshitz scaling symmetries)
- Other z, e.g. $z = 3$ for the heavy fermion compounds.

How do we analyze strongly interacting, scale invariant field theories?

The role of AdS/CFT

The AdS/CFT correspondence provides a tool to study a class of strongly interacting field theories with Lorentzian symmetry in d dimensions by mapping the field theories to classical gravity in $d+1$ dimensions.

- equation of state
- real time correlation functions
- transport properties — conductivities, diffusion constants, etc.

The ambitious program: There may be an example in this class of field theories which describes the quantum critical region of a real world material such as a high T_c superconductor.

The less ambitious program: By learning about this class of field theories, we may find universal features that could hold more generally for QCPs ($\eta/s = \hbar/4\pi k_B$).

The Basic Holographic Setup

$$S = \frac{1}{2\kappa^2} \int d^{d+1}x\sqrt{-g}(R - 2\Lambda) - \frac{1}{4g^2} \int d^{d+1}x\sqrt{-g}F_{\mu\nu}F^{\mu\nu} + S_\Psi$$

- Assume a diagonal metric with radial coordinate z of the form

$$ds^2 = g_{tt}(z)dt^2 + g_{xx}(z)(dx^2 + dy^2 + \ldots) + g_{zz}(z)dz^2 \ .$$

- The presence of a black hole horizon implies $g_{tt}(z_h) = 0$. Hawking temperature of the black hole maps to the temperature T of the field theory.
- The presence of an electric field implies $A_t(z) \neq 0$. Electric field of the black hole maps to charge density ρ of the field theory.
- We will probe the system with fermions Ψ; field theory from solving the classical Dirac equation.

The Dirac equation revisited

$$S_\Psi = -i \int d^{d+1}x \sqrt{-g}\, \overline{\Psi}(\gamma^\mu D_\mu - m)\Psi$$

$$D_\mu = \partial_\mu + \frac{i}{4}\omega_{\mu,\underline{ab}}\gamma^{\underline{ab}} - iqA_\mu,$$

In $d = 3$, the Dirac equation decouples into equations for the two spin components $\psi_\pm$:

$$\left[\sqrt{-g^{tt}}\sigma_2(\omega + qA_t) + \sqrt{g^{zz}}\sigma_3\partial_z \pm i\sqrt{g^{xx}}\sigma_1 k - m\right]\psi_\pm = 0,$$

where $e^{-i\omega t + ikx}\psi(z) = (-\det(g)g^{zz})^{1/4}\Psi$.

From a holographic point of view, the game is to solve these coupled differential equations.

Recent History

Much has already been done (numerically) with this system.

- Faulkner, McGreevy, Liu, Vegh et al. have shown that the fermionic Green's function at $T = 0$, $\rho \neq 0$ can exhibit non-Fermi liquid properties.
- Control parameter $\nu = \sqrt{m^2 + k^2 - \mu^2}$ where k is the momentum and μ the chemical potential.
  - $\nu > 1/2$: Fermi liquid
  - $\nu = 1/2$: marginal Fermi liquid
  - $\nu < 1/2$: non-Fermi liquid
  - ν imaginary: oscillatory region, fermions condense to form an electron star Hartnoll et al.
- Non-Fermi liquid means a dispersion relation $\omega \sim (k - k_F)^{1/2\nu}$. For $\nu = 1/2$, the contribution of the fermions to the resistivity is linear in T.

Objections

The $T \to 0$ limit is delicate.

- If there are additional scalar fields, they may condense at low temperature (holographic superconductors) [Gubser, Hartnoll, H, Horowitz].
- A four fermion interaction term can lead to a BCS phase transition at low temperatures [Hartman and Hartnoll].
- For $|m| < \mu$, a Fermi sea in the bulk modifies the geometry (electron star) and leads to the usual Fermi liquid behavior [Hartnoll et al].
- The non-Fermi liquid winds up in a small corner of available phase space.

This Talk

What remains to be done?

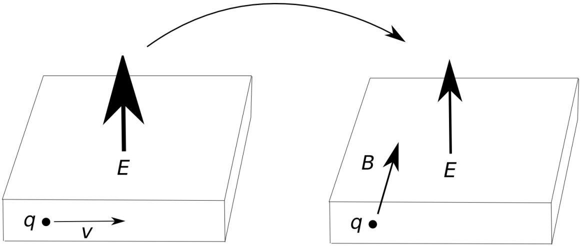
Little attention has been paid to the spin of the fermions.

- The role of spin orbit coupling in the dispersion relation.
- Improving analytic intuition for the damping term in the dispersion relation.

Part I: Spin Orbit Coupling

Spin Orbit Coupling

Nonrelativistic 2D electron gas in a transverse E field:

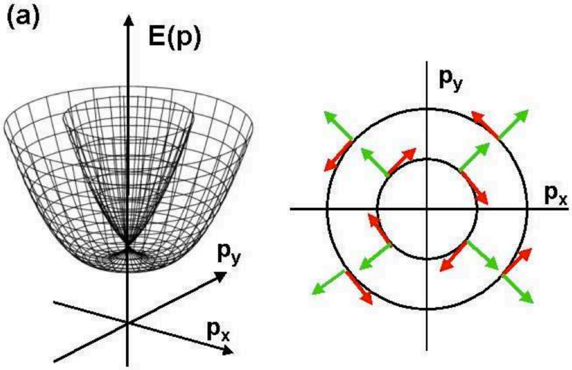


Spin orbit coupling: The electron feels a magnetic field in its own rest frame that produces an energy splitting between the two spin states.

The Rashba Hamiltonian

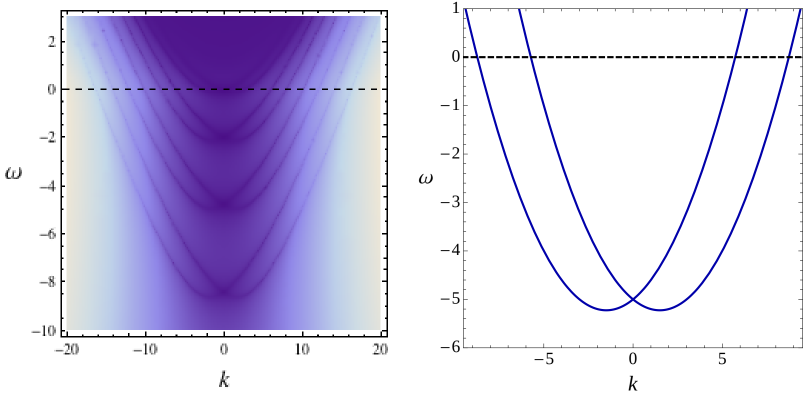
$$H = \frac{k^2}{2m_{\text{eff}}} - \lambda \vec{\sigma} \cdot (\hat{z} \times \vec{k}) - \mu \ ,$$

λ is the Rashba coupling constant, μ a chemical potential, $\vec{\sigma}$ the Pauli matrices, m_{eff} the electron effective mass, and $\hat{z}$ the unit vector perpendicular to the gas.



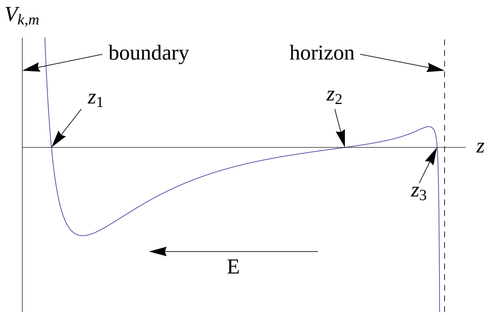
green arrows: momentum

red arrows: spin



(a) The dispersion relation in the boundary theory for a bulk fermion with charge times chemical potential divided by temperature $\mu q/4\pi T = 25$ and mass times AdS radius $mL = 2$. (b) The dispersion relation for a fermion with a Rashba type coupling. The Fermi surface in both cases is indicated by the dashed line.

Bulk Perspective



- z_1, z_1, and z_3 are classical turning points for the electron trajectory.
- There exists a quasibound state with a small imaginary part given by the tunneling probability through the potential barrier to the horizon.
- Explain the origins of this potential later.

The Point: An electron moving into or out of the page in the potential well will feel a spin orbit coupling.

Translating the Bulk into the Boundary

We have discussed the gravitational (bulk) behavior. How does this behavior map to field theory (boundary) quantities?

- These quasibound states are called quasinormal modes (QNMs). QNMs describe the ring down of excited black holes.
- An entry in the AdS/CFT dictionary (Son and Starinets): The location of the QNMs in the complex frequency plane is also the location of poles in the field theory Green's functions. The dispersion relation for a QNM with a small imaginary part can be interpreted as the dispersion relation for a quasiparticle in the field theory.

Field Theory Interpretation

The field theory interpretation is troubling.

- Field theory should be intrinsically 2+1 dimensional with no electric field, nonzero density ρ and nonzero temperature T.
- If spin is related to angular momentum, it's meaningless to say the spin points in the x or y direction.
- For strange metals, one naively anticipates that spin should be essentially an internal $SU(2)$ symmetry of the electrons, that the Fermi surface should be spin degenerate. There is no strong E field.

A Way Out

Natural to expect a modified Dirac equation

$$\left[(1 + F(\omega, k))p_\mu + (-\mu + G(\omega, k))u_\mu\right]\gamma^\mu\psi = 0 \ ,$$

where $p^\mu = (\omega, k_x, k_y)$ and $k = \sqrt{k_x^2 + k_y^2}$.

- Dirac equation for free massless 2+1 dimensional electron recovered by setting $G = F = \mu = 0$.
- $\mu \neq 0$ and $T \neq 0$ identifies a preferred frame $u^\mu = (1, 0, 0)$.
- Underlying theory is conformal, $m_{\rm bare} = 0$.

Further Along the Way Out

$$[(1 + F(\omega, k))p_\mu + (-\mu + G(\omega, k))u_\mu]\,\gamma^\mu\psi = 0 \ .$$

vs.

$$H = \frac{k^2}{2m_{\rm eff}} - \lambda\vec{\sigma}\cdot(\hat{z}\times\vec{k}) - \mu \ ,$$

- Choosing gamma matrices

$$\gamma^t = i\sigma_z \ , \quad \gamma^x = \sigma_x \ , \quad \gamma^y = \sigma^y \ ,$$

  and setting $F = 0 = G$ recovers the Rashba Hamiltonian without the the k^2 term and with $\lambda = 1$.
- To add the k^2 term, guess $G(\omega, k) \sim k^2$ which is allowed by the symmetries.
- For more bands, posit several species of massless fermions with different charges, $\mu \to \mu q_i$.

Where did this Schrödinger potential come from?

One can rewrite the Dirac equation as a Schrödinger equation for a single (in this case upper) component of the spinor:

$$-\phi''_{\pm}(z) + V_{\pm k,m}(z)\phi_{\pm}(z) = 0$$

where

$$\begin{aligned} V_{k,m}(z) &= \frac{1}{\hbar^2}\left(g_{zz}m^2 - Z_{-k}^2 Z_k^2\right) \\ &\quad -\frac{1}{\hbar} m\sqrt{g_{zz}}\,\partial_z \ln\left[\sqrt{g^{zz}}Z_k^2\right] + Z_k \partial_z^2 \frac{1}{Z_k}\ , \end{aligned}$$

and

$$Z_k = (g_{zz})^{1/4}\left[\sqrt{-g^{tt}}(\omega + qA_t) - \sqrt{g^{xx}}k\right]^{1/2}\ .$$

The Charged Scalar for Comparison

The Laplacian operator:

$$(D_\mu D^\mu - m^2)\Phi = 0 \ ,$$

Let $\Phi = e^{-i\omega t + ikx} Z(z)\phi(z)$ where $Z = \sqrt{g_{zz}}(-g)^{-1/4}$, then we obtain

$$-\phi''(z) + V_s(z)\phi(z) = 0$$

where

$$V_s(z) = \frac{1}{\hbar^2}\left(g_{zz} m^2 - Z_{-k}^2 Z_k^2\right) + Z\partial_z^2 \frac{1}{Z} \ .$$

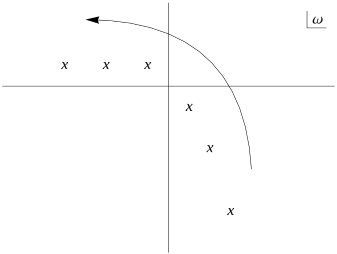
Same as the fermion potential to leading order in $\hbar$.

Part II: Spin and Damping

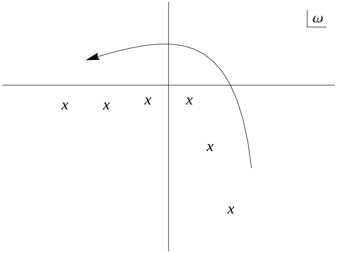
Thus far we have focused on the real part of the dispersion relation. The imaginary part gives the quasiparticles a lifetime, hopefully.

We are assuming a time dependence of the form $e^{-i\omega t}$ so ω had better be in the lower half plane.

The Position of Quasinormal Modes

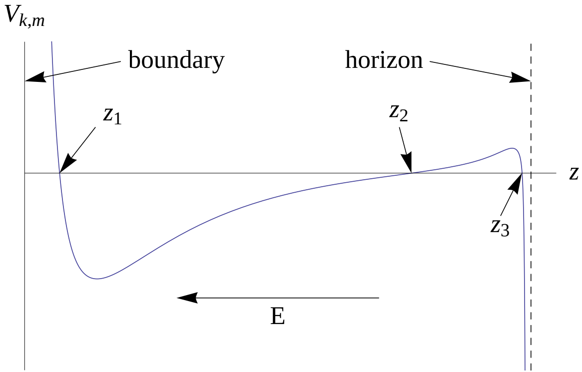


scalar



spinor

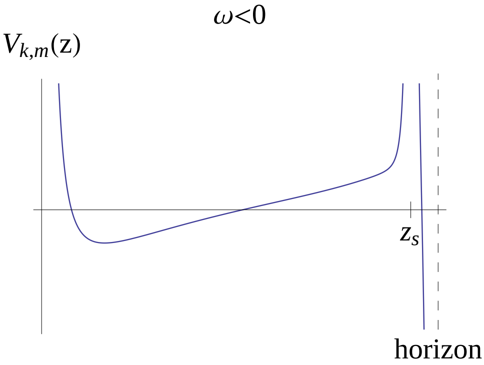
- Holographic superconducting instability caused by the motion of a scalar QNM into the upper half plane. This instability is called super radiance, gravitational analog of BEC.
- Fermions do not super radiate. QNM's in lower half plane.
- McGreevy et al. were able to see this numerically and show it in the $\omega \to 0$ limit at $T = 0$. Can we see this distinction more generally from a WKB type analysis of the Schrödinger problem?



- For the scalar, WKB analysis implies QNM's are in the lower half plane when $\mathrm{Re}\,\omega > 0$ and in the upper half plane when $\mathrm{Re}\,\omega < 0$.
- For the fermion, WKB would seem to indicate the same.

Fermions vs. Scalars

However, for the fermion, Z_k will vanish when $\omega < 0$. If $Z_k = 0$ at $z = z_s$, then $V_{k,m} \sim \frac{3/4}{(z-z_s)^2}$.



The connection matrix at $z = z_s$ forces the QNM's into the lower half plane.

Conclusion and Discussion

- Spin-orbit coupling leaves a strong and recognizable imprint on the dispersion relation of the quasiparticles in the boundary field theory.
- Similar dispersion relations can be found in quantum wells and topological insulators. Can we compute a spin Hall conductivity?
- Are these spin effects important in heavy fermions and high T_c superconductors? The anti-ferromagnetic parent state suggests maybe yes.
- WKB is a useful tool for gaining a better qualitative understanding of the fermions (and scalars) in the dual gravity description.
- AdS/QCD is a game played exclusively with bosonic fields. Can we / should we add fermions to the mix?