Holographic fermions

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Brookhaven National Laboratory – March 29, 2012

Introduction
- Landau Fermi liquids
- Non-Fermi liquids

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Holographic fermions at tree-level
- Charged black hole in AdS_4
- Near-horizon AdS_2
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Holographic fermions at one-loop
- Fermionic self-energy
- Conductivity
- Summary

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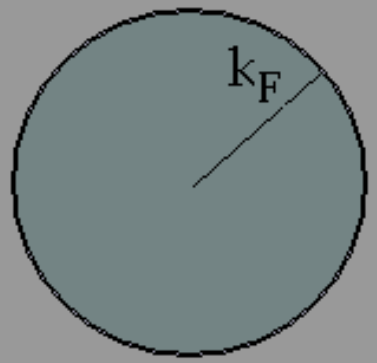

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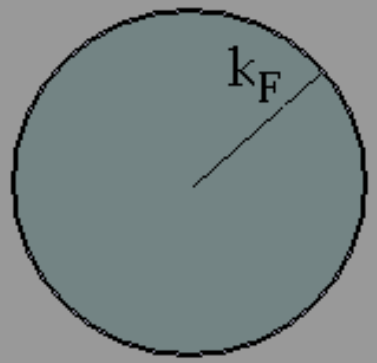

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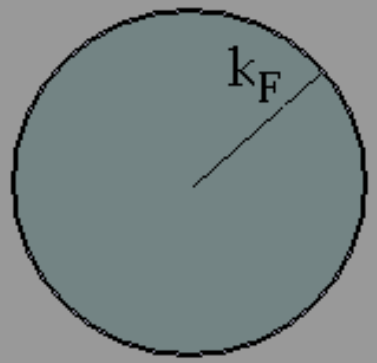

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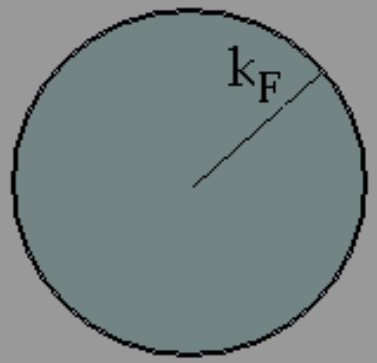

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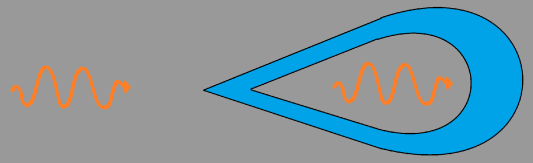

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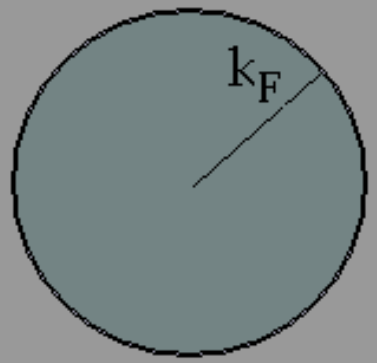

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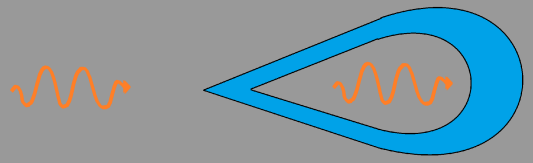

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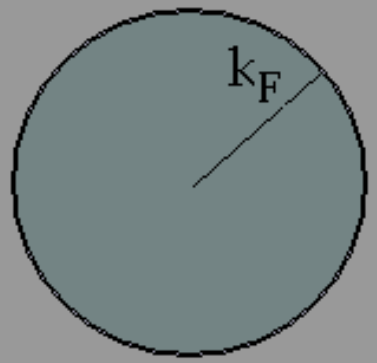

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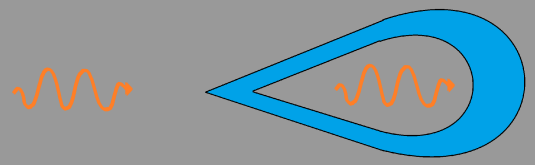

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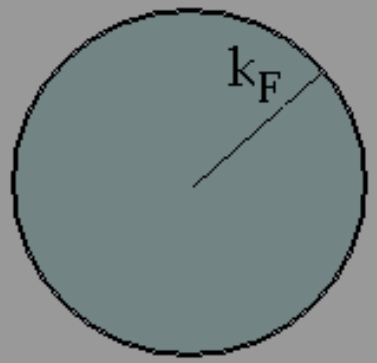

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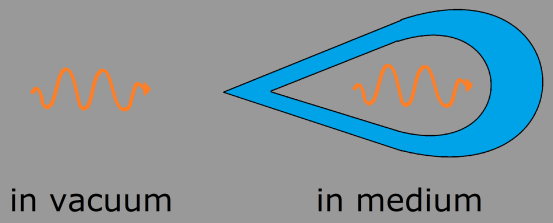

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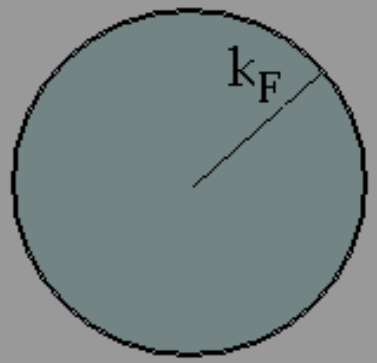

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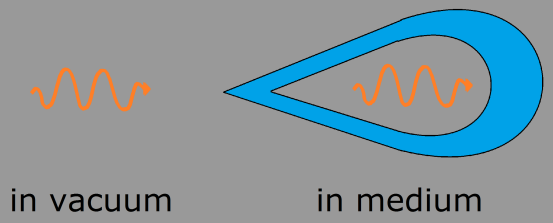

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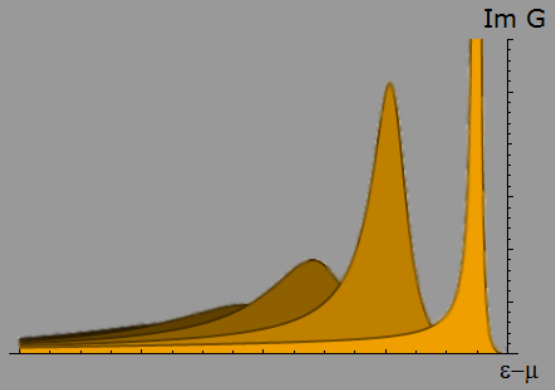
$$\rho(\omega,\vec{k}) \equiv \mathrm{Im}\, G_R(\omega,\vec{k}) \overset{k_\perp\to 0}{\longrightarrow} Z\delta(\omega - v_F k_\perp)$$



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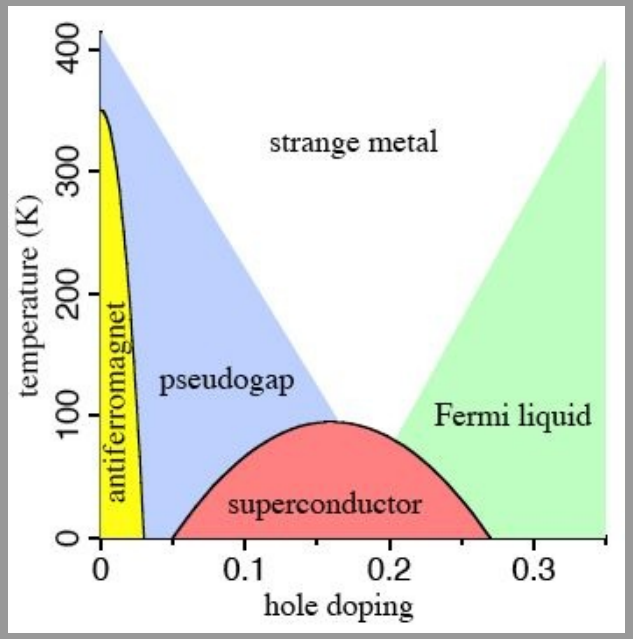
generic phase diagram for hole-doped cuprates



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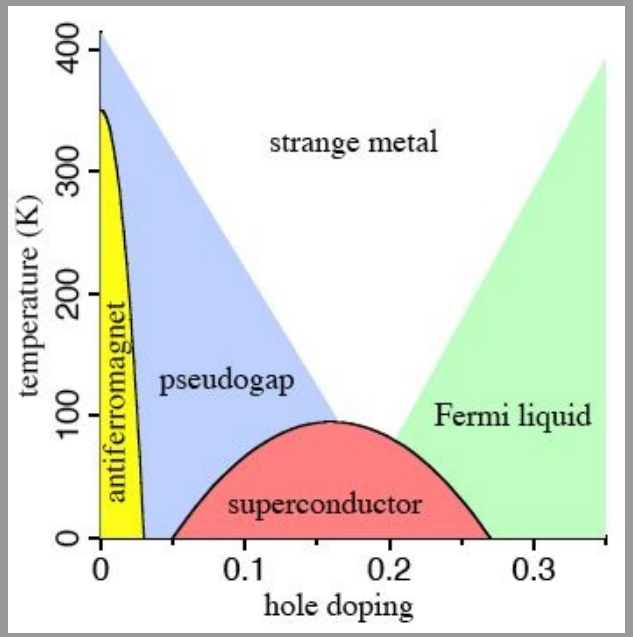

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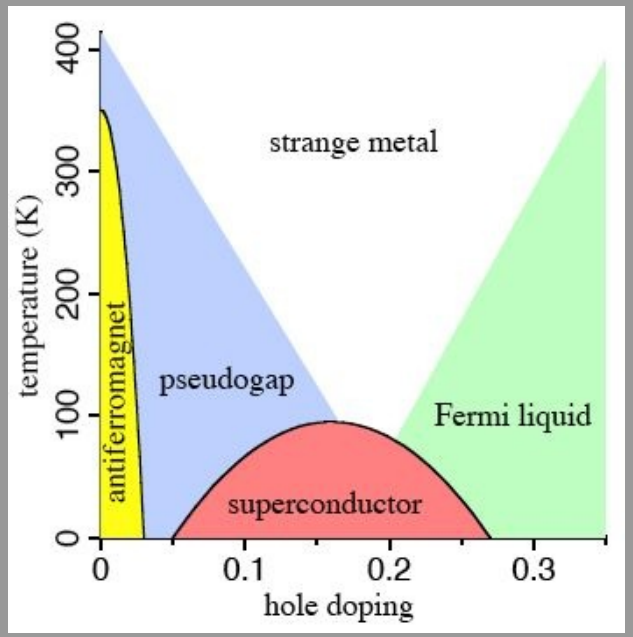

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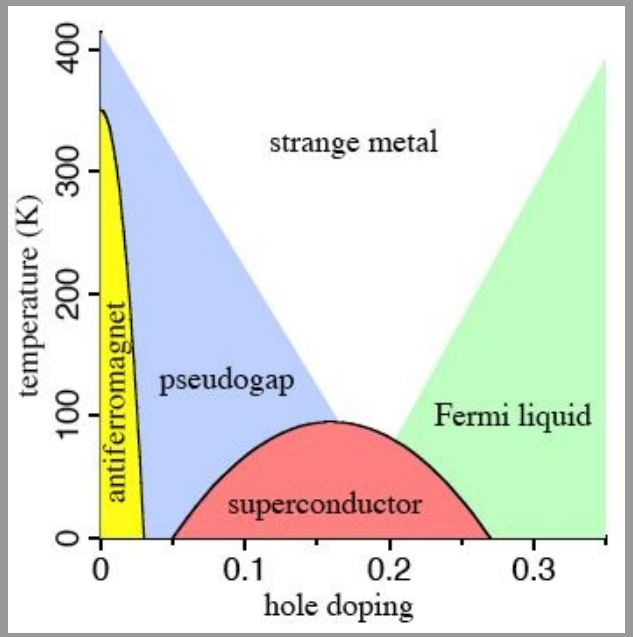

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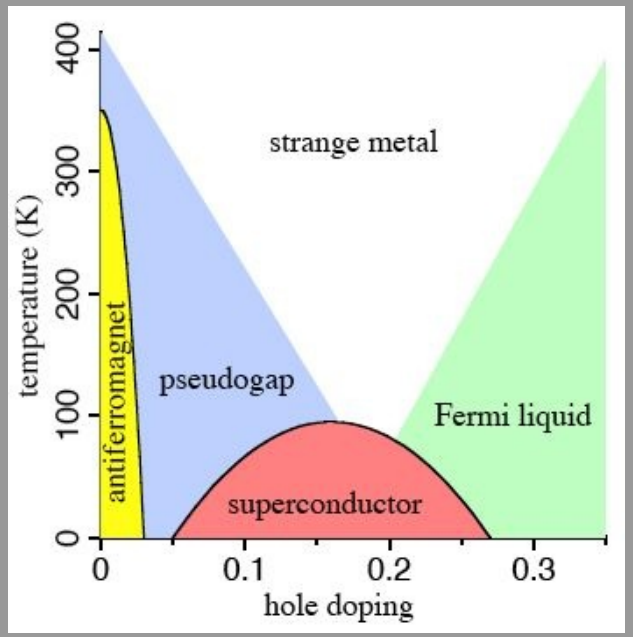

Fermion-boson system in 2+1 dimensions

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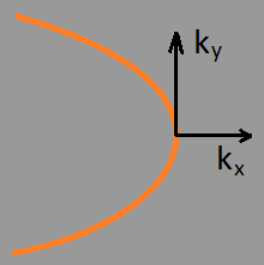

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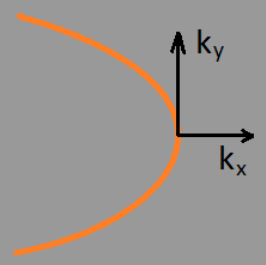

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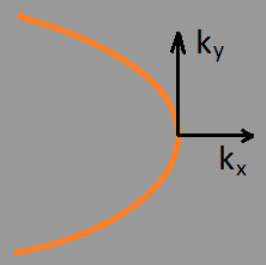

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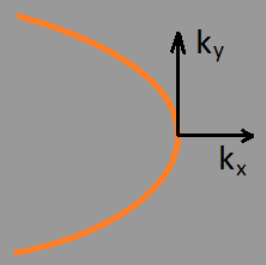

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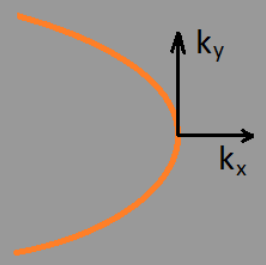

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finite chemical potential	$\Longleftrightarrow$	electrically charged black hole

Charged black hole in AdS_4

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$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left(\mathcal{R} + \frac{6}{R^2} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \dots \right)$$

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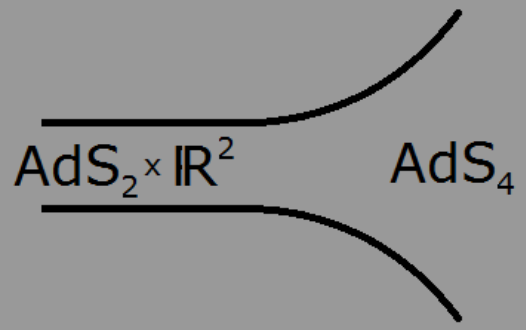
Charged black hole solution



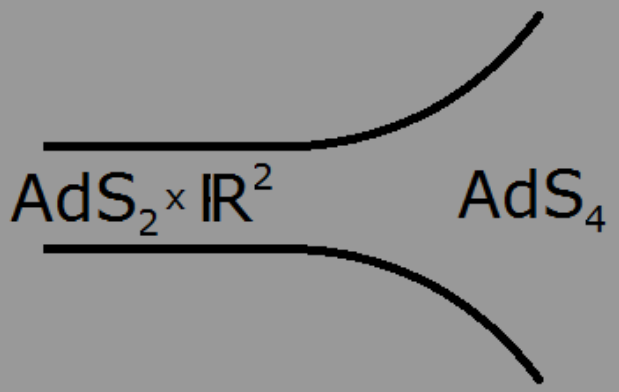
$$ds^2 = \frac{r^2}{R^2}\left(-f(r)dt^2 + d\vec{x}^2\right) + R^2\frac{dr^2}{f(r)r^2}$$

$$f(r) = 1 + \frac{Q^2}{r^4} - \frac{M}{r^3} \qquad A = \mu\left(1 - \frac{r_0}{r}\right)dt$$

μ: chemical potential, horizon at $r = r_0$.

Low-energy physics

- *Emergent "IR CFT"*
  $AdS_2 \Leftrightarrow$ "(0+1)-d CFT"



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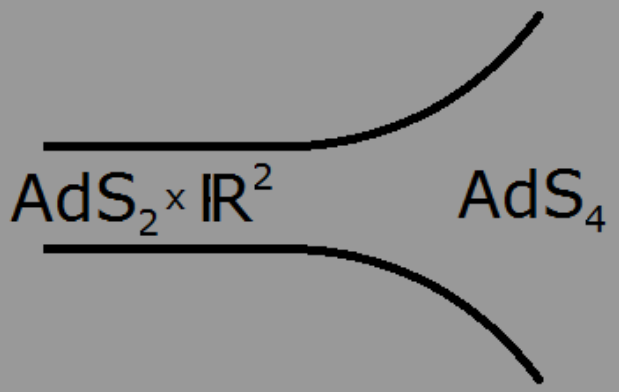

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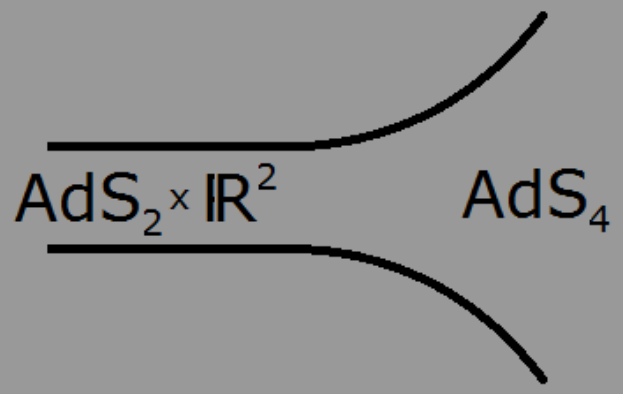

$$\delta_{\vec{k}} = \frac{1}{2} + \nu_{\vec{k}} \qquad \nu_{\vec{k}} = \frac{1}{\sqrt{6}}\sqrt{\frac{3}{2} + m^2R^2 + \frac{k^2}{\mu_*^2} - \frac{q^2}{2}}$$

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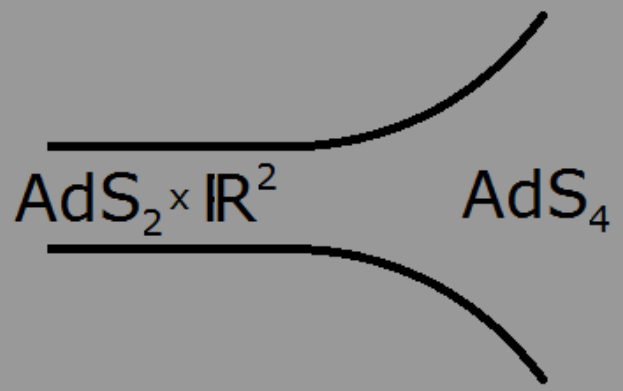

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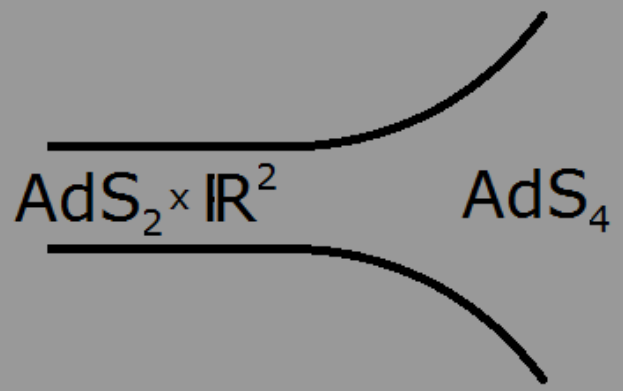

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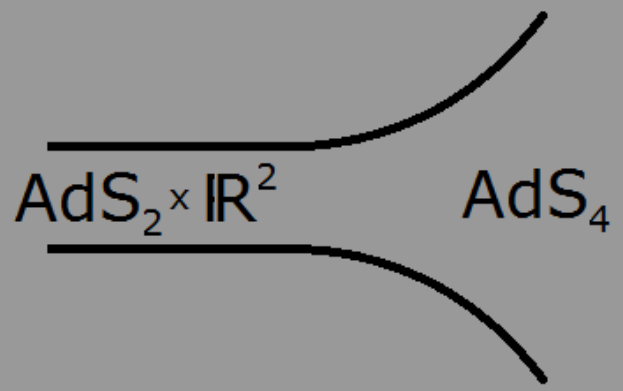

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$$\psi = (-gg^{rr})^{-1/4}e^{-i\omega t+ikx}\Psi \qquad \Phi_\alpha = \frac{1}{2}(1-(-1)^\alpha\Gamma^r\Gamma^t\Gamma^x)\Psi$$

$$\Phi_\alpha \overset{r\to\infty}{\approx} a_\alpha r^m\begin{pmatrix}0\\1\end{pmatrix} + b_\alpha r^{-m}\begin{pmatrix}1\\0\end{pmatrix} \qquad G_\alpha(\omega,k) = \frac{b_\alpha}{a_\alpha} \qquad \alpha = 1,2$$

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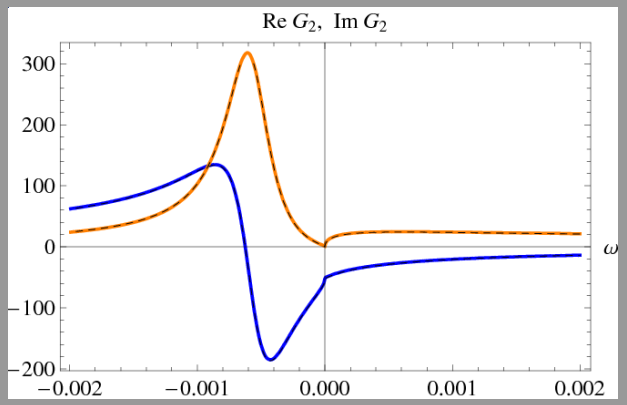
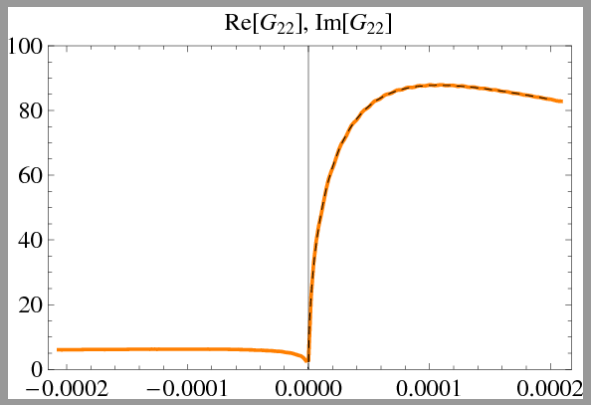
At $q = 1, \Delta = 3/2$ the numerical computation gives [Liu-McGreevy-DV] [Zaanen et al.]





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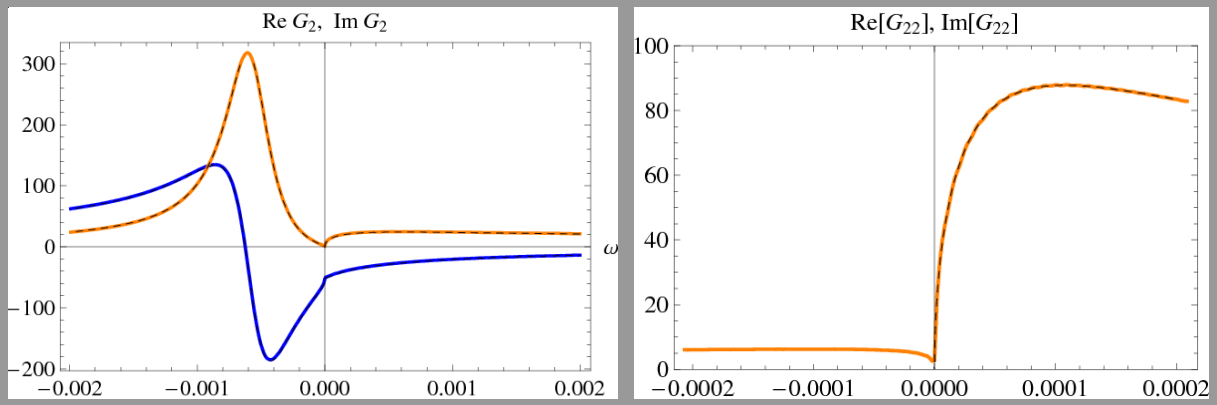
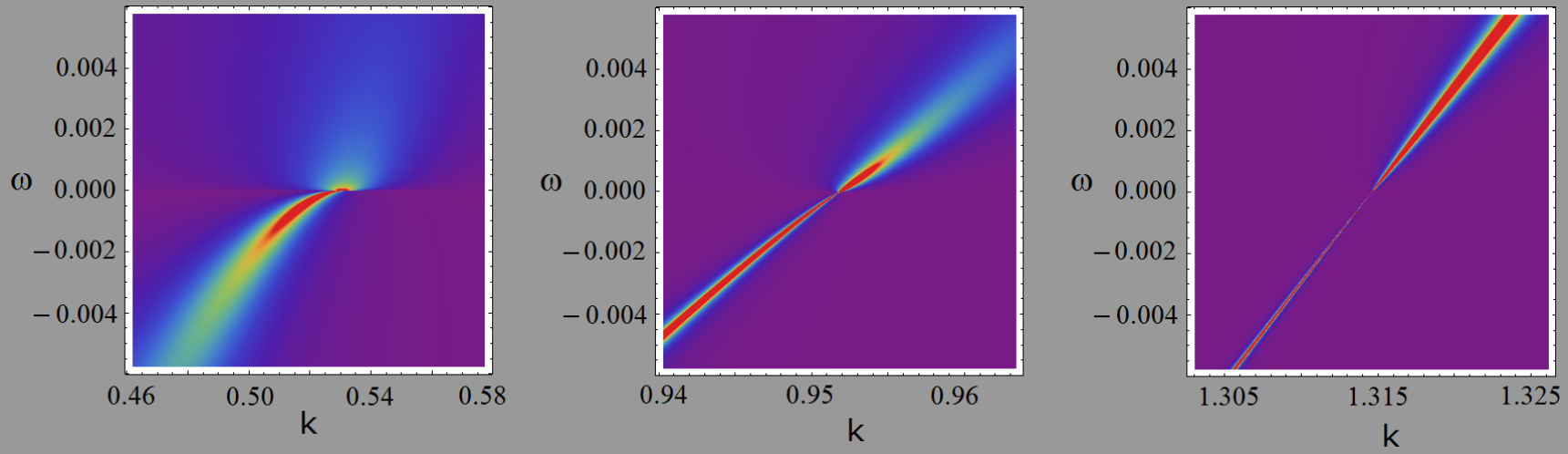


$\Delta = 3/2,\ q = 1.0,\ 1.56$ and 2.0



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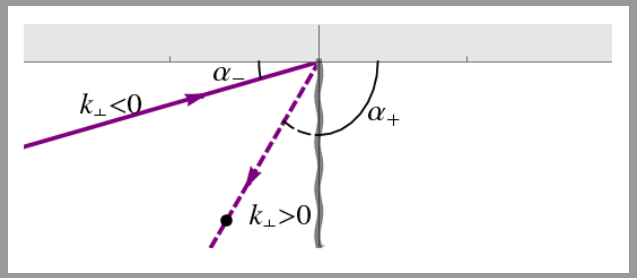

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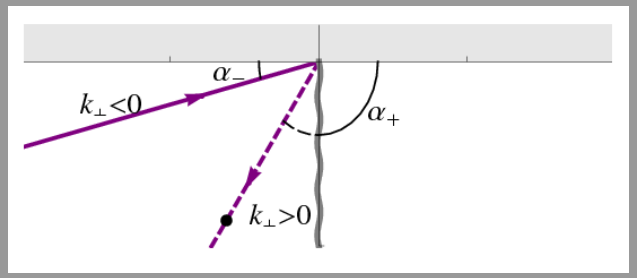

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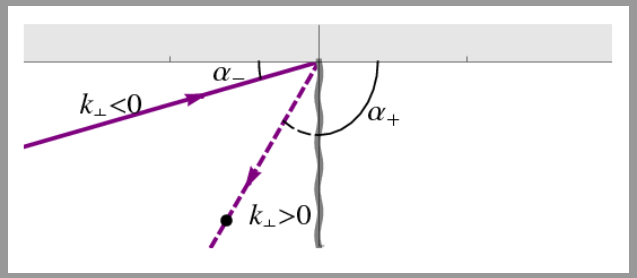

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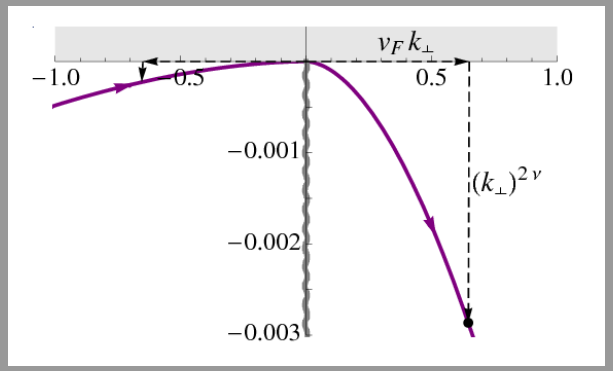

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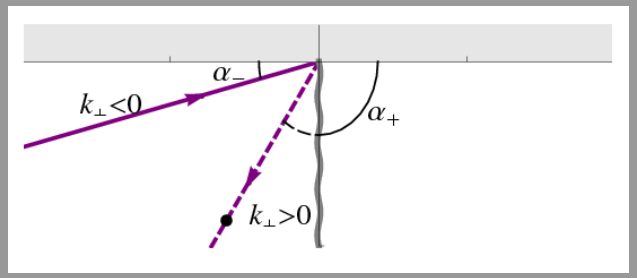

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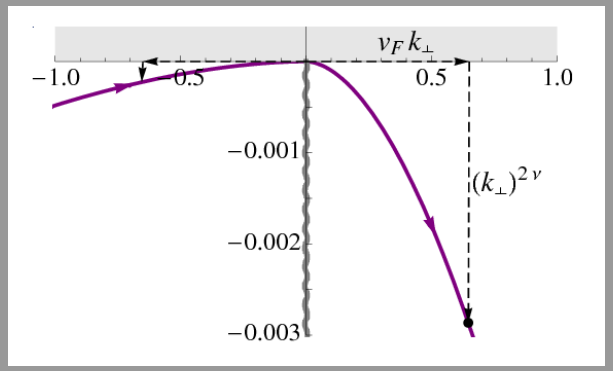

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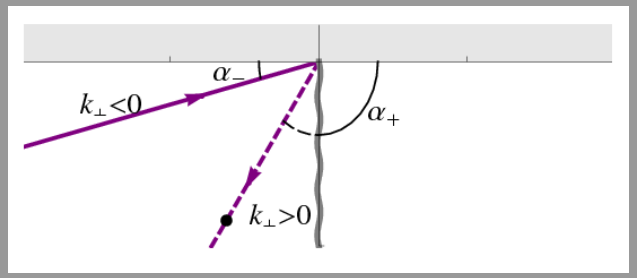

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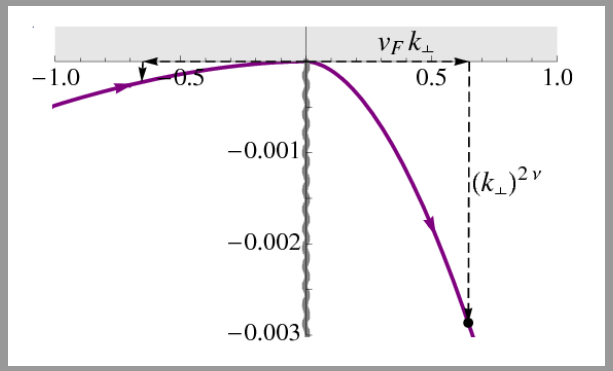

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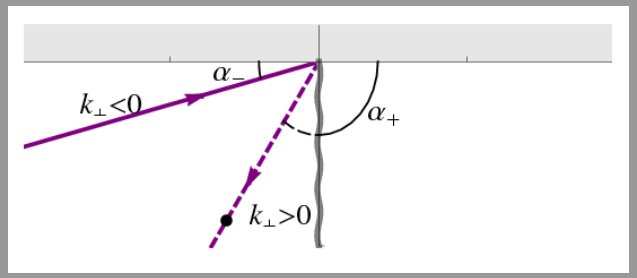

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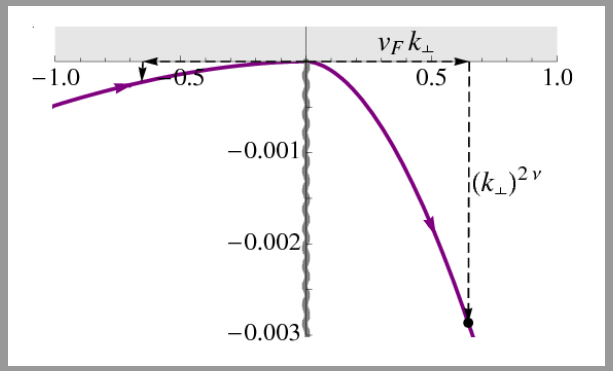

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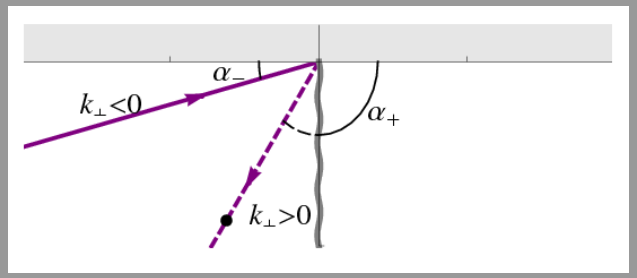

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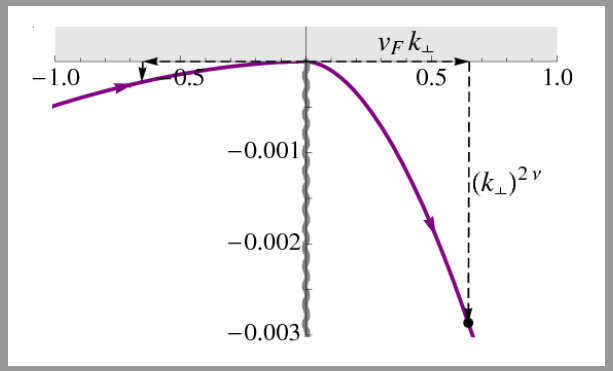

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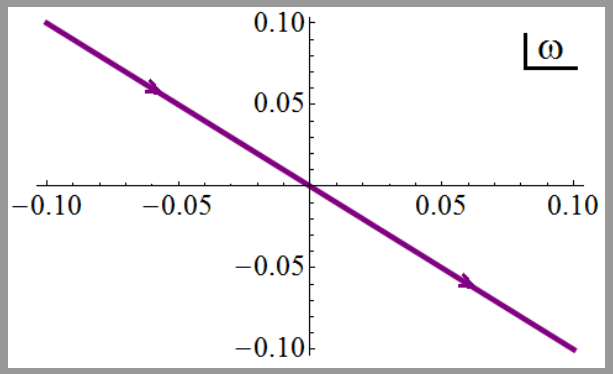
Bosonic normalizable mode: instability or QCP

instability at finite k_B momentum



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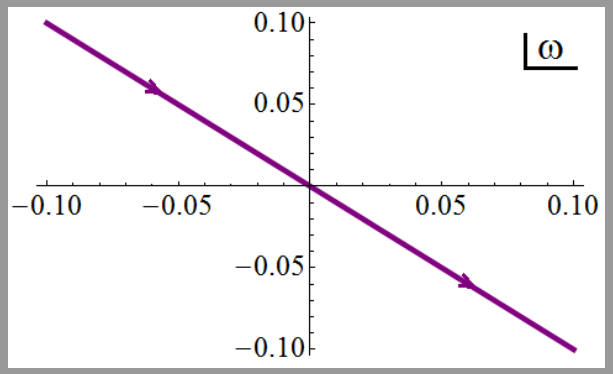

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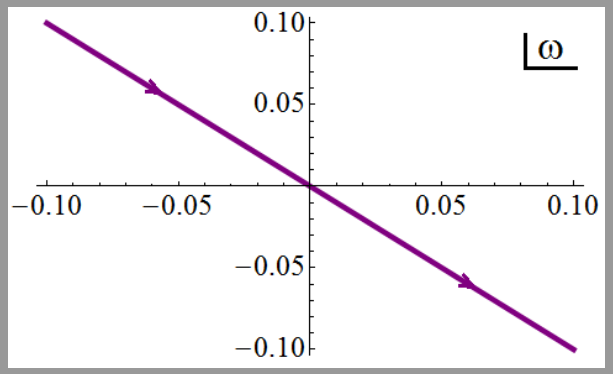

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This gives the "hybridized" QCP [Faulkner-Horowitz-Roberts] [Iqbal-Liu-Mezei]

$$\chi(\omega, k) = \frac{Z}{g - g_c + k^2 + \omega^2 + c(k)\omega^{2\nu}}$$

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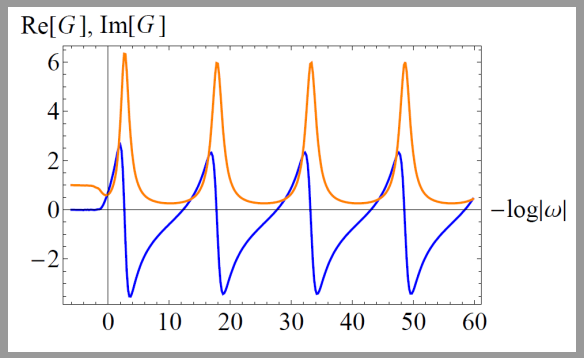

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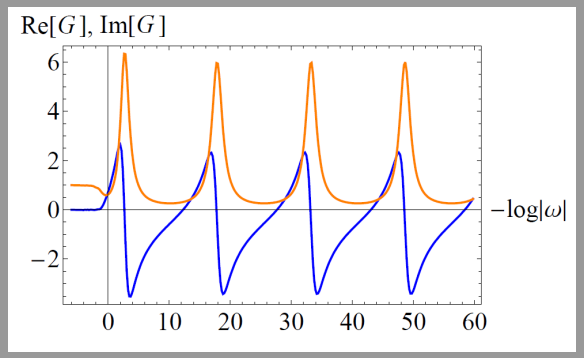

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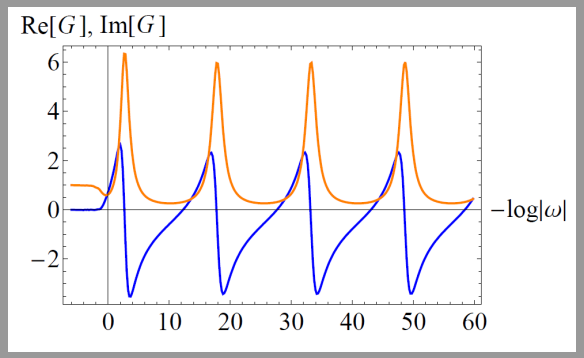

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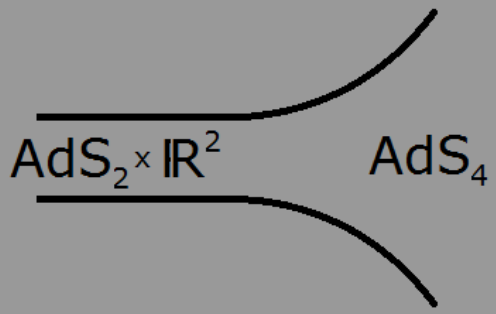

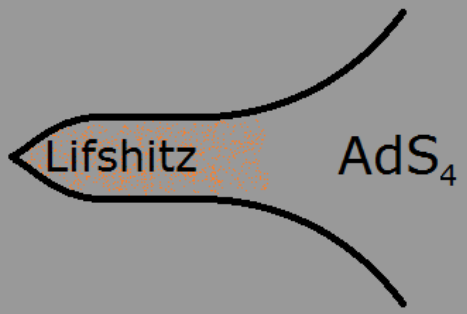

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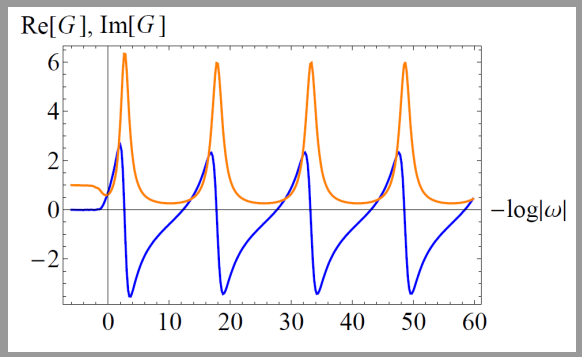

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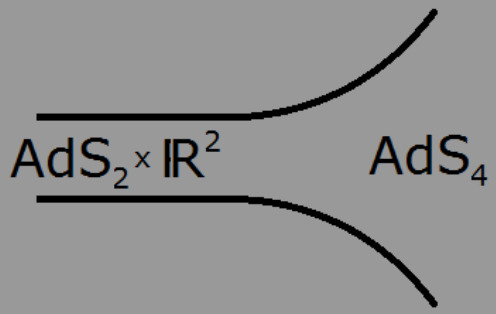
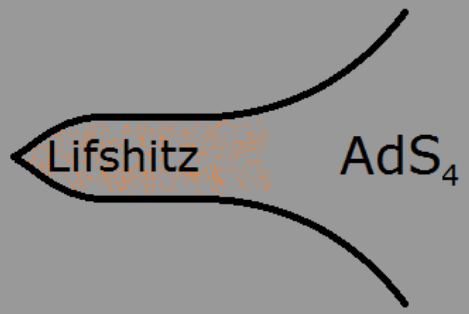
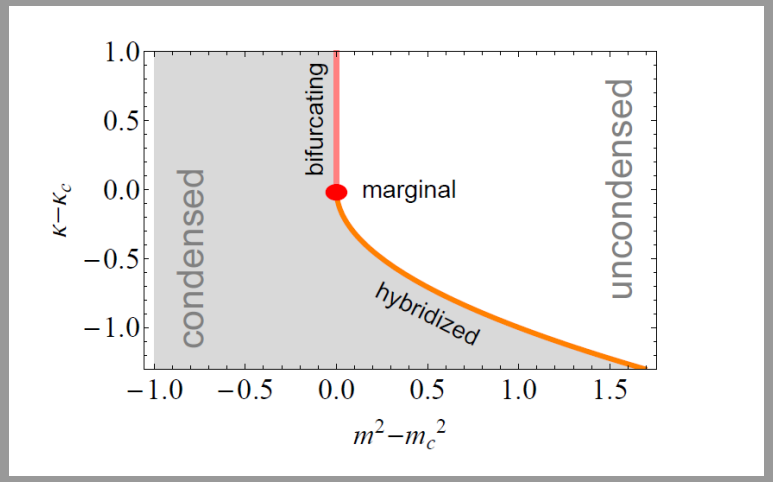




[Hartnoll et al.] [Liu et al.] [Zaanen et al.]

heavy fermions, many Fermi surfaces

Complex IR dimensions: neutral boson



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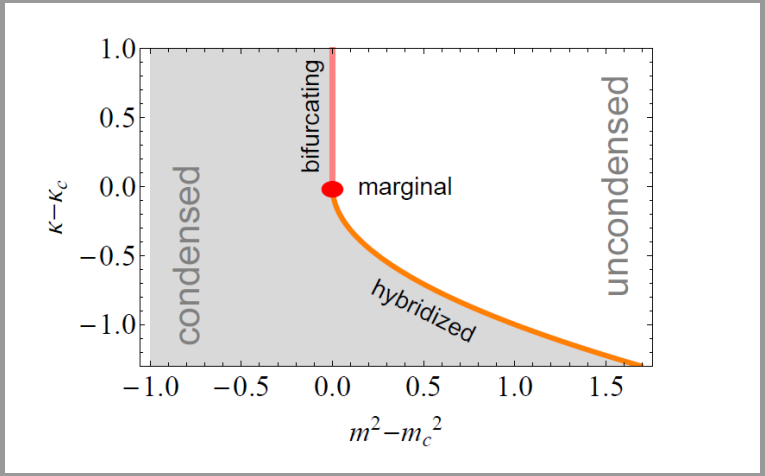

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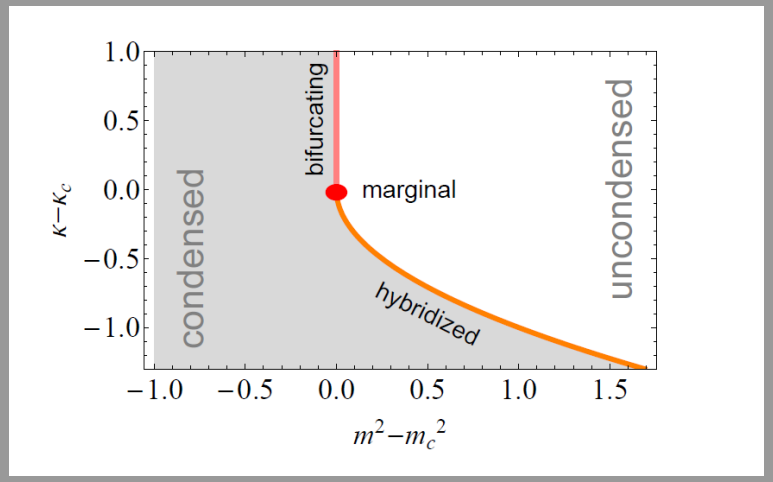

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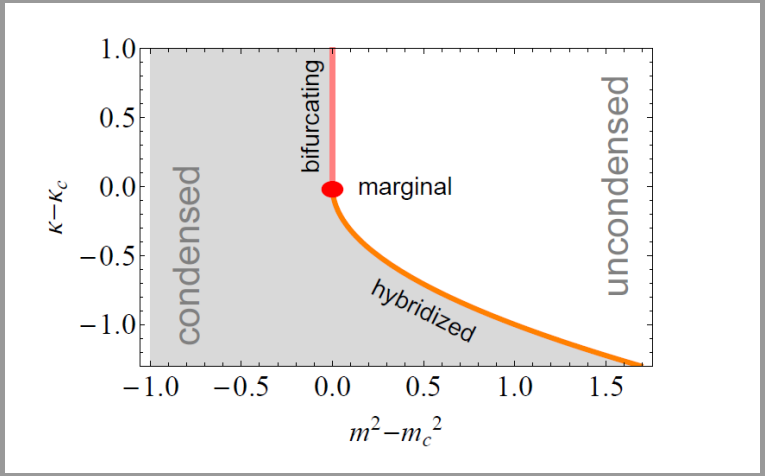

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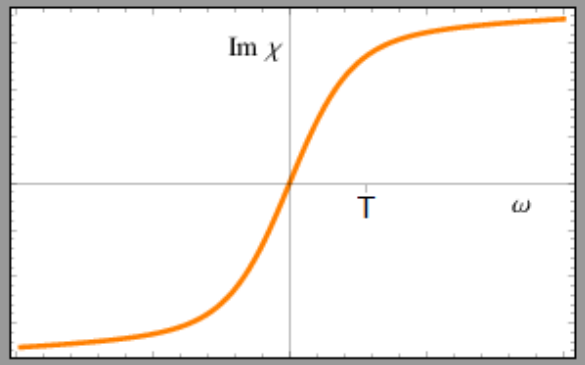
(Varma's boson spectrum)



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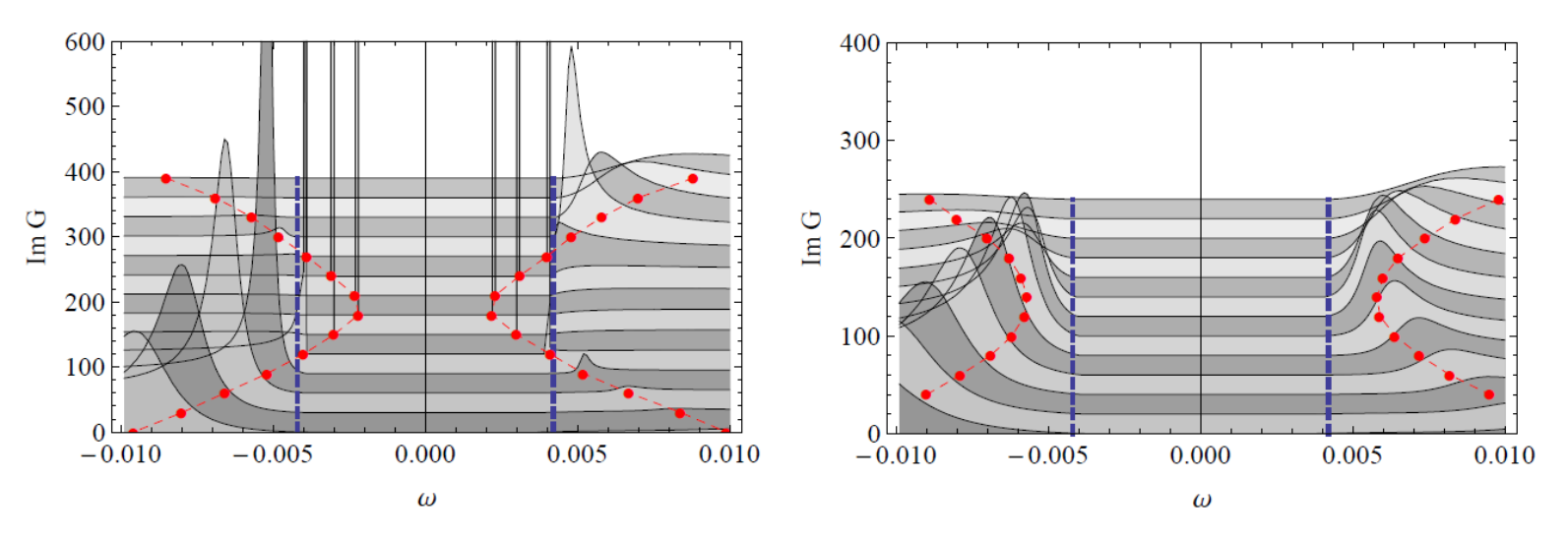

Holographic marginal Fermi liquid

Two ways of obtaining holographic non-Fermi liquids

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- at tree level

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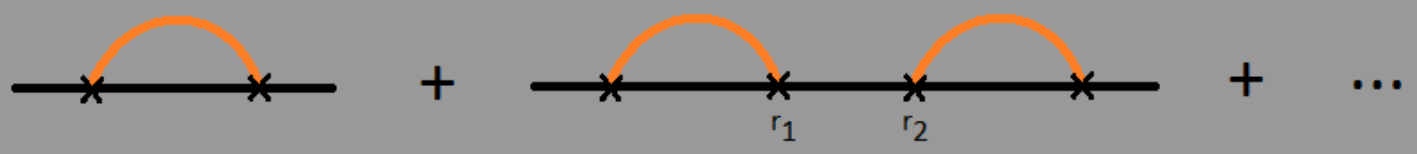
Fermionic self-energy



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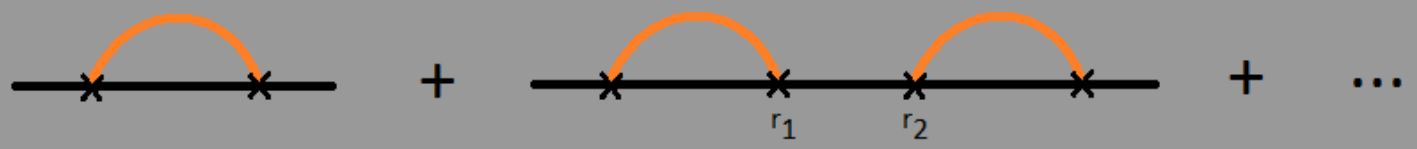

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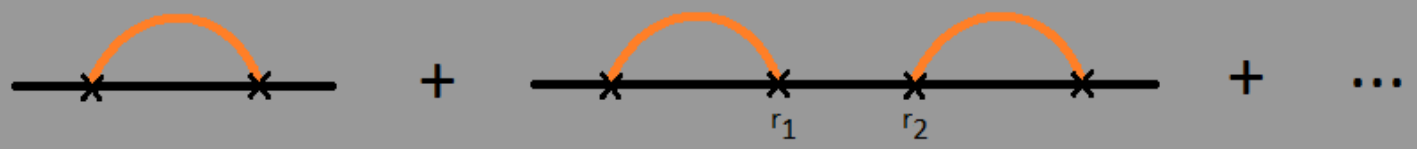

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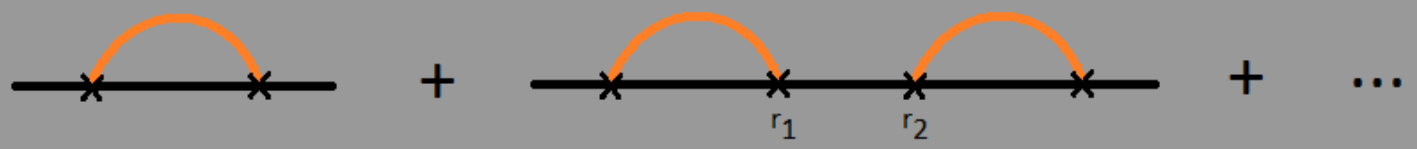

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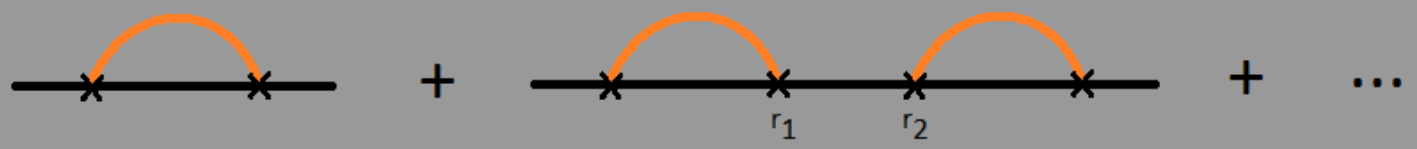

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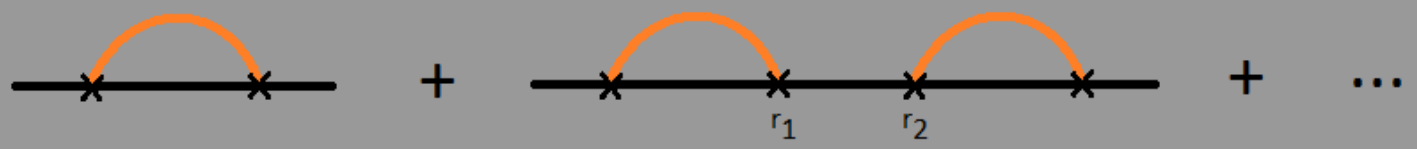

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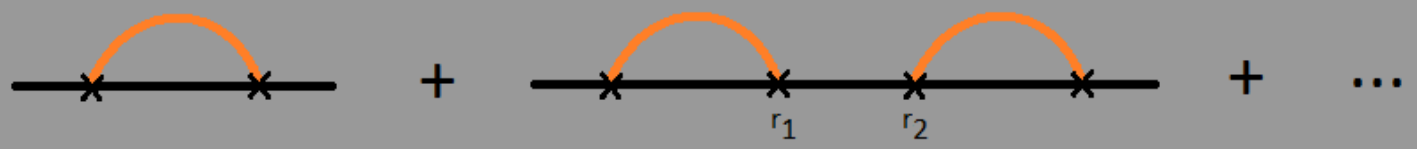

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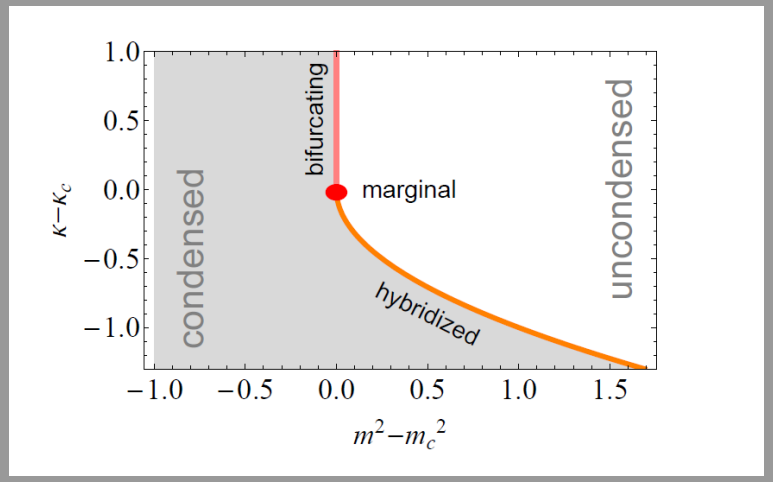
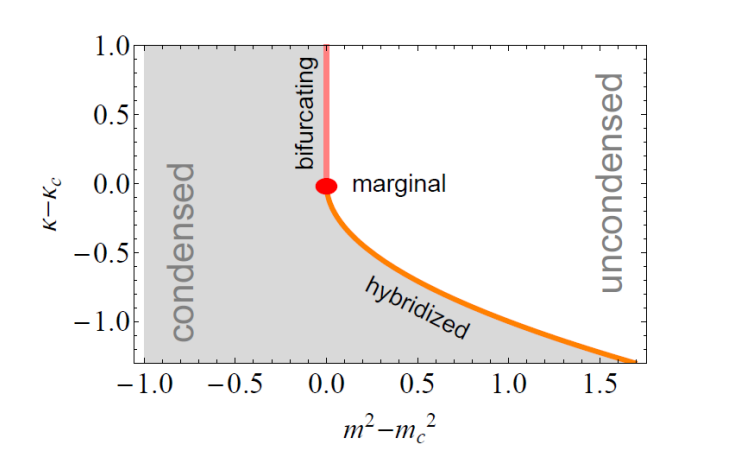
$$D_{ret.}(r_1, r_2; \omega, k_F + k_\perp) \approx \frac{G(\omega, k)}{W} \varphi_{norm.}(r_1; \omega, k_F) \varphi_{norm.}(r_2; \omega, k_F)$$

Thus, one gets a geometric series which can be summed up just like in QFT.


1.0
0.5
0.0
−0.5
−1.0
κ−κc
condensed
bifurcating
marginal
hybridized
uncondensed
−1.0
−0.5
0.0
0.5
1.0
1.5
m²−mc²

$$\nu^2 = m^2 - m_c^2$$


bifurcating
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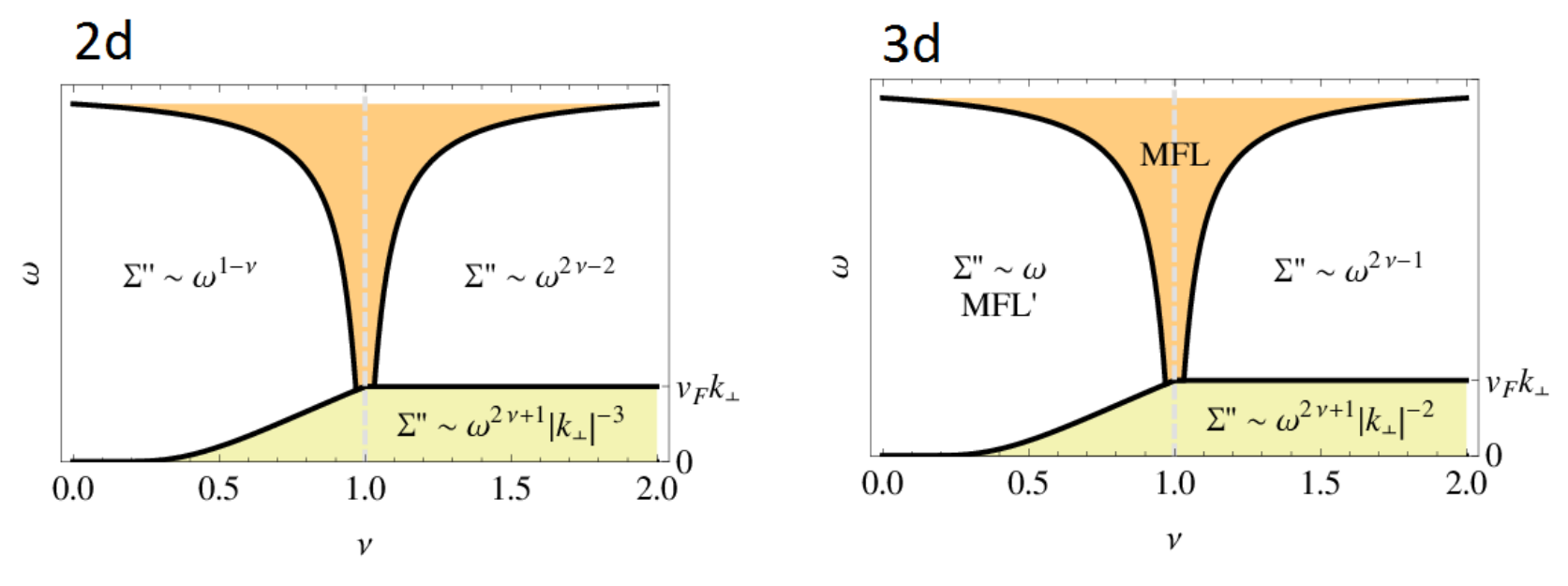
2d
$\Sigma'' \sim \omega^{1-\nu}$
$\Sigma'' \sim \omega^{2\nu-2}$
$\Sigma'' \sim \omega^{2\nu+1}|k_\perp|^{-3}$
$v_F k_\perp$
ω
ν
3d
MFL
$\Sigma'' \sim \omega$
MFL'
$\Sigma'' \sim \omega^{2\nu-1}$
$\Sigma'' \sim \omega^{2\nu+1}|k_\perp|^{-2}$
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ω
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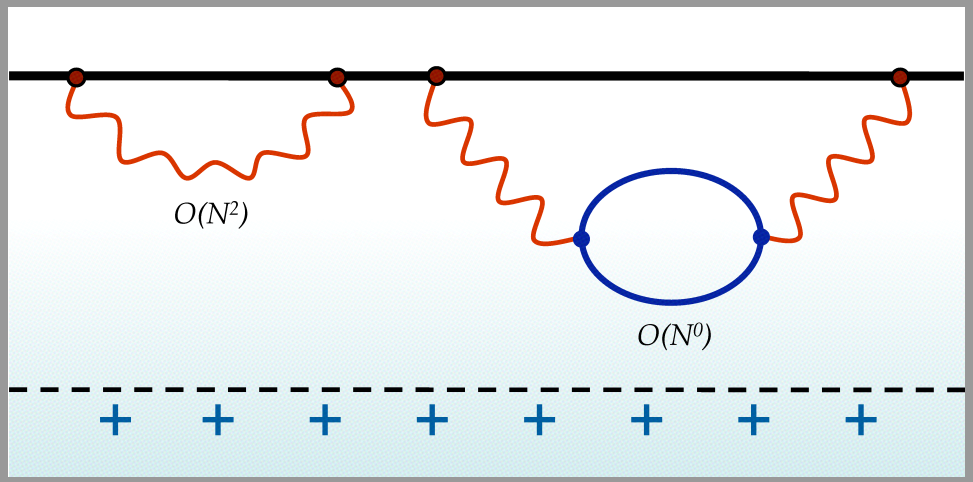

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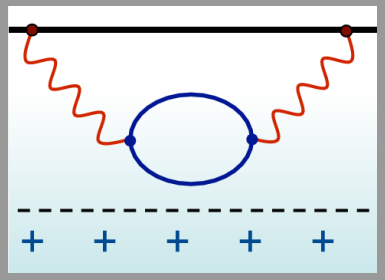

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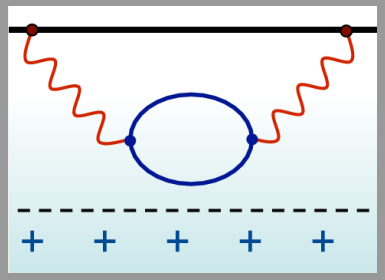

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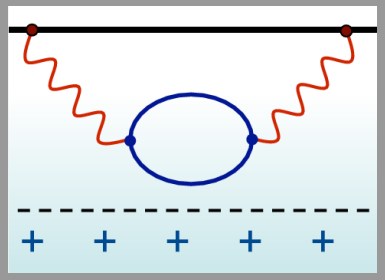

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