Hard Probes of QGP in strong magnetic field

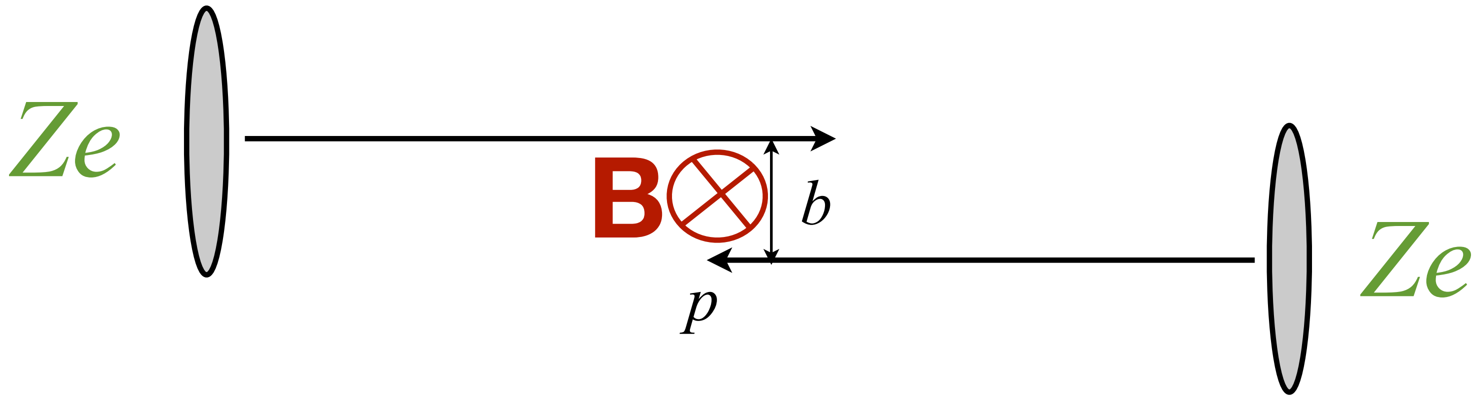
Kirill Tuchin

IOWA STATE UNIVERSITY
OF SCIENCE AND TECHNOLOGY

RIKEN lunch seminar, 08/02/2012

Origin and properties of magnetic field

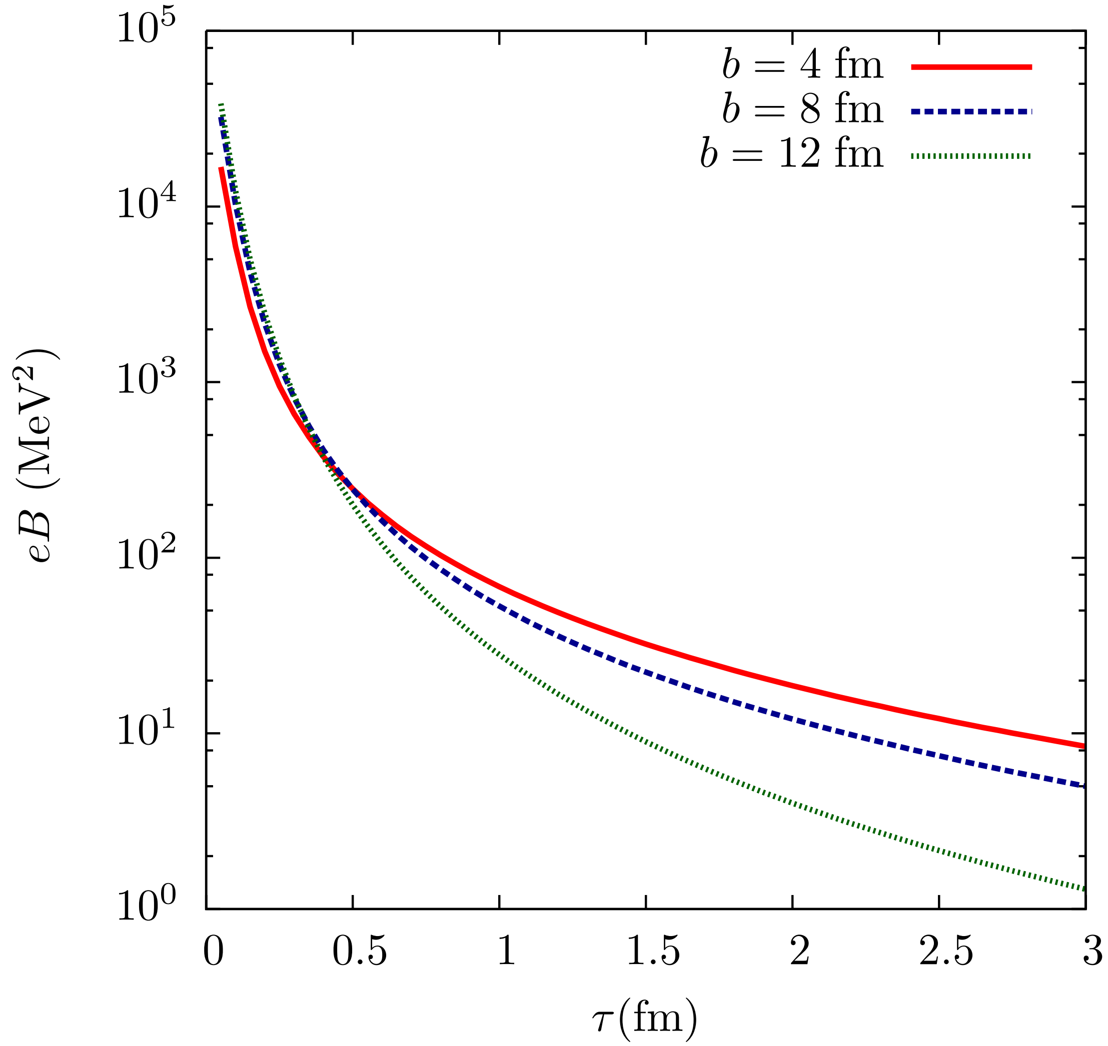
ORIGIN



Order of magnitude estimate: $B \sim Ze\dfrac{b}{R^3}\gamma$

For $Z=79,\ b=7\ fm,\ \gamma=100$ we get $eB = (200\ MeV)^2 \approx m_\pi^2$

A better estimate: take Lienard-Wiechert potentials, integrate over the charge distribution in nuclei.



Kharzeev, McLerran, Warringa (2007)

Fig. A.2. Magnetic field at the center of a gold-gold collision, for different impact parameters. Here the center of mass energy is 200 GeV per nucleon pair ($Y_0 = 5.4$).

Similar results:

UrQMD based calculation: Skokov, Illarionov, Toneev (2009)

Hadron String Dynamic transport code: Voronyuk, Toneev, Cassing, Bratkovskaya, Konchakovski, Voloshin (2011)

LIFETIME: MEDIUM EFFECTS

Medium is formed at a very early stage after a Heavy Ion Collision: Glasma (t~0.2 fm) gives way to Quark Gluon Plasma (t~0.5 fm) according to the state-of-the-art phenomenology.

$$\nabla \times \mathbf{E} = -\dot{\mathbf{B}}$$
$$\nabla \times \mathbf{B} = \mathbf{j} = \sigma \mathbf{E}$$

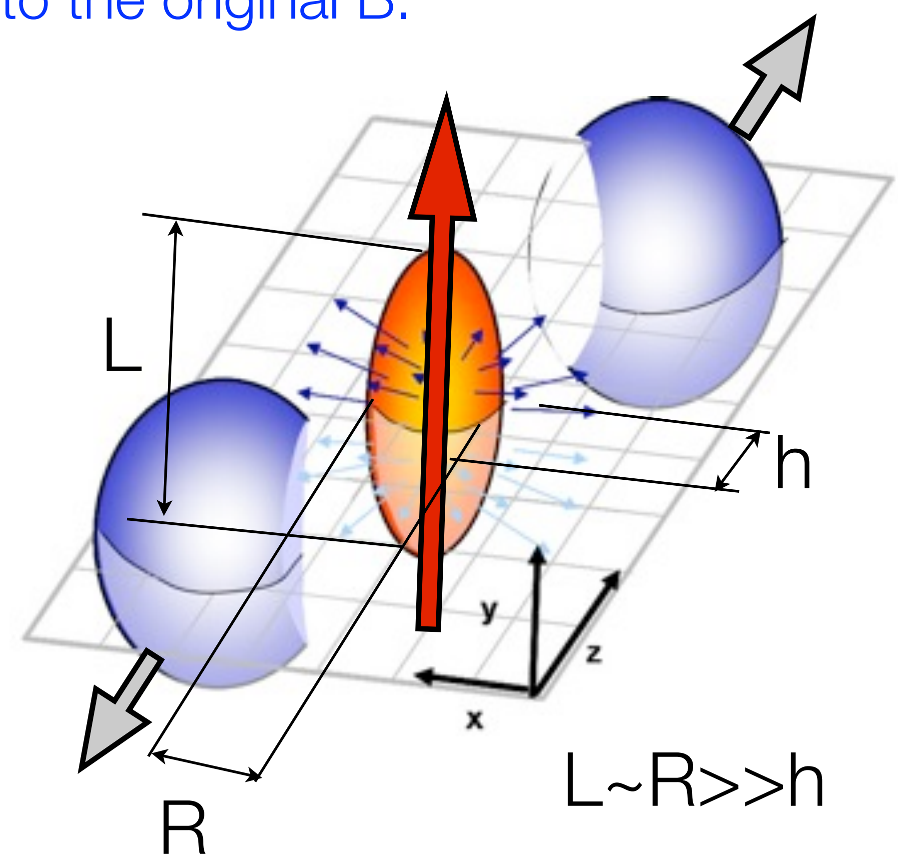
Lenz's law: induced B is parallel to the original B.

$$\nabla^2 \boldsymbol{B} = \sigma \dot{\boldsymbol{B}}$$

$$\tau = \sqrt{t^2 - z^2} \qquad \eta = \frac{1}{2} \ln \frac{t+z}{t-z}$$

At $\eta \ll 1$
$$\frac{\partial^2 \boldsymbol{B}}{\partial \eta^2} \frac{1}{\tau^2} + \nabla_\perp^2 \boldsymbol{B} = \sigma \frac{\partial B}{\partial \tau}$$

$$\boldsymbol{B}\big|_{\eta=\eta_\pm} = 0 \qquad \boldsymbol{B}\big|_{\tau=0} = \boldsymbol{B}_0$$



B time-dependence is characterized by the scale

$$\tau \sim \sigma R^2$$

Conductivity: $\sigma = 0.37\, C_{\text{em}}\, T \;=\; 4.5\,\text{MeV}\, \frac{T}{T_c}$

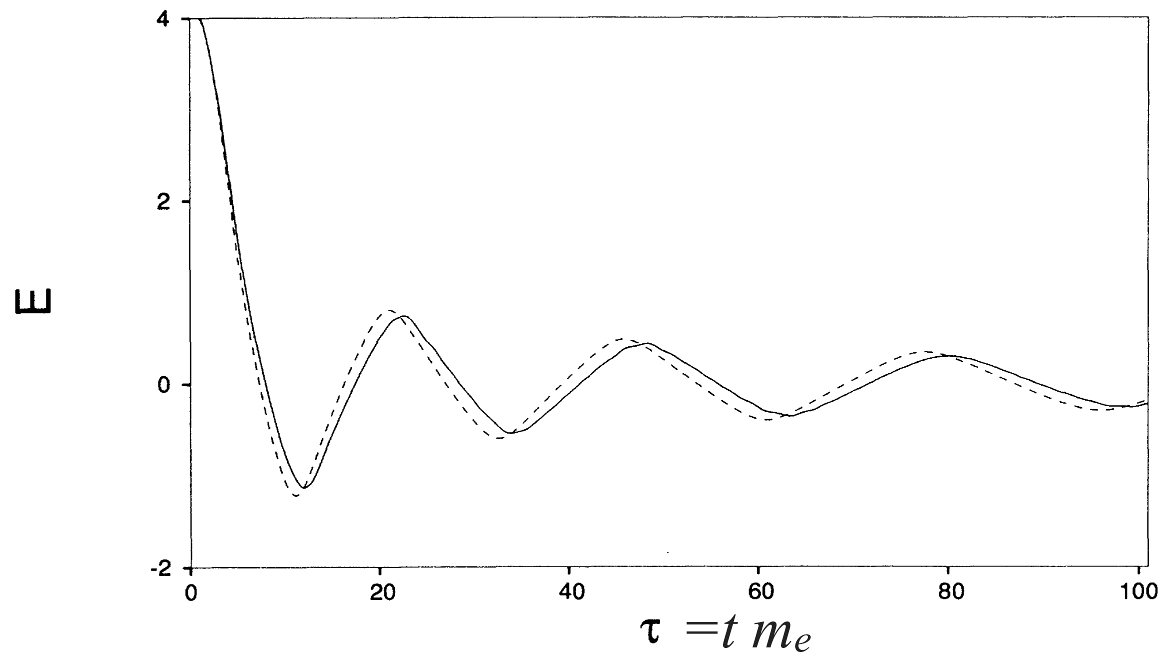
Aarts at al (2007), Ding at al (2010):
only gluon contribution!

for R=5 fm ⇒ **τ≈1-2 fm/c** ≫R/γ

LIFE TIME: BACK-REACTION ?

Once E-field is induced by time-decreasing B, it starts **generating electron-positron pairs via the Schwinger mechanism**.

Cooper at al argued that E(t) is adiabatic with relaxation time *~1/m_e=400 fm*



Cooper, Eisenberg, Kluger, Mottola, Svetitsky (1992)

similar results by Tanji (2009)

Number density of electrons $n \sim \frac{E^2}{m_e} = \frac{m_\pi^4}{e^2 m_e} \sim 10^6\,\mathrm{fm}^{-3}$

However, this mechanism doesn't seem realistic since the electron Compton wavelength is ~ 20 times larger than the QGP size.

APPLICATIONS

- ✳ Synchrotron radiation — KT (2010,2012)
- ✳ Photon/dilepton production — KT (2010)
- ✳ Azimuthal anisotropy of QGP — Mohaparta, Saumia, Srivastava (2011), KT (2011)
- ✳ Ionization of bound states (e.g. J/ψ) — Marasinghe, KT (2011)
- ✳ Chiral Magnetic Effect — Kharzeev (2006), Kharzeev, Zhitnitsky (2007), Kharzeev, McLerran, Warringa (2008), ...
- ✳ QCD phase diagram

Quarkonium dissociation

QUARKONIUM IN MAGNETIC FIELD

✳ **Lorentz ionization**: in the quarkonium rest frame there are perpendicular electric and magnetic fields. Electric field renders quarkonium unstable with respect to decay into q and anti-q.

✳ **Zeeman effect**. Quarkonium state of total angular momentum J splits (in weak field) into states of different mass: $\Delta M = (eB/2m) g J_z$, where $J_z = -J, -J+1, \ldots, J$. For example J/ψ (S=1,L=0, J=1) $J_z = 0, \pm 1 \Rightarrow$

$\Delta M = 0.15$ GeV (at LHC)

✳ **Distortion of the quarkonium potential** due to high order effects. This is important $B \sim 3\pi m^2/e$ which is $3\pi/\alpha$ stronger than the Schwinger's field.

Machet, Vysotsky (2010)

J/ψ IN MAGNETIC FIELD

Marasinghe, KT (2011)

Parameters: binding energy ε_b and quarkonium radius α_s/ε_b

Quasi-static approximation is justified if the field does not change much over the quarkonium radius. For J/ψ:

$$\varepsilon_b \tau / \alpha_s \approx 10$$

i.e. B varies over scales much larger than the charmonium radius.

Inhomogeneity of space distribution of B is ~ $1/m_N >> \alpha_s/\varepsilon_b$

Thanks to the small size of J/ψ we can confidently set B ≈ const

(Note: as $T \to T_c$ this approximation breaks down because J/ψ becomes "fatter")

BOOST FROM LAB TO J/Ψ REST FRAME

$$E_\parallel = 0\,, \qquad \boldsymbol{E}_\perp = \gamma_L \boldsymbol{V}\times\boldsymbol{B}_0\,,$$

$$B_\parallel = \frac{\boldsymbol{B}_0\cdot\boldsymbol{V}}{V}\,, \qquad \boldsymbol{B}_\perp = \gamma_L\frac{(\boldsymbol{V}\times\boldsymbol{B}_0)\times\boldsymbol{V}}{V^2}\,,$$

$$\gamma_L = (1-V^2)^{-1/2}.$$

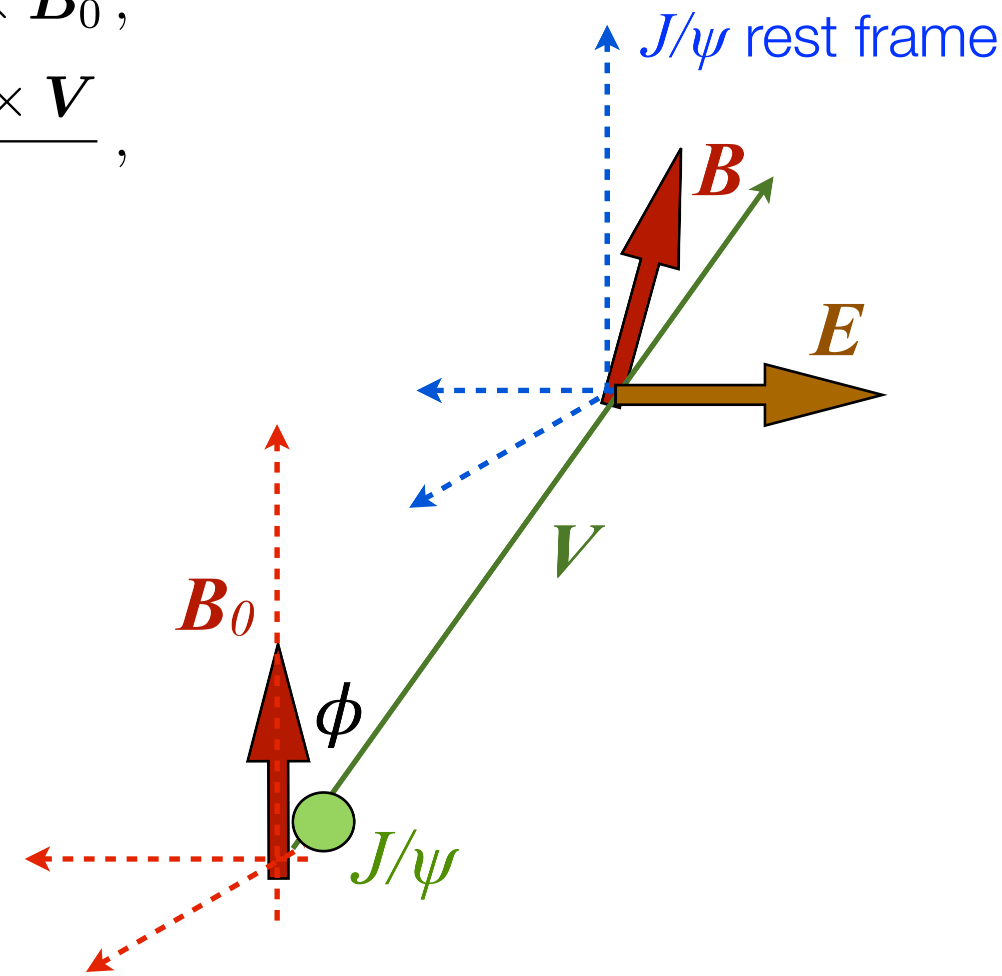
$$B = B_0\sqrt{\cos^2\phi(1-\gamma_L^2)+\gamma_L^2}\,,$$

$$E = B_0\gamma_L V\sin\phi\,.$$

$$\rho = \frac{E}{B} = \frac{\gamma_L V\sin\phi}{\sqrt{\cos^2\phi(1-\gamma_L^2)+\gamma_L^2}}\,.$$

✳ $0 \leq \boldsymbol{\rho} \leq 1$

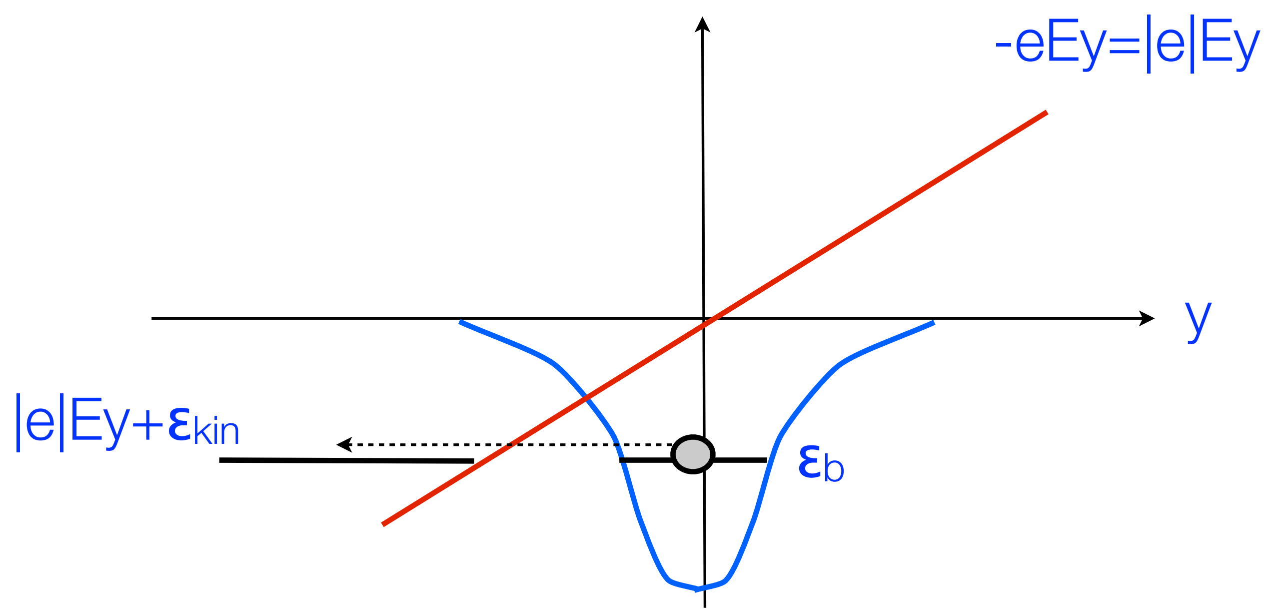
✳ For $V \perp B_0$: $\boldsymbol{\rho} = V$



IONIZATION MECHANISM

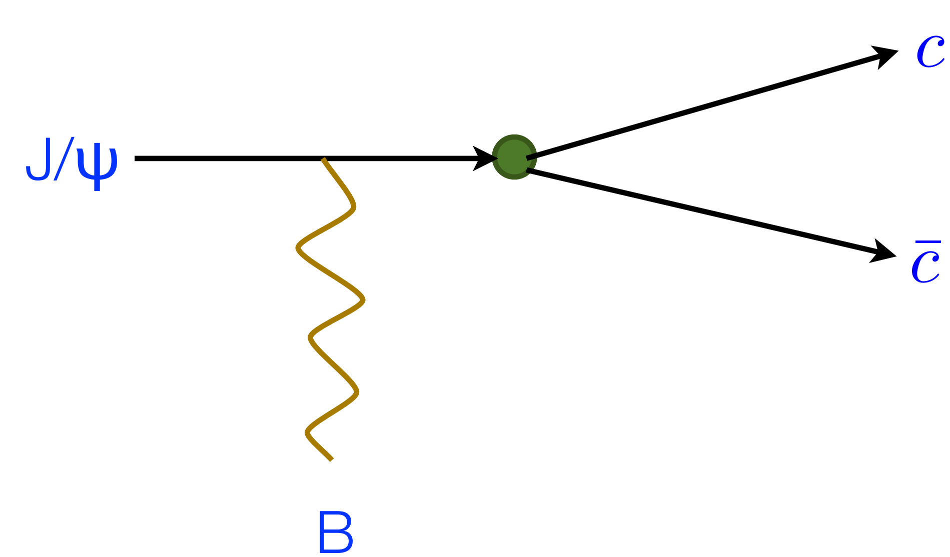
Quarkonium rest frame

There is finite quantum probability for the anti-quark ($e<0$) to tunnel through the potential barrier and go to $y \to -\infty$.



This is $J/\psi + E \to D^+D^-$ decay.

Lab frame



Magnetic field supplies momentum,
while moving quarkonium supplies energy.

J/Ψ REST FRAME: ROLE OF B

In the rest frame decay happens only due to electric field (magnetic field does no work). What is the role of magnetic field?

$$\varepsilon_0 = \sqrt{m^2 + (\boldsymbol{p} - e\boldsymbol{A})^2} + e\varphi = \sqrt{m^2 + (p_x + eBy)^2 + p_y^2 + p_z^2} - eEy$$

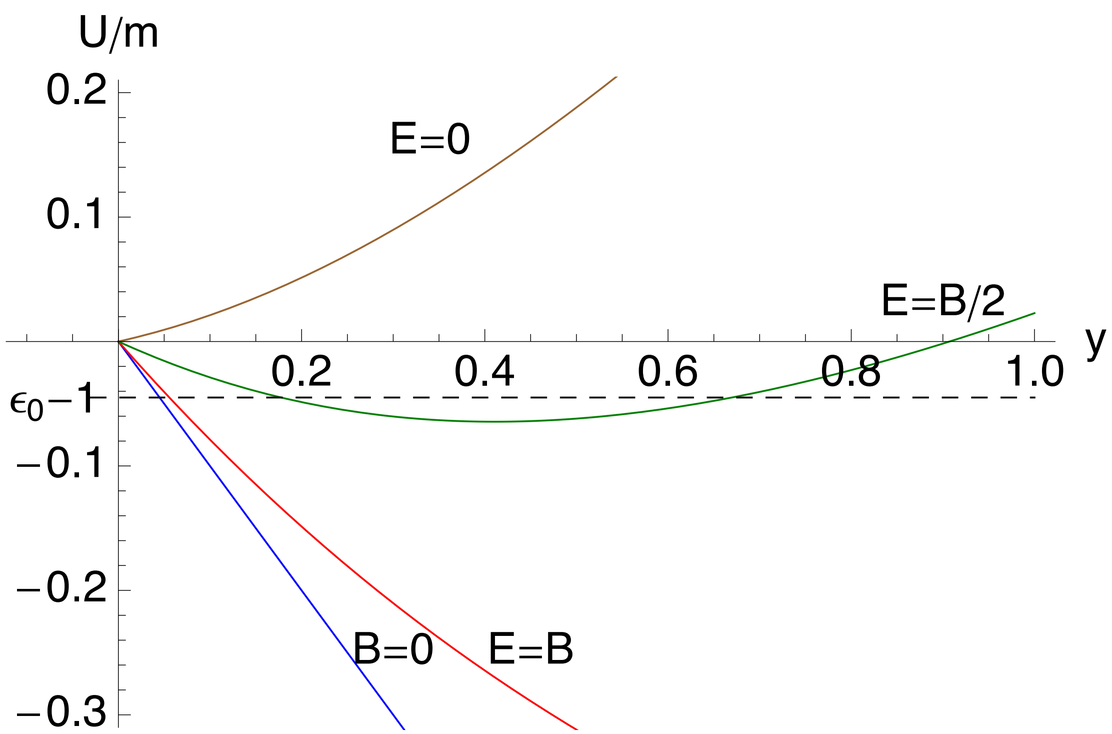


FIG. 1: Effective potential $U(y) = \sqrt{m^2 + (p_x + eBy)^2 + p_y^2} - eEy - \sqrt{m^2 + p_x^2 + p_y^2}$ for $p_y = 0$, $p_x = m/6$, $B = m$ (except the blue line where $B = 0$). The width of the potential barrier decreases with E and increases with B. $1 - \epsilon_0$ corresponds to the binding energy in units of m.

DETAILS OF THE WKB CALCULATION

"0"th approximation: if $(M\varepsilon_b)^{1/2}R \ll 1$, we can treat the potential as a short-range one.

(Radius of quarkonium $R \propto \alpha_s$)

Transmission coefficient $w = e^{-2\int_0^{y_1}\sqrt{-p_y^2}dy} \equiv e^{-f}$ where y_1 is the turning point.

$$p_y^2 = -e^2(B^2 - E^2)(y - y_1)(y - y_2)$$

Transmission coefficient depends on transverse momentum $q=p_x/m$. In the saddle-point approximation only contribution at $q=q_m$ contributes such that

$$\frac{\partial f(q_m)}{\partial q_m} = 0$$

The leading exponent:

$$f_m = \frac{m^2\tau_0\rho}{eE\sqrt{1-\rho^2}}[1 - \epsilon_0(\epsilon_0 - q_m\rho)]$$

Popov, Karnakov, Mur (1998)
Marasinghe, Tuchin (2011)

τ is a certain known function of ρ and q_m

NON-RELATIVISTIC APPROXIMATION

- If quark's momentum is small $p \ll m$, then $\varepsilon_0 \approx m$ or $\varepsilon_b = m - \varepsilon_0 \ll m$

Also $v \sim E/B = \rho \ll 1$

- Expanding in ρ and ε_b/m we get

$$f_m = \frac{2m^2(2\epsilon_b)^{3/2}}{3eE} g(\gamma)$$

Keldysh (1965)

Keldysh function $g(\gamma) = \frac{3\tau_0}{2\gamma}\left[1 - \frac{1}{\gamma}\left(\frac{\tau_0^2}{\gamma^2} - 1\right)^{1/2}\right]$

where $\gamma = \frac{\sqrt{2\epsilon_b}}{\rho}$ is the adiabaticity parameter

Weak binding: $\varepsilon_b \ll \rho^2 \Rightarrow \gamma \ll 1$ $\quad w = \exp\left\{-\frac{2}{3}\frac{(2\varepsilon_b m)^{3/2}}{meE}\right\}$

If $\gamma << 1$ the field can be considered as adiabatic (slowly varying)

NR APPROXIMATION: SPIN

Spin contribution can be calculated from Dirac equation

$$\left[(\varepsilon - e\varphi)^2 - (\mathbf{p} - e\mathbf{A})^2 - m^2 + e\mathbf{\Sigma}\cdot\mathbf{B} - ie\alpha\cdot\mathbf{E}\right]\psi = 0$$

Problem: Σ_z and α_y do not commute ⇒ need to square the above equation to apply WKB

- Much easier in **non-relativistic** approximation:

$$\frac{1}{2m}\left[(p_x + eBy)^2 + p_y^2\right] - eEy - \frac{\mu}{s}\mathbf{s}\cdot\mathbf{B} = -\varepsilon_b \qquad \mu = \frac{e\hbar}{2mc}$$

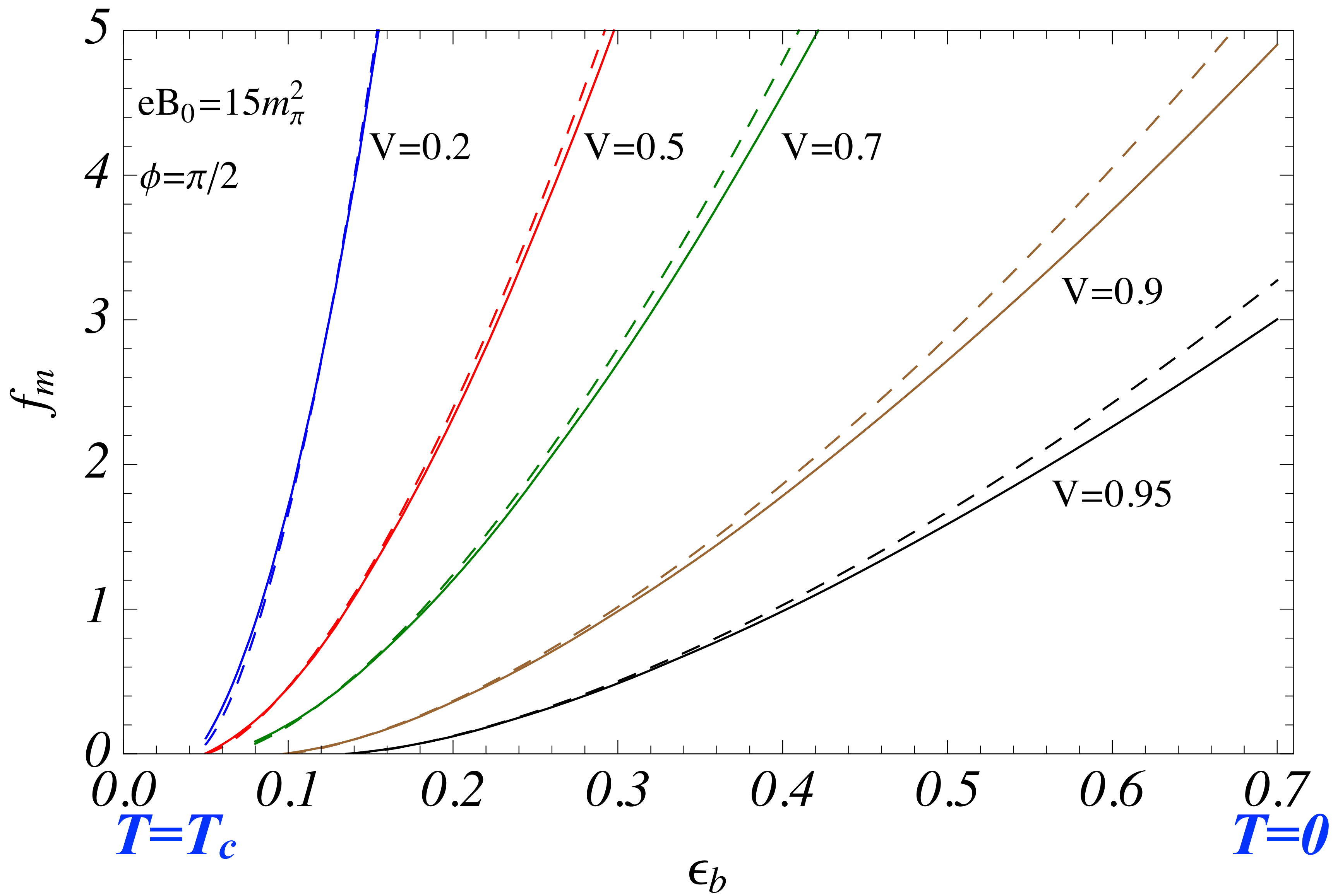
$$p_y^2 = 2m\left(-\varepsilon_b + \frac{\mu}{s}\mathbf{s}\cdot\mathbf{B} + eEy\right) - (p_x + eBy)^2$$

Simple replacement takes spin into account $\varepsilon_b \to \varepsilon_b' = \varepsilon_b - \frac{\mu}{s}\mathbf{s}\cdot\mathbf{B}$

How good is non-relativistic approximation?

$w = e^{-f_m}$

NR APPROXIMATION IS VERY GOOD!



In NR approximation we can take into account the finite width of the quarkonium wave function and the Coulomb interaction between the q and anti-q

FIG. 3: f_m versus ϵ_b for different values of J/ψ velocity V. Dissociation probability is $w = \exp\{-f_m\}$. Magnetic field in the lab frame is taken to be $eB_0 = 15m_\pi^2$. J/ψ moves perpendicularly to the field (i.e. in the reaction plane). Solid lines correspond to the full relativistic calculation, dashed lines to the non-relativistic approximation. J/ψ binding energy in vacuum corresponds to $\epsilon_b = 0.68$.

CHIRAL-MAGNETIC EFFECT

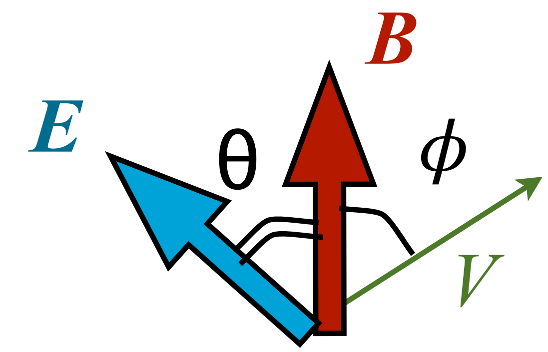
If metastable P and CP-odd bubbles are induced by axial anomaly in hot nuclear matter, then in the presence of external magnetic field **B_0 the bubble generates an electric field E_0** which is parallel to the magnetic one.

$$\boldsymbol{E}_0 = -N_c \sum_f \frac{e_f^2}{4\pi^2}\,\frac{\Theta}{N_f}\,\boldsymbol{B}_0 = -\frac{2}{3}\frac{\alpha\,\Theta}{\pi}\boldsymbol{B}_0$$

Θ fluctuates from event to event. CME is a macroscopic manifestation of this effect - separation of electric charges with respect to the reaction plane.

In the presence of finite E_0 decay rate of quarkonia increases. What is the value of Θ which can have an observable effect on J/ψ dissociation?

BOOST FROM LAB TO QUARKOINUM REST FRAME II



Lab frame: E_0 , B_0

J/ψ rest frame: E, B

$$\boldsymbol{E} = E_0 \left\{ \gamma_L(\boldsymbol{b}_0 + \rho_0^{-1}\boldsymbol{V} \times \boldsymbol{b}_0) - (\gamma_L - 1)\boldsymbol{V}\frac{\boldsymbol{V}\cdot\boldsymbol{b}_0}{V^2} \right\}$$

$$\boldsymbol{B} = B_0 \left\{ \gamma_L(\boldsymbol{b}_0 - \rho_0\boldsymbol{V} \times \boldsymbol{b}_0) - (\gamma_L - 1)\boldsymbol{V}\frac{\boldsymbol{V}\cdot\boldsymbol{b}_0}{V^2} \right\}$$

$$\rho_0 = E_0/B_0 = 2\alpha|\Theta|/3\pi$$

$$\cos\theta = \frac{\boldsymbol{E}\cdot\boldsymbol{B}}{EB} = \frac{1}{\sqrt{[1+\gamma_L^2(\boldsymbol{b}_0\times\boldsymbol{V})^2(1+\rho_0^{-2})][1+\gamma_L^2(\boldsymbol{b}_0\times\boldsymbol{V})^2(1+\rho_0^2)]}}$$

DISSOCIATION RATE

$$w = \frac{8\varepsilon_b}{\epsilon}\, P(\gamma,\theta)\, C^2(\gamma,\theta)\, e^{-\frac{2}{3\epsilon} g(\gamma,\theta)}$$

where the leading quasi-classical exponent is

$$g = \frac{3\tau_0}{2\gamma}\left[1 - \frac{1}{\gamma}\left(\frac{\tau_0^2}{\gamma^2} - 1\right)^{1/2}\sin\theta - \frac{\tau_0^2}{3\gamma^2}\cos^2\theta\right]$$

- P is a prefactor for the S-wave state of quarkonium accounting for a deviation from the quasi-classics.
- C accounts for the Coulomb interaction between the valence quark and anti-quark.

Explicit expression for P and C in NR approximation was derived by Popov, Karnakov, Mur (1998)

AVERAGING

- We assumed that the dissociation process happens entirely inside a bubble and that Θ=const inside the bubble. However, many bubbles are produced in a single heavy-ion collision, such that $\langle\Theta\rangle=0$. Can we neglect other bubbles?

- Bubble size R_0 ~ sphaleron size ~ chromo-magnetic screening length ~ $1/g^2T$

  Quarkonium size $R_J \sim \alpha_s/\varepsilon_b \Rightarrow R_J/R_0 \sim \alpha_s^2\ T/\varepsilon_b \ll 1$

  provided that T is not too high and not too close to T_c

- The rate w depends on $|\rho_0|^2$ and is not sensitive to the sign of E_0 and Θ. Thus after averaging $w(\Theta)$ over all events with some distribution of Θ's will get $w(\Theta_{eff})$.

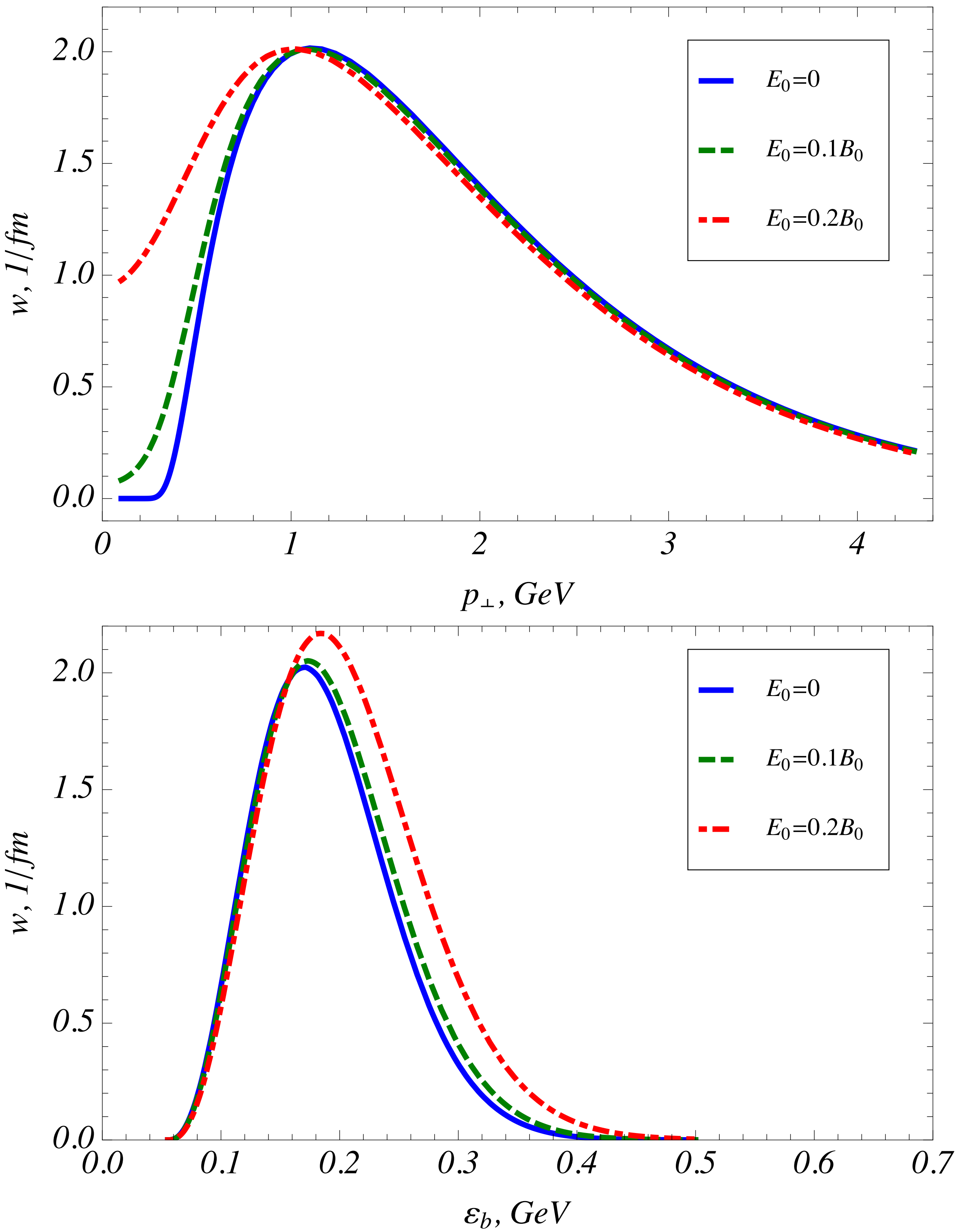


FIG. 1: Dissociation rate of J/ψ at $eB_0 = 15m_\pi^2$, $\phi = \pi/2$ (in the reaction plane), $\eta = 0$ (midrapidity) as a function of (a) $P_\perp$ at $\varepsilon_b = 0.16$ GeV and (b) ε_b at $P_\perp = 1$ GeV.

- *J/ψ*'s with $p_T > 0.5$ *GeV* are not stable

- There is a significant (measurable) effect of CP-odd bubbles only if $E_0 > 0.1\ B_0 \Rightarrow$

$$|\Theta|/\pi \sim 0.1/\alpha$$

This is about an order of magnitude larger than Θ required to explain CME.

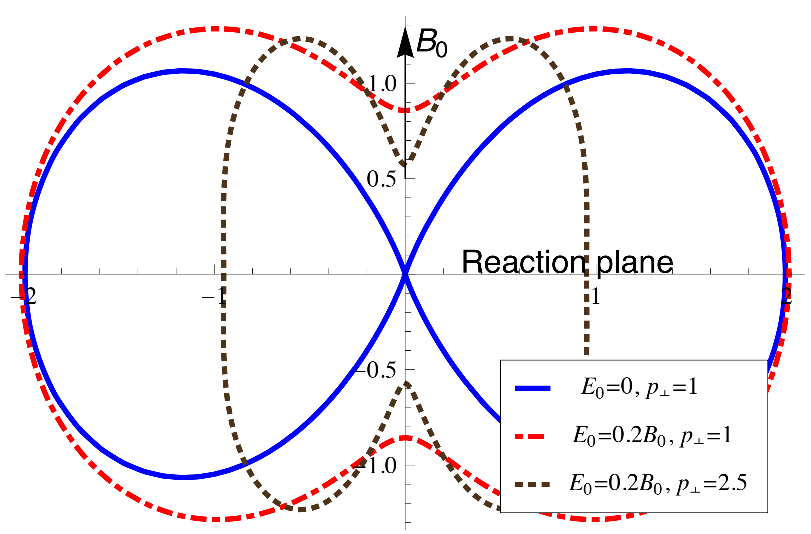
AZIMUTHAL ASYMMETRY

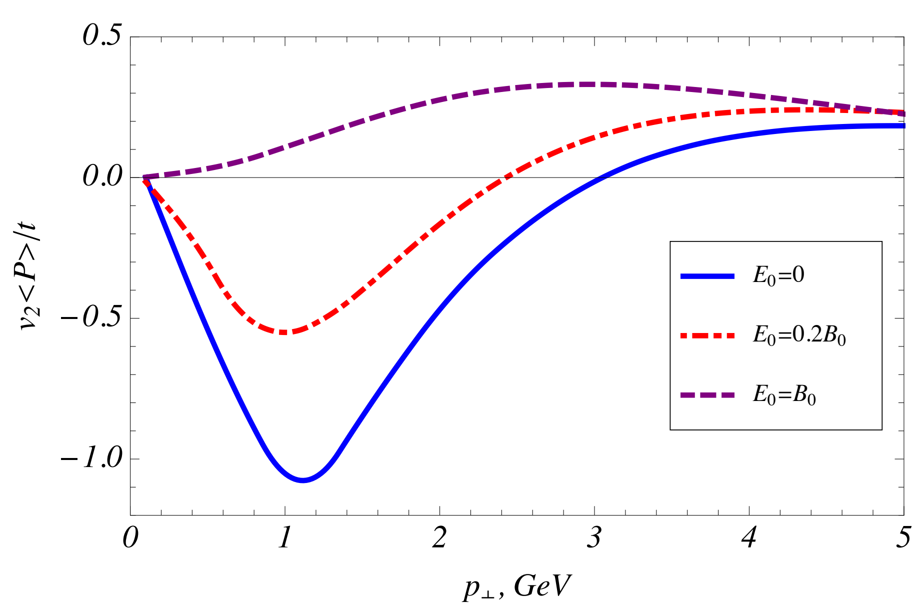
Spectrum of quarkonia surviving in EM field is proportional to survival probability $P=1-wt$

Let $\chi=\pi/2-\varphi$ be the angle between the quarkonium velocity and the reaction plane

$$P(\chi)=\frac{1}{2}P_0+\sum_{n=1}^{\infty}P_n\,\cos(n\chi)\,,\quad P_n=\frac{1}{\pi}\int_{-\pi}^{\pi}P(\chi)\,\cos(n\chi)\,d\chi$$

Ellipticity of the distribution $$v_2=\frac{P_2}{\frac{1}{2}P_0}=\frac{\int_{-\pi}^{\pi}(1-wt)\,\cos 2\chi\,d\chi}{\pi\,\langle P\rangle}=-\frac{t}{\pi\,\langle P\rangle}\int_{-\pi}^{\pi}w\,\cos 2\chi\,d\chi$$





AZIMUTHAL ASYMMETRY

Experimental result for v_2 of J/ψ is consistent with zero. Possible reasons:

1.Magnetic field is significantly weaker and short-lived than we assumed, which however contradicts observations of the CME.

2. Almost none of J/ψ's produced in the center of QGP survive. Rather they originate from the peripheral regions.

CONCLUSIONS I

Magnetic field destroys J/ψ's. This effect grows with p_T and strongly depends on azimuthal angle.

CHIRAL MAGNETIC EFFECT FROM QED+ ?

What if no strong QED effect is observed, but the Chiral Magnetic Effect survives??

No physical principle prohibits appearance of “irrelevant” terms in the QED lagrangian. E.g.:

$$\mathcal{L}_{\mathrm{QED+}} = \frac{1}{M^4}(F\tilde{F})F^2$$

This term disappears in weak fields and/or long distances, but can be large if $F \sim M$, where M comes from some exotic small-distance physics.

This term would produce the CME even at $\theta=0$

High intensity fields is an opportunity to search for a new physics.

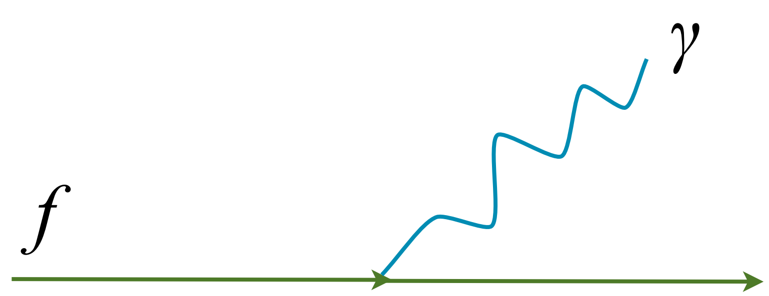
Synchrotron radiation

-source of photons (shinning of QGP in magnetic field)

-contributes to quark energy loss

ANGULAR DISTRIBUTION OF RADIATION

Synchrotron radiation:



$$f(e_f, j, p) \to f(e_f, k, q) + \gamma(\boldsymbol{k})$$

QGP is transparent to the emitted electromagnetic radiation because its absorption coefficient is suppressed by α^2.

Spacing between the Landau levels ~ ***eB/ε***, while their thermal width ~ ***T***. When ***eB/ε≳T*** it is essential to account for quantization of fermion spectra.

Fermion spectrum quantization is important not only for hard and electromagnetic probes but also for the bulk properties of QGP.

KINEMATICS

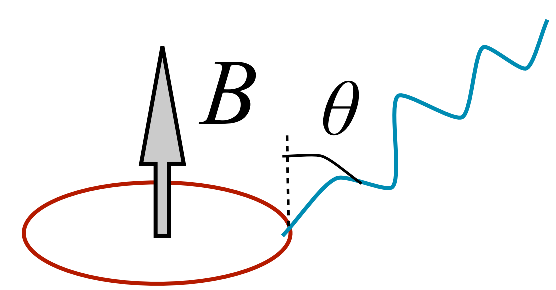
$$\varepsilon_j = \sqrt{m^2 + p^2 + 2je_f B}\,, \quad \varepsilon_k = \sqrt{m^2 + q^2 + 2ke_f B}$$

j (k) is the quantum number of Landau orbit of *initial (final)* charged fermion.

p (q) is the projection of *initial (final)* fermion momentum on the direction of *B*

Magnetic filed does no work, thus energy is conserved. Magnetic Lorentz force has no component along the *B*-direction:

$$\varepsilon_j = \omega + \varepsilon_k\,, \quad p = q + \omega\cos\theta$$



Angular distribution of the power spectrum:

$$\frac{dI^j}{d\omega d\Omega} = \sum_f \frac{z_f^2\alpha}{\pi}\omega^2 \sum_{k=0}^{j} \Gamma_{jk}\left\{|\mathcal{M}_\perp|^2 + |\mathcal{M}_\parallel|^2\right\}\delta(\omega - \varepsilon_j + \varepsilon_k)$$

Matrix elements for synchrotron transitions corresponding to photon polarization perpendicular and parallel to B

Sokolov, Ternov (1968) and others

$$4\varepsilon_j\varepsilon_k|\mathcal{M}_\perp|^2 = (\varepsilon_j\varepsilon_k - pq - m^2)[I^2_{j,k-1} + I^2_{j-1,k}] + 2\sqrt{2je_fB}\sqrt{2ke_fB}[I_{j,k-1}I_{j-1,k}]\,.$$

$$\begin{aligned}4\varepsilon_j\varepsilon_k|\mathcal{M}_\parallel|^2 = &\cos^2\theta\big\{(\varepsilon_j\varepsilon_k - pq - m^2)[I^2_{j,k-1} + I^2_{j-1,k}] - 2\sqrt{2je_fB}\sqrt{2ke_fB}[I_{j,k-1}I_{j-1,k}]\big\}\\ &- 2\cos\theta\sin\theta\big\{p\sqrt{2ke_fB}[I_{j-1,k}I_{j-1,k-1} + I_{j,k-1}I_{j,k}]\\ &+ q\sqrt{2je_fB}[I_{j,k}I_{j-1,k} + I_{j-1,k-1}I_{j,k-1}]\big\}\\ &+ \sin^2\theta\big\{(\varepsilon_j\varepsilon_k + pq - m^2)[I^2_{j-1,k-1} + I^2_{j,k}] + 2\sqrt{2je_fB}\sqrt{2ke_fB}(I_{j-1,k-1}I_{j,k})\big\}\end{aligned}$$

$$I_{j,k} \equiv I_{j,k}(x) = (-1)^{j-k}\sqrt{\frac{k!}{j!}}e^{-\frac{x}{2}}x^{\frac{j-k}{2}}L_k^{j-k}(x).$$

Laguerre polynomials

$$x = \frac{\omega^2}{2e_fB}\sin^2\theta$$

ANGULAR DISTRIBUTION OF RADIATION

Integrate over photon energies ω keeping in mind that ε_k is a function of ω.

$$\frac{dI^j}{d\Omega} = \sum_f \frac{z_f^2 \alpha}{\pi} \sum_{k=0}^{j} \frac{\omega^*(\varepsilon_j - \omega^*)}{\varepsilon_j - p\cos\theta - \omega^* \sin^2\theta} \Gamma_{jk} \left\{ |\mathcal{M}_\perp|^2 + |\mathcal{M}_\parallel|^2 \right\}$$

where the photon energy is fixed by the delta-function as

$$\omega^* = \frac{1}{\sin^2\theta} \left\{ (\varepsilon_j - p\cos\theta) - \left[(\varepsilon_j - p\cos\theta)^2 - 2e_f B(j-k)\sin^2\theta \right]^{1/2} \right\}$$

PHOTON NUMBER SPECTRUM

We are interested in the photon number spectrum radiated from QGP

$$\frac{dN^{\text{synch}}}{dtd\Omega d\omega} = \sum_f \int_{-\infty}^{\infty} dp \frac{e_f B(2N_c)V}{2\pi^2} \sum_{j=0}^{\infty} \sum_{k=0}^{j} \frac{dI^j}{\omega d\omega d\Omega}(2-\delta_{j,0}) f(\varepsilon_j)[1-f(\varepsilon_k)]$$

$$f(\varepsilon) = \frac{1}{e^{\varepsilon/T}+1}$$

To take integral over p write

$$\delta(\omega - \varepsilon_j + \varepsilon_k) = \sum_{\pm} \frac{\delta(p - p_\pm^*)}{\left|\frac{p}{\varepsilon_j} - \frac{q}{\varepsilon_k}\right|}$$

$$p_\pm^* = \Big\{ \cos\theta(m_j^2 - m_k^2 + \omega^2 \sin^2\theta) \pm \sqrt{[(m_j+m_k)^2 - \omega^2\sin^2\theta][(m_j-m_k)^2 - \omega^2 \sin^2\theta]} \Big\} / (2\omega \sin^2\theta)$$

$$m_j^2 = m^2 + 2je_f B\,, \quad m_k^2 = m^2 + 2ke_f B$$

$p_\pm$ is real in two cases:

$$\text{(i)}\; m_j - m_k \geq \omega \sin\theta\,, \quad \text{or} \quad \text{(ii)}\; m_j + m_k \leq \omega \sin\theta$$

synchrotron radiation

one-photon pair annihilation

In case (i) the $j \to k$ transition must satisfy

$$\omega \leq \omega_{s,jk} \equiv \frac{m_j - m_k}{\sin\theta} = \frac{\sqrt{m^2 + 2je_fB} - \sqrt{m^2 + 2ke_fB}}{\sin\theta}$$

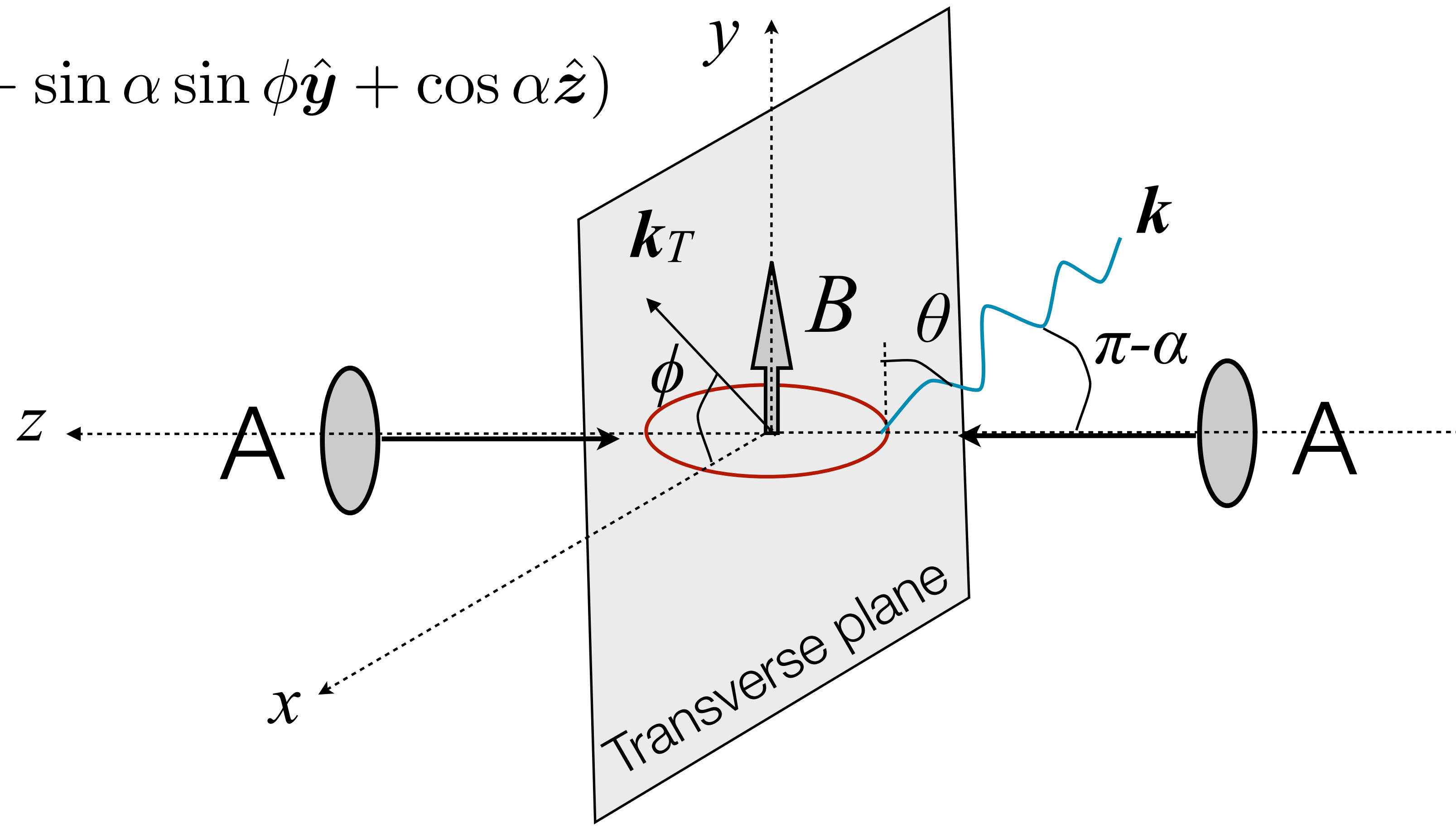
in particular $j=k$ transition is forbidden.

Spectral distribution of the synchrotron radiation rate per unit volume:

$$\frac{dN^{\text{synch}}}{V\,dt\,d\Omega\,d\omega} = \sum_f \frac{2N_c z_f^2 \alpha}{\pi^3} e_f B \sum_{j=0}^{\infty} \sum_{k=0}^{j} \omega (1 + \delta_{k0})\, \vartheta(\omega_{s,ij} - \omega) \int dp \sum_{\pm} \frac{\delta(p - p_\pm^*)}{\left|\frac{p}{\varepsilon_j} - \frac{q}{\varepsilon_k}\right|}$$
$$\times \left\{ |\mathcal{M}_\perp|^2 + |\mathcal{M}_\parallel|^2 \right\} f(\varepsilon_j)[1 - f(\varepsilon_k)]\,,$$

HIGH-ENERGY REFERENCE FRAME

$$\boldsymbol{k} = \omega(\sin\alpha\cos\phi\hat{\boldsymbol{x}} + \sin\alpha\sin\phi\hat{\boldsymbol{y}} + \cos\alpha\hat{\boldsymbol{z}})$$



$$\hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{y}} = \cos\theta \;\Rightarrow$$

$$\cos\theta = \sin\alpha\sin\phi$$

Thus, azimuthal dependence (ϕ) of the spectrum is an artifact of the frame choice!

$$k_\perp = \sqrt{k_x^2 + k_y^2} = \frac{\omega\cos\theta}{\sin\phi}\,, \quad y = -\ln\tan\frac{\alpha}{2}$$

$$\frac{dN^{\rm synch}}{dV\,dt\,d^2k_\perp dy} = \omega\frac{dN^{\rm synch}}{dV\,dt\,d^3k} = \frac{dN^{\rm synch}}{dV\,dt\,\omega d\omega d\Omega}$$

SYNCHROTRON SPECTRUM

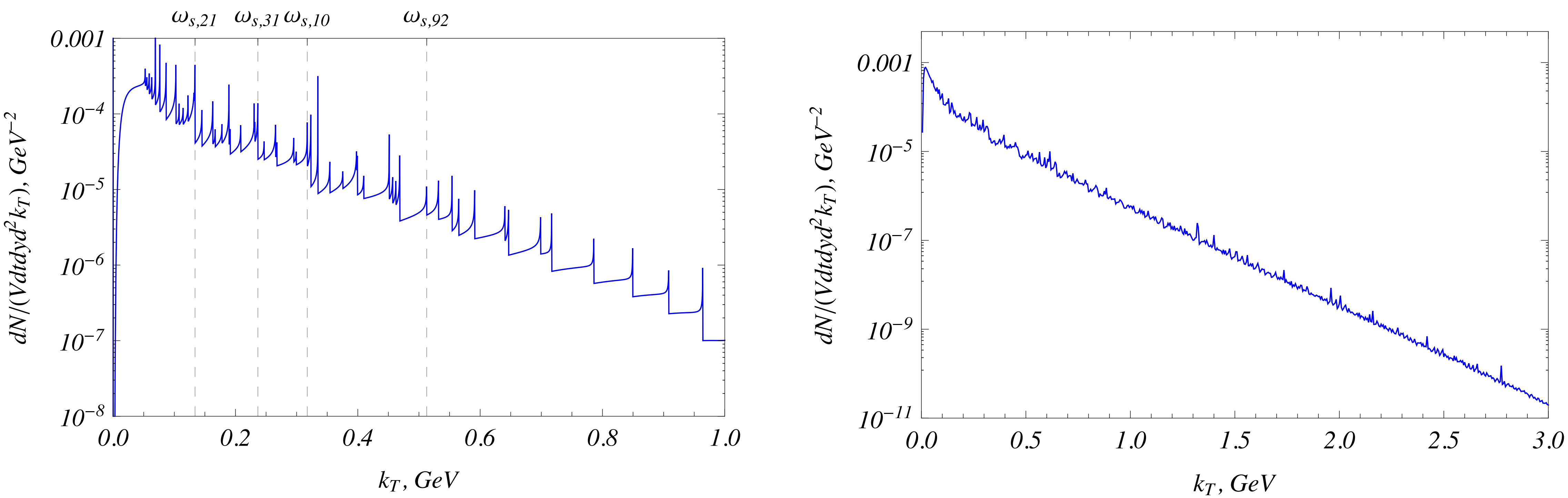


FIG. 1: Spectrum of synchrotron radiation by u quarks at $eB = m_\pi^2$, $y = 0$, $\phi = \pi/3$: (a) contribution of 10 lowest Landau levels $j \leq 10$; several cutoff frequencies are indicated; (b) summed over all Landau levels. $m_u = 3$ MeV, $T = 200$ MeV.

ANGULAR DISTRIBUTION OF SR

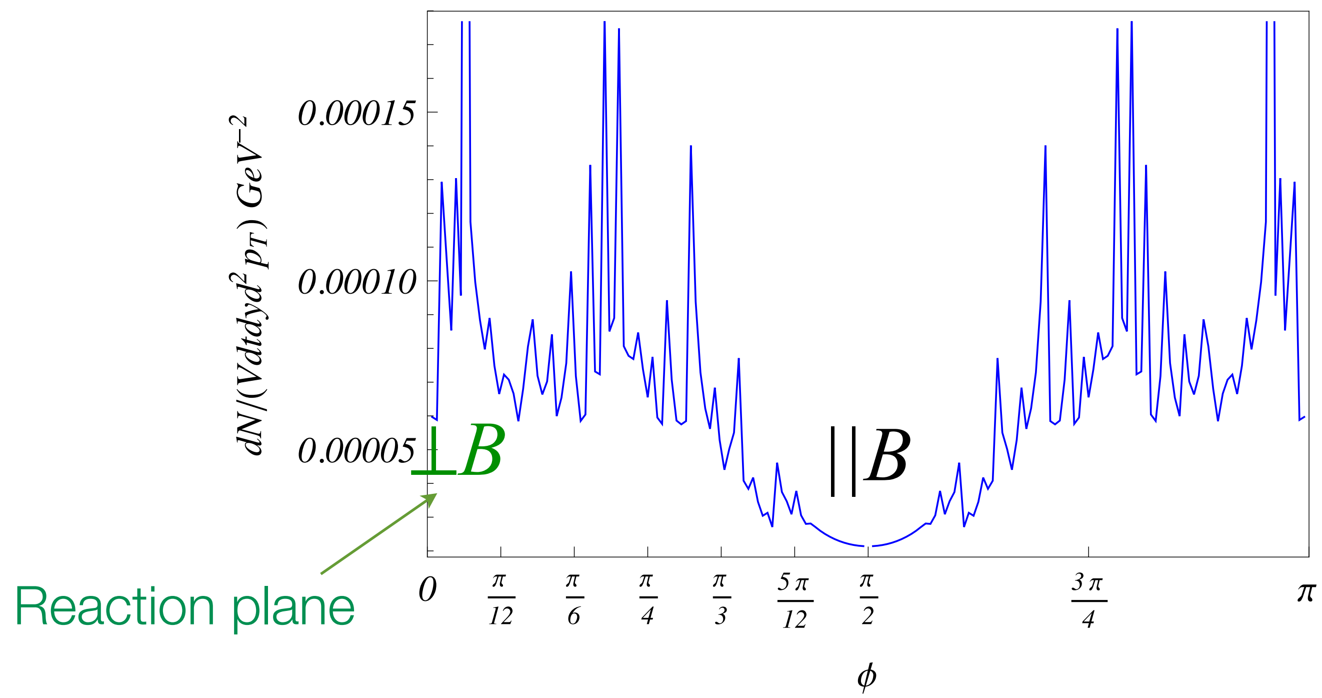


FIG. 2: Azimuthal distribution of synchrotron radiation by u-quarks at $k_\perp = 0.2$ GeV, $eB = m_\pi^2$, $y = 0$. $m_u = 3$ MeV.

This distribution implies that $v_2 > 0$ (to be calculated)

SYNCHROTRON SPECTRUM

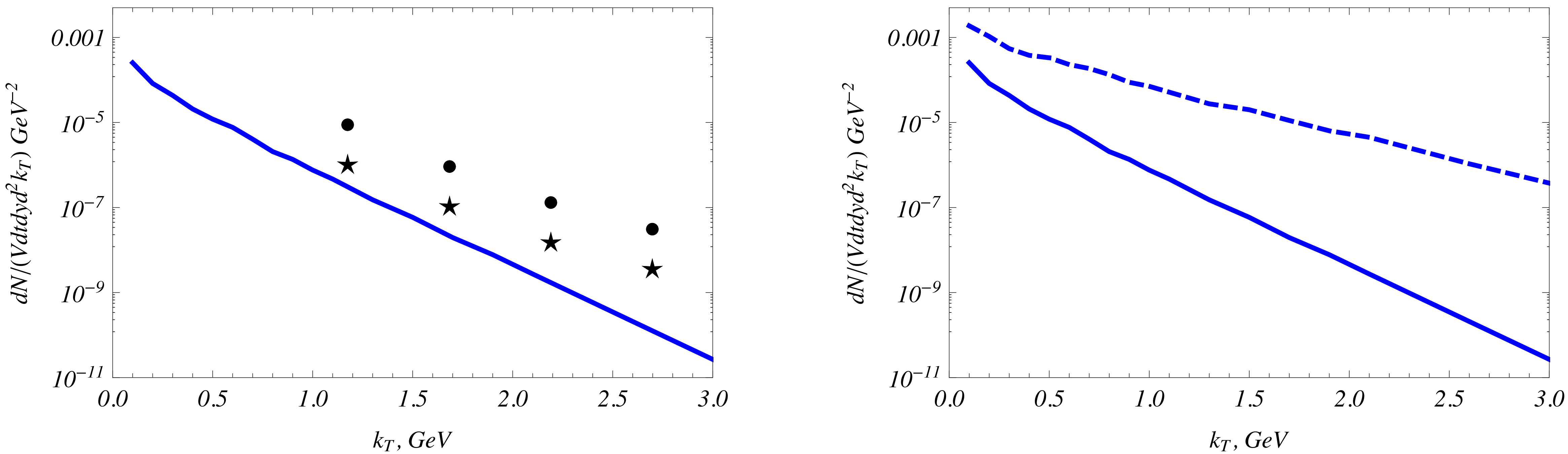


FIG. 3: Azimuthal average of the synchrotron radiation spectrum of u,d,s quarks and their corresponding antiquarks. (a) $eB = m_\pi^2$, $y = 0$ compared to the experimental data from [39] divided by $Vt = 25\pi$ fm^4 (dots) and $Vt = 9 \times 25\pi$ fm^4 (stars), (b) $eB = m_\pi^2$, $T = 200$ MeV, $y = 0$ (solid line) compared to $eB = 15m_\pi^2$, $T = 400$ MeV, $y = 0$ (dashed line). $m_u = 3$ MeV, $m_d = 5$ MeV, $m_s = 92$ MeV.

HOW MANY LANDAU LEVELS CONTRIBUTE?

$$\frac{dN^{\text{synch}}}{dt d\Omega d\omega} = \sum_f \int_{-\infty}^{\infty} dp \frac{e_f B(2N_c)V}{2\pi^2} \sum_{j=0}^{j_{\max}} \sum_{k=0}^{j} \frac{dI^j}{\omega d\omega d\Omega}(2-\delta_{j,0}) f(\varepsilon_j)[1-f(\varepsilon_k)]$$

f	u	u	u	u	u	u	s	u	u	s
eB/m_π^2	1	1	1	1	1	1	1	15	15	15
T, GeV	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.4	0.4	0.4
ϕ	$\frac{\pi}{3}$	$\frac{\pi}{3}$	$\frac{\pi}{3}$	$\frac{\pi}{3}$	$\frac{\pi}{6}$	$\frac{\pi}{12}$	$\frac{\pi}{3}$	$\frac{\pi}{3}$	$\frac{\pi}{3}$	$\frac{\pi}{3}$
$k_\perp$, GeV	0.1	1	2	3	1	1	1	1	2	1
x	0.096	9.6	38	86	29	35	19	0.64	2.6	1.3
$j_{\max}$	30	40	90	150	120	200	90	8	12	16

TABLE I: The upper summation limit in (18) that yields the 5% accuracy. $j_{\max}$ is the highest Landau level of the initial quark that is taken into account at this accuracy. Throughout the table $y = 0$.

Large j,k correspond to quasi-classical limit.

HIGH PHOTON ENERGY APPROXIMATION

$$\omega \gg m\sqrt{mT/e_f B \sin\theta}$$

Photon spectrum in HIC satisfies this condition, except very close to the B-direction ($\theta \approx 0$). Fortunately it contributes little to the average, but causes numerical trouble when calculating harmonics.

$$\frac{dN^{\rm synch}}{V\,dt\,d\Omega\,d\omega} = \sum_f \frac{z_f^2\alpha}{\pi}\frac{n_f \omega m^2}{4T^3}\sqrt{\frac{e_f BT\sin\theta}{m^3}}e^{-\omega/T}$$

Quark number density/T^3 is T-independent $n_f = \frac{2\cdot 2N_c\, e_f B}{4\pi^2}\sum_{j=0}^{\infty}\int_{-\infty}^{\infty} dp\, e^{-\varepsilon_j/T} \approx \frac{4N_c}{\pi^2}T^3$

☞ This part of the spectrum increases with magnetic field strength as $B^{1/2}$ and and with temperature as $T^{1/2}e^{\omega/T}$. Therefore, variation of the spectrum with T is much stronger than with B.

TEMPERATURE DEPENDENCE

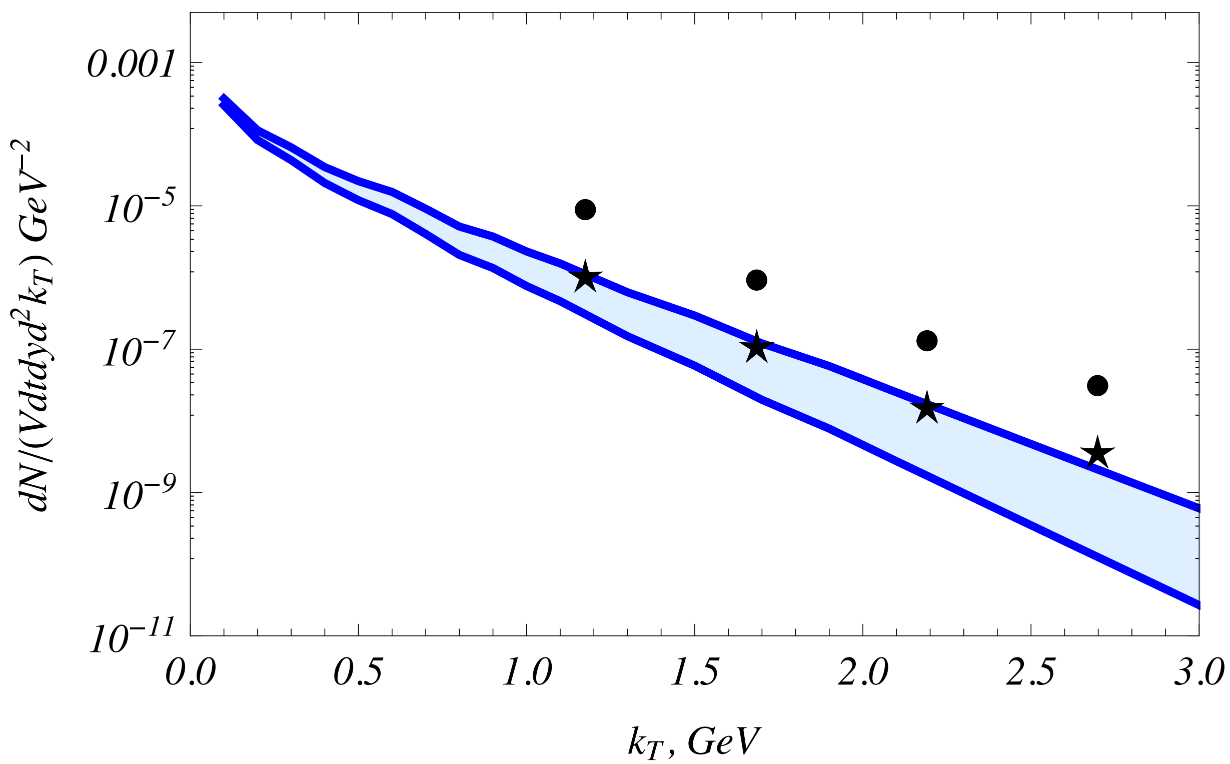
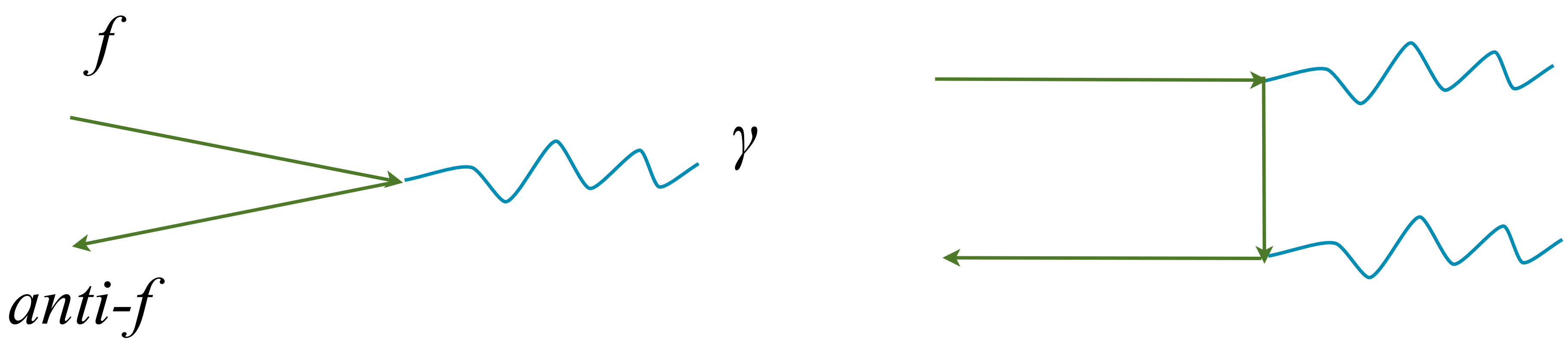


FIG. 4: Variation of the synchrotron spectrum with plasma temperature. Lower line: $T = 200$ MeV, upper line: $T = 250$ MeV. Other parameters are the same as in Fig. 3(a).

Photon spectrum is very sensitive to the QGP temperature in the first 1-2 fm of its existence.

PAIR ANNIHILATION



One and two-photon annihilation: At $eB \gg m^2$ one-photon annihilation dominates.

One-photon annihilation is a cross-channel of synchrotron radiation. The corresponding matrix elements are straightforward to calculate.

$$\frac{dN^{\text{annih}}}{V\,dt\,d\omega\,d\Omega} = \sum_f \frac{\alpha z_f^2 \omega N_c}{4\pi e_f B} \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \int dp\, \frac{2e_f B}{2\pi^2} f(\varepsilon_j) \int dq\, \frac{2e_f B}{2\pi^2} f(\varepsilon_k)$$

$$\times \delta(p + q - \omega\cos\theta)\,\delta(\varepsilon_j + \varepsilon_k - \omega)\{|\mathcal{T}_\perp|^2 + |\mathcal{T}_\parallel|^2\}$$

PAIR ANNIHILATION SPECTRUM

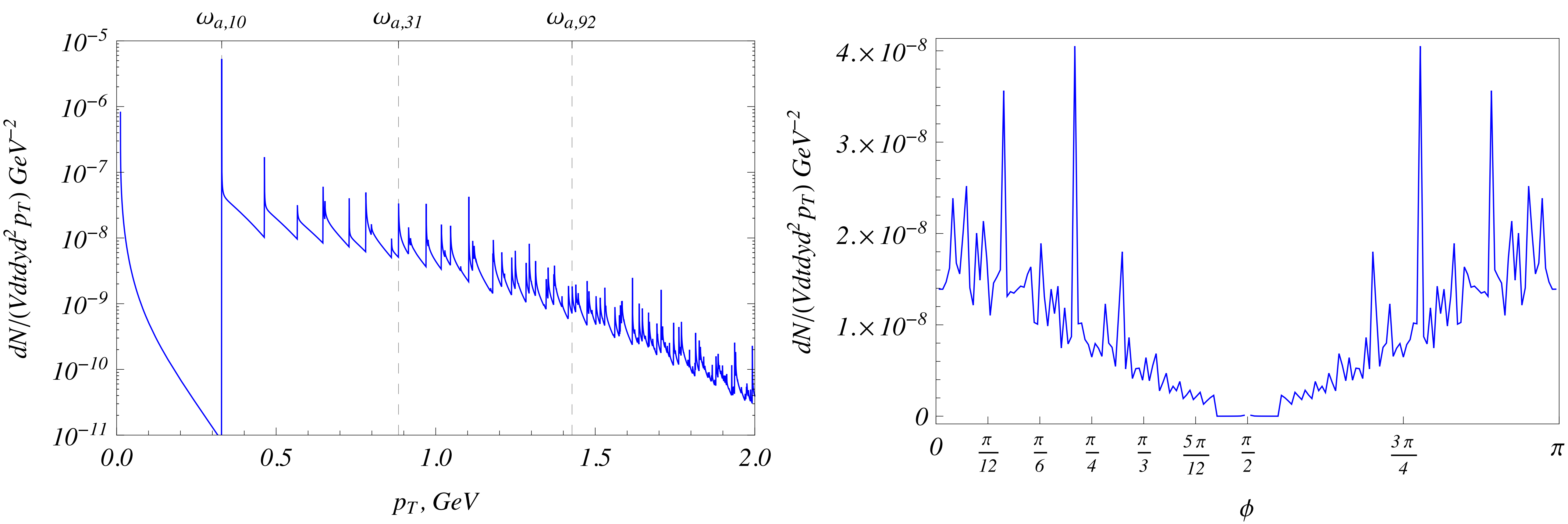


FIG. 5: Photon spectrum in one-photon annihilation of u and $\bar{u}$ quarks. $eB = m_\pi^2$, $y = 0$. (a) $k_\perp$-spectrum at $\phi = \pi/3$, (b) azimuthal angule distribution at $k_\perp = 1$ GeV.

Pair annihilation is numerically much smaller than synchrotron radiation.

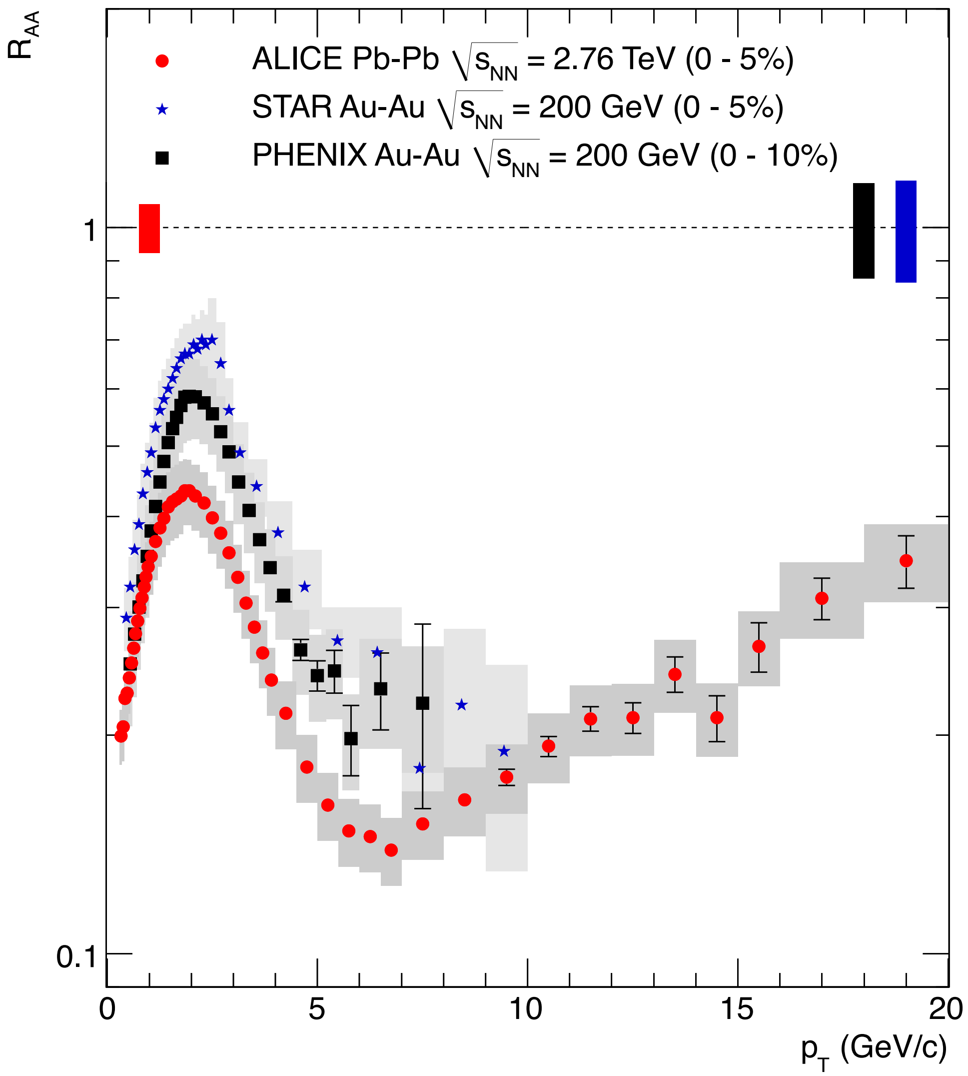
CONCLUSIONS II

Photon production by QGP due to its interaction with external magnetic field give a considerable contribution to the total photon multiplicity in heavy-ion collisions.

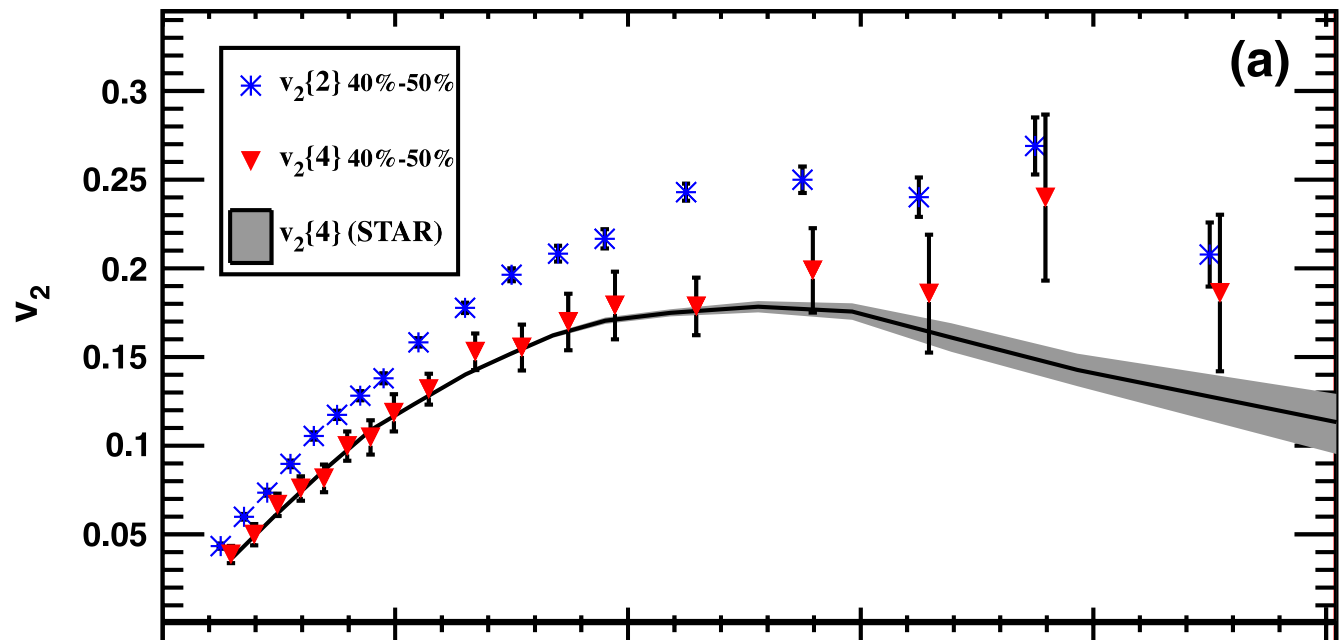
The two processes were considered: synchrotron radiation and pair annihilation. In the kinematic region relevant for the current high energy heavy-ion experiments, contribution of the synchrotron radiation is about two orders of magnitude larger than that of pair annihilation.

One possible way to ascertain the contribution of electromagnetic radiation in external magnetic field is to isolate the azimuthally symmetric component with respect to the direction of the magnetic field by rotating the reference frame, so that z-axis coincides with B-direction.

ENERGY LOSS AND AZIMUTHAL ANISOTROPY



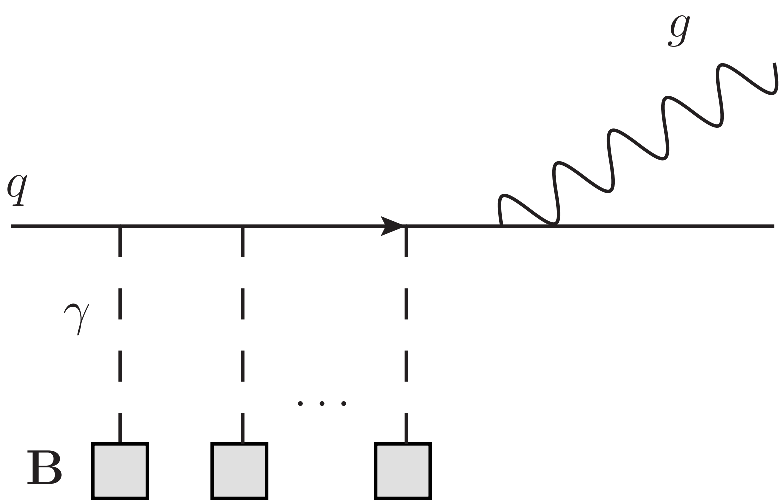
arXiv:1012.1004 [nucl-ex]



Alice, PRL 105 (2010)
arXiv:1011.3914 [nucl-ex]

Synchrotron radiation contributes to quark anergy loss and azimuthal asymmetry.

GLUON SYNCHROTRON RADIATION



- General formulas for synchrotron radiation simplify if quark is **ultra-relativistic** $\varepsilon >> m$ *before* and *after* gluon radiation.

  This always holds in week fields $eB \ll m^2$

  In strong fields $eB \gg m^2$ this approximation breaks down at the threshold $\omega \sim \varepsilon$, i.e. gluon carries away almost all quark energy ⇒ energy loss in this approximation must satisfy $\Delta\varepsilon \ll \varepsilon$

- Synchrotron radiation is **quasi-classical** if

  1.Spacing between Landau levels eB/ε is much smaller than ε => $\varepsilon^2 \gg eB$

  2. Recoil due to gluon emission is small: $\omega \ll \varepsilon$ (i.e. far from the threshold)

ULTRA-RELATIVISTIC + QUASI-CLASSICAL LIMIT

In the quasi-classical approximation $j \gg 1$, $k \gg 1$. Taking also the UR limit Laugerre polynomials reduce to Airy functions:

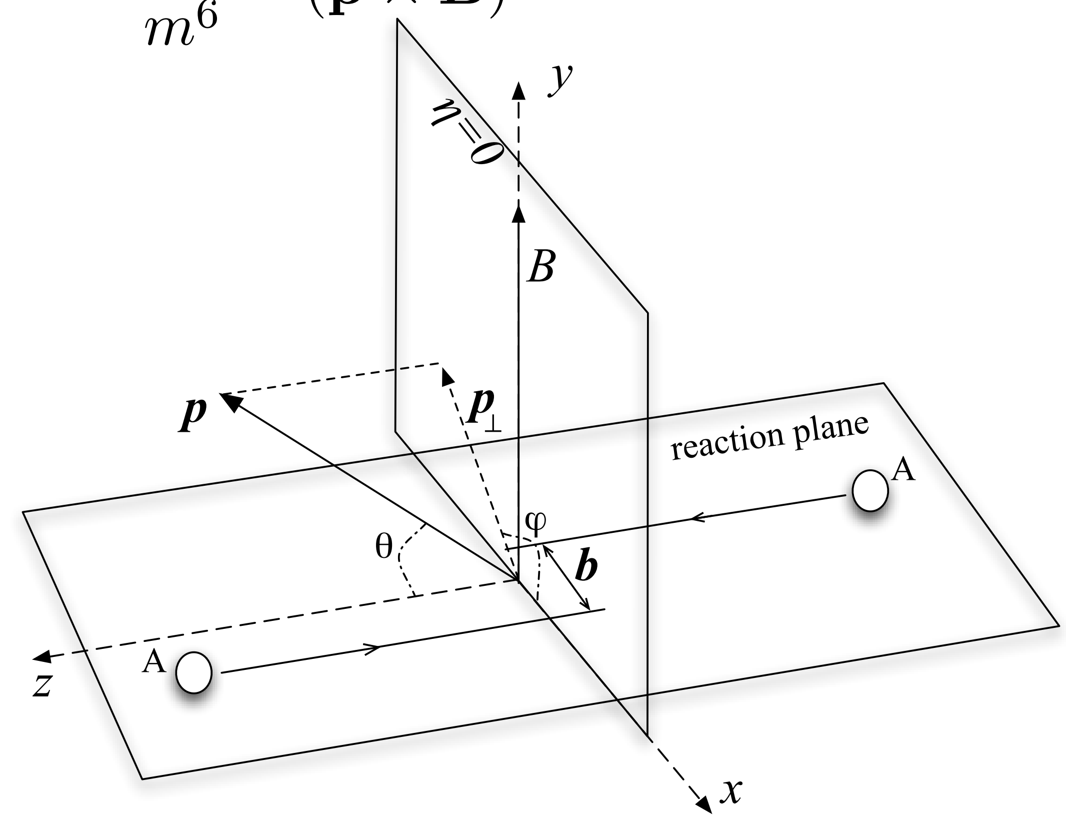
$$\frac{dI}{d\omega} = -\alpha_s C_F \frac{m^2\,\omega}{\varepsilon^2}\left\{\int_x^\infty \mathrm{Ai}(\xi)\,d\xi + \left(\frac{2}{x} + \frac{\omega}{\varepsilon}\,\chi\,x^{1/2}\right)\mathrm{Ai}'(x)\right\}$$

Invariant parameter $\chi^2 = -\frac{\alpha_{\mathrm{em}} Z_q^2 \hbar^3}{m^6}(F_{\mu\nu}p^\nu)^2 = \frac{\alpha_{\mathrm{em}} Z_q^2 \hbar^3}{m^6}(\mathbf{p}\times\mathbf{B})^2$

At high energies $\chi \approx \frac{Z_q B\varepsilon}{B_c m}$

Strong fields $\chi >> 1$, weak fields: $\chi << 1$.

In our case $\chi \sim (m_\pi/m_u)^2 \gg 1$



$$\chi^2 = \frac{\hbar^2 (eB)^2}{m^6}\, p_\perp^2 (\sinh^2\eta + \cos^2\varphi)$$

ENERGY LOSS

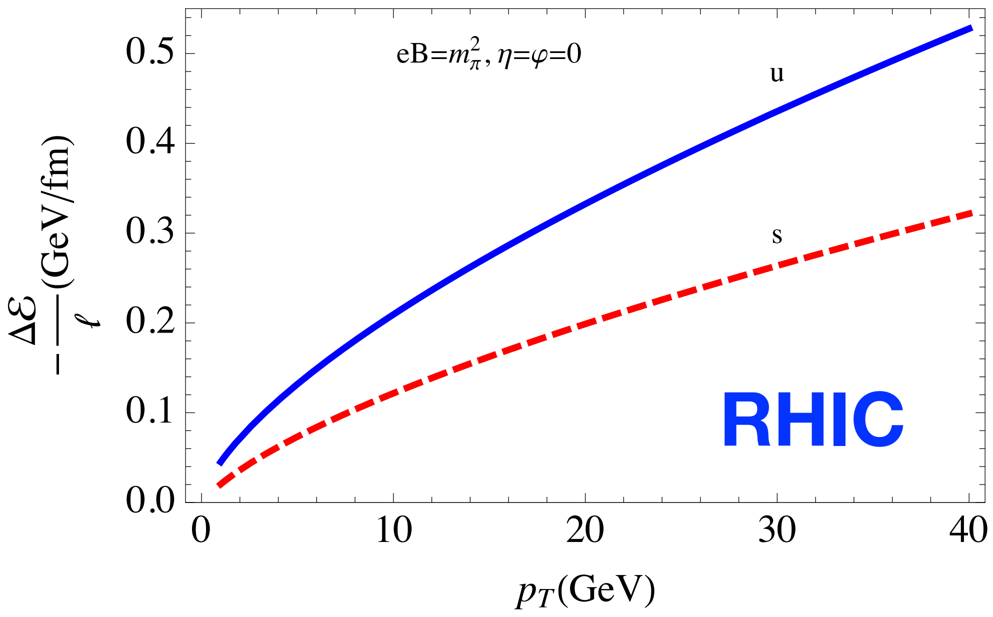
$$\frac{d\varepsilon}{dl} = -\int_0^\infty d\omega \frac{dI}{d\omega} = \alpha_s C_F \, \frac{m^2\,\chi^2}{2} \int_0^\infty \frac{4 + 5\chi\, x^{3/2} + 4\chi^2\, x^3}{(1+\chi\, x^{3/2})^4} \, \mathrm{Ai}'(x)\, x\, dx$$

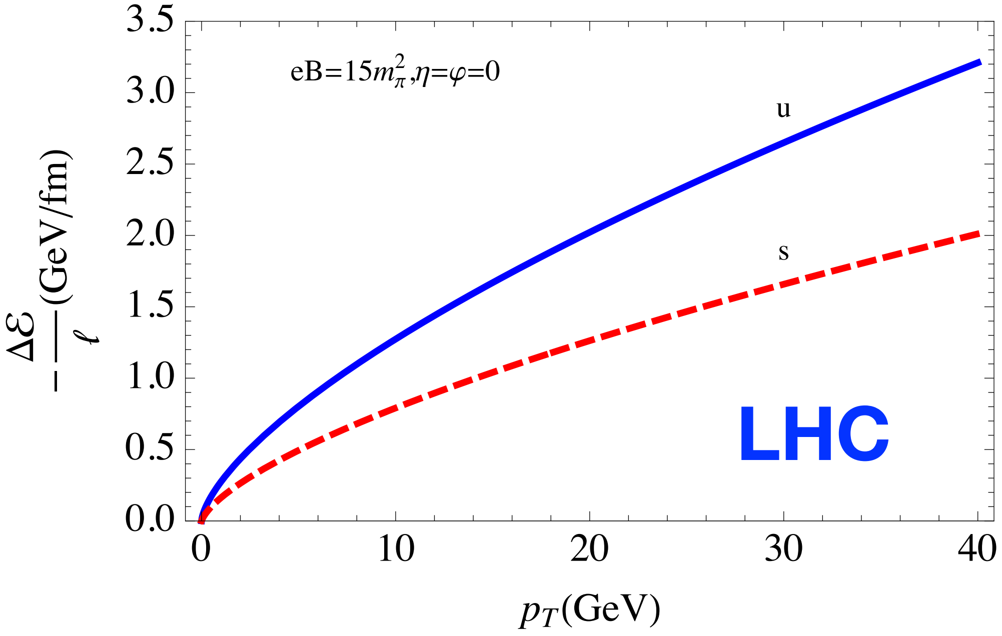
At η=φ=0:

$$\frac{d\varepsilon}{dl} = -\frac{2\,\alpha_s\,\hbar\, C_F\,(Z_q e B)^2 \varepsilon^2}{3m^4}\,, \quad \chi \ll 1\,,$$ Weak fields

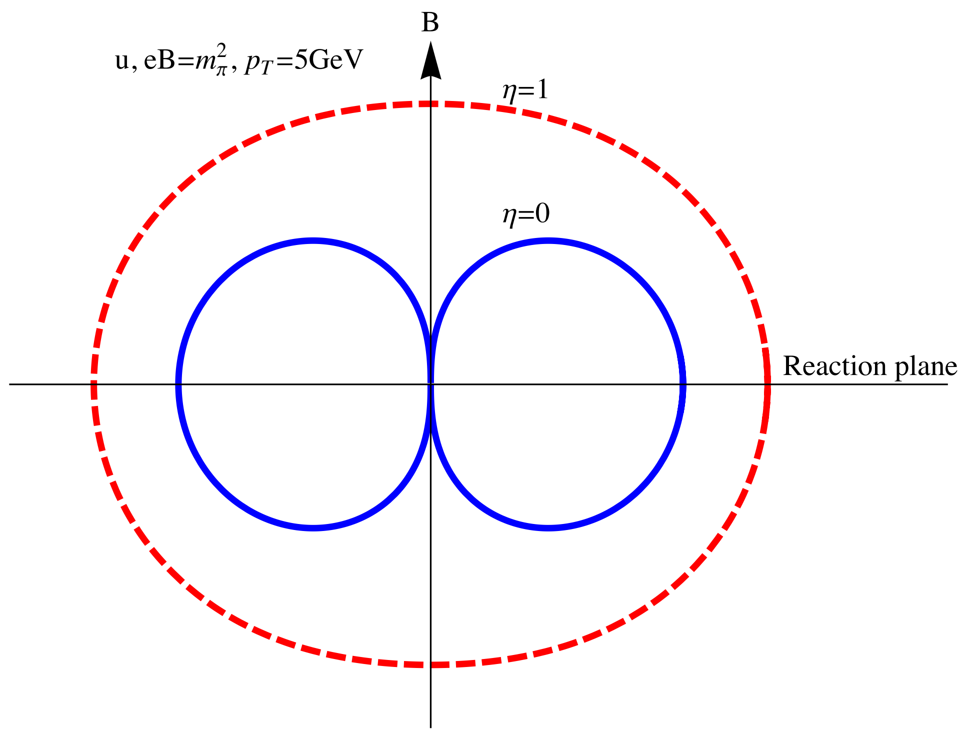
$$\frac{d\varepsilon}{dl} = -0.37\,\alpha_s\,\hbar^{-1/3}\, C_F\,\left(Z_q e B\,\varepsilon\right)^{2/3}\,, \quad \chi \gg 1$$ Strong fields

Energy loss in magnetic field





Azimuthal asymmetry:

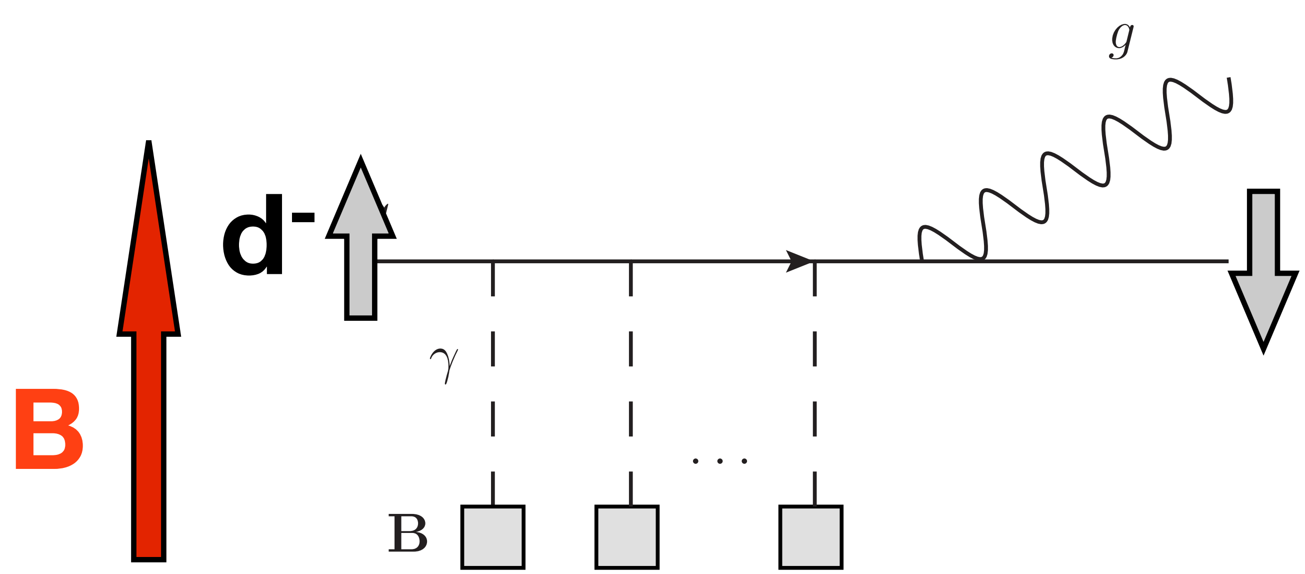


POLARIZATION OF LIGHT QUARKS

Spin-flip probability per unit time

$$w = \frac{5\sqrt{3}\alpha_s C_F}{16}\frac{\hbar^2}{m^2}\left(\frac{\varepsilon}{m}\right)^5\left(\frac{Z_q e\,|\boldsymbol{v}\times\boldsymbol{B}|}{\varepsilon}\right)^3\left(1-\frac{2}{9}\,(\boldsymbol{\zeta}\cdot\boldsymbol{v})^2-\frac{8\sqrt{3}}{15}\mathrm{sign}\,(e_q)\,(\boldsymbol{\zeta}\cdot\boldsymbol{b})\right)$$

Sokolov, Ternov (1964)



NR case: $H = -\boldsymbol{\mu}\cdot\boldsymbol{B} = -\left(\frac{geZ_q\hbar}{2m}\right)\boldsymbol{s}\cdot\boldsymbol{B}$

SPIN-ASYMMETRY

- Let n(↑)/n(↓) be the number of fermions with given momentum and spin direction parallel /anti-parallel to the field in a given event.

Spin-asymmetry: $$A = \frac{n_\uparrow - n_\downarrow}{n_\uparrow + n_\downarrow}$$

- Immediately after the collision at t_0 A=0, however at $t>t_0$

$$A = \frac{8\sqrt{3}}{15}\left(1 - e^{-\frac{t-t_0}{\tau}}\right)$$

$$\tau = \frac{8}{5\sqrt{3}\, m\, \alpha_s C_F}\left(\frac{m}{\varepsilon}\right)^2\left(\frac{m^2}{Z_q e|\boldsymbol{v}\times\boldsymbol{B}|}\right)^3$$

For muon with p_T=1 GeV, $\eta=\varphi=0 \Rightarrow \tau \sim 0.004$ fm = 0 on the relevant scale

$$A = \frac{8}{5\sqrt{3}} = 92\%$$

A very strong polarization of quarks and leptons

BEYOND THE QUASI-CLASSICAL APPROXIMATION

- **In strong fields** $B \gg e/m^2$ **near the threshold** $\omega = \varepsilon$:

  Quark looses almost all its energy due to synchrotron radiation and falls on one of the lowest Landau levels.

  This brakes both the quasi-classical and ultra-relativistic approximation.

- Transition to the ground state occurs with probability

$$w_{n0} = \frac{\alpha_s}{2}\frac{m^2}{\varepsilon}\frac{B}{B_c}e^{-B_c/B}$$

Sokolov, Borisov, Zhukovskii (1975)

where $B_c = e/m^2$

- In heavy-ion collisions B is stronger than B_c, so such transitions must be taken into account.

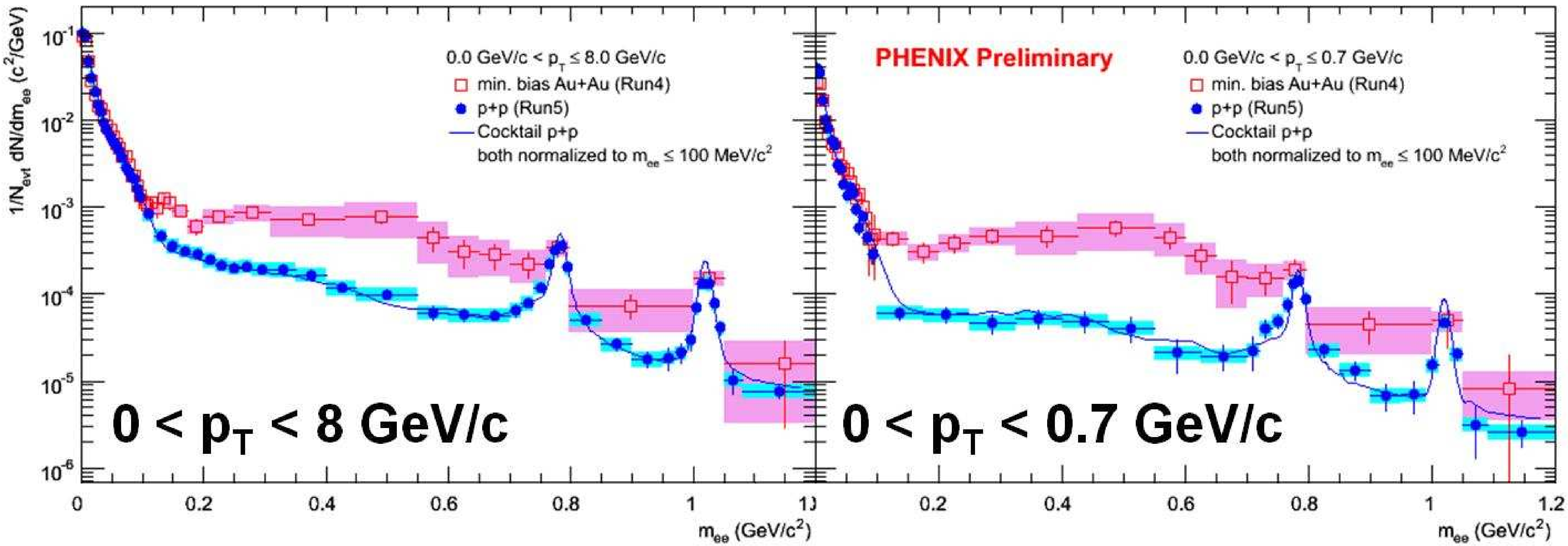
  ☞ Future project.

CONCLUSIONS III

Synchrotron radiation of gluons contributes to the quark energy loss and is azimuthally asymmetric.

Polarization of leptons escaping the QGP is a very good probe of the QED in magnetic field.

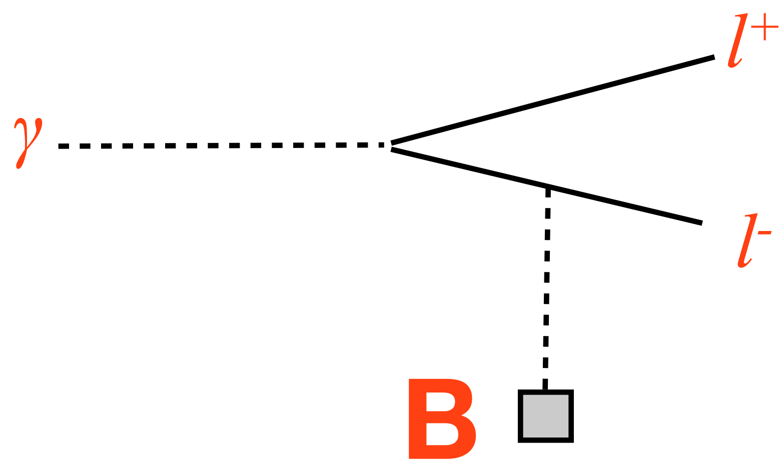
Dilepton production



PHENIX (Y. Akiba)
arXiv:0907.4794 [nucl-ex]

PHOTON DECAY

Photon decay is another cross-channel of the synchrotron radiation

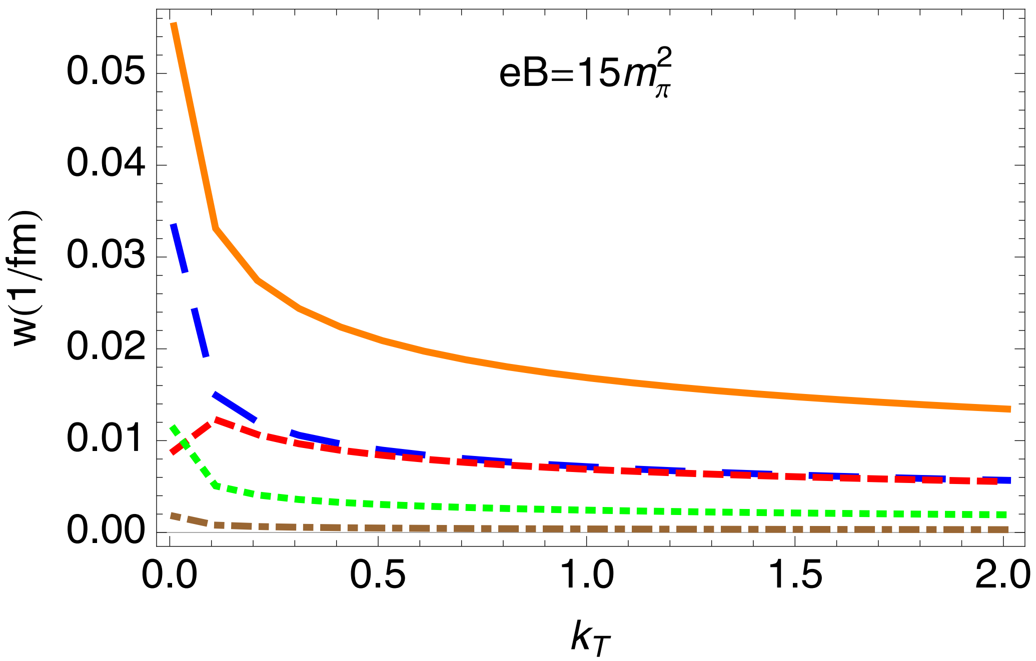


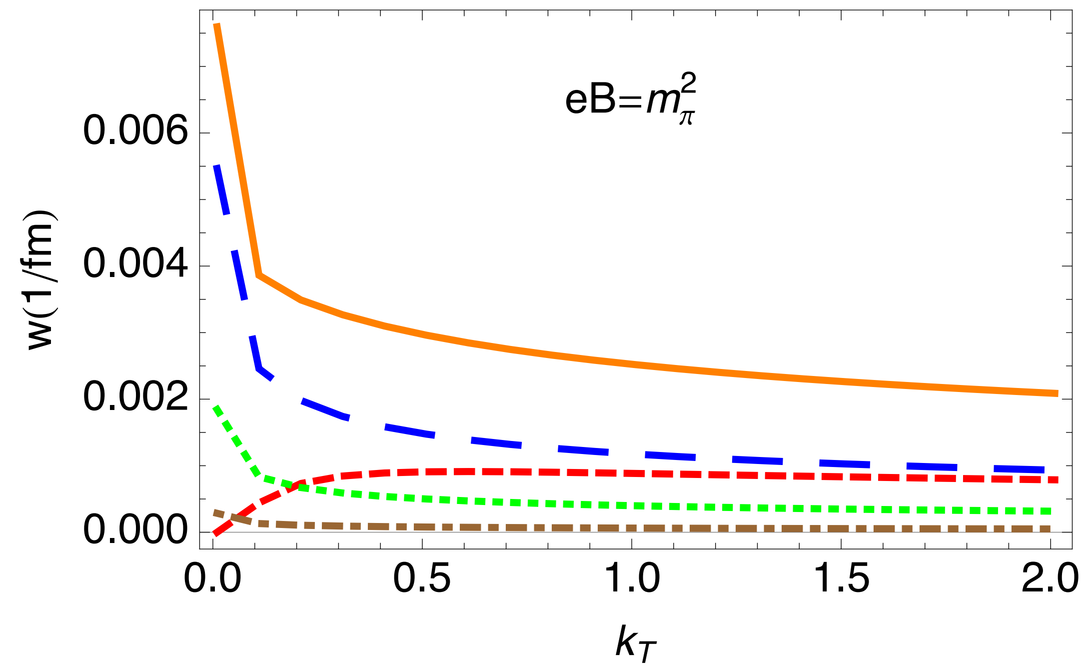
Rate

$$w = -\sum_a \frac{\alpha_{\rm em}\, z_a^3\, eB}{m_a \varkappa_a} \int_{(4/\varkappa_a)^{2/3}}^{\infty} \frac{2(x^{3/2} + 1/\varkappa_a)\, \mathrm{Ai}'(x)}{x^{11/4}(x^{3/2} - 4/\varkappa_a)^{3/2}}$$

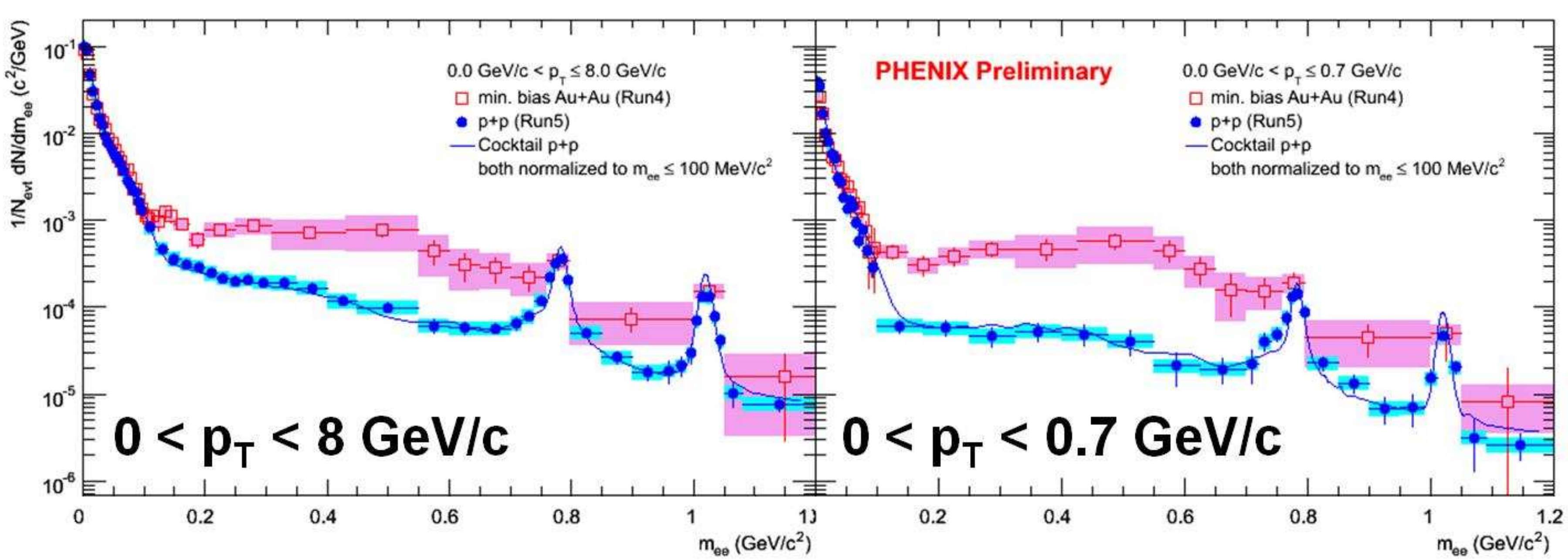
$$\varkappa_a^2 = -\frac{\alpha_{\rm em} z_a^2 \hbar^3}{m_a^6} (F_{\mu\nu} k^\nu)^2 = \frac{\alpha_{\rm em} z_a^2 \hbar^3}{m_a^6} (\boldsymbol{k} \times \boldsymbol{B})^2$$

PHOTON DECAY RATE



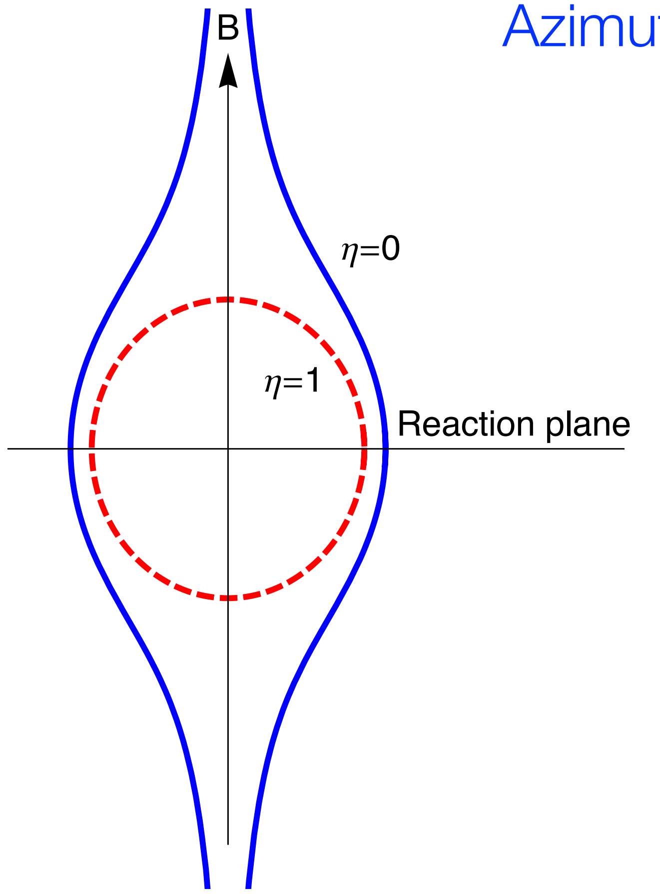


Survival probability: P=1-wΔt, where Δt ≈ 5fm ⇒ P(RHIC)=98.5%, P(LHC)=90%



PHENIX (Y. Akiba)
arXiv:0907.4794 [nucl-ex]

AZIMUTHAL ASYMMETRY



Azimuthal asymmetry of the decay rate w: $w(\varphi) = \frac{1}{2} w_0 + \sum_{n=1}^{\infty} w_n \cos(n\varphi)$

$$w_{2k} = \frac{3\, 2^{1/3}\, A}{B\left(\frac{5}{6}+k, \frac{5}{6}-k\right)}\,, \quad w_{2k+1} = 0\,, \quad k = 0, 1, 2, \ldots$$

$$w = \frac{1}{2} w_0 \left[1 - \sum_{k=1}^{\infty} \frac{\sqrt{\pi}\Gamma\left(-\frac{1}{6}\right)}{2^{2/3} B\left(\frac{5}{6}+k, \frac{5}{6}-k\right)} \cos(2k\varphi)\right]$$

Azimuthal asymmetry of the survival probability P:

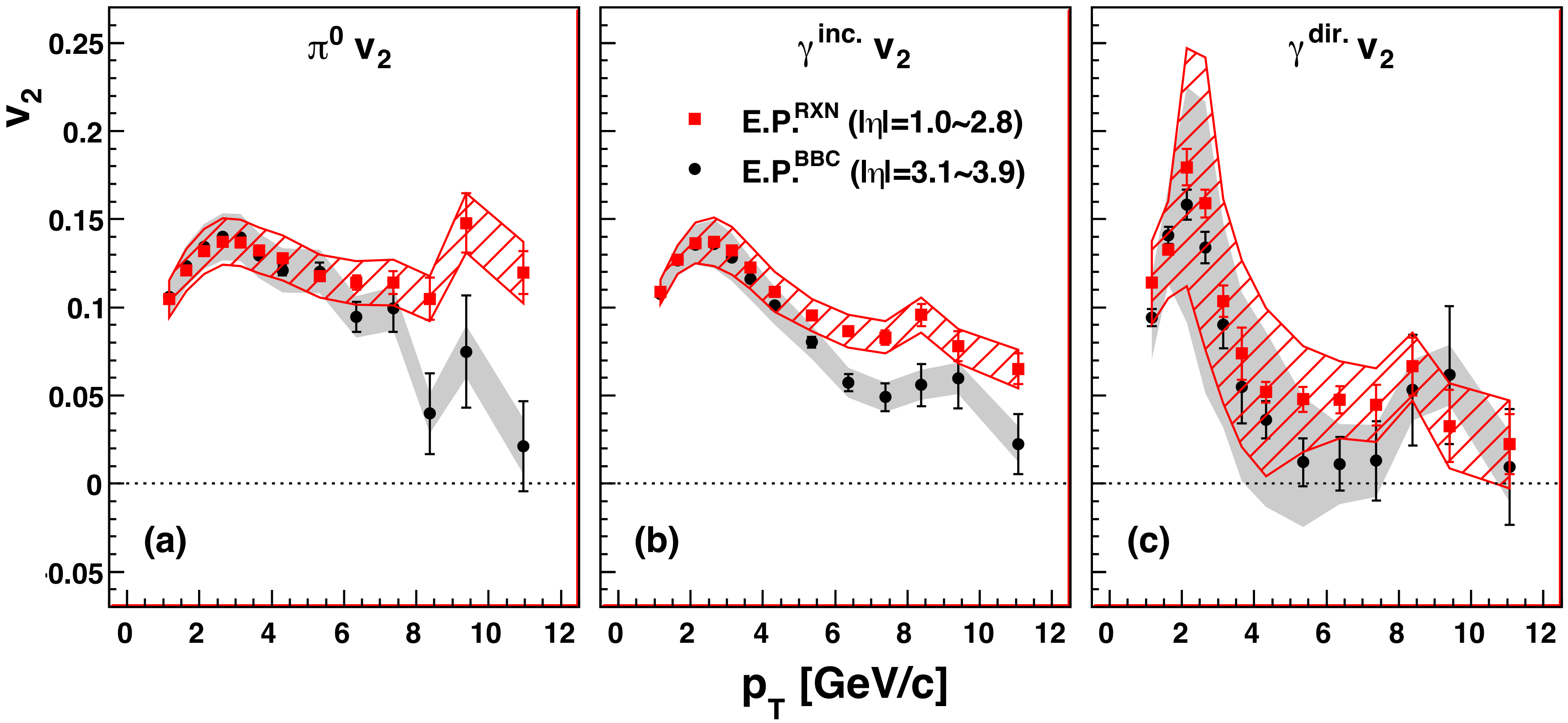
$$P = \bar{P}\left(1 + \sum_{k=1}^{\infty} v_{2k} \cos(2\varphi k)\right)\,, \quad v_{2k} = -\frac{1-\bar{P}}{\bar{P}} \frac{2\, w_{2k}}{w_0}$$

where azimuthal average is $\bar{P} = \langle 1 - w\Delta t\rangle_{\varphi} = 1 - w_0 \Delta t \approx 1$

$$v_{2k} \approx -\frac{2w_{2k}}{w_0}\, w_0 \Delta t = -\frac{2w_{2k}}{w_0} \Delta t\, \frac{5\, 6^{2/3}\Gamma\left(\frac{2}{3}\right)}{7\pi} \sum_a \frac{\alpha_{\rm em}(eB)^{2/3} z_a^{8/3}}{(k_T)^{1/3}}$$

$$v_2 = \Delta t\, \frac{2\, 6^{2/3}\Gamma\left(\frac{2}{3}\right)}{7\pi} \sum_a \frac{\alpha_{\rm em}(eB)^{2/3} z_a^{8/3}}{(k_T)^{1/3}}$$

For k_T=1GeV and Δt ≈ 5fm ⇒ v_2(RHIC)=1%, v_2(LHC)=7%



PHENIX
arXiv:1105.4126 [nucl-ex]

CONCLUSIONS IV

Dileptons produced in magnetic field significantly contribute to the overall dilepton rate.

More theoretical work is needed to calculate

$$q \to q + \gamma \to q + \ell^+ + \ell^-$$

(in progress)

SUMMARY

- Magnetic field in relativistic heavy-ion collisions exceeds the critical value at least for a short interval of time.

- Back-reaction of the medium in which the magnetic field decays significantly inhibits its decay.

- Strong magnetic field can trigger a lot of new phenomena; some have never been observed before.

- **Abundance of possible effects calls for a detailed experimental investigation.**